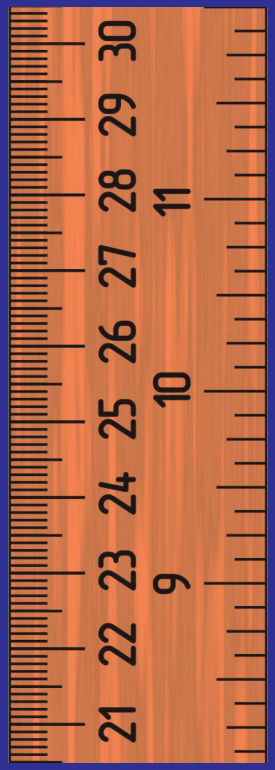
Massive Gravity:
Trouble with Metrics

metric



imperial

Wayne Hu

JGRG, December 2015

Massive Gravity:
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NASA's metric confusion caused Mars orbiter loss

September 30, 1999
Web posted at: 1:46 p.m. EDT (1746 GMT)

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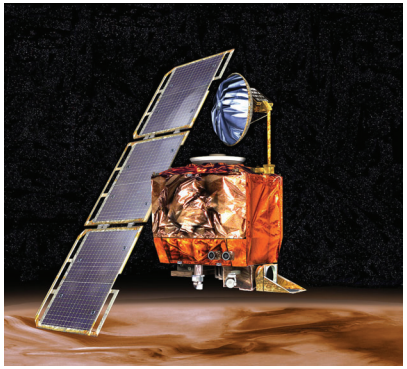

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Wayne Hu JSPS Fellow

JGRG, December 2015

Pierre Gratia, Austin Joyce, Hayato Motohashi, Pavel Motloch, Mark Wyman

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Wayne Hu PhD Student

JGRG, December 2015

Pierre Gratia, Austin Joyce, Hayato Motohashi, Pavel Motloch, Mark Wyman

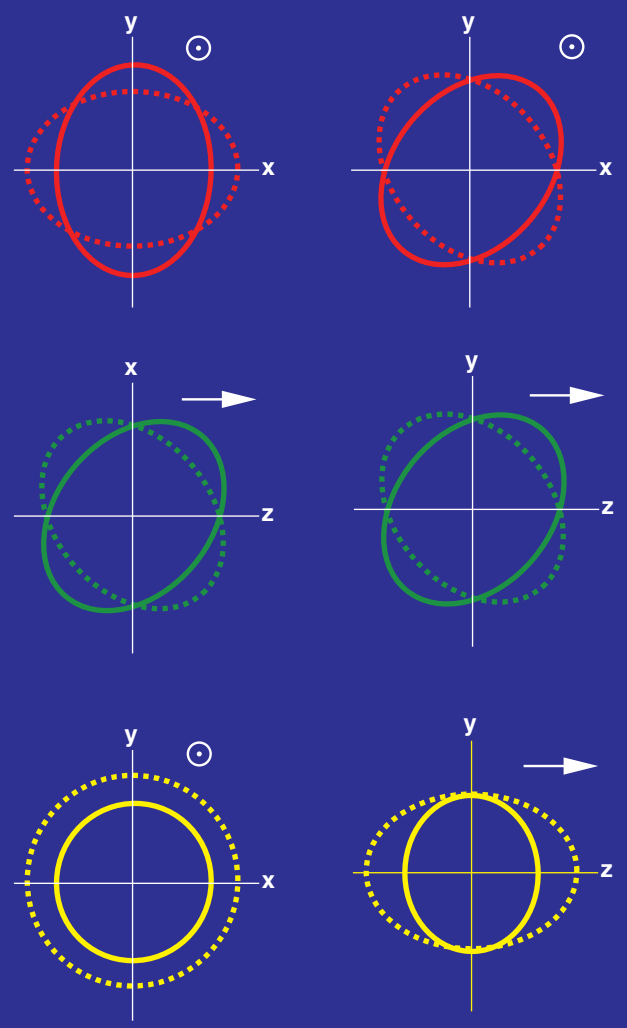
Massive Gravity

- A generic theory of massive gravity propagates 6 polarization states: 5 for a massive spin-2 and 1ghost

Tensor

Vector

Scalar



General Relativity

New Degrees of Freedom

Massive Gravity

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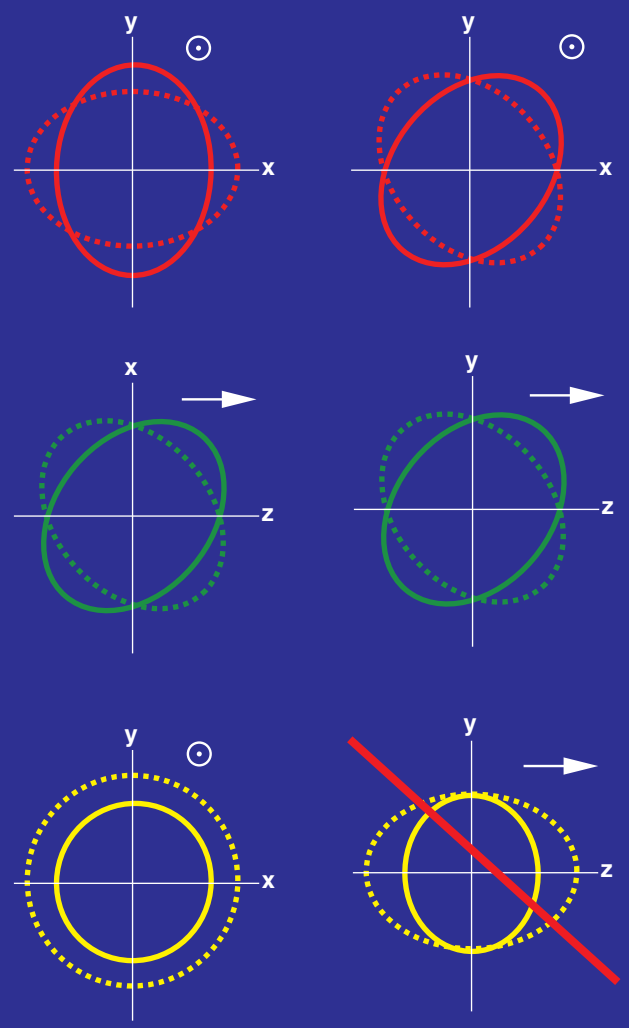
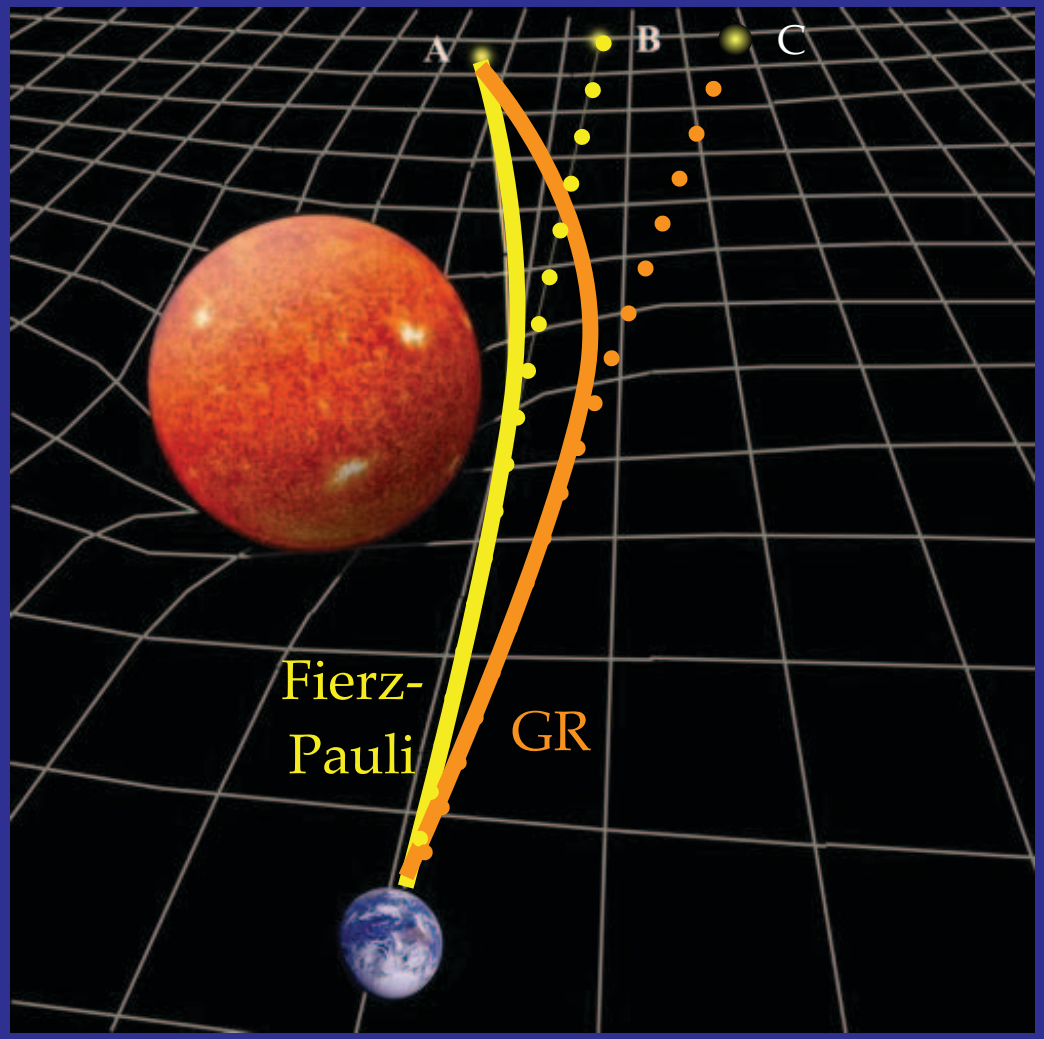


Fierz & Pauli (1939):
Ghost free linearized theory

$$m^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right)$$

vDVZ Discontinuity

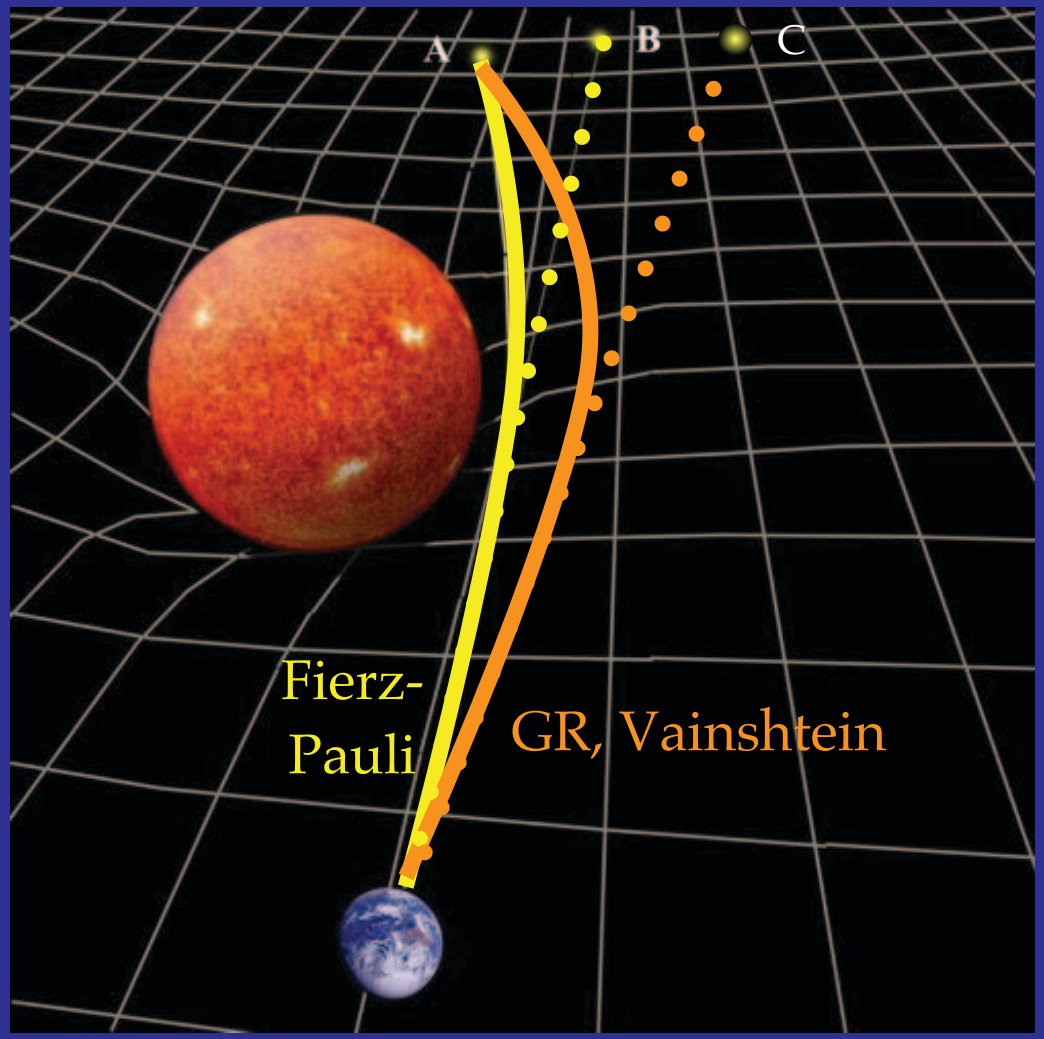
- Scalar mode coupled to matter changes space curvature per unit dynamical mass violating solar system lensing even as $m \to 0$



van Dam & Veltman (1970)
Zakharov (1970)

Vainshtein Mechanism

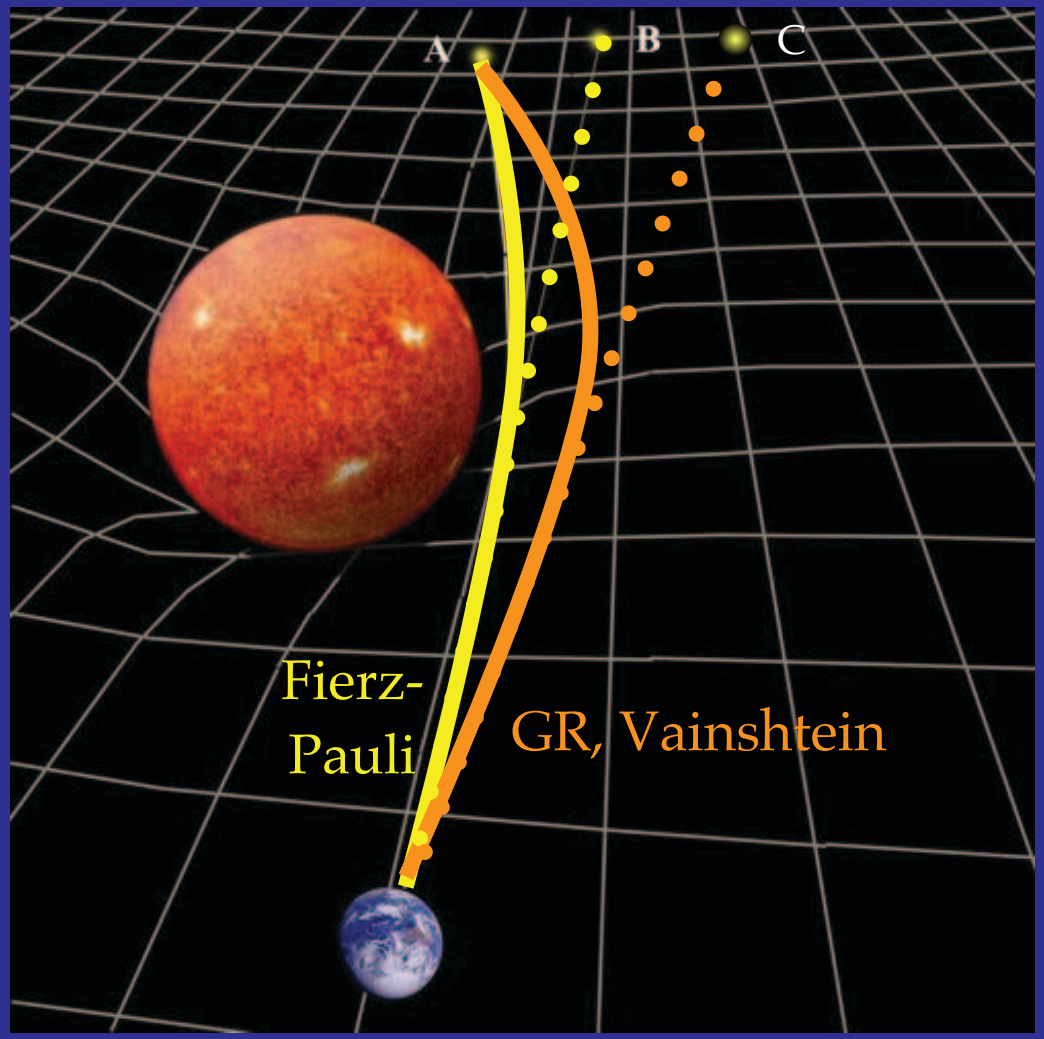
- Around massive sources, nonlinear interactions suppress scalar force

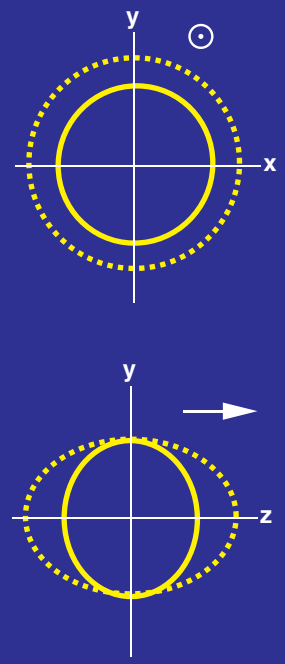


Vainshtein (1972)

Boulware-Deser Ghost

- But a generic nonlinear completion restores the 6th ghostly polarization





Bad trade!

Boulware & Deser (1972)

Massive Gravity

- de Rham, Gabadadze, Tolley (dRGT 2011) provided nonlinear completion to Fierz-Pauli that evades the Boulware-Deser ghost

$$S = \frac{M_p}{2} \int d^4X \sqrt{-g} \left[R - \frac{m^2}{2} \sum_{n=0}^{4} \frac{\beta_n}{n!} F_n(\sqrt{\mathbf{g}^{-1}\boldsymbol{\eta}}) \right]$$

where $\boldsymbol{\eta}$ is a fiducial metric, taken to be non-dynamical flat

$$ds_g^2 = g_{ab} dX^a dX^b, \quad ds_f^2 = \eta_{ab} dX^a dX^b = -dT^2 + dX_i^2$$

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- Presence of fiducial metric breaks diffeomorphism invariance: a preferred unitary gauge where metric is standard Minkowski
- Diffeomorphism invariance can be restored by transforming from these preferred coordinates

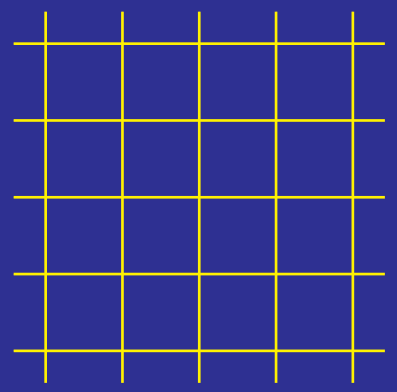
$$\mathbf{g}^{-1}\boldsymbol{\eta} \to g^{\alpha\mu} \partial_\mu X^a \partial_\nu X^b \eta_{ab} = g^{\alpha\mu} f_{\mu\nu}$$

- Jacobian transformation represents fiducial metric covariantly $f_{\mu\nu}$
- Unitary gauge coordinates become 4 scalar Stückelberg fields

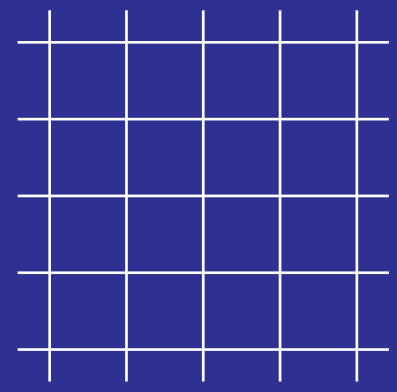
Spacetime Evolves from Minkowski

- Using Minkowski coordinates to chart the expanding spacetime

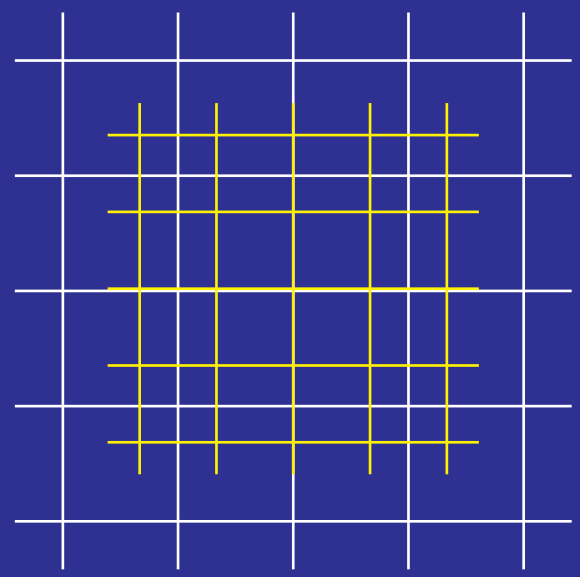
Minkowski Space



FRW Space



$a(t)$



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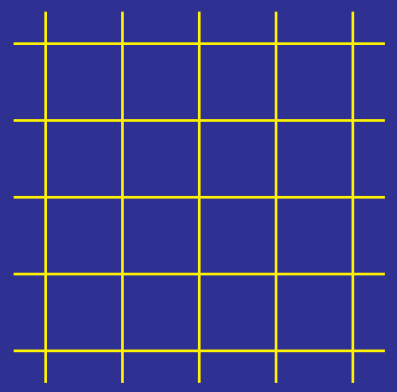

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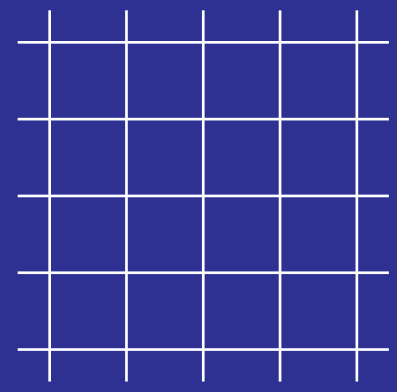
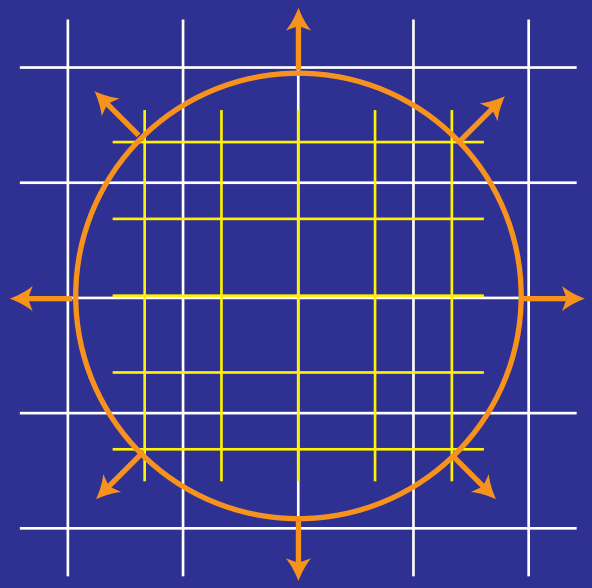


$a(t)$

In spatially flat Minkowski coordinates the spacetime metric is superficially inhomogeneous but isotropic (H^2R^2 terms; static/physical vs comoving coordinates)



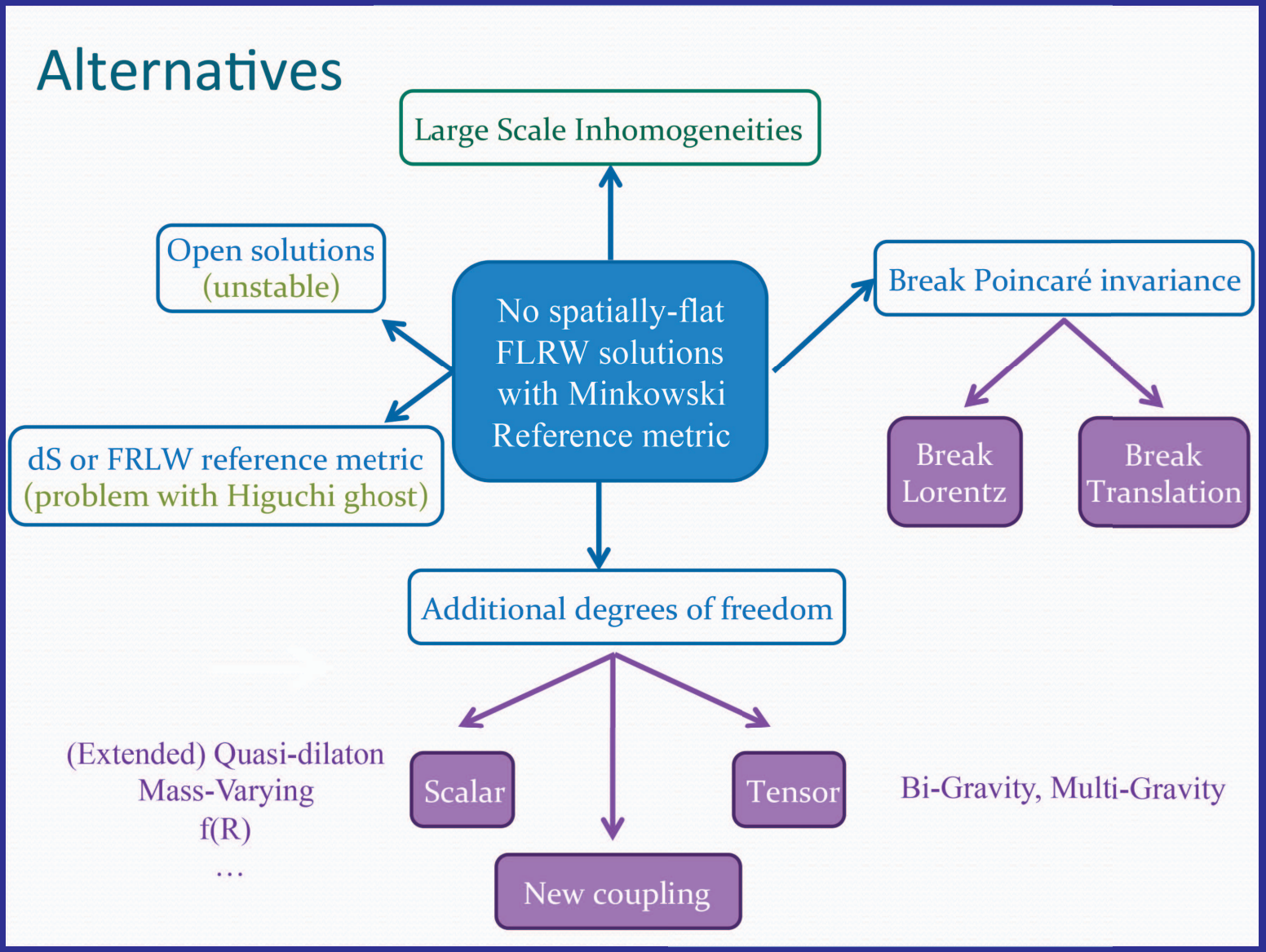
Homogeneity and Isotropy

- Coordinate problems take on geometric significance with two metrics
- Spatially flat slicing of Minkowski incompatible with homogeneous and isotropic FRW slicing of spacetime
  "no spatially flat FRW cosmologies" d'Amico et al (2011)

  = no single coordinates where both the spacetime and fiducial metric are simultaneously homogeneous and isotropic
- Open slicing of Minkowski (Milne) compatible with homogeneous and isotropic slicing of an open FRW spacetime
  Gumrukcuoglu, Lin, Mukohyama (2011)

  ...but these are generally are generally unstable
  Gumrukcuoglu, Lin, Mukohyama (2011); DeFelice, Gumrukcuoglu, Mukohyama (2012)
- Note: this does not preclude homogeneous and isotropic FRW spacetimes of any curvature or address their stability

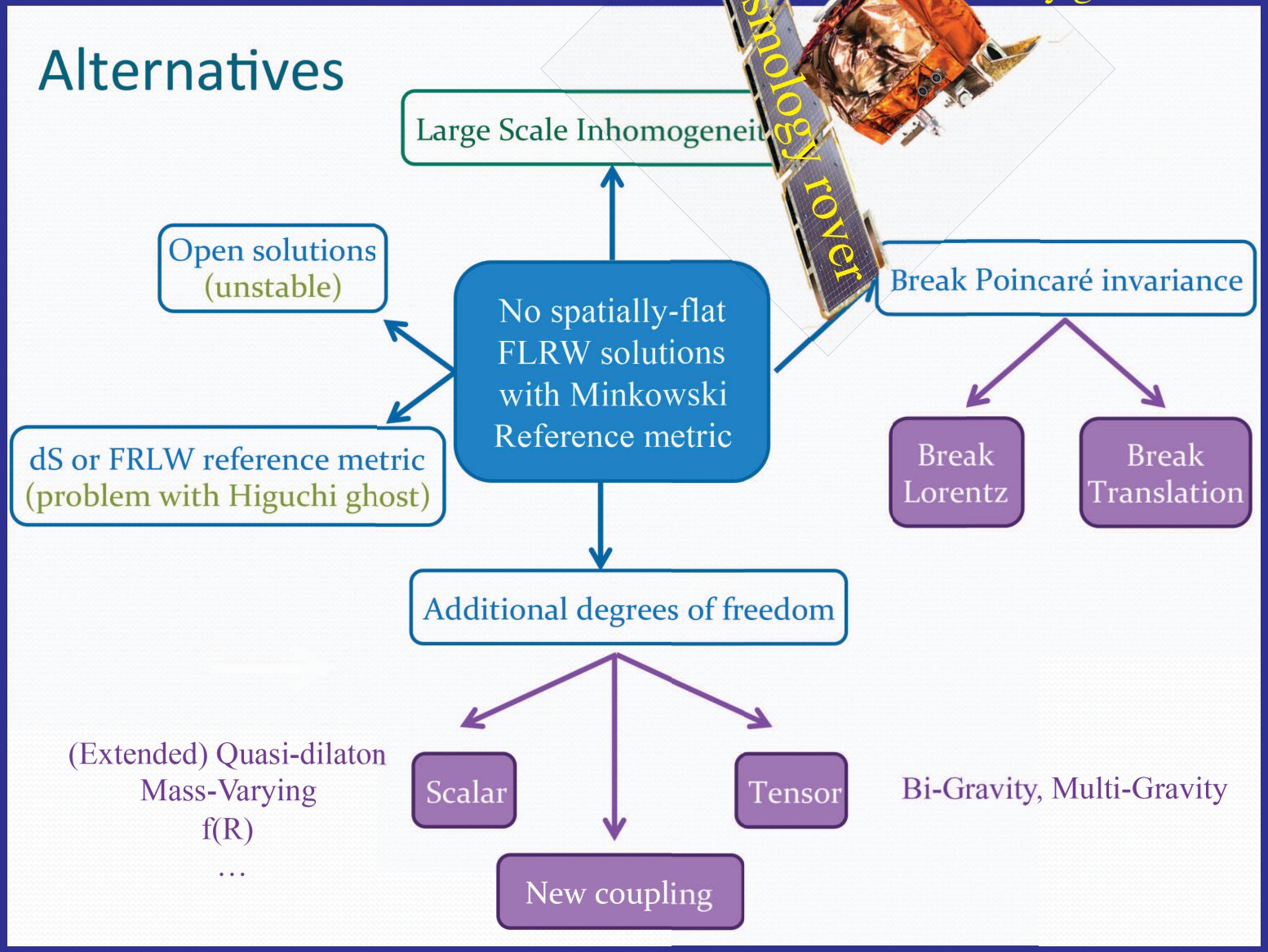
Massive Multiverse

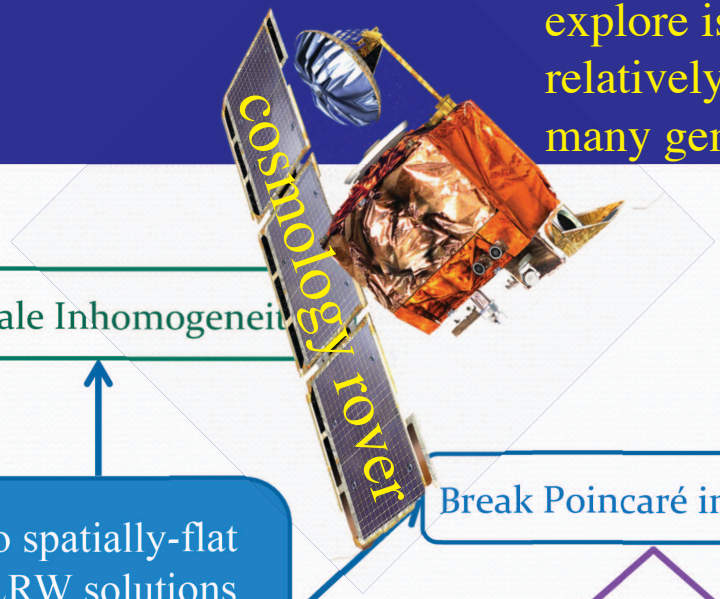


de Rham (2015)

Massive Multiverse

explore issues with 2 metrics relatively simply; common to many generalizations



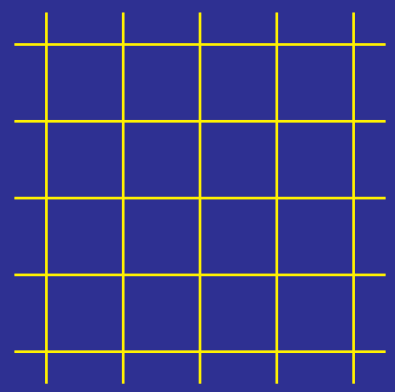


de Rham (2015)

Self-Accelerating Solutions

- Allow the Minkowski coordinates T, R or Stuckelberg field to be inhomogeneous in isotropic FRW coordinates

Minkowski Space



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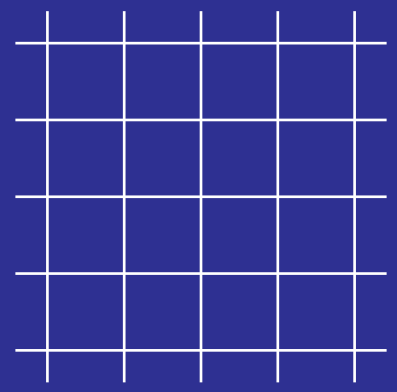
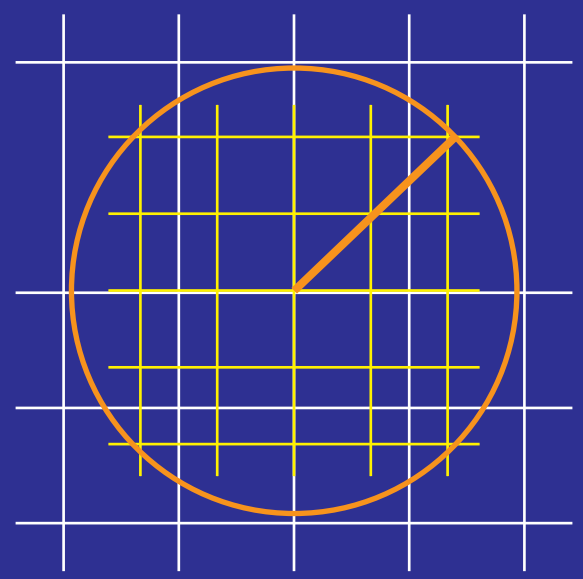


$a(t)$



$R = x_0 a(t) r$

x_0 constant determined by MG Parameters

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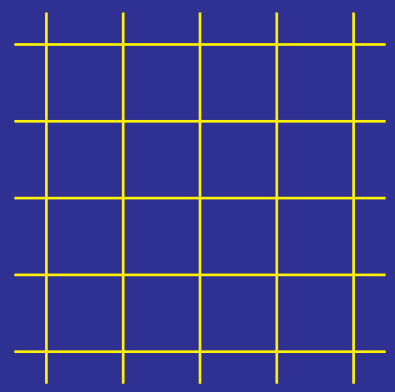
Minkowski Space



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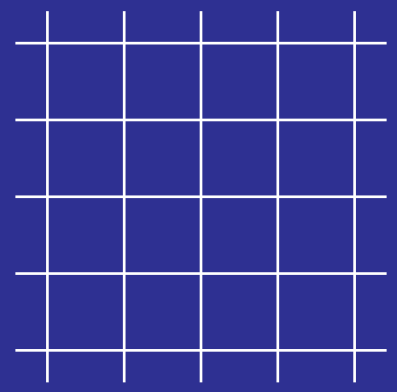
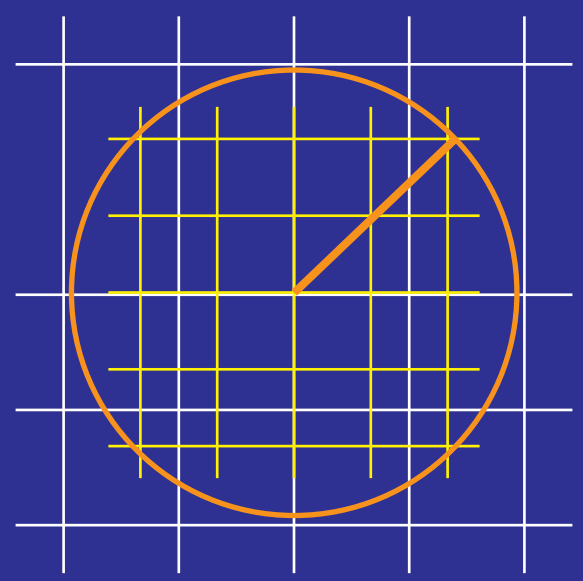


$a(t)$

All such constructions lead to an effective stress energy of a cosmological constant leaving remaining freedom in choosing Minkowski time $T(t,r)$



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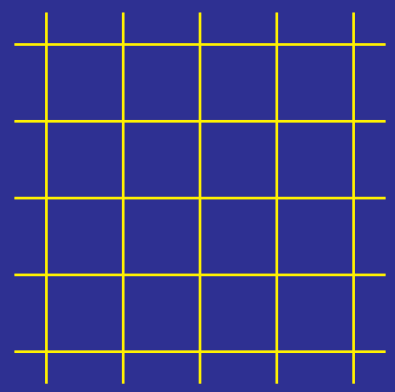

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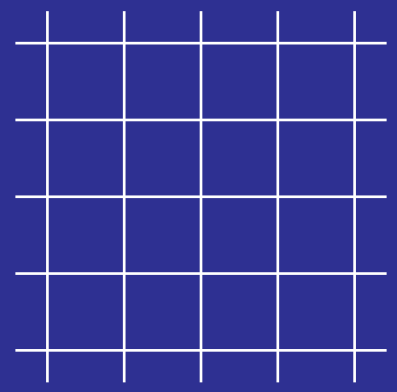
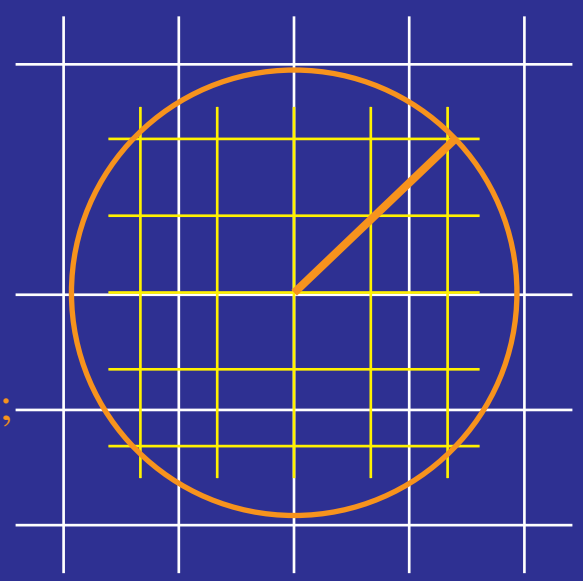


$a(t)$

applies to any isotropic distribution of matter and unifies the description of all self-accelerating solutions
Gratia, Hu, Wyman (2012)

generalizes Koyama, Niz, Tasinato (2011); d'Amico et al (2012); Gumrukcouglu et al (2012); Berezhiani et al (2011);...



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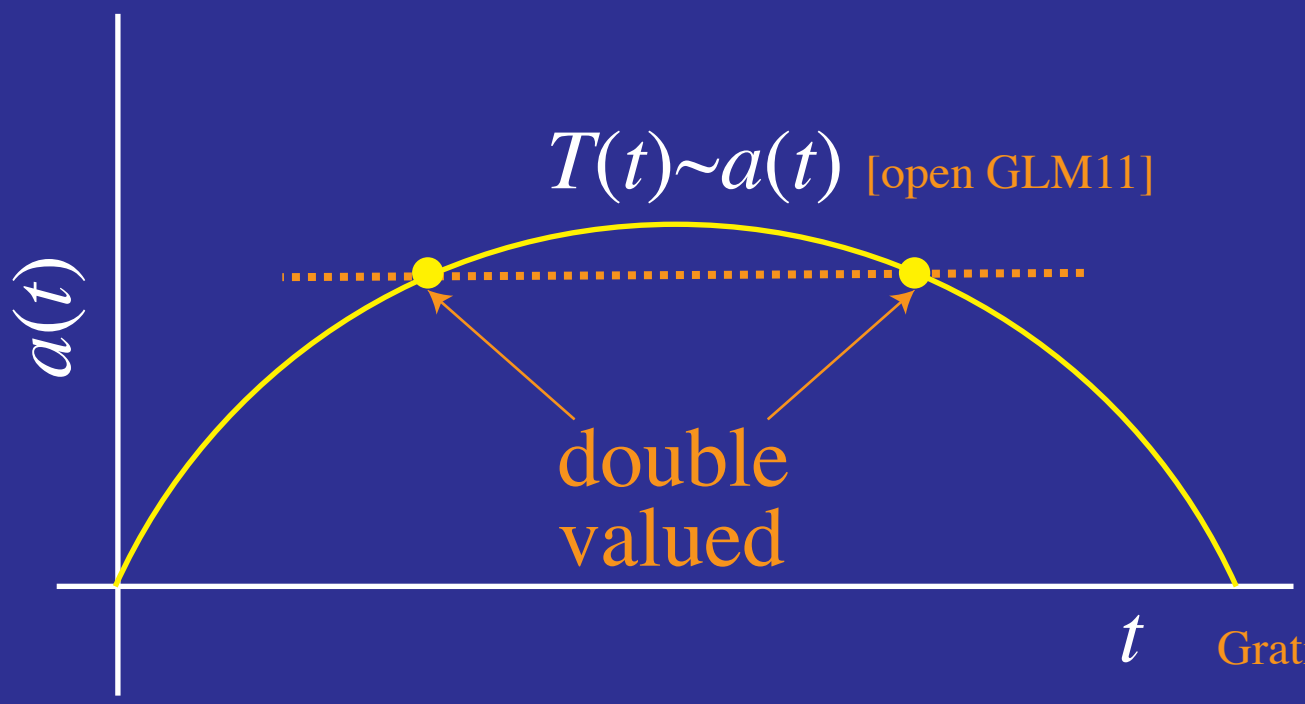
Determinant Singularities

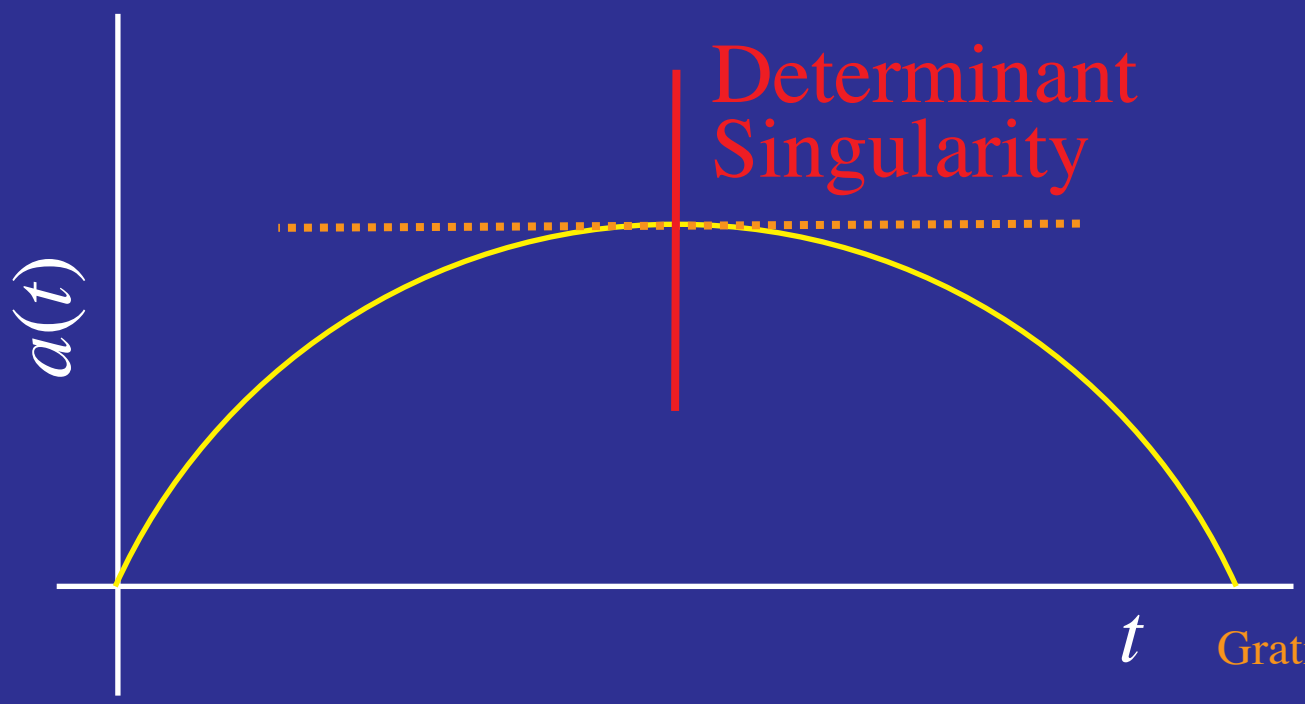
- Minkowski coordinates may not uniquely chart the whole spacetime - Jacobian between Minkowski and spacetime coordinates singular
- Fiducial metric has a determinant singularity where the spacetime metric does not or vice versa - ratio of determinants is a diffeomorphism invariant spacetime scalar
- Example: evolution to a det singularity



Gratia, Hu, Wyman (2013)

Determinant Singularities

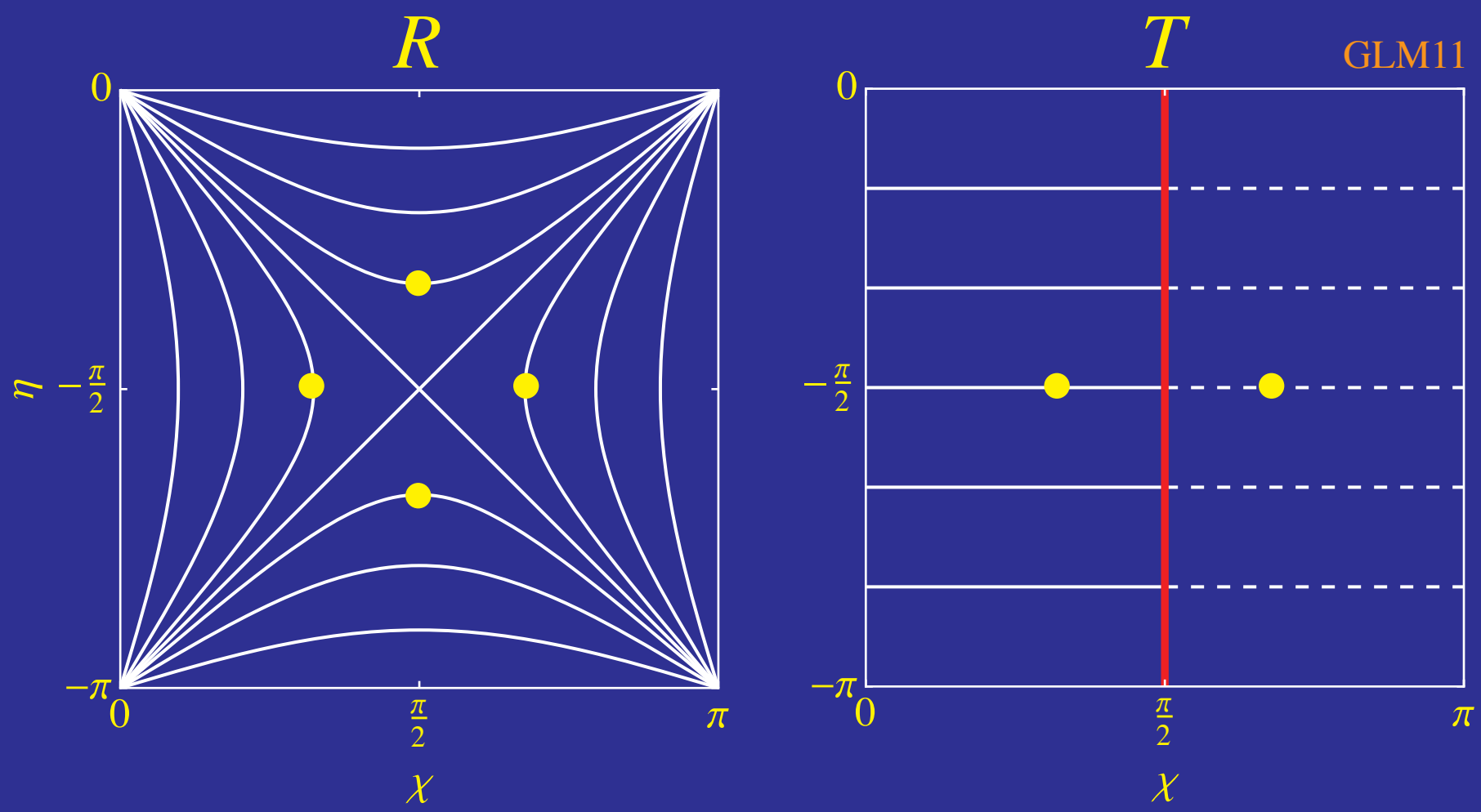
- No curvature singularity in the spacetime, normal matter sees only spacetime metric
- But requires ad hoc rules for smoothly joining charts for the massive gravity degrees of freedom; evolves into a singularity
- Occurs in more general bi-gravity models Gratia, Hu, Wyman (2014); Lagos & Ferreira (2014); Johnson & Terrana (2015) and extended quasi dilaton model (where smooth continuation fails) Motohashi & Hu (2014)



Gratia, Hu, Wyman (2013)

DeSitter Solutions

- Conformal diagram of de Sitter self-accelerating solutions
- Det=0 singularity when coordinates double valued



4 fold symmetric

Motloch, Hu, Joyce, Motohashi (2015)

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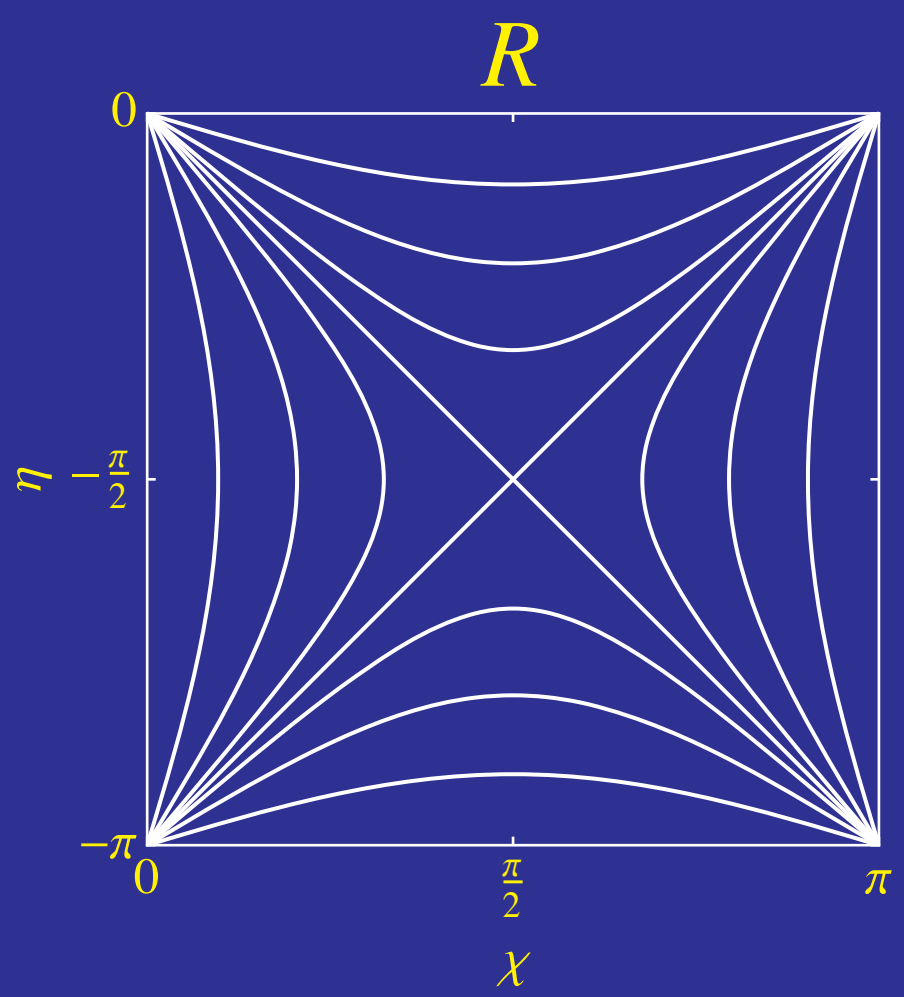

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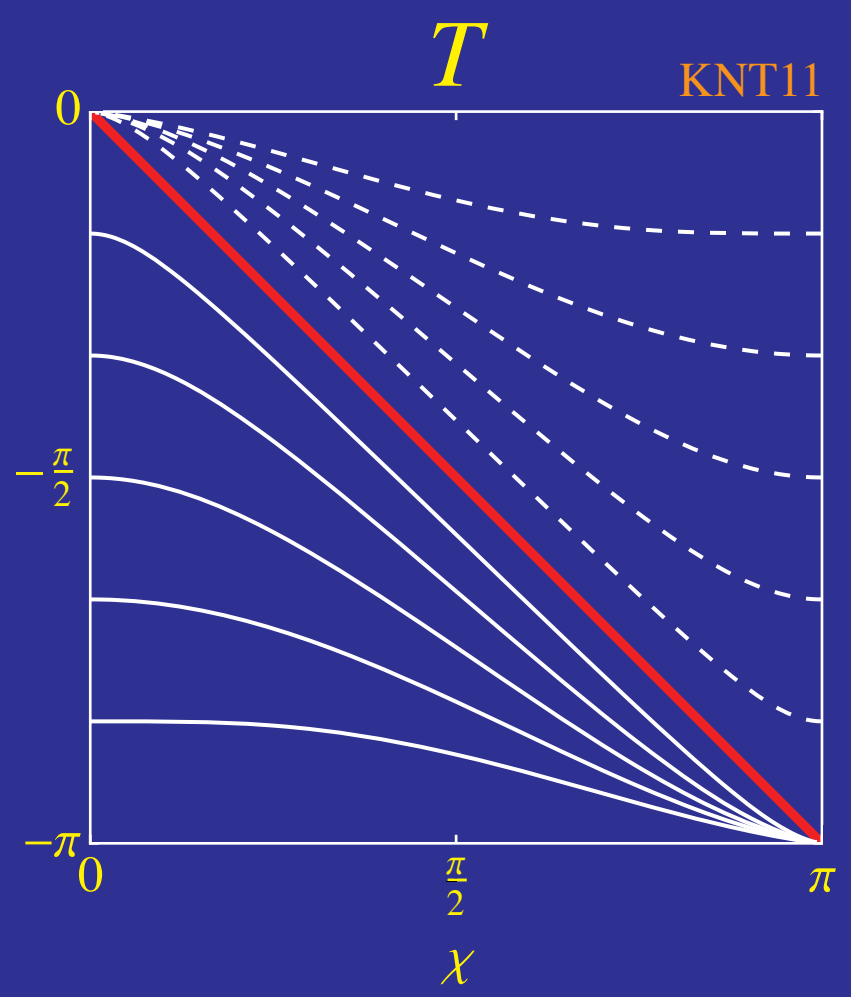
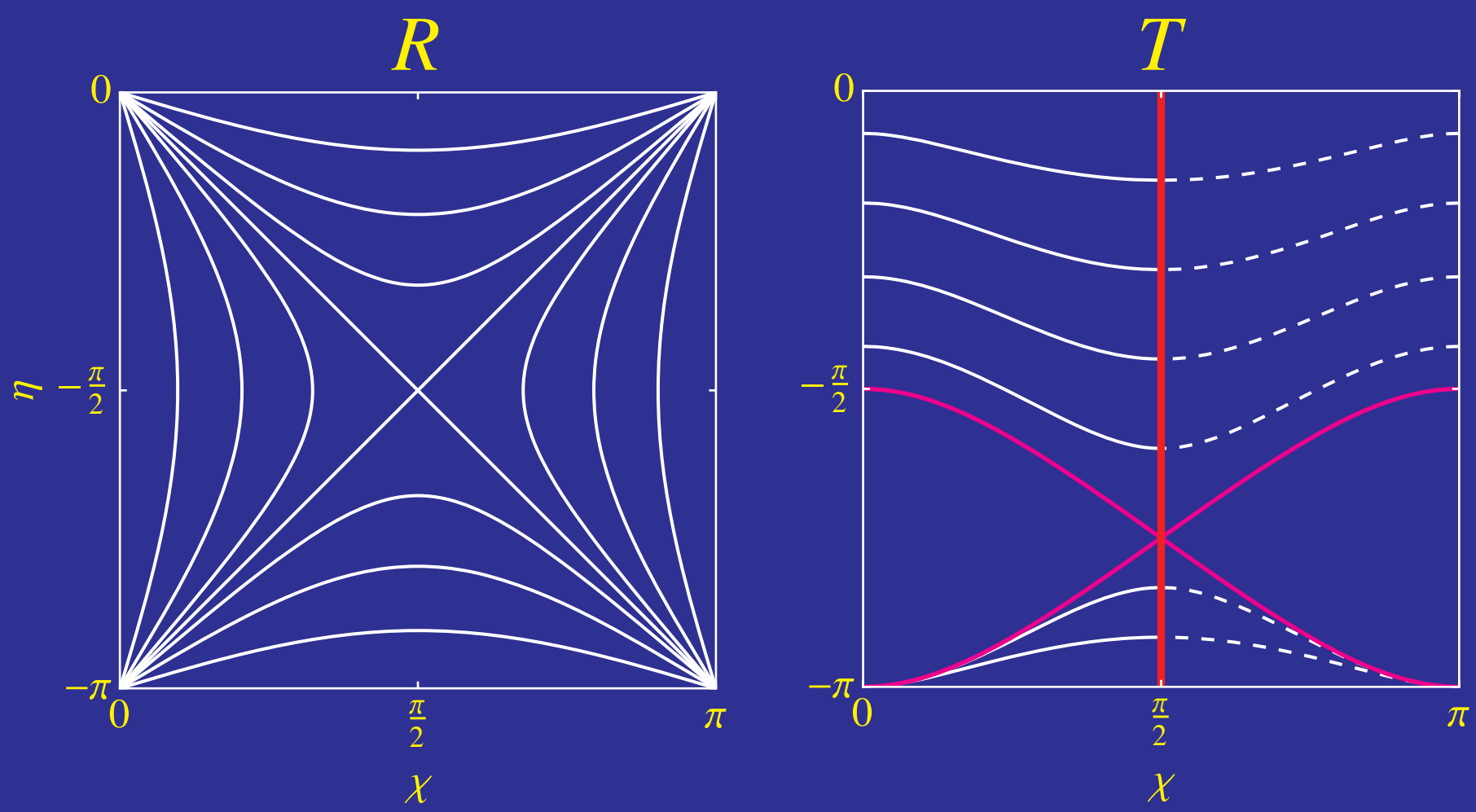


Motloch, Hu, Joyce, Motohashi (2015)

DeSitter Solutions

- Conformal diagram of de Sitter self-accelerating solutions
- Det=±∞ singularity where continuation flips signature



4 fold symmetric

Motloch, Hu, Joyce, Motohashi (2015)

Perturbations

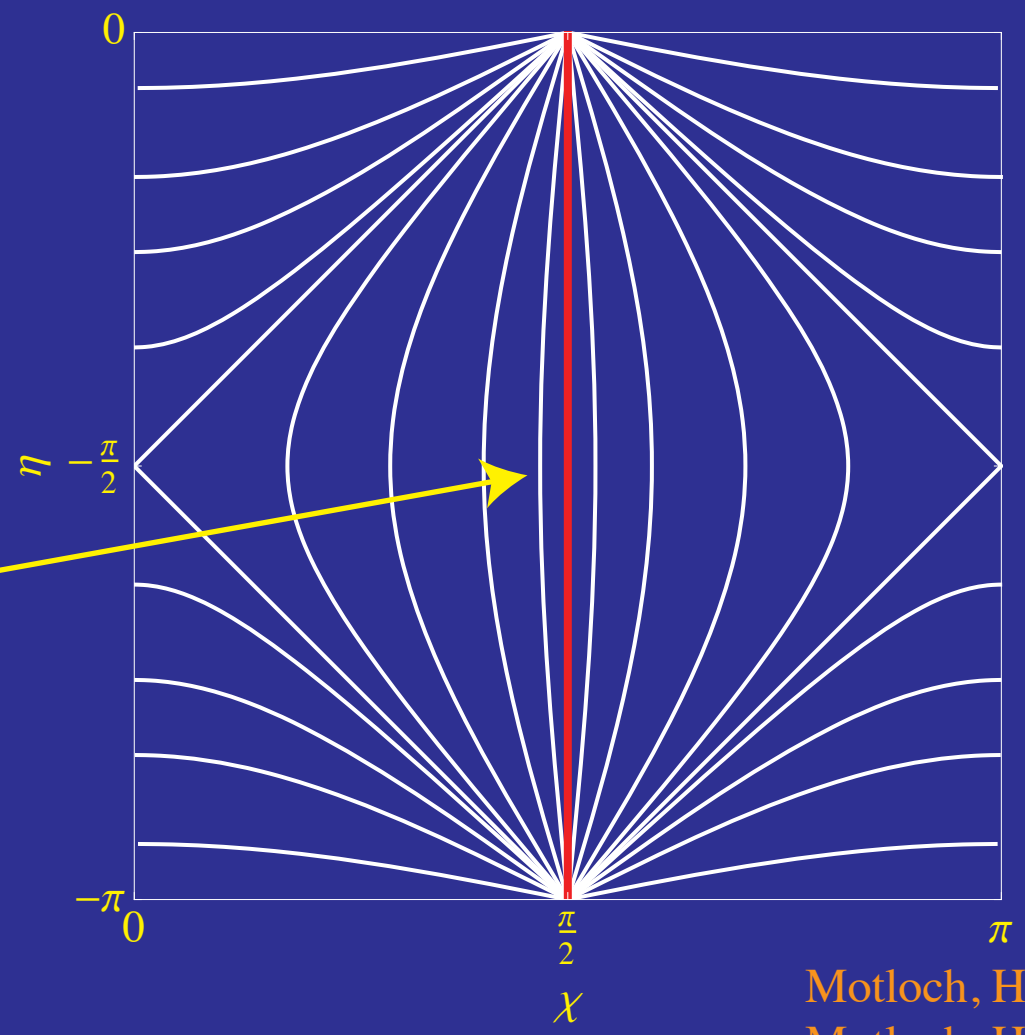
- Inhomogeneous Stuckelberg background complicates analysis
- Isotropic mode (scalar) not sourced by matter, carries stress energy, obeys first order equation of motion Wyman, Hu, Gratia (2011)

  simple system, analytic solutions
- Decoupling limit expectations for the helicity 0 and ±1 modes not obeyed, kinetic terms only at order curvature
  d'Amico (2011); Motloch & Hu (2014)

  In general 5 degrees of freedom (including open GLM solution, but 3 parabolic not hyperbolic)
- Fully covariant Stuckelberg-metric quadratic Lagrangian
  Motloch & Hu (2014)
- Specialize to vacuum unitary perturbation gauge: metric perts only Regge-Wheeler analysis of gw polarizations
  Motloch, Hu, Motohashi (2015)

Characteristics

- Characteristic curves of new degrees of freedom
- Example: "open FRW" solution of GLM11



characteristics run tangent to determinant singularities

Motloch, Hu, Joyce, Motohashi (2015)
Motloch, Hu, Motohashi (2015)

see also: Deser, Waldron, etal (2012-15); Izumi & Ong (2013)

Characteristics

- Characteristic curves of new degrees of freedom
- Example: "open FRW" solution of GLM11

- Characteristics coincide with constant open time slices [no dynamics in open frame]
- Superluminal characteristics
- For monopole & dipole mode first order system: characterstics give all smooth and discontinuous front solutions
- Superluminal front and group velocity

Characteristics

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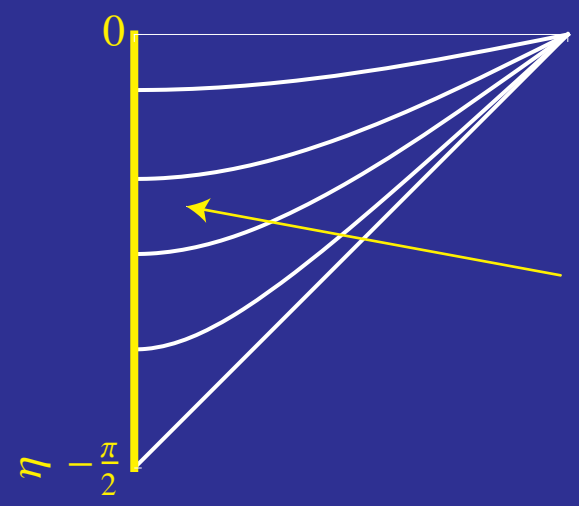

No Spacelike Cauchy Surface:
Spatial Boundary Conditions
Motloch, Hu, Joyce, Motohashi (2015)

- No spacelike surface intersect all characteristics
- For isotropic & dipole modes, second order system decouples into two first order systems, where a conditions on a single spatial boundary defines unique solution

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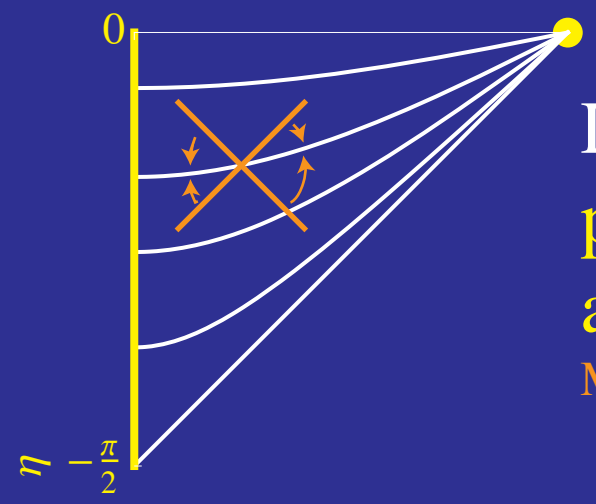


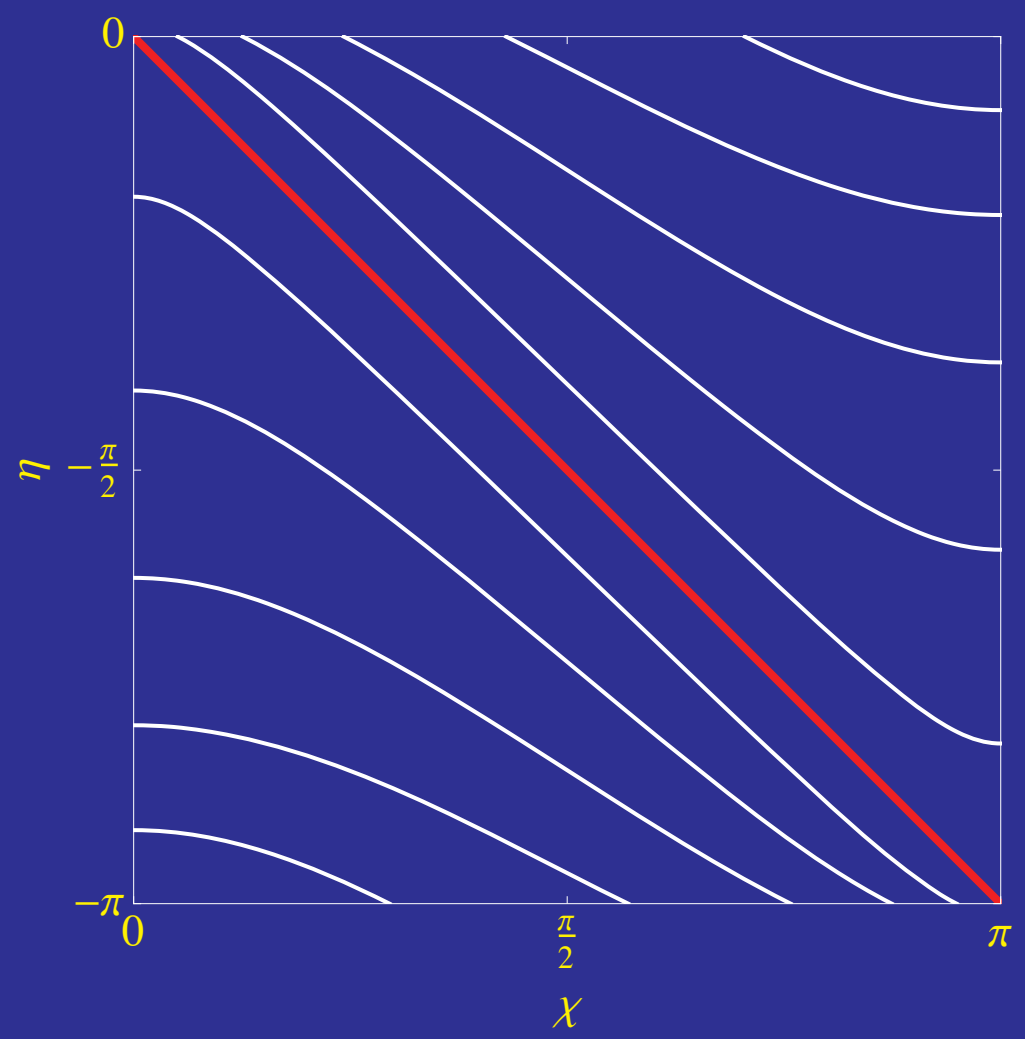
Lightcone degenerates: parabolic equation for anisotropic modes

Motloch, Hu, Motohashi (2015)

- Anisotropic $l \geq 2$ odd modes are second order and parabolic, not hyperbolic
- No wavelike solutions, similar to heat equation
- Requires two spatial boundary conditions to define unique solution

Characteristics

- Example: "SdS" solution of KNT11: characteristic curves run tangent to det singularities - information doesn't cross
- Spacelike surface do intersect characteristics defining initial value problem for isotropic & dipole modes
- Special case with luminal characteristics
- But $l \geq 2$ odd parity modes are still parabolic, requiring two boundary conditions: true of all self accelerating solutions



Motloch, Hu, Joyce, Motohashi (2015)
Motloch, Hu, Motohashi (2015)

Summary: Trouble with Metrics

- Self-accelerating dRGT massive gravity provides a relatively simple arena where Cauchy breakdown occurs at linear order in cosmological perturbations
  (det singularities, parabolic/elliptic equations, no joint spacelike surface)
- In other cases where modes propagate on a separate metric similar problems occur on nonlinear backgrounds

  Cosmological voids with cubic galileon [Barreira et al. 2013; Winther & Ferreira 2015] [hyperbolic turns to elliptic]

  Spherical collapse far from quasistatic approximation with DGP [Babichev et al. 2011] [no joint spacelike Cauchy surface]

- Can be viewed as a strong coupling problem which may be solved by a UV completion of effective theory but occurs at relatively low densities and large scales from non pathological initial conditions

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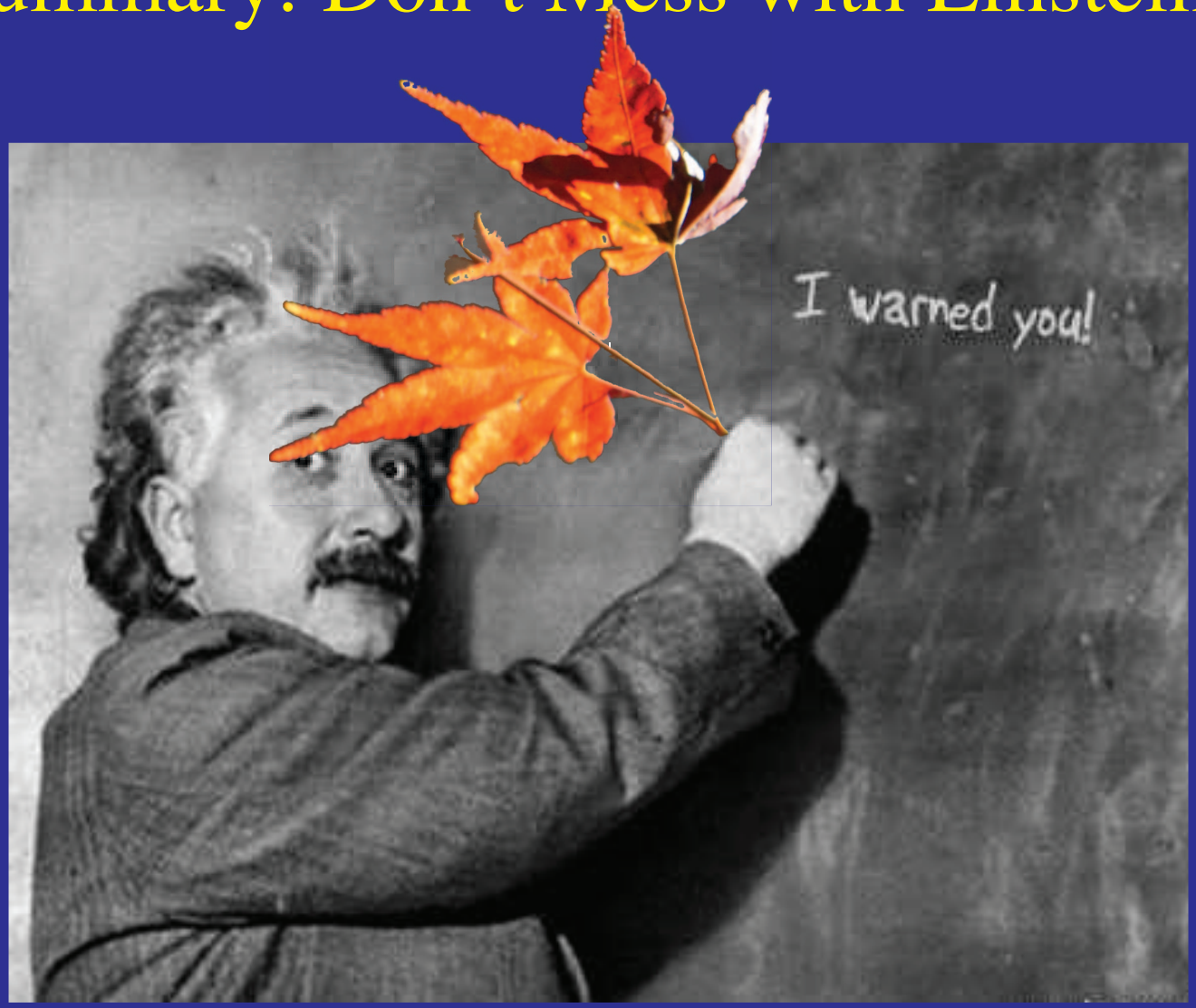
Summary: Don’t Mess with Einstein!



Happy 100th Birthday
GR