Anisotropy Suppression

- A fraction $\tau \sim 0.1$ of photons rescattered during reionization out of line of sight and replaced statistically by photon with random temperature fluctuation - suppressing anisotropy as $e^{-\tau}$
Why Are Secondaries So Small?

- Original anisotropy replaced by new secondary sources
- Late universe more developed than early universe
  - Density fluctuations nonlinear not $10^{-5}$
  - Velocity field $10^{-3}$ not not $10^{-5}$
- Shouldn’t $\Delta T/T \sim \tau v \sim 10^{-4}$?
- Limber says no!
- Spatial and angular dependence of sources contributing and cancelling broadly in redshift
Integral Solution

• Formal solution to the radiative transfer or Boltzmann equation involves integrating sources across line of sight.

• Linear solution describes the decomposition of the source $S^{(m)}_\ell$ with its local angular dependence and plane wave spatial dependence as seen at a distance $x = D\hat{n}$.

• Proceed by decomposing the angular dependence of the plane wave

$$e^{ik \cdot x} = \sum_{\ell} (-i)^{\ell} \sqrt{4\pi(2\ell + 1)} j_{\ell}(kD) Y^0_{\ell}(\hat{n})$$

• Recouple to the local angular dependence of $G^m_{\ell}$

$$G^m_{\ell_s} = \sum_{\ell} (-i)^{\ell} \sqrt{4\pi(2\ell + 1)} \alpha^{(m)}_{\ell_s \ell}(kD) Y^m_{\ell}(\hat{n})$$
Integral Solution

- **Projection kernels** (monopole, temperature; dipole, doppler):
  
  \[ \ell_s = 0, \quad m = 0 \quad \quad \alpha^{(0)}_{0\ell} \equiv j_\ell \]
  
  \[ \ell_s = 1, \quad m = 0 \quad \quad \alpha^{(0)}_{1\ell} \equiv j'_\ell \]

- **Integral solution**: for \( \Theta = \Delta T/T \)
  
  \[ \Theta^{(m)}_\ell(k, 0) = \frac{2}{2\ell + 1} \int_0^\infty dDe^{-\tau} \sum_{\ell_s} S^{(m)}_{\ell_s} \alpha^{(m)}_{\ell_s \ell} (kD) \]

- **Power spectrum**:
  
  \[ C_\ell = \frac{2}{\pi} \int \frac{dk}{k} \sum_m k^3 \langle \Theta^{(m)*}_\ell \Theta^{(m)}_\ell \rangle \frac{1}{(2\ell + 1)^2} \]

- Solving for \( C_\ell \) reduces to solving for the behavior of a handful of sources. Straightforward generalization to polarization.
Anisotropy Suppression and Regeneration

- Recombination sources obscured and replaced with secondary sources that suffer Limber cancellation from integrating over many wavelengths of the source.
- Net suppression despite substantially larger sources due to growth of structure except beyond damping tail <10’.

\[ j_l(kd)Y^0_l \]

\[ e^{-\tau} \]

\[ S \]

\[ (2l+1)j_l(100) \]

Temperature

\[ Y^0_0 \]
Scattering Secondaries

$\Delta T$ (µK)

$\mu K$

$Doppler$

$suppression$

$SZ$

$linear$

$density-mod$

$ion-mod$
Doppler Effect in Limber Approximation

- Only fluctuations transverse to line of sight survive in Limber approx but linear Doppler effect has no contribution in this direction.
Cancellation of the Linear Effect

Reionization Surface

Observer

Cancellation

e−velocity

overdensity

redshifted γ

blueshifted γ
Modulated Doppler Effect

Observer

Reionization Surface

e— velocity

overdensity, ionization patch, cluster...

unscattered $\gamma$

blueshifted $\gamma$
Ostriker–Vishniac Effect

Hu & White (1996)

see Shirley Ho's talk
Inhomogeneous Ionization

- As reionization completes, ionization regions grow and fill the space

Zahn et al. (2006) [Mortonson et al (2009)]
Inhomogeneous Ionization

- Provides a source for modulated Doppler effect that appears on the scale of the ionization region
Polarization from Thomson Scattering

- Differential cross section depends on polarization and angle

$$\frac{d\sigma}{d\Omega} = \frac{3}{8\pi} |\hat{e}' \cdot \hat{e}|^2 \sigma_T$$
Polarization from Thomson Scattering

- Isotropic radiation scatters into unpolarized radiation
Polarization from Thomson Scattering

- Quadrupole anisotropies scatter into linear polarization aligned with cold lobe
Quadrupoles from Gravitational Waves

- Transverse-traceless distortion provides temperature quadrupole
- Gravitational wave polarization picks out direction transverse to wavevector
How do Scalars Differ?

- Temperature inhomogeneities in a medium
- Photons arrive from different regions producing an anisotropy

Azimuthally symmetric around wavevector
Whence Polarization Anisotropy?

• Observed photons scatter into the line of sight
• Polarization arises from the projection of the quadrupole on the transverse plane
Polarization Multipoles

- Mathematically pattern is described by the tensor (spin-2) spherical harmonics [eigenfunctions of Laplacian on trace-free 2 tensor]
- Correspondence with scalar spherical harmonics established via Clebsch-Gordon coefficients (spin x orbital)
- Amplitude of the coefficients in the spherical harmonic expansion are the multipole moments; averaged square is the power

\[ l=2, \ m=0 \]
Gravitational Wave Pattern

- Projection of the quadrupole anisotropy gives polarization pattern
- Transverse polarization of gravitational waves breaks azimuthal symmetry
Electric & Magnetic Polarization
(a.k.a. gradient & curl)

• Alignment of principal vs polarization axes
  (curvature matrix vs polarization direction)

Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)
Recombination B-Modes

- Rescattering of quadrupoles at recombination yield a peak in B-modes
Two scattering epochs: recombination and reionization leave two imprints on B-modes

maximum amplitude!
Integrated Sachs-Wolfe Effect
ISW Effect

- **Gravitational blueshift** on infall does not cancel redshift on climbing out.
- Contraction of **spatial metric** doubles the effect: $\Delta T/T = 2\Delta \Phi$.
- Effect from potential **hills** and **wells** cancel on small scales.
ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: \( \Delta T/T = 2 \Delta \Phi \)
- Effect from potential hills and wells cancel on small scales
Smooth Energy Density & Potential Decay

- Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature $\zeta$)
Smooth Energy Density & Potential Decay

• Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature $\zeta$)

• A smooth component contributes density $\rho$ to the expansion but not density fluctuation $\delta\rho$ to the Poisson equation

• Imbalance causes potential to decay once smooth component dominates the expansion
ISW Spatial Modes

- ISW effect comes from *nearby* acceleration regime
- Shorter wavelengths project onto same angle
- Broad source kernel: *Limber cancellation* out to quadrupole
Quadrupole Origins

- Transfer function for the quadrupole