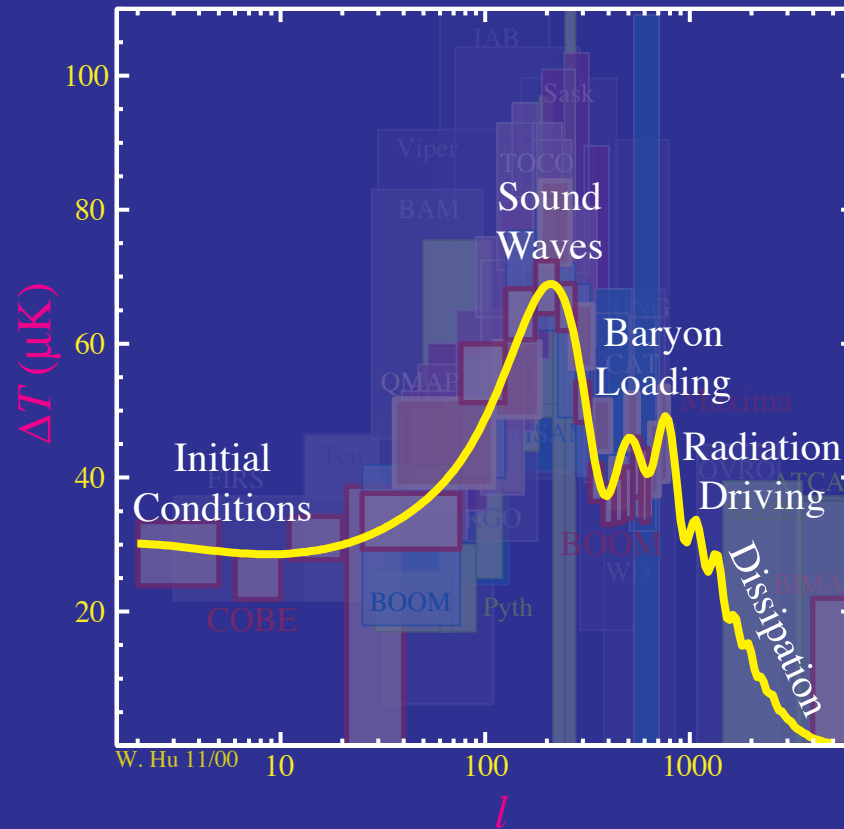


# Lecture II

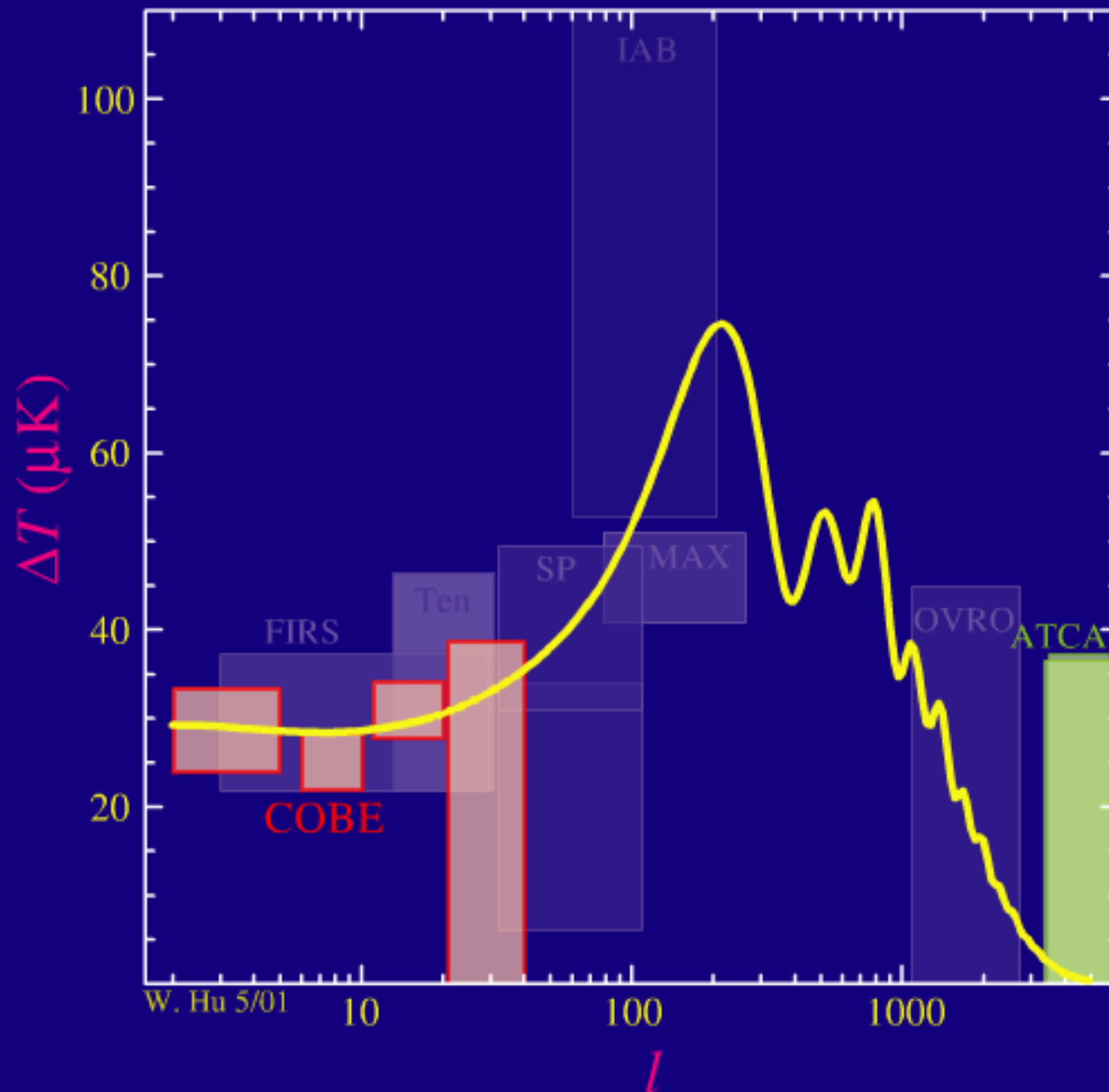


## Acoustic Dynamics

*Wayne Hu*

Tenerife, November 2007

# Theorist's Time-Ordered Data



# Restoring Gravity: Continuity

- Take a simple **photon dominated** system **with gravity**
- **Continuity** altered since a gravitational potential represents a **stretching** of the **spatial fabric** that dilutes number densities – formally a spatial **curvature perturbation**
- Think of this as a perturbation to the **scale factor**  $a \rightarrow a(1 + \Phi)$  so that the cosmological redshift is generalized to

$$\frac{\dot{a}}{a} \rightarrow \frac{\dot{a}}{a} + \dot{\Phi}$$

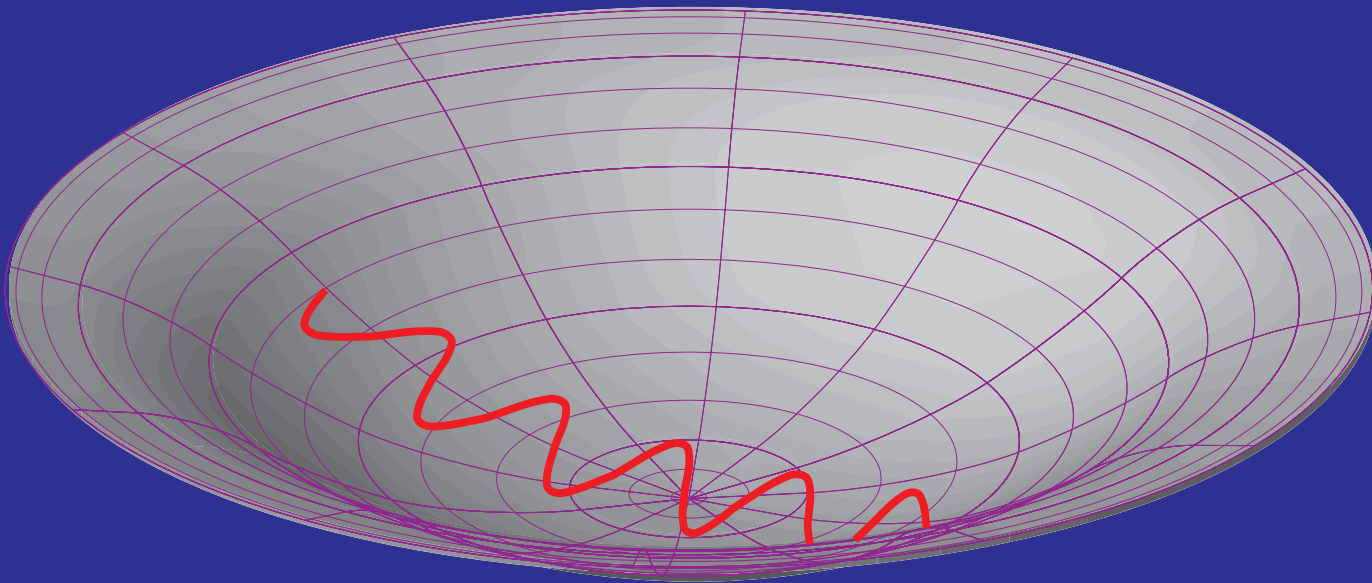
$$(\delta n_\gamma)' = -3\delta n_\gamma \frac{\dot{a}}{a} - 3n_\gamma \dot{\Phi} - n_\gamma \nabla \cdot \mathbf{v}_\gamma$$

so that the **continuity equation** becomes

$$\dot{\Theta} = -\frac{1}{3}k v_\gamma - \dot{\Phi}$$

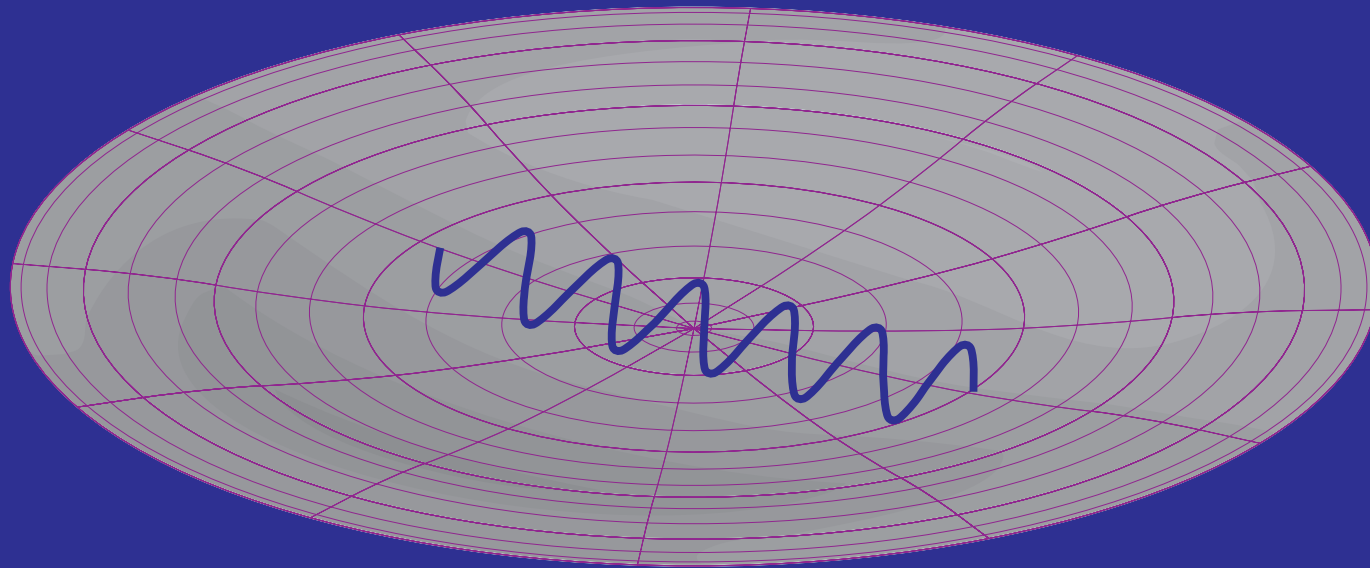
# Metric Stretch

- Potential wells curve or stretch space
- Like the expansion of the universe, changes in the potential change the wavelength of photons



# Metric Stretch

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# Restoring Gravity: Euler

- Gravitational force in momentum conservation  $\mathbf{F} = -m\nabla\Psi$  generalized to momentum density modifies the Euler equation to

$$\dot{v}_\gamma = k(\Theta + \Psi)$$

- General relativity says that  $\Phi$  and  $\Psi$  are the relativistic analogues of the Newtonian potential and that  $\Phi \approx -\Psi$ .
- In our matter-dominated approximation,  $\Phi$  represents matter density fluctuations through the cosmological Poisson equation

$$k^2\Phi = 4\pi G a^2 \rho_m \Delta_m$$

where the difference comes from the use of comoving coordinates for  $k$  ( $a^2$  factor), the removal of the background density into the background expansion ( $\rho_m \Delta_m$ ) and finally a coordinate subtlety that enters into the definition of  $\Delta_m$

# Constant Potentials

- In the matter dominated epoch potentials are constant because infall generates velocities as  $v_m \sim k\eta\Psi$
- Velocity divergence generates density perturbations as  $\Delta_m \sim -k\eta v_m \sim -(k\eta)^2\Psi$
- And density perturbations generate potential fluctuations as  $\Phi \sim \Delta_m/(k\eta)^2 \sim -\Psi$ , keeping them constant. Note that because of the expansion, density perturbations must grow to keep potentials constant.
- Here we have used the Friedman equation  $H^2 = 8\pi G\rho_m/3$  and  $\eta = \int d\ln a/(aH) \sim 1/(aH)$
- More generally, if stress perturbations are negligible compared with density perturbations (  $\delta p \ll \delta\rho$  ) then potential will remain roughly constant – more specifically a variant called the Bardeen or comoving curvature  $\zeta$  is constant

# Oscillator: Take Two

- Combine these to form the **simple harmonic oscillator** equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = -\frac{k^2}{3} \Psi - \ddot{\Phi}$$

- In a **CDM dominated** expansion  $\dot{\Phi} = \dot{\Psi} = 0$ . Also for **photon domination**  $c_s^2 = 1/3$  so the oscillator equation becomes

$$\ddot{\Theta} + \ddot{\Psi} + c_s^2 k^2 (\Theta + \Psi) = 0$$

- Solution is just an **offset version** of the original

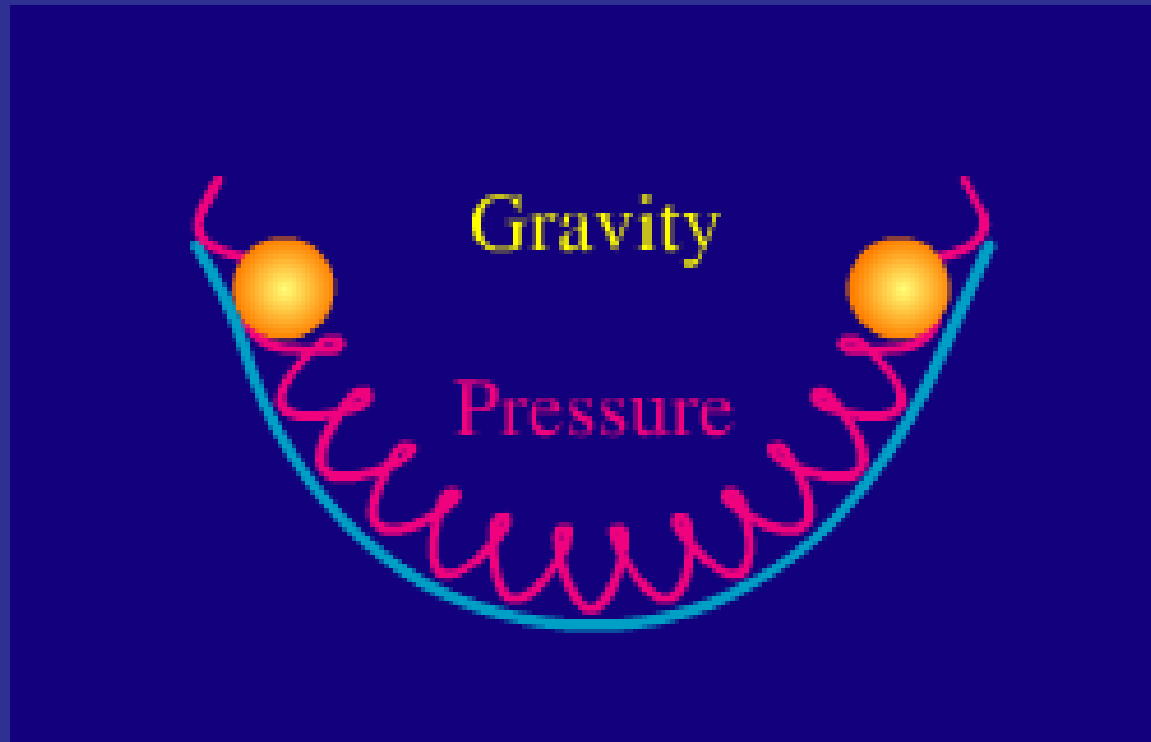
$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0) \cos(ks)$$

- $\Theta + \Psi$  is also the **observed temperature fluctuation** since photons lose energy climbing out of **gravitational potentials** at recombination



# Gravitational Ringing

- Potential wells = inflationary seeds of structure
- Fluid falls into wells, pressure resists: acoustic oscillations



# Effective Temperature

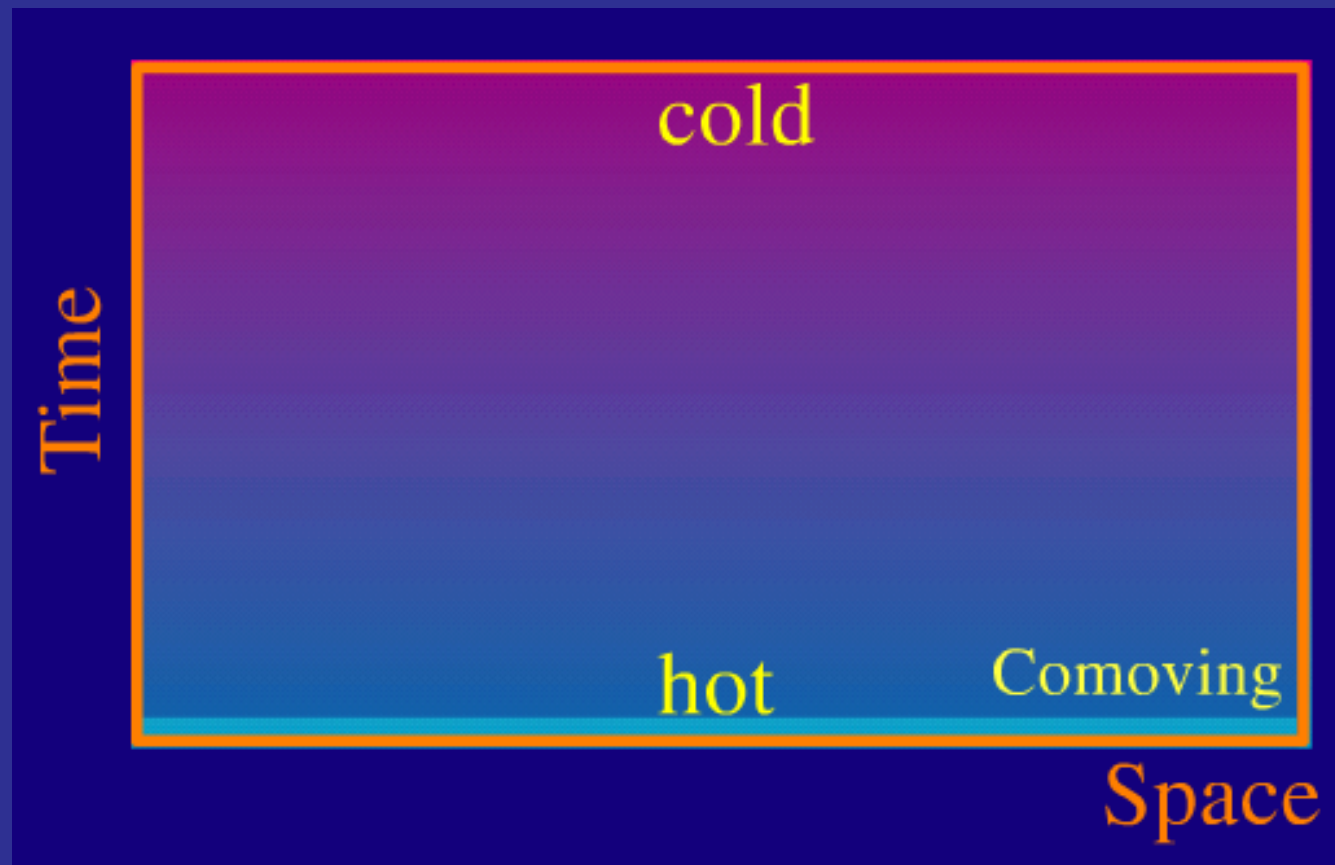
- Photons climb out of potential wells at last scattering
- Lose energy to gravitational redshifts
- Observed or **effective temperature**

$$\Theta + \Psi$$

- Effective temperature oscillates around **zero** with amplitude given by the **initial conditions**
- Note: initial conditions are set when the perturbation is **outside of horizon**, need inflation or other modification to matter-radiation FRW universe.
- GR says that **initial temperature** is given by **initial potential**

# Inflation and the Initial Conditions

- Inflation: (nearly) scale-invariant curvature (potential) perturbations
- Superluminal expansion  $\rightarrow$  superhorizon scales  $\rightarrow$  "initial conditions"
- Accompanying temperature perturbations due to cosmological redshift



- Potential perturbation  $\Psi$  = time-time metric perturbation  
 $\delta t/t = \Psi \quad \rightarrow \quad \delta T/T = -\delta a/a = -2/3 \delta t/t = -2/3 \Psi$

# Sachs-Wolfe Effect and the Magic 1/3

- A **gravitational potential** is a perturbation to the temporal coordinate [formally a **gauge transformation**]

$$\frac{\delta t}{t} = \Psi$$

- Convert this to a perturbation in the **scale factor**,

$$t = \int \frac{da}{aH} \propto \int \frac{da}{a\rho^{1/2}} \propto a^{3(1+w)/2}$$

where  $w \equiv p/\rho$  so that during **matter domination**

$$\frac{\delta a}{a} = \frac{2}{3} \frac{\delta t}{t}$$

- CMB temperature is **cooling** as  $T \propto a^{-1}$  so

$$\Theta + \Psi \equiv \frac{\delta T}{T} + \Psi = -\frac{\delta a}{a} + \Psi = \frac{1}{3}\Psi$$

# Sachs-Wolfe Effect and the Magic 1/3

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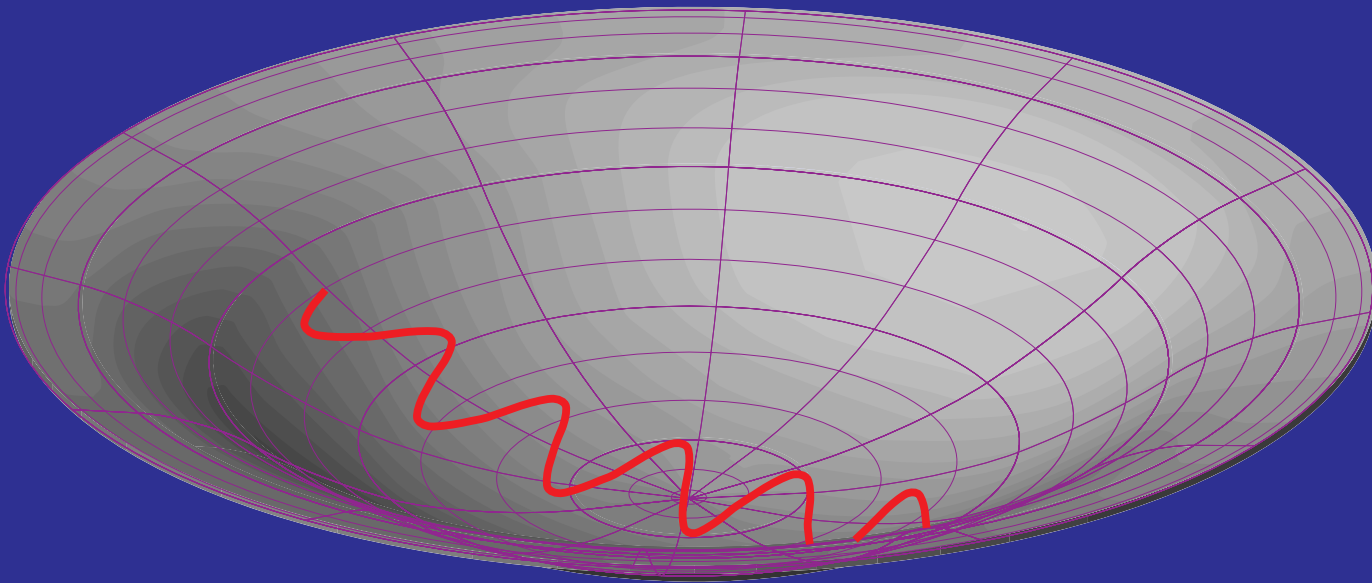
$$\Theta + \Psi \equiv \frac{\delta T}{T} + \Psi = -\frac{\delta a}{a} + \Psi = \frac{1}{3}\Psi$$

# Smooth Energy Density & Potential Decay

- A smooth component contributes density  $\rho$  to the expansion but not density fluctuation  $\delta\rho$  to the Poisson equation
- Imbalance causes potential to decay once smooth component dominates the expansion

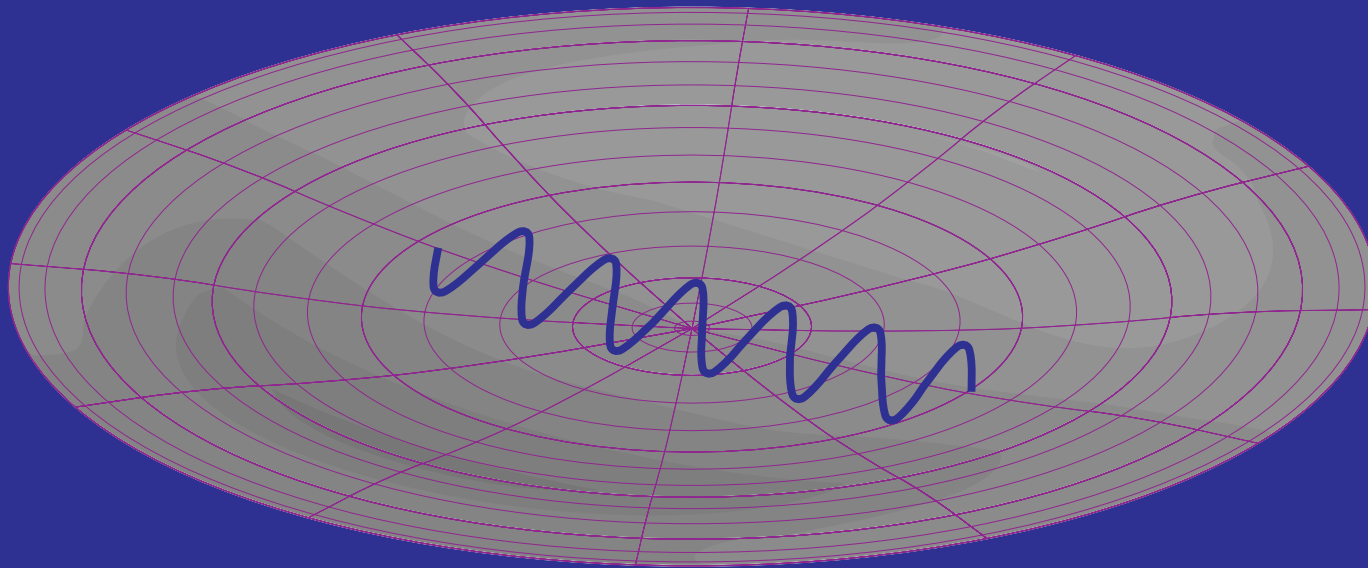
# ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect:  $\Delta T/T = 2\Delta\Phi$
- Effect from potential hills and wells cancel on small scales



# ISW Effect

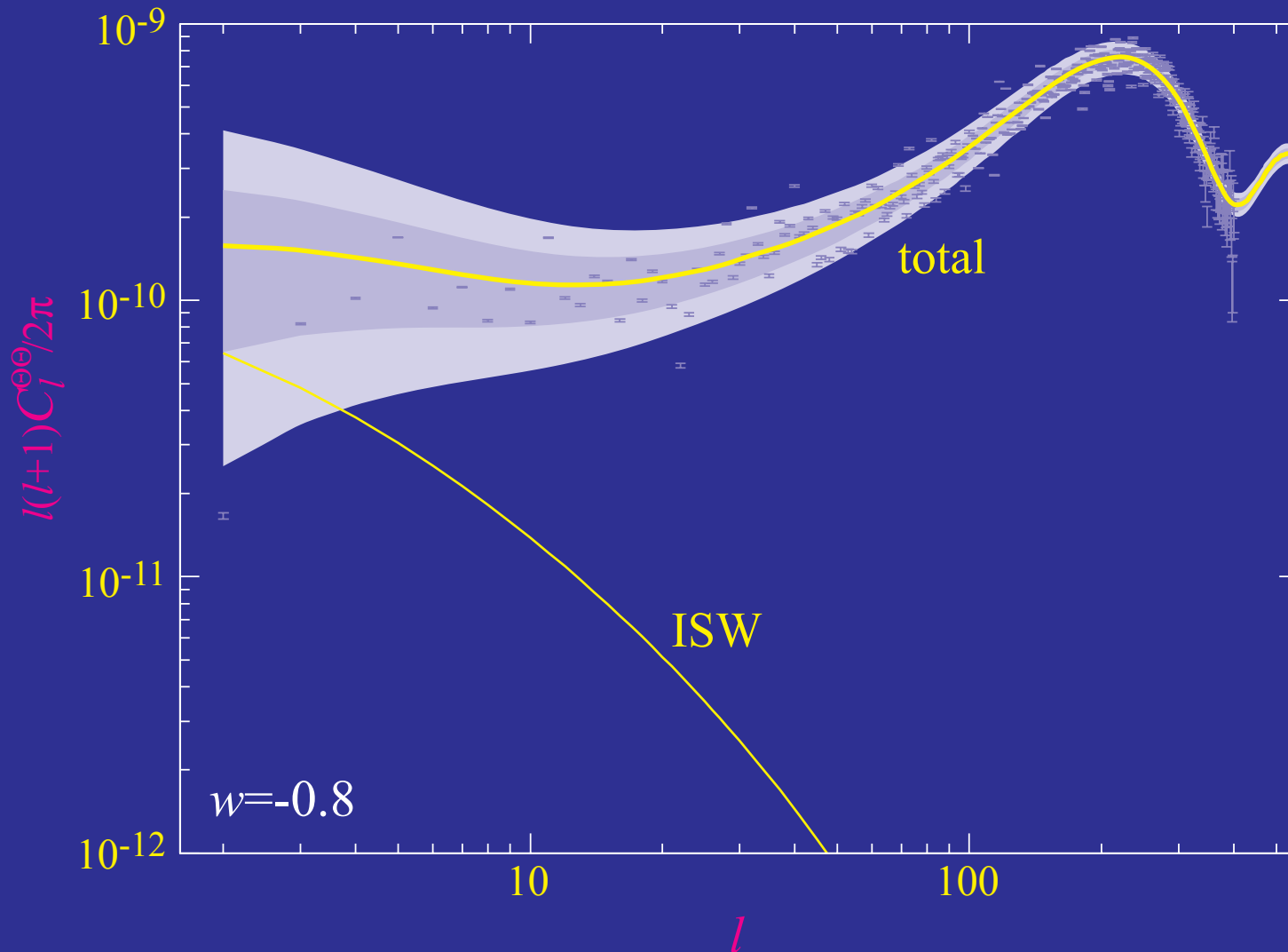
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# ISW Effect

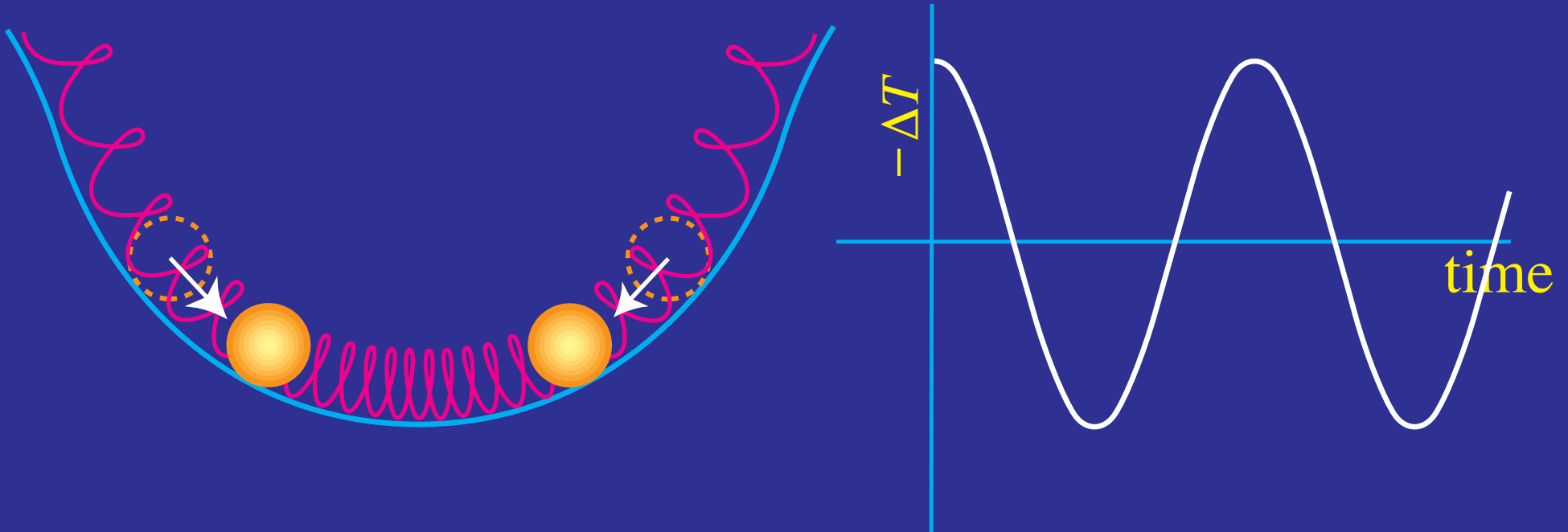
- ISW effect **hidden** in the temperature power spectrum by **primary anisotropy** and **cosmic variance**



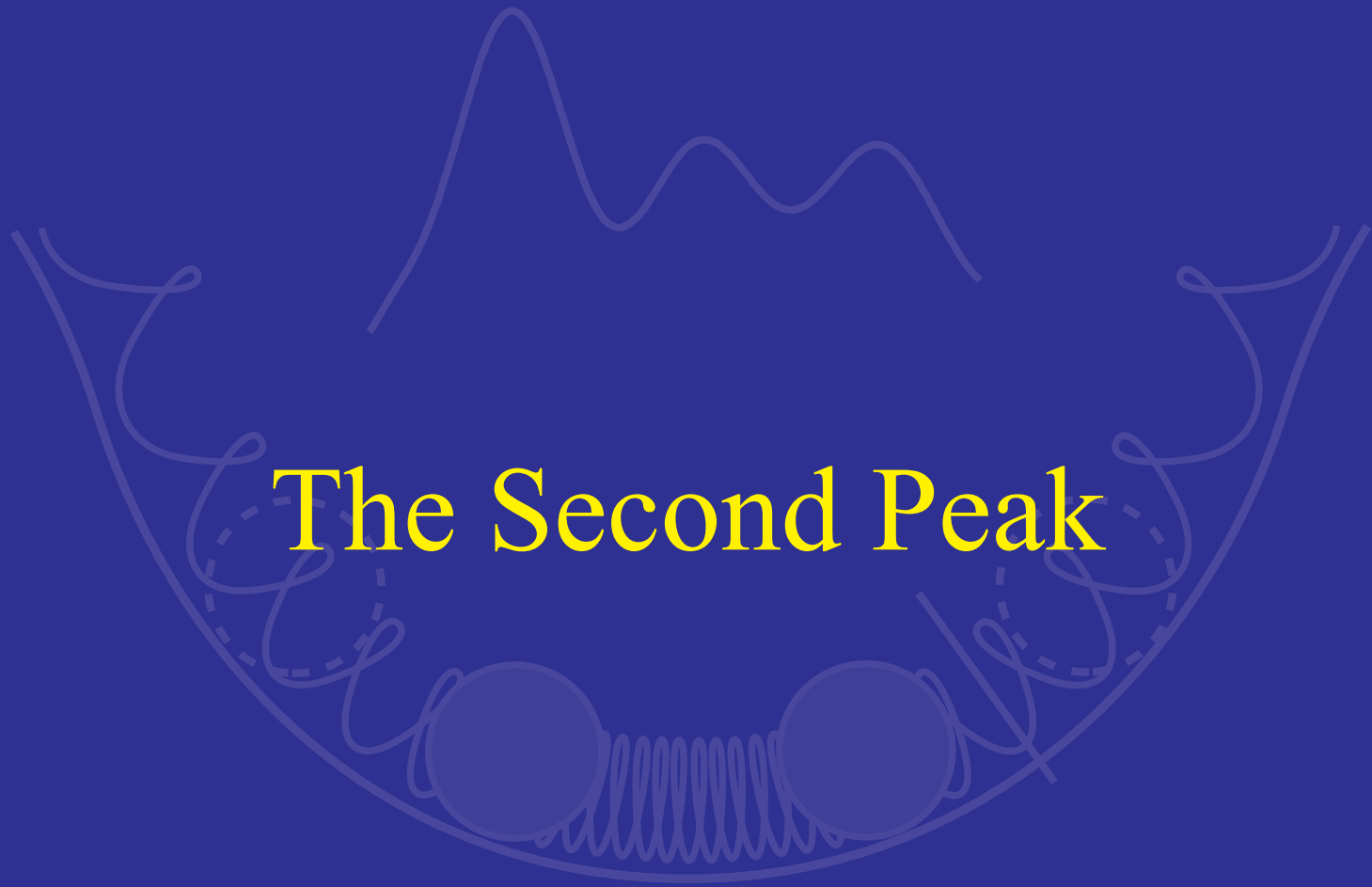
[plot: Hu & Scranton (2004)]

# Effective Temperature

- Effective temperature initially  $\Theta + \Psi = \Psi/3$  and is negative in an overdensity
- Effective temperature oscillates around zero
- Effective temperature becomes observed temperature after gravitational redshift



# The Second Peak



# Baryon Loading

- Baryons add extra **mass** to the photon-baryon fluid
- Controlling parameter is the **momentum density ratio**:

$$R \equiv \frac{p_b + \rho_b}{p_\gamma + \rho_\gamma} \approx 30\Omega_b h^2 \left( \frac{a}{10^{-3}} \right)$$

of order **unity** at recombination

- Momentum density of the **joint system** is conserved

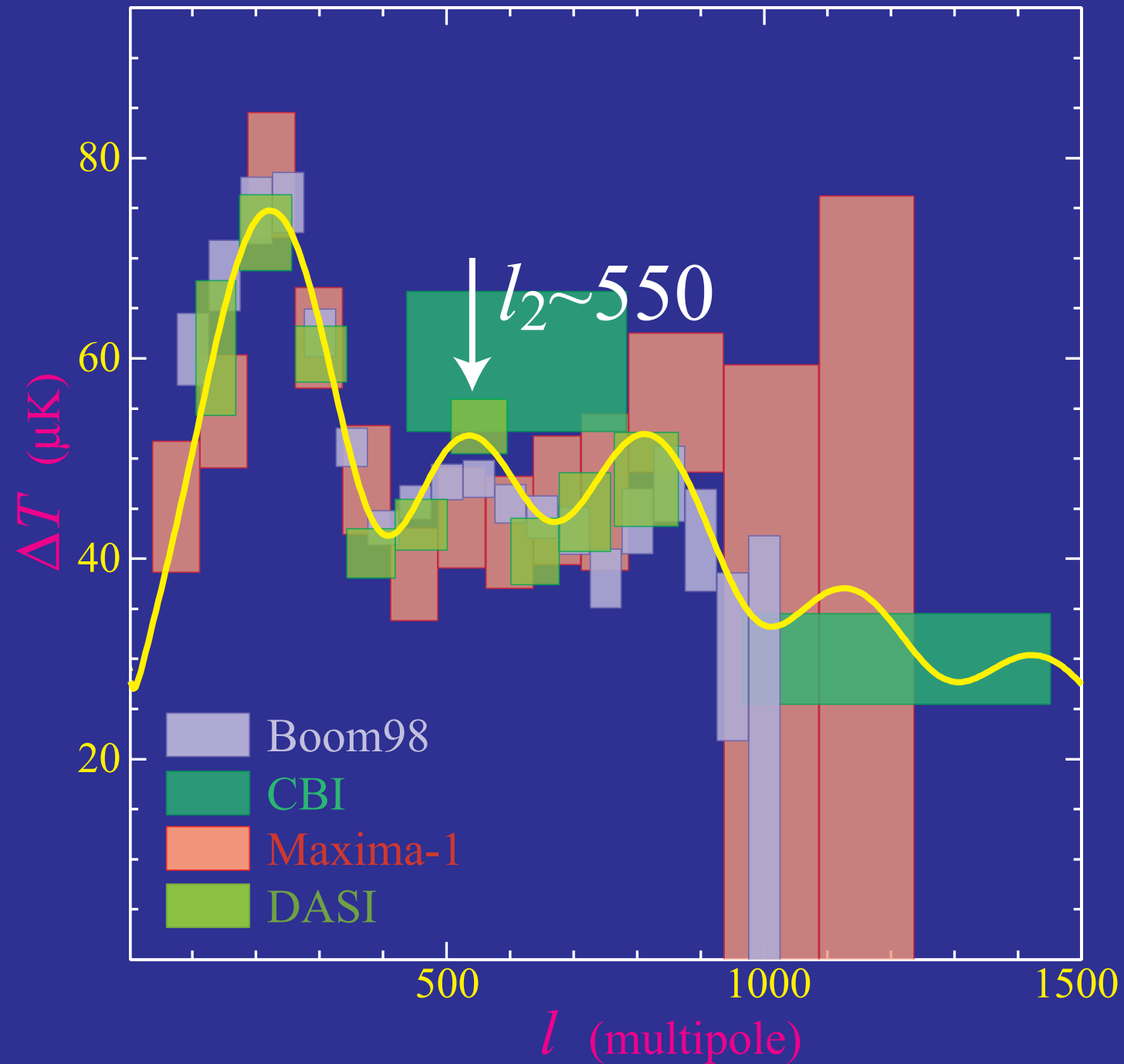
$$\begin{aligned} (\rho_\gamma + p_\gamma)v_\gamma + (\rho_b + p_b)v_b &\approx (p_\gamma + p_\gamma + \rho_b + \rho_\gamma)v_\gamma \\ &= (1 + R)(\rho_\gamma + p_\gamma)v_{\gamma b} \end{aligned}$$

where the controlling parameter is the **momentum density ratio**:

$$R \equiv \frac{p_b + \rho_b}{p_\gamma + \rho_\gamma} \approx 30\Omega_b h^2 \left( \frac{a}{10^{-3}} \right)$$

of order **unity** at recombination

# Second Peak First Measured



# New Euler Equation

- Momentum density ratio enters as

$$[(1 + R)(\rho_\gamma + p_\gamma)\mathbf{v}_{\gamma b}]^\cdot = -4\frac{\dot{a}}{a}(1 + R)(\rho_\gamma + p_\gamma)\mathbf{v}_{\gamma b} - \nabla p_\gamma - (1 + R)(\rho_\gamma + p_\gamma)\nabla\Psi$$

same as before except for  $(1 + R)$  terms so

$$[(1 + R)v_{\gamma b}]^\cdot = k\Theta + (1 + R)k\Psi$$

- Photon continuity remains the same

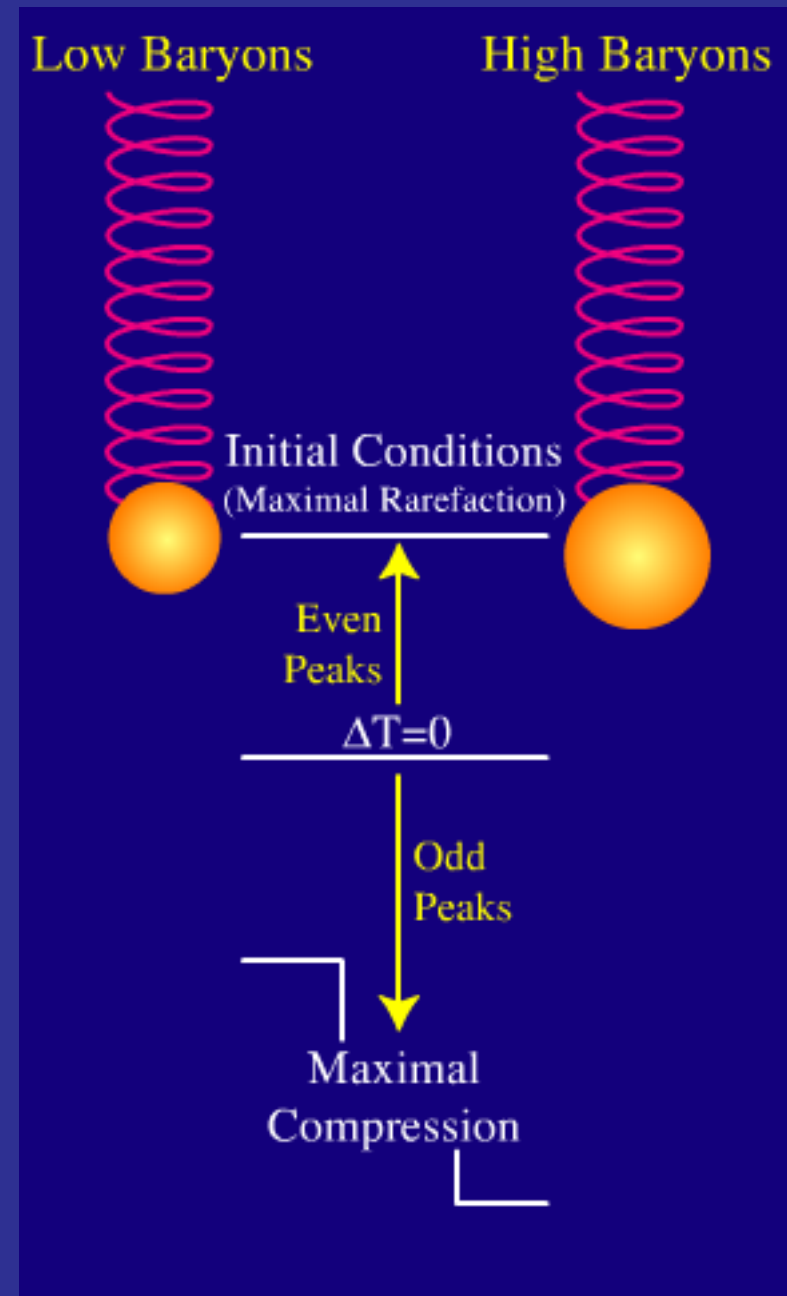
$$\dot{\Theta} = -\frac{k}{3}v_{\gamma b} - \dot{\Phi}$$

- Modification of oscillator equation

$$[(1 + R)\dot{\Theta}]^\cdot + \frac{1}{3}k^2\Theta = -\frac{1}{3}k^2(1 + R)\Psi - [(1 + R)\dot{\Phi}]^\cdot$$

# Baryon & Inertia

- Baryons add inertia to the fluid
- Equivalent to adding mass on a spring
- Same initial conditions
- Same null in fluctuations
- Unequal amplitudes of extrema



# Oscillator: Take Three

- Combine these to form the not-quite-so **simple harmonic oscillator** equation

$$c_s^2 \frac{d}{d\eta} (c_s^{-2} \dot{\Theta}) + c_s^2 k^2 \Theta = -\frac{k^2}{3} \Psi - c_s^2 \frac{d}{d\eta} (c_s^{-2} \dot{\Phi})$$

where  $c_s^2 \equiv \dot{p}_{\gamma b} / \dot{\rho}_{\gamma b}$

$$c_s^2 = \frac{1}{3} \frac{1}{1 + R}$$

- In a **CDM dominated** expansion  $\dot{\Phi} = \dot{\Psi} = 0$  and the **adiabatic approximation**  $\dot{R}/R \ll \omega = kc_s$

$$[\Theta + (1 + R)\Psi](\eta) = [\Theta + (1 + R)\Psi](0) \cos(k s)$$



# Baryon Peak Phenomenology

- Photon-baryon ratio enters in **three** ways
- Overall larger **amplitude**:

$$[\Theta + (1 + R)\Psi](0) = \frac{1}{3}(1 + 3R)\Psi(0)$$

- Even-odd peak **modulation** of effective temperature

$$[\Theta + \Psi]_{\text{peaks}} = [\pm(1 + 3R) - 3R] \frac{1}{3}\Psi(0)$$

$$[\Theta + \Psi]_1 - [\Theta + \Psi]_2 = [-6R] \frac{1}{3}\Psi(0)$$

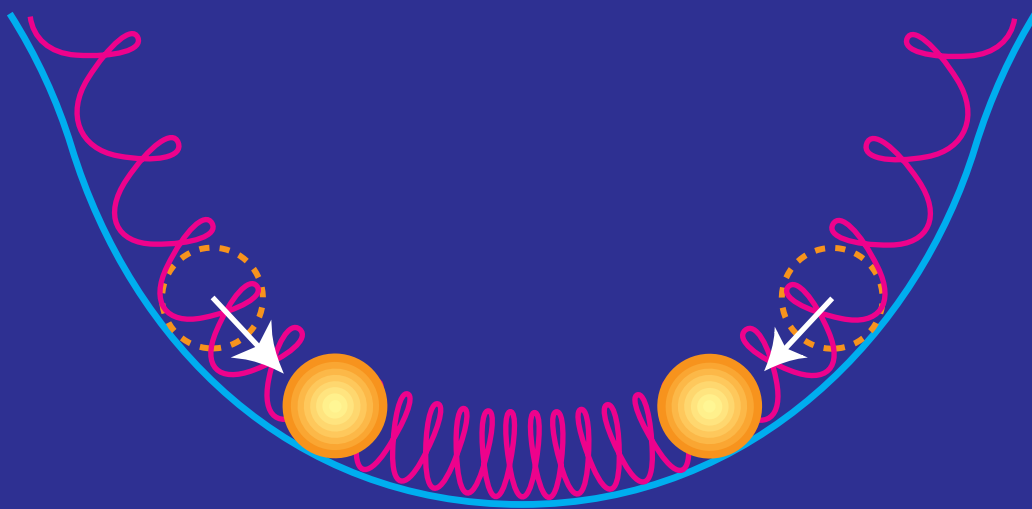
- Shifting of the **sound horizon** down or  $\ell_A$  up

$$\ell_A \propto \sqrt{1 + R}$$

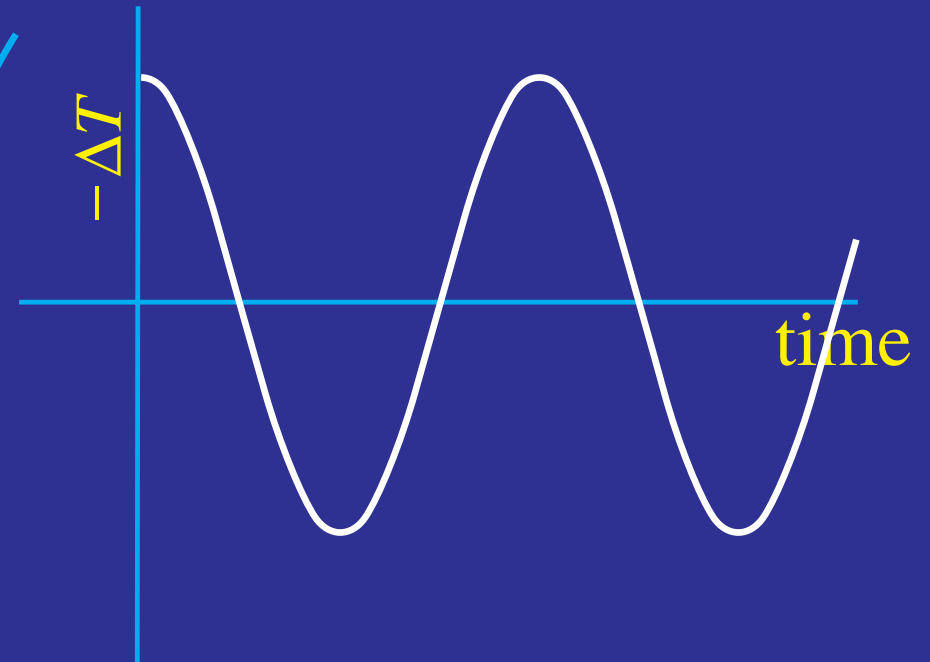
- Actual effects **smaller** since  $R$  evolves

# A Baryon-meter

- **Low baryons:** symmetric compressions and rarefactions

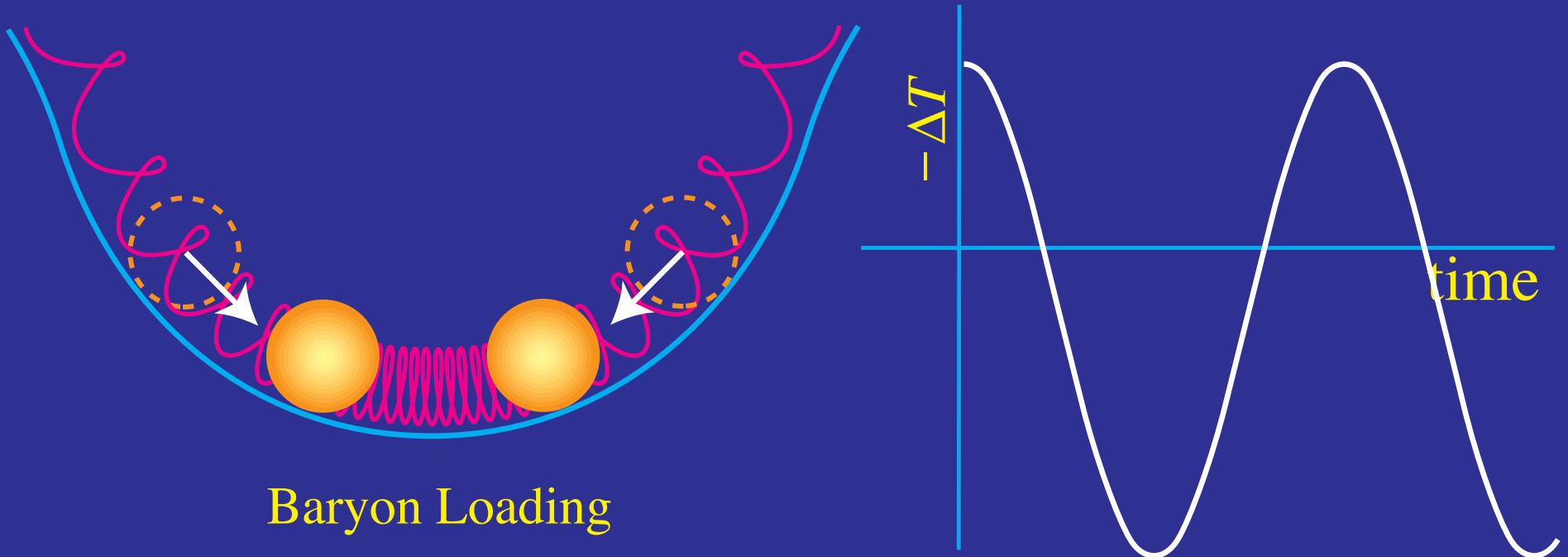


Low Baryons



# A Baryon-meter

- Load the fluid adding to gravitational force
- Enhance compressional peaks (odd) over rarefaction peaks (even)



# A Baryon-meter

- Enhance **compressional peaks** (odd) over **rarefaction peaks** (even)

e.g. relative suppression of **second peak**



# Photon Baryon Ratio Evolution

- Oscillator equation has time evolving mass

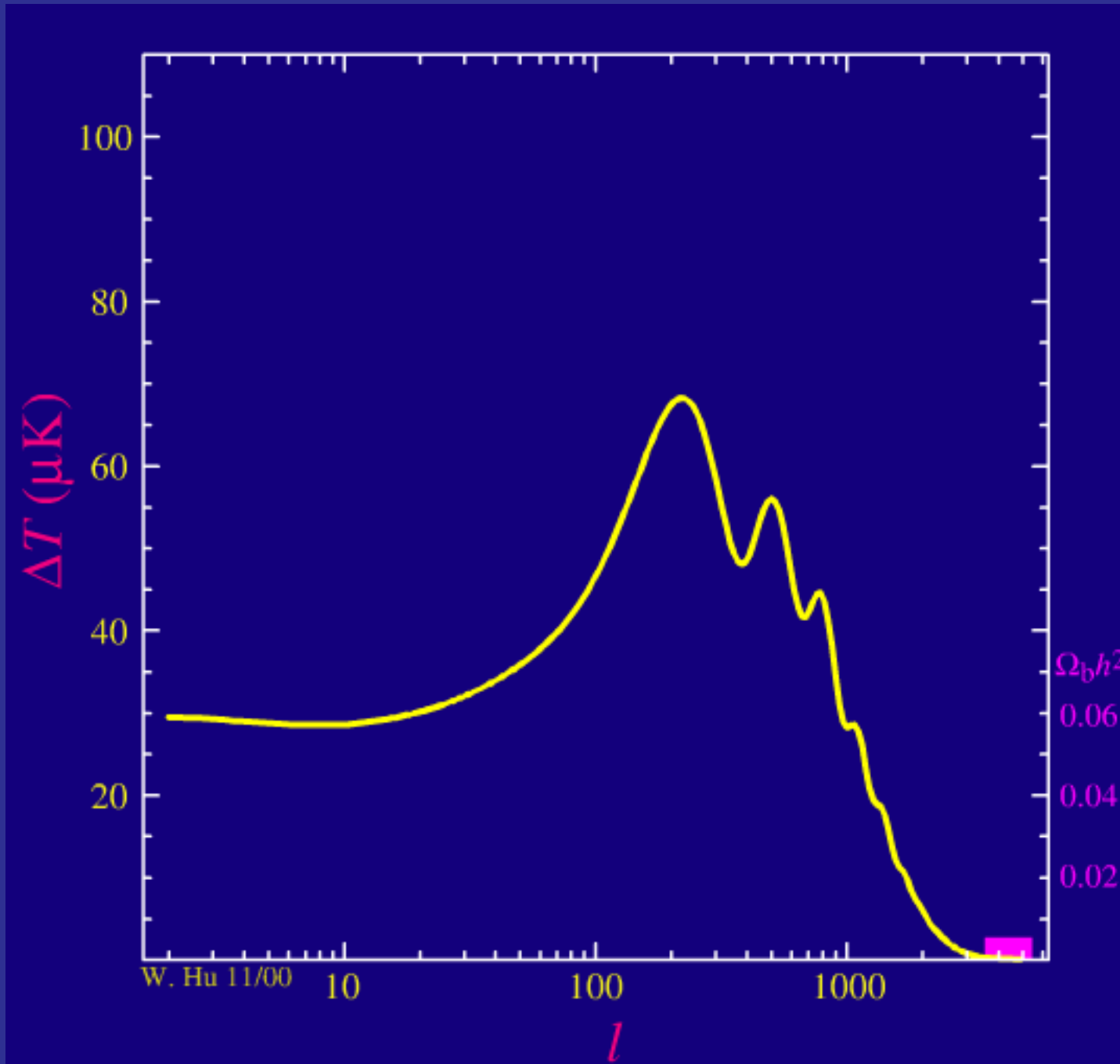
$$c_s^2 \frac{d}{d\eta} (c_s^{-2} \dot{\Theta}) + c_s^2 k^2 \Theta = 0$$

- Effective mass is  $m_{\text{eff}} = 3c_s^{-2} = (1 + R)$
- Adiabatic invariant

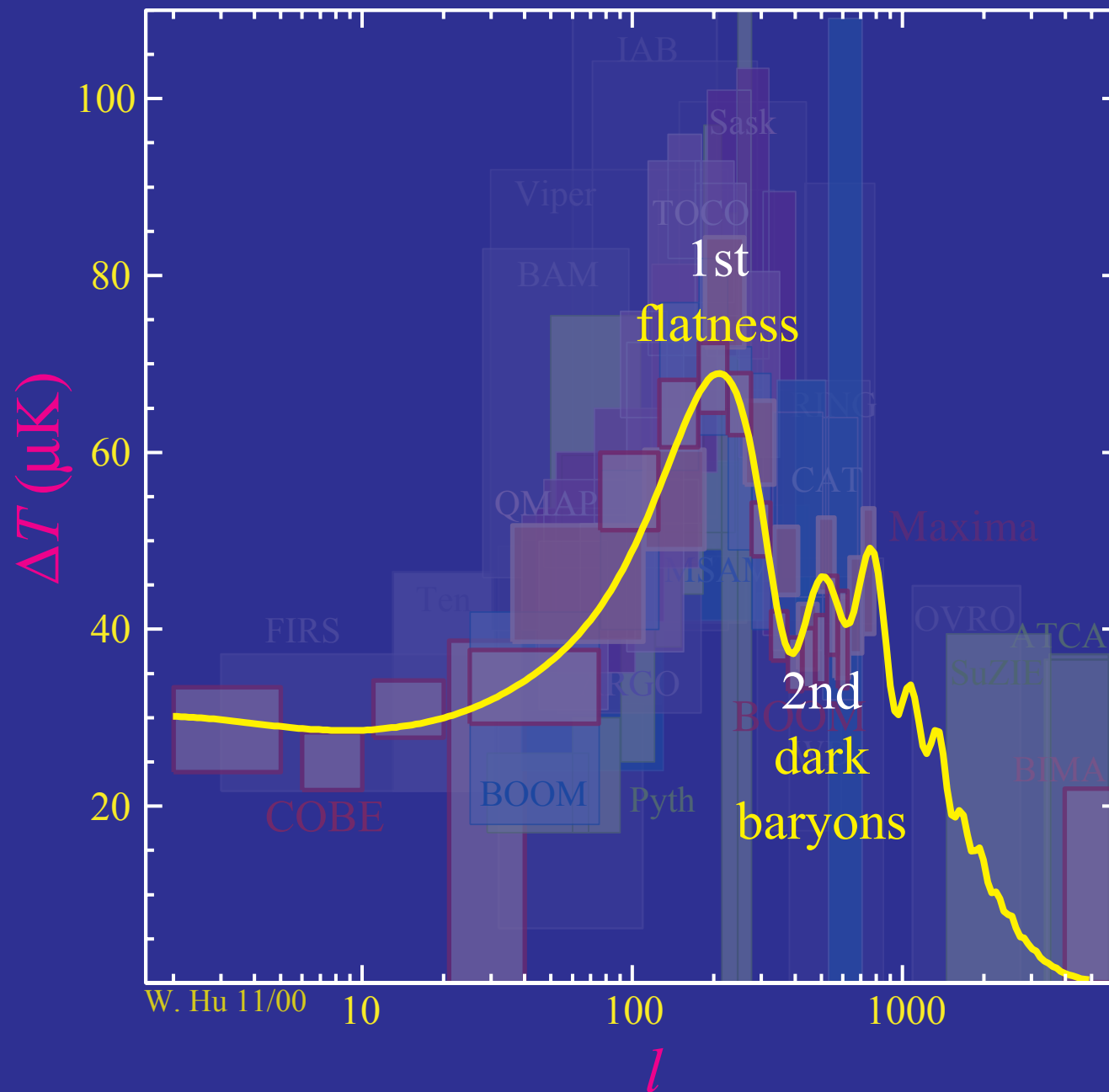
$$\frac{E}{\omega} = \frac{1}{2} m_{\text{eff}} \omega A^2 = \frac{1}{2} 3c_s^{-2} k c_s A^2 \propto A^2 (1 + R)^{1/2} = \text{const.}$$

- Amplitude of oscillation  $A \propto (1 + R)^{-1/4}$  decays adiabatically as the photon-baryon ratio changes

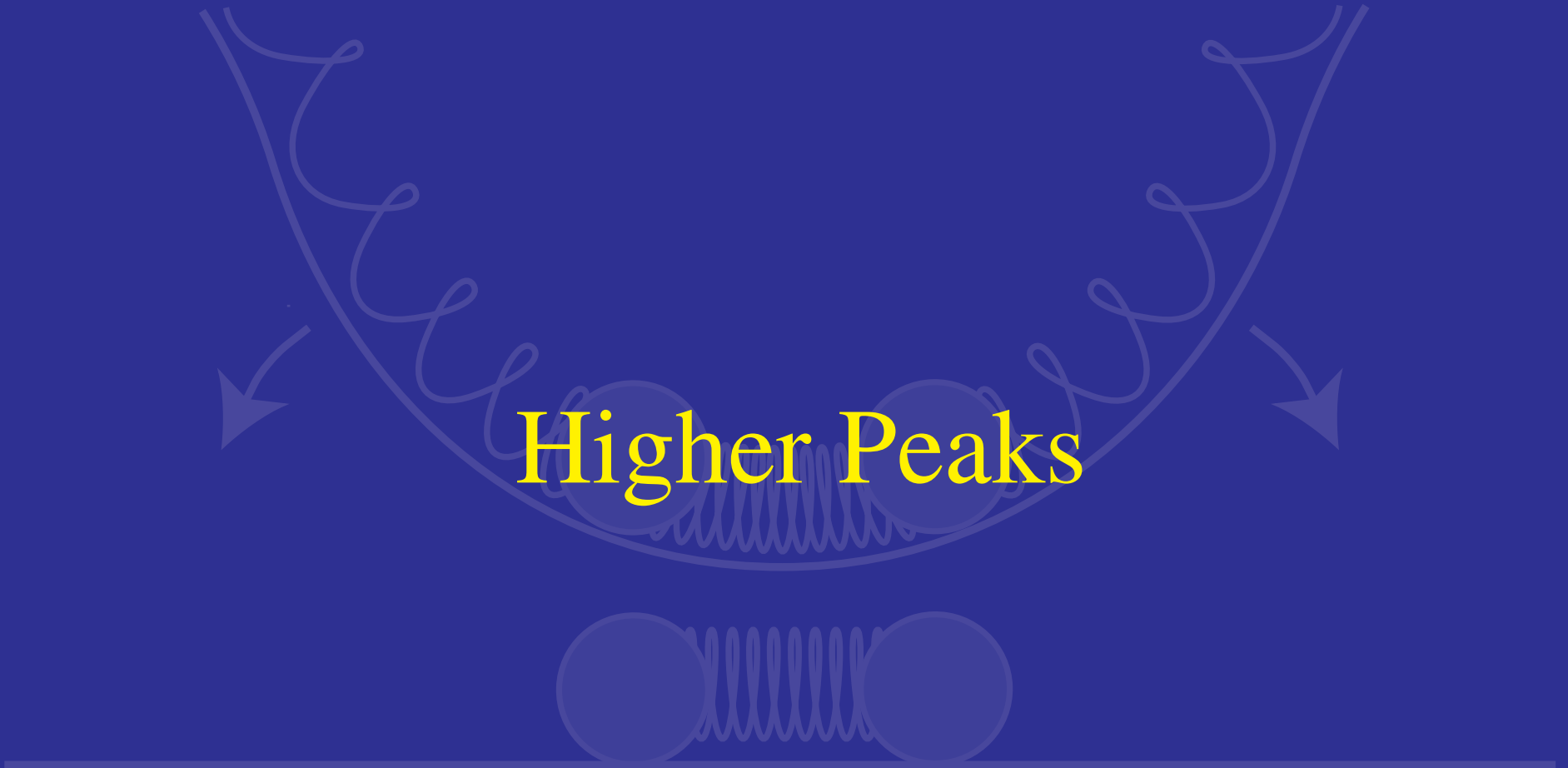
# Baryons in the Power Spectrum



# Score Card



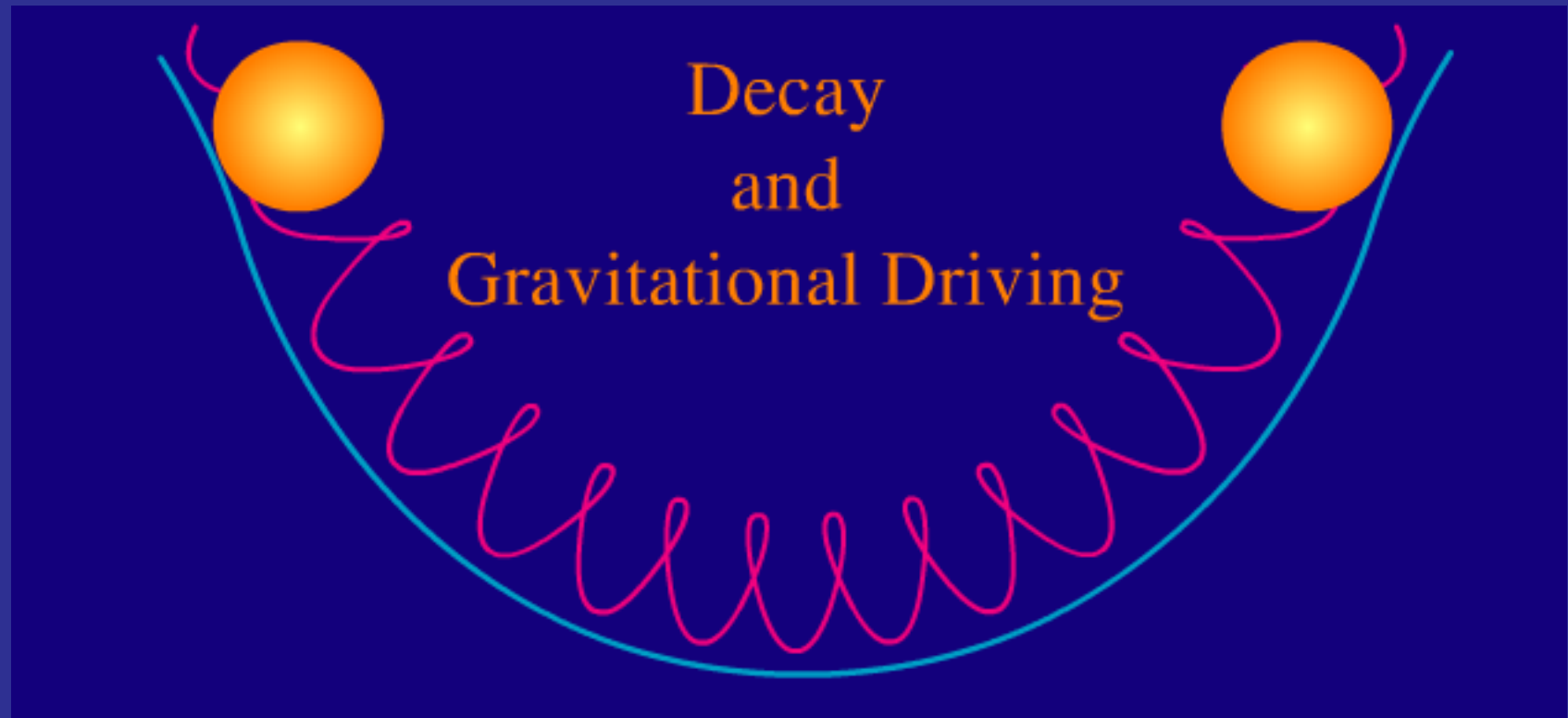
Higher Peaks





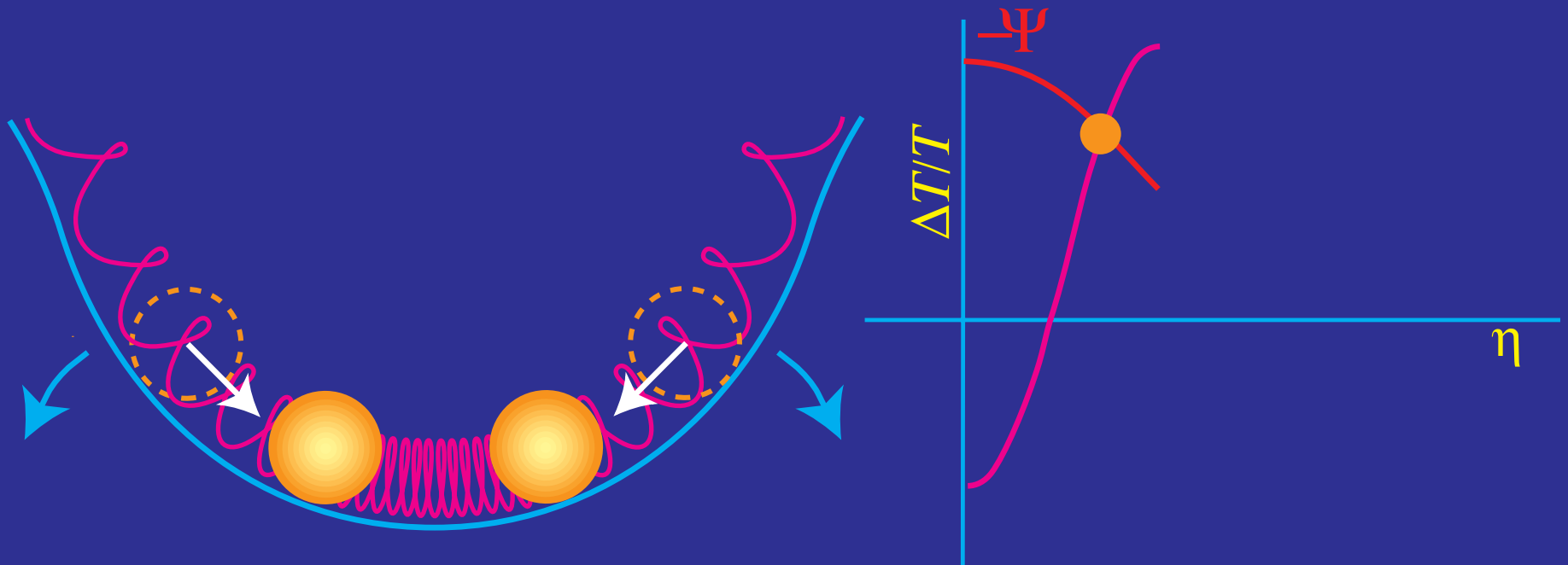
# Radiation and Dark Matter

- Radiation domination:  
potential wells created by CMB itself
- Pressure support  $\Rightarrow$  potential decay  $\Rightarrow$  driving
- Heights measures when dark matter dominates



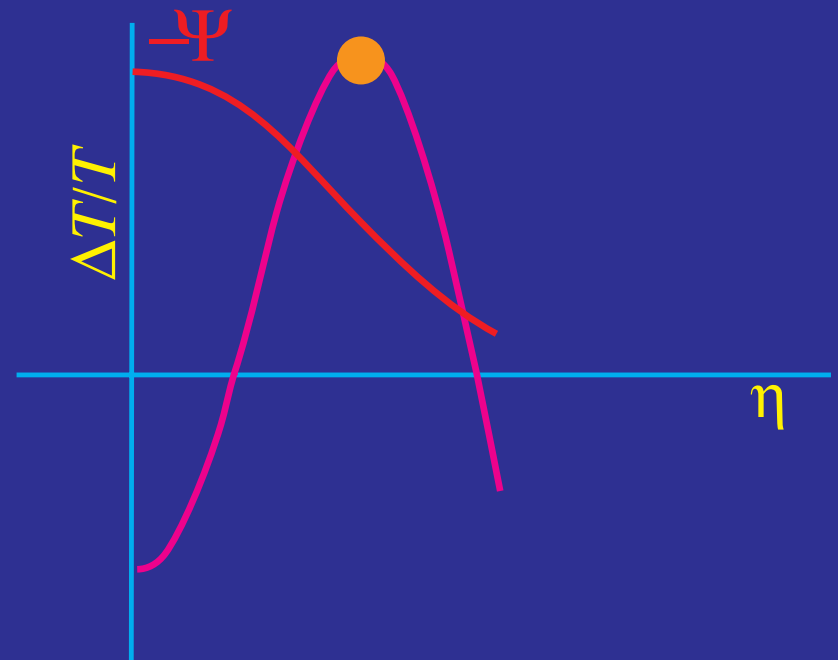
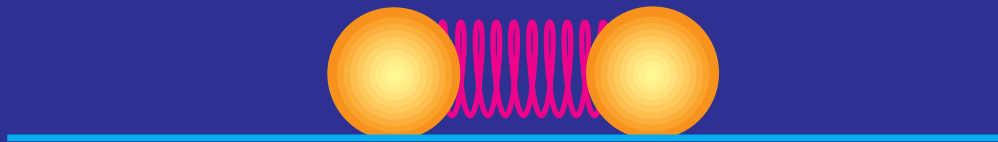
# Driving Effects and Matter/Radiation

- Potential perturbation:  $k^2\Psi = -4\pi G a^2 \delta\rho$  generated by radiation
- **Radiation**  $\rightarrow$  Potential: inside sound horizon  $\delta\rho/\rho$  **pressure supported**  
 $\delta\rho$  hence  $\Psi$  **decays** with expansion



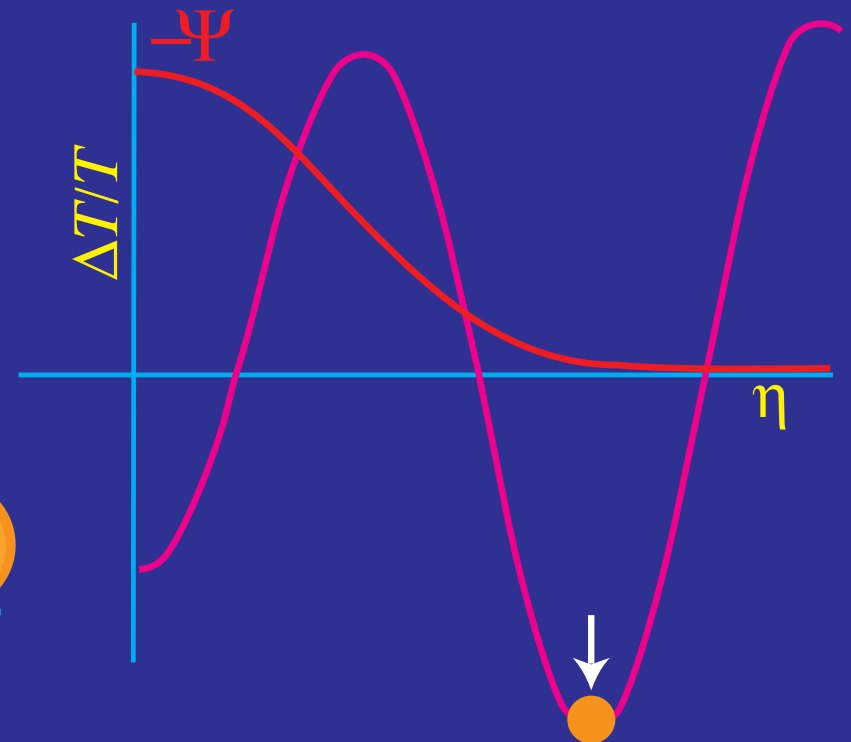
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- Potential  $\rightarrow$  Radiation:  $\Psi$ -decay timed to drive oscillation  
 $-2\Psi + (1/3)\Psi = -(5/3)\Psi \rightarrow 5x$  boost
- Feedback stops at matter domination



# Driving Effects and Matter/Radiation

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- Feedback stops at matter domination



# Oscillator: Take Three and a Half

- The not-quite-so **simple harmonic oscillator** equation is a **forced harmonic oscillator**

$$c_s^2 \frac{d}{d\eta} (c_s^{-2} \dot{\Theta}) + c_s^2 k^2 \Theta = -\frac{k^2}{3} \Psi - c_s^2 \frac{d}{d\eta} (c_s^{-2} \Phi)$$

changes in the **gravitational potentials** alter the form of the acoustic oscillations

- If the forcing term has a **temporal structure** that is related to the **frequency** of the oscillation, this becomes a **driven harmonic oscillator**
- Term involving  $\Psi$  is the ordinary **gravitational force**
- Term involving  $\Phi$  involves the  $\dot{\Phi}$  term in the **continuity equation** as a (curvature) perturbation to the **scale factor**

# Potential Decay

- Matter-to-radiation ratio

$$\frac{\rho_m}{\rho_r} \approx 24 \Omega_m h^2 \left( \frac{a}{10^{-3}} \right)$$

of order **unity** at recombination in a low  $\Omega_m$  universe

- Radiation is not stress free and so **impedes** the growth of structure

$$k^2 \Phi = 4\pi G a^2 \rho_r \Delta_r$$

$\Delta_r \sim 4\Theta$  **oscillates** around a constant value,  $\rho_r \propto a^{-4}$  so the Newtonian **curvature decays**.

- General rule: potential decays if the dominant energy component has substantial stress fluctuations, i.e. below the generalized sound horizon or Jeans scale

# Radiation Driving

- Decay is timed precisely to **drive** the oscillator - close to fully **coherent**

$$\begin{aligned} [\Theta + \Psi](\eta) &= [\Theta + \Psi](0) + \Delta\Psi - \Delta\Phi \\ &= \frac{1}{3}\Psi(0) - 2\Psi(0) = -\frac{5}{3}\Psi(0) \end{aligned}$$

- **5** $\times$  the amplitude of the Sachs-Wolfe effect!
- Coherent approximation is **exact** for a photon-baryon fluid but reality is reduced to  $\sim 4\times$  because of **neutrino contribution** to radiation
- Actual **initial conditions** are  $\Theta + \Psi = \Psi/2$  for radiation domination but comparison to matter dominated SW correct

# External Potential Approach

- Solution to homogeneous equation

$$(1 + R)^{-1/4} \cos(ks), \quad (1 + R)^{-1/4} \sin(ks)$$

- Give the general solution for an external potential by propagating impulsive forces

$$(1 + R)^{1/4} \Theta(\eta) = \Theta(0) \cos(ks) + \frac{\sqrt{3}}{k} \left[ \dot{\Theta}(0) + \frac{1}{4} \dot{R}(0) \Theta(0) \right] \sin ks \\ + \frac{\sqrt{3}}{k} \int_0^\eta d\eta' (1 + R')^{3/4} \sin[ks - ks'] F(\eta')$$

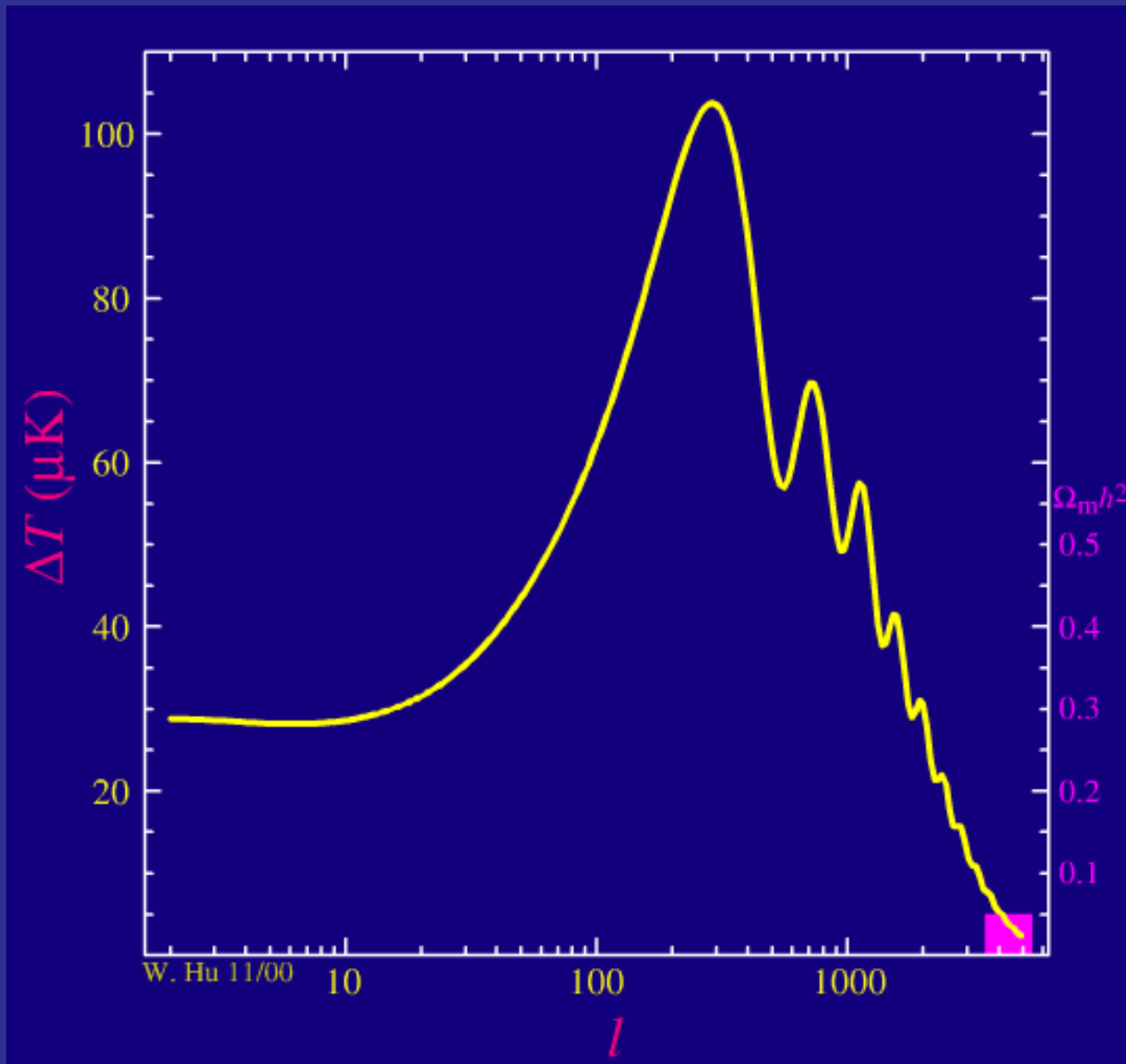
where

$$F = -\ddot{\Phi} - \frac{\dot{R}}{1 + R} \dot{\Phi} - \frac{k^2}{3} \Psi$$

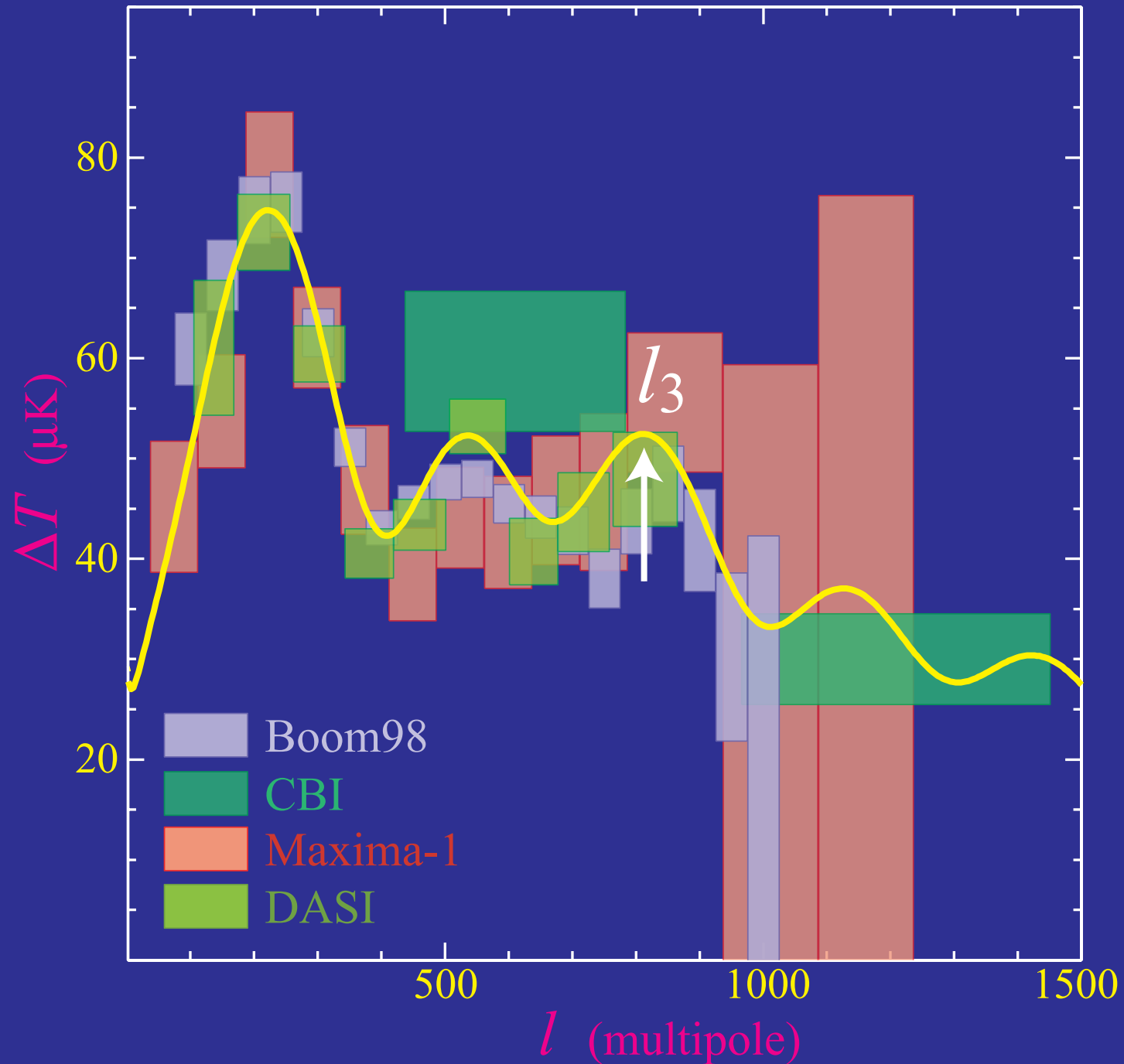
- Useful if general form of potential evolution is known



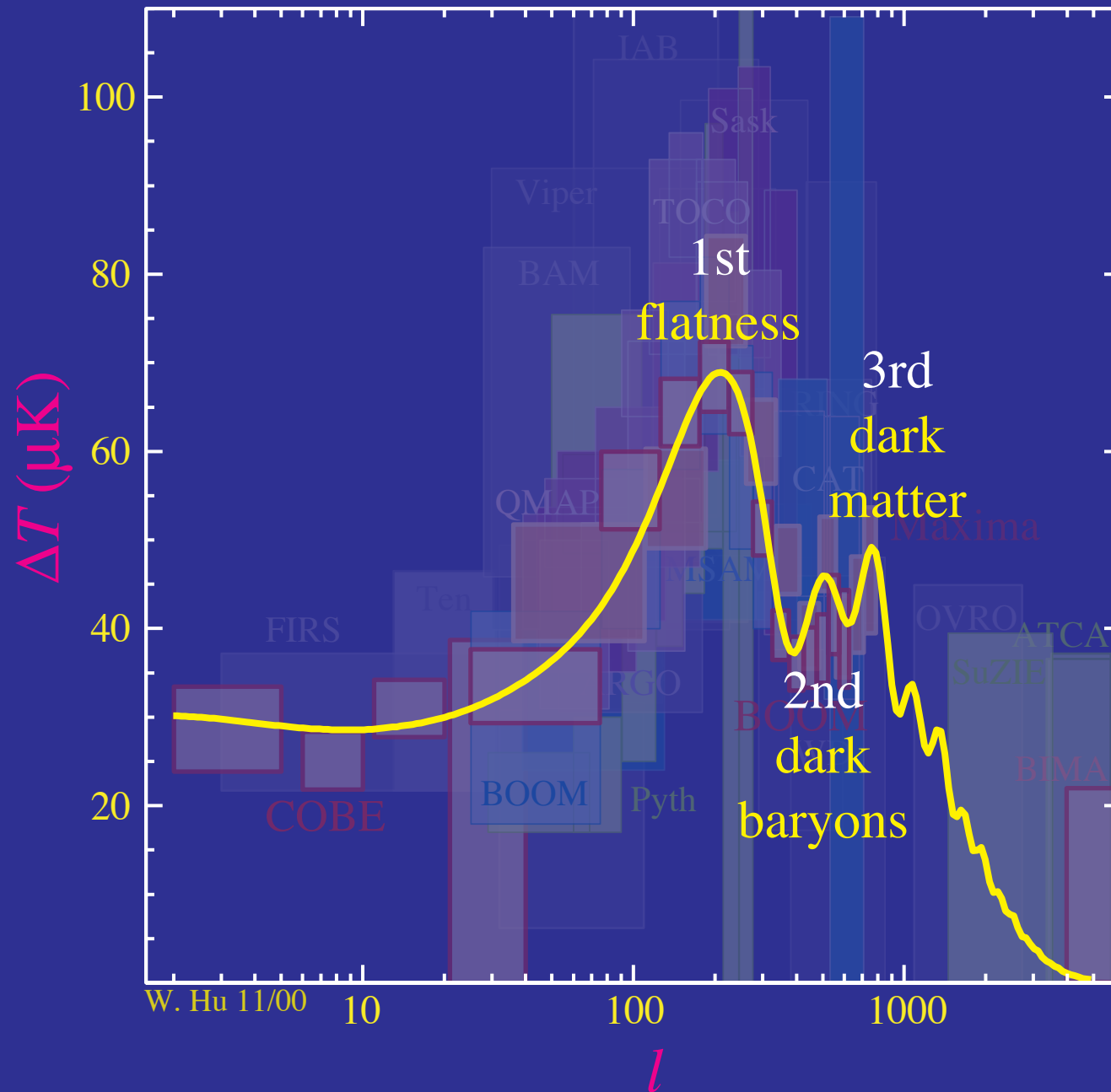
# Dark Matter in the Power Spectrum



# Third Peak First Measured



# Score Card

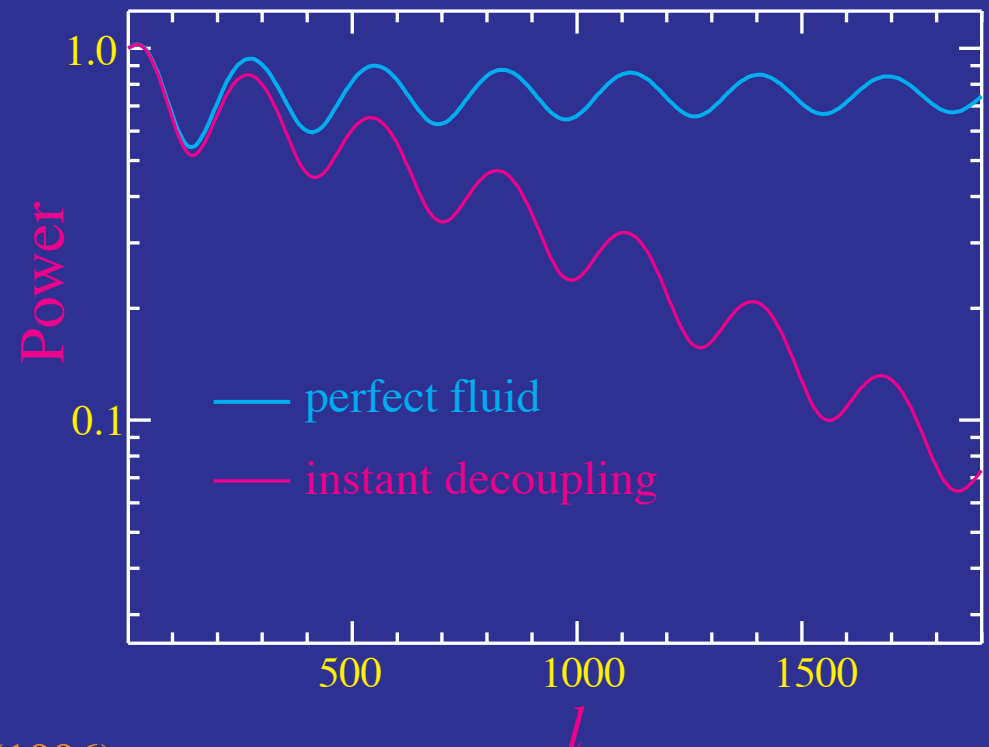
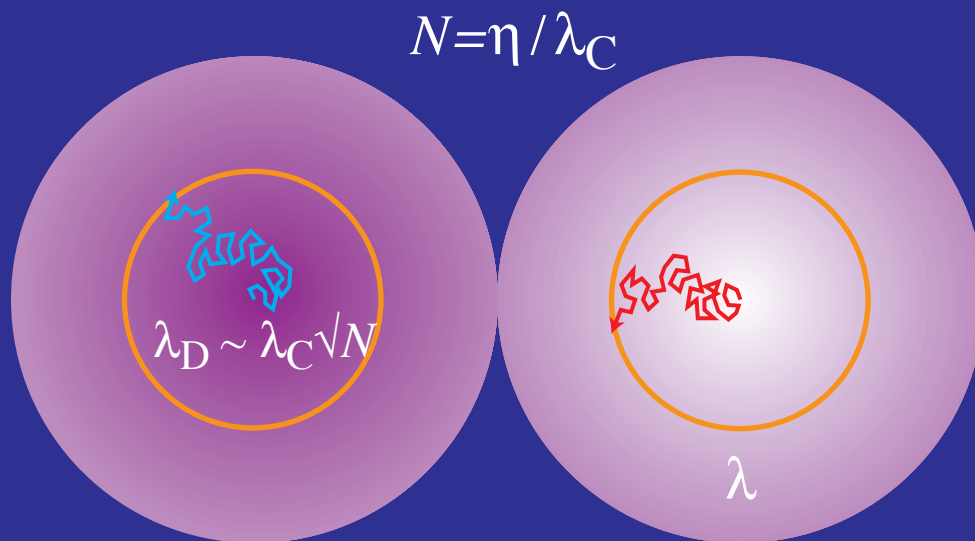




# Damping Tail

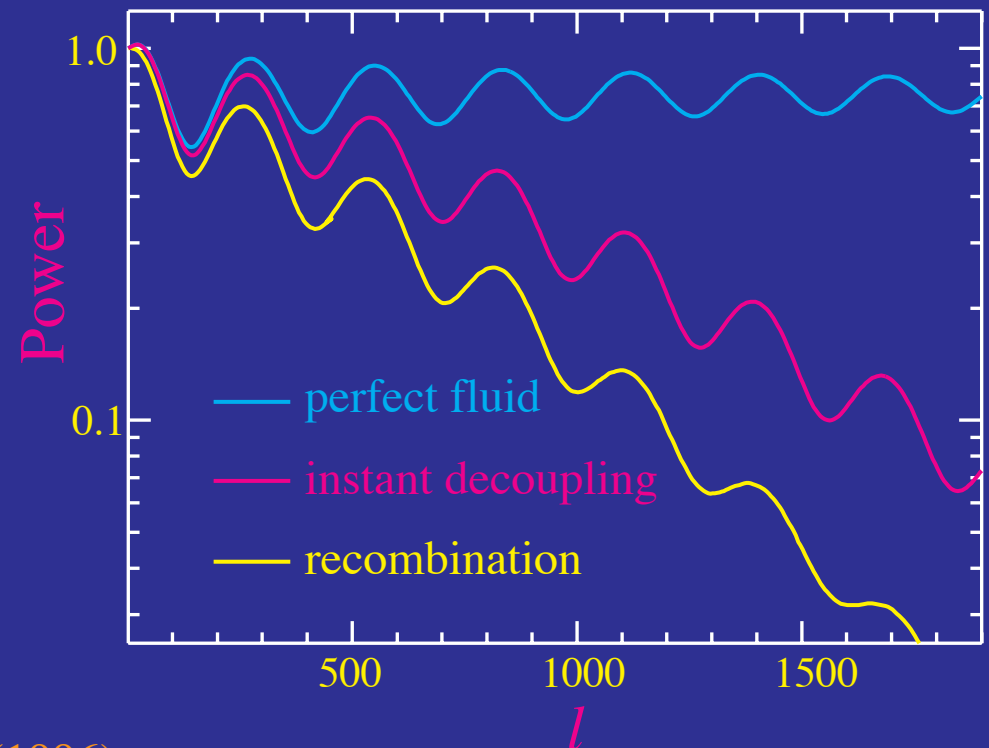
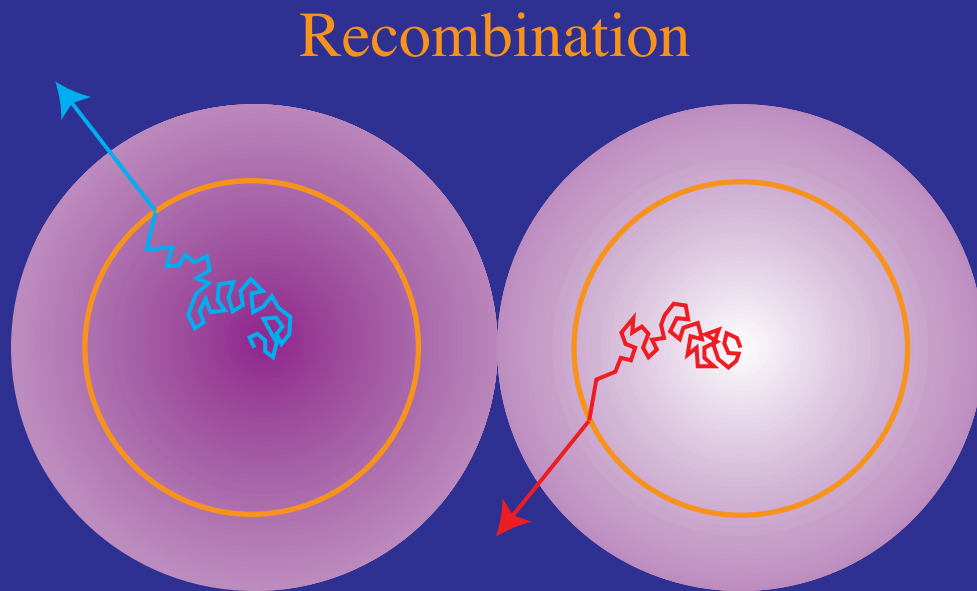
# Dissipation / Diffusion Damping

- Imperfections in the coupled fluid  $\rightarrow$  mean free path  $\lambda_C$  in the baryons
- Random walk over diffusion scale: geometric mean of mfp & horizon  
 $\lambda_D \sim \lambda_C \sqrt{N} \sim \sqrt{\lambda_C \eta} \gg \lambda_C$
- Overtake wavelength:  $\lambda_D \sim \lambda$ ; second order in  $\lambda_C/\lambda$
- Viscous damping for  $R < 1$ ; heat conduction damping for  $R > 1$



# Dissipation / Diffusion Damping

- Rapid increase at **recombination** as  $mfp \uparrow$
- Independent of (robust to changes in) **perturbation spectrum**
- Robust **physical scale** for **angular diameter distance** test ( $\Omega_K, \Omega_\Lambda$ )



Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)

# Damping

- Tight coupling equations assume a **perfect fluid**: no **viscosity**, no **heat conduction**
- Fluid imperfections are related to the **mean free path of the photons in the baryons**

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

is the conformal opacity to **Thomson scattering**

- Dissipation is related to the **diffusion length**: random walk approximation

$$\lambda_D = \sqrt{N} \lambda_C = \sqrt{\eta / \lambda_C} \lambda_C = \sqrt{\eta \lambda_C}$$

the **geometric mean** between the horizon and mean free path

- $\lambda_D / \eta_* \sim$  **few %**, so expect the **peaks  $> 3$**  to be affected by **dissipation**

# Equations of Motion

- Continuity

$$\dot{\Theta} = -\frac{k}{3}v_{\gamma} - \dot{\Phi}, \quad \dot{\delta}_b = -kv_b - 3\dot{\Phi}$$

where the photon equation remains unchanged and the baryons follow number conservation with  $\rho_b = m_b n_b$

- Euler

$$\begin{aligned}\dot{v}_{\gamma} &= k(\Theta + \Psi) - \frac{k}{6}\pi_{\gamma} - \dot{\tau}(v_{\gamma} - v_b) \\ \dot{v}_b &= -\frac{\dot{a}}{a}v_b + k\Psi + \dot{\tau}(v_{\gamma} - v_b)/R\end{aligned}$$

where the photons gain an anisotropic stress term  $\pi_{\gamma}$  from **radiation viscosity** and a **momentum exchange** term with the baryons and are compensated by the **opposite term** in the baryon Euler equation



# Viscosity

- **Viscosity** is generated from radiation **streaming** from hot to cold regions
- Expect

$$\pi_\gamma \sim v_\gamma \frac{k}{\dot{\tau}}$$

generated by streaming, suppressed by **scattering** in a wavelength of the fluctuation. **Radiative transfer** says

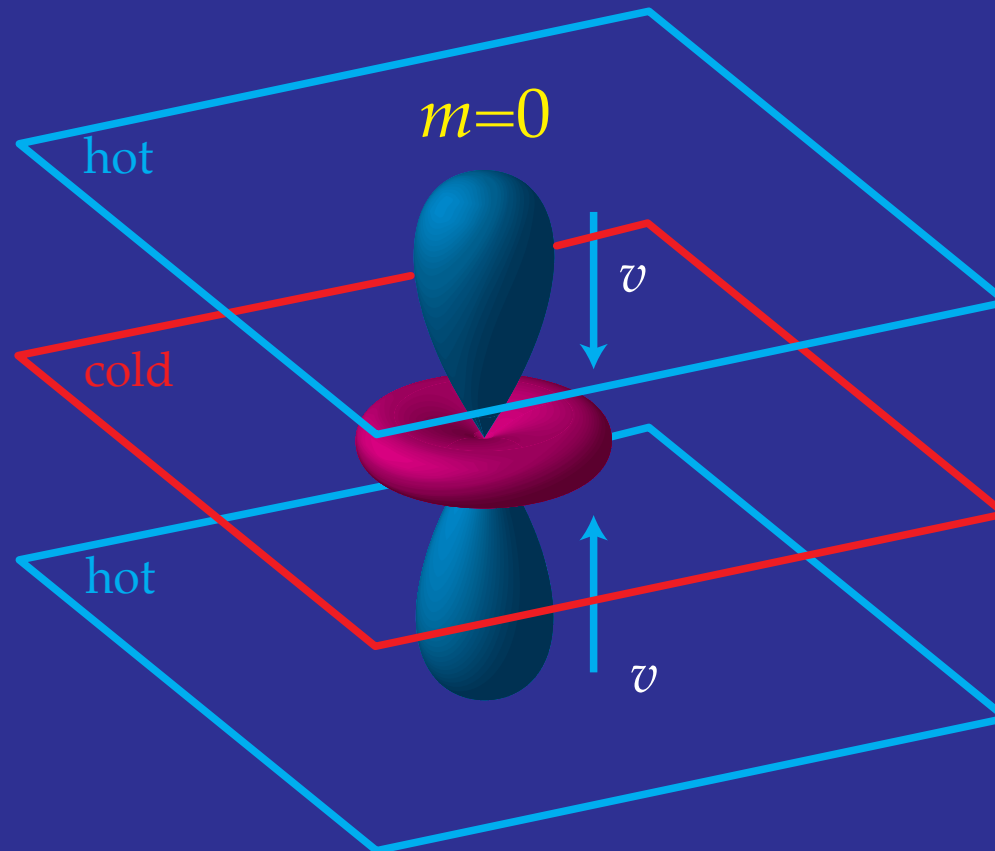
$$\pi_\gamma \approx 2A_v v_\gamma \frac{k}{\dot{\tau}}$$

where  $A_v = 16/15$

$$\dot{v}_\gamma = k(\Theta + \Psi) - \frac{k}{3} A_v \frac{k}{\dot{\tau}} v_\gamma$$

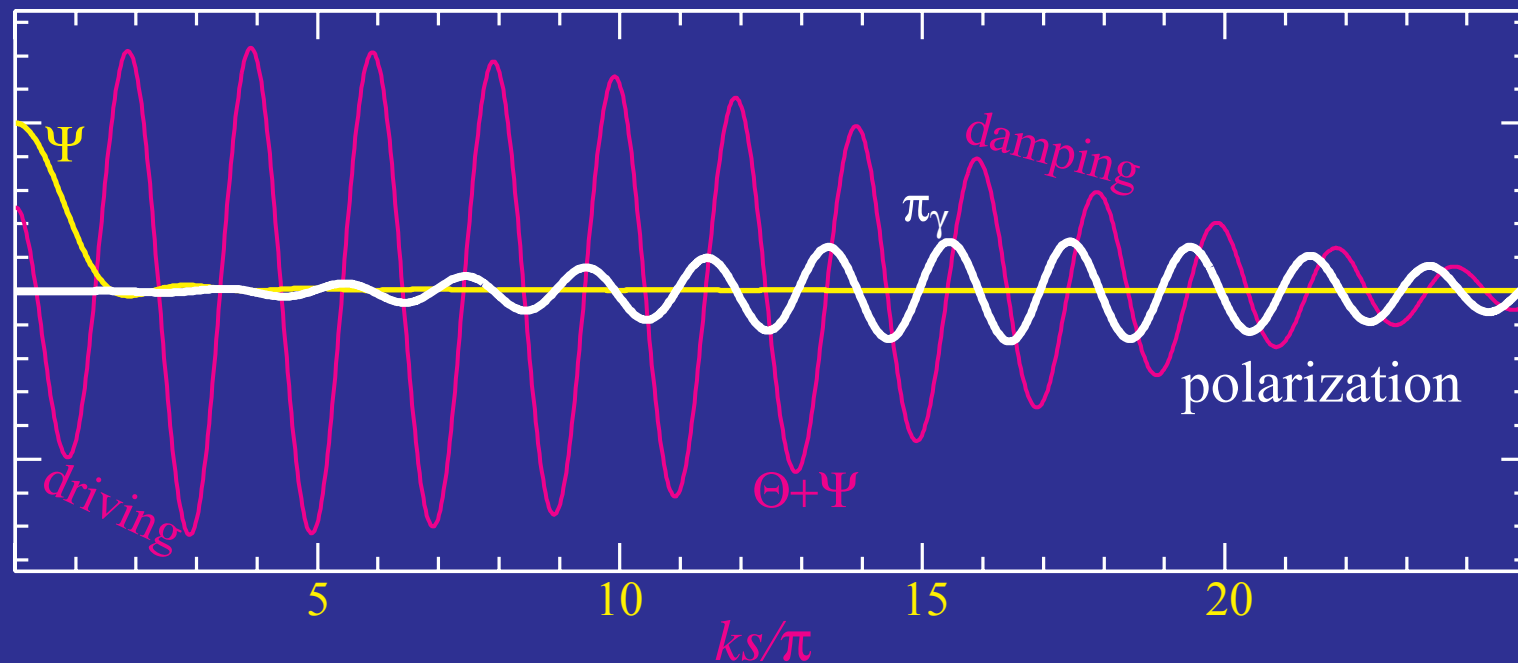
# Viscosity & Heat Conduction

- Both fluid imperfections are related to the gradient of the velocity  $kv_\gamma$  by opacity  $\dot{\tau}$ : slippage of fluids  $v_\gamma - v_b$ .
- **Viscosity** is an anisotropic stress or **quadrupole moment** formed by radiation **streaming** from hot to cold regions



# Damping & Viscosity

- Quadrupole moments:
  - damp** acoustic oscillations from fluid viscosity
  - generates **polarization** from scattering (next lecture)
- Rise in polarization **power** coincides with fall in temperature power –  $l \sim 1000$



# Oscillator: Penultimate Take

- Adiabatic approximation ( $\omega \gg \dot{a}/a$ )

$$\dot{\Theta} \approx -\frac{k}{3}v_\gamma$$

- Oscillator equation contains a  $\dot{\Theta}$  damping term

$$c_s^2 \frac{d}{d\eta} (c_s^{-2} \dot{\Theta}) + \frac{k^2 c_s^2}{\dot{\tau}} A_v \dot{\Theta} + k^2 c_s^2 \Theta = -\frac{k^2}{3} \Psi - c_s^2 \frac{d}{d\eta} (c_s^{-2} \dot{\Phi})$$

- Heat conduction term similar in that it is proportional to  $v_\gamma$  and is suppressed by scattering  $k/\dot{\tau}$ . Expansion of Euler equations to leading order in  $k/\dot{\tau}$  gives

$$A_h = \frac{R^2}{1 + R}$$

since the effects are only significant if the baryons are dynamically important

# Oscillator: Final Take

- Final oscillator equation

$$c_s^2 \frac{d}{d\eta} (c_s^{-2} \dot{\Theta}) + \frac{k^2 c_s^2}{\dot{\tau}} [A_v + A_h] \dot{\Theta} + k^2 c_s^2 \Theta = -\frac{k^2}{3} \Psi - c_s^2 \frac{d}{d\eta} (c_s^{-2} \dot{\Phi})$$

- Solve in the adiabatic approximation

$$\Theta \propto \exp(i \int \omega d\eta)$$

$$-\omega^2 + \frac{k^2 c_s^2}{\dot{\tau}} (A_v + A_h) i\omega + k^2 c_s^2 = 0$$

# Dispersion Relation

- Solve

$$\begin{aligned}\omega^2 &= k^2 c_s^2 \left[ 1 + i \frac{\omega}{\dot{\tau}} (A_v + A_h) \right] \\ \omega &= \pm k c_s \left[ 1 + \frac{i}{2} \frac{\omega}{\dot{\tau}} (A_v + A_h) \right] \\ &= \pm k c_s \left[ 1 \pm \frac{i}{2} \frac{k c_s}{\dot{\tau}} (A_v + A_h) \right]\end{aligned}$$

- Exponentiate

$$\begin{aligned}\exp(i \int \omega d\eta) &= e^{\pm i k s} \exp\left[-k^2 \int d\eta \frac{1}{2} \frac{c_s^2}{\dot{\tau}} (A_v + A_h)\right] \\ &= e^{\pm i k s} \exp\left[-(k/k_D)^2\right]\end{aligned}$$

- Damping is **exponential** under the scale  $k_D$

# Diffusion Scale

- Diffusion wavenumber

$$k_D^{-2} = \int d\eta \frac{1}{\dot{\tau}} \frac{1}{6(1+R)} \left( \frac{16}{15} + \frac{R^2}{(1+R)} \right)$$

- Limiting forms

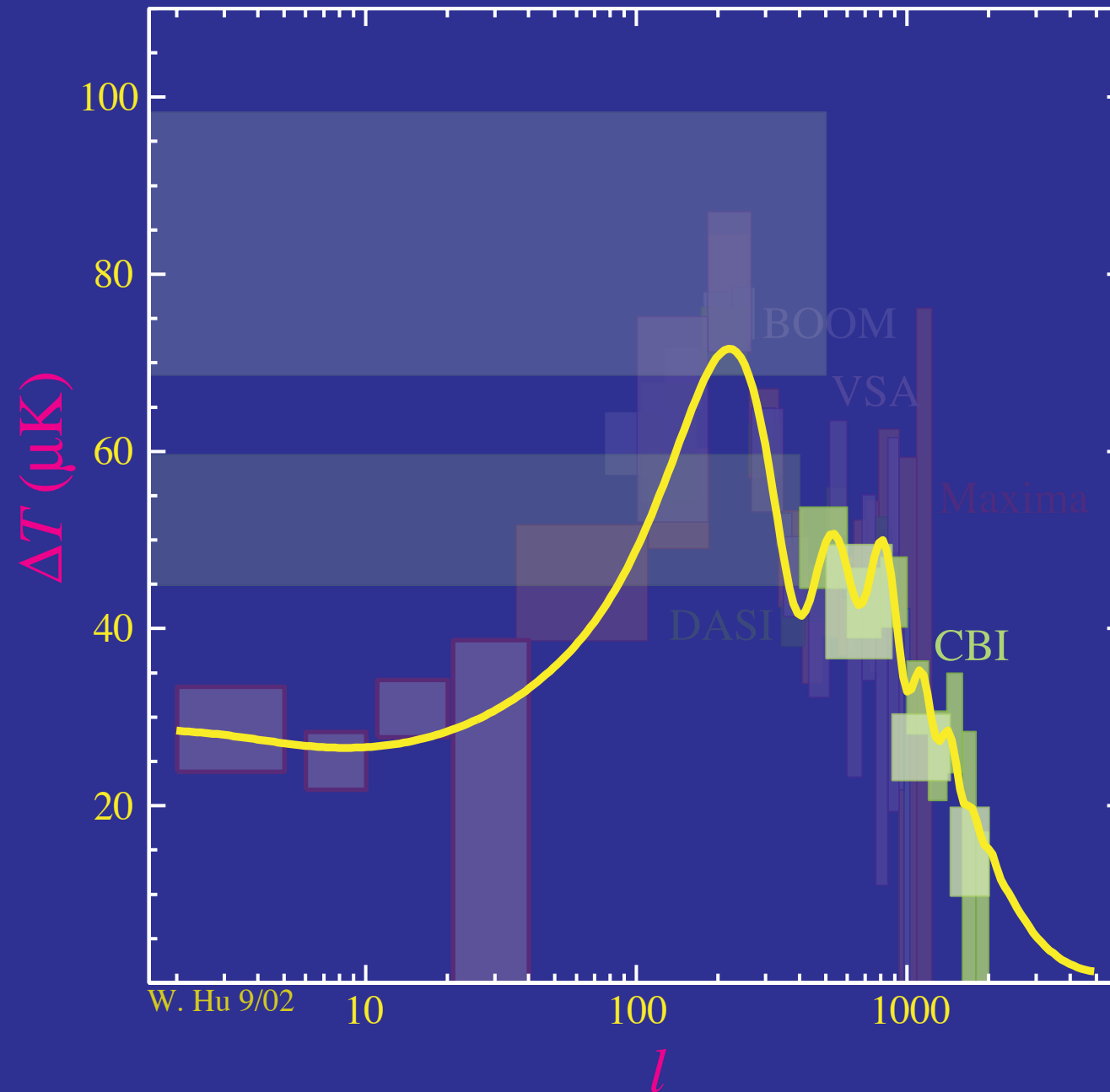
$$\lim_{R \rightarrow 0} k_D^{-2} = \frac{1}{6} \frac{16}{15} \int d\eta \frac{1}{\dot{\tau}}$$

$$\lim_{R \rightarrow \infty} k_D^{-2} = \frac{1}{6} \int d\eta \frac{1}{\dot{\tau}}$$

- Geometric mean between horizon and mean free path as expected from a random walk

$$\lambda_D = \frac{2\pi}{k_D} \sim \frac{2\pi}{\sqrt{6}} (\eta \dot{\tau}^{-1})^{1/2}$$

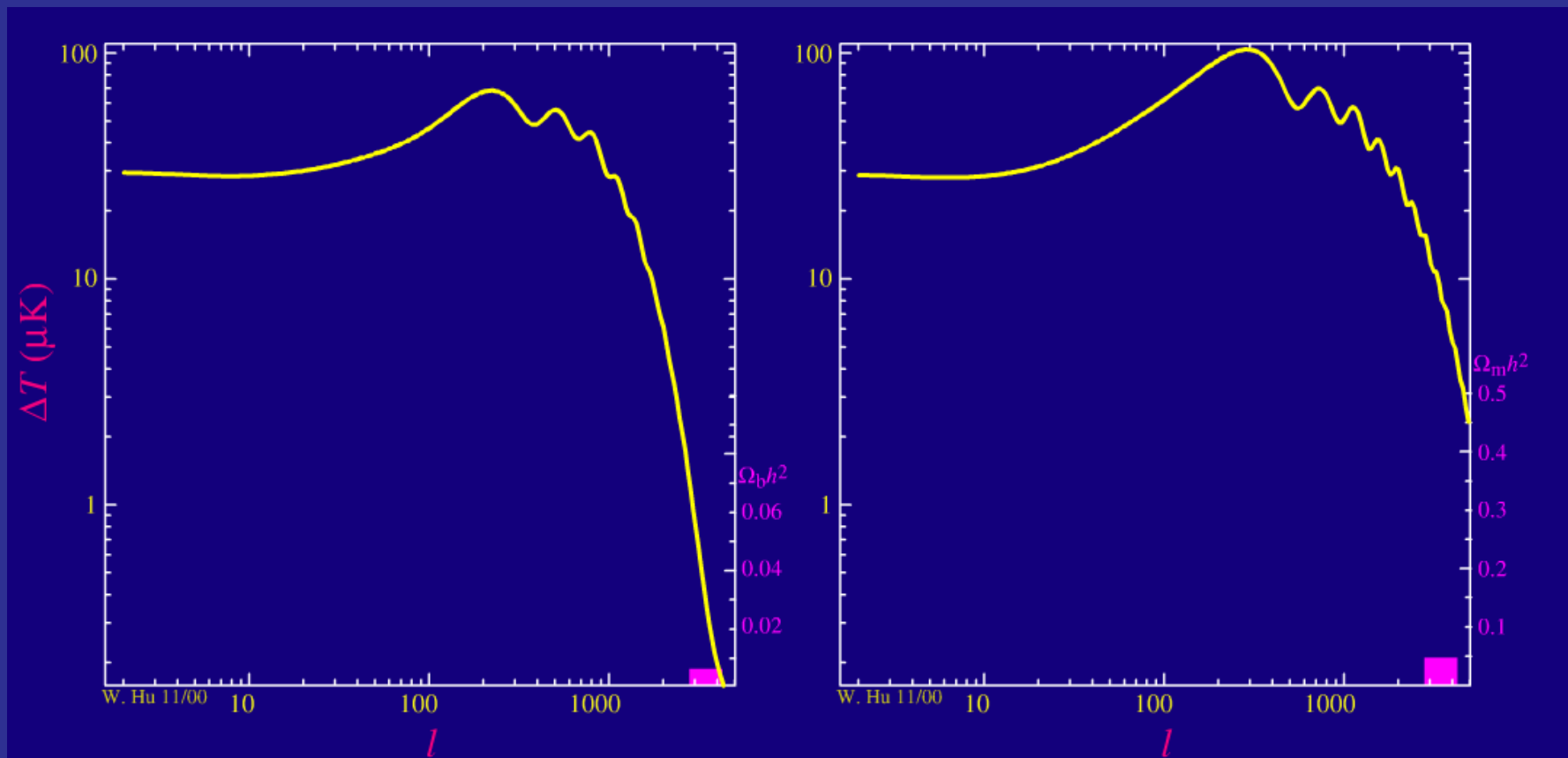
# Damping Tail Measured





# Standard Ruler

- **Damping length** is a fixed **physical scale** given properties at recombination
- Geometric mean of **mean free path** and **horizon**: depends on **baryon-photon ratio** and **matter-radiation ratio**



# Standard Rulers

- Calibrating the Standard Rulers
- Sound Horizon



← Baryons  
Matter/Radiation →

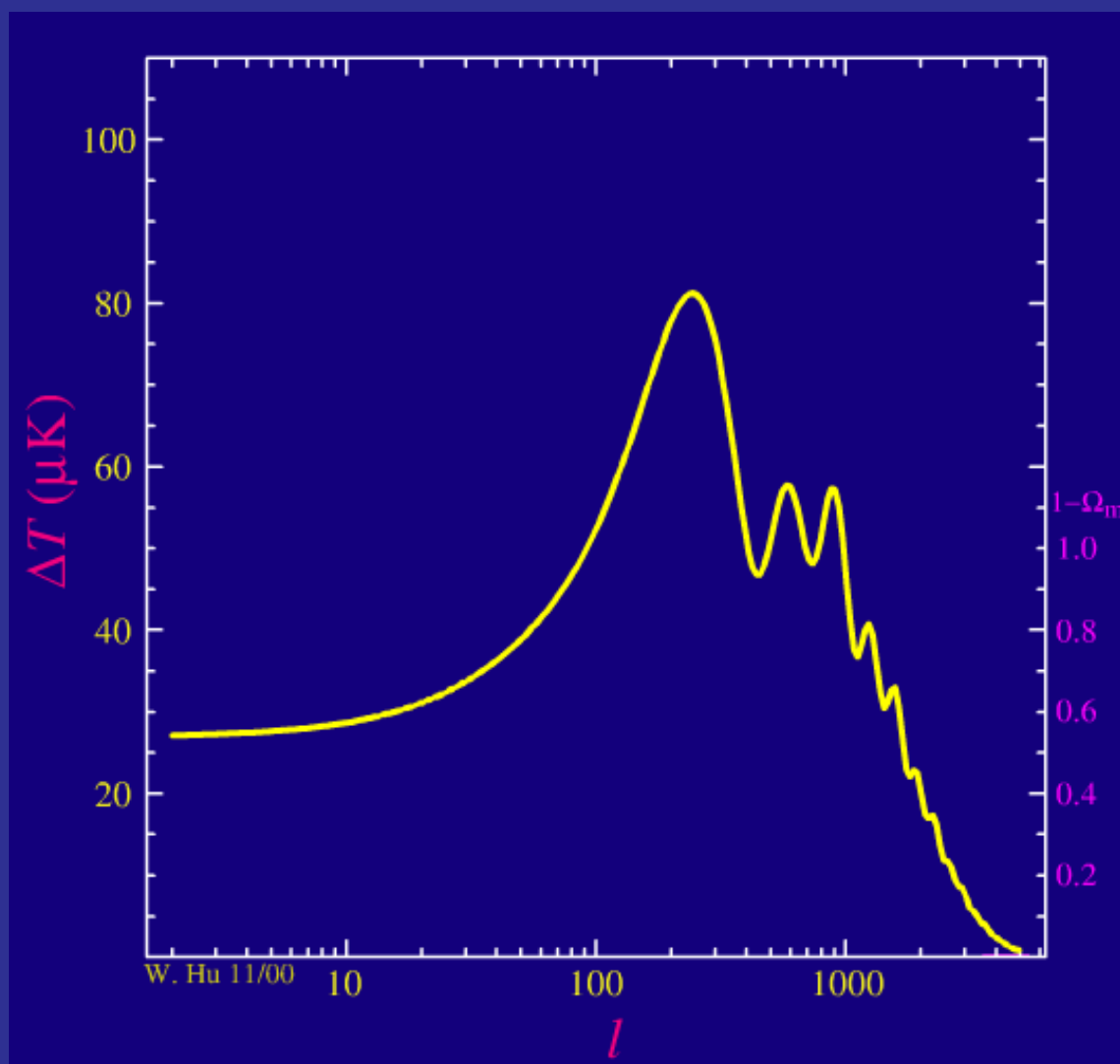
- Damping Scale



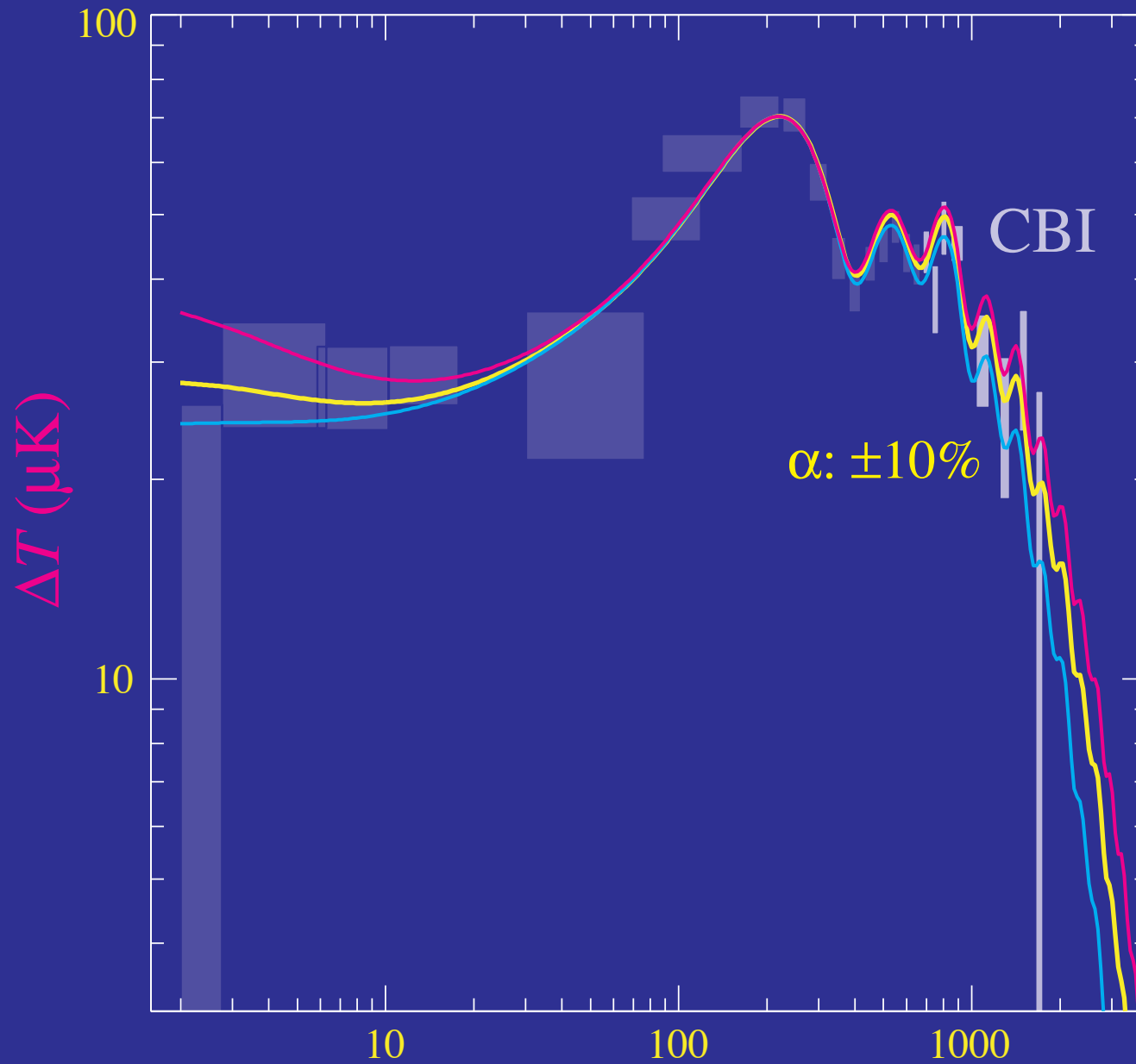
← Baryons  
Matter/Radiation →

# Curvature

- Calibration from lower peaks of  $\Omega_b h^2$  and  $\Omega_m h^2$  allows measurement of **curvature** from damping scale
- **Independently** of peak scale, confirms **flat geometry**

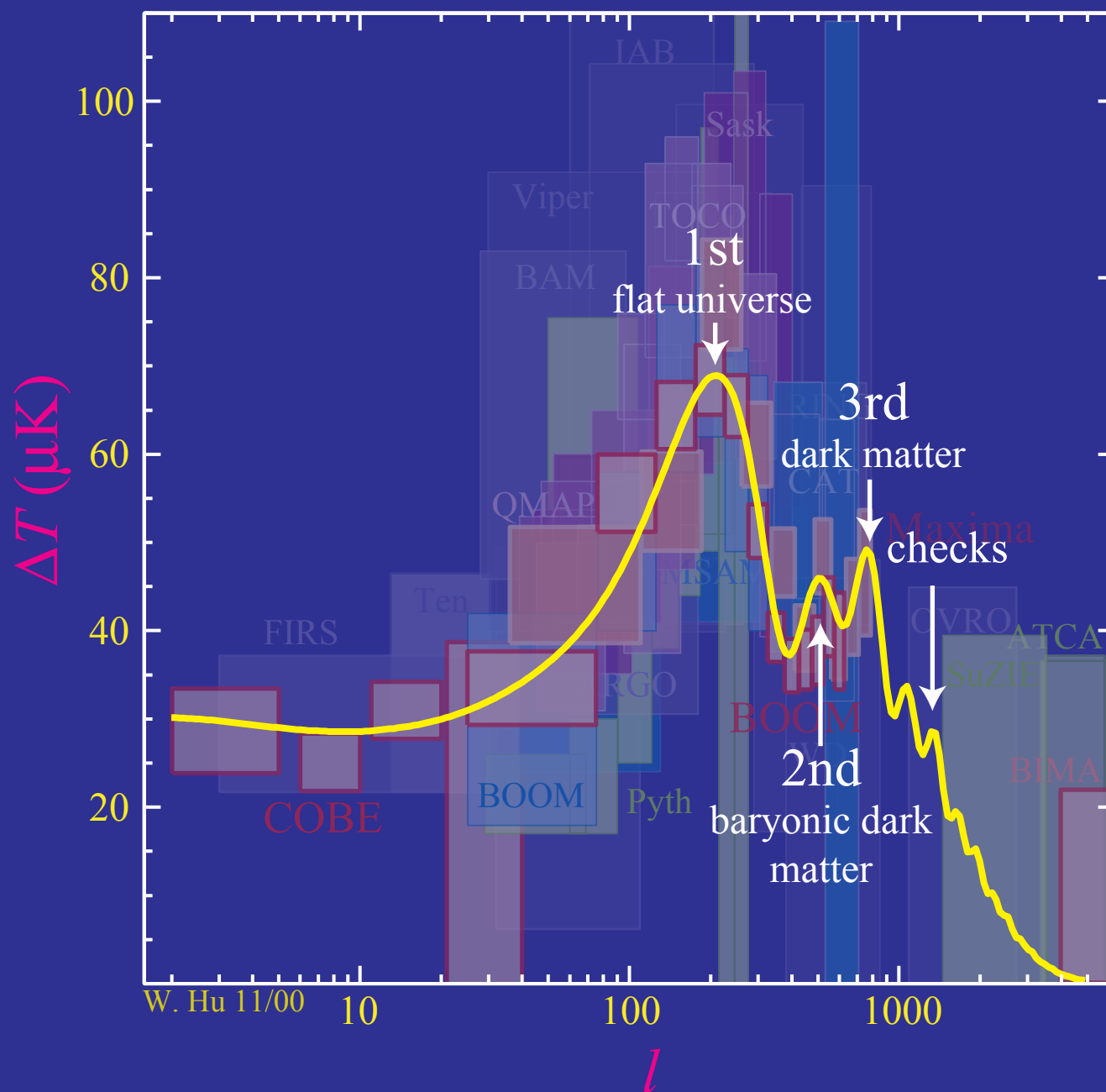


# Consistency Check on Recombination



fixed  $l_A$ ,  $\rho_b/\rho_\gamma$ ,  $\rho_m/\rho_r$

# The Peaks



# Lecture II: Summary

- Gravitational **potential redshift** combines with gravitationally induced initial perturbation to form the **Sachs-Wolfe effect**
- **Baryon loading** enhances **odd numbered peaks** so that the ratio of first to **second peak** height determines the **baryon density**
- Decay of potentials during **radiation domination** drives oscillations so that the relative peak heights across the first **three peaks** determines the **matter-radiation ratio**
- Fluid imperfections due to **viscosity** (quadrupole stresses) and **heat conduction** dissipate acoustic waves in a manner **predicted** by baryon density and matter-radiation ratio
- Strong **consistency checks** for recombination physics, angular diameter distance and source of acoustic **polarization**