Testing

Gravity

on Cosmological Scales

Wayne Hu

HKUST, June 2011

Testing

on Cosmological Scales

Wayne Hu

HKUST, June 2011

Outline

- Modified Gravity vs Missing Energy
- Falsifiability of Smooth Dark Energy
- Worked Examples

  Modified Action $f(R)$

  DGP Braneworld / Galileon

Outline

- Modified Gravity vs Missing Energy
- Falsifiability of Smooth Dark Energy
- Worked Examples

    Modified Action $f(R)$

    DGP Braneworld / Galileon

    Sundrum Theorem: It's Λ, Stupid

Outline

- Modified Gravity vs Missing Energy
- Falsifiability of Smooth Dark Energy
- Worked Examples

  Modified Action $f(R)$

  DGP Braneworld / Galileon

  Sundrum Theorem: It's Λ, Stupid

  Sundrum Corollary: and Messing with Einstein is in Bad Taste

Outline

- Modified Gravity vs Missing Energy
- Falsifiability of Smooth Dark Energy
- Worked Examples

    Modified Action $f(R)$

    DGP Braneworld / Galileon

    Sundrum Theorem: It's Λ, Stupid

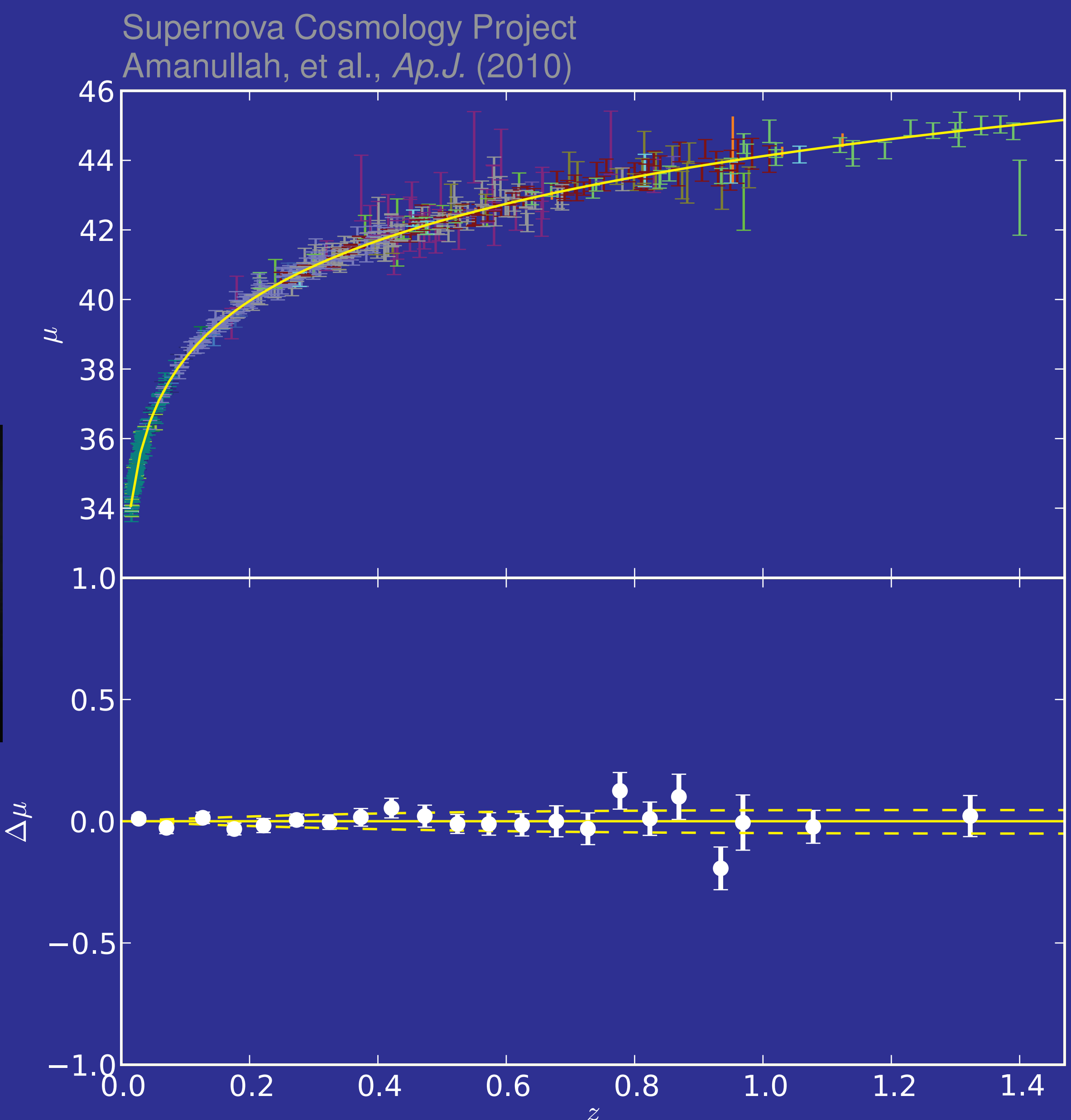
    Sundrum Corollary: and Messing with Einstein is in Bad Taste

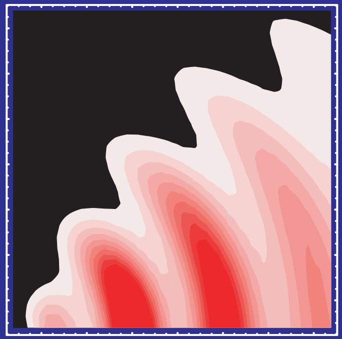
Upcoming surveys will qualitative increase your confidence in your own brilliance and good taste!

Cosmic Acceleration

- Geometric measures of distance redshift from SN, CMB, BAO

Standard(izable)
Candle
Supernovae
Luminosity v Flux

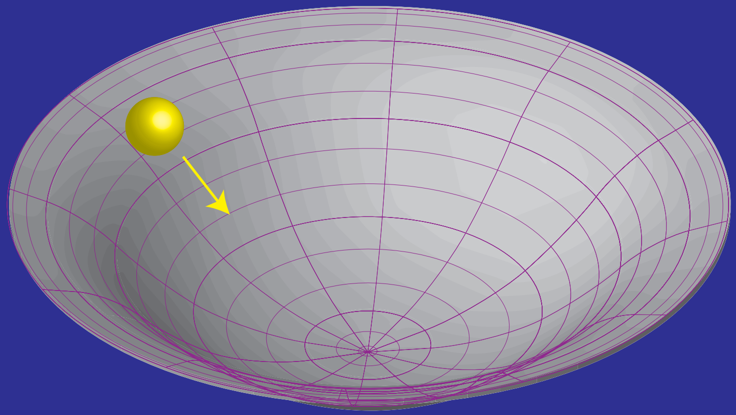




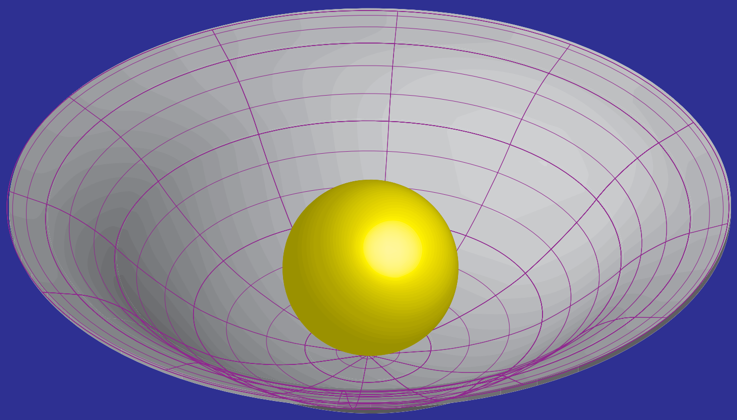
Standard Ruler
Sound Horizon
v CMB, BAO angular
and redshift separation

Mercury or Pluto?

- General relativity says Gravity = Geometry



- And Geometry = Matter-Energy



- Could the missing energy required by acceleration be an incomplete description of how matter determines geometry?

Two Potentials

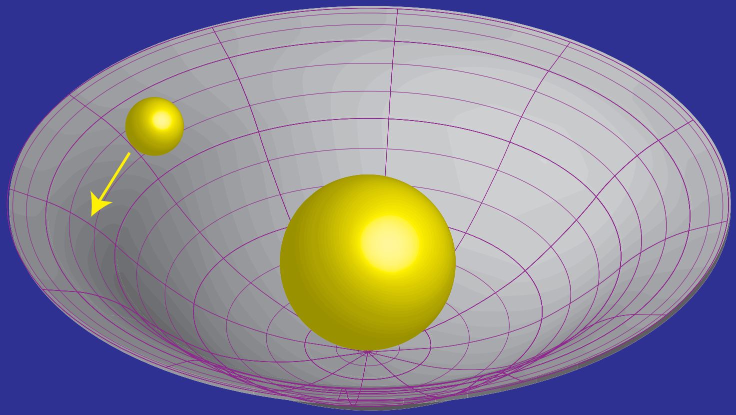
- Line Element

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2$$

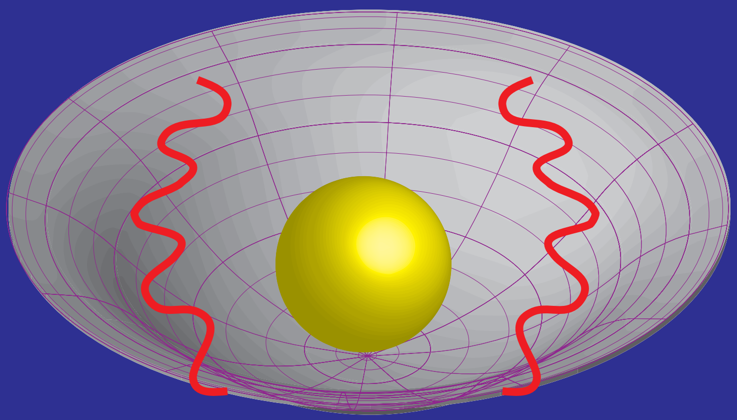
- Newtonian dynamical potential Ψ
- Space curvature potential Φ
- As in the parameterized post Newtonian approach, cosmological tests of the Φ/Ψ
- Space curvature per unit dynamical mass
- Given parameterized metric, matter falls on geodesics

Dynamical vs Lensing Mass

- Newtonian potential: $\Psi=\delta g_{00}/2g_{00}$ which non-relativistic particles feel



- Space curvature: $\Phi=\delta g_{ii}/2g_{ii}$ which also deflects photons



- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass

Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content
- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy

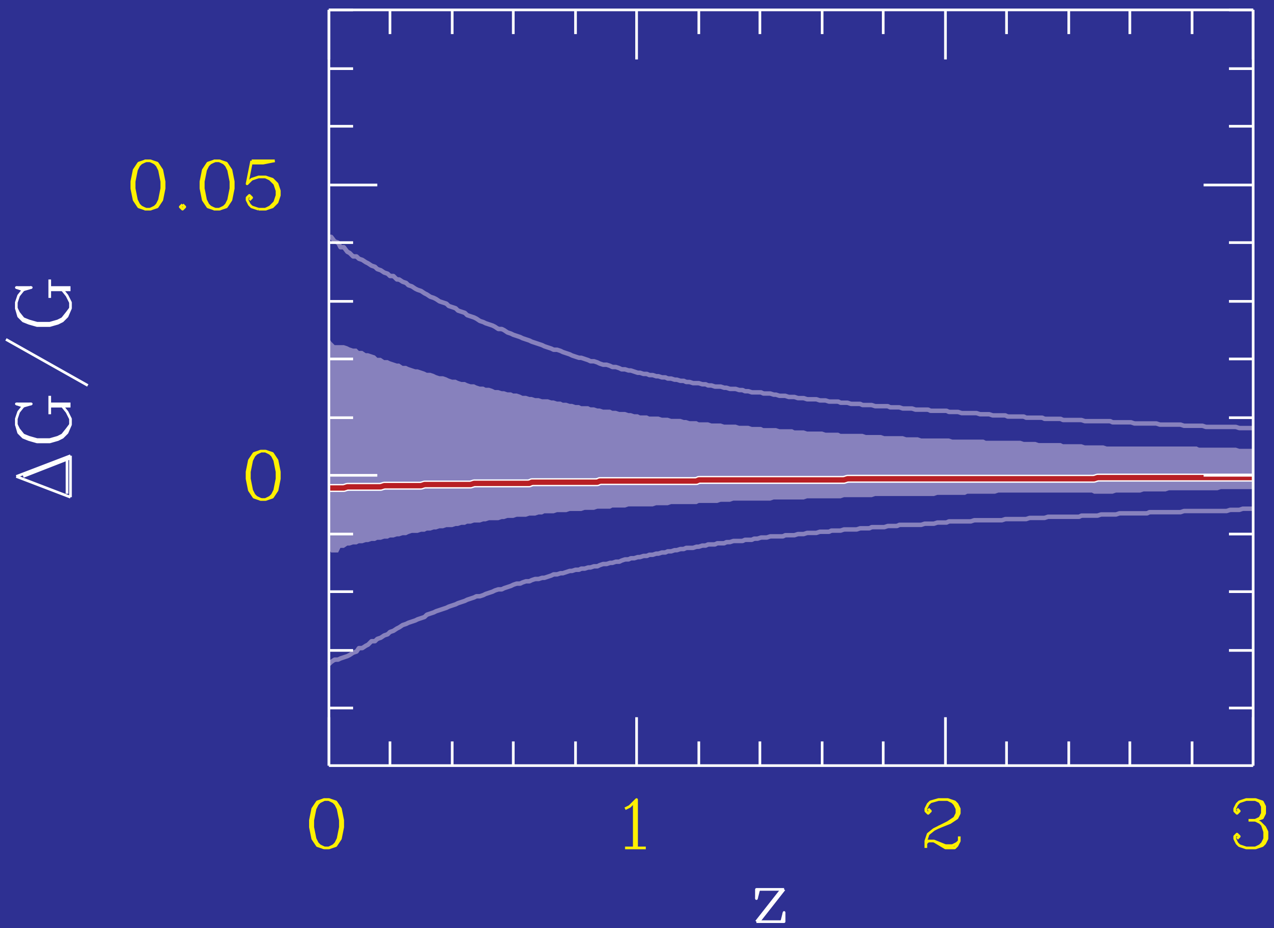
$$\begin{aligned} F(g_{\mu\nu}) + G_{\mu\nu} &= 8\pi G T^{\rm M}_{\mu\nu} \\ G_{\mu\nu} &= 8\pi G [T^{\rm M}_{\mu\nu} + T^{\rm DE}_{\mu\nu}] \end{aligned} \qquad \boxed{-F(g_{\mu\nu}) = 8\pi G T^{\rm DE}_{\mu\nu}}$$

and the Bianchi identity guarantees $\nabla^\mu T^{\rm DE}_{\mu\nu} = 0$

- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor
- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress

Falsifying ΛCDM

- Λ slows growth of structure in highly predictive way



Cosmological Constant

Modified Gravity $\neq$ "Smooth DE"

- Scalar field dark energy has $\delta p = \delta \rho$ (in constant field gauge) – relativistic sound speed, no anisotropic stress
- Jeans stability implies that its energy density is spatially smooth compared with the matter below the sound horizon

$$\begin{aligned} ds^2 &= -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2 \\ \nabla^2(\Phi - \Psi) &\propto \text{matter density fluctuation} \end{aligned}$$

- Anisotropic stress changes the amount of space curvature per unit dynamical mass

$$\nabla^2(\Phi + \Psi) \propto \text{anisotropic stress}$$

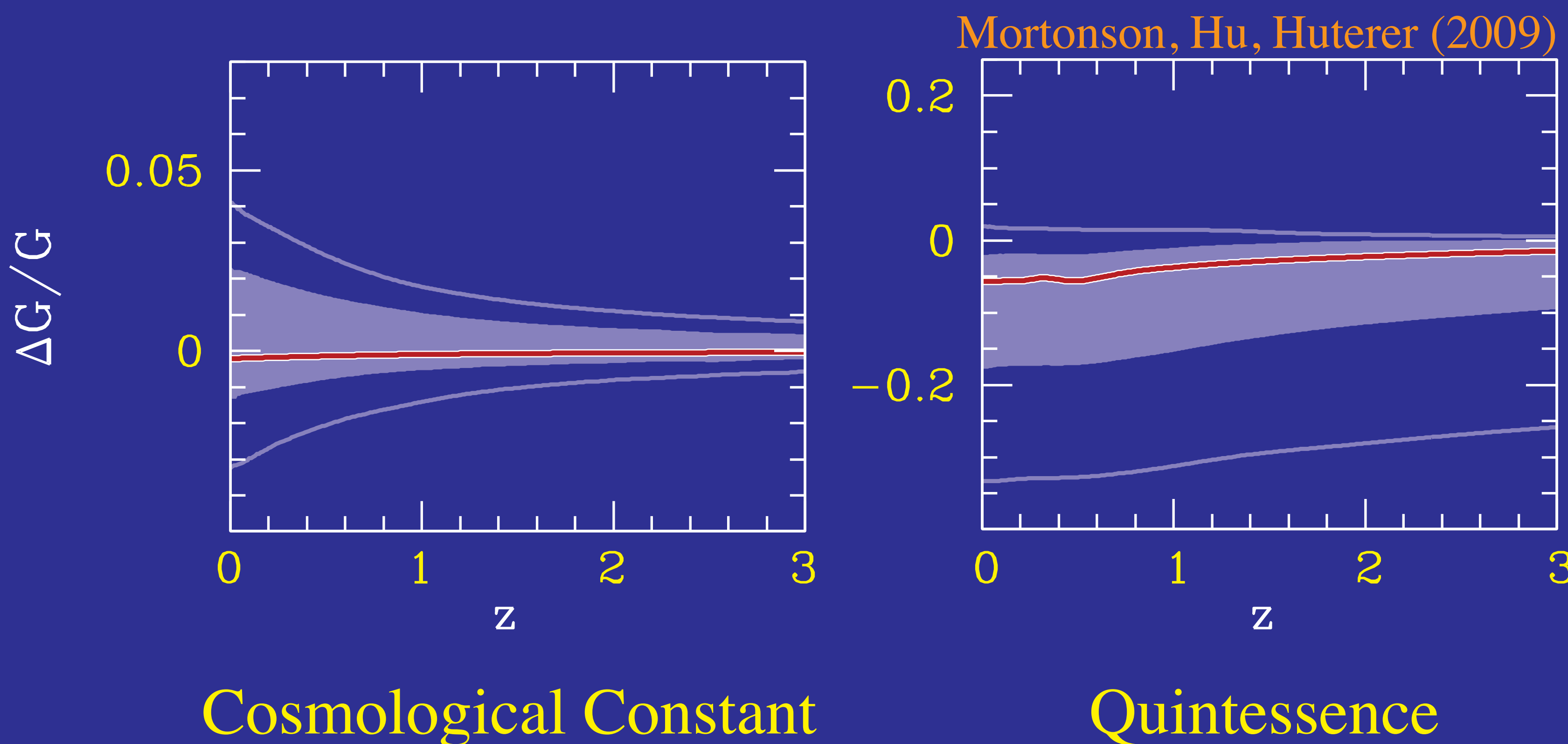
but its absence in a smooth dark energy model makes $g = (\Phi + \Psi)/(\Phi - \Psi) = 0$ for non-relativistic matter

Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate
- Hence geometric measurements of the expansion rate predict the growth of structure
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations
- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)

Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way

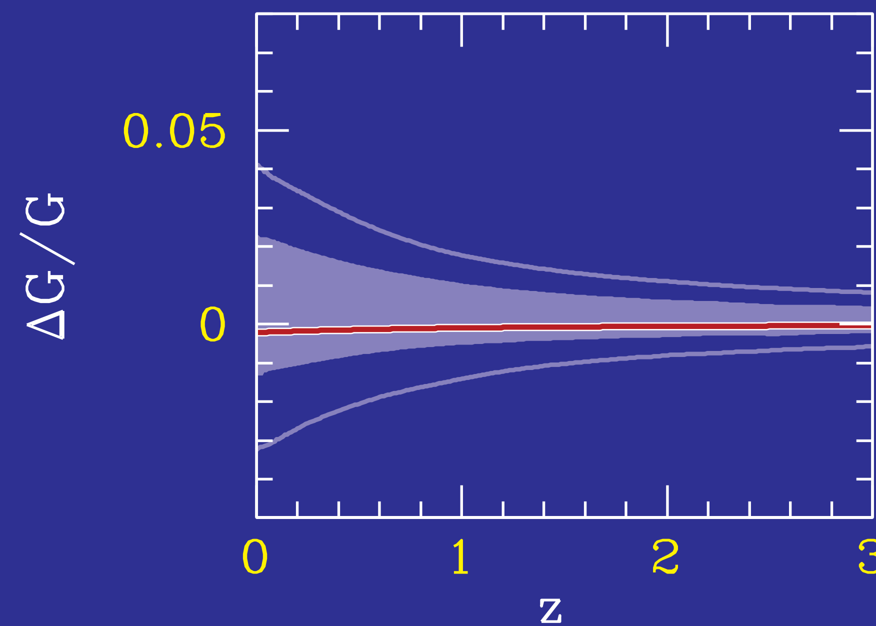


- Deviation significantly >2% rules out Λ with or without curvature
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in H_0]

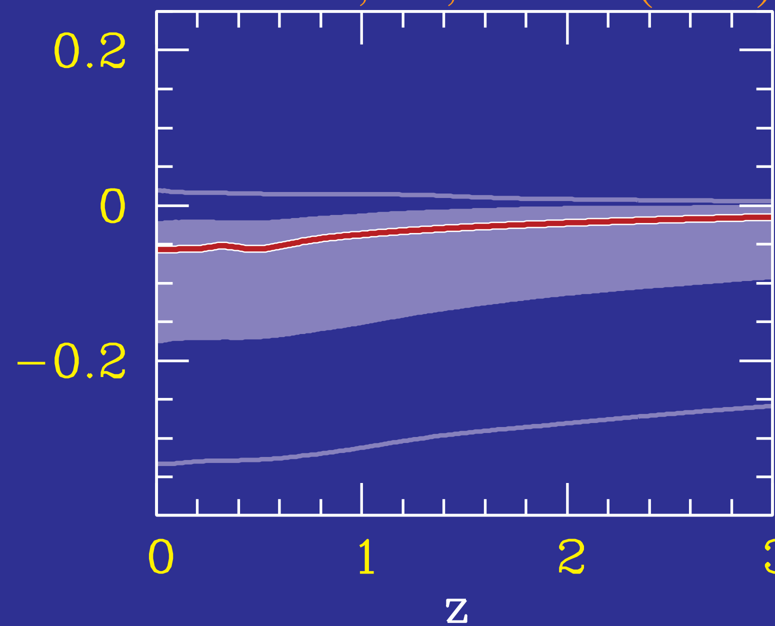
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

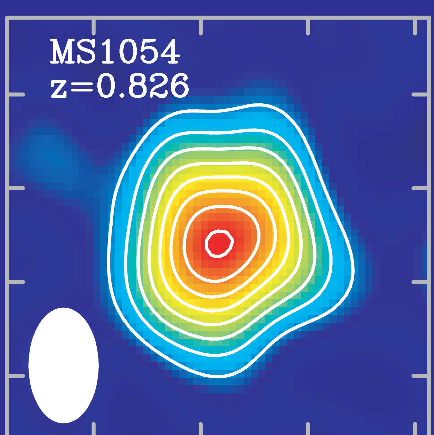
Mortonson, Hu, Huterer (2009)

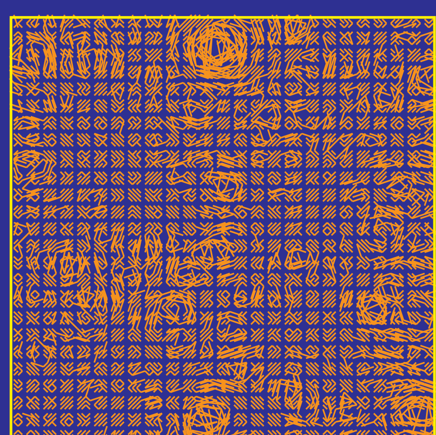


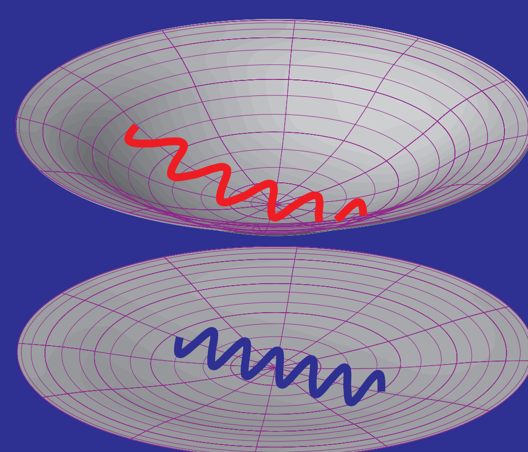
Cosmological Constant

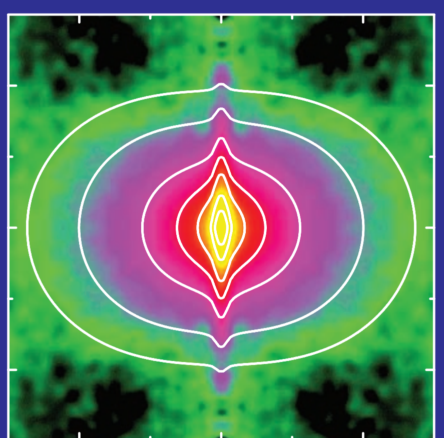


Quintessence





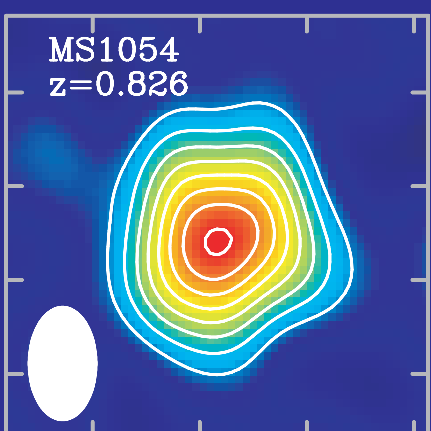


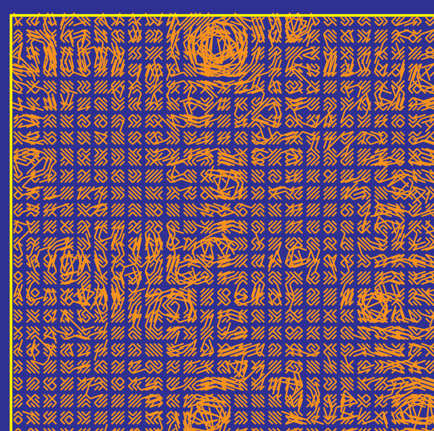


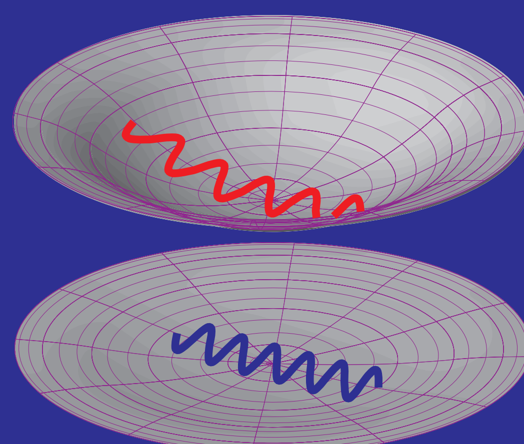
Quintessence Falsified?

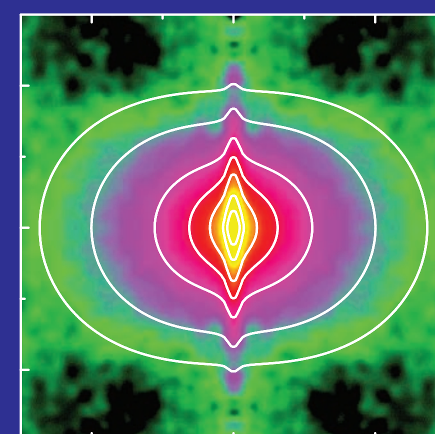
- No excess numbers of massive z>1 X-ray or SZ clusters with Gaussian initial conditions (Jee et al 2009, Brodwin et al 2010)
- No excess power in gravitational lensing at high z relative to low z (Bean 0909.3853)

- But would such violations favor modified gravity?
- Given astrophysical systematics, expect purported 2σ violations of smooth dark energy predictions will be common in coming years!



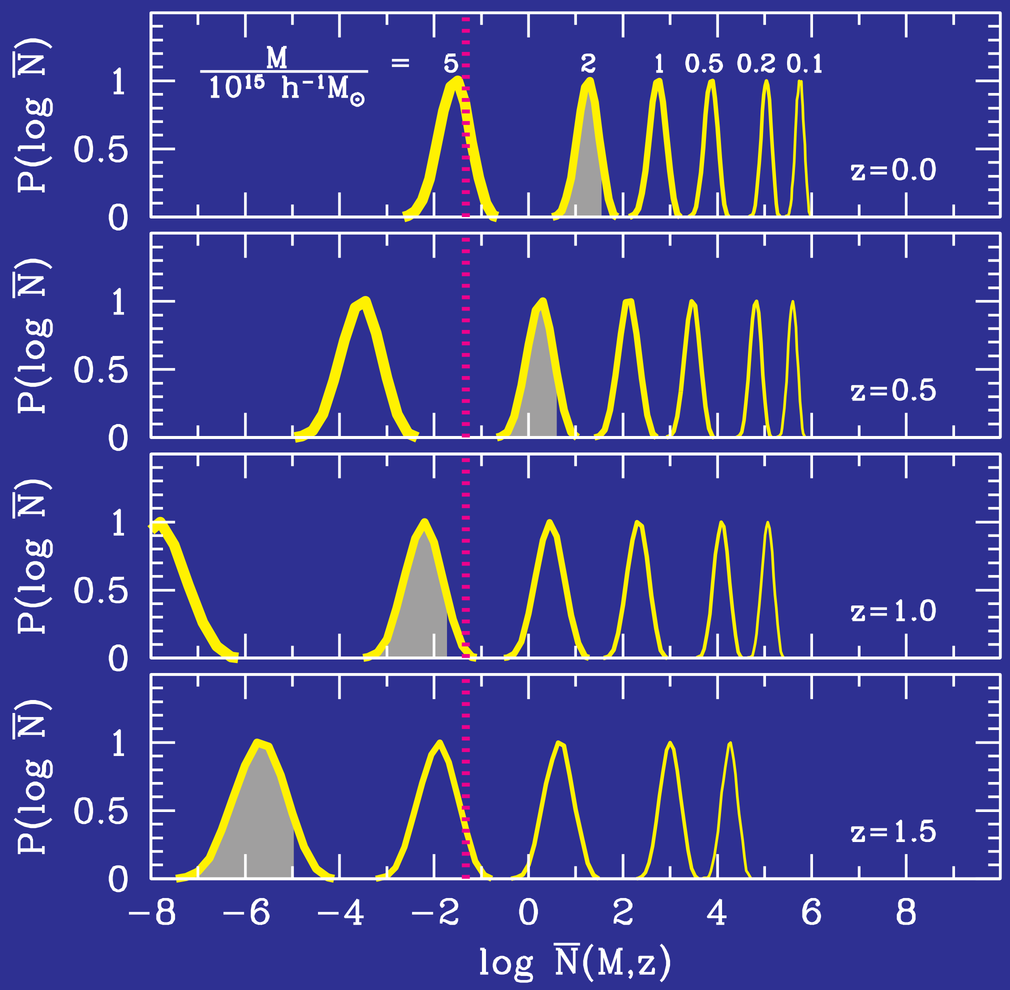






Elephantine Predictions

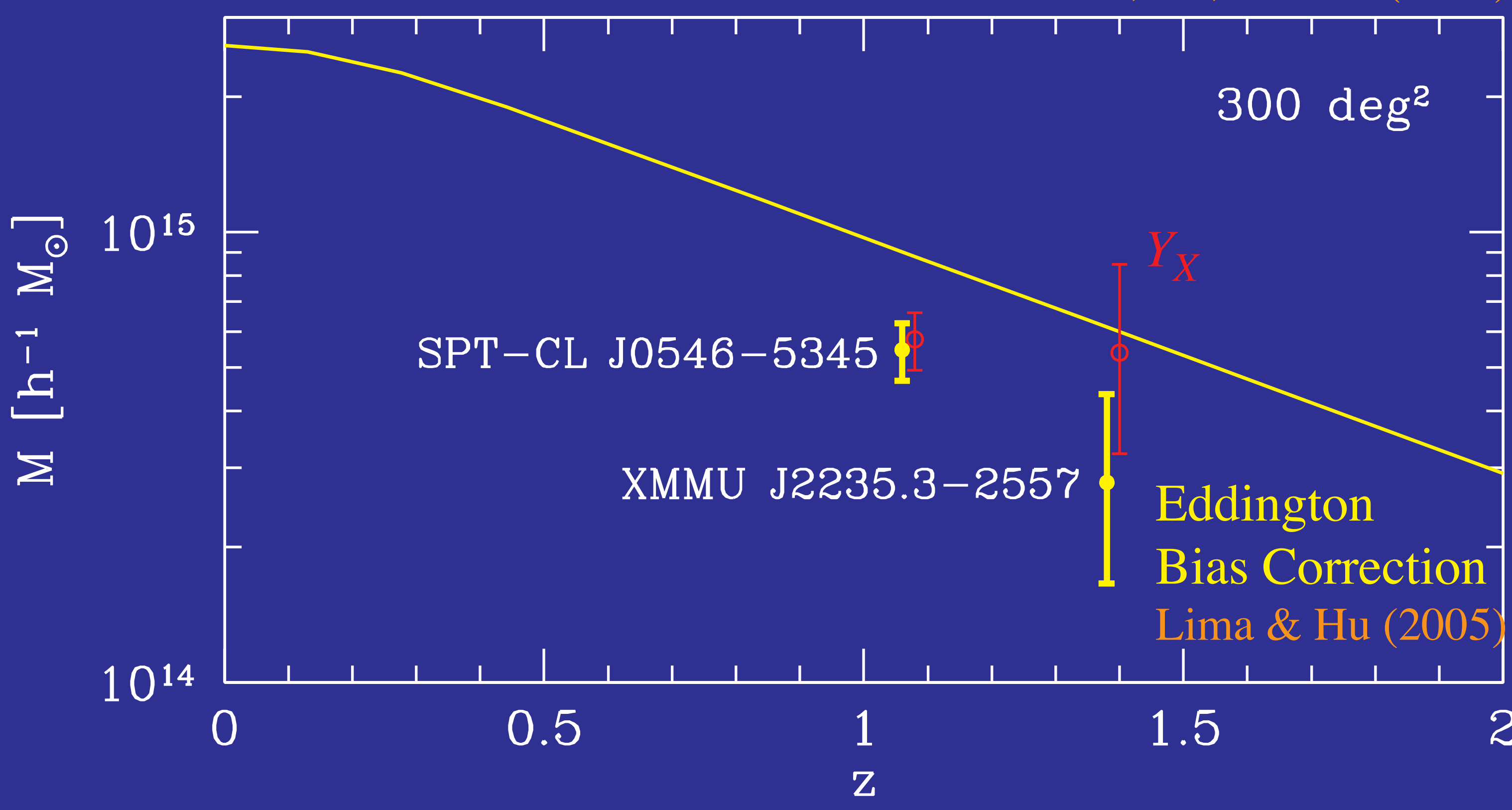
- Geometric constraints on the cosmological parameters of ΛCDM
- Convert to distributions for the predicted average number of clusters above a given mass and redshift



ΛCDM Falsified?

- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the $M(z)$ curve
- No currently known high mass, high redshift cluster violates this bound

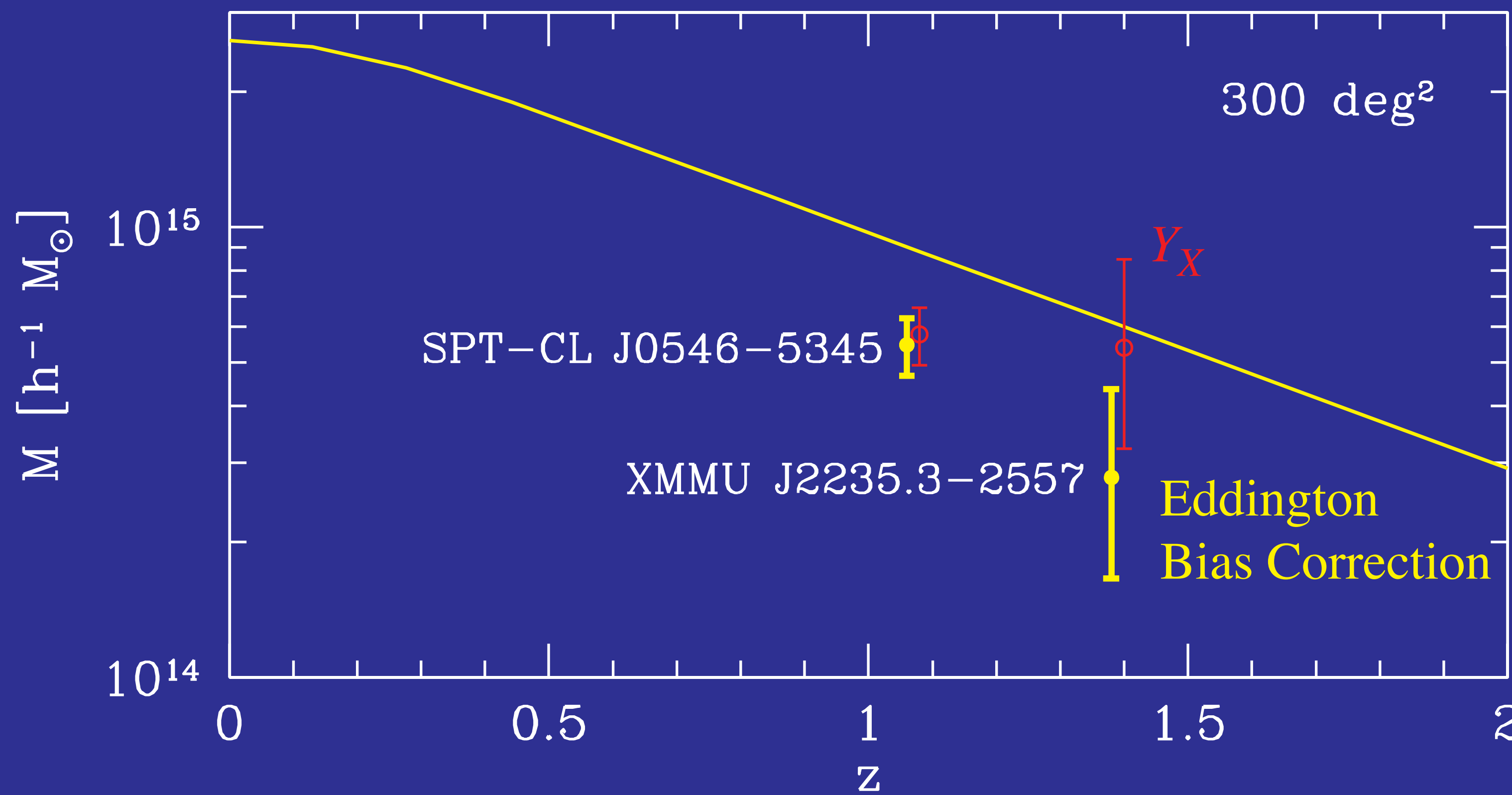
Mortonson, Hu, Huterer (2010)



ΛCDM Falsified?

- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the $M(z)$ curve
- Convenient fitting formulae for future elephants:
  http://background.uchicago.edu/abundance

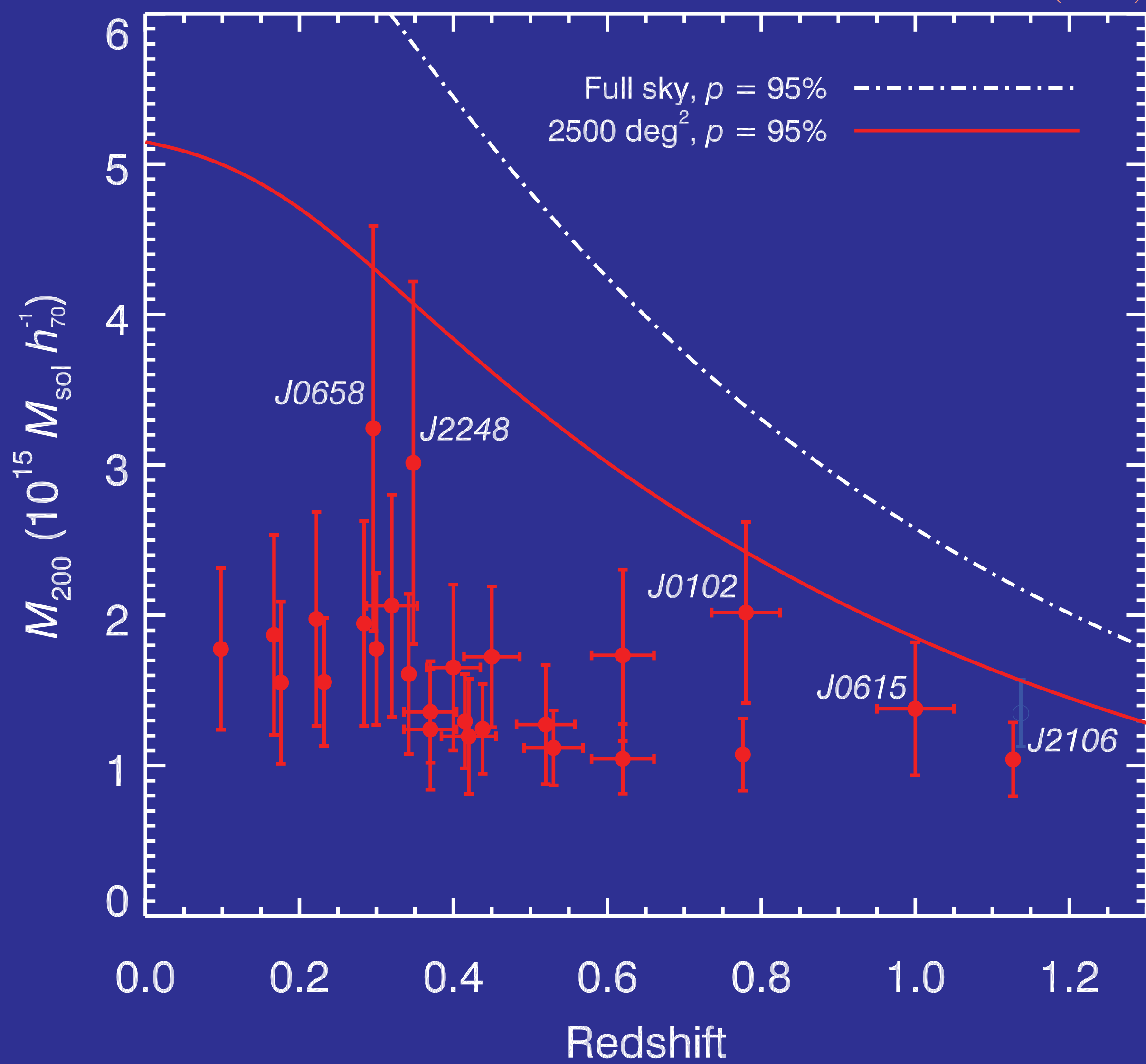
Mortonson, Hu, Huterer (2010)



Pink Elephant Parade

- SPT catalogue on 2500 sq degrees

Williamson et al (2011)



Falsify in Favor of What?
some toy examples

Modified Action $f(R)$ Model

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_{\rm m} \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B: Compton wavelength of f_R squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate

DGP Braneworld Acceleration

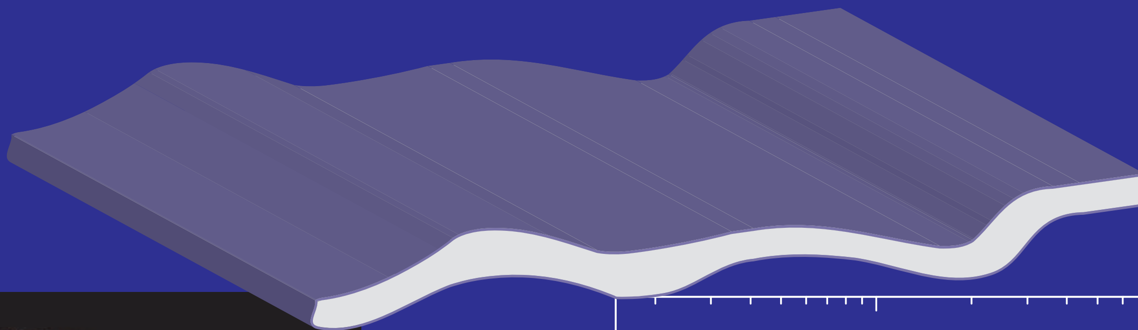
- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

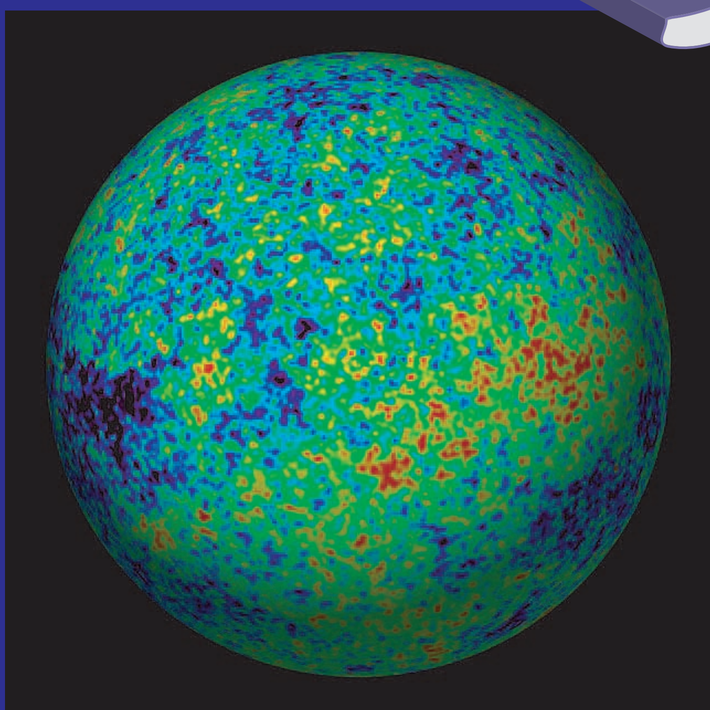
$$S = \int d^5x \sqrt{-g} \left[\frac{{}^{(5)}R}{2\kappa^2} + \delta(\chi) \left(\frac{{}^{(4)}R}{2\mu^2} + \mathcal{L}_m \right) \right]$$

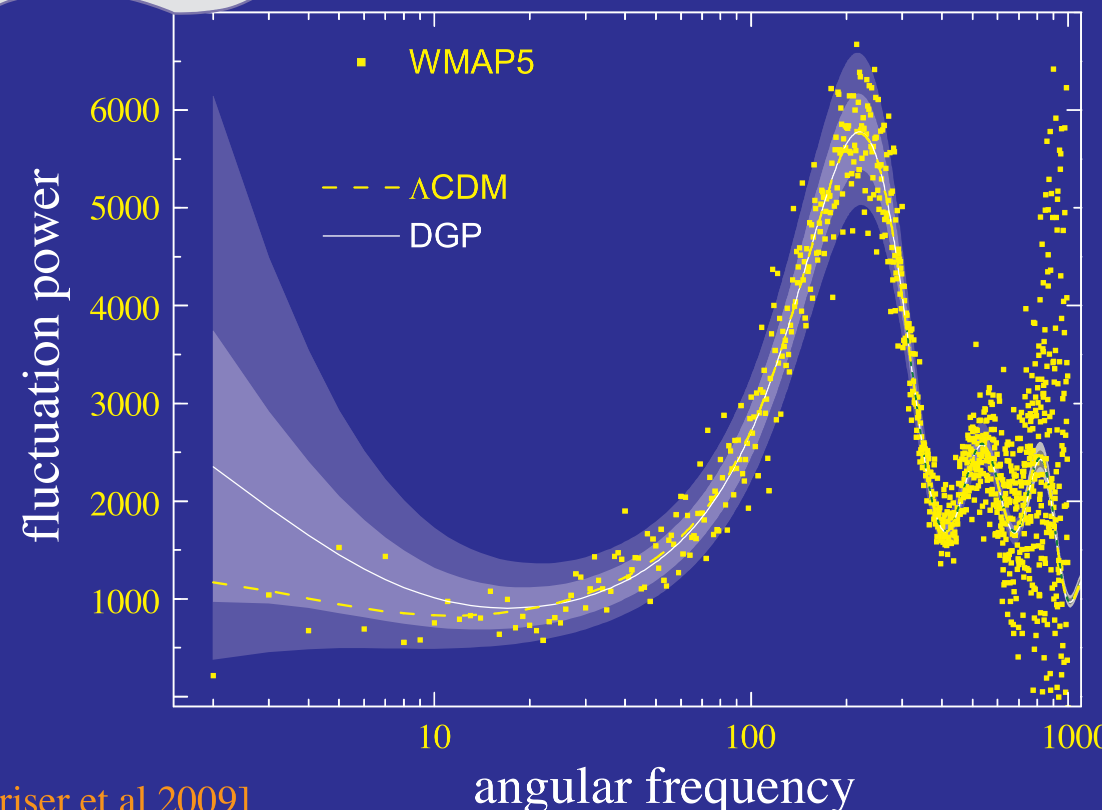
  with crossover scale $r_c = \kappa^2 / 2\mu^2$
- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)
- Matter still minimally coupled and conserved
- Exhibits the 3 regimes of modified gravity
- Weyl tensor anisotropy dominated conserved curvature regime $r > r_c$ (Sawicki, Song, Hu 2006; Cardoso et al 2007)
- Brane bending scalar tensor regime $r_* < r < r_c$ (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)
- Strong coupling General Relativistic regime $r < r_* = (r_c^2 r_g)^{1/3}$ where $r_g = 2GM$ (Dvali 2006)

DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background







Fang et al (2008) [Seahra & Hu 2010; Lombriser et al 2009]

Three Regimes

- Fully worked $f(R)$ and DGP examples show 3 regimes
- Superhorizon regime: ζ =const., $g(a)$
- Linear regime - closure condition - analogue of "smooth" dark energy density:

$$\begin{aligned} \nabla^2(\Phi - \Psi)/2 &= -4\pi G a^2 \Delta\rho \\ g(a,\mathbf{x}) &\leftrightarrow g(a,k) \end{aligned}$$

G can be promoted to $G(a)$ but conformal invariance relates fluctuations to field fluctuation that is small

- Non-linear regime:

$$\begin{aligned} \nabla^2(\Phi - \Psi)/2 &= -4\pi G a^2 \Delta\rho \\ \nabla^2\Psi &= 4\pi G a^2 \Delta\rho - \frac{1}{2}\nabla^2\phi \end{aligned}$$

Nonlinear Interaction

Nonlinearity in field equation recovers linear theory if $N[\phi] \to 0$

$$\nabla^2 \phi = g_{\rm lin}(a) a^2 \left(8\pi G \Delta\rho - N[\phi] \right)$$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

  a nonlinear function of the field

  Linked to gravitational potential

- For DGP, ϕ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4} \left[(\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$

  a nonlinear function of second derivatives of the field

  Linked to density fluctuation - Galileon invariance - no self-shielding of external forces

Non-Linear Chameleon

- For $f(R)$ the field equation
$$\nabla^2 f_R \approx \frac{1}{3}(\delta R(f_R) - 8\pi G \delta\rho)$$
is the non-linear equation that returns general relativity
- High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$
- Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if
$$\Delta f_R \leq \frac{2}{3}\Phi\,,$$
else required field gradients too large despite $\delta R = 8\pi G \delta\rho$ being the local minimum of effective potential

Non-Linear Dynamics

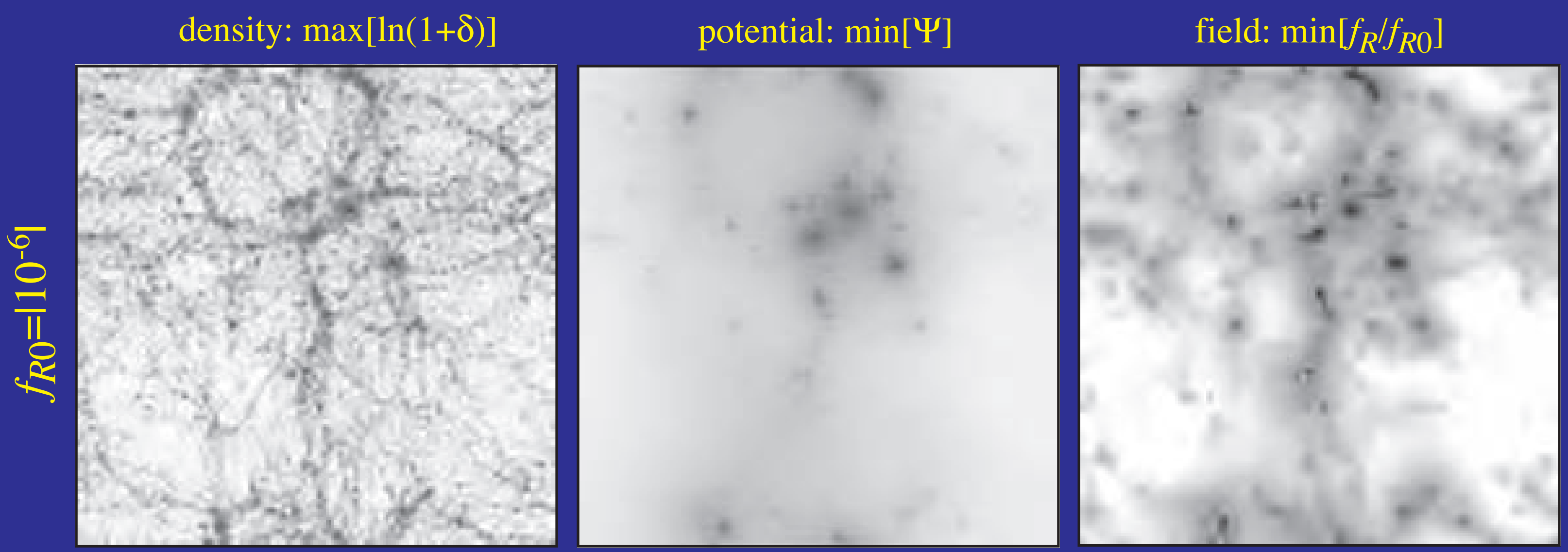
- Supplement that with the modified Poisson equation

$$\nabla^2\Psi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R(f_R)$$

- Matter evolution given metric unchanged: usual motion of matter in a gravitational potential Ψ
- Prescription for N-body code
- Particle Mesh (PM) for the Poisson equation
- Field equation is a non-linear Poisson equation: relaxation method for f_R
- Initial conditions set to GR at high redshift

Environment Dependent Force

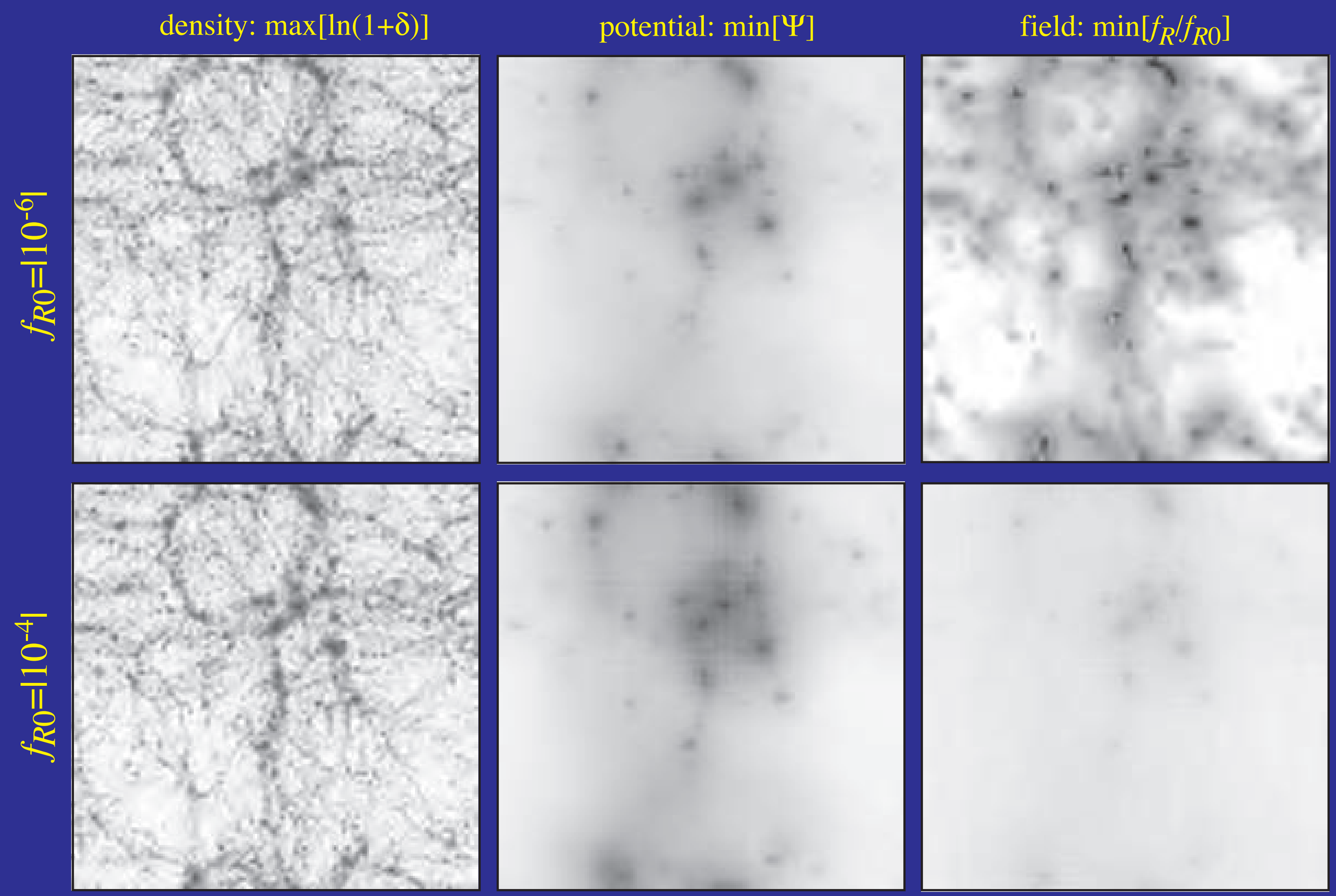
- Chameleon suppresses extra force (scalar field) in high density, deep potential regions



Oyaizu, Lima, Hu (2008)

Environment Dependent Force

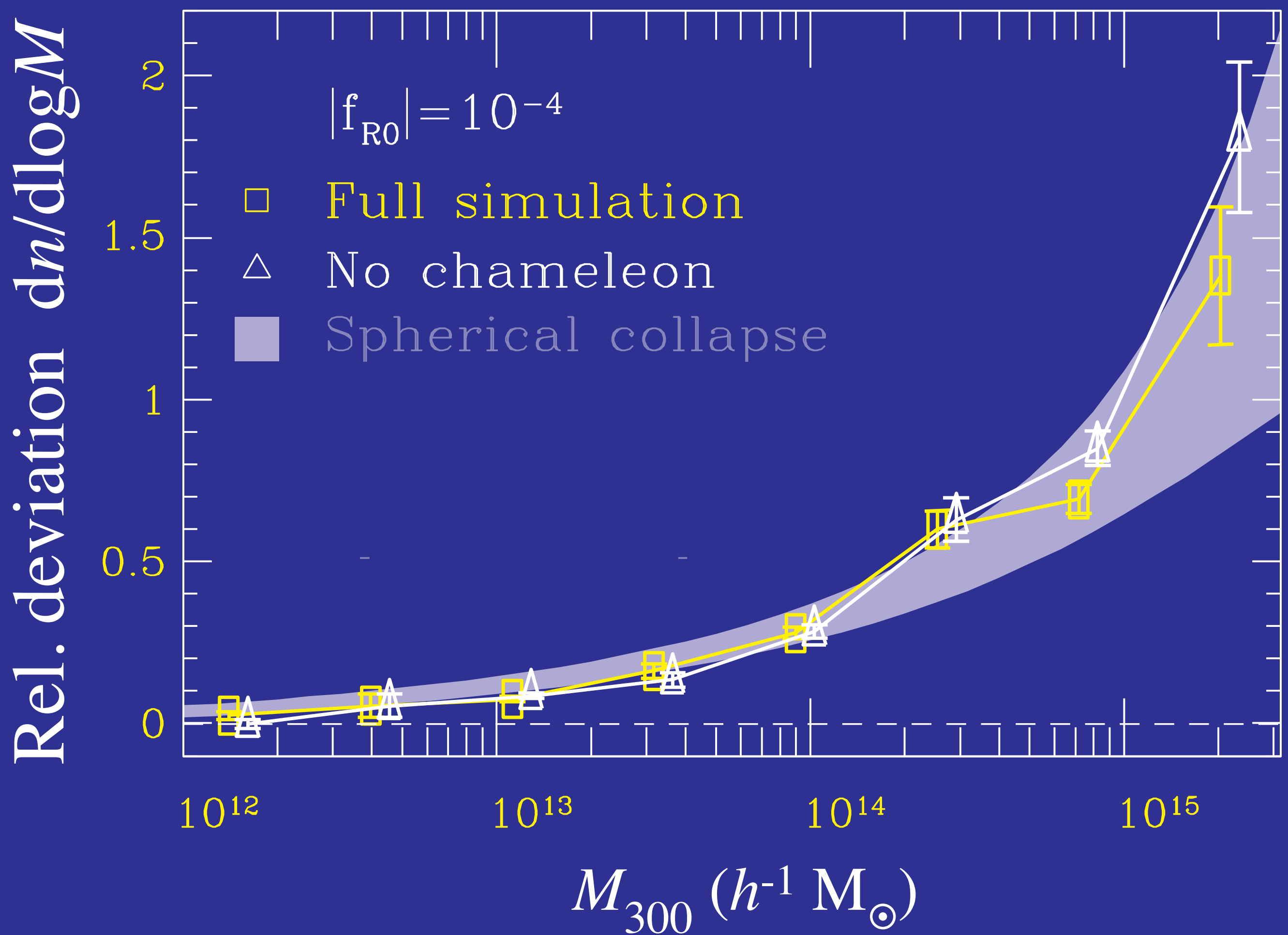
- For large background field, gradients in the scalar prevent the chameleon from appearing



Oyaizu, Lima, Hu (2008)

Cluster Abundance

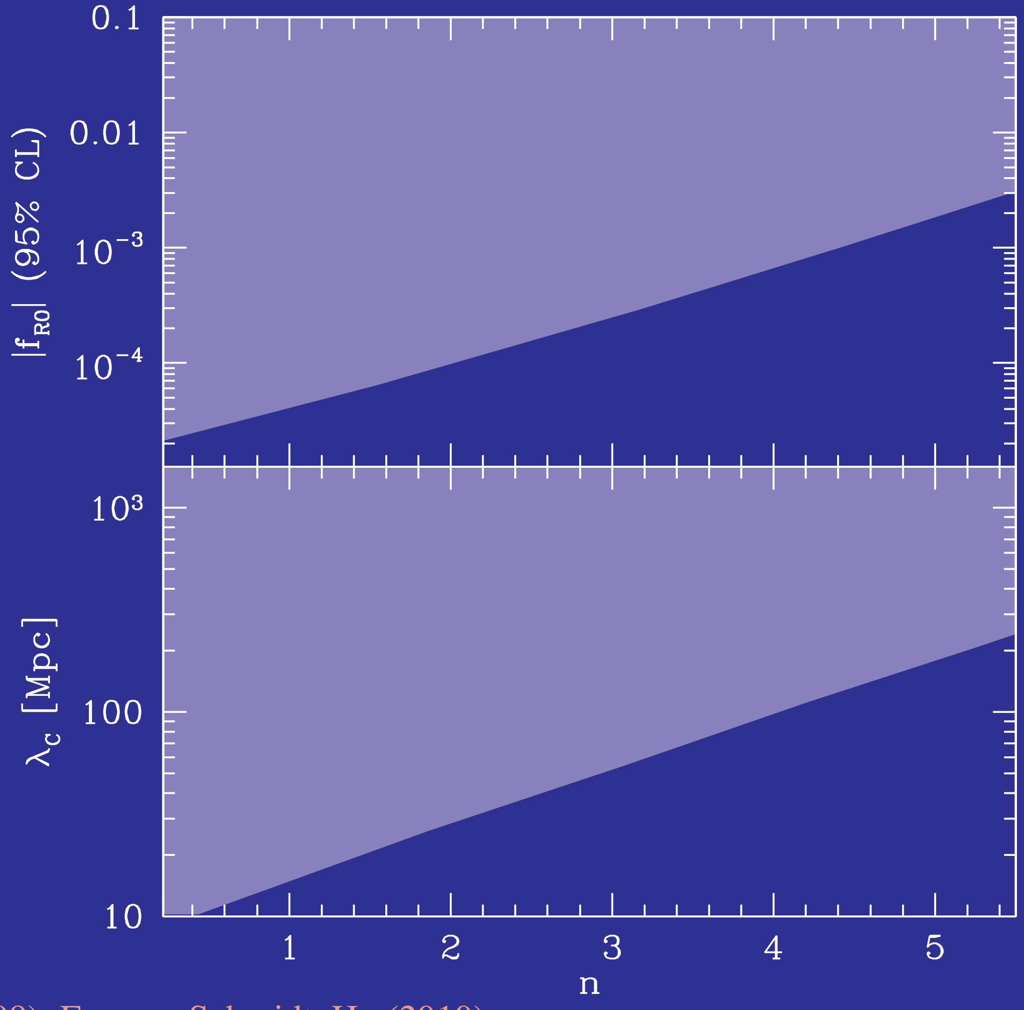
- Enhanced abundance of rare dark matter halos (clusters) with extra force



Lima, Schmidt, Oyaizu, Hu (2008)

Cluster *f*(*R*) Constraints

- Clusters provide best current cosmological constraints on *f*(*R*) models
- Spherical collapse rescaling to place constraints on full range of inverse power law models of index *n*



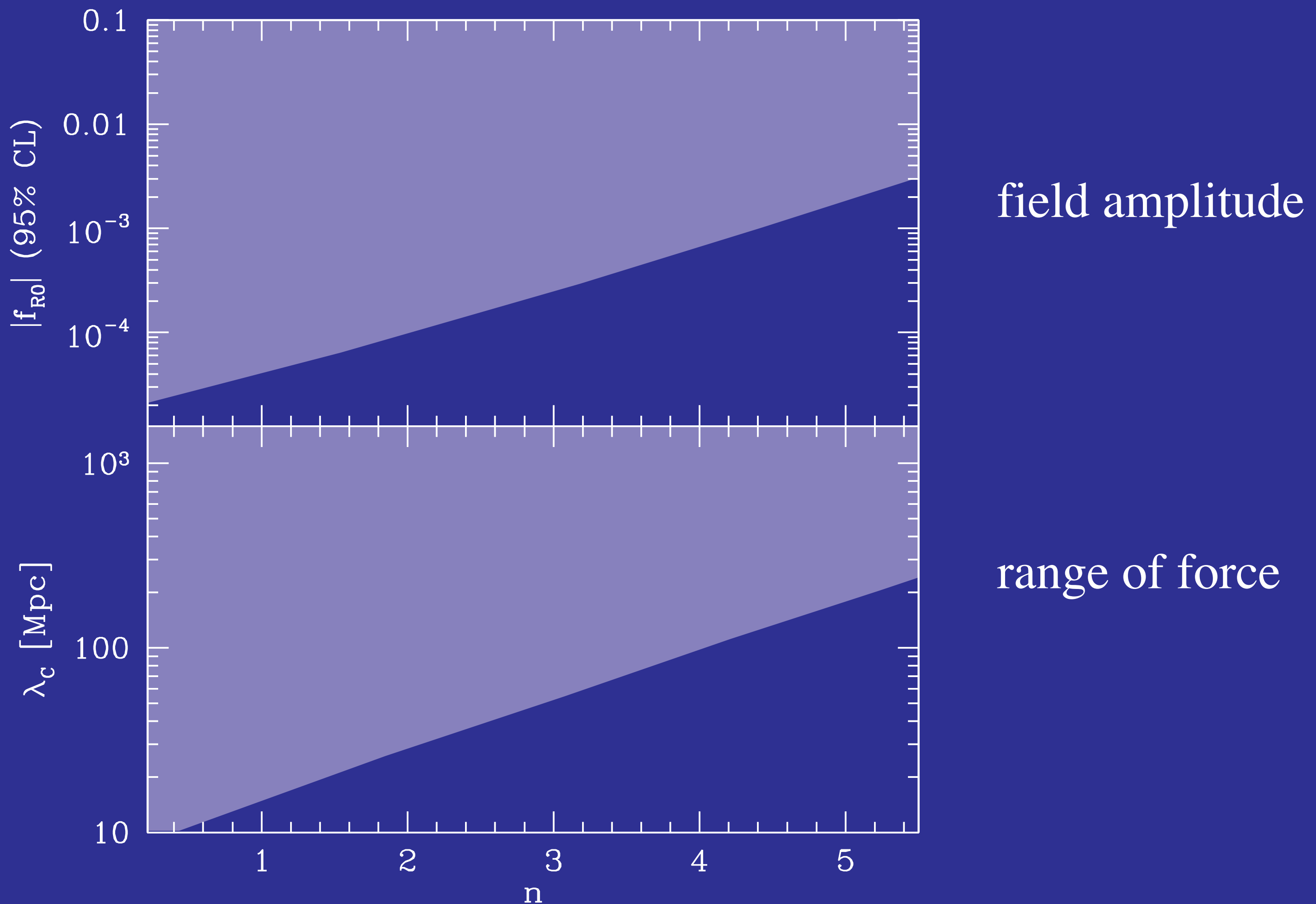
field amplitude

range of force

Schmidt, Vikhlinin, Hu (2009); Ferraro, Schmidt, Hu (2010)

Cluster $f(R)$ Constraints

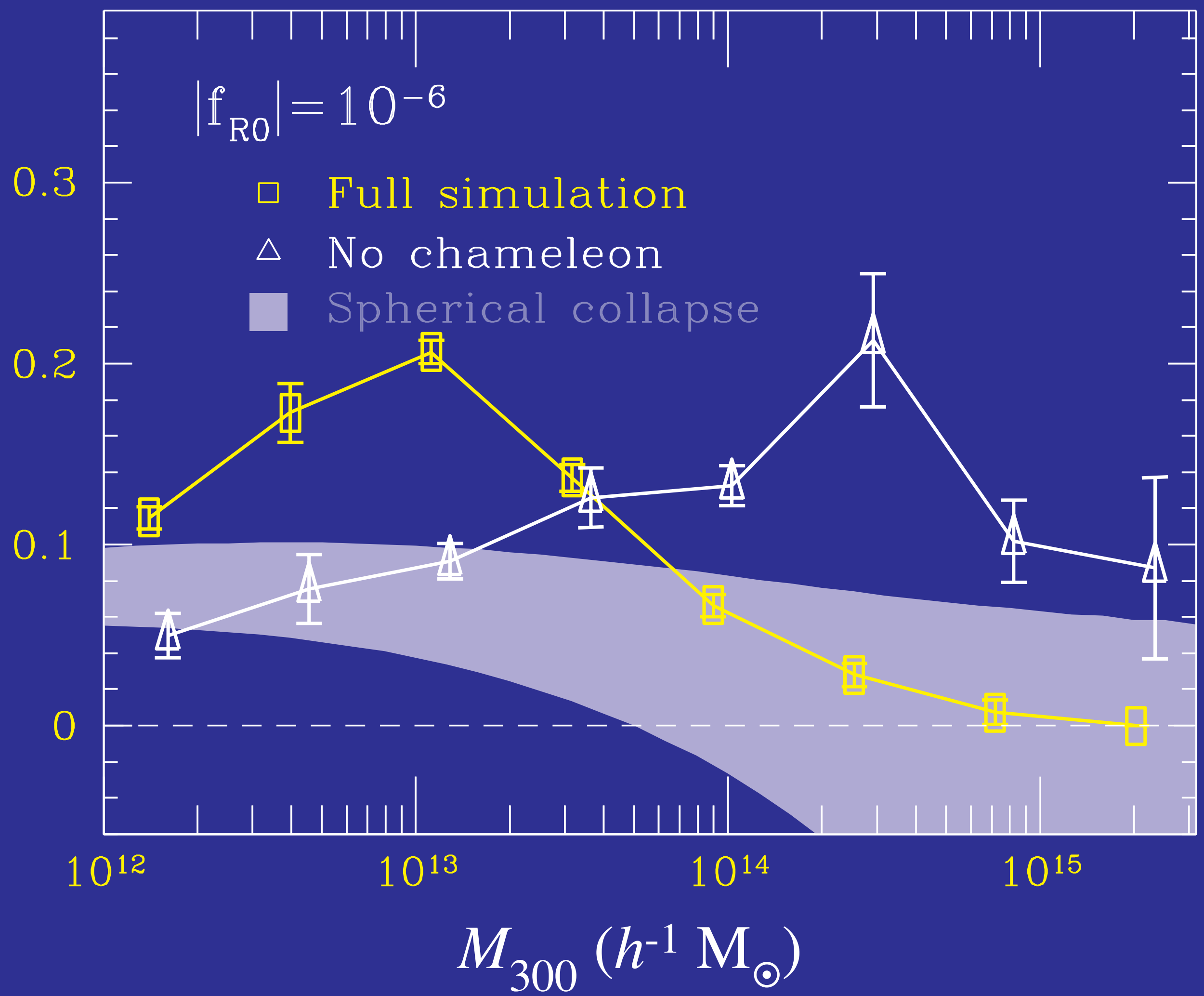
- Approaching competitiveness with solar system + Galaxy constraints of few 10^{-6} at low n
- Vastly different scale



Schmidt, Vikhlinin, Hu (2009); Ferraro, Schmidt, Hu (2010)

Chameleon Mass Function

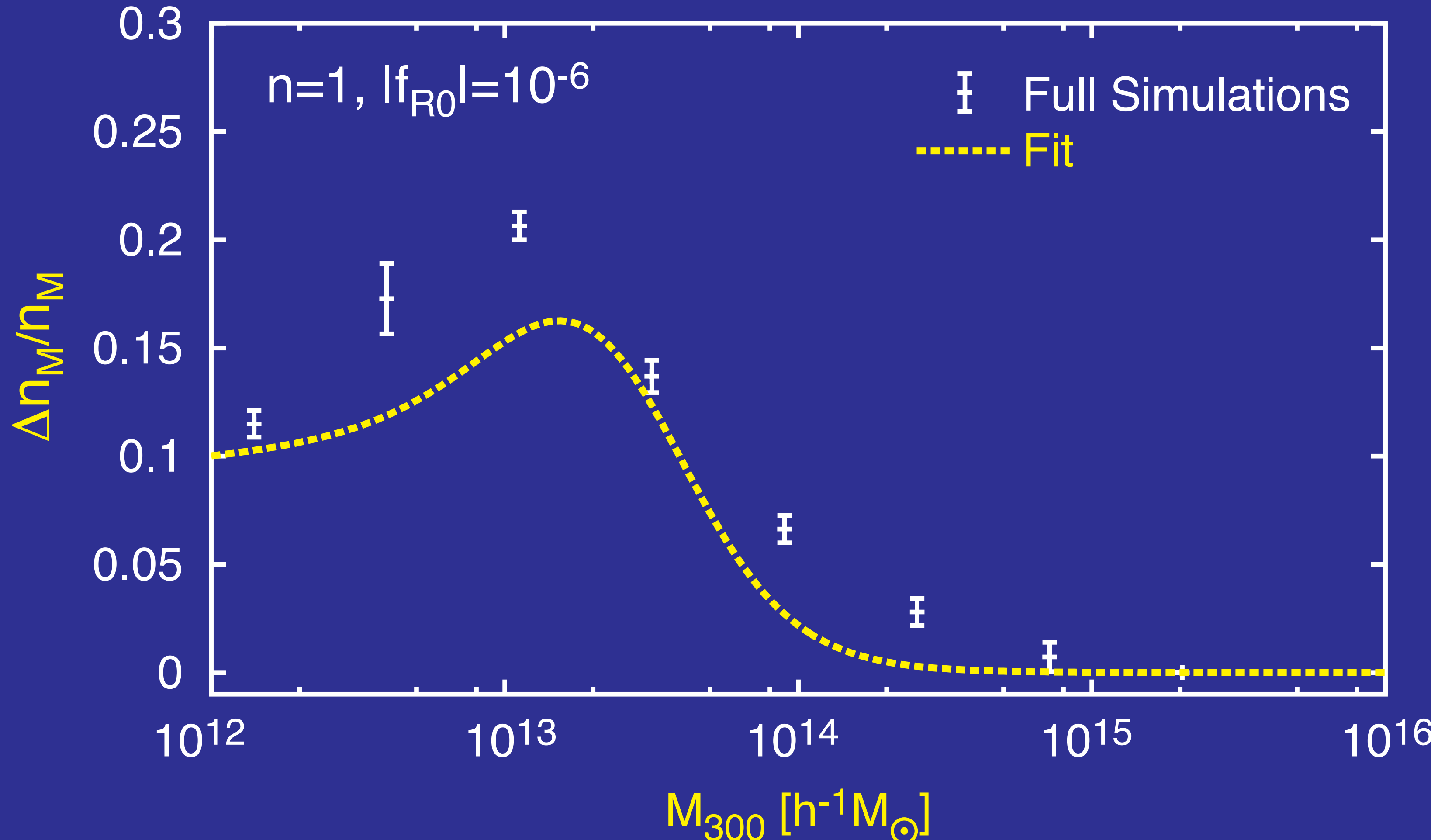
- Chameleon effect suppresses the enhancement at high masses
- Pile up of abundance at intermediate group scale



Lima, Schmidt, Oyaizu, Hu (2008)

Chameleon Mass Function

- Simple single parameter extention covers variety of models
- Basis of a halo model based post Friedmann parameterization of chameleon



Li & Hu (2011)

Nonlinear Interaction

Nonlinearity in field equation recovers linear theory if $N[\phi] \to 0$

$$\nabla^2 \phi = g_{\rm lin}(a) a^2 \left(8\pi G \Delta\rho - N[\phi] \right)$$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a nonlinear function of the field

Linked to gravitational potential

- For DGP, ϕ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4} \left[(\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$

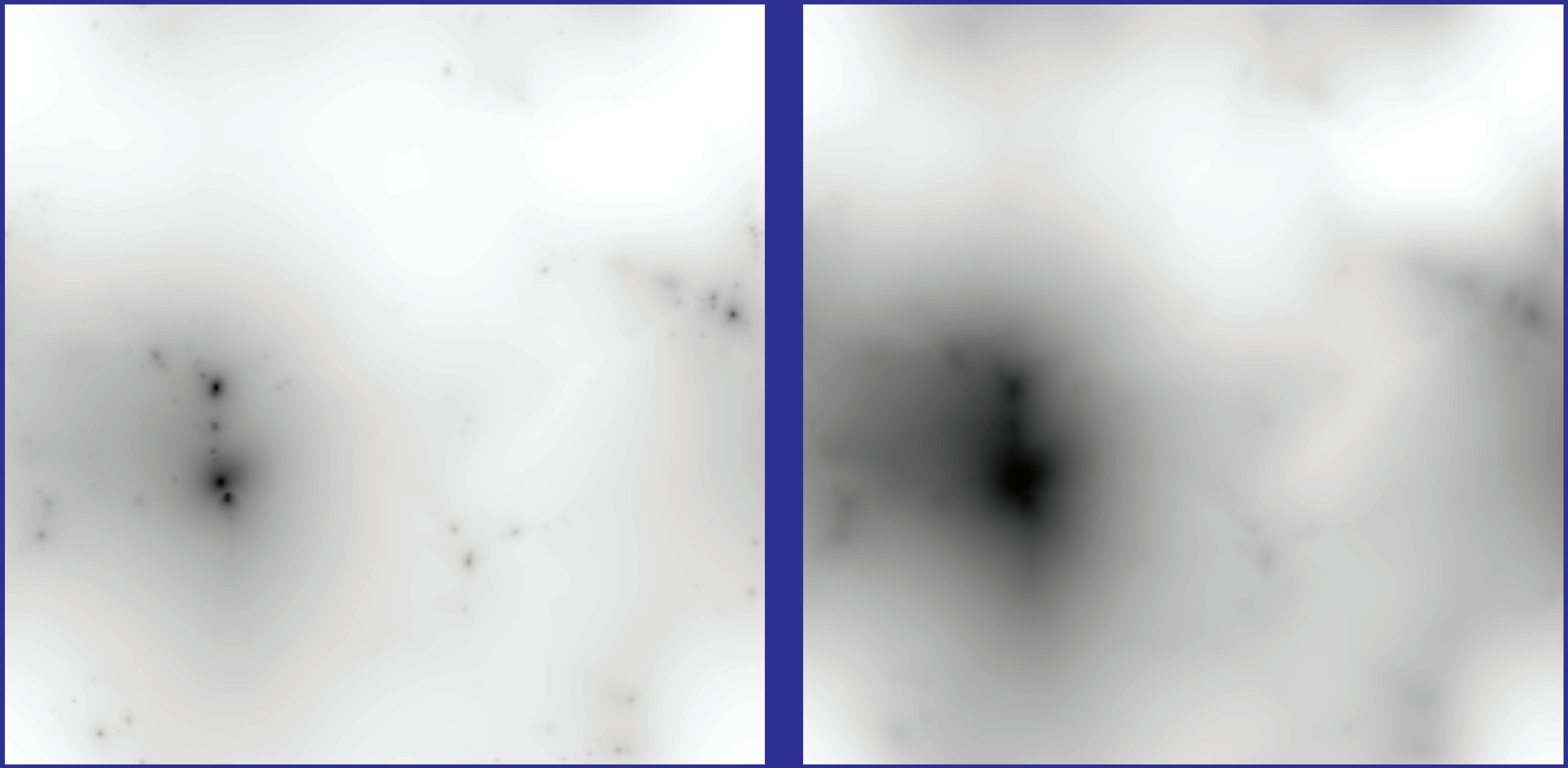
a nonlinear function of second derivatives of the field

Linked to density fluctuation - Galileon invariance - no self-shielding of external forces

DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential | Brane Bending Mode



Schmidt (2009); Chan & Scoccimarro (2009) (cf. Khoury & Wyman 2009)

Summary

- Formal equivalence between dark energy and modified gravity
- Practical inequivalence of smooth dark energy and extra propagating scalar fifth force
- Appears as difference between dynamical mass and lensing mass or dark energy anisotropic stress
- Smooth dark energy (e.g. quintessence) highly falsifiable
- Three regimes of modified gravity
- Nonlinearity in field equations return to ordinary gravity

    Chameleon: deep potential well

    Vainshtein: high local density
- $f(R)$ modified action and DGP braneworld fully-worked examples
- Insights on how cosmology does and does not complement solar system tests


謝謝