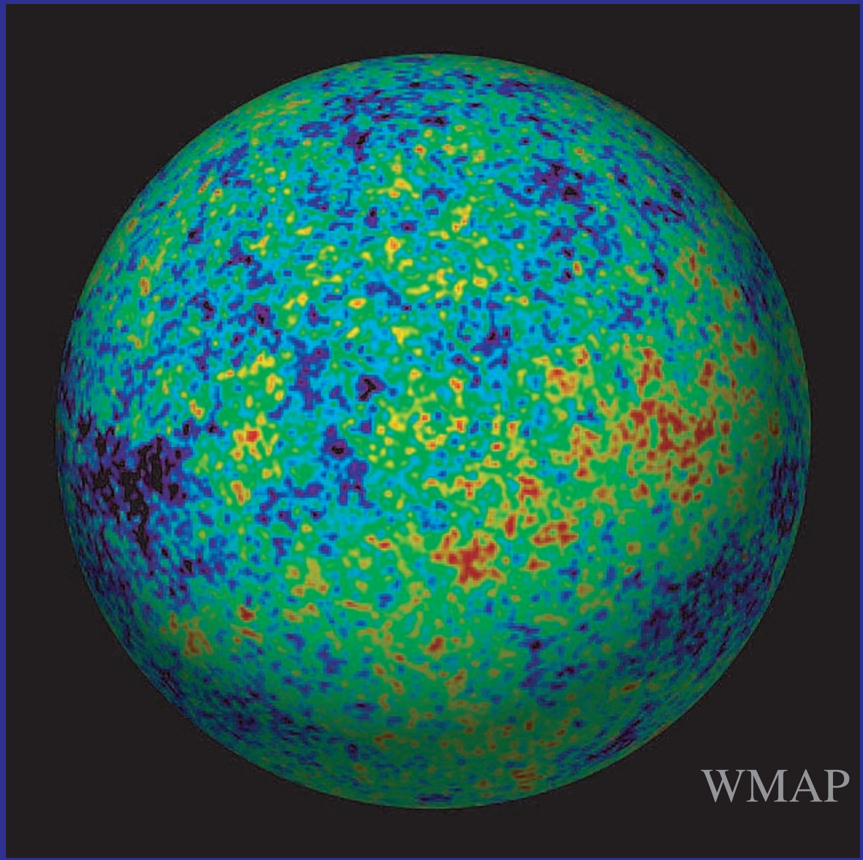
Lecture I

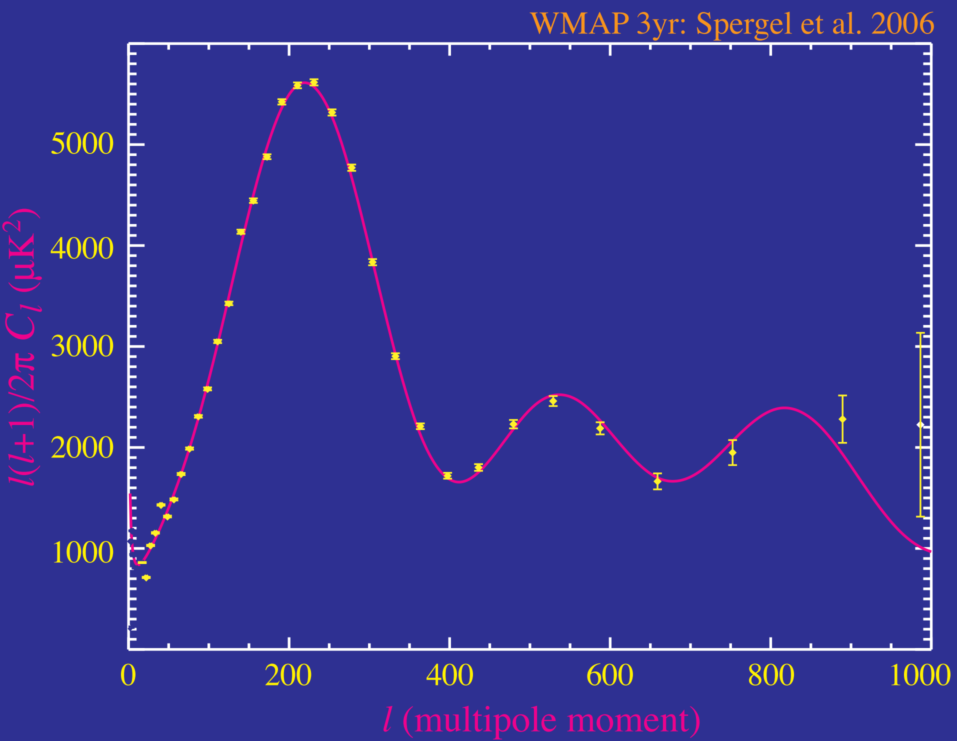


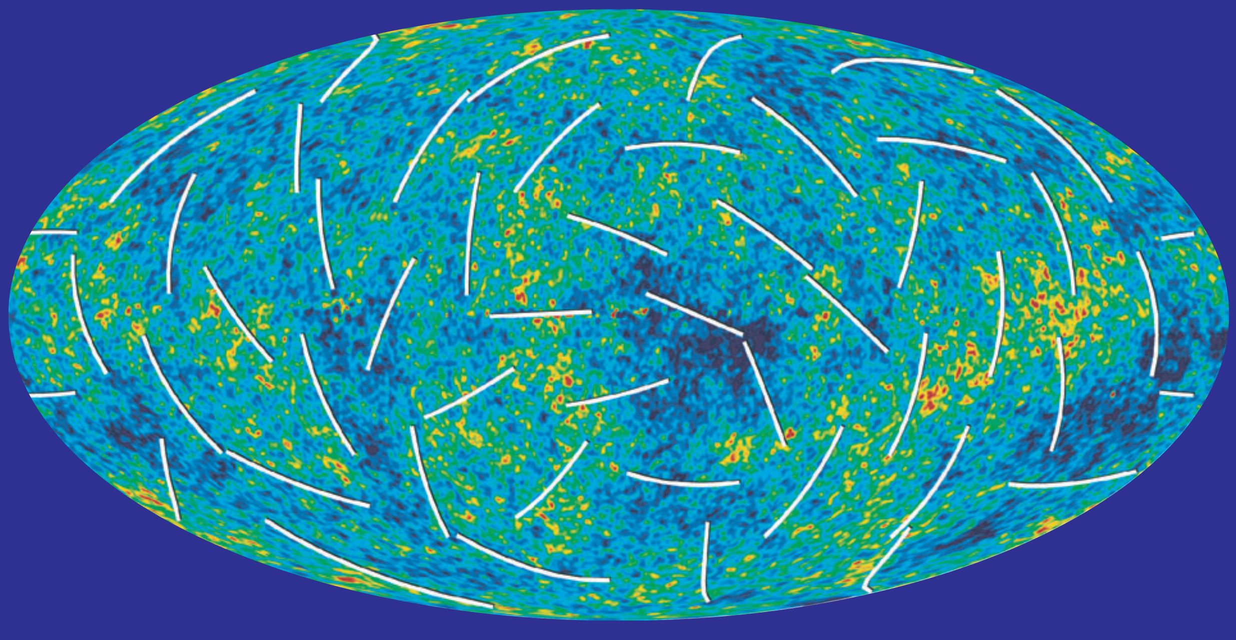
Thermal History and Acoustic Kinematics

Wayne Hu

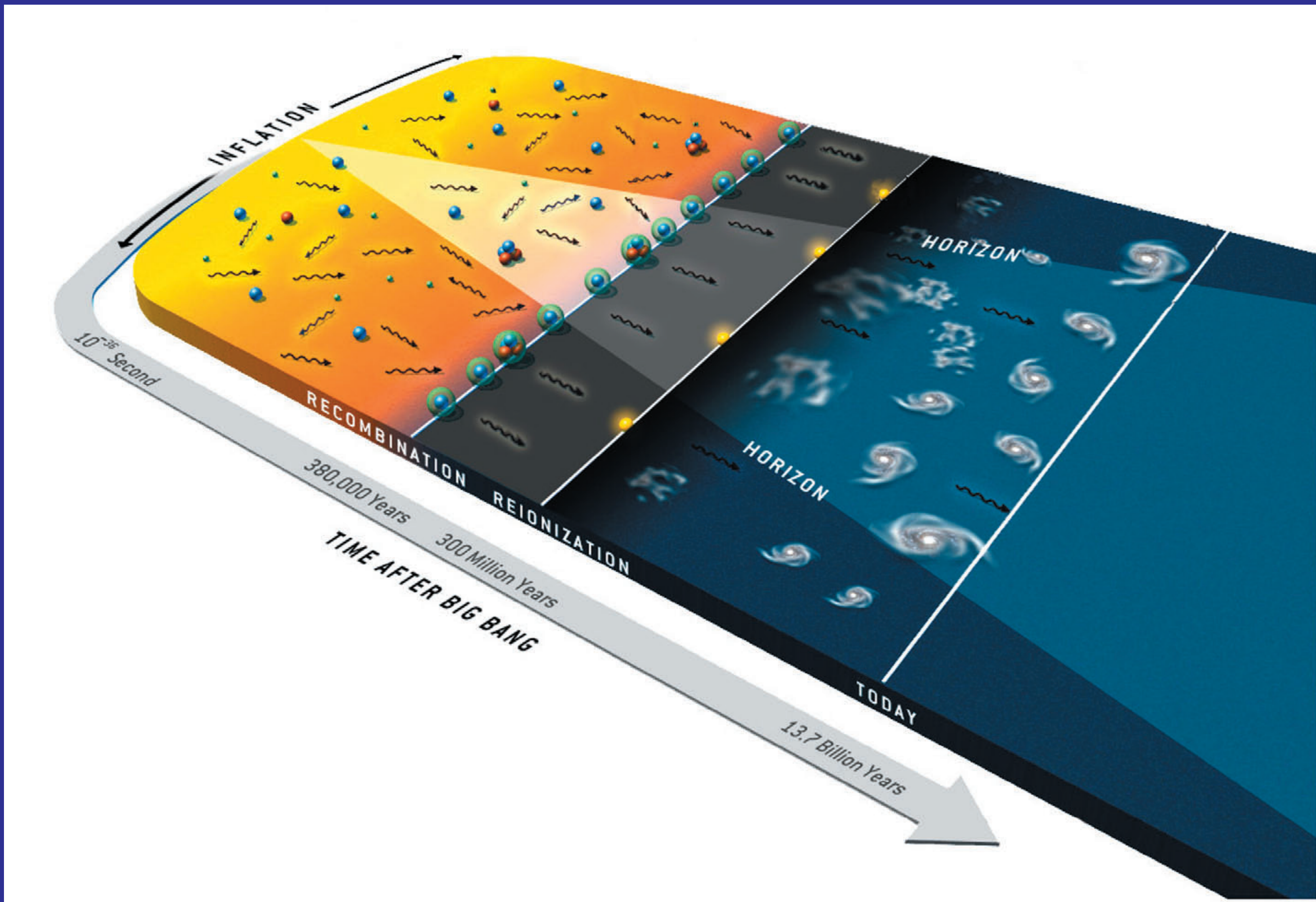
Tenerife, November 2007

WMAP 3yr Data





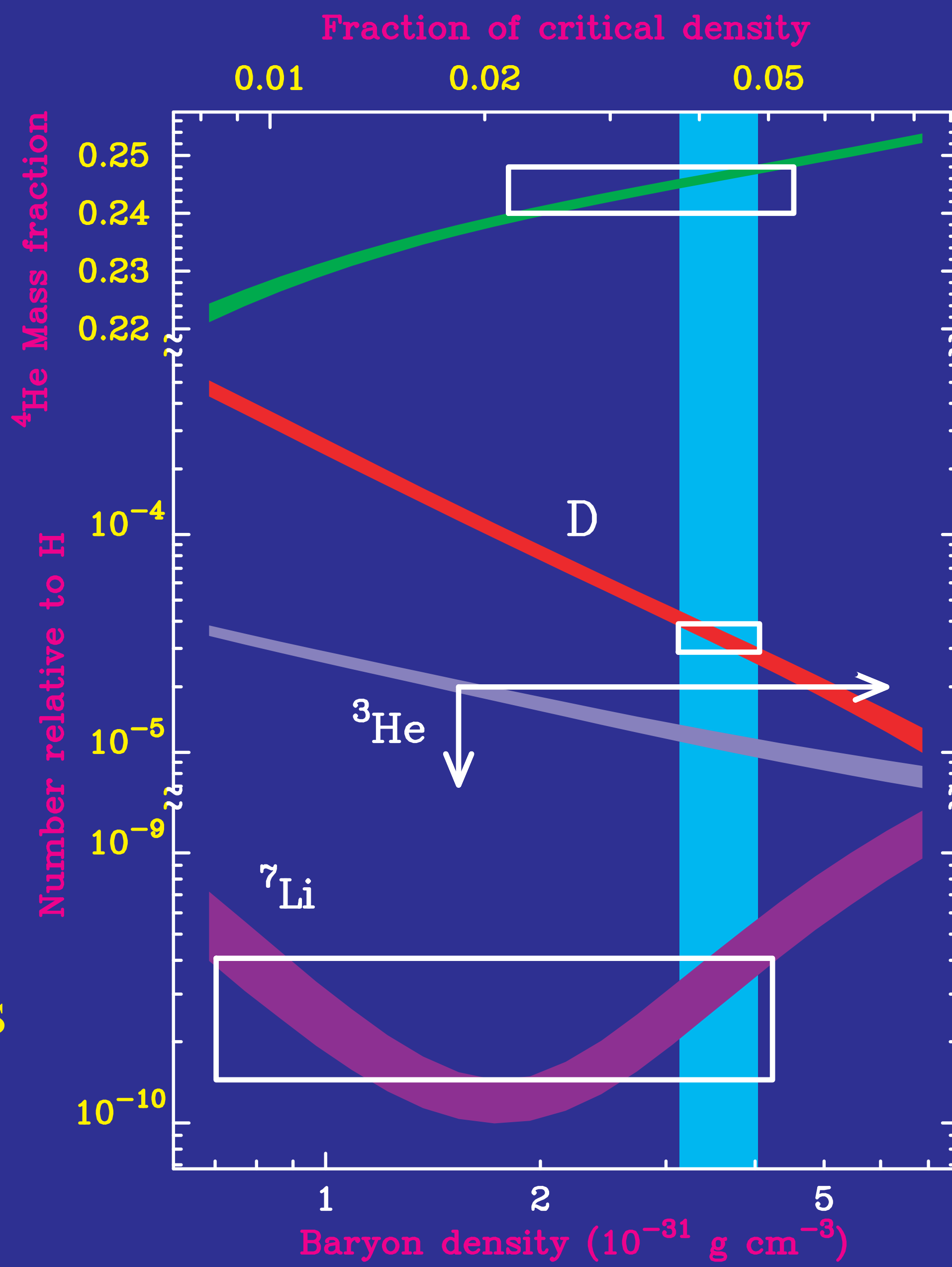
In the Beginning...



Hu & White (2004); artist:B. Christie/SciAm; available at http://background.uchicago.edu

Predicting the CMB: Nucleosynthesis

- Light element abundance depends on baryon/photon ratio
- Existence and temperature of CMB originally predicted (Gamow 1948) by light elements + visible baryons
- With the CMB photon number density fixed by the temperature light elements imply dark baryons
- Peaks say that photon-baryon ratio at MeV and eV scales are same



Burles, Nollett, Turner (1999)

Thermalization

- Compton scattering conserves the number of photons

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

  hence cannot create a blackbody, only Bose-Einstein spectrum

- Above $z \sim 10^5 - 10^{-6}$ creation/absorption processes: bremmstrahlung and double (or radiative) Compton scattering

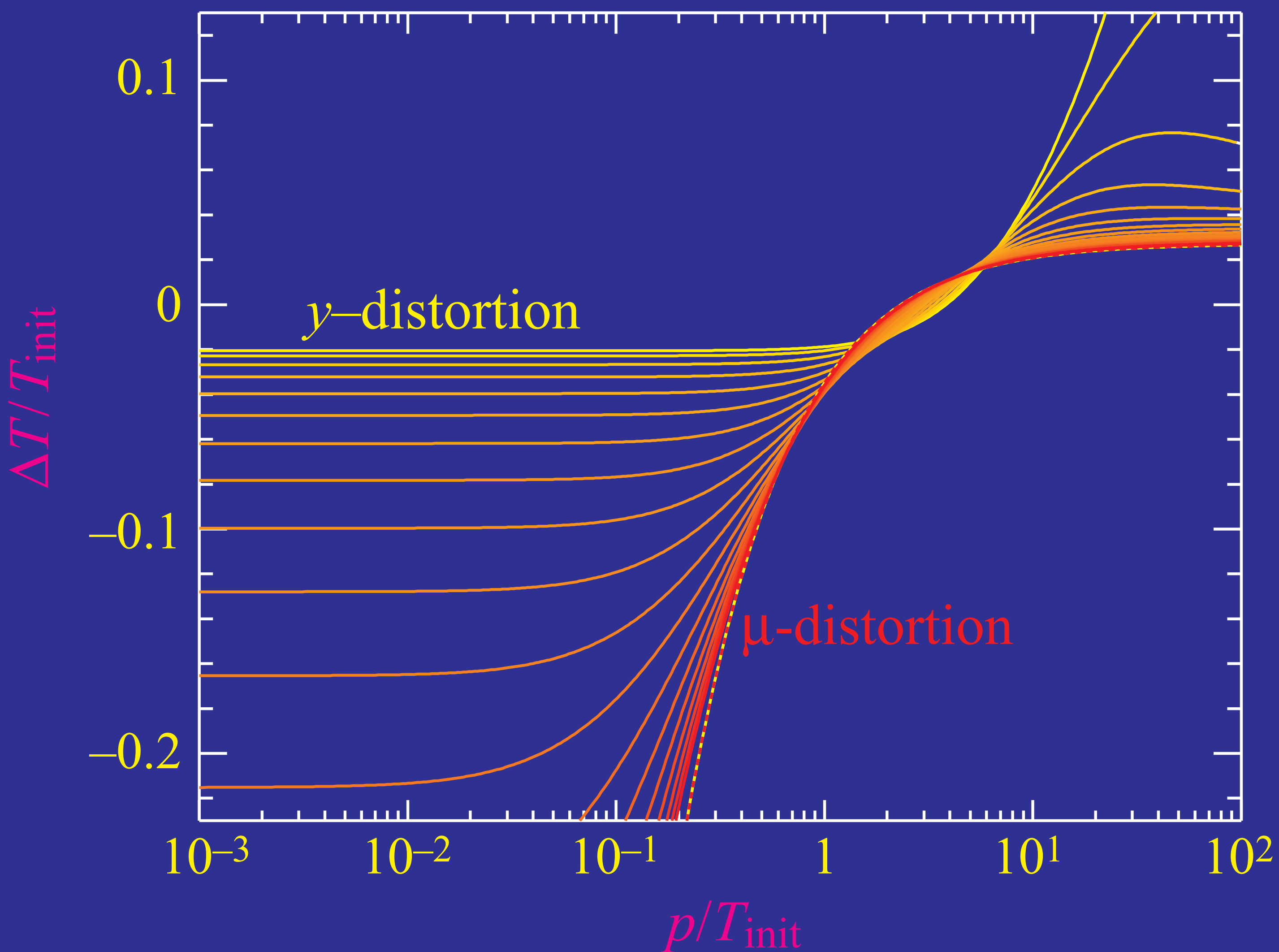
$$e^- + p \leftrightarrow e^- + p + \gamma$$
$$e^- + \gamma \leftrightarrow e^- + \gamma + \gamma$$

  sufficently rapid to bring spectrum to a blackbody

- Below this redshift, only low frequency spectrum thermalized leaving a μ or y distortion near and above the peak
- Consequently energy injection into the plasma after $\sim$ few months strongly constrained

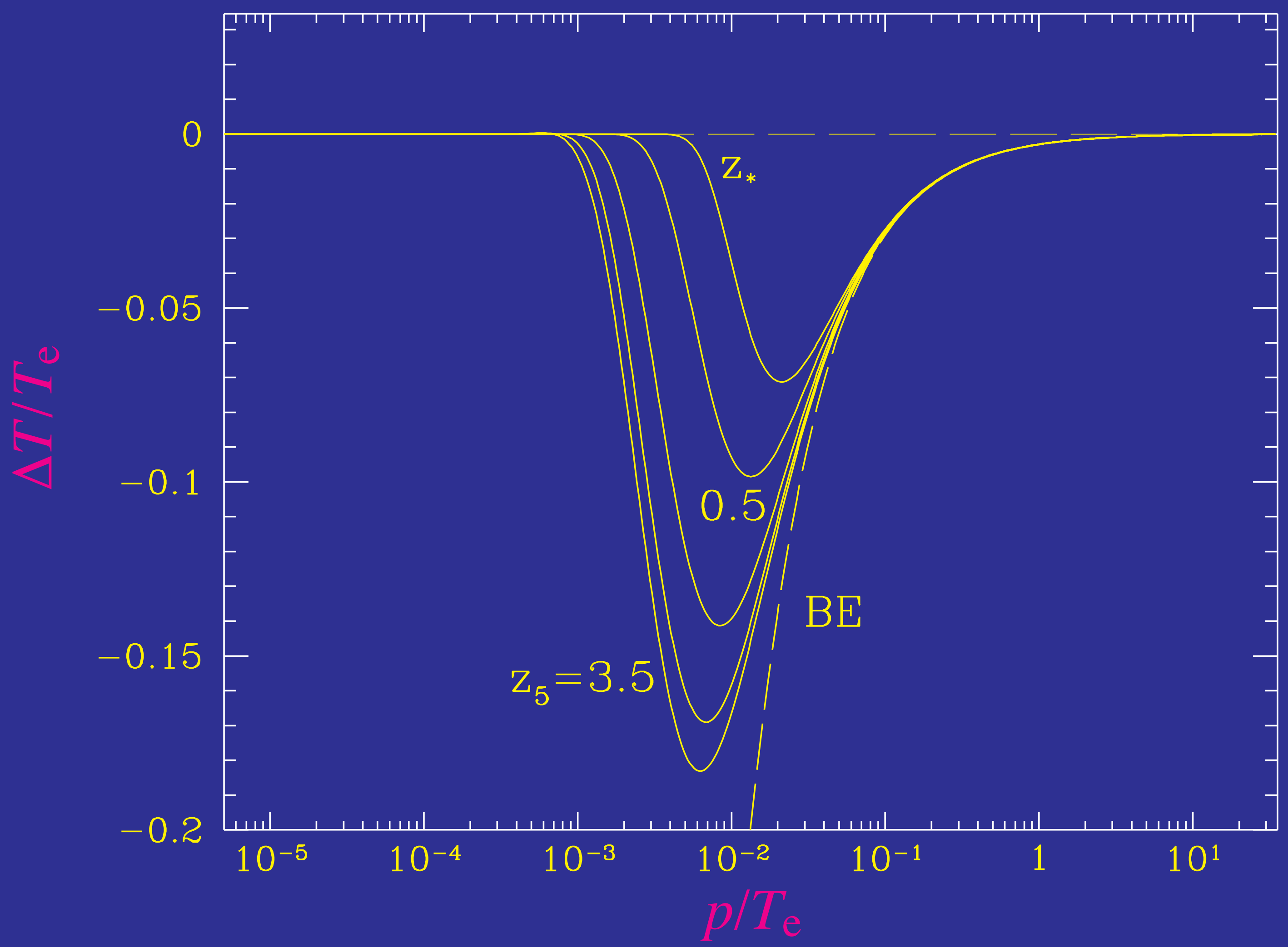
Comptonization

- Compton upscattering: *y*–distortion - seen in Galaxy clusters
- Redistribution: μ-distortion



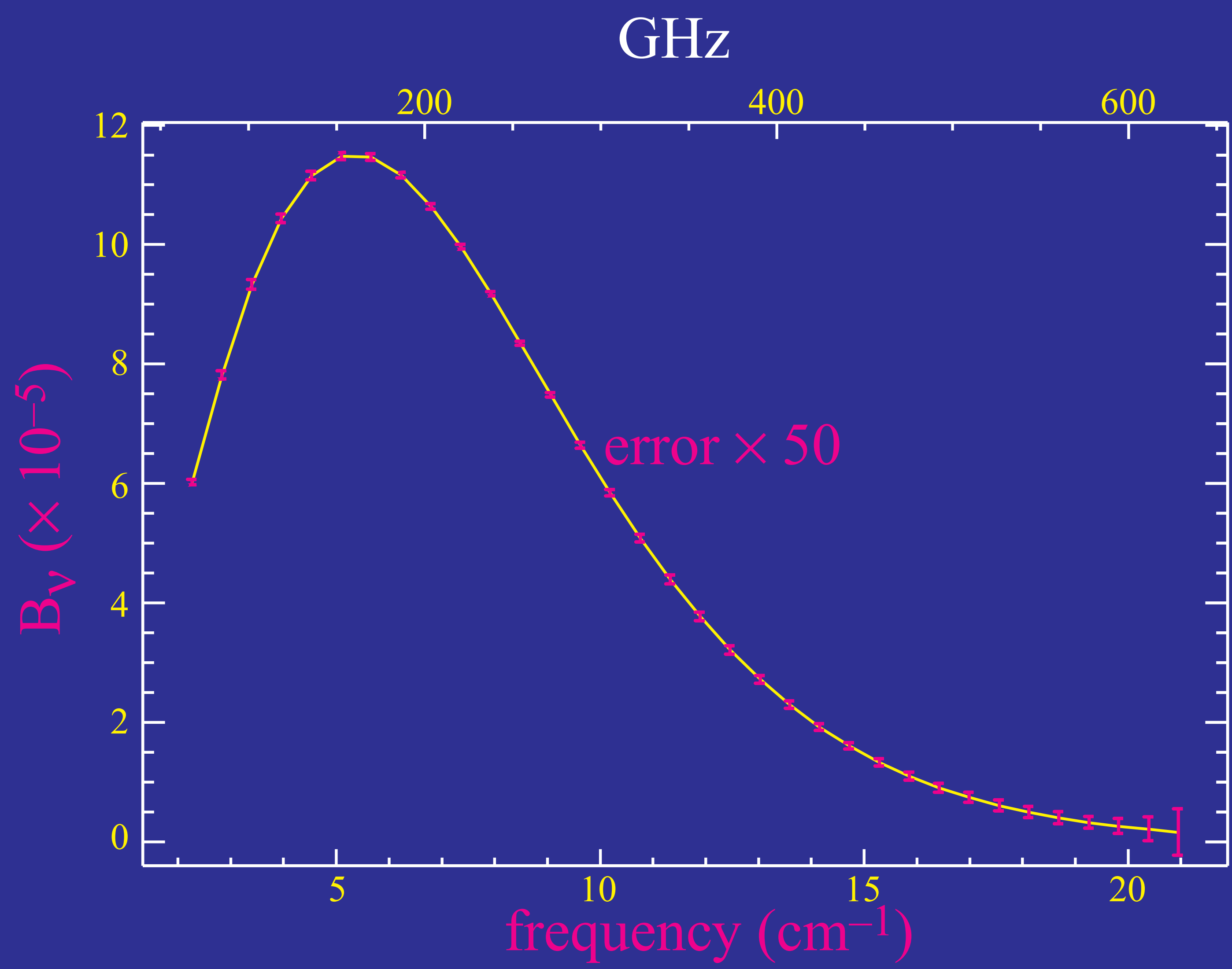
Thermalization

- Photon creation processes effective at low frequency and high redshift; in conjunction with Compton scattering, thermalizes the spectrum



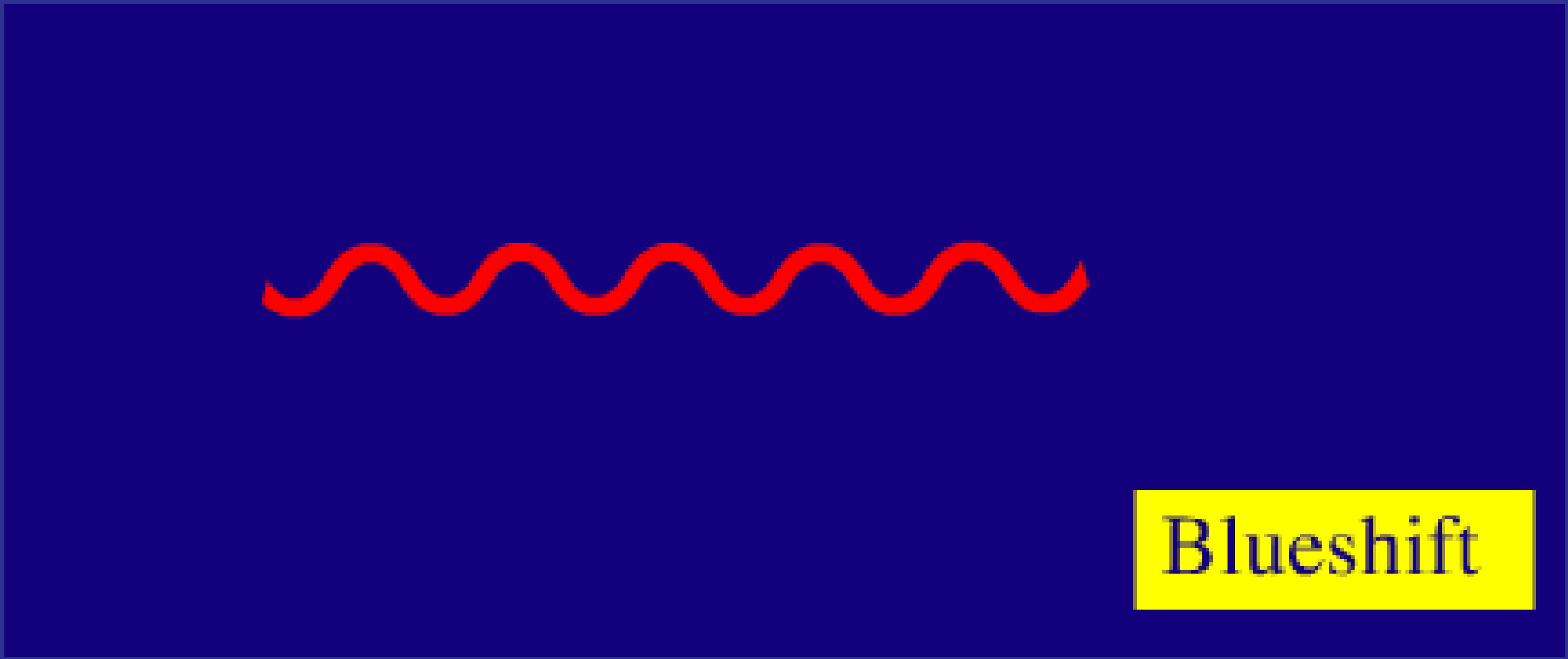
Spectrum

- FIRAS Spectrum
- Perfect Blackbody



Darkness from Light: Recombination

- Reversing the expansion, CMB photons got hotter and hotter into the past
- When the universe was 1000 times smaller and the CMB photons were at 3000K they were energetic enough disintingrate atoms into electrons and protons.



Recombination

- Maxwell-Boltzmann distribution

$$n = g e^{-(m-\mu)/T} \left(\frac{mT}{2\pi} \right)^{3/2}$$

determines the equilibrium distribution for species in reaction and hence the equilibrium ionization:

$$p + e^- \leftrightarrow H + \gamma$$

$$\frac{n_p n_e}{n_H} \approx e^{-B/T} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{(\mu_p + \mu_e - \mu_H)/T}$$

where $B = m_p + m_e - m_H = 13.6\mathrm{eV}$ is the binding energy, $g_p = g_e = \frac{1}{2} g_H = 2$, and $\mu_p + \mu_e = \mu_H$ in equilibrium

Recombination

- Define ionization fraction

$$n_p = n_e = x_e n_b$$

$$n_H = n_{\rm tot} - n_b = (1 - x_e) n_b$$

- Saha Equation

$$\frac{n_e n_p}{n_H n_b} = \frac{x_e^2}{1 - x_e} = \frac{1}{n_b}\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-B/T}$$

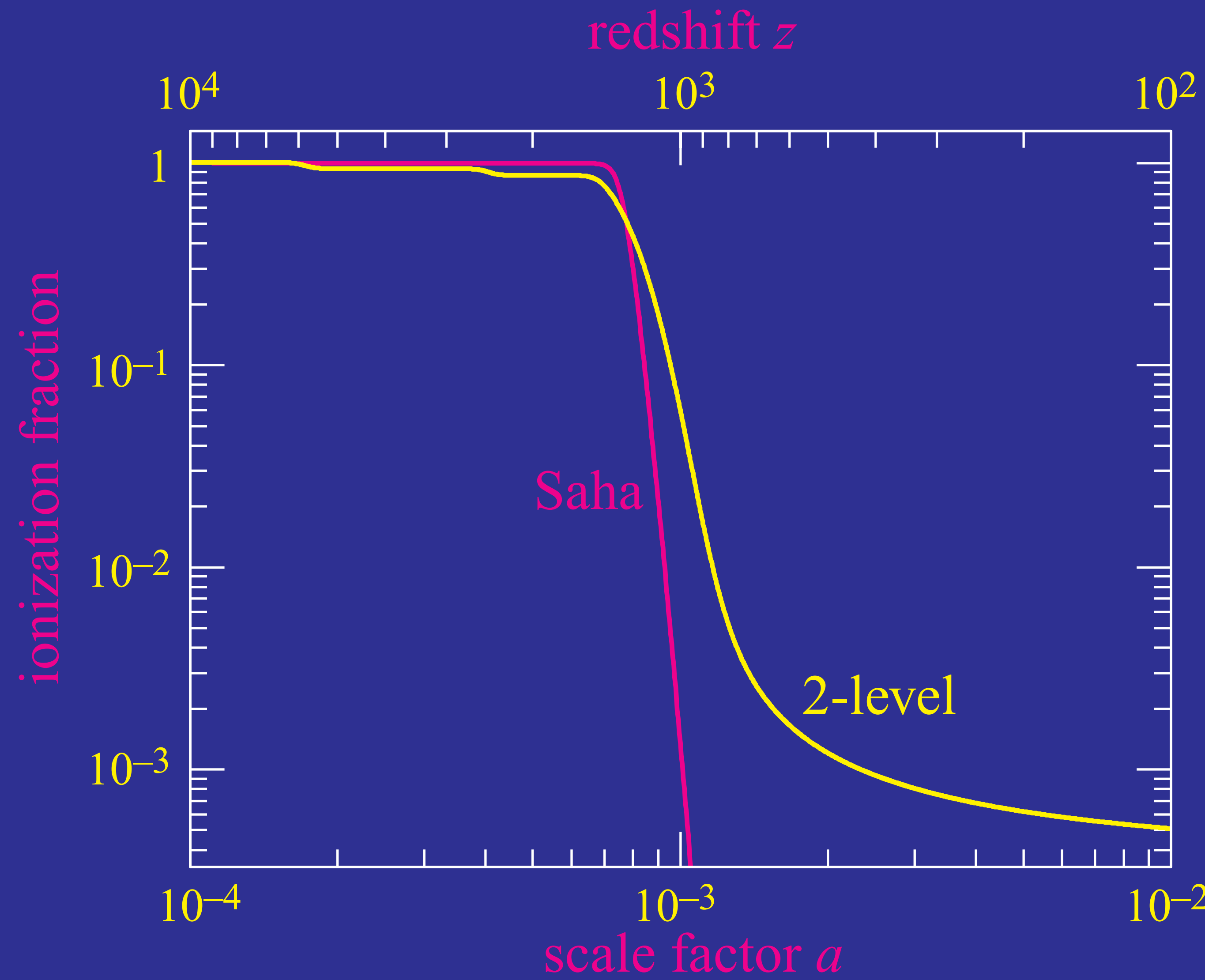
- Naive guess of $T_* = B$ wrong due to the low baryon-photon ratio

$$\eta_{b\gamma} \equiv n_b/n_\gamma \approx 3 \times 10^{-8} \Omega_b h^2$$

- Sufficient number of ionizing photons in Wien tail until $T_* \approx 0.3$eV so recombination at $z_* \approx 1000$

Recombination

- Hung up by Lyα opacity (2γ forbidden transition + redshifting)
- Frozen out with a finite residual ionization fraction



Anisotropy Formation

- Temperature inhomogeneities at recombination become anisotropy

Temperature Fluctuations

- Observe blackbody radiation with a temperature that differs at 10^{-5} coming from the surface of recombination

$$f(\nu, \hat{\mathbf{n}}) = [\exp(2\pi\nu/T(\hat{\mathbf{n}})) - 1]^{-1}$$

- Decompose the temperature perturbation in spherical harmonics

$$T(\hat{\mathbf{n}}) = \sum_{\ell m} T_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$$

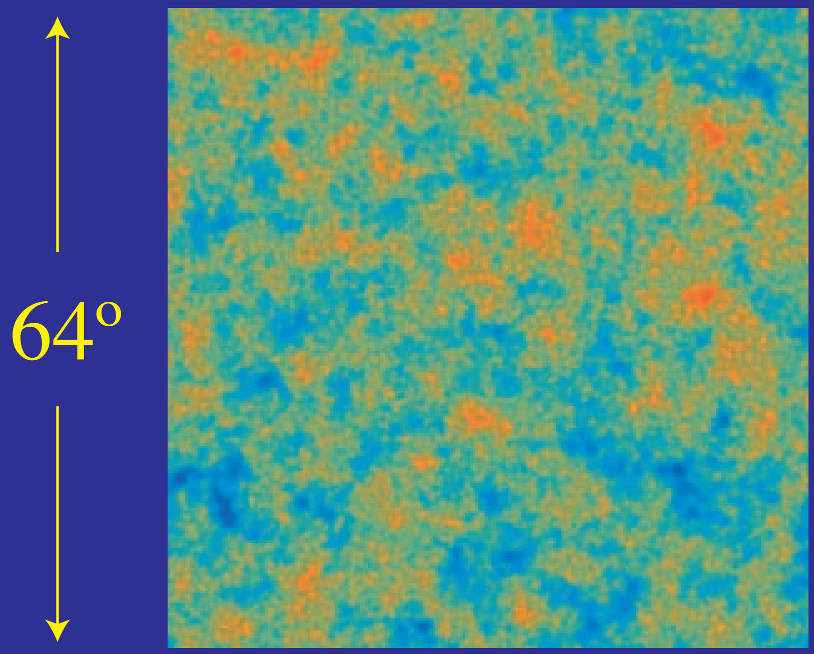
- For Gaussian random statistically isotropic fluctuations, the statistical properties of the temperature field are determined by the power spectrum

$$\langle T_{\ell m}{}^* T_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$$

in units of $\mu\mathrm{K}^2$ or in terms of dimensionless temperature $\Theta = \Delta T/T \sim 10^{-5}$

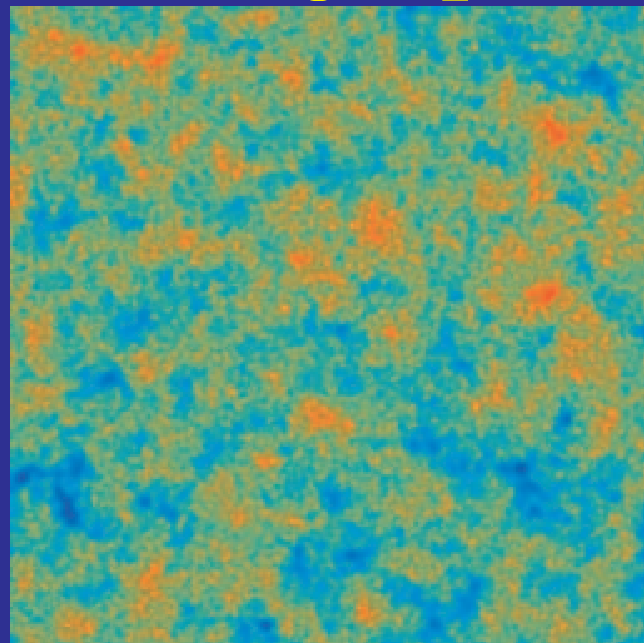
Seeing Spots

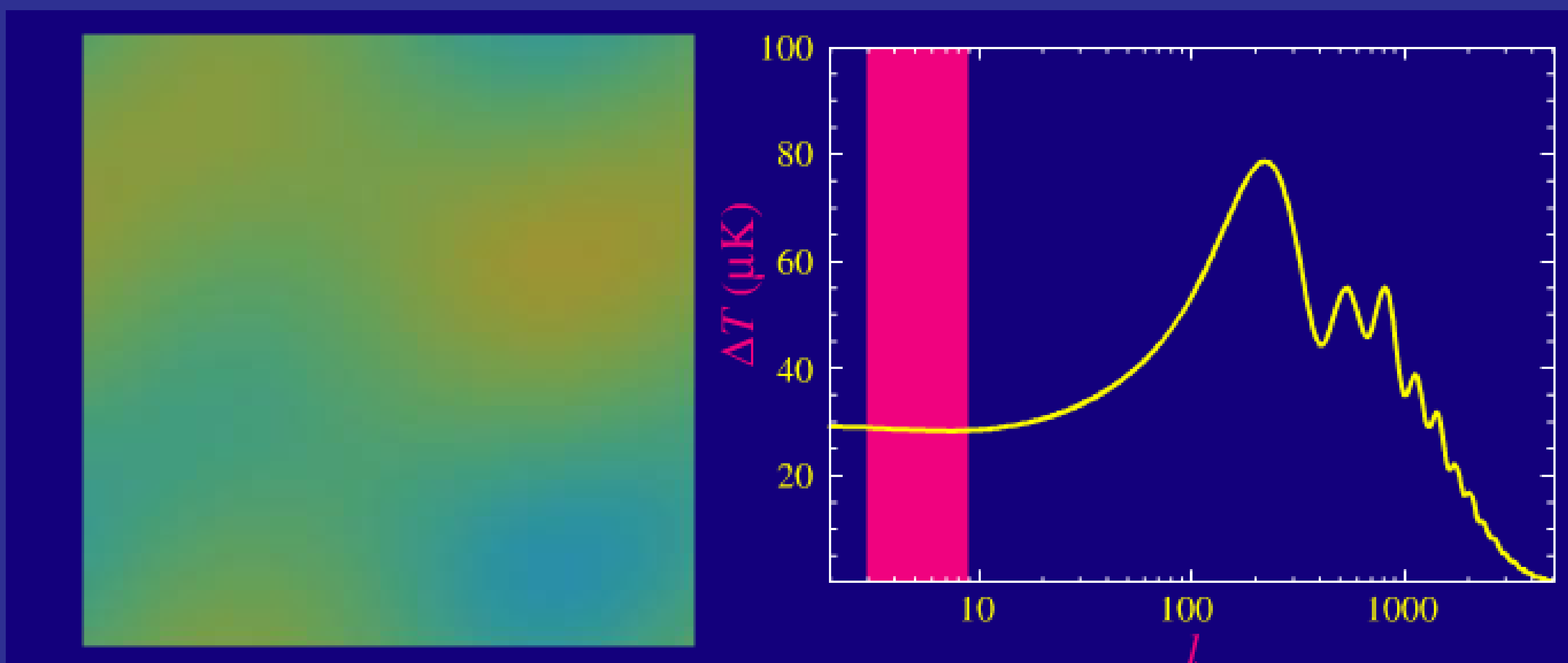
- 1 part in 100000 variations in temperature
- Spot sizes ranging from a fraction of a degree to 180 degrees



- Selecting only spots of a given range of sizes gives a power spectrum or frequency spectrum of the variations much like a graphic equalizer for sound.

Seeing Spots





Spatial vs Angular Power

- Take the radiation distribution at recombination to be described by an isotropic temperature field $T(\mathbf{x})$ and recombination to be instantaneous

$$T(\hat{\mathbf{n}}) = \int dD\, T(\mathbf{x})\delta(D - D_*)$$

where D is the comoving distance and D_* denotes recombination

- Describe the temperature field by its Fourier moments

$$T(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} T(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

with a power spectrum

$$\langle T(\mathbf{k})^* T(\mathbf{k}')\rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') P_T(k)$$

Spatial vs Angular Power

- Note that the variance of the field

$$\langle T(\mathbf{x})T(\mathbf{x})\rangle = \int \frac{d^3k}{(2\pi)^3} P(k)$$
$$= \int d\ln k\, \frac{k^3 P(k)}{2\pi^2} \equiv \int d\ln k\, \Delta_T^2(k)$$

so it is more convenient to think in the log power spectrum $\Delta_T^2(k)$

- Angular temperature field

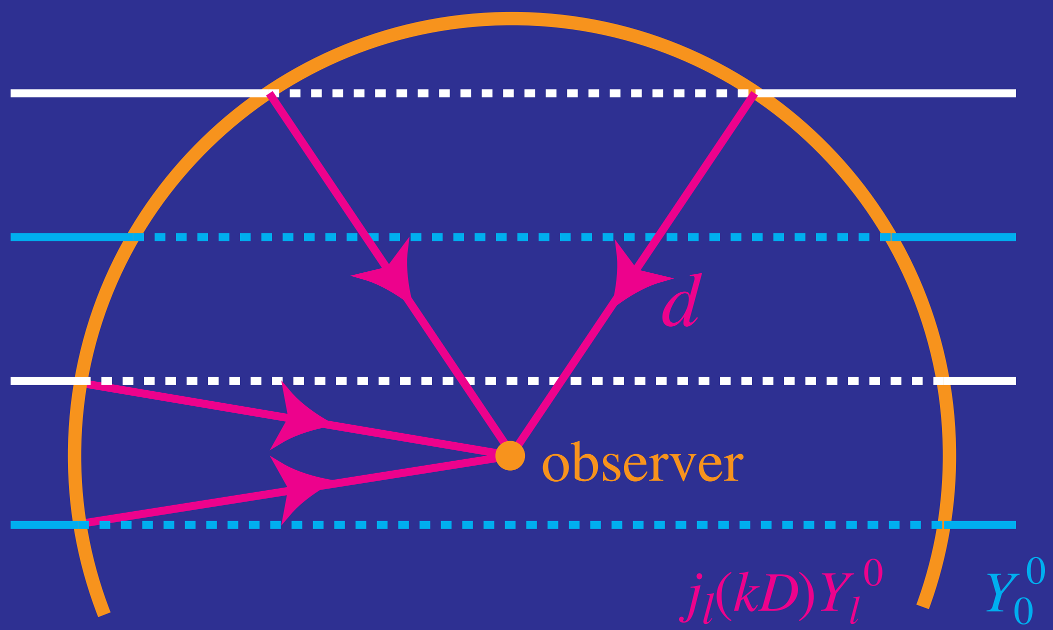
$$T(\hat{\mathbf{n}}) = \int \frac{d^3k}{(2\pi)^3} T(\mathbf{k}) e^{i\mathbf{k}\cdot D_* \hat{\mathbf{n}}}$$

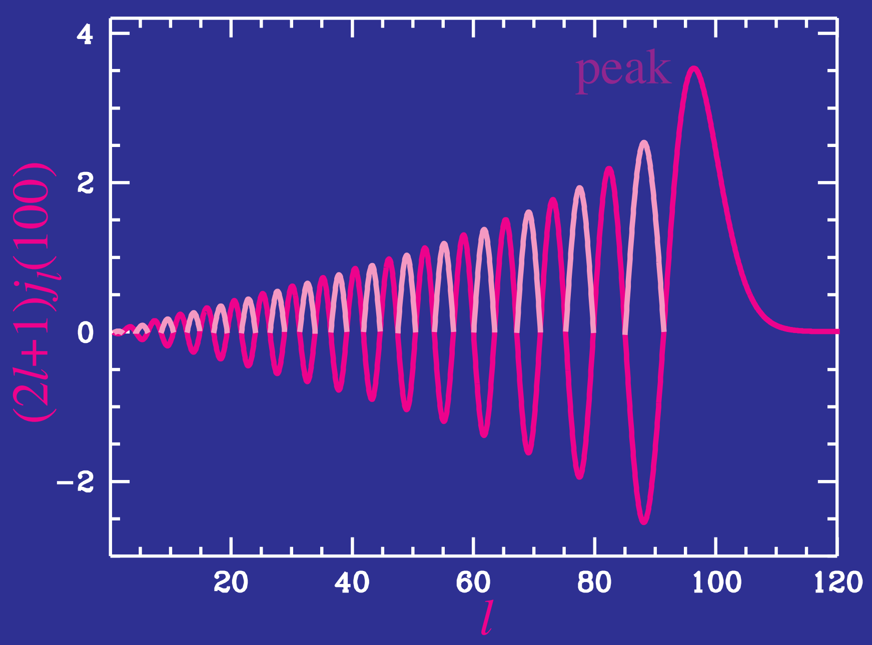
- Expand out plane wave in spherical coordinates

$$e^{i\mathbf{k}D_*\cdot\hat{\mathbf{n}}} = 4\pi \sum_{\ell m} i^\ell j_\ell(kD_*) Y_{\ell m}^*(\mathbf{k}) Y_{\ell m}(\hat{\mathbf{n}})$$

Angular Projection

- Angular projection comes from the spherical harmonic decomposition of plane waves
- Angular field is an integral over source shells with Bessel function weights
- Bessel function peaks near $l=kD$ with a long tail to lower multipoles





Spatial vs Angular Power

- Multipole moments

$$T_{\ell m} = \int \frac{d^3k}{(2\pi)^3} T(\mathbf{k}) 4\pi i^\ell j_\ell(kD_*) Y_{\ell m}(\mathbf{k})$$

- Power spectrum

$$\langle T^*_{\ell m} T_{\ell' m'} \rangle = \delta_{\ell\ell'}\delta_{mm'} 4\pi \int d\ln k \, j_\ell^2(kD_*) \Delta_T^2(k)$$

with $\int_0^\infty j_\ell^2(x) d\ln x = 1/(2\ell(\ell+1))$, slowly varying Δ_T^2

$$C_\ell = \frac{4\pi \Delta_T^2(\ell/D_*)}{2\ell(\ell+1)} = \frac{2\pi}{\ell(\ell+1)} \Delta_T^2(\ell/D_*)$$

so $\ell(\ell+1)C_\ell/2\pi = \Delta_T^2$ is commonly used log power

Spatial vs Angular Power

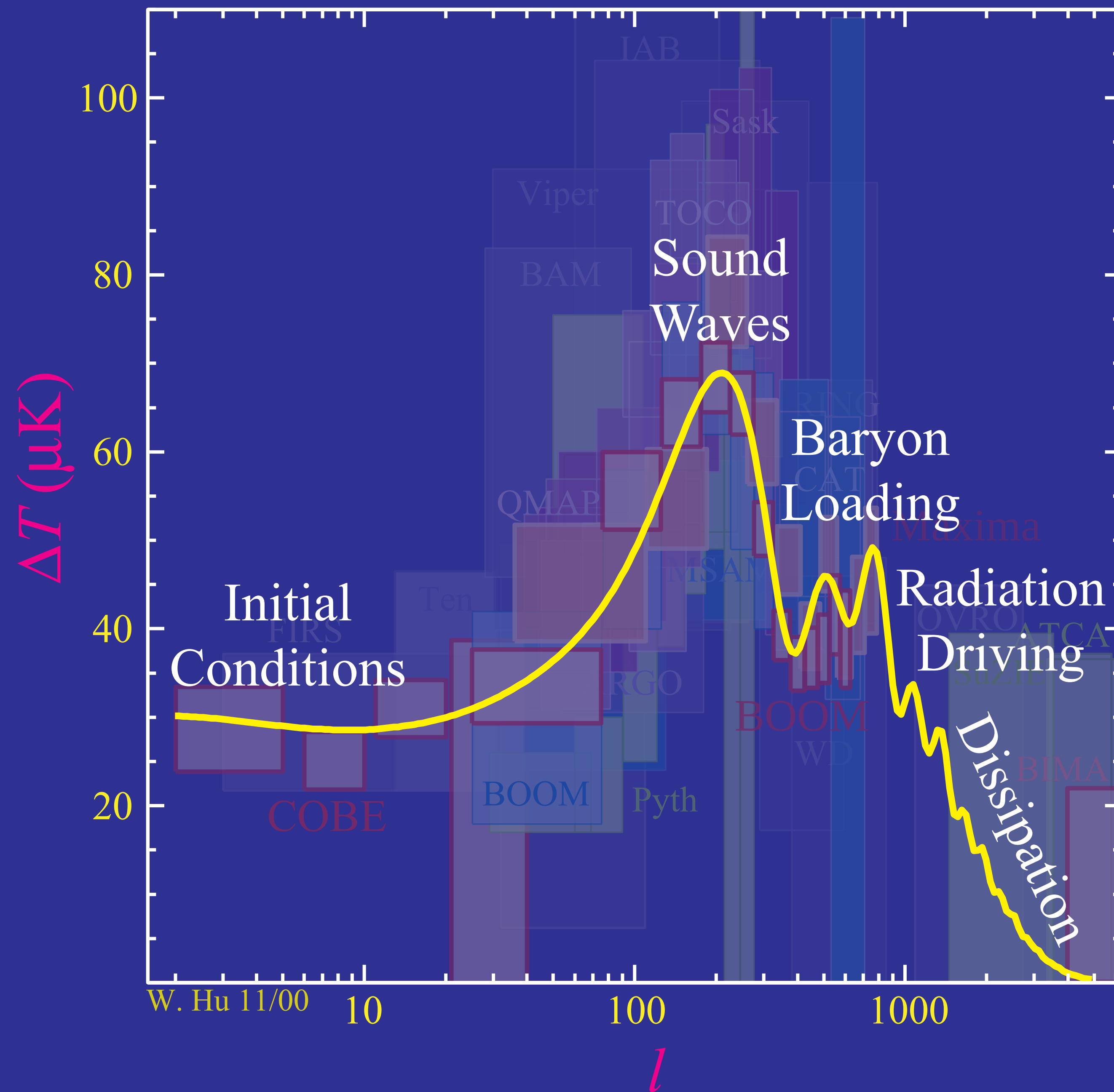
- Closely related to the variance per log interval in multipole space:

$$\begin{aligned}
\langle T(\hat{\mathbf{n}})T(\hat{\mathbf{n}})\rangle &= \sum_{\ell m}\sum_{\ell' m'}\langle T^*_{\ell m}T_{\ell' m'}\rangle Y^*_{\ell m}(\hat{\mathbf{n}})Y_{\ell' m'}(\hat{\mathbf{n}}) \\
&= \sum_{\ell} C_\ell \sum_{m} Y^*_{\ell m}(\hat{\mathbf{n}})Y_{\ell' m'}(\hat{\mathbf{n}}) \\
&= \sum_{\ell} \frac{2\ell+1}{4\pi} C_\ell \\
&\approx \int d\ln\ell\, \frac{\ell(2\ell+1)}{4\pi} C_\ell
\end{aligned}$$

- In particular: scale invariant in physical space becomes scale invariant in multipole space at $\ell \gg 1$

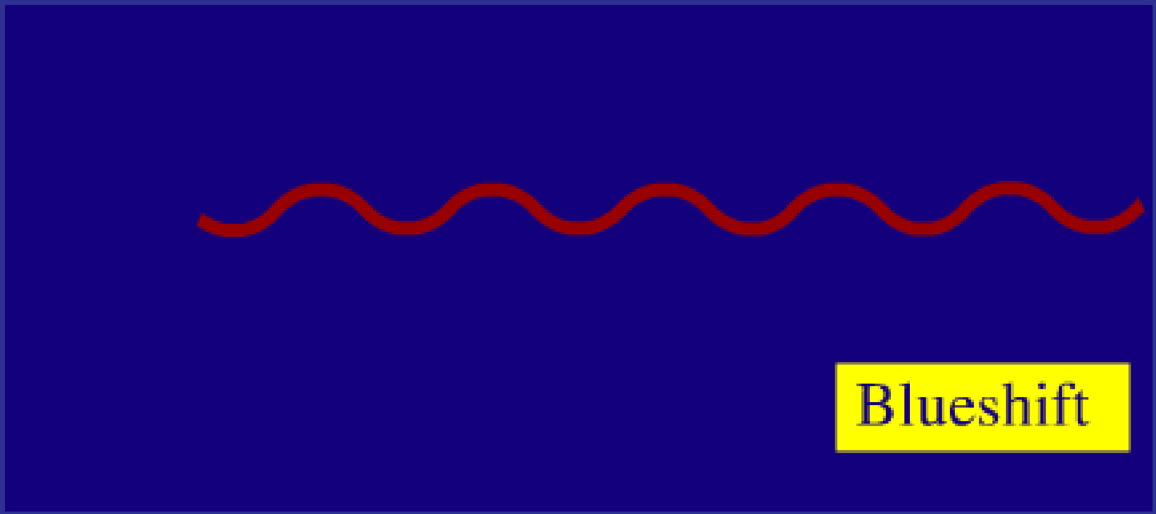
Angular Peaks

Physical Landscape



Seeing Sound

- Colliding electrons, protons and photons forms a plasma
- Acts as gas just like molecules in the air
- Compressional disturbance propagates in the gas through particle collisions



- Unlike sound in the air, we see the sound in the CMB
- Compression heats the gas resulting in a hot spot in the CMB

Thomson Scattering

- Thomson scattering of photons off of free electrons is the most important CMB process with a cross section (averaged over polarization states) of

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} = 6.65 \times 10^{-25}\mathrm{cm}^2$$

- Density of free electrons in a fully ionized $x_e = 1$ universe

$$n_e = (1 - Y_p/2)x_e n_b \approx 10^{-5}\Omega_b h^2 (1+z)^3 \mathrm{cm}^{-3}\,,$$

where $Y_p \approx 0.24$ is the Helium mass fraction, creates a high (comoving) Thomson opacity

$$\dot\tau \equiv n_e \sigma_T a$$

where dots are conformal time $\eta \equiv \int dt/a$ derivatives and τ is the optical depth.

Tight Coupling Approximation

- Near recombination $z \approx 10^3$ and $\Omega_b h^2 \approx 0.02$, the (comoving) mean free path of a photon

$$\lambda_C \equiv \frac{1}{\dot{\tau}} \sim 2.5\text{Mpc}$$

  small by cosmological standards!

- On scales $\lambda \gg \lambda_C$ photons are tightly coupled to the electrons by Thomson scattering which in turn are tightly coupled to the baryons by Coulomb interactions
- Specifically, their bulk velocities are defined by a single fluid velocity $v_\gamma = v_b$ and the photons carry no anisotropy in the rest frame of the baryons
- $\rightarrow$ No heat conduction or viscosity (anisotropic stress) in fluid

Zeroth Order Approximation

- Momentum density of a fluid is $(\rho + p)v$, where p is the pressure
- Neglect the momentum density of the baryons

$$R \equiv \frac{(\rho_b + p_b)v_b}{(\rho_\gamma + p_\gamma)v_\gamma} = \frac{\rho_b + p_b}{\rho_\gamma + p_\gamma} = \frac{3\rho_b}{4\rho_\gamma}$$

$$\approx 0.6 \left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{a}{10^{-3}}\right)$$

since $\rho_\gamma \propto T^4$ is fixed by the CMB temperature $T = 2.73(1+z)$K – OK substantially before recombination

- Neglect radiation in the expansion

$$\frac{\rho_m}{\rho_r} = 3.6 \left(\frac{\Omega_m h^2}{0.15}\right)\left(\frac{a}{10^{-3}}\right)$$

Number Continuity

- Photons are not created or destroyed. Without expansion

$$\dot{n}_\gamma + \nabla \cdot (n_\gamma \mathbf{v}_\gamma) = 0$$

but the expansion or Hubble flow causes $n_\gamma \propto a^{-3}$ or

$$\dot{n}_\gamma + 3n_\gamma \frac{\dot{a}}{a} + \nabla \cdot (n_\gamma \mathbf{v}_\gamma) = 0$$

- Linearize $\delta n_\gamma = n_\gamma - \bar{n}_\gamma$

$$(\delta n_\gamma)^{\cdot} = -3\delta n_\gamma \frac{\dot{a}}{a} - n_\gamma \nabla \cdot \mathbf{v}_\gamma$$

$$\left(\frac{\delta n_\gamma}{n_\gamma}\right)^{\cdot} = -\nabla \cdot \mathbf{v}_\gamma$$

Continuity Equation

- Number density $n_\gamma \propto T^3$ so define temperature fluctuation Θ

$$\frac{\delta n_\gamma}{n_\gamma} = 3\frac{\delta T}{T} \equiv 3\Theta$$

- Real space continuity equation

$$\dot{\Theta} = -\frac{1}{3}\nabla \cdot \mathbf{v}_\gamma$$

- Fourier space

$$\dot{\Theta} = -\frac{1}{3}i\mathbf{k} \cdot \mathbf{v}_\gamma$$

Momentum Conservation

- No expansion: $\dot{\mathbf{q}} = \mathbf{F}$
- De Broglie wavelength stretches with the expansion

$$\dot{\mathbf{q}} + \frac{\dot{a}}{a}\mathbf{q} = \mathbf{F}$$

for photons this the redshift, for non-relativistic particles expansion drag on peculiar velocities

- Collection of particles: momentum $\rightarrow$ momentum density $(\rho_\gamma + p_\gamma)\mathbf{v}_\gamma$ and force $\rightarrow$ pressure gradient

$$\begin{aligned}
[(\rho_\gamma + p_\gamma)\mathbf{v}_\gamma]^{\cdot} &= -4\frac{\dot{a}}{a}(\rho_\gamma + p_\gamma)\mathbf{v}_\gamma - \nabla p_\gamma \\
\frac{4}{3}\rho_\gamma \dot{\mathbf{v}}_\gamma &= \frac{1}{3}\nabla\rho_\gamma \\
\dot{\mathbf{v}}_\gamma &= -\nabla\Theta
\end{aligned}$$

Euler Equation

- Fourier space

$$\dot{\mathbf{v}}_\gamma = -ik\Theta$$

- Pressure gradients (any gradient of a scalar field) generates a curl-free flow
- For convenience define velocity amplitude:

$$\mathbf{v}_\gamma \equiv -iv_\gamma\hat{\mathbf{k}}$$

- Euler Equation:

$$\dot{v}_\gamma = k\Theta$$

- Continuity Equation:

$$\dot{\Theta} = -\frac{1}{3}kv_\gamma$$

Oscillator: Take One

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

where the adiabatic sound speed is defined through

$$c_s^2 \equiv \frac{\dot{p}_\gamma}{\dot{\rho}_\gamma}$$

here $c_s^2 = 1/3$ since we are photon-dominated

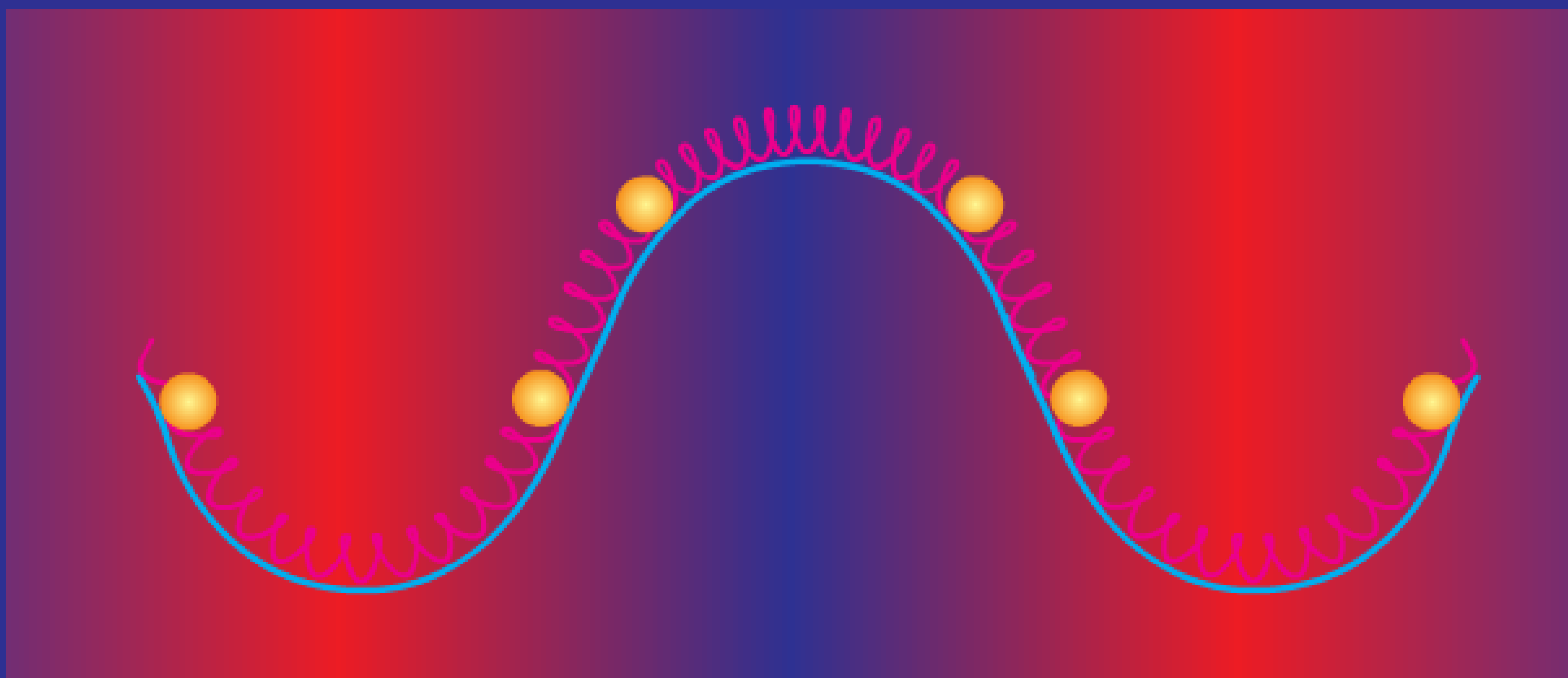
- General solution:

$$\Theta(\eta) = \Theta(0)\cos(ks) + \frac{\dot{\Theta}(0)}{kc_s}\sin(ks)$$

where the sound horizon is defined as $s \equiv \int c_s d\eta$

Seeing Sound

- Oscillations frozen at recombination
- Compression=hot spots, Rarefaction=cold spots



Harmonic Extrema

- All modes are frozen in at recombination (denoted with a subscript $*$) yielding temperature perturbations of different amplitude for different modes. For the adiabatic (curvature mode) $\dot\Theta(0) = 0$

$$\Theta(\eta_*) = \Theta(0)\cos(ks_*)$$

- Modes caught in the extrema of their oscillation will have enhanced fluctuations

$$k_n s_* = n\pi$$

yielding a fundamental scale or frequency, related to the inverse sound horizon

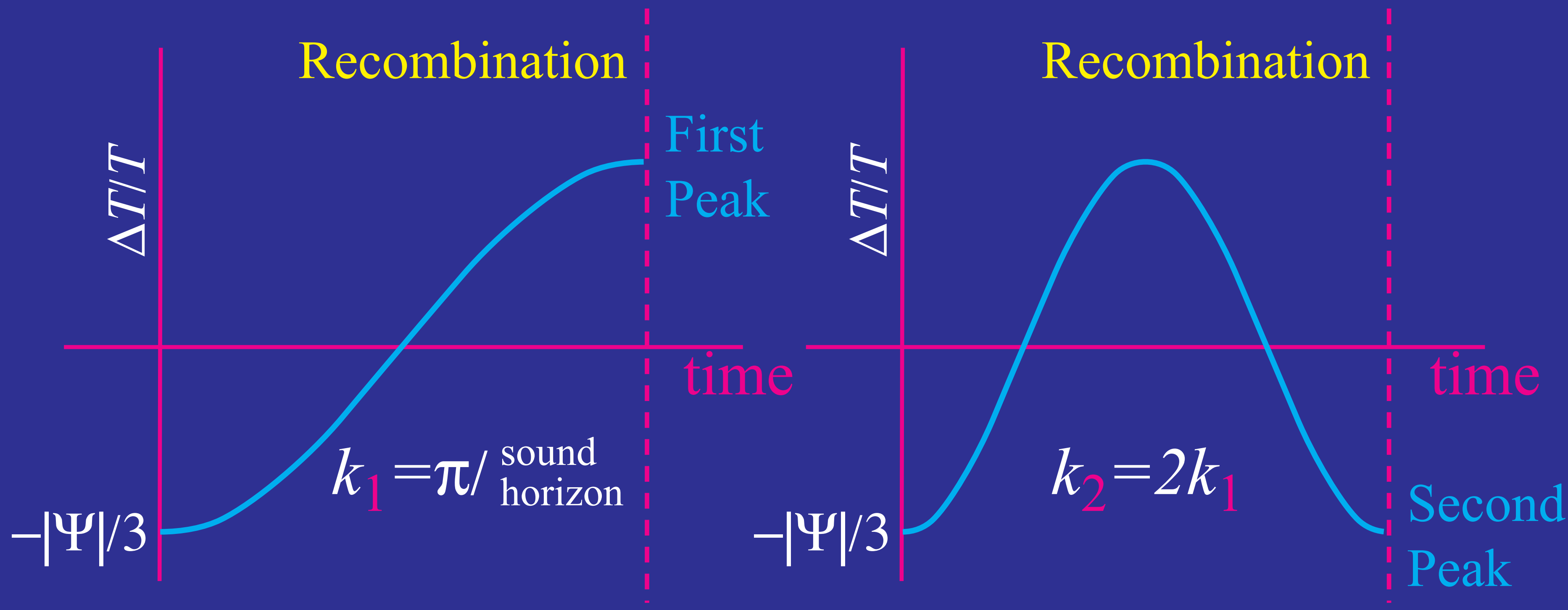
$$k_A = \pi/s_*$$

and a harmonic relationship to the other extrema as $1:2:3\ldots$

The First Peak

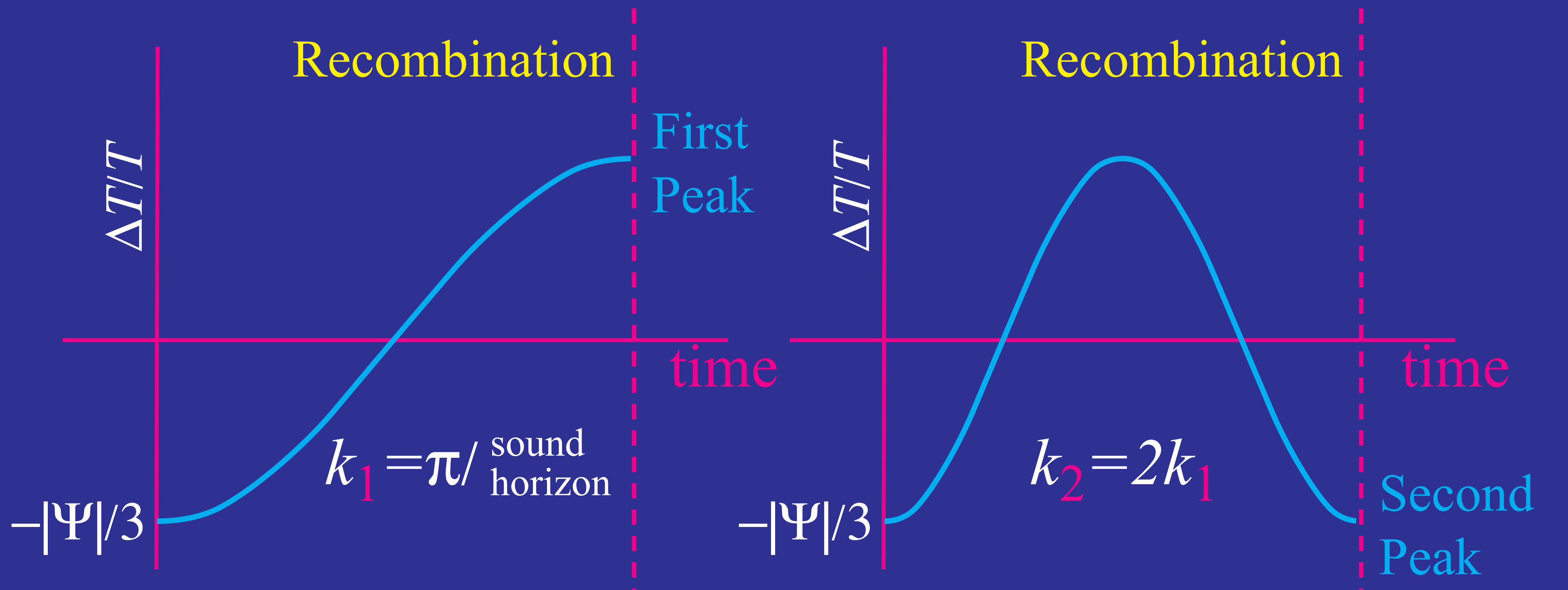
Extrema=Peaks

- First peak = mode that just compresses
- Second peak = mode that compresses then rarefies: twice the wavenumber



Extrema=Peaks

- First peak = mode that just compresses
- Second peak = mode that compresses then rarefies: twice the wavenumber
- Harmonic peaks: 1:2:3 in wavenumber



Peak Location

- The fundmental physical scale is translated into a fundamental angular scale by simple projection according to the angular diameter distance D_A

$$
\begin{aligned}
\theta_A &= \lambda_A / D_A \\
\ell_A &= k_A D_A
\end{aligned}
$$

- In a flat universe, the distance is simply $D_A = D \equiv \eta_0 - \eta_* \approx \eta_0$, the horizon distance, and $k_A = \pi / s_* = \sqrt{3}\pi/\eta_*$ so

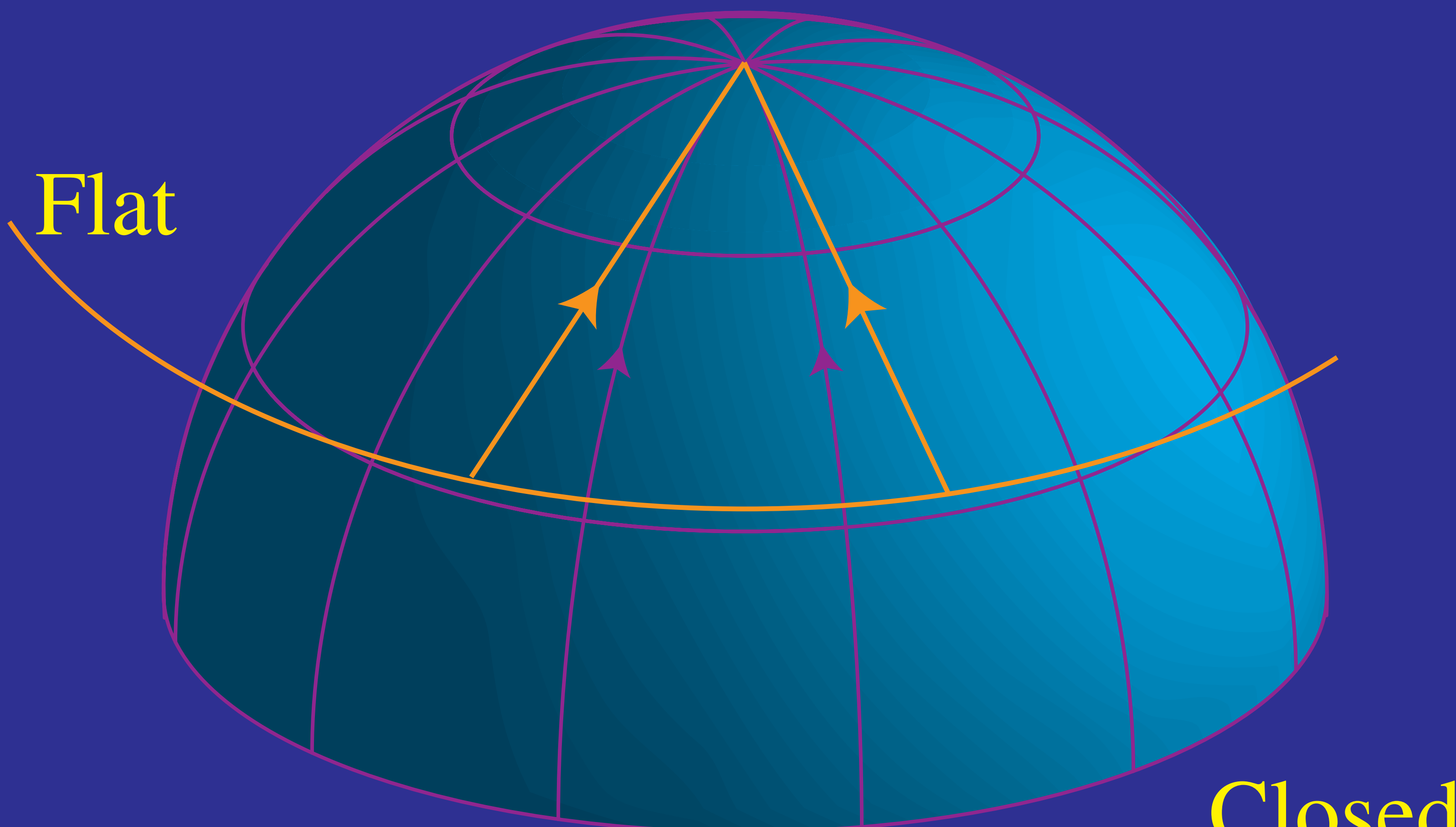
$$\theta_A \approx \frac{\eta_*}{\eta_0}$$

- In a matter-dominated universe $\eta \propto a^{1/2}$ so $\theta_A \approx 1/30 \approx 2^\circ$ or

$$\ell_A \approx 200$$

Spatial Curvature

- Physical scale of peak = distance sound travels
- Angular scale measured: comoving angular diameter distance test for curvature

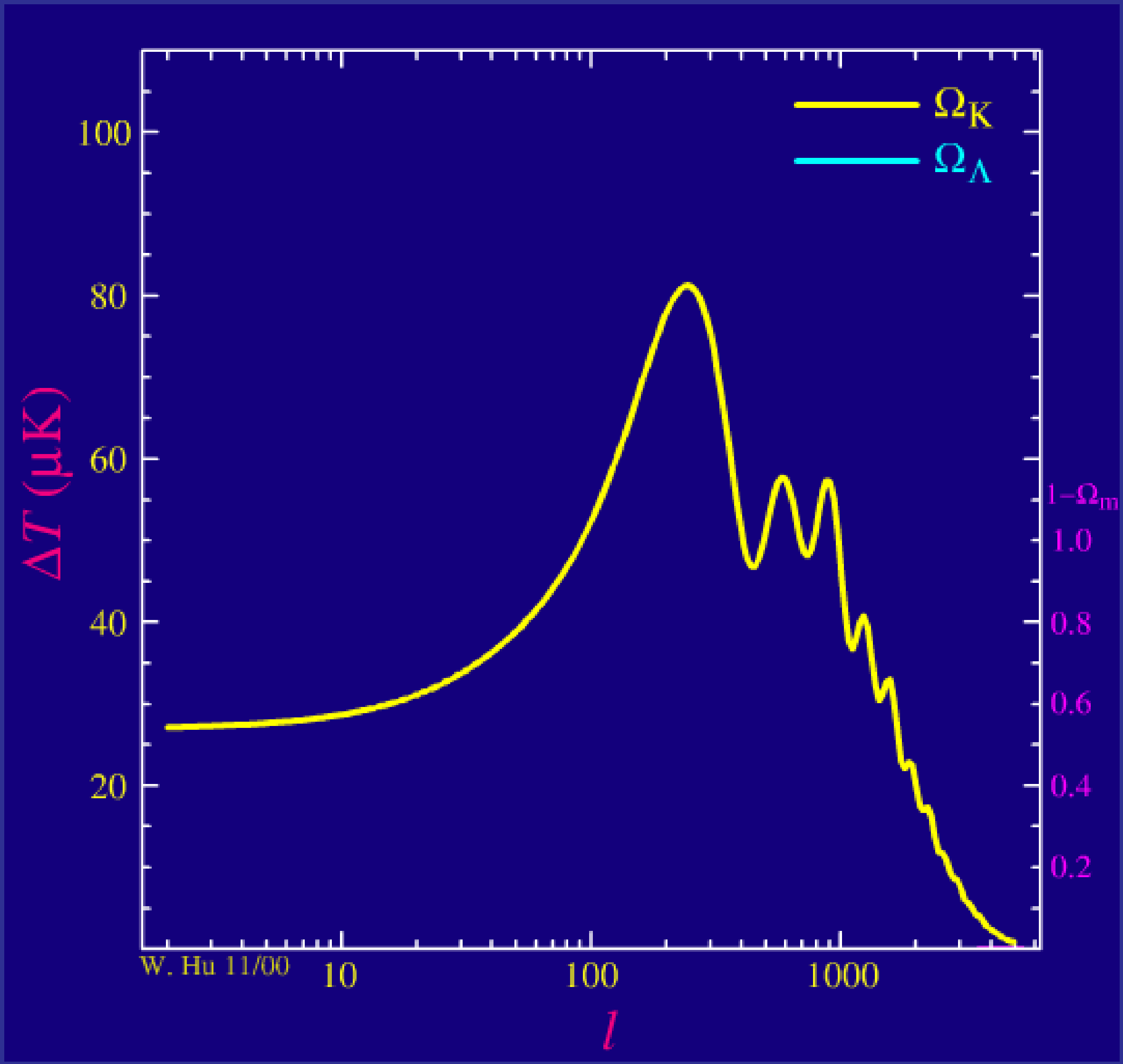


Curvature

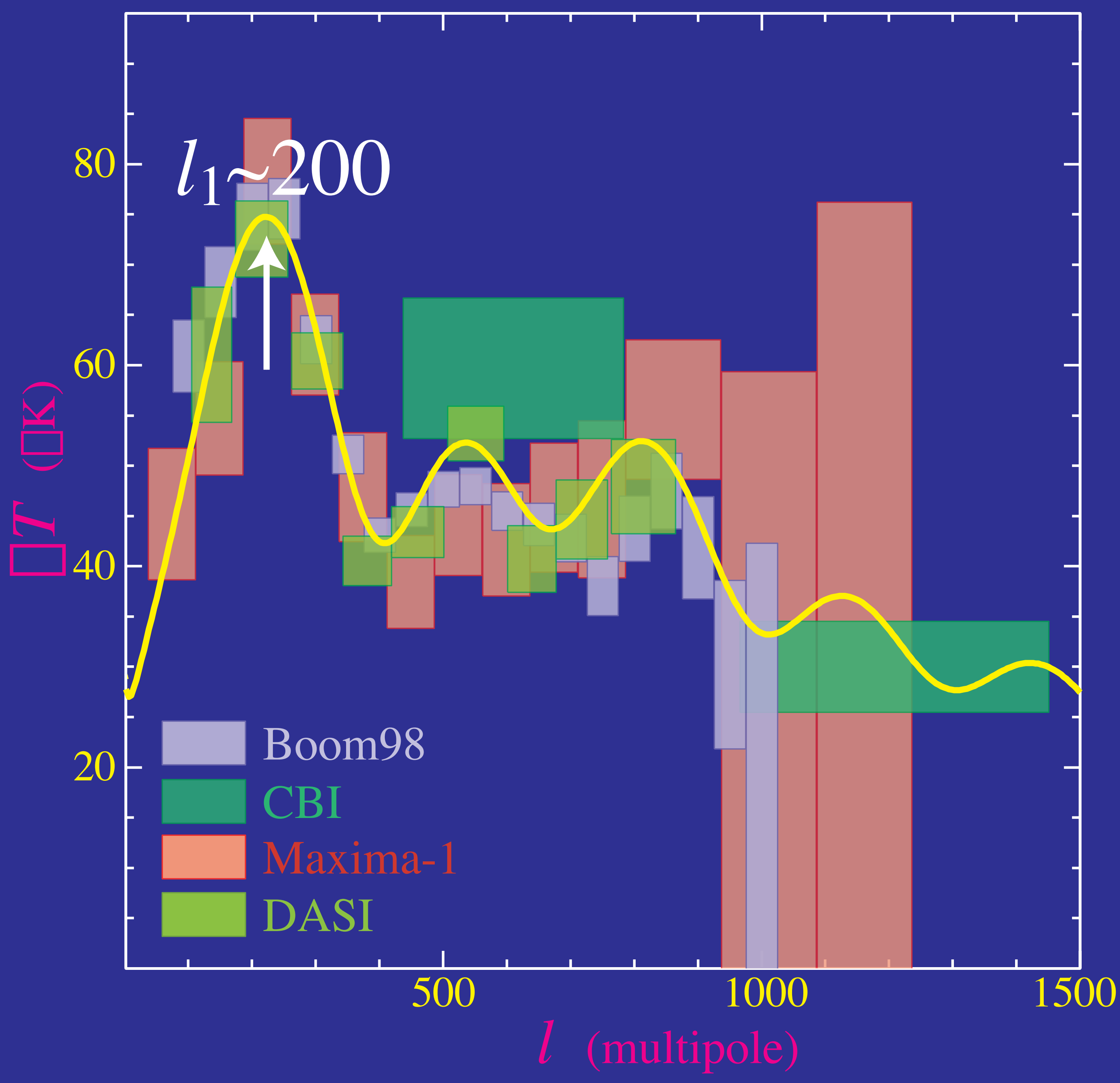
- In a curved universe, the apparent or angular diameter distance is no longer the conformal distance $D_A = R\sin(D/R) \neq D$
- Objects in a closed universe are further than they appear! gravitational lensing of the background...
- Curvature scale of the universe must be substantially larger than current horizon
- Flat universe indicates critical density and implies missing energy given local measures of the matter density “dark energy”
- D also depends on dark energy density $\Omega_{\rm DE}$ and equation of state $w = p_{\rm DE}/\rho_{\rm DE}$.
- Expansion rate at recombination or matter-radiation ratio enters into calculation of k_A.

Curvature in the Power Spectrum

- Features scale with angular diameter distance
- Angular location of the first peak



First Peak Precisely Measured



Doppler Effect

- Bulk motion of fluid changes the observed temperature via Doppler shifts

$$\left(\frac{\Delta T}{T}\right)_{\rm dop} = \hat{\mathbf{n}} \cdot \mathbf{v}_\gamma$$

- Averaged over directions

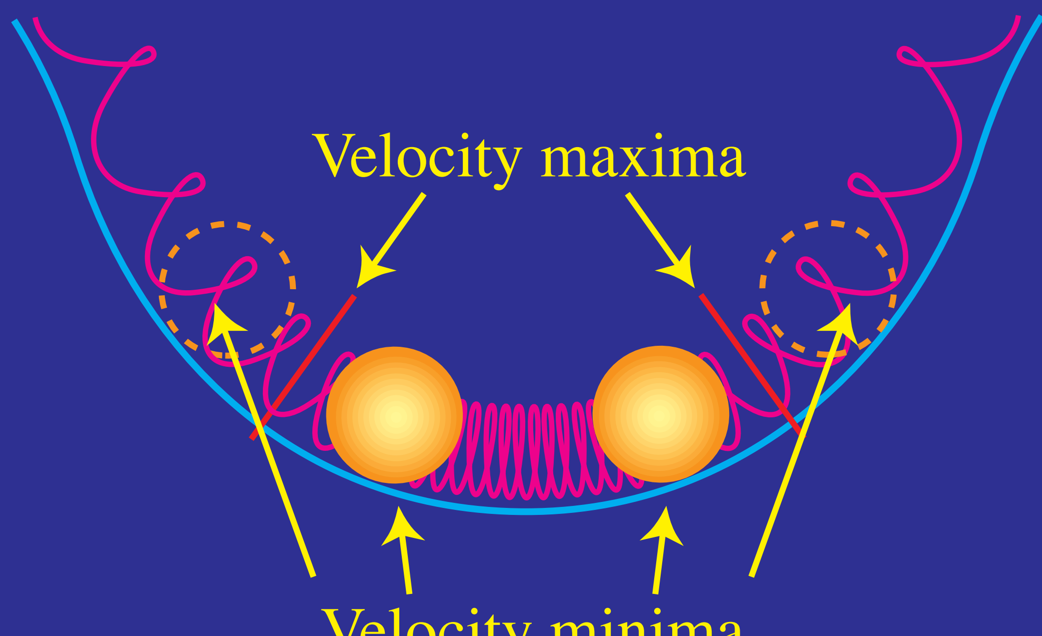
$$\left(\frac{\Delta T}{T}\right)_{\rm rms} = \frac{v_\gamma}{\sqrt{3}}$$

- Acoustic solution

$$\begin{aligned}\frac{v_\gamma}{\sqrt{3}} &= -\frac{\sqrt{3}}{k}\dot{\Theta} = \frac{\sqrt{3}}{k} k c_s\, \Theta(0) \sin(ks) \\ &= \Theta(0)\sin(ks)\end{aligned}$$

Doppler Effect

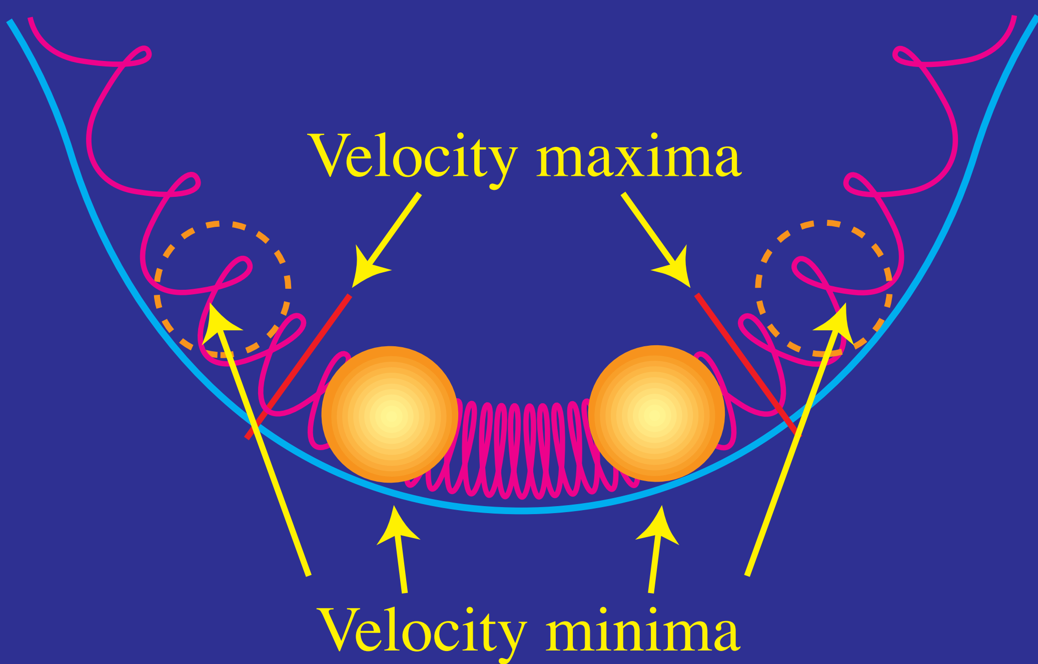
- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature

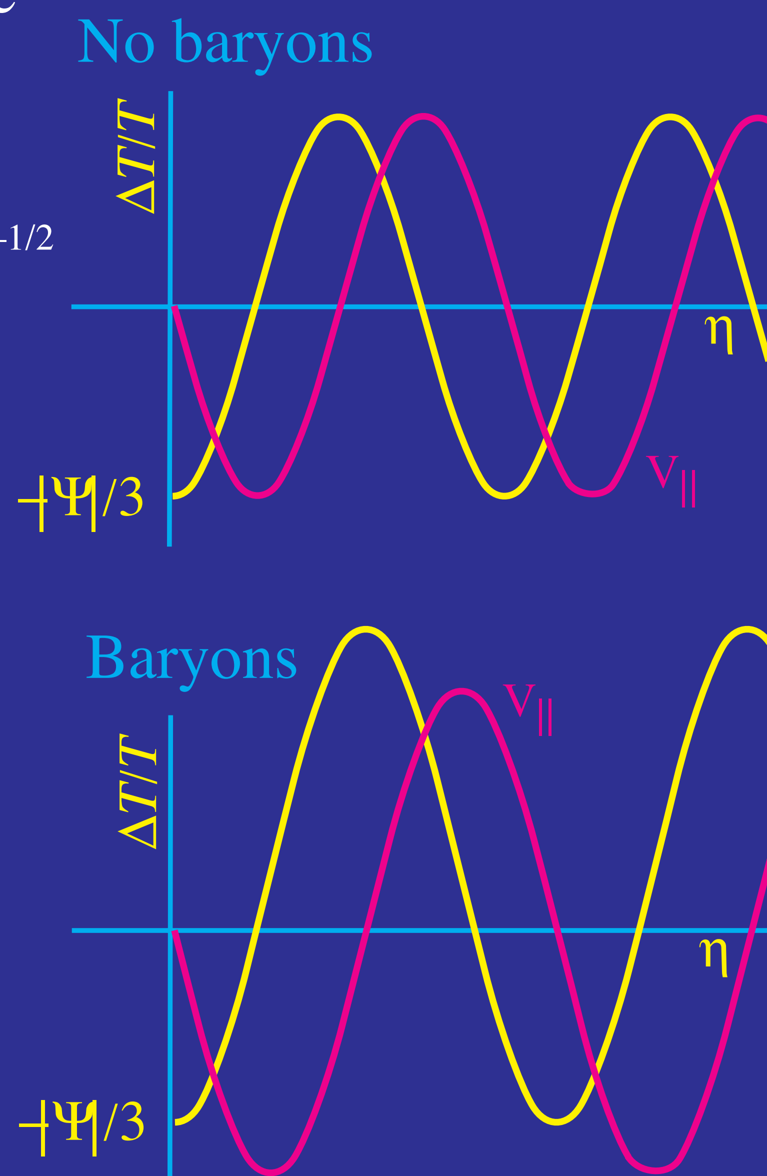


Doppler Effect

- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature
- Zero point not shifted by baryon drag
- Increased baryon inertia decreases effect

$$m_{\rm eff} V^2 = \text{const.} \qquad V \propto m_{\rm eff}^{-1/2} = (1+R)^{-1/2}$$





Doppler Peaks?

- Doppler effect for the photon dominated system is of equal amplitude and $\pi/2$ out of phase: extrema of temperature are turning points of velocity
- Effects add in quadrature:

$$\left(\frac{\Delta T}{T}\right)^2 = \Theta^2(0)[\cos^2(ks) + \sin^2(ks)] = \Theta^2(0)$$

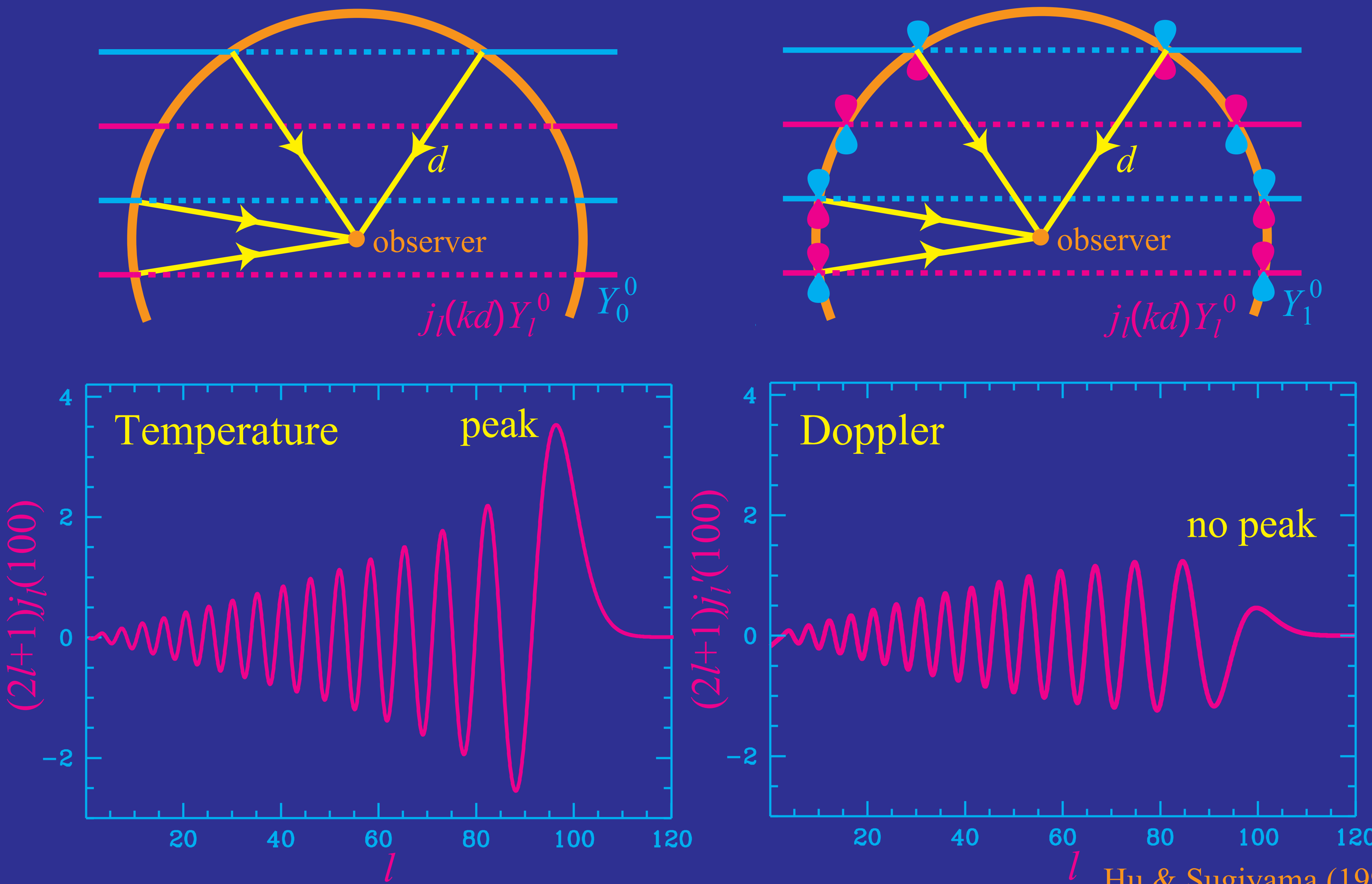
- No peaks in k spectrum! However the Doppler effect carries an angular dependence that changes its projection on the sky $\hat{\mathbf{n}} \cdot \mathbf{v}_\gamma \propto \hat{\mathbf{n}} \cdot \hat{\mathbf{k}}$
- Coordinates where $\hat{\mathbf{z}} \parallel \hat{\mathbf{k}}$

$$Y_{10} Y_{\ell 0} \rightarrow Y_{\ell \pm 1\, 0}$$

recoupling $j_\ell' Y_{\ell 0}$: no peaks in Doppler effect

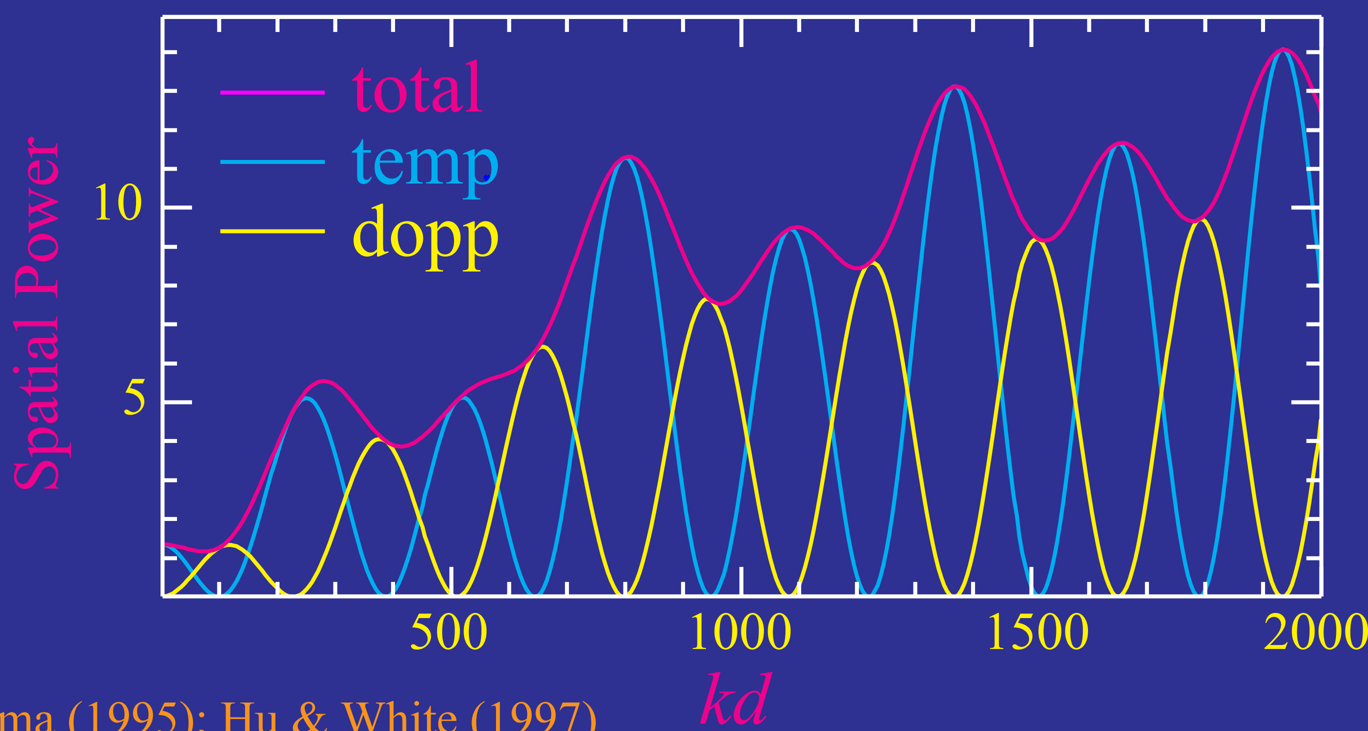
Doppler Peaks?

- Doppler effect has lower amplitude and weak features from projection



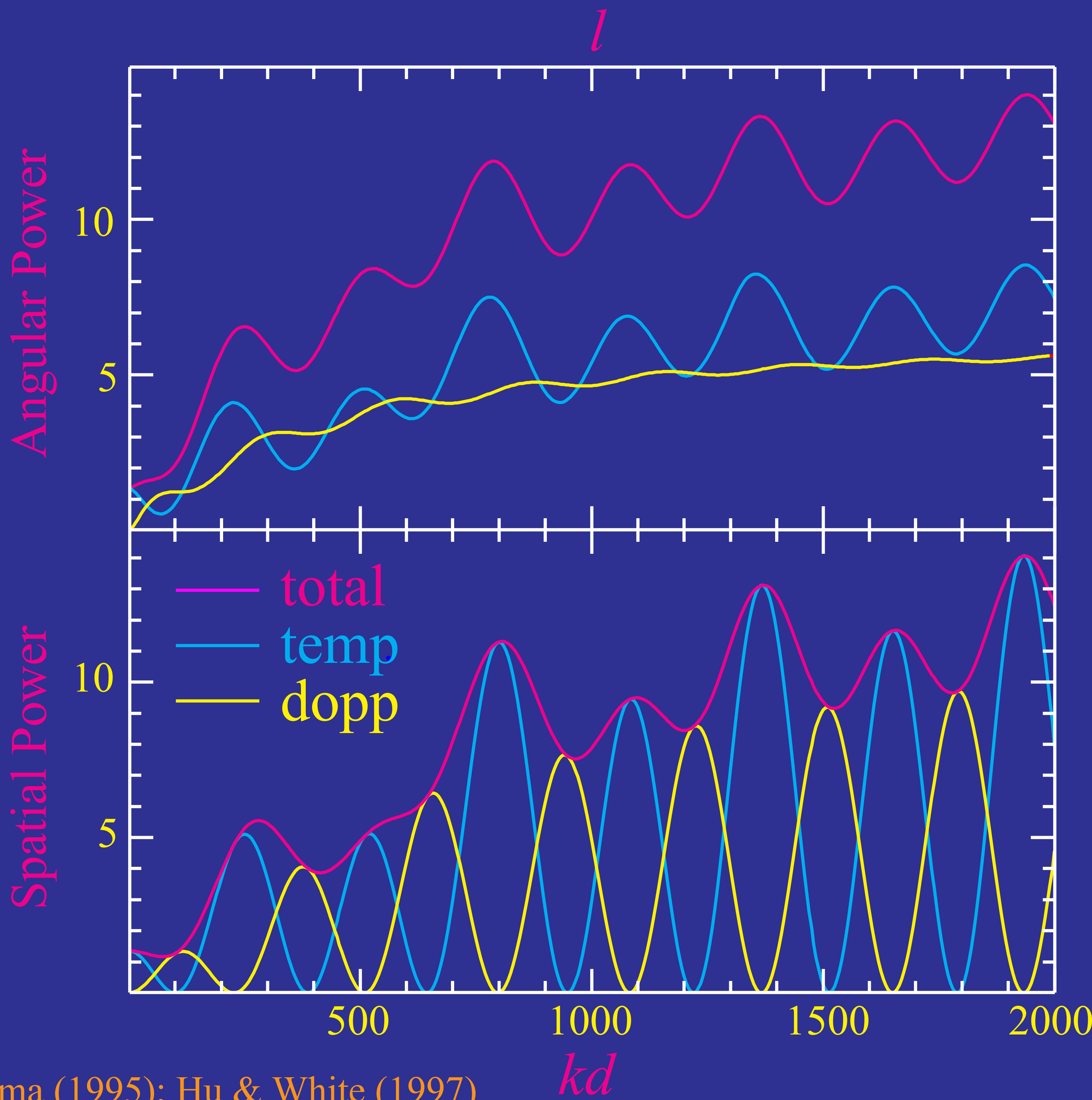
Hu & Sugiyama (1995)

Relative Contributions



Hu & Sugiyama (1995); Hu & White (1997)

Relative Contributions



Hu & Sugiyama (1995); Hu & White (1997)

Lecture I: Summary

- CMB photons emerge from the cosmic photosphere at $z \sim 10^3$ when the universe (re)combines
- Temperature inhomogeneity at recombination becomes anisotropy to the observer at present
- Initial temperature inhomogeneities oscillate as sound waves in the plasma
- Harmonic series of peaks based on the distance sound travels by recombination
- Distance can be calibrated if expansion history is known and baryon content known
- Angular scale measures the angular diameter distance to recombination involving the curvature and to a lesser extent the dark energy