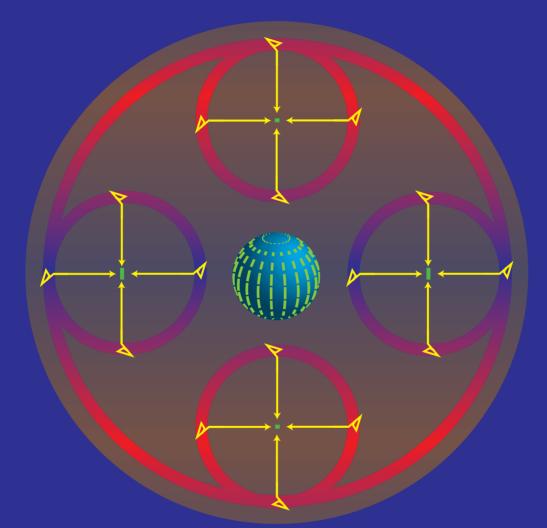
CMB Polarization Theory



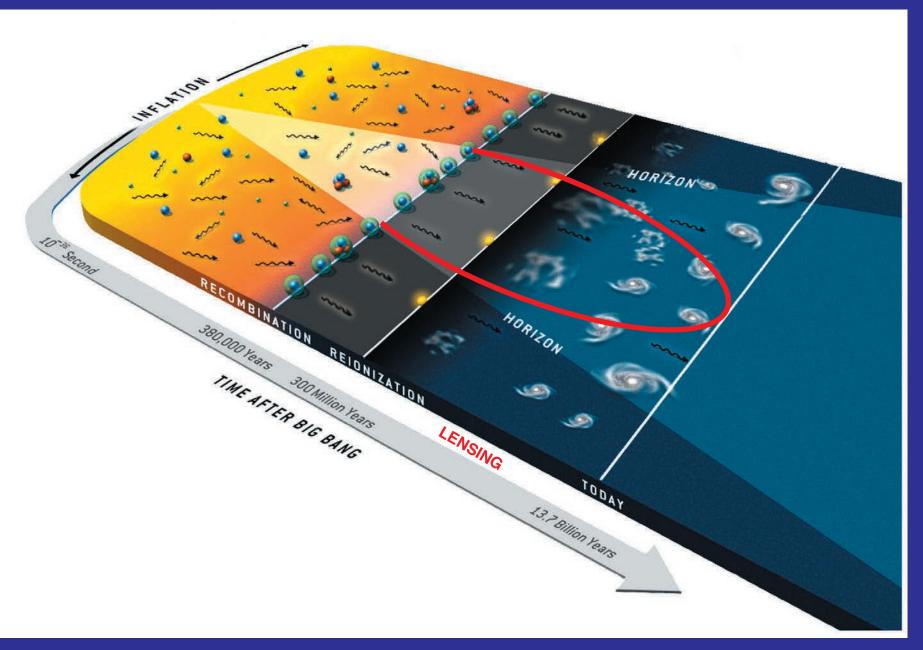
Wayne Hu Varenna, July 2017

Polarization Trinity



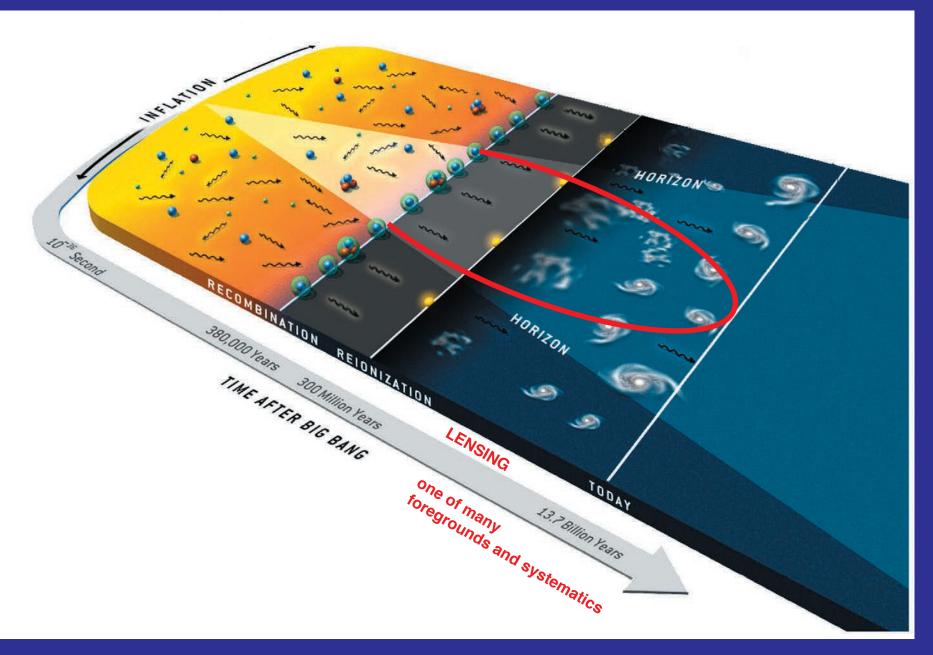
Isolating Three Cosmological Epochs

Polarization 4 Noble (Nobel?) Truths



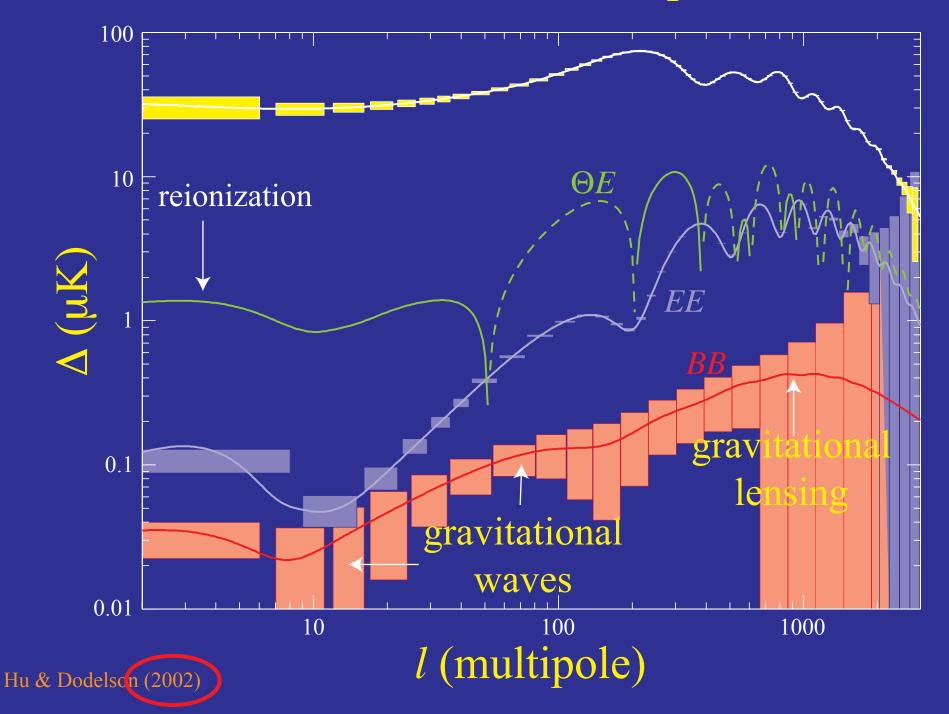
And one integrated probe

Polarization 4 Noble (Nobel?) Truths

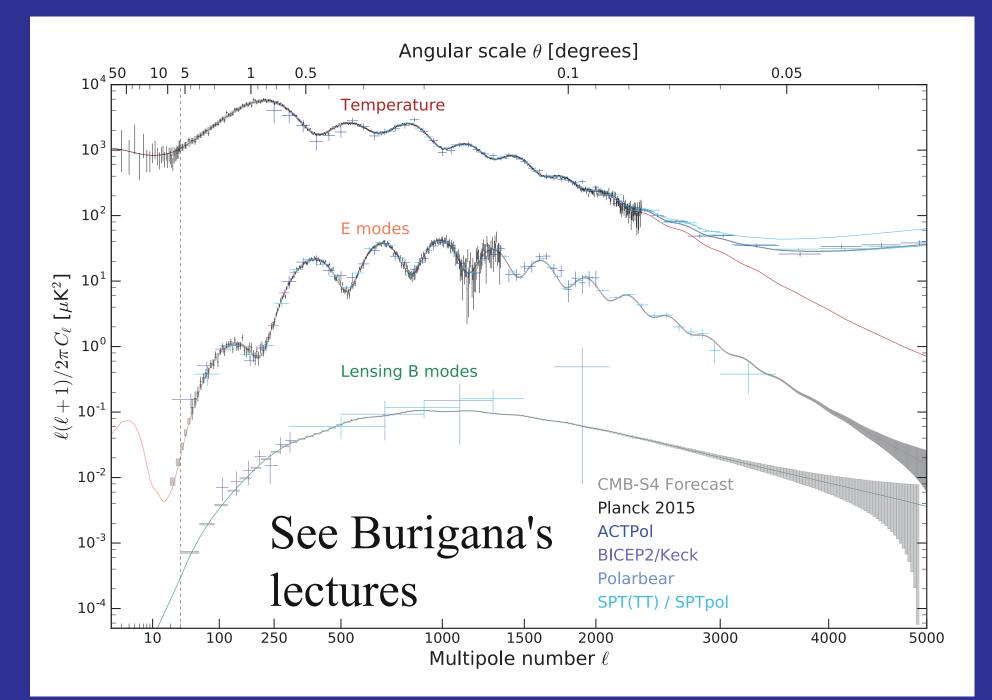


polarization is suffering... but cessation of suffering is nirvana

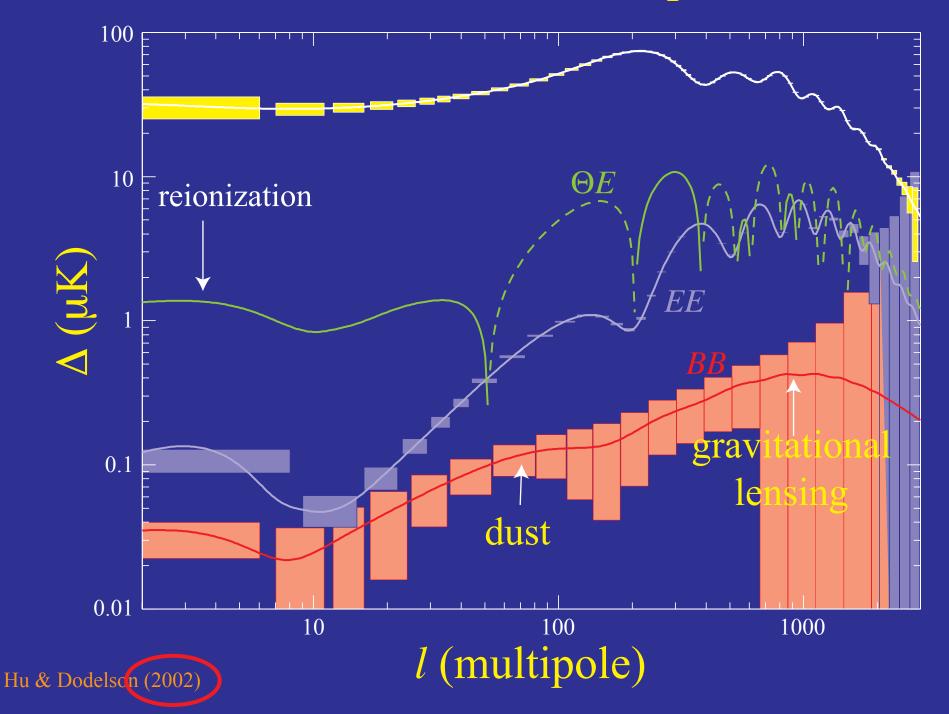
Polarized Landscape



CMB Power Spectra Measurements



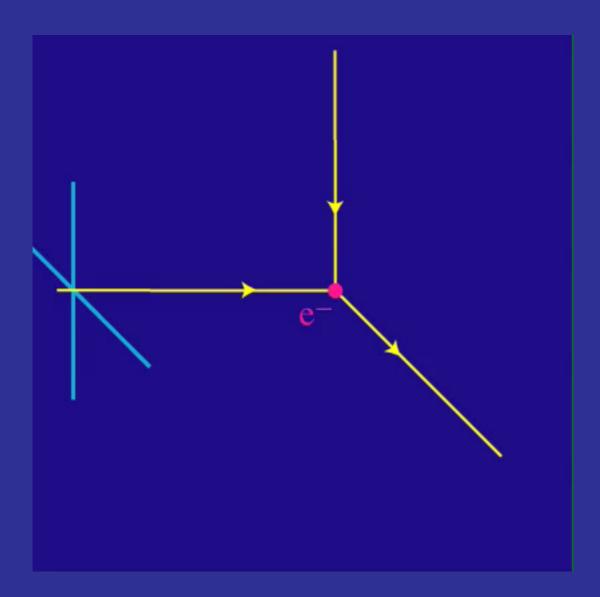
Polarized Landscape



Why is the CMB polarized?

Polarization from Thomson Scattering

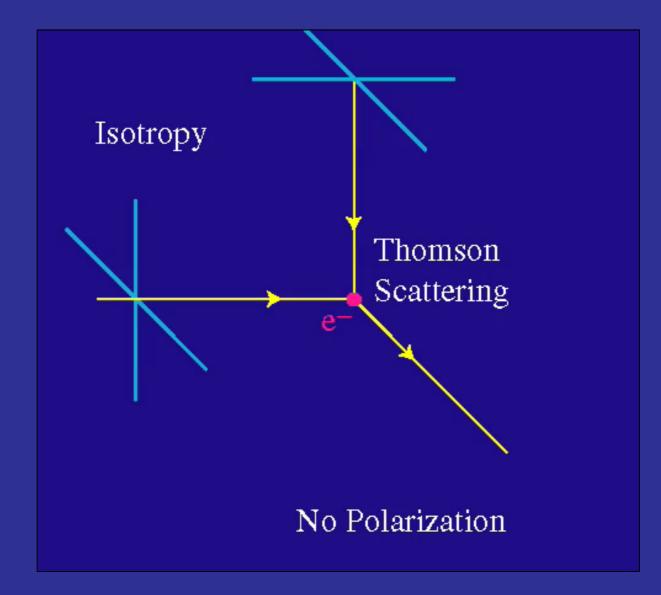
• Differential cross section depends on polarization and angle



 $\frac{d\sigma}{d\Omega} = \frac{3}{8\pi} |\hat{\varepsilon}' \cdot \hat{\varepsilon}|^2 \sigma_T$

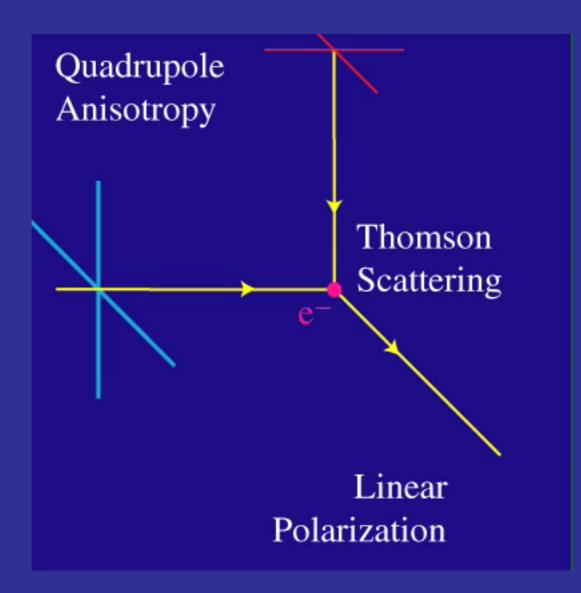
Polarization from Thomson Scattering

Isotropic radiation scatters into unpolarized radiation



Polarization from Thomson Scattering

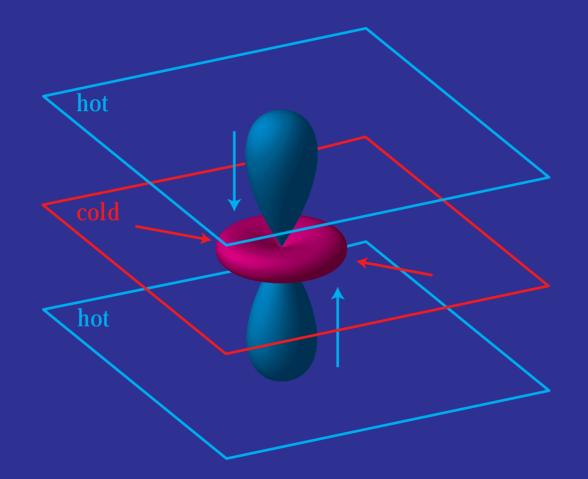
Quadrupole anisotropies scatter into linear polarization



aligned with cold lobe

Whence Quadrupoles?

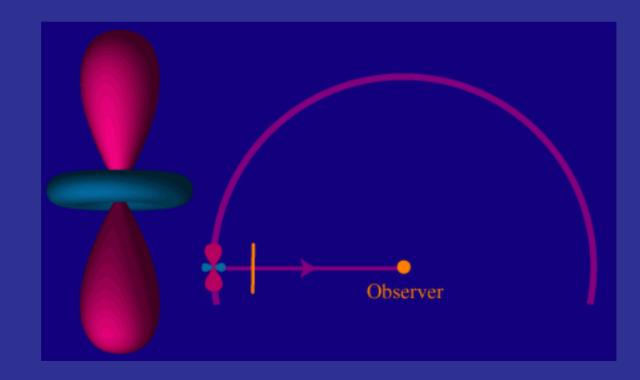
- Temperature inhomogeneities in a medium
- Photons arrive from different regions producing an anisotropy



(Scalar) Temperature Inhomogeneity

Whence Polarization Anisotropy?

- Observed photons scatter into the line of sight
- Polarization arises from the projection of the quadrupole on the transverse plane



E and B Modes

Polarization Multipoles

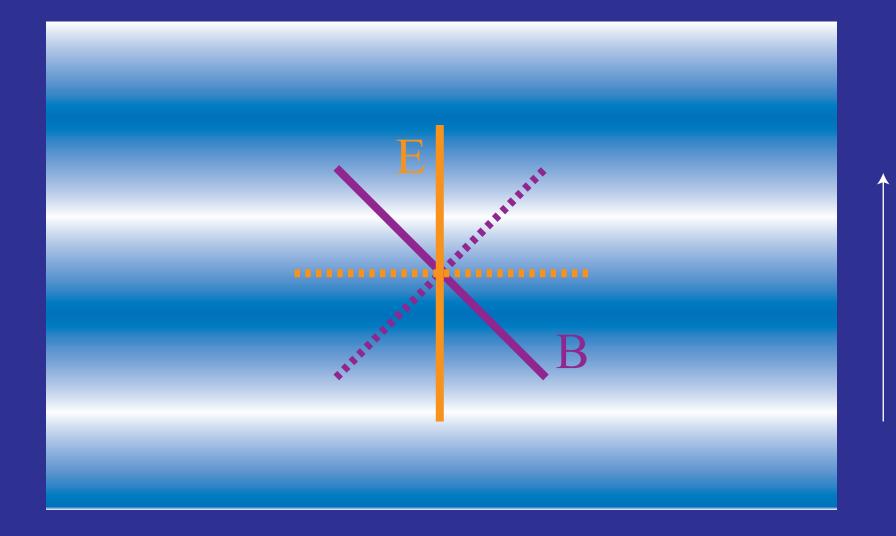
- Mathematically pattern is described by the tensor (spin-2) spherical harmonics [eigenfunctions of Laplacian on trace-free 2 tensor]
- Correspondence with scalar spherical harmonics established via Clebsch-Gordan coefficients (spin x orbital)
- Amplitude of the coefficients in the spherical harmonic expansion are the multipole moments; averaged square is the power



E-tensor harmonic l=2, m=0

E and B modes

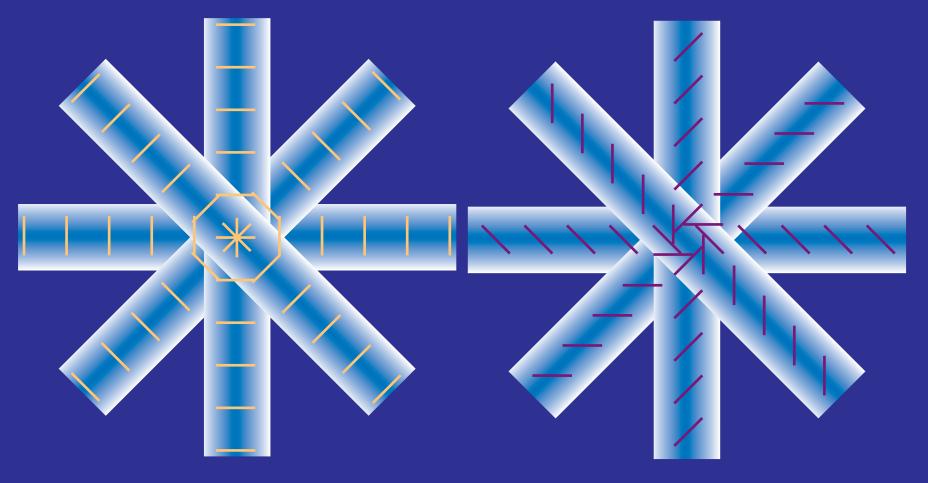
- E-modes are Stokes Q polarization in wavenumber basis
- B-modes are Stokes U polarization



wavevector

E and B modes

- Superimposing wavevectors
- B-modes have handedness or odd parity

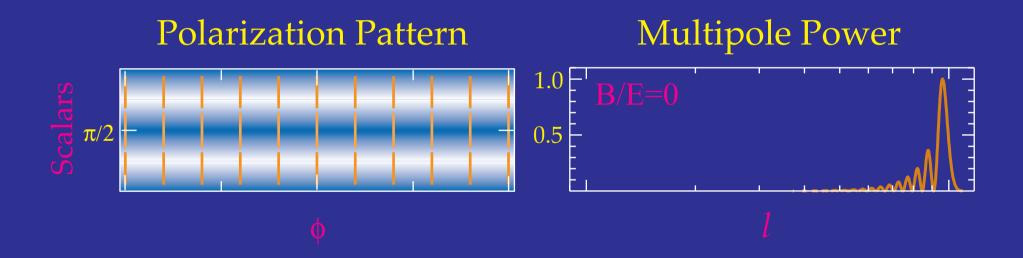


E-modes

B-modes

Modulation by Plane Wave

- Amplitude modulated by plane wave → higher multipole moments
- Direction detemined by perturbation type \rightarrow E-modes

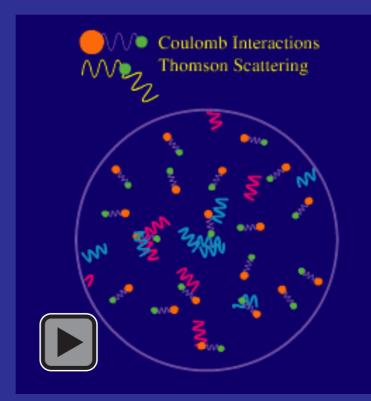


edge on orientation dominates: nearly single *l* per *k*

Polarization Peaks

A Catch-22

- Polarization is generated by scattering of anisotropic radiation
- Scattering isotropizes radiation
- Polarization only arises in optically thin conditions: reionization and end of recombination
- Polarization fraction is at best a small fraction of the 10^{-5} anisotropy: $\sim 10^{-6}$ or μK in amplitude



Pros: Polarization Isolates Scattering Epoch

Acoustic Polarization

- Perfect fluid: no anisotropic stresses due to scattering isotropization; baryons and photons move as single fluid
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1}$$
 where $\dot{\tau} = n_e \sigma_T a$

is the conformal opacity to Thomson scattering

• Dissipation is related to the diffusion length: random walk approximation

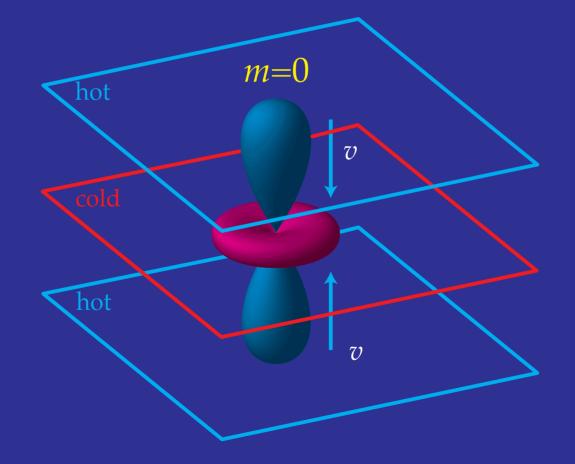
$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

the geometric mean between the horizon and mean free path

λ_D/η_{*} ~ few %, so expect the peaks >3 to be affected by dissipation

Viscosity & Heat Conduction

- Both fluid imperfections are related to the gradient of the velocity kv_{γ} by opacity $\dot{\tau}$: slippage of fluids $v_{\gamma} v_b$.
- Viscosity is an anisotropic stress or quadrupole moment formed by radiation streaming from hot to cold regions



Back of the Envelope

• Viscosity= quadrupole anisotropy that follows the fluid velocity

$$\pi_{\gamma} \approx \frac{k}{\dot{\tau}} v_{\gamma}$$

- Mean free path related to the damping scale via the random walk $k_D = (\dot{\tau}/\eta_*)^{1/2} \rightarrow \dot{\tau} = k_D^2 \eta_*$
- Damping scale at $\ell \sim 1000$ vs horizon scale at $\ell \sim 100$ so $k_D \eta_* \approx 10$
- Polarization amplitude rises to the damping scale to be $\sim 10\%$ of anisotropy

$$\pi_{\gamma} \approx \frac{k}{k_D} \frac{1}{10} v_{\gamma} \qquad \Delta_P \approx \frac{\ell}{\ell_D} \frac{1}{10} \Delta_T$$

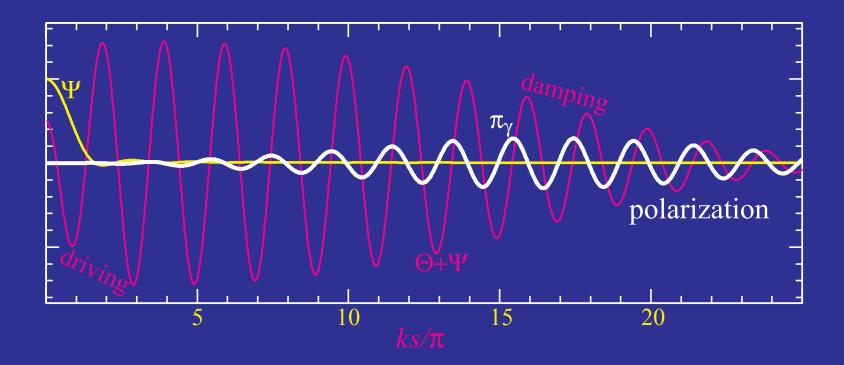
Polarization phase follows fluid velocity

Damping & Polarization

• Quadrupole moments:

damp acoustic oscillations from fluid viscosity generates polarization from scattering

• Rise in polarization power coincides with fall in temperature power $-l \sim 1000$



Acoustic Polarization

- Gradient of velocity is along direction of wavevector, so polarization is pure *E*-mode
- Velocity is 90° out of phase with temperature turning points of oscillator are zero points of velocity:

 $\Theta + \Psi \propto \cos(ks); \quad v_{\gamma} \propto \sin(ks)$

• Polarization peaks are at troughs of temperature power

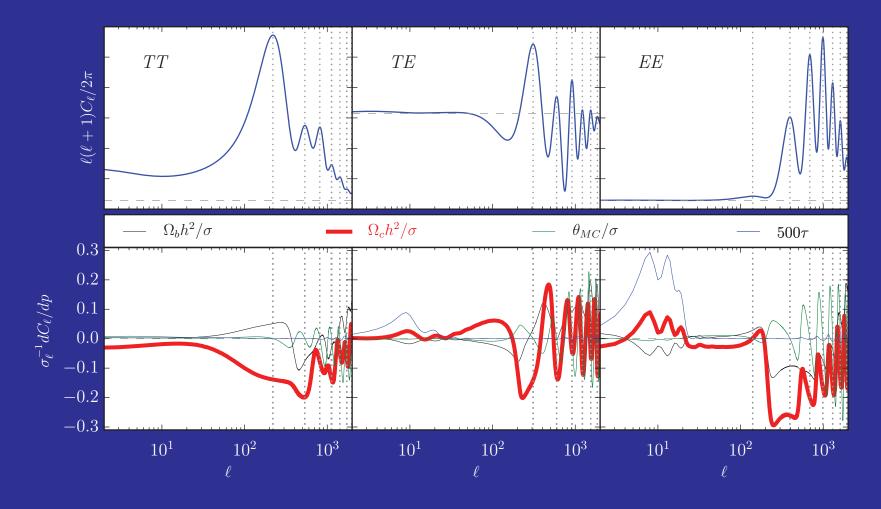
Cross Correlation

• Cross correlation of temperature and polarization

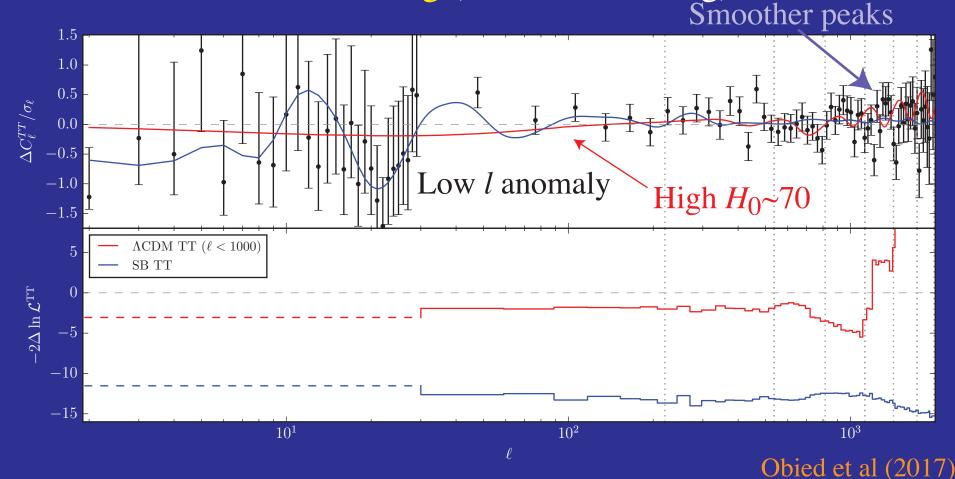
$(T)(v_{\gamma}) \propto \cos(ks) \sin(ks) \propto \sin(2ks)$

- Oscillation at twice the frequency
- Correlation: radial or tangential around hot spots
- Partial correlation: easier to measure if polarization data is noisy
- Good check for systematics and foregrounds
- Comparison of temperature and polarization is proof against features in initial conditions mimicking acoustic features
- Polarization isolates scattering leading to reduced projection effects

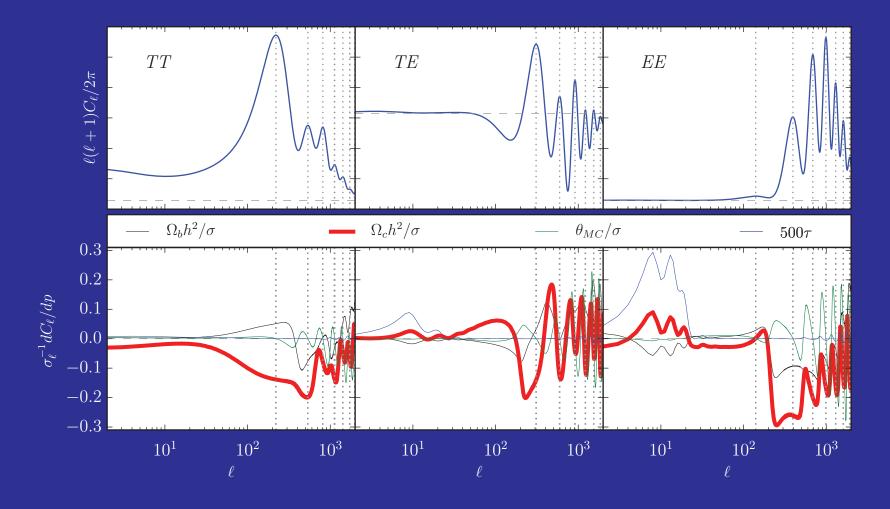
- Shift to lower *H*⁰ from changes in the shape of peaks indicating more CDM relative to radiation
- Increased angular scale of sound horizion compensated by larger distance to recombination through lower H_0



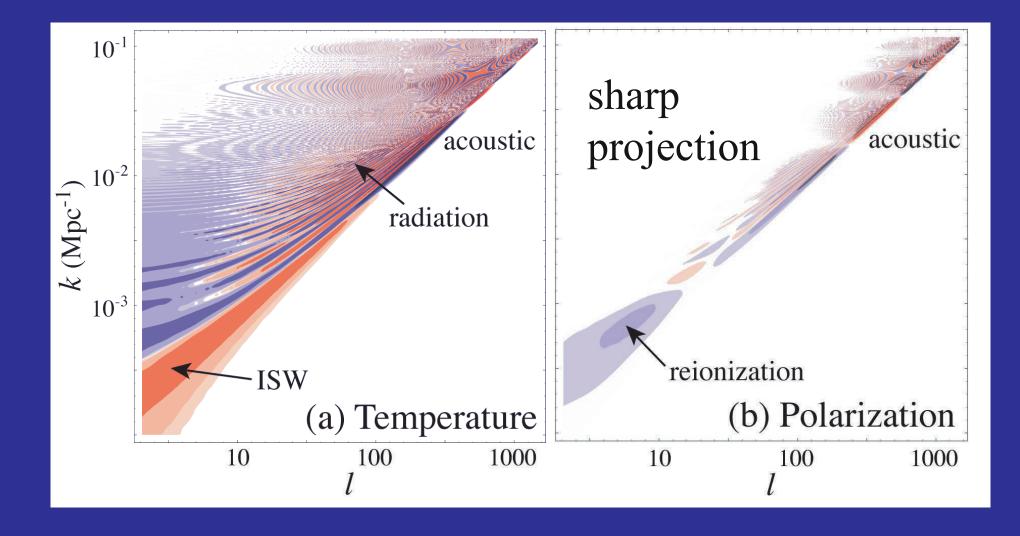
- Residuals from the best fit $H_0 \sim 67$ km/s/Mpc Λ CDM solution
- High H_0 at l < 1000 driven by low l anomaly Addison et al (2015)
- Low H_0 at l>1000 driven by smoother peaks Aghanim et al (2016) from less radiation driving (and more lensing)



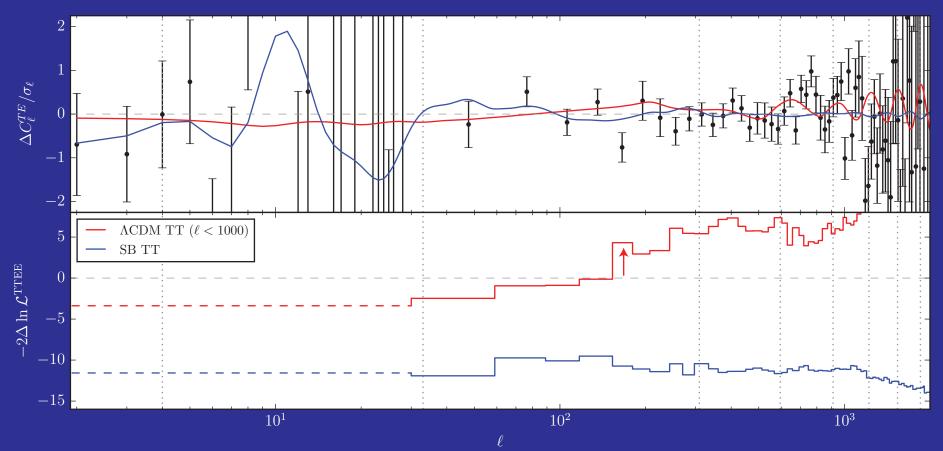
- Polarization response to parameter shifts very sharp around first temperature peak: no intervening ISW sources, geometry of projection
- Powerful cross check in a different observable and scale



Transfer of Initial Power

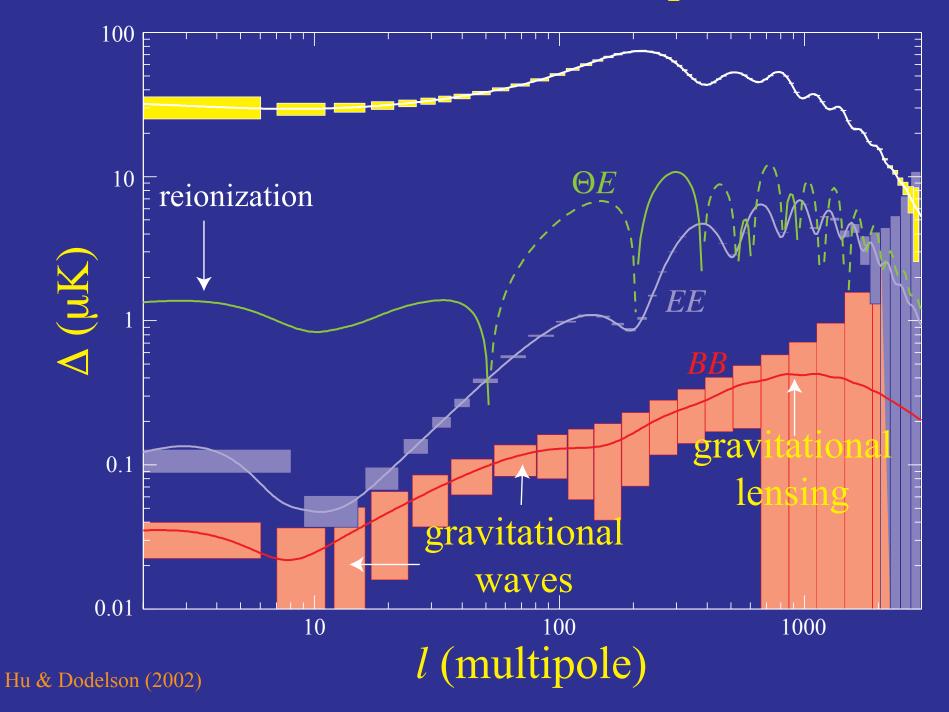


- TE residuals favor $H_0 \sim 67$ km/s/Mpc but at l < 1000
- As sensitive as all of TT
- Anomalous sensitivity from a 2σ outlier at *l*~165 near the first polarization trough



Obied et al (2017)

Polarized Landscape

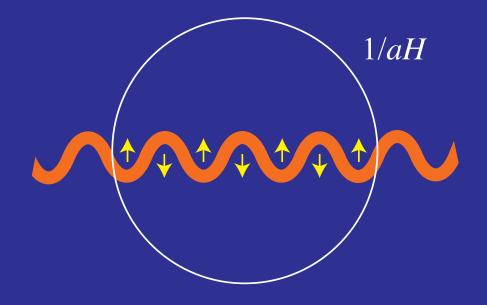


Gravitational Waves

Gravitational Waves in Cosmology

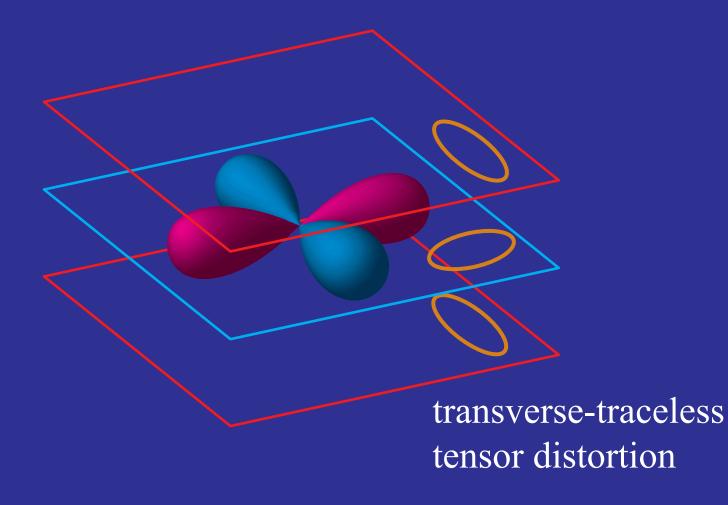
- During deceleration epoch gravity waves are frozen outside the horizon
- Oscillate inside the horizon and decay or redshift as radiation





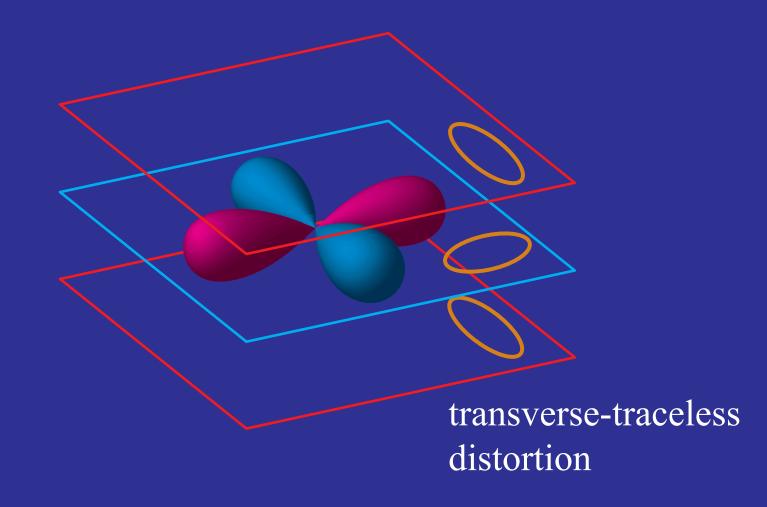
Quadrupoles from Gravitational Waves

- Changing transverse traceless distortion of space, aka gravitational waves, creates quadrupole CMB anisotropy
- Gravitational waves are frozen when larger than the horizon and oscillate and decay as radiation inside horizon



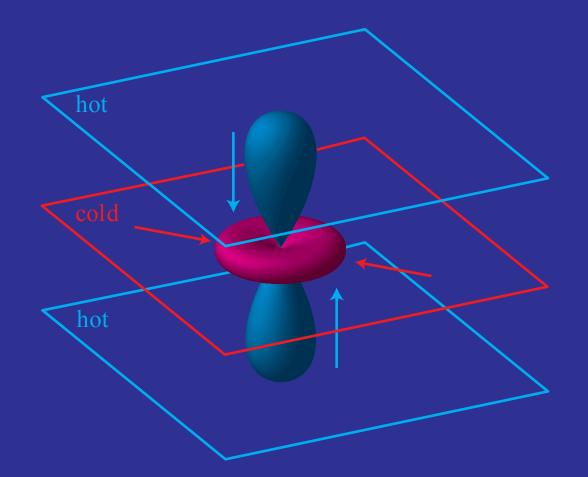
Quadrupoles from Gravitational Waves

- Transverse-traceless distortion provides temperature quadrupole
- Gravitational wave polarization picks out direction transverse to wavevector



How do Scalars Differ?

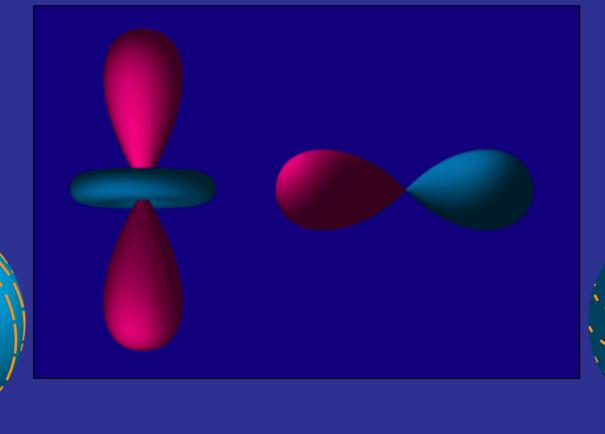
- Temperature inhomogeneities in a medium
- Photons arrive from different regions producing an anisotropy

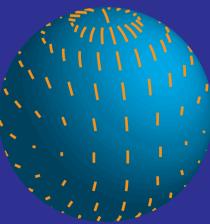


Azimuthally symmetric around wavevector

Gravitational Wave Pattern

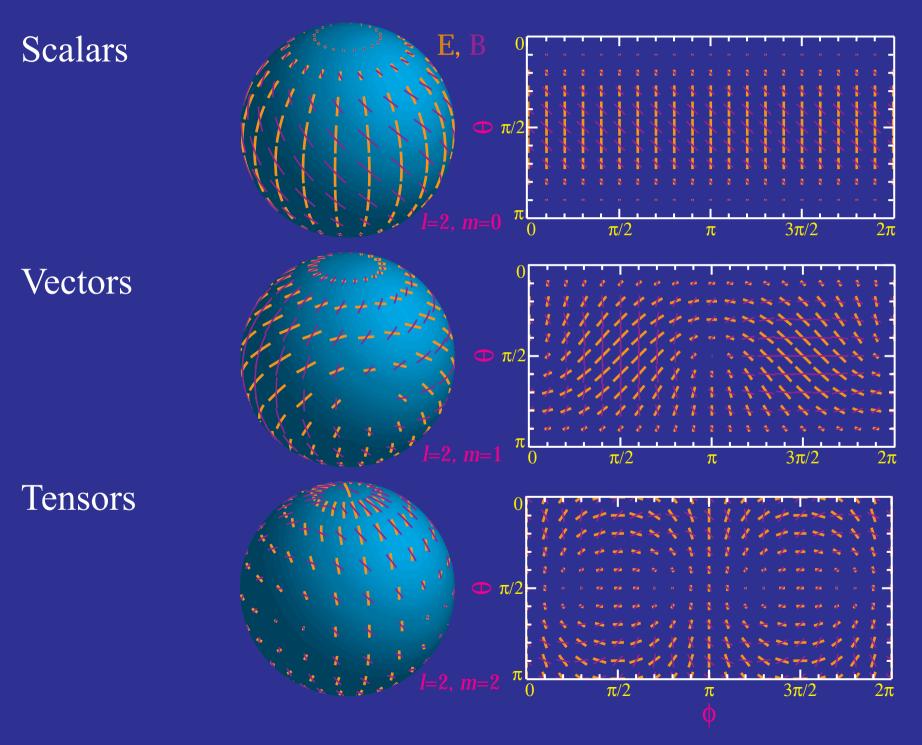
- Projection of the quadrupole anisotropy gives polarization pattern
- Transverse polarization of gravitational waves breaks azimuthal symmetry





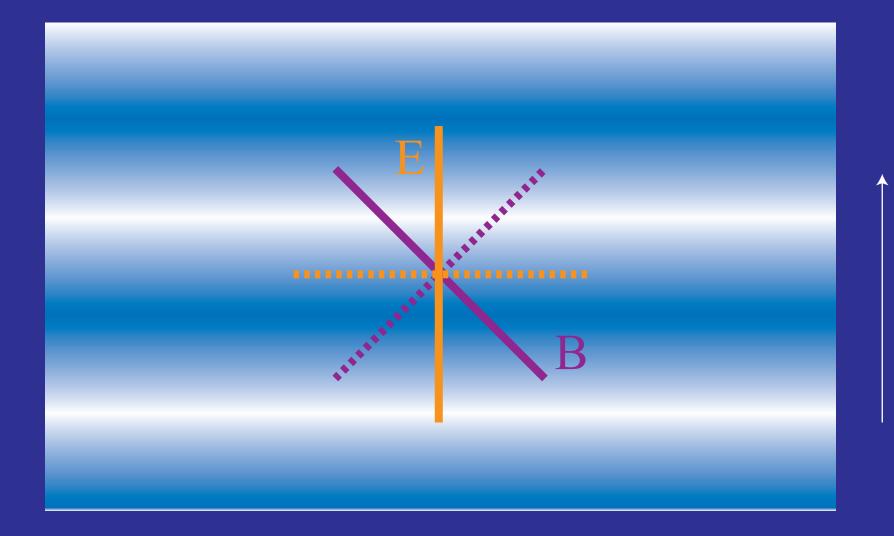
density perturbation gravitational wave

Polarization Patterns



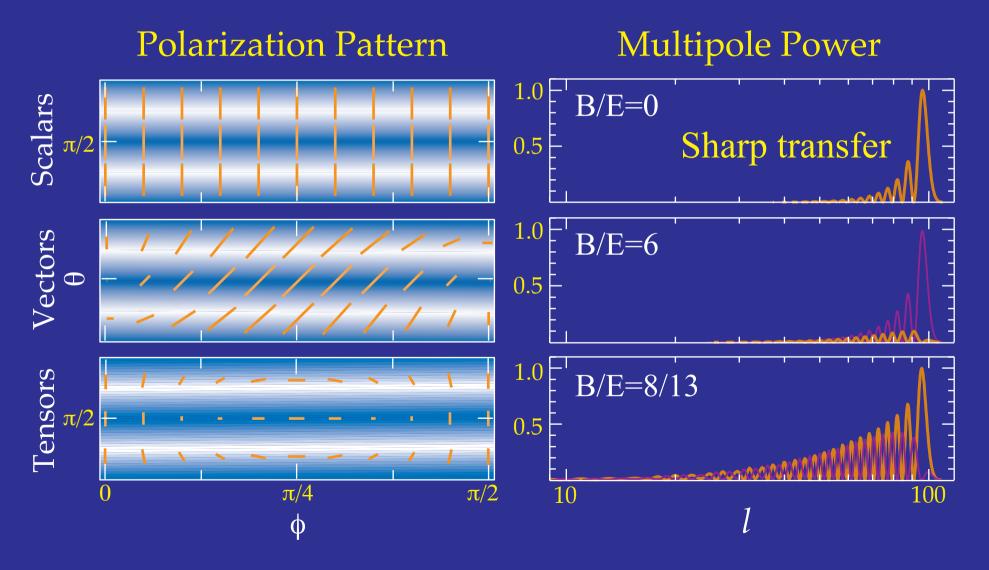
E and B modes

- E-modes are Stokes Q polarization in wavenumber basis
- B-modes are Stokes U polarization



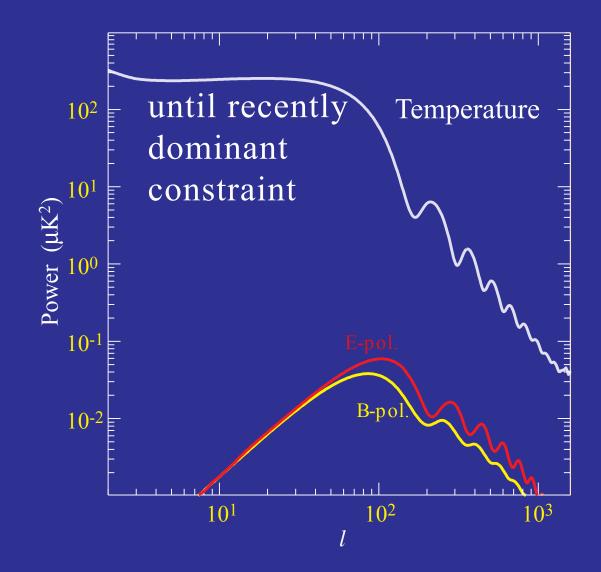
Patterns and Perturbation Types

- Amplitude modulated by plane wave \rightarrow Principle axis
- Direction detemined by perturbation type \rightarrow Polarization axis



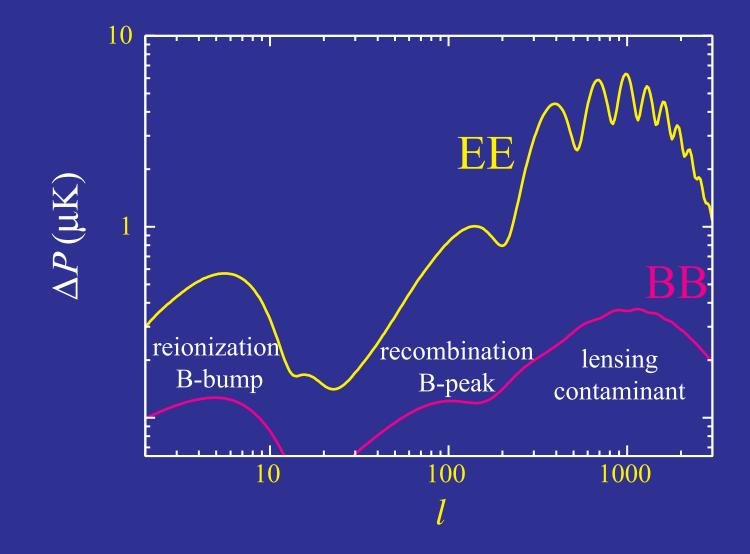
Recombination B-Modes

 Rescattering of quadrupoles at recombination yield a peak in B-modes



Polarized Landscape

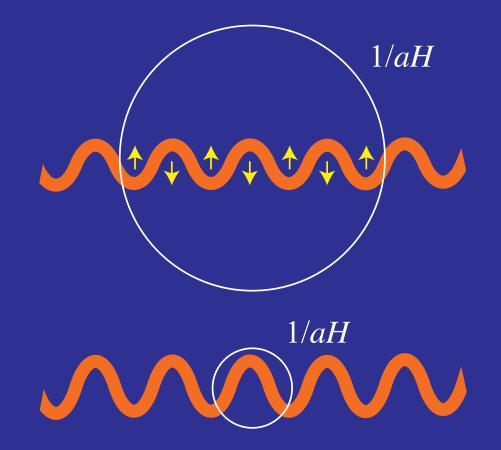
• Two scattering epochs: recombination and reionization leave two imprints on B-modes



Inflation

Gravitational Waves during Inflation

- During acceleration epoch gravity waves behave oppositely to deceleration epoch
- Oscillate inside the horizon and freeze when crossing horizon



Gravitational Waves

- Gravitational wave amplitude h_{+,×} satisfies same Klein-Gordon equation as scalars
- Just like inflaton \u03c6, quantum fluctuations freeze out at horizon crossing with power per ln k given by the Hubble scale H

$$\Delta_{\delta\phi}^2 = \frac{H^2}{(2\pi)^2}; \quad \Delta_{+,\times}^2 = \frac{2}{M_{\rm pl}^2} \frac{H^2}{(2\pi)^2}$$

• By the Friedmann equation

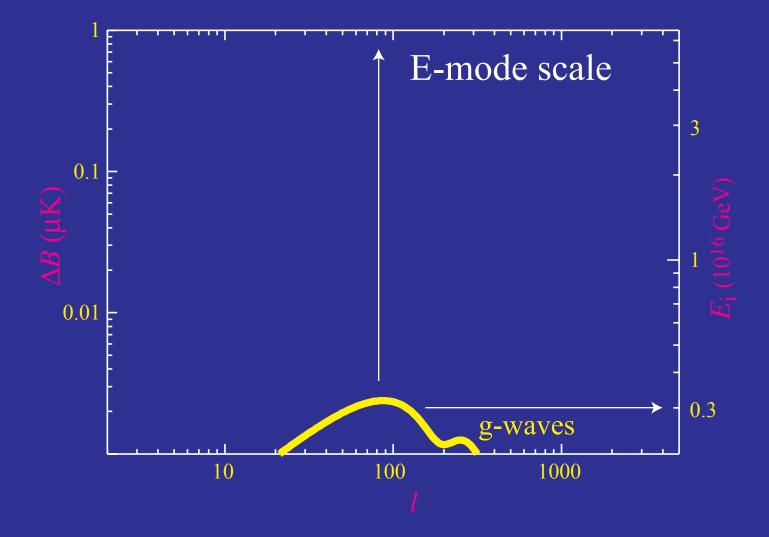
$$H^2 = \frac{\rho}{3M_{\rm pl}^2} \approx \frac{V(\phi)}{3M_{\rm pl}^2}$$

Measurement of *B*-modes determines energy scale $E_i = V^{1/4}$

$$B_{\text{peak}} \approx 0.024 \left(\frac{E_i}{10^{16} \text{GeV}}\right)^2 \mu \text{K}$$

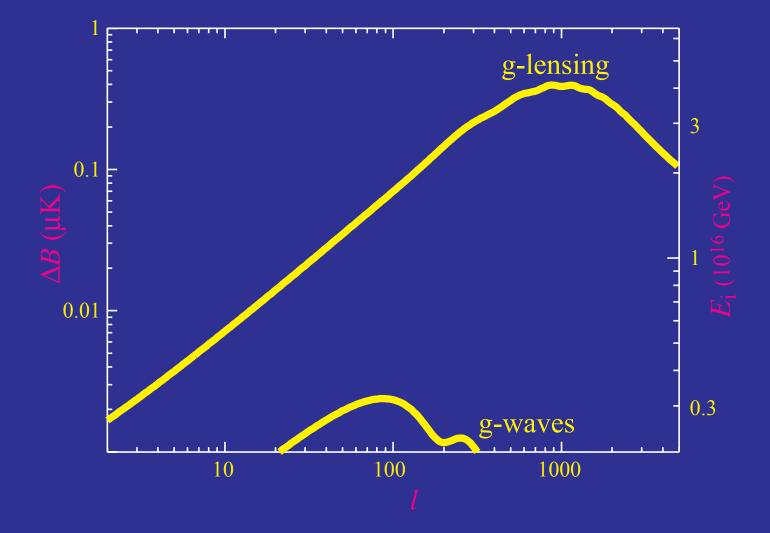
Scaling with Inflationary Energy Scale

• RMS B-mode signal scales with inflationary energy scale squared E_i^2



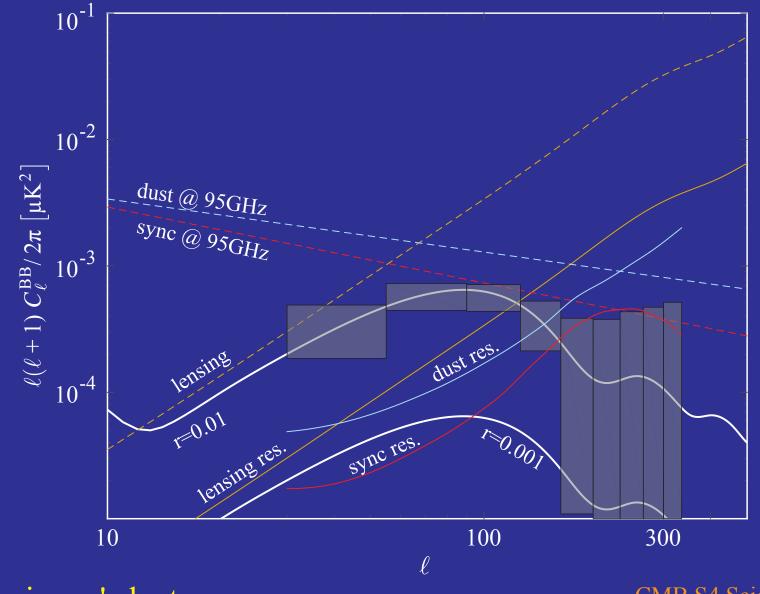
Contamination for Gravitational Waves

 Gravitational lensing contamination of B-modes from gravitational waves cleaned to *E*_i~0.3 x 10¹⁶ GeV



Polarized Foregrounds

• Dust and synchrotron

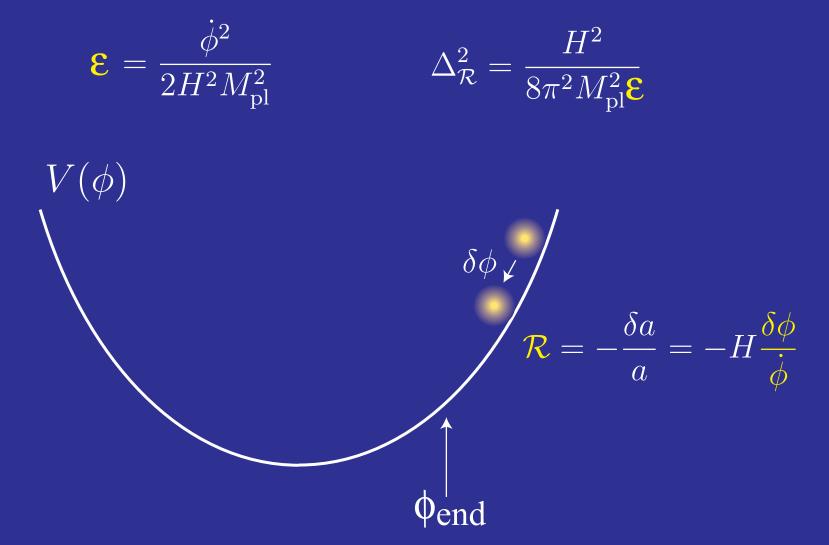


See Burigana's lectures

CMB S4 Science Book

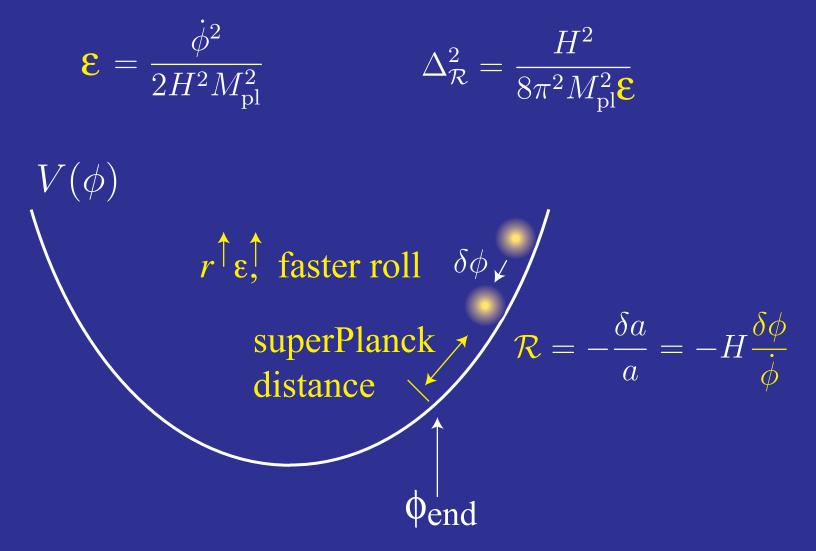
Tensor-Scalar Ratio

- Unlike gravitational waves, inflaton fluctuations determine when inflation ends in a given patch, changing the scale factor or curvature
- Curvature power is enhanced by the slowness of the roll



Tensor-Scalar Ratio

- Unlike gravitational waves, inflaton fluctuations determine when inflation ends in a given patch, changing the scale factor or curvature
- Curvature power is enhanced by the slowness of the roll



Tensor-Scalar Ratio

• Tensor-scalar ratio *r*

$$r \equiv 4 \frac{\Delta_+^2}{\Delta_R^2} = 16\epsilon$$

• A large r implies a large ϵ and a large roll

$$\epsilon = \frac{1}{2M_{\rm pl}^2} \left(\frac{d\phi}{d\ln a}\right)^2$$

• Observable scales span $d \ln a = d \ln k \sim 5$ so

$$\Delta \phi \approx 5 \frac{d\phi}{d\ln a} = 5(r/8)^{1/2} M_{\rm pl} \approx 0.6(r/0.1)^{1/2} M_{\rm pl}$$

- For r = 0.2 the field must roll by at least $M_{\rm pl}$
- Difficult to protect the flat potential across this large a range in field space

$n_S - r$ Plane

• Scalar power spectrum depends on both H and ε , so its tilt:

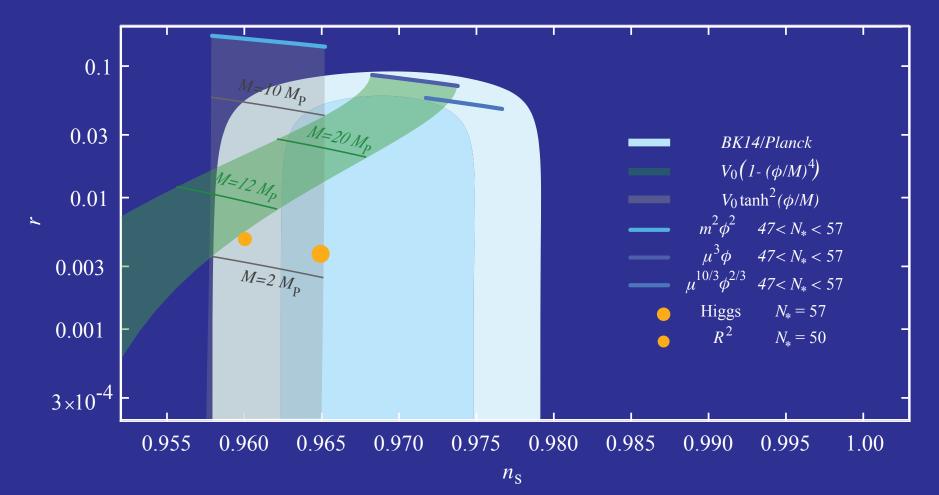
$$\frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k} \equiv n_S - 1$$
$$= 2\frac{d\ln H}{d\ln k} - \frac{d\ln\epsilon}{d\ln k} = -2\epsilon - \frac{d\ln\epsilon}{d\ln k}$$

- Measuring both $n_S 1$ and r constrain the inflationary model
- In slow roll, related to derivatives of potential

$$\epsilon \approx \frac{M_{\rm pl}^2}{2} \left(\frac{V'}{V}\right)^2$$
$$\frac{d\ln\epsilon}{d\ln k} = 4\epsilon - 2M_{\rm pl}^2 \frac{V''}{V}$$

r-*n*_s Trajectories and Constraints

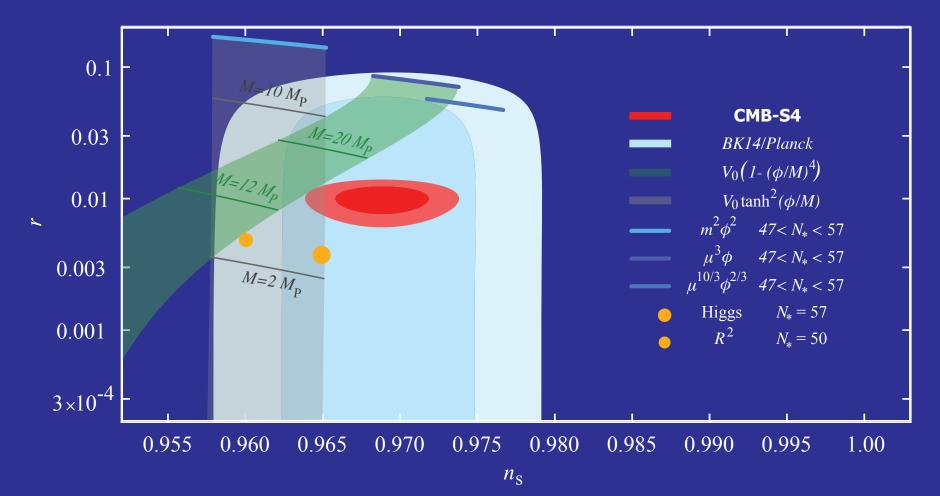
- Each inflationary model executes a trajectory in the plane
- Scale free models predict large tensors and large field excursions



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r-*n*_s Trajectories and Constraints

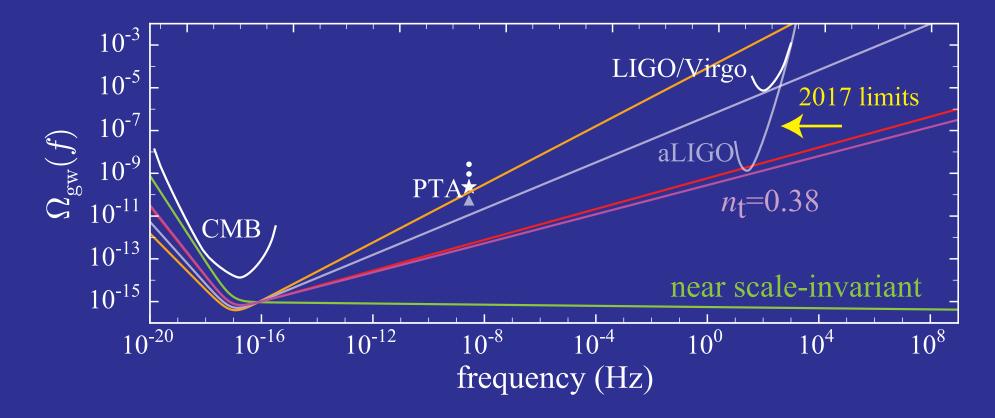
- Each inflationary model executes a trajectory in the plane
- Large improvements in *r* limits from B-modes, moderate improvement in *n*_s possible



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Inflationary GW Background

- Near scale invariant spectrum gives flat Ω contributions in radiation domination
- Blue tilted spectra directly constrained

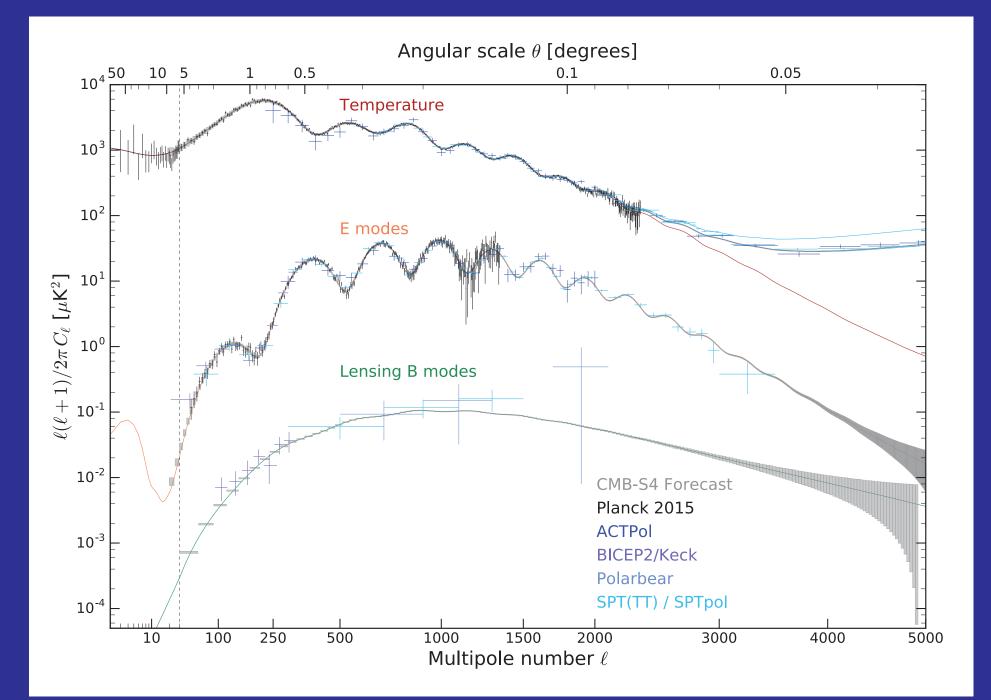


See Christensen's lectures

Lasky et al (2015)

Gravitational Lensing

CMB Power Spectra



Gravitational Lensing

• Lensing is a surface brightness conserving remapping of source to image planes by the gradient of the projected potential

$$\phi(\hat{\mathbf{n}}) = 2 \int \frac{dz}{H(z)} \frac{D_A(D_s - D)}{D_A(D) D_A(D_s)} \Phi(D_A \hat{\mathbf{n}}, D),$$

such that the fields are remapped as

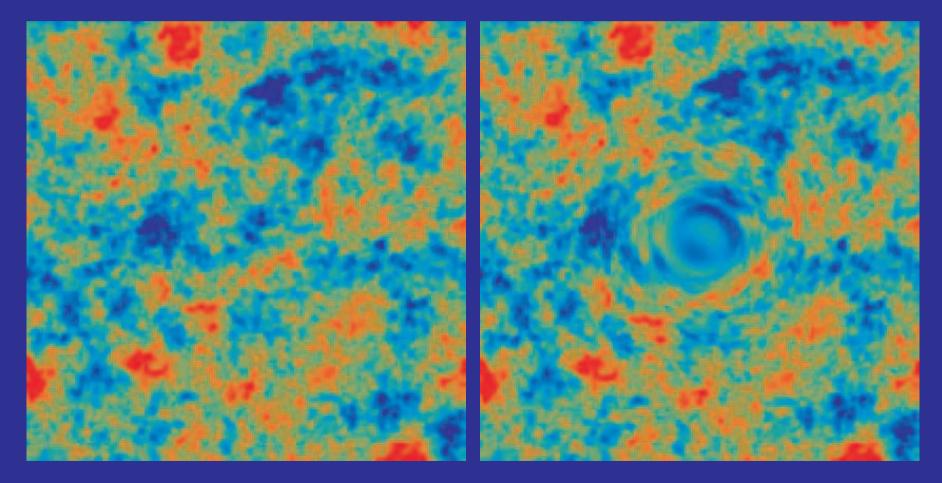
 $x(\hat{\mathbf{n}}) \to x(\hat{\mathbf{n}} + \nabla \phi),$

where $x \in \{T, Q, U\}$ temperature and polarization.

- Taylor expansion leads to product of fields and Fourier mode-coupling
- Appears in the power spectrum as a convolution kernel for T and E and an $E \rightarrow B$.

Lensing of a Gaussian Random Field

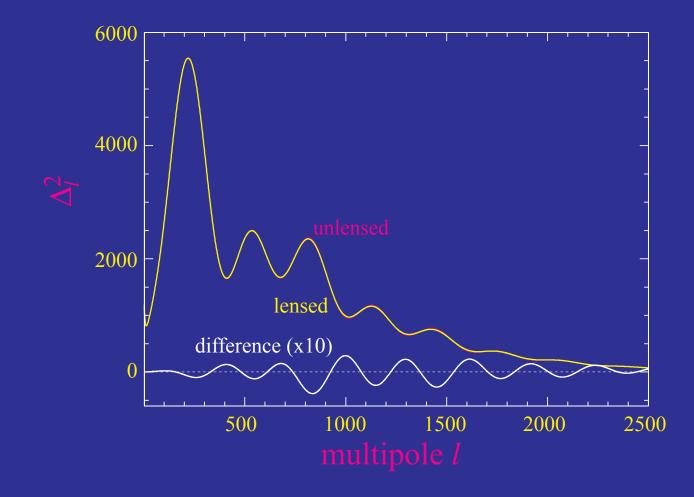
- CMB temperature and polarization anisotropies are Gaussian random fields unlike galaxy weak lensing
- Average over many noisy images like galaxy weak lensing



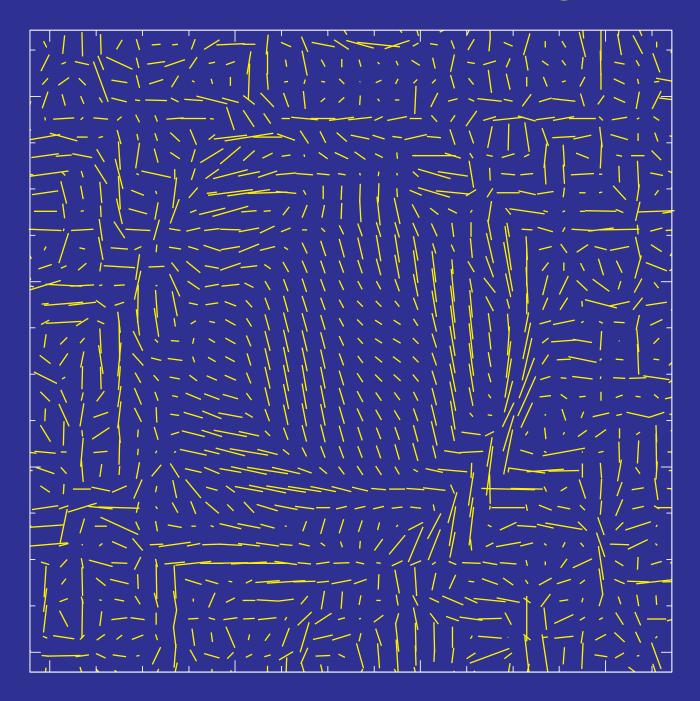
highly exaggerated: see Burigana's talk for realism

Temperature Power Spectrum

- Lensing acts to smooth temperature (and E polarization)peaks)
- Subtle effect reaches 10% deep in the damping tail
- Statistically detected at high significance

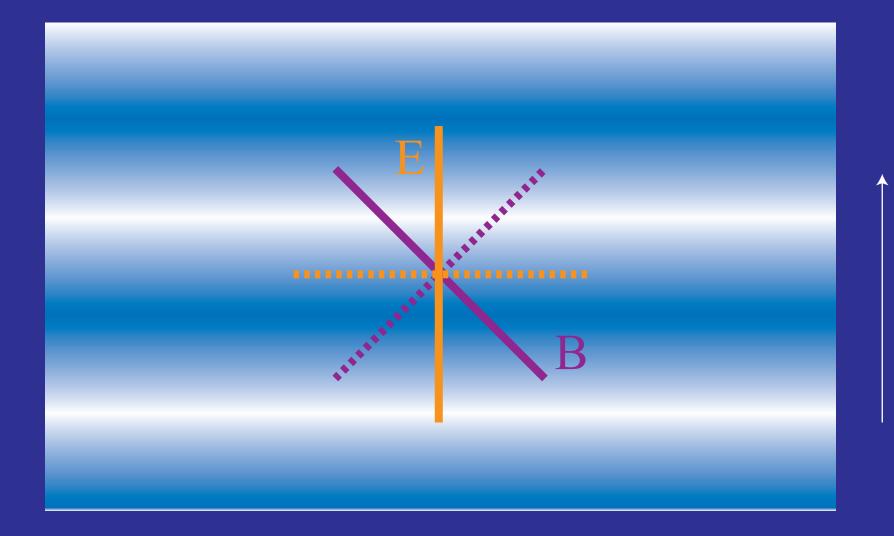


Polarization Lensing



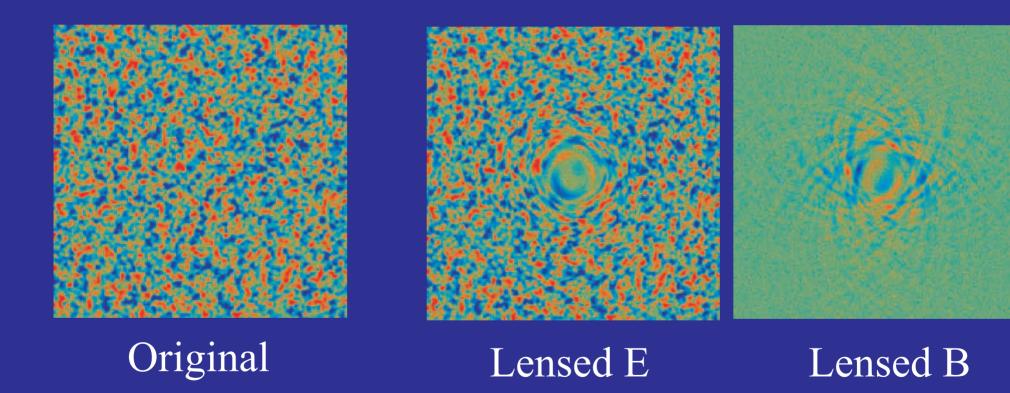
E and B modes

- E-modes are Stokes Q polarization in wavenumber basis
- B-modes are Stokes U polarization



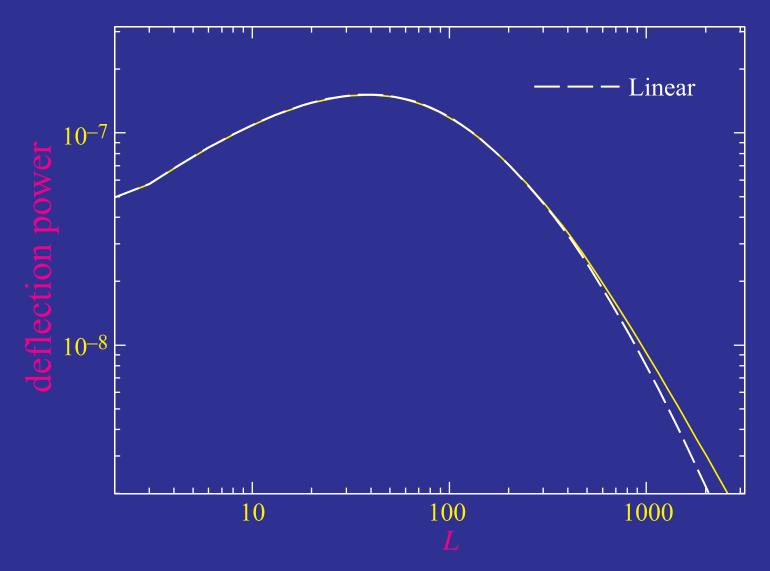
Polarization Lensing

• Since E and B denote the relationship between the polarization amplitude and direction, warping due to lensing creates B-modes



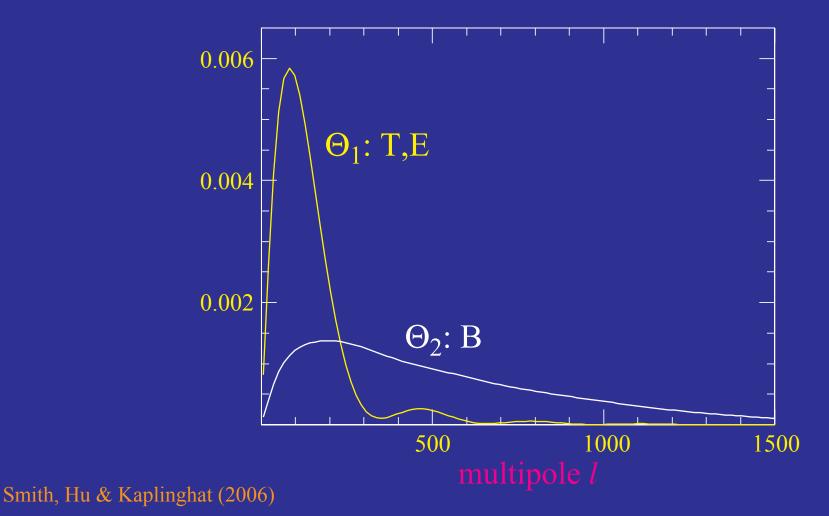
Deflection Power Spectrum

- Fundamental observable is deflection power spectrum (or convergence / l²)
- Nearly entirely in linear regime



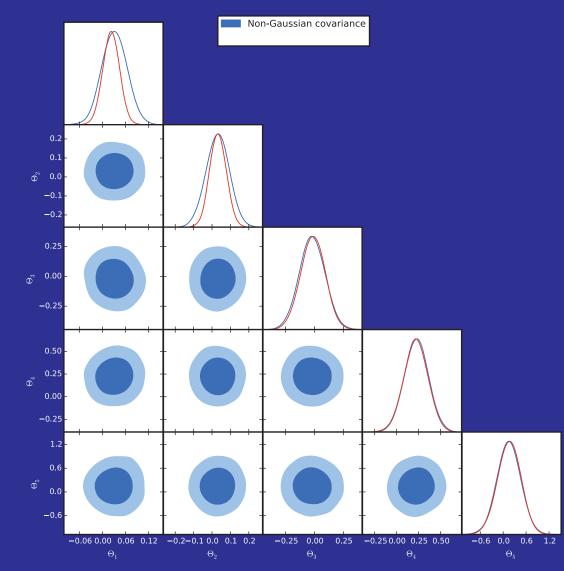
Lensed Power Spectrum Observables

- Principal components show two observables in lensed power spectra
- Temperature and E-polarization: deflection power at *l*~100
 B-polarization: deflection power at *l*~500
- Normalized so that observables error = fractional lens power error



Principal in Practice

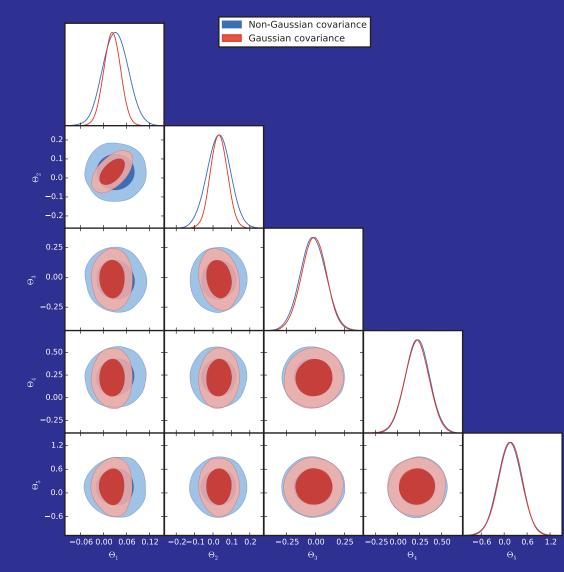
• Extracting principal components from LensPix simulated CMB temperature and polarization maps



Motloch & Hu (2017)

Principal in Practice

• Treating CMB maps as Gaussian leads to overly tight constraints and potentially misleading tension



Motloch & Hu (2017)

Mass Reconstruction

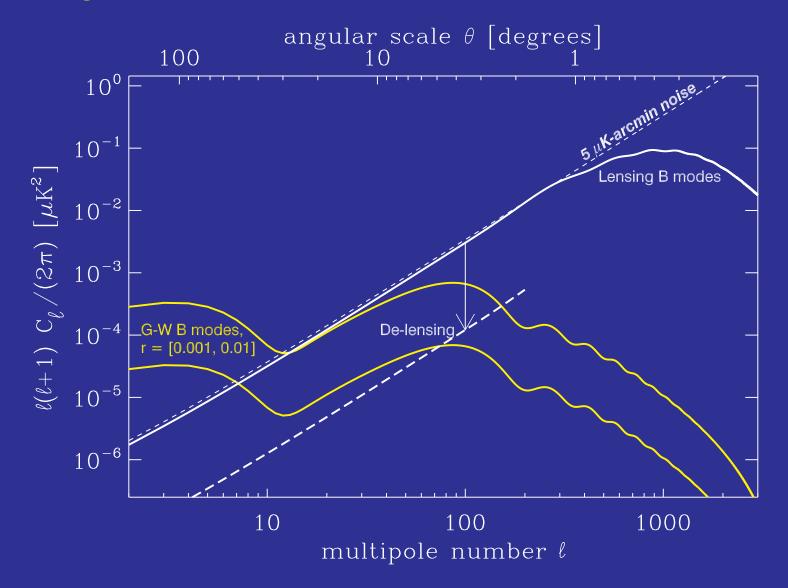
Why Care

- Gravitational lensing sensitive to amount and hence growth of structure
- Examples: massive neutrinos $d \ln C_{\ell}^{BB}/dm_{\nu} \approx -1/3$ eV, dark energy $d \ln C_{\ell}^{BB}/dw \approx -1/8$
- Mass reconstruction measures the large scale structure on large scales and the mass profile of objects on small scales
- Large scale delensing of the gravitational wave
- Lensing by high-z dark matter halos: mass calibration of clusters and cosmography (same lens, different sources)

See Simon White's Lectures

Lensing Contamination

• Lensing acts as cosmic noise that isn't Gaussian - delensing



CMB S4 Science Book

Quadratic Estimator

• Taylor expand mapping

$$T(\hat{\mathbf{n}}) = \tilde{T}(\hat{\mathbf{n}} + \nabla\phi)$$

= $\tilde{T}(\hat{\mathbf{n}}) + \nabla_i \phi(\hat{\mathbf{n}}) \nabla^i \tilde{T}(\hat{\mathbf{n}}) + \dots$

● Fourier decomposition → mode coupling of harmonics

$$T(\mathbf{l}) = \int d\hat{\mathbf{n}} T(\hat{\mathbf{n}}) e^{-il\cdot\hat{\mathbf{n}}}$$
$$= \tilde{T}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} (\mathbf{l} - \mathbf{l}_1) \cdot \mathbf{l}_1 \tilde{T}(\mathbf{l}_1) \phi(\mathbf{l} - \mathbf{l}_1)$$

Consider fixed lens and Gaussian random CMB realizations: each pair is an estimator of the lens at L = l₁ + l₂:

$$\langle T(\mathbf{l})T'(\mathbf{l}')\rangle_{\mathrm{CMB}} \approx \left[\tilde{C}_{l_1}^{TT}(\mathbf{L}\cdot\mathbf{l}_1) + \tilde{C}_{l_2}^{TT}(\mathbf{L}\cdot\mathbf{l}_2)\right]\phi(\mathbf{L}) \quad (\mathbf{l}\neq -\mathbf{l}')$$

Reconstruction from the CMB

 Generalize to polarization: each quadratic pair of fields estimates the lensing potential

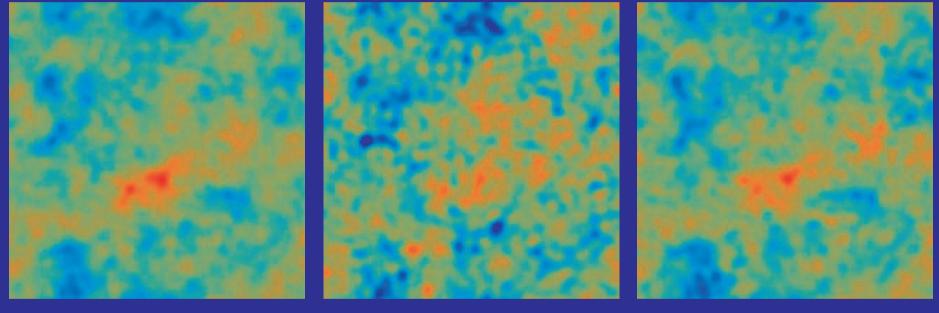
 $\langle x(\mathbf{l})x'(\mathbf{l}')\rangle_{\text{CMB}} = f_{\alpha}(\mathbf{l},\mathbf{l}')\phi(\mathbf{l}+\mathbf{l}'),$

where $x \in$ temperature, polarization fields and f_{α} is a fixed weight that reflects geometry

- Each pair forms a noisy estimate of the potential or projected mass
 just like a pair of galaxy shears
- Minimum variance weight all pairs to form an estimator of the lensing mass
- Generalize to inhomogeneous noise, cut sky and maximum likelihood by iterating the quadratic estimator

High Signal-to-Noise B-modes

- Cosmic variance of CMB fields sets ultimate limit for *T*,*E*
- *B*-polarization allows mapping to finer scales and in principle is not limited by cosmic variance of *E*



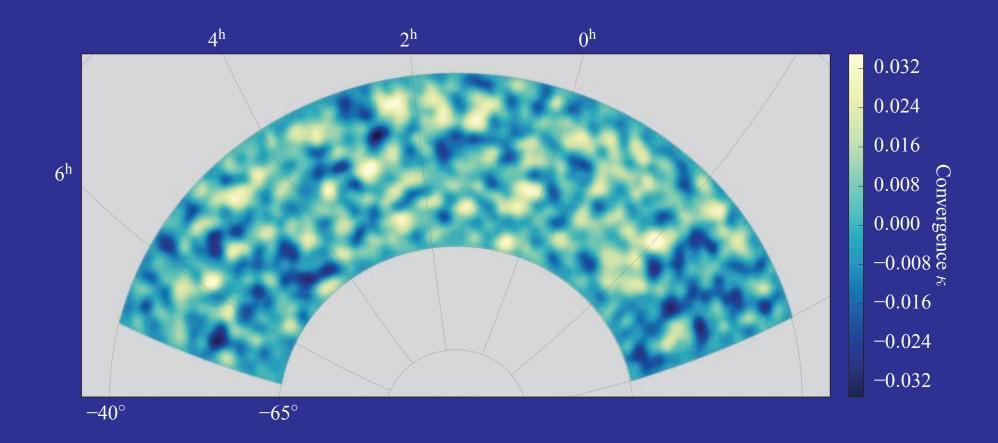
mass

temp. reconstruction EB pol. reconstruction 100 sq. deg; 4' beam; 1µK-arcmin

Hu & Okamoto (2001)

Lensing Reconstruction

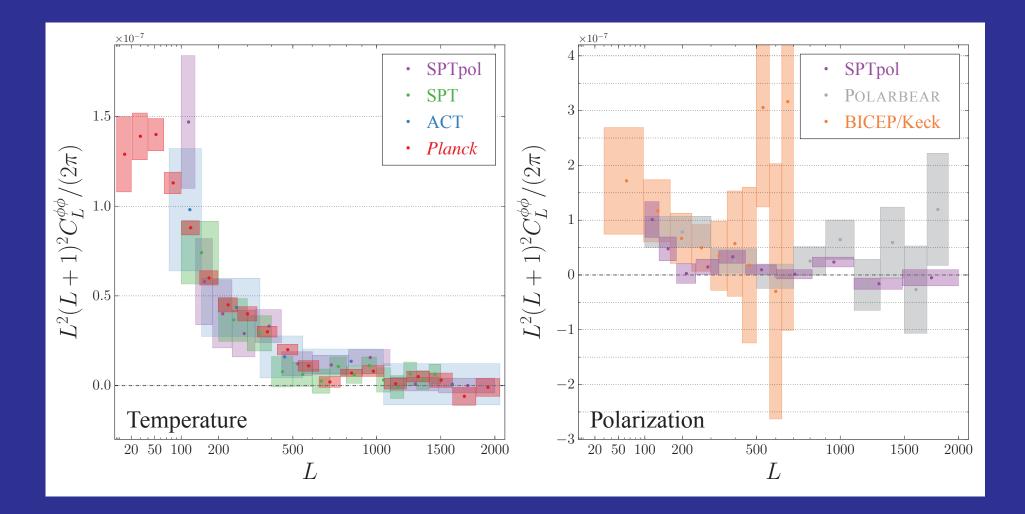
• SPT+Planck example



Omori et al (2017)

Lens Power Spectra

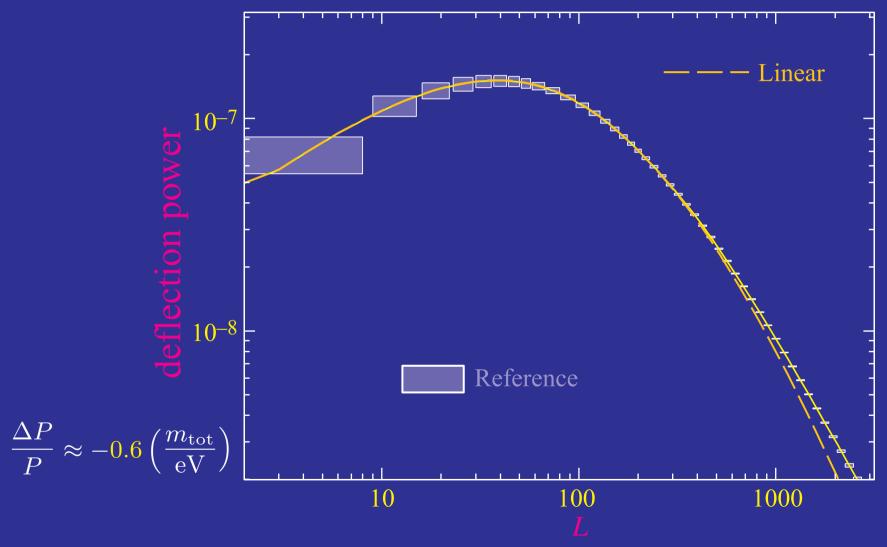
• Temperature and polarization reconstruction



CMB S4 Science Book

Matter Power Spectrum

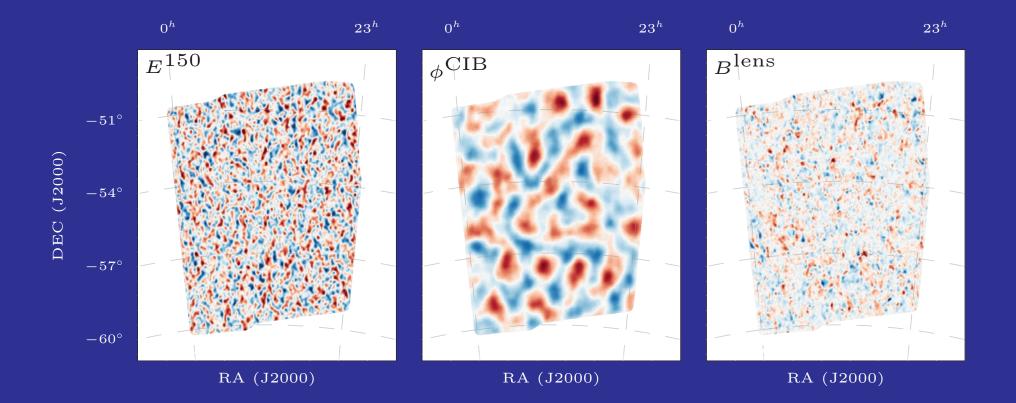
 Measuring projected matter power spectrum to cosmic variance limit across whole linear regime 0.002< k < 0.2 h/Mpc



Hu & Okamoto (2001)

Delensing with External Template

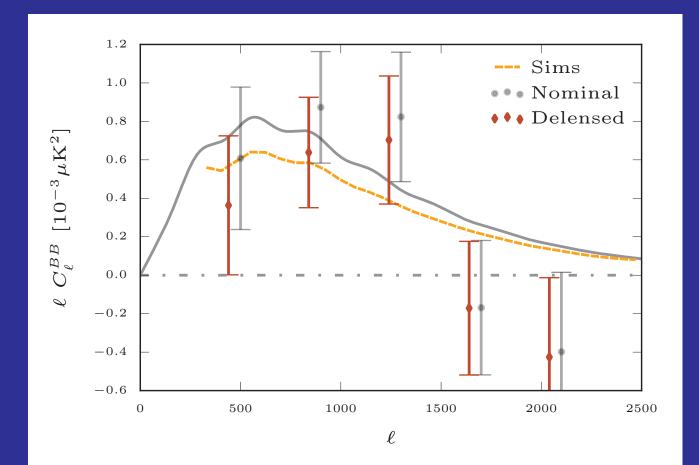
- Herschel CIB data as tracer of lensing
- Predict and subtract B-mode contamination SPT example



Manzotti et al (2017)

Delensing with External Template

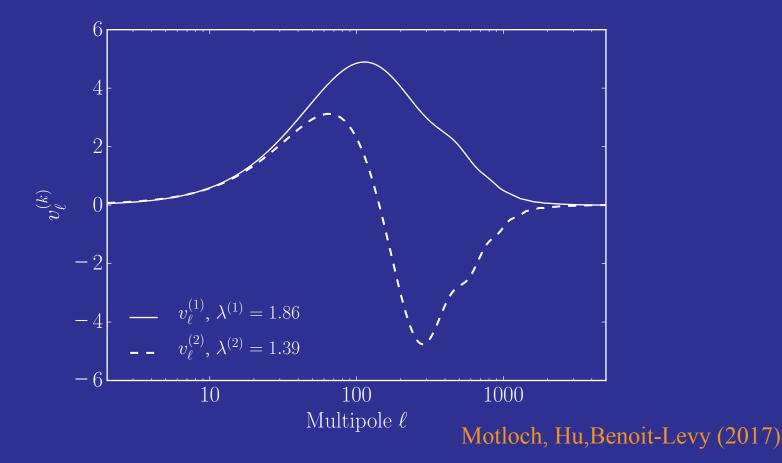
- Herschel CIB data as tracer of lensing
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Manzotti et al (2017)

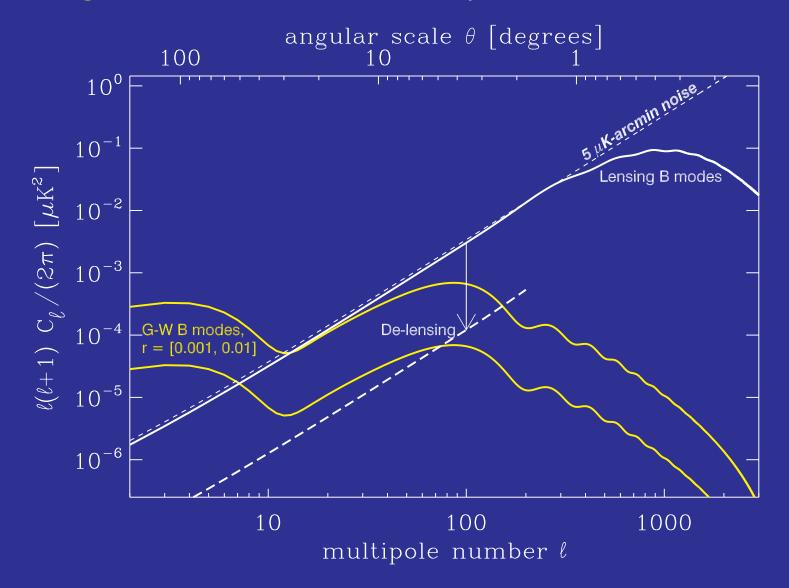
Consistency in Lens Observables

- Consistency between lensed CMB power spectra and lensing reconstruction critical for delensing
- Compare directly lens power spectrum information in model independent and nearly sample variance free way (consistency modes: a more precise Alens test)



Delensing Goals

• Lensing noise isn't Gaussian, may be removed to uncover *r*

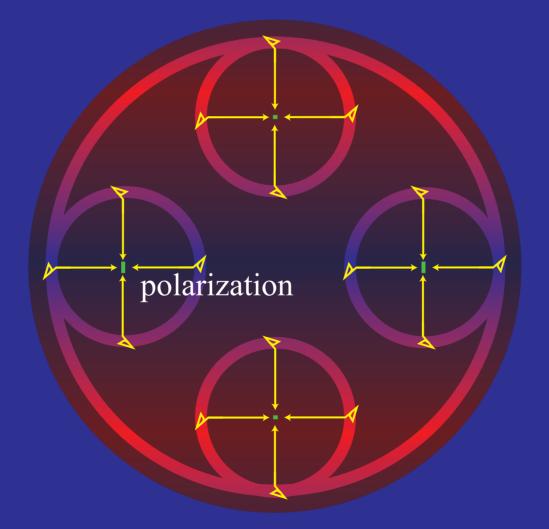


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Reionization

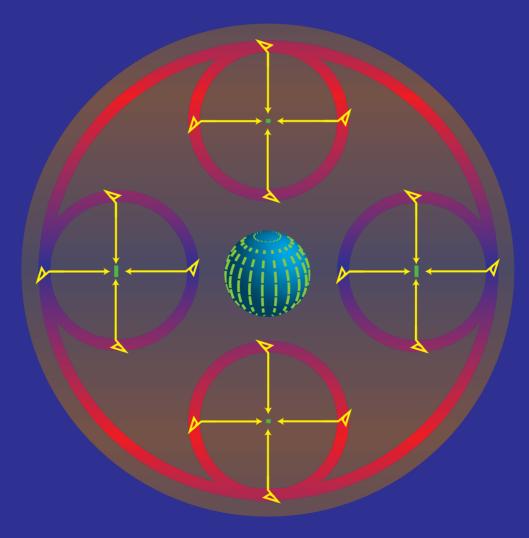
Polarization Anisotropy

• Electron sees the temperature anisotropy on its recombination surface and scatters it into a polarization



Temperature Correlation

• Pattern correlated with the temperature anisotropy that generates it; here an *m*=0 quadrupole

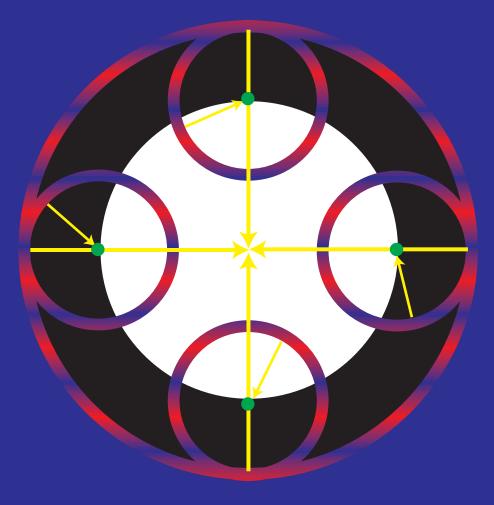


Why Care?

- Early ionization would imply more exotic astrophysics (Pop-III stars) or physics (dark matter annihilation)
- Reionization screens temperature anisotropy on small scales making the true amplitude of initial fluctuations larger by e^τ
- Measuring the growth of fluctuations is one of the best ways of determining the neutrino masses and the dark energy limits lensing information if not substantially better than 1%
- Offers an opportunity to study the origin of the low multipole statistical anomalies
- Presents a second, and statistically cleaner, window on gravitational waves from the early universe

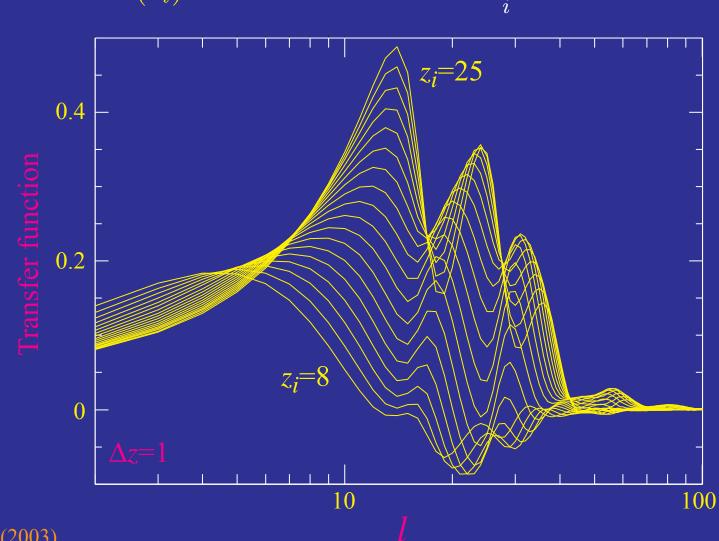
Anisotropy Suppression

 A fraction τ of photons rescattered during reionization out of line of sight and replaced statistically by photon with random temperature flucutuation - suppressing anisotropy as e^{-τ}



Transfer Function

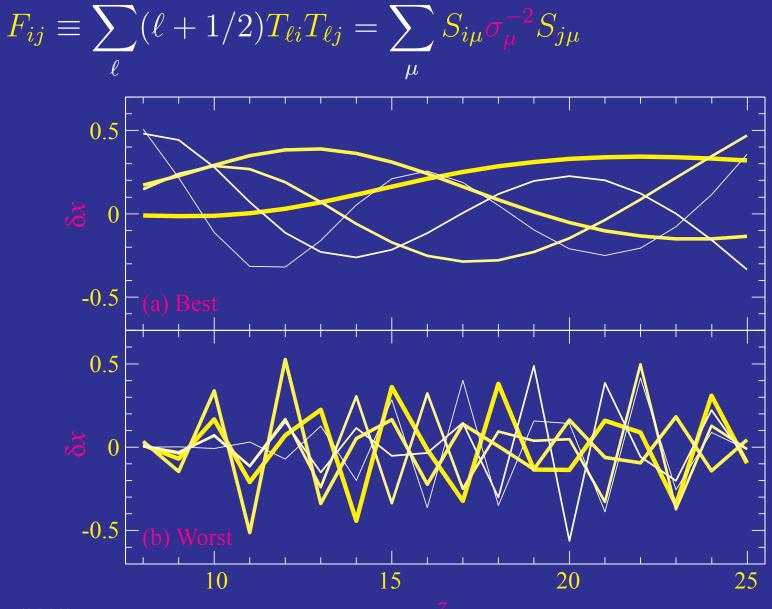
• Linearized response to delta function ionization perturbation $T_{\ell i} \equiv \frac{\partial \ln C_{\ell}^{EE}}{\partial x(z_{i})}, \qquad \delta C_{\ell}^{EE} = C_{\ell}^{EE} \sum_{i} T_{\ell i} \delta x(z_{i})$



Hu & Holder (2003)

Principal Components

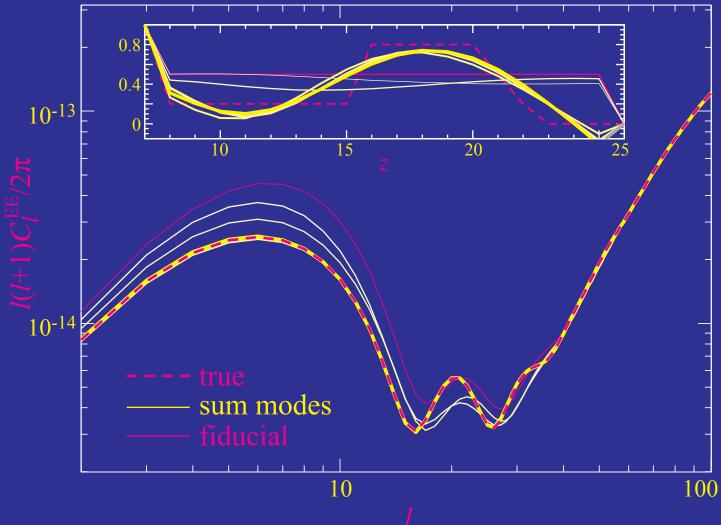
• Eigenvectors of the Fisher Matrix



Hu & Holder (2003)

Representation in Modes

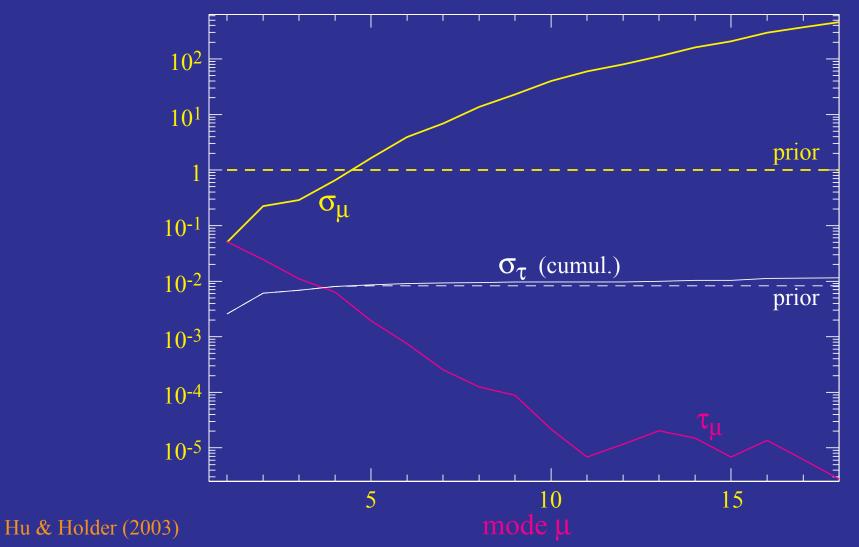
 Reproduces the power spectrum with sum over >3 modes more generally 5 modes suffices: e.g. total τ=0.1375 vs 0.1377



Hu & Holder (2003)

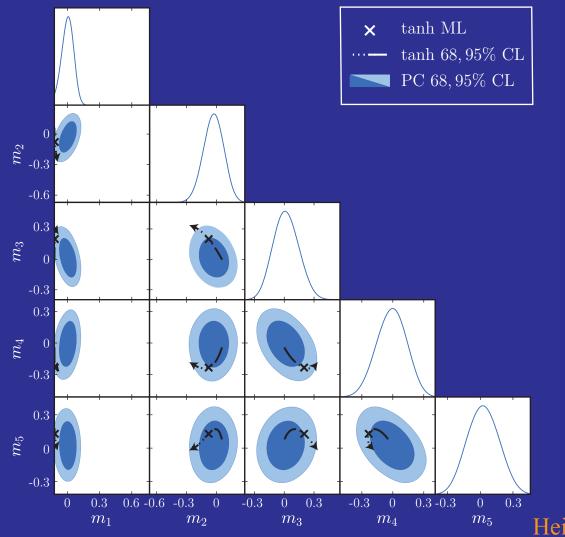
Total Optical Depth

- Optical depth measurement unbiased
- Ultimate errors set by cosmic variance here 0.01
- Equivalently 1% measure of initial amplitude, impt for dark energy



Complete Planck 2015 Reionization

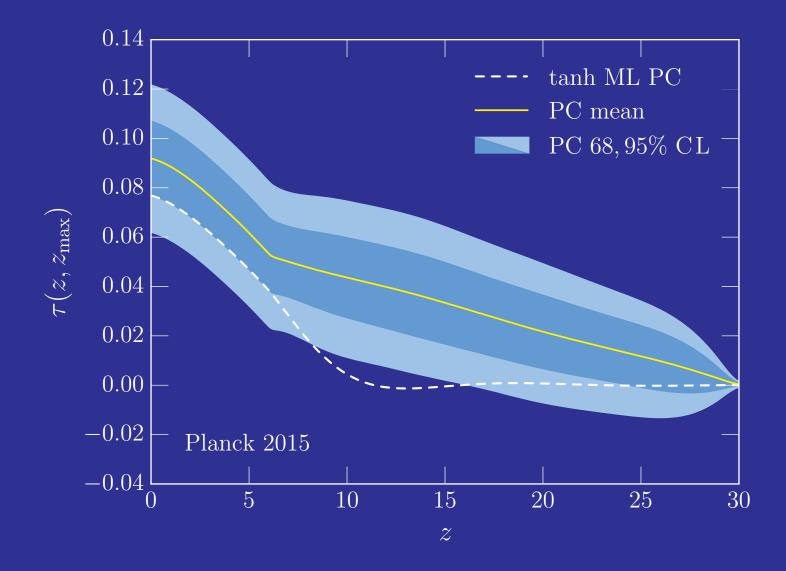
- 5PCs completely span z < 30 reionization observables
- Step function models only skirt the favored regions



Heinrich, Miranda, Hu (2016)

Complete Planck 2015 Reionization

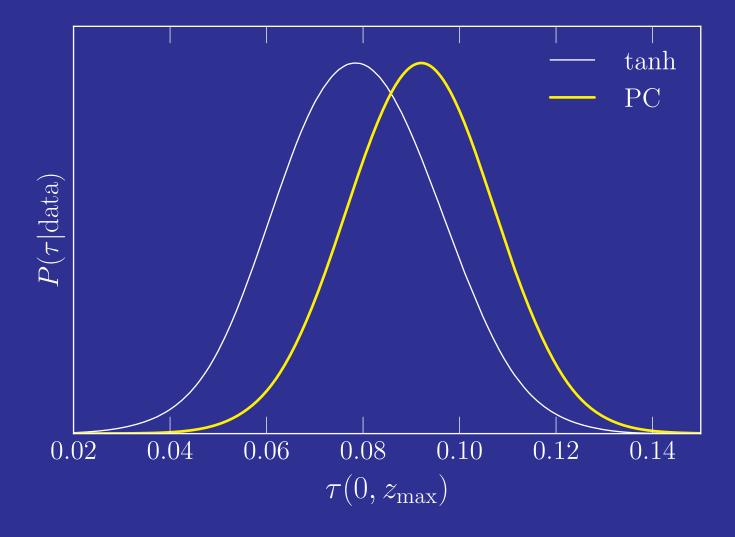
• Allows for a high redshift component of ionization



Heinrich, Miranda, Hu (2016)

Complete Planck 2015 Reionization

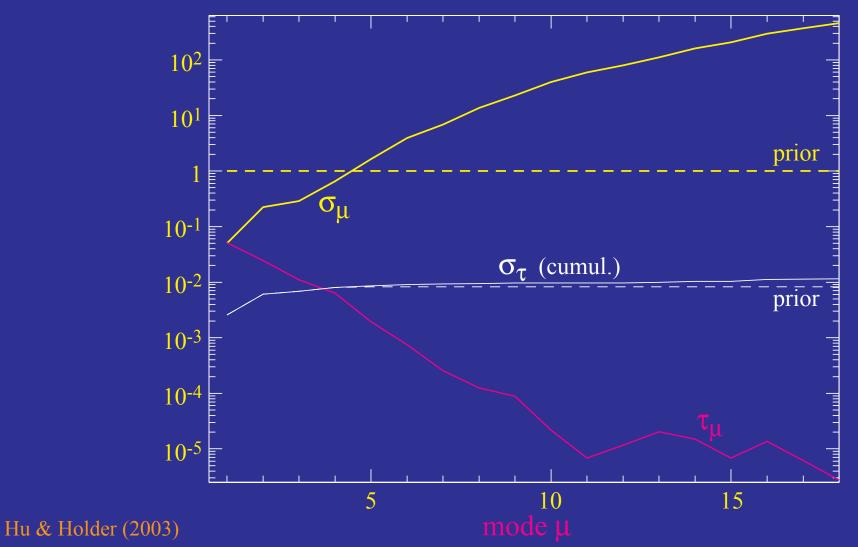
• Shifts optical depth higher



Heinrich, Miranda, Hu (2016)

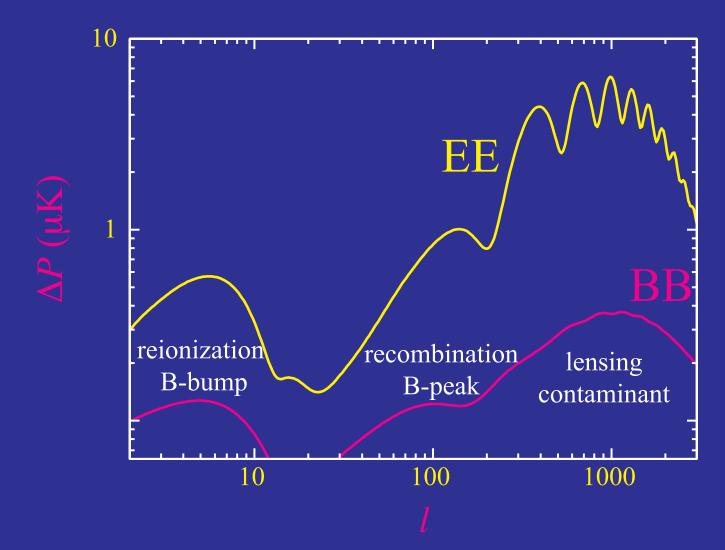
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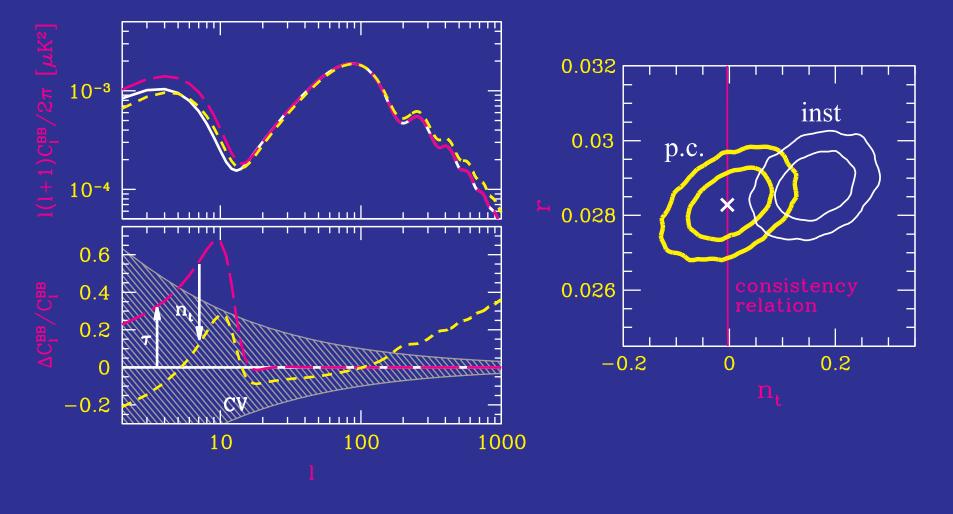
The B-Bump

- Rescattering of gravitational wave anisotropy generates the B-bump
- If *r* is near current upper limit, motivates next generation satellite
- Potentially enables test consistency test of canonical inflation

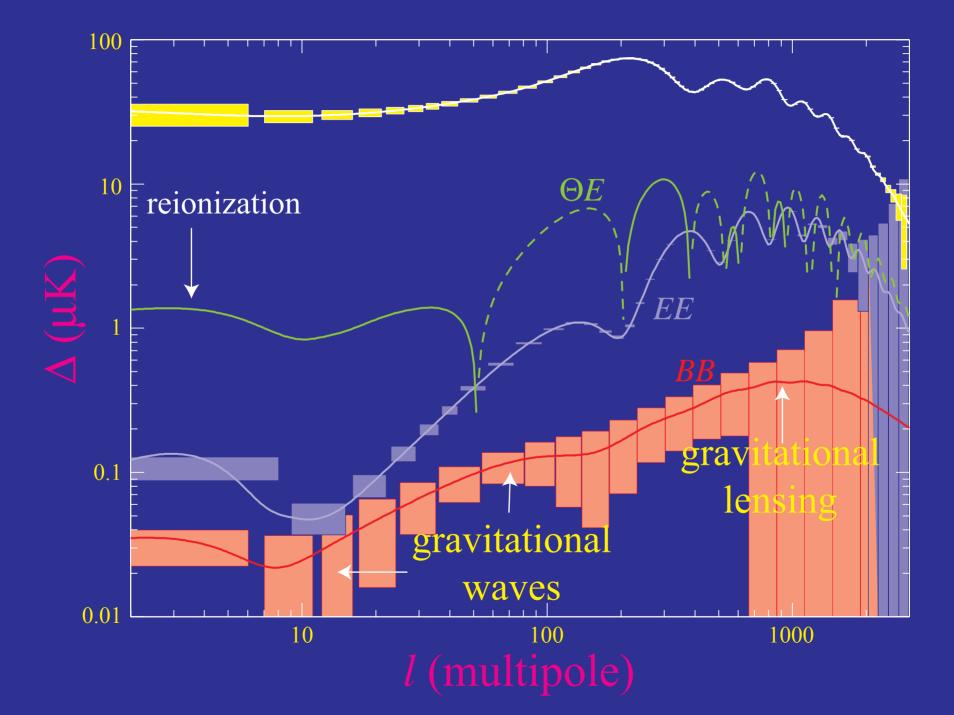


Slow Roll Consistency Relation

- Consistency relation between tensor-scalar ratio and tensor tilt $r = -8n_t$ tested by reionization
- Reionization uncertainties controlled by a complete p.c. analysis



Temperature and Polarization Spectra



Summary

- CMB polarized by Thomson scattering of quadrupole anisotropy: isolates recombination, reionization with little projection effects in transfer
- Linear scalar fluctuations generate E-modes where polarization direction (anti)aligned with amplitude change
- Linear tensor fluctuations also generate B-modes where polarization direction (anti)crossed with amplitude change
- B-mode gravitational wave amplitude measures the inflation energy scale: if observably large imply superPlanckian roll
- Beyond linear theory, scalar fluctuations generate B-modes
- Gravitational lensing B-modes measure amplitude of structure at *z*~2, neutrino mass and can be quadratically reconstructed
- Delensing of the CMB can enable measurements to $r \sim 10^{-3}$

Ciao!

