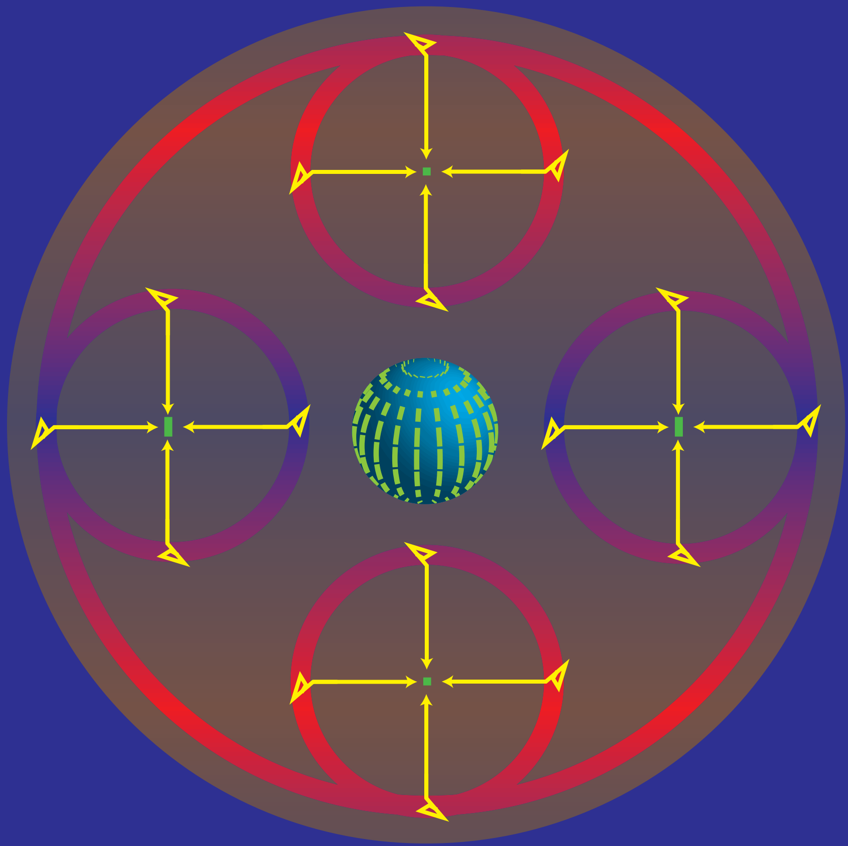
Lecture III

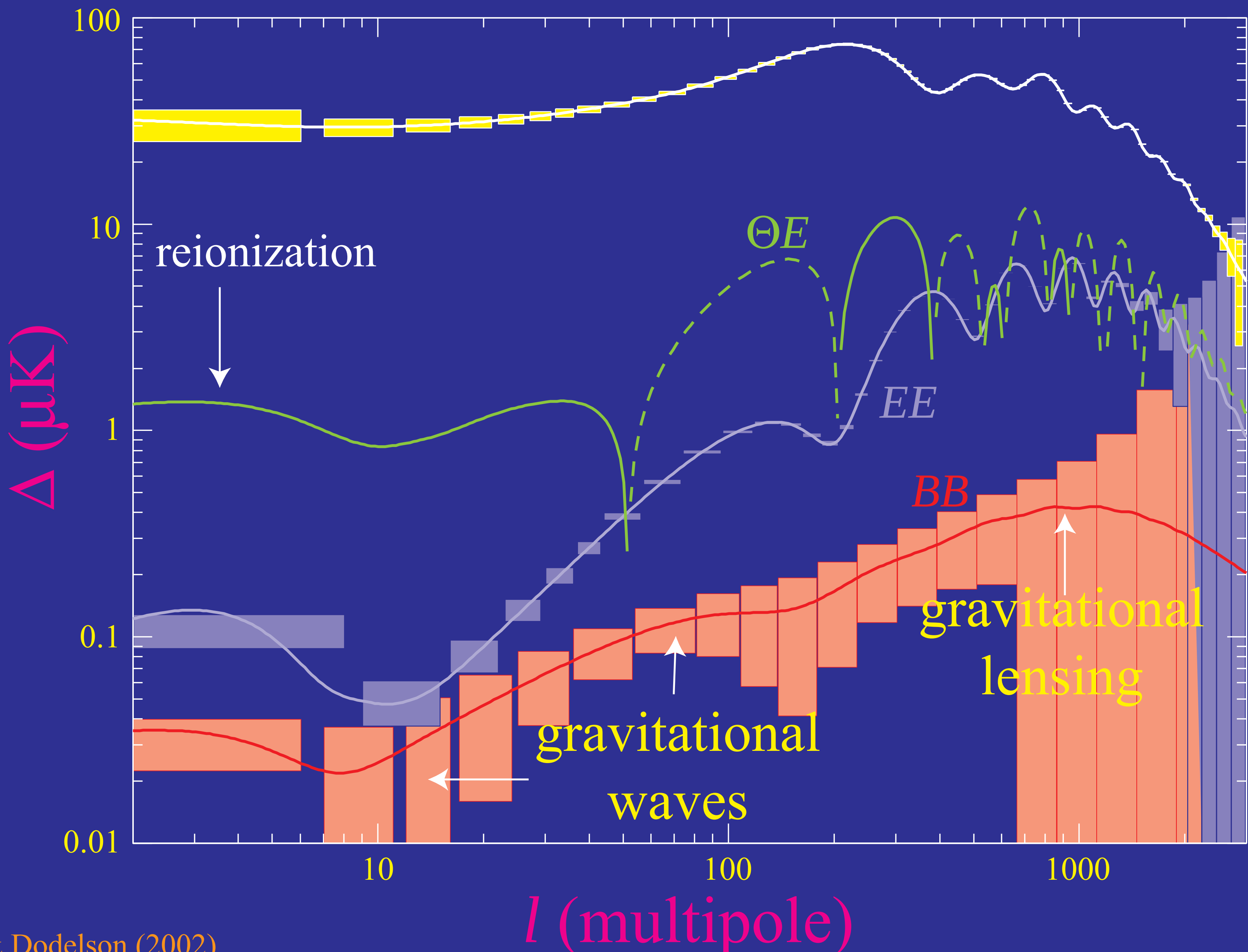


CMB Polarization

Wayne Hu

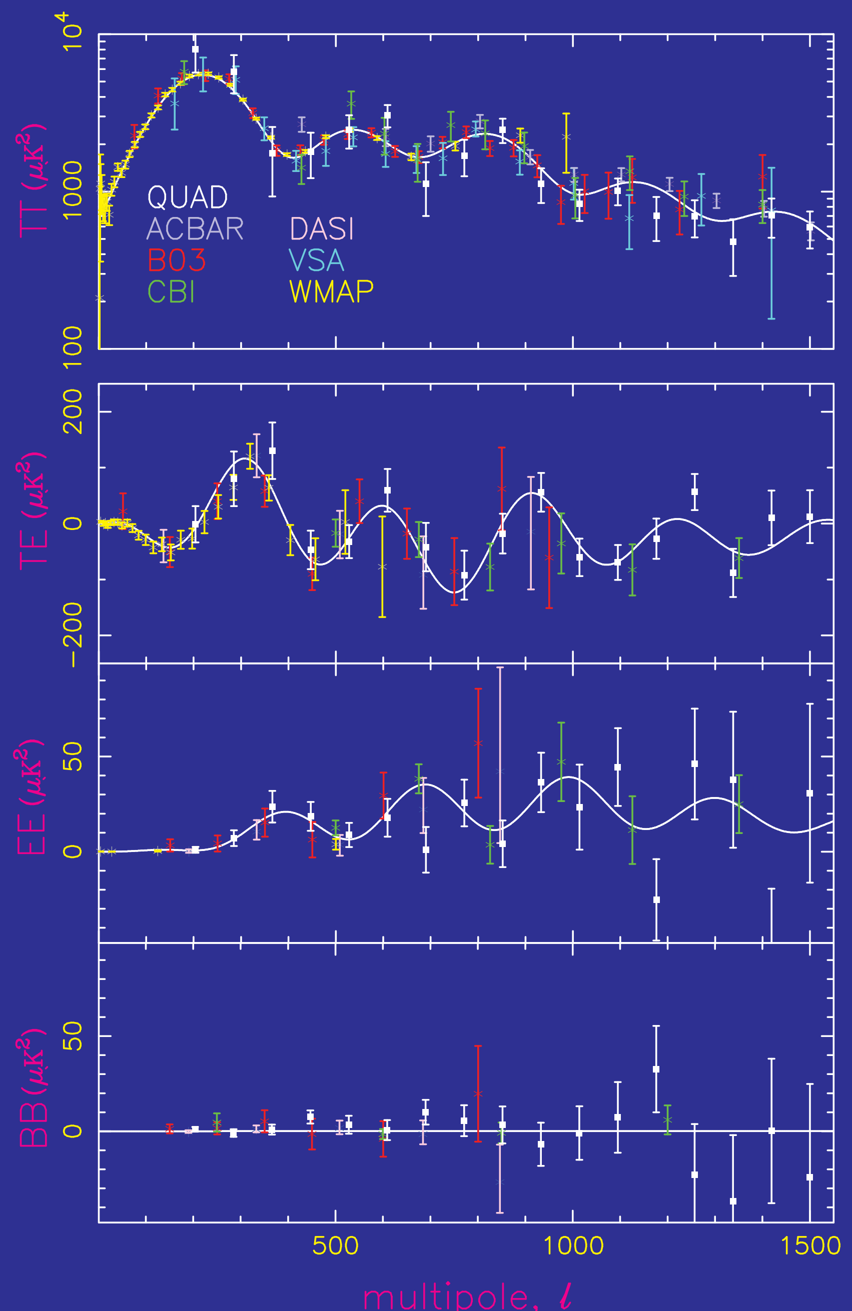
Tenerife, November 2007

Polarized Landscape



Hu & Dodelson (2002)

Recent Data

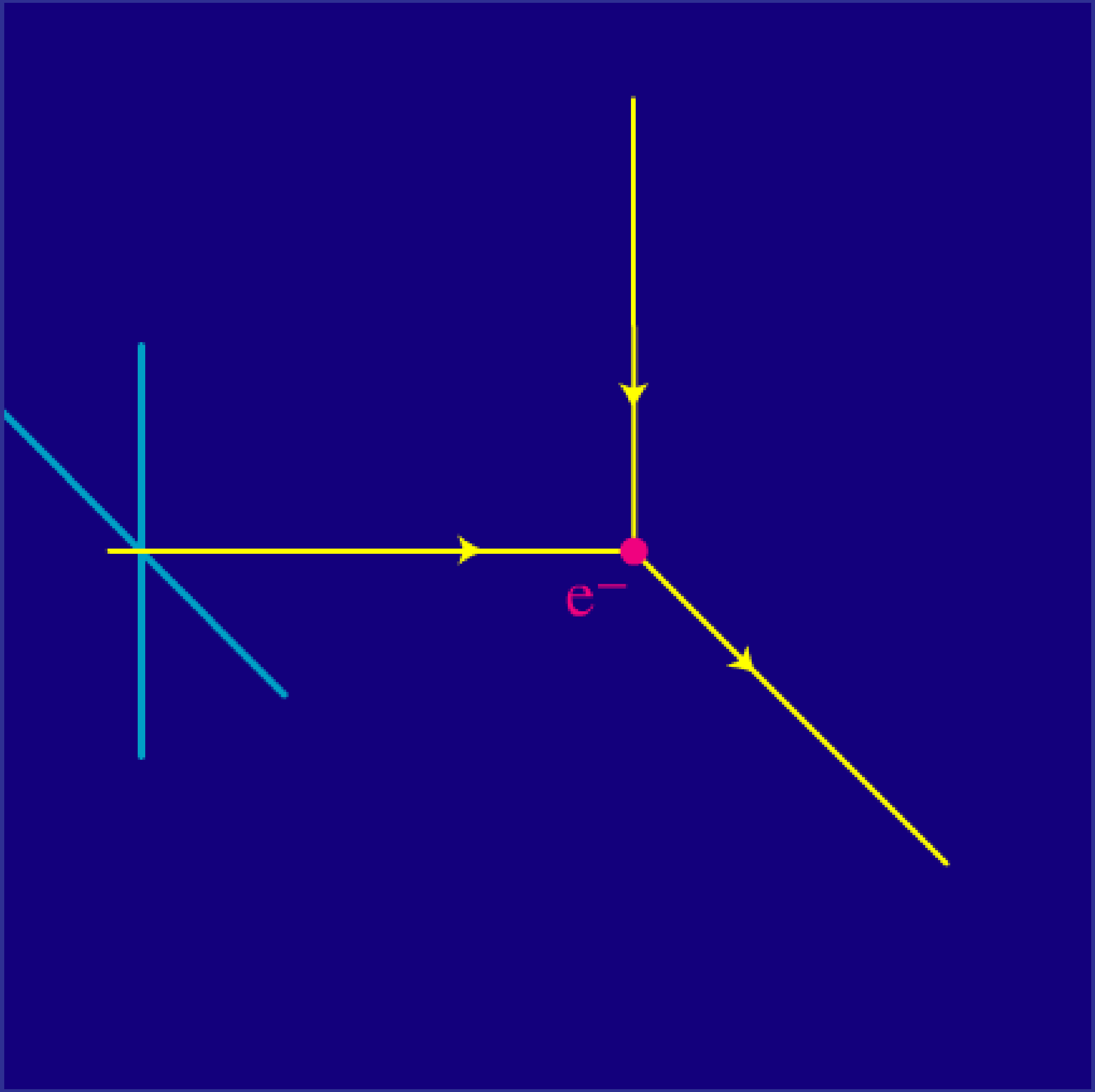


Ade et al (QUAD, 2007)

Why is the CMB polarized?

Polarization from Thomson Scattering

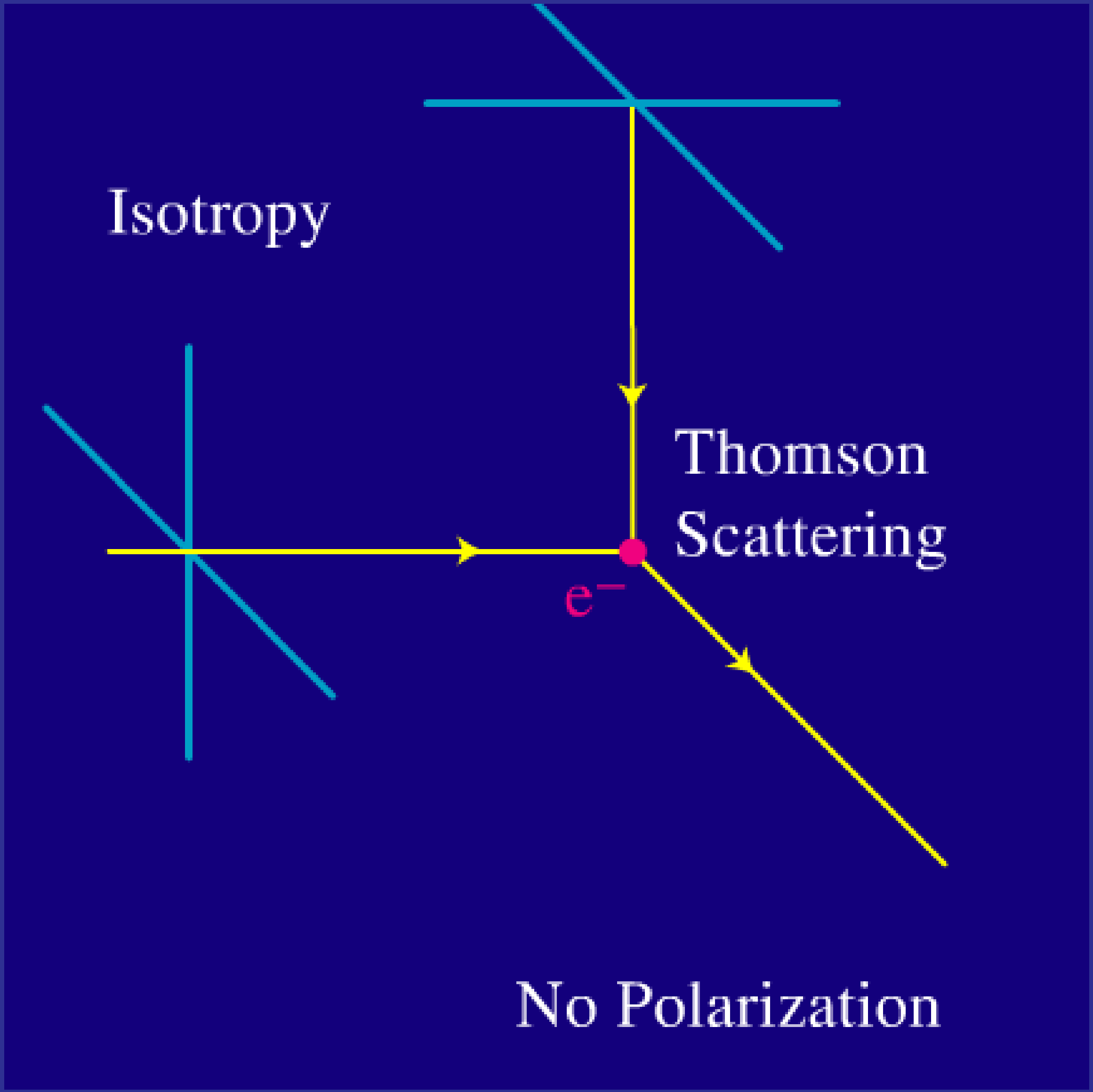
- Differential cross section depends on polarization and angle



$$\frac{d\sigma}{d\Omega} = \frac{3}{8\pi}|\hat{\varepsilon}' \cdot \hat{\varepsilon}|^2 \sigma_T$$

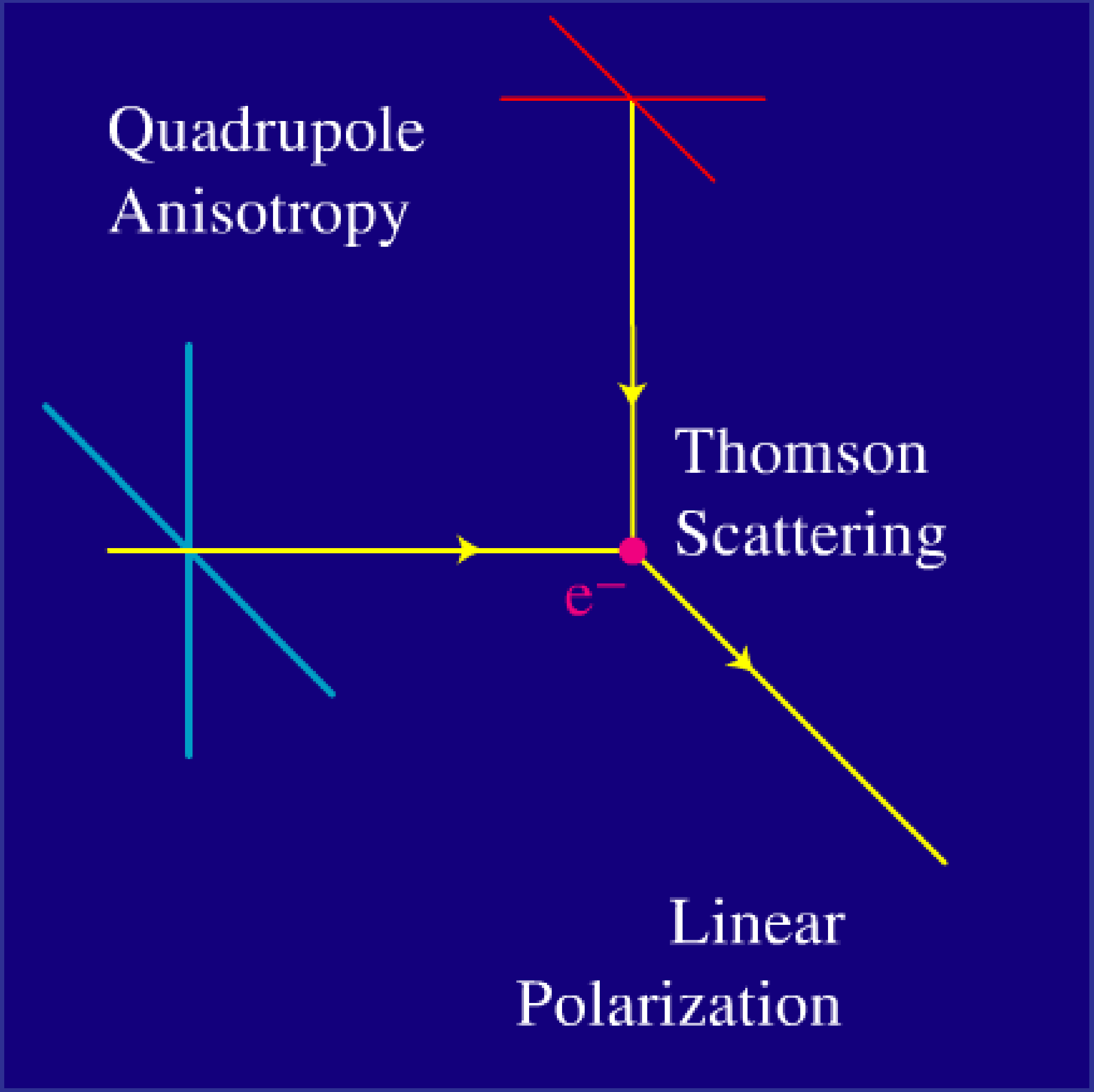
Polarization from Thomson Scattering

- Isotropic radiation scatters into unpolarized radiation



Polarization from Thomson Scattering

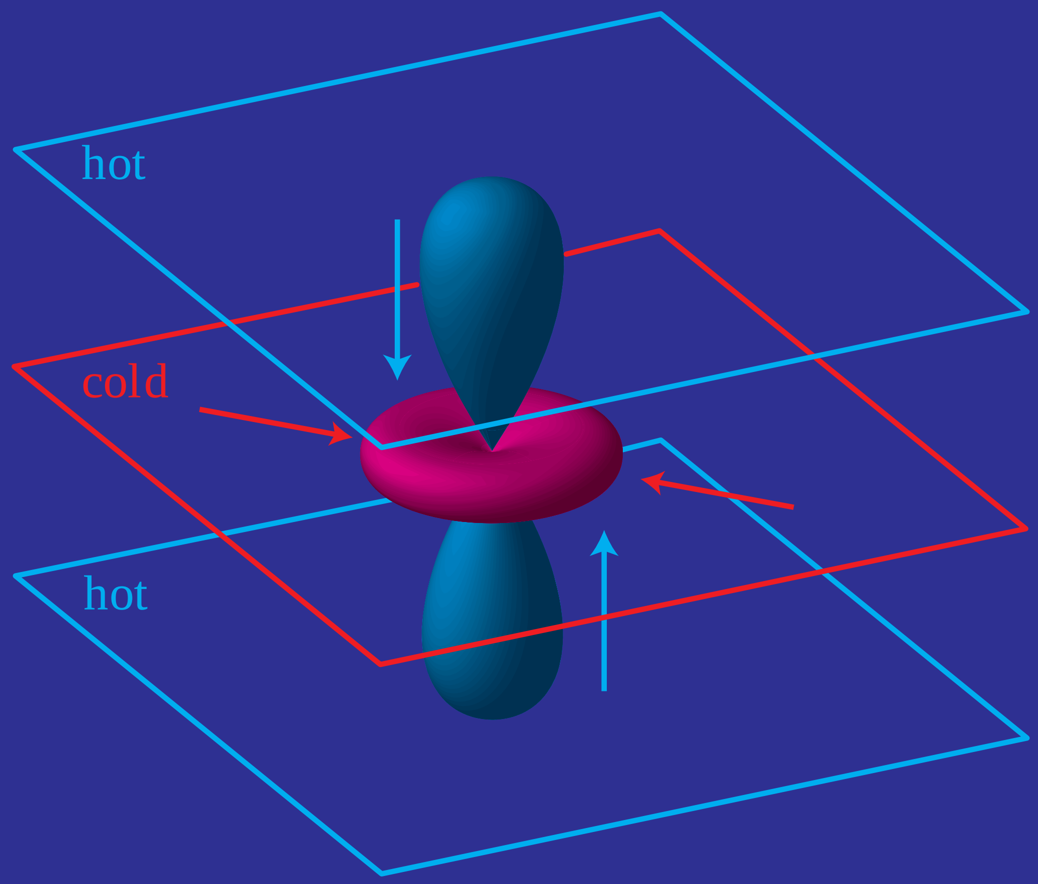
- Quadrupole anisotropies scatter into linear polarization



aligned with
cold lobe

Whence Quadrupoles?

- Temperature inhomogeneities in a medium
- Photons arrive from different regions producing an anisotropy

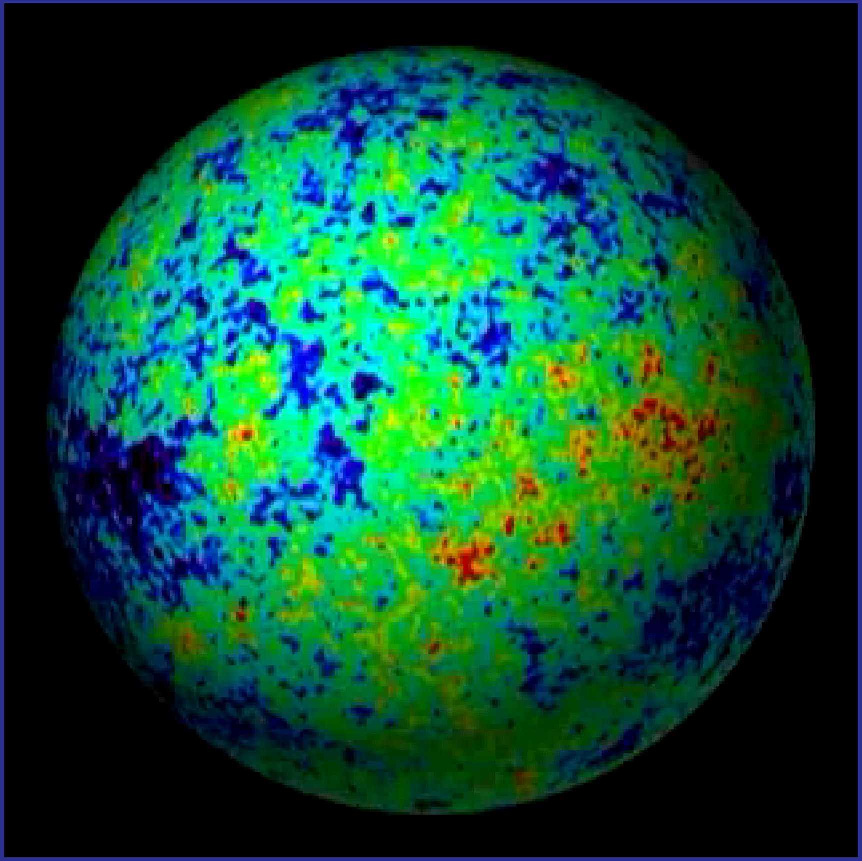


(Scalar) Temperature Inhomogeneity

Hu & White (1997)

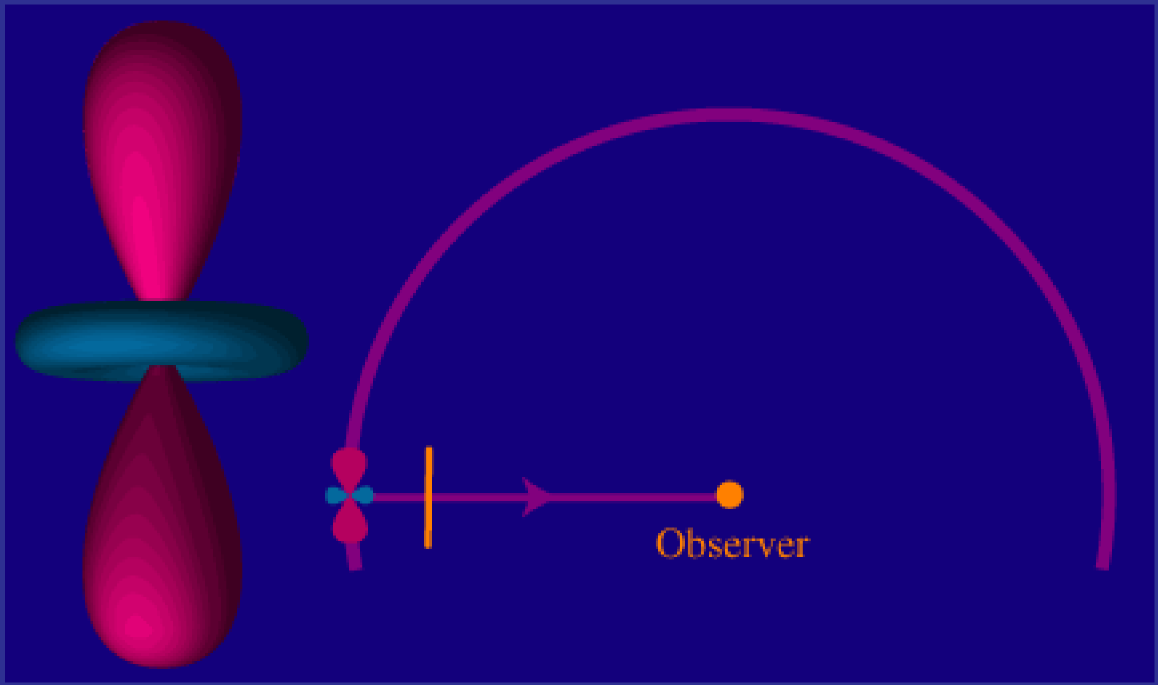
CMB Anisotropy

- WMAP map of the CMB temperature anisotropy



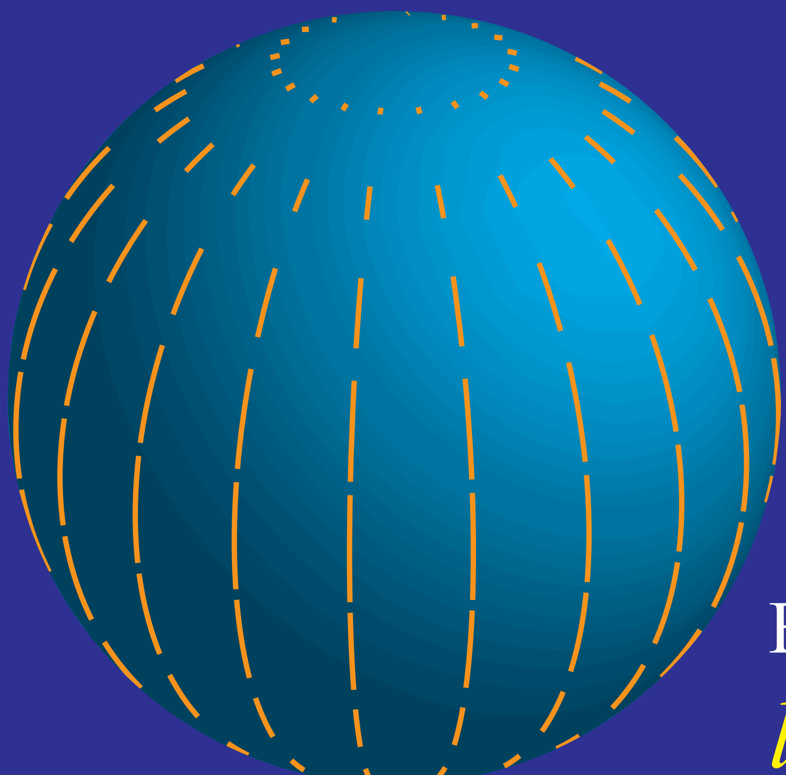
Whence Polarization Anisotropy?

- Observed photons scatter into the line of sight
- Polarization arises from the projection of the quadrupole on the transverse plane



Polarization Multipoles

- Mathematically pattern is described by the tensor (spin-2) spherical harmonics [eigenfunctions of Laplacian on trace-free 2 tensor]
- Correspondence with scalar spherical harmonics established via Clebsch-Gordan coefficients (spin x orbital)
- Amplitude of the coefficients in the spherical harmonic expansion are the multipole moments; averaged square is the power

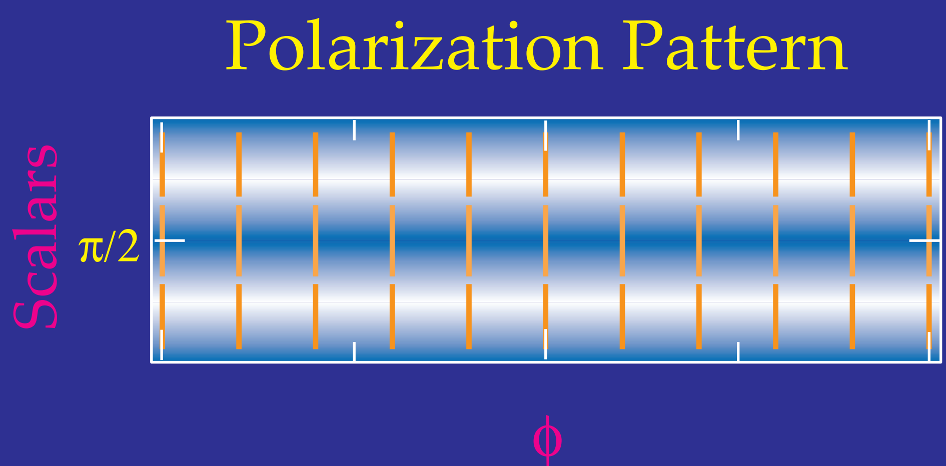


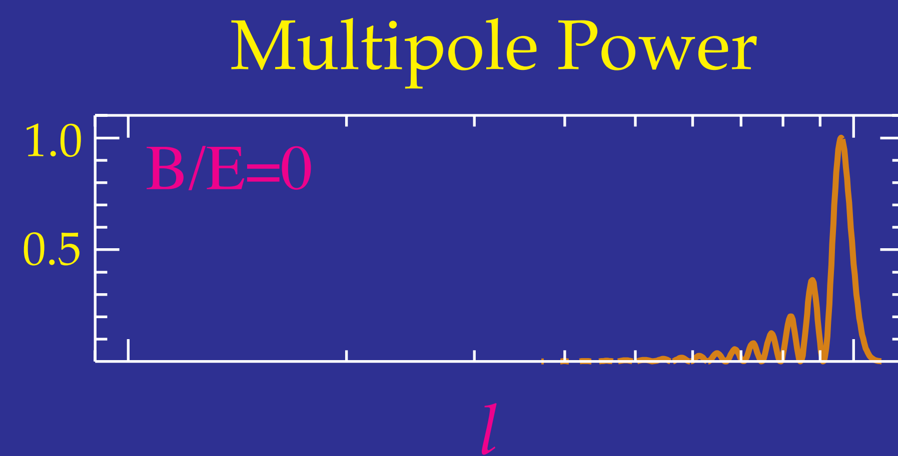
E-tensor harmonic

$l=2,\ m=0$

Modulation by Plane Wave

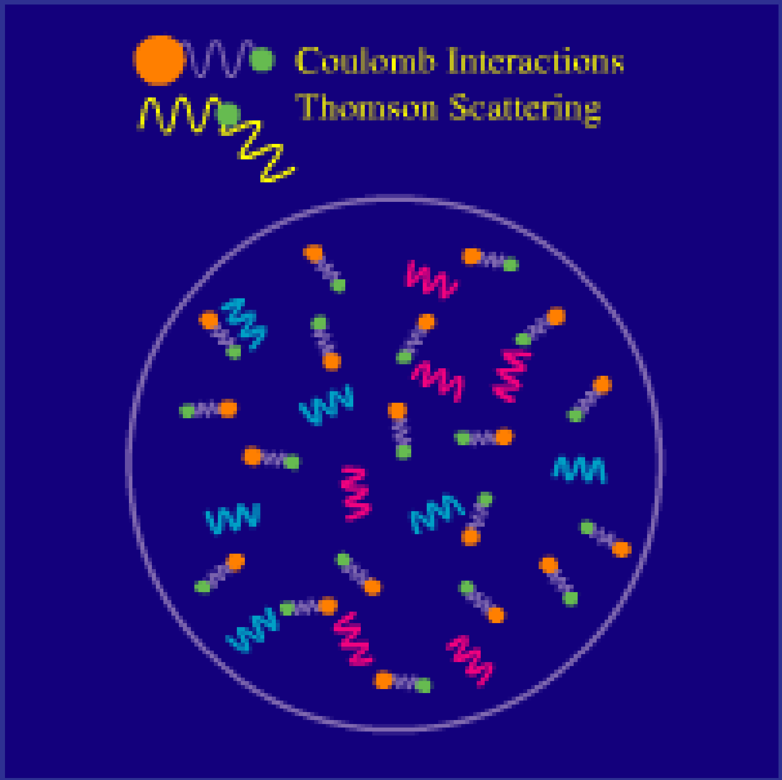
- Amplitude modulated by plane wave → higher multipole moments
- Direction detemined by perturbation type → E-modes





A Catch-22

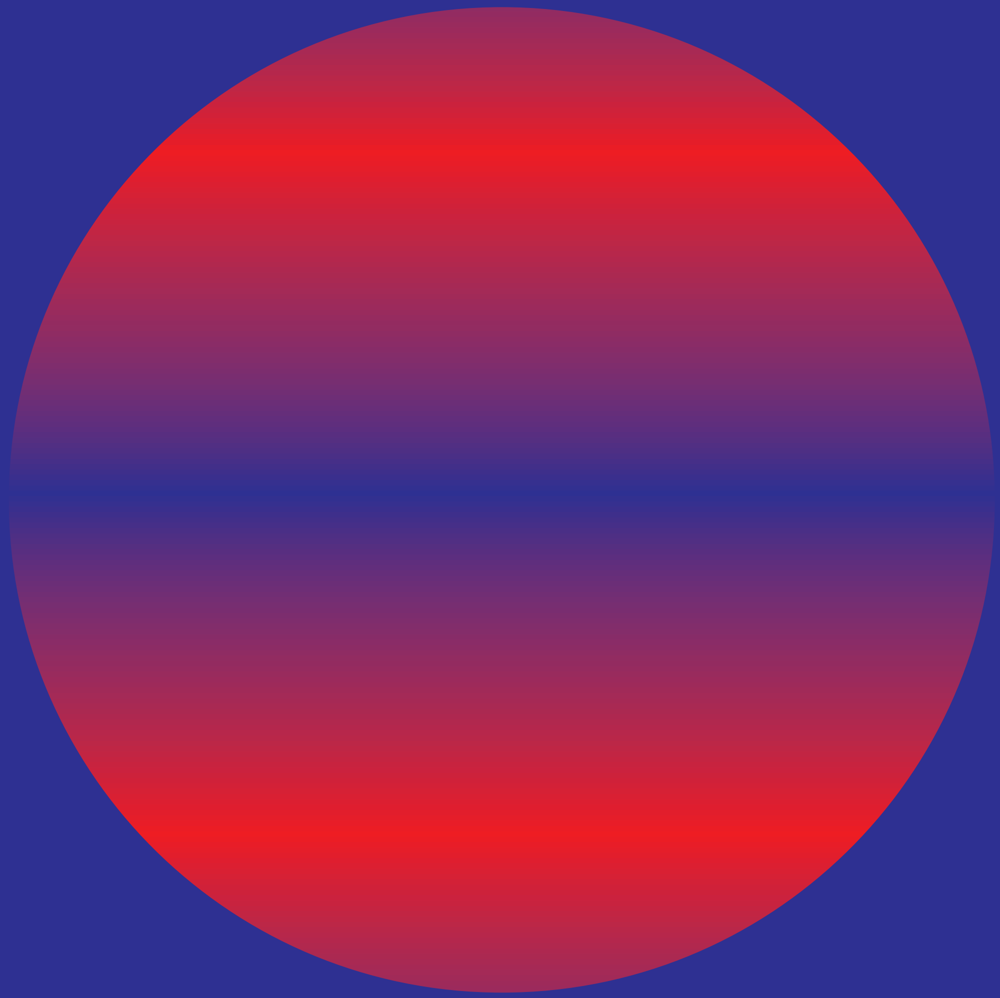
- Polarization is generated by scattering of anisotropic radiation
- Scattering isotropizes radiation
- Polarization only arises in optically thin conditions: reionization and end of recombination
- Polarization fraction is at best a small fraction of the 10^{-5} anisotropy: $\sim 10^{-6}$ or μK in amplitude



Reionization

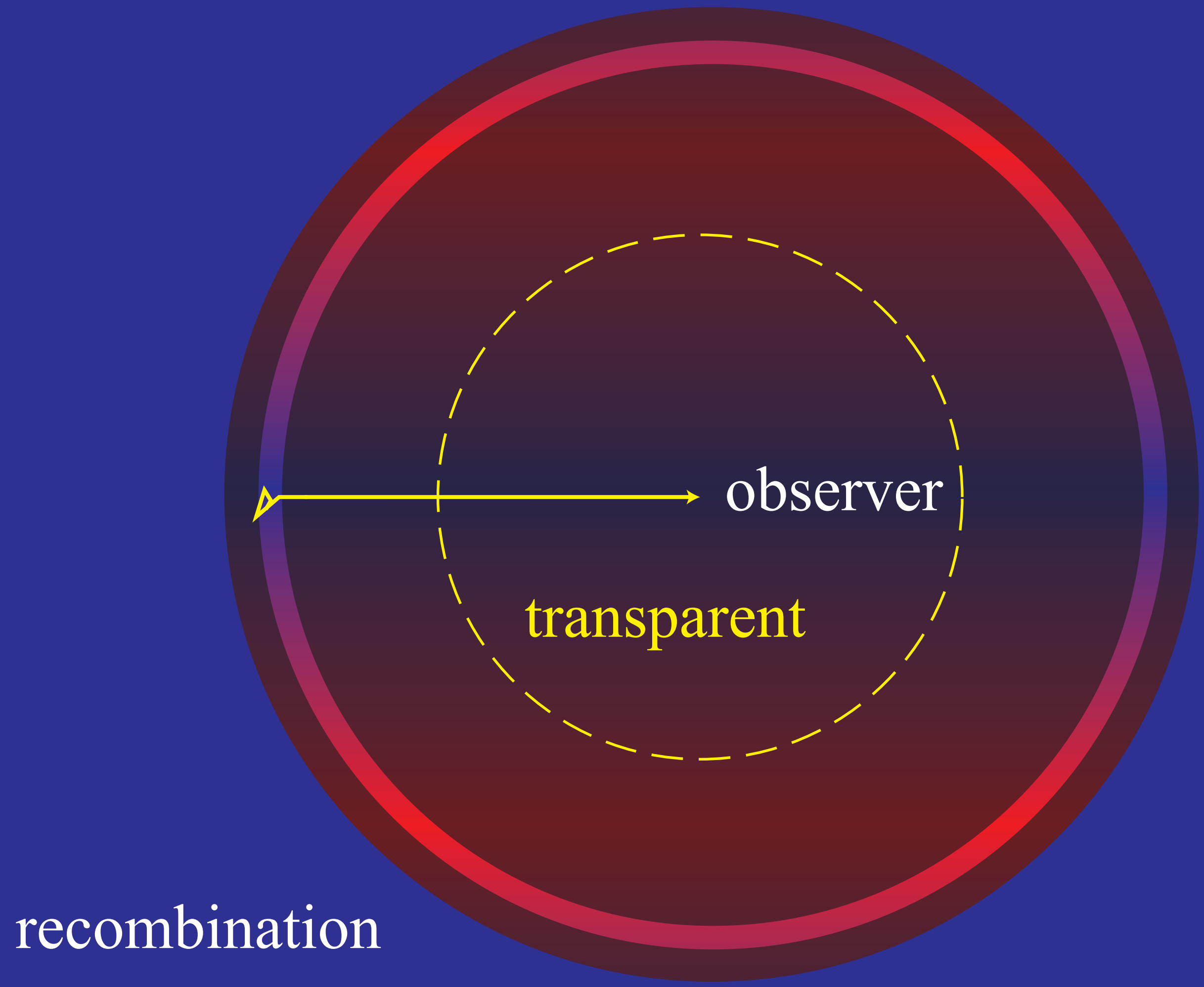
Temperature Inhomogeneity

- Temperature inhomogeneity reflects initial density perturbation on large scales
- Consider a single Fourier moment:



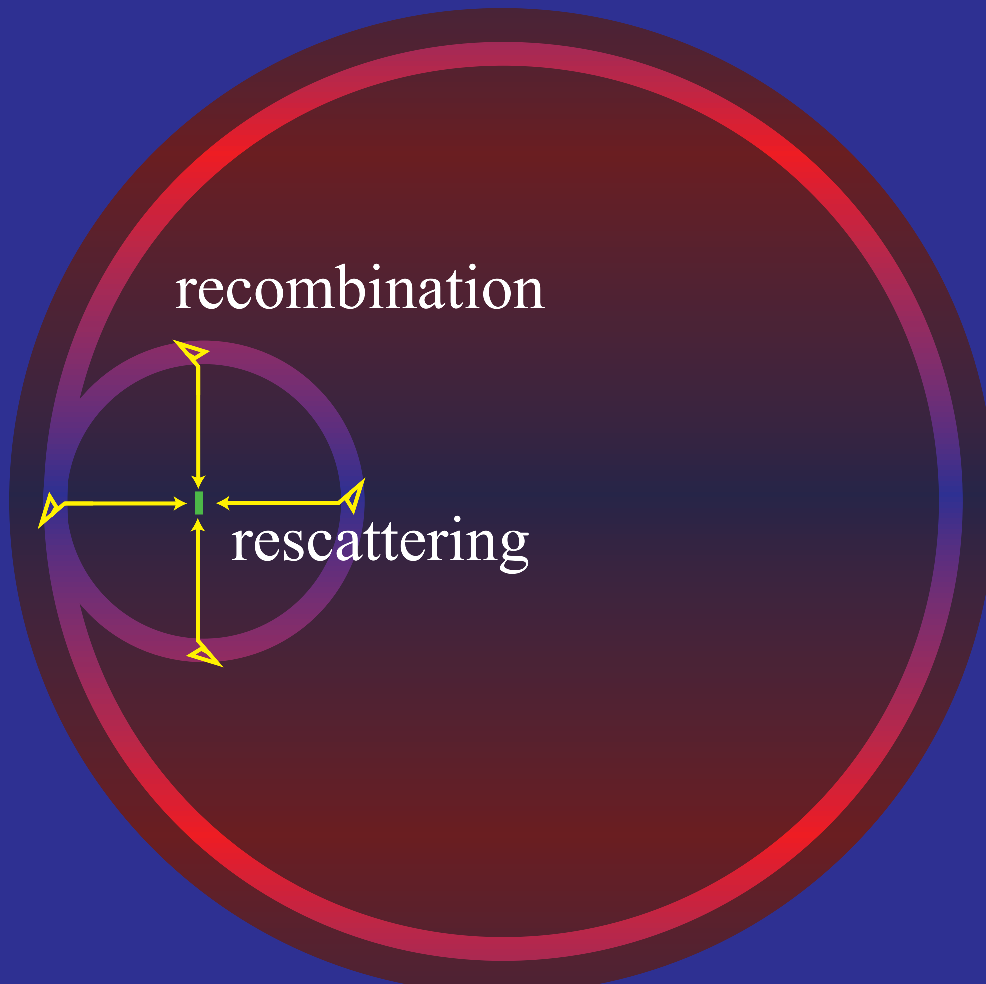
Locally Transparent

- Presently, the matter density is so low that a typical CMB photon will not scatter in a Hubble time (~age of universe)



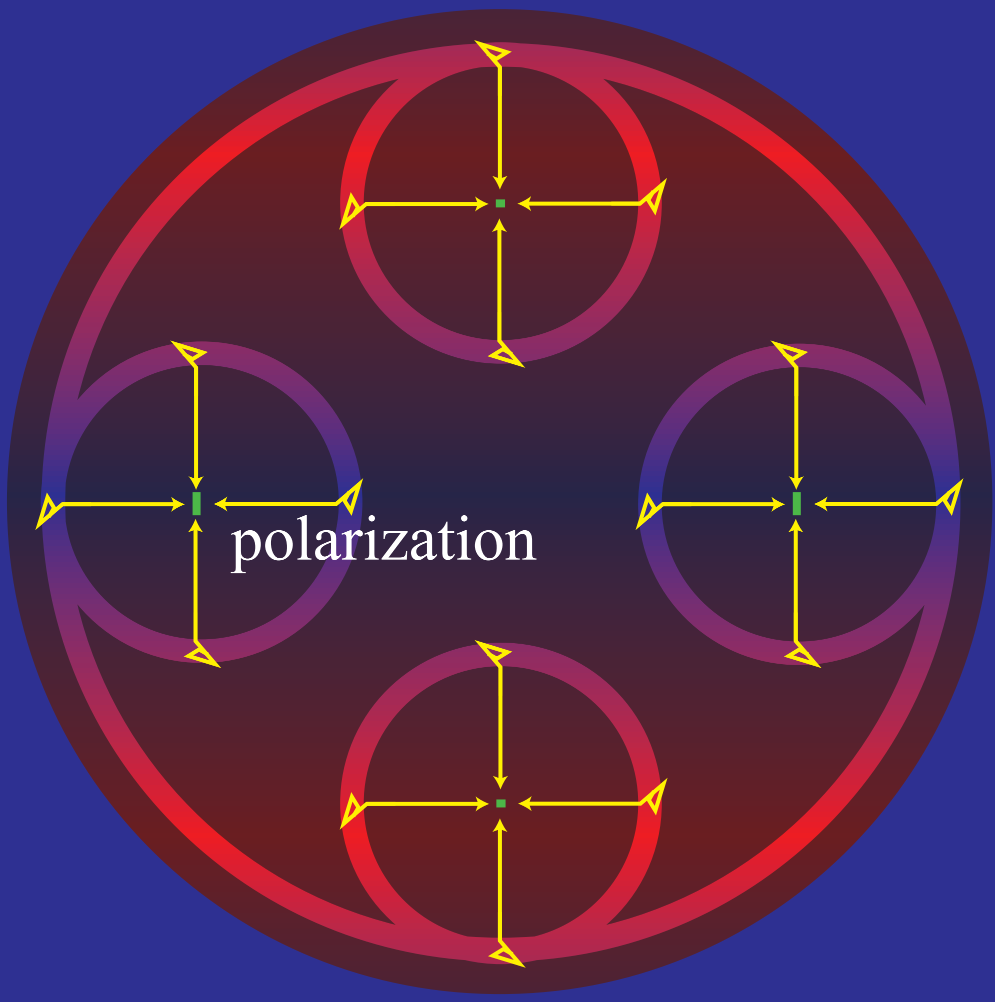
Reversed Expansion

- Free electron density in an ionized medium increases as scale factor a^{-3}; when the universe was a tenth of its current size CMB photons have a finite (~10%) chance to scatter



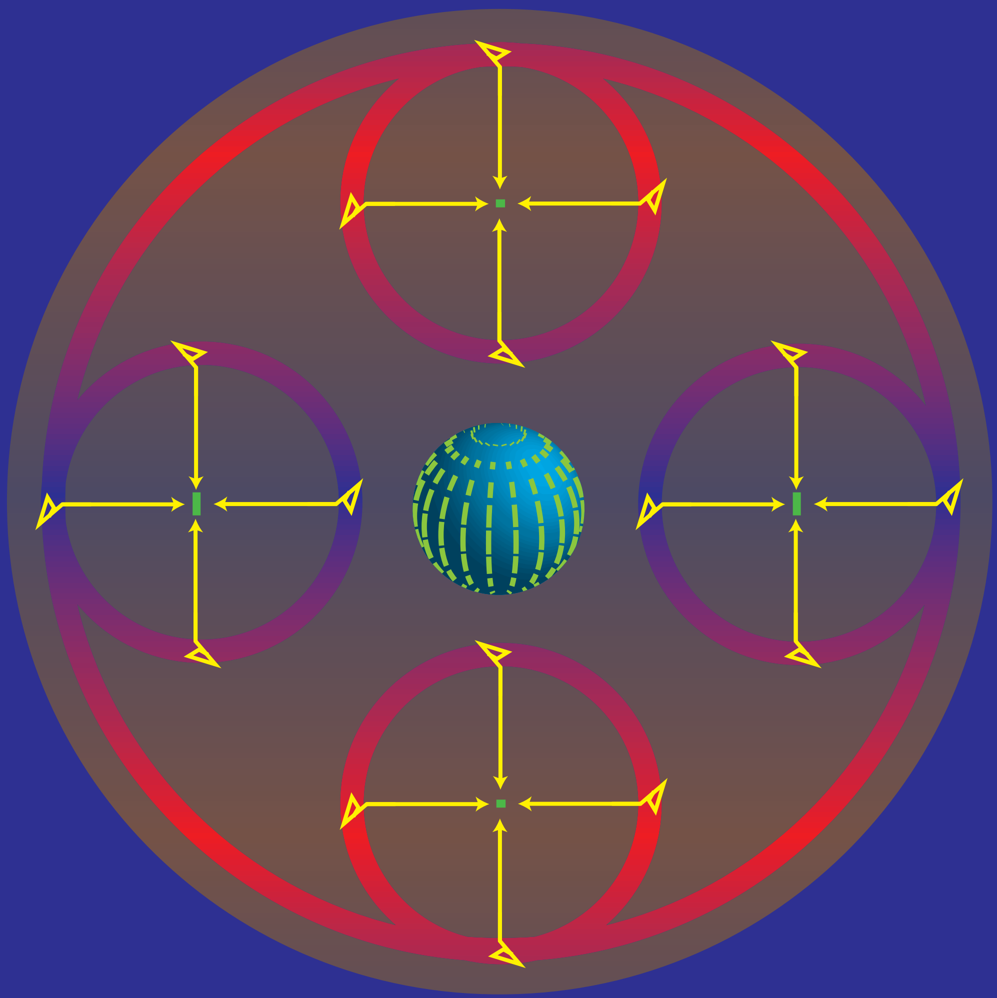
Polarization Anisotropy

- Electron sees the temperature anisotropy on its recombination surface and scatters it into a polarization

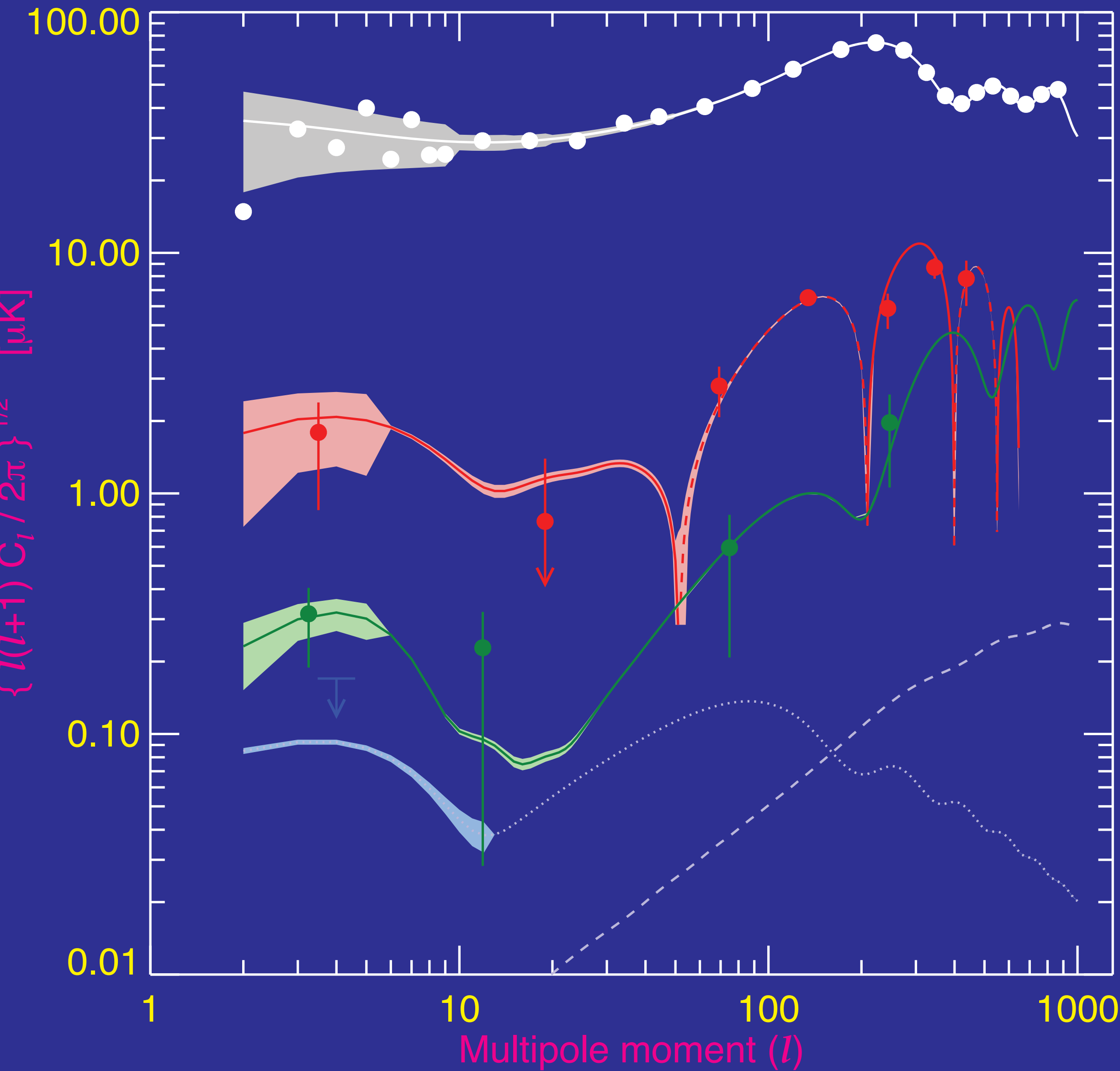


Temperature Correlation

- Pattern correlated with the temperature anisotropy that generates it; here an m=0 quadrupole



WMAP 3year



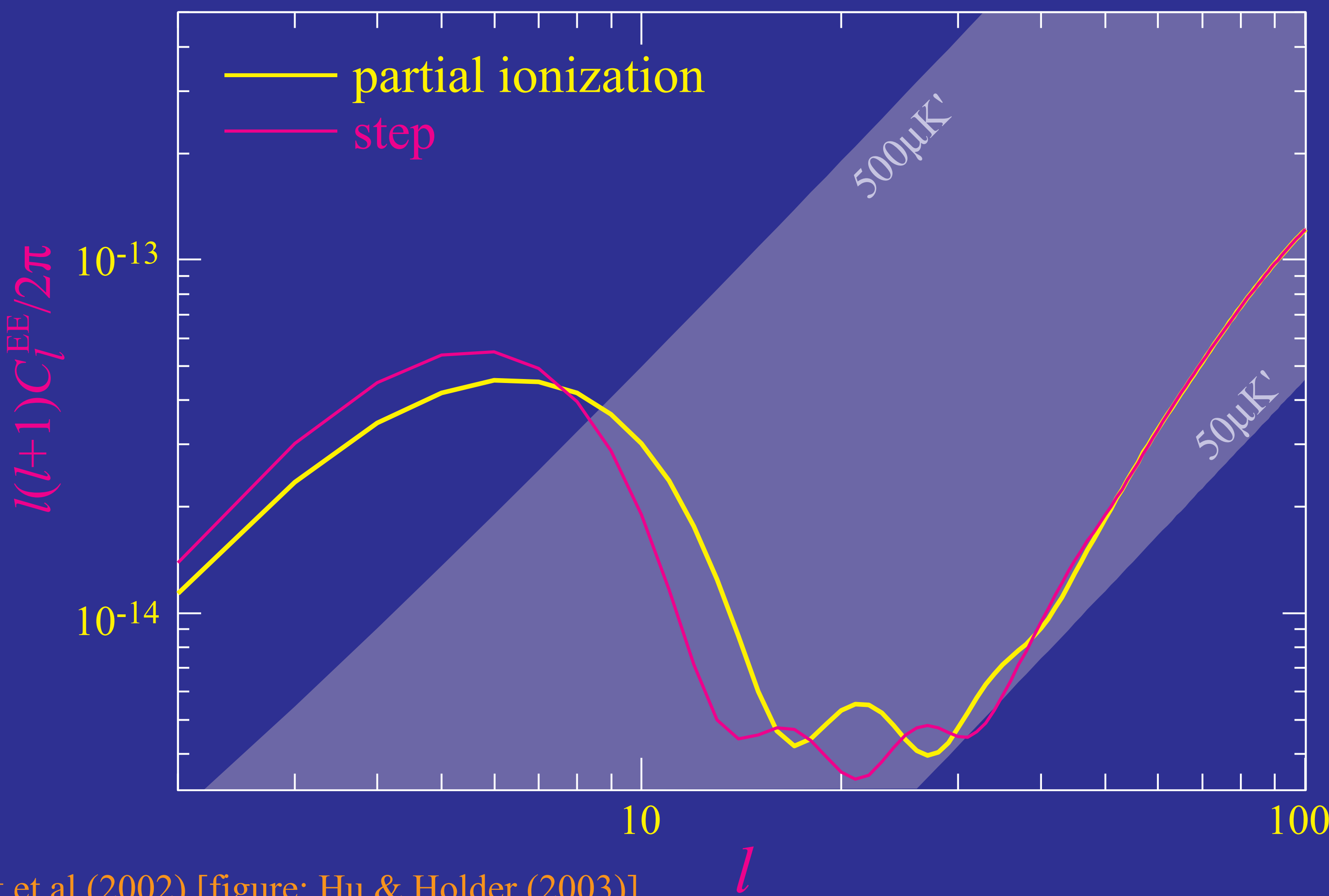
Page et al (WMAP, 2006)

Why Care?

- Early ionization is puzzling if due to ionizing radiation from normal stars; may indicate more exotic physics is involved
- Reionization screens temperature anisotropy on small scales making the true amplitude of initial fluctuations larger by e^{τ}
- Measuring the growth of fluctuations is one of the best ways of determining the neutrino masses and the dark energy
- Offers an opportunity to study the origin of the low multipole statistical anomalies
- Presents a second, and statistically cleaner, window on gravitational waves from the early universe

Polarization Power Spectrum

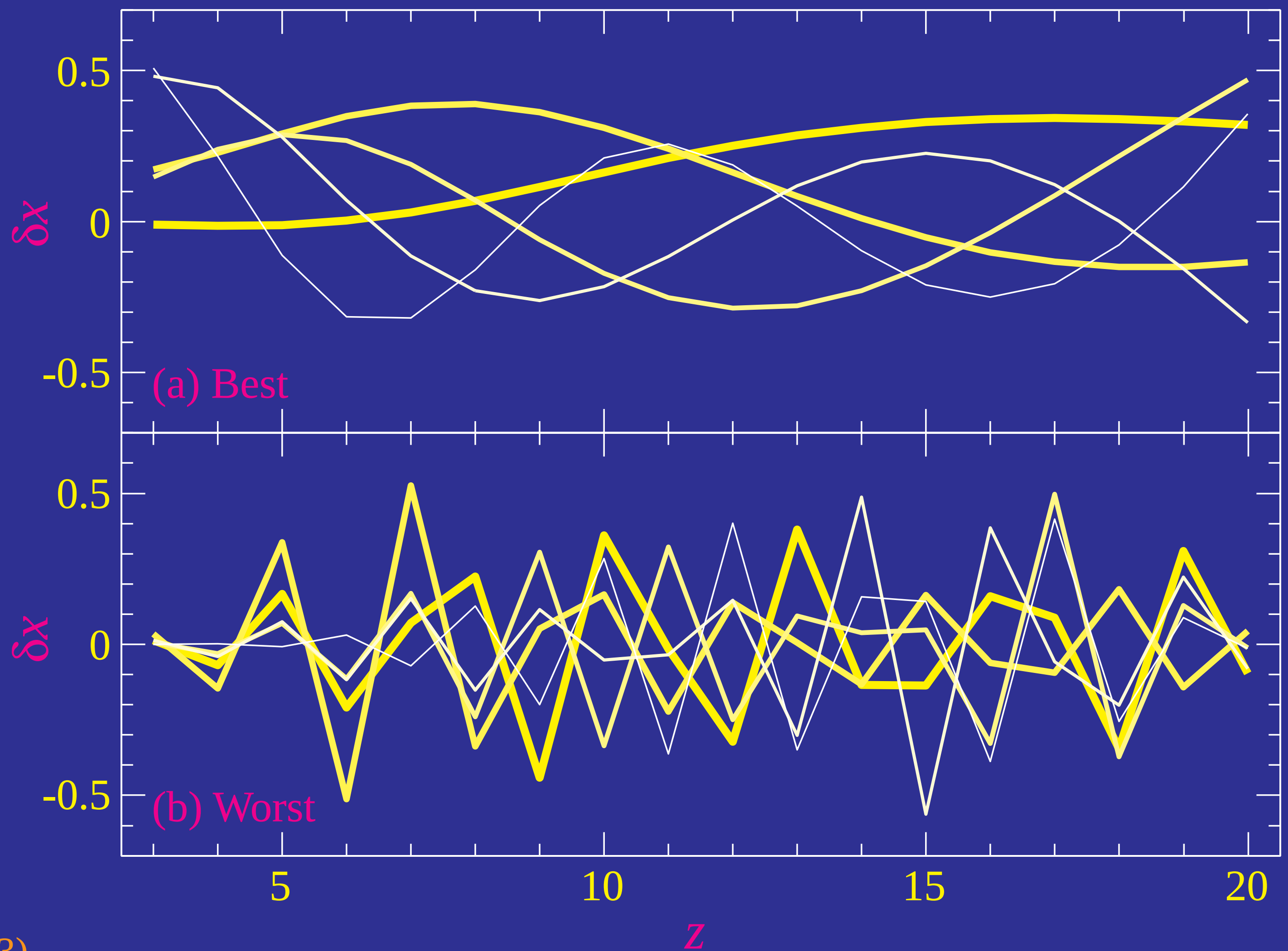
- Most of the information on ionization history is in the polarization (auto) power spectrum - two models with same optical depth but different ionization fraction



Kaplinghat et al (2002) [figure: Hu & Holder (2003)]

Principal Components

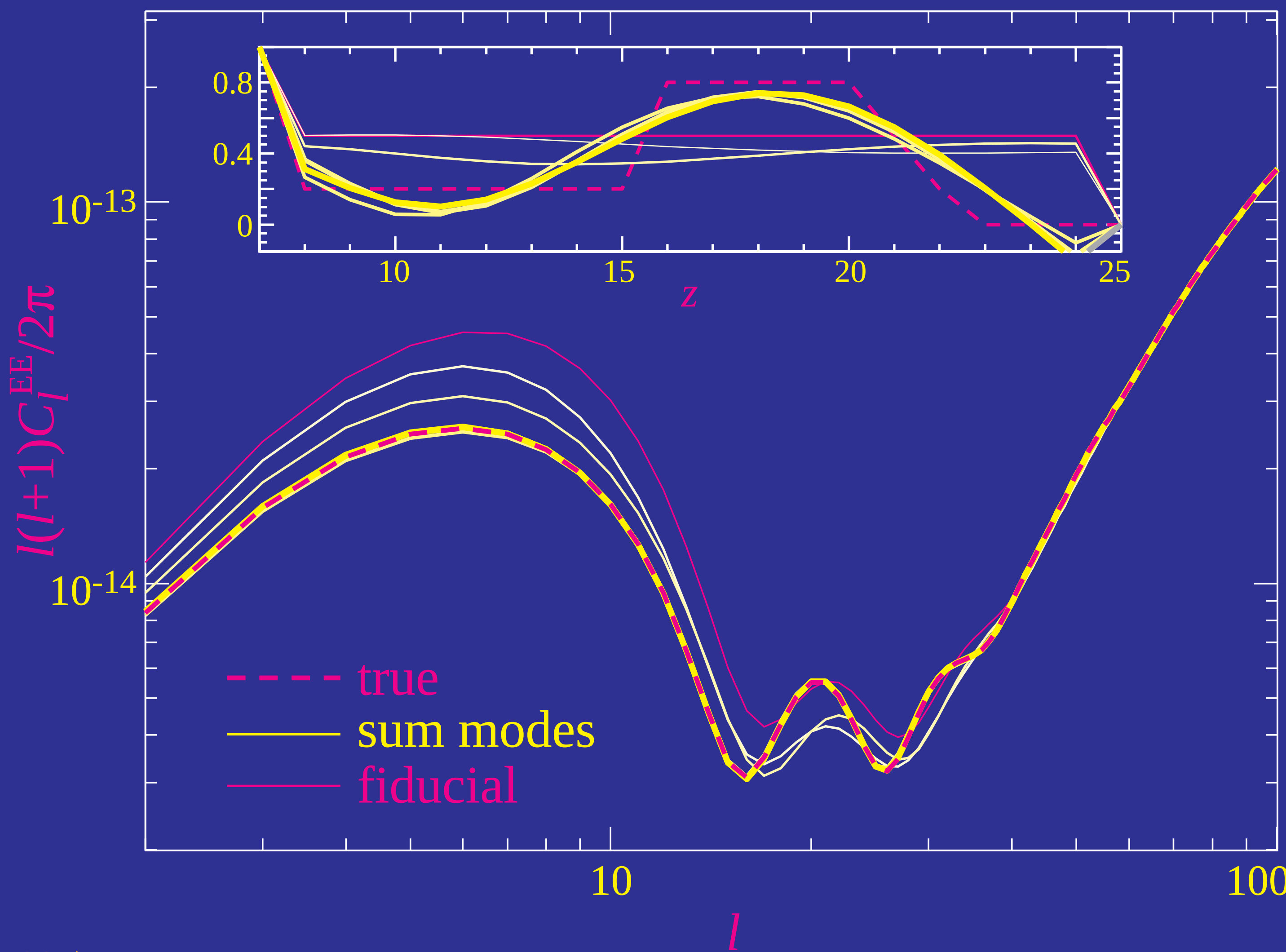
- Information on the ionization history is contained in ~5 numbers
  - essentially coefficients of first few Fourier modes



Hu & Holder (2003)

Representation in Modes

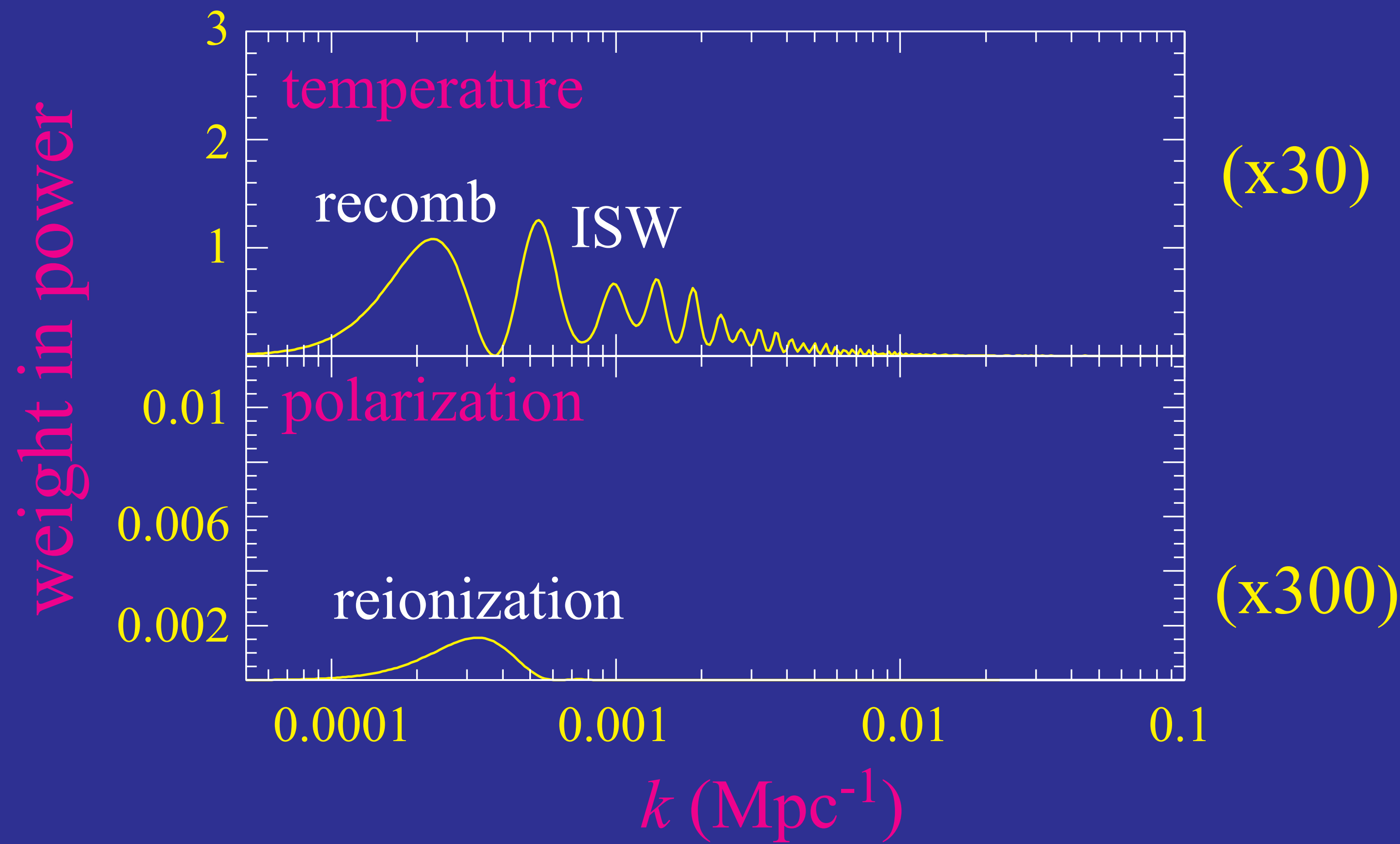
- Reproduces the power spectrum and net optical depth (actual τ=0.1375 vs 0.1377); indicates whether multiple physical mechanisms suggested



Hu & Holder (2003)

Temperature v. Polarization

- Quadrupole in polarization originates from a tight range of scales around the current horizon
- Quadrupole in temperature gets contributions from 2 decades in scale

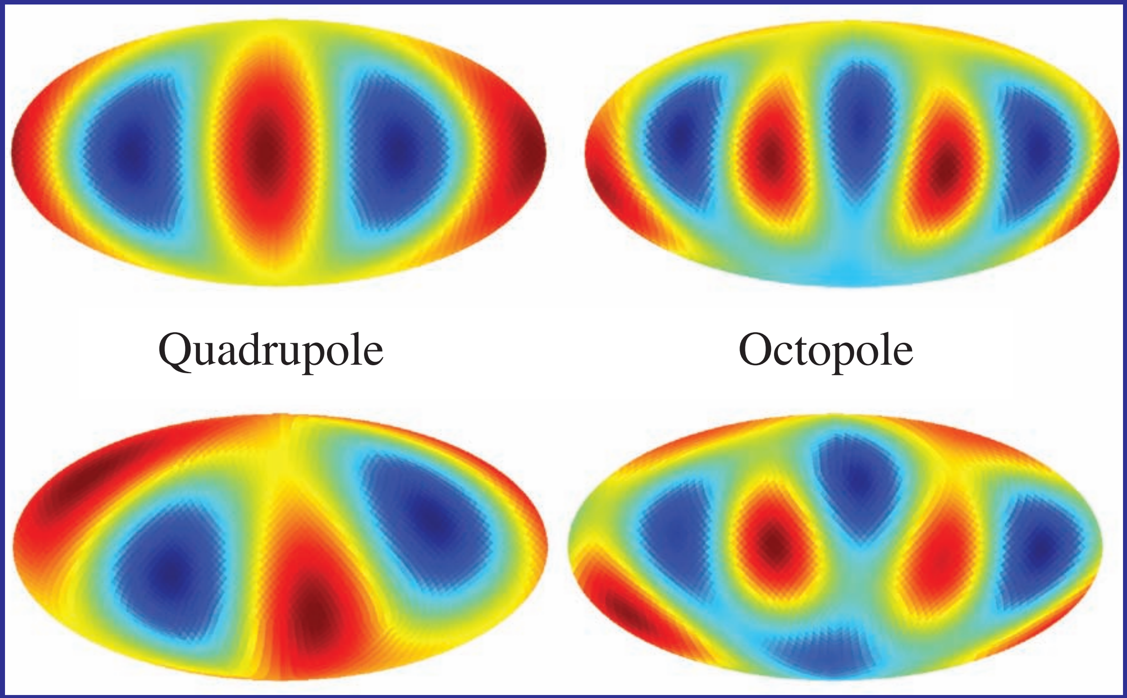


Hu & Okamoto (2003)

Alignments

Temperature

E-polarization

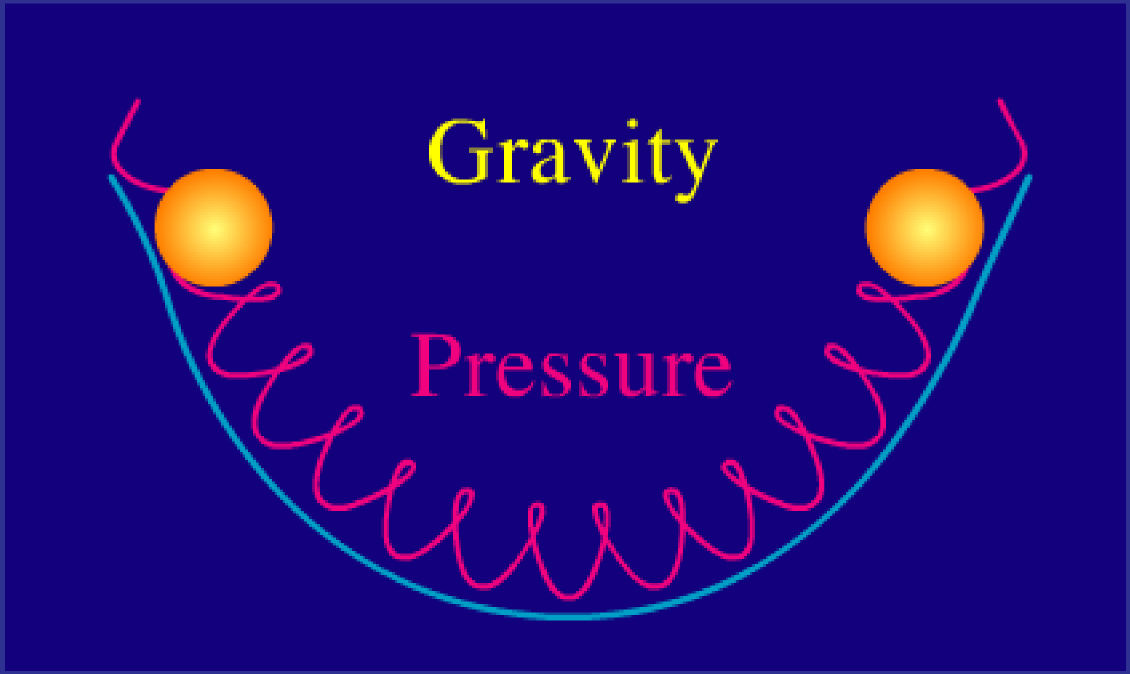


Dvorkin, Peiris, Hu (2007)

Polarization Peaks

Acoustic Oscillations

- When T>3000K, medium ionized
- Photons tightly coupled to free electrons via Thomson scattering; electrons to protons via Coulomb interactions
- Medium behaves as a perfect fluid
- Radiation pressure competes with gravitational attraction causing perturbations to oscillate



Quadrupoles at Recombination's End

- Acoustic inhomongeneities become anisotropies by streaming/diffusion

Quadrupoles at Recombination's End

- Electron "observer" sees a quadrupole anisotropy
- Polarization pattern is a projection quadrupole anisotropy

Fluid Imperfections

- Perfect fluid: no anisotropic stresses due to scattering isotropization; baryons and photons move as single fluid
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

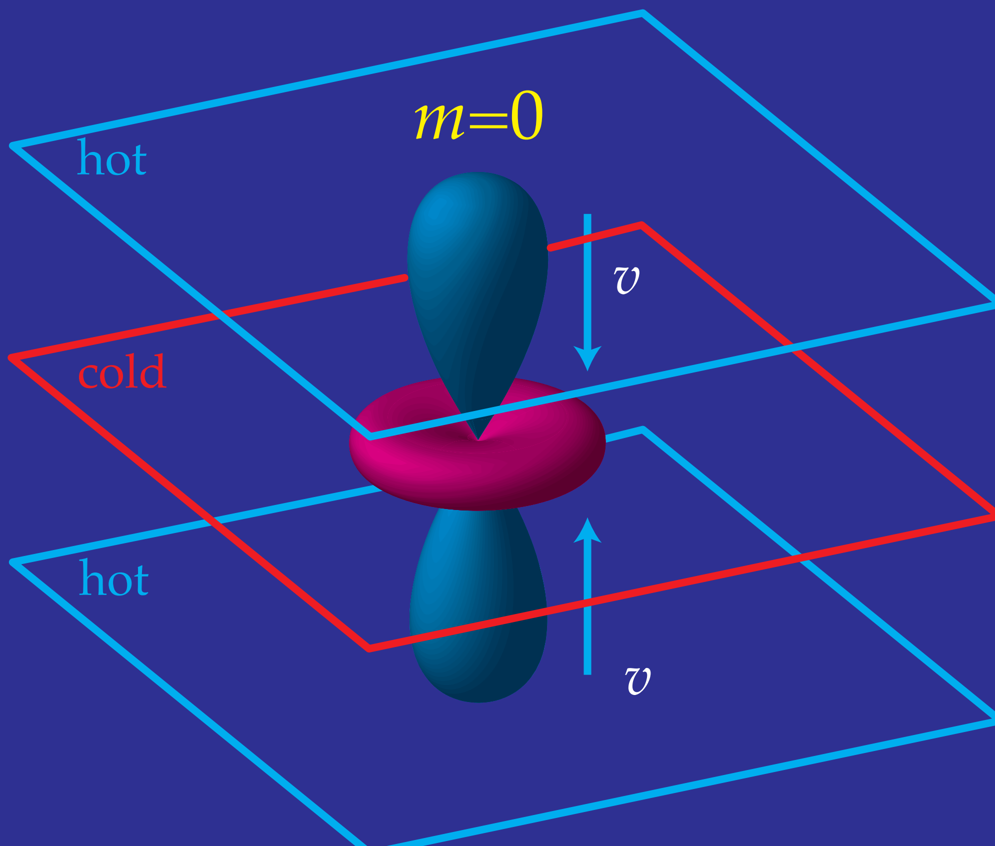
  is the conformal opacity to Thomson scattering
- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

  the geometric mean between the horizon and mean free path
- $\lambda_D/\eta_* \sim$ few %, so expect the peaks >3 to be affected by dissipation

Viscosity & Heat Conduction

- Both fluid imperfections are related to the gradient of the velocity kv_γ by opacity $\dot{\tau}$: slippage of fluids $v_\gamma - v_b$.
- Viscosity is an anisotropic stress or quadrupole moment formed by radiation streaming from hot to cold regions



Dimensional Analysis

- Viscosity= quadrupole anisotropy that follows the fluid velocity

$$\pi_\gamma \approx \frac{k}{\dot\tau} v_\gamma$$

- Mean free path related to the damping scale via the random walk $k_D = (\dot\tau/\eta_*)^{1/2} \to \dot\tau = k_D^2 \eta_*$
- Damping scale at $\ell \sim 1000$ vs horizon scale at $\ell \sim 100$ so $k_D \eta_* \approx 10$
- Polarization amplitude rises to the damping scale to be $\sim 10\%$ of anisotropy

$$\pi_\gamma \approx \frac{k}{k_D} \frac{1}{10} v_\gamma \qquad \Delta_P \approx \frac{\ell}{\ell_D} \frac{1}{10} \Delta_T$$

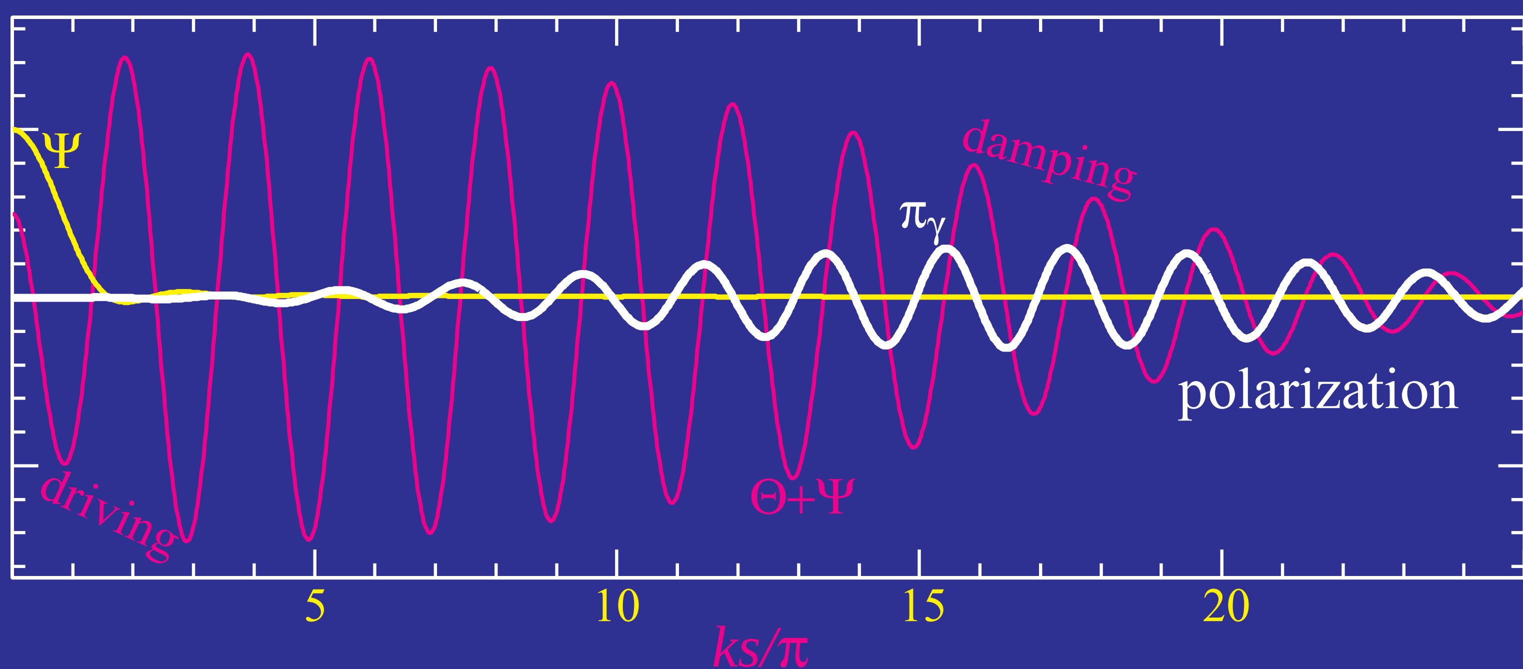
- Polarization phase follows fluid velocity

Damping & Polarization

- Quadrupole moments:

  damp acoustic oscillations from fluid viscosity
  generates polarization from scattering

- Rise in polarization power coincides with fall in temperature power – $l \sim 1000$



Acoustic Polarization

- Gradient of velocity is along direction of wavevector, so polarization is pure E-mode
- Velocity is 90° out of phase with temperature – turning points of oscillator are zero points of velocity:

$$\Theta + \Psi \propto \cos(ks); \quad v_\gamma \propto \sin(ks)$$

- Polarization peaks are at troughs of temperature power

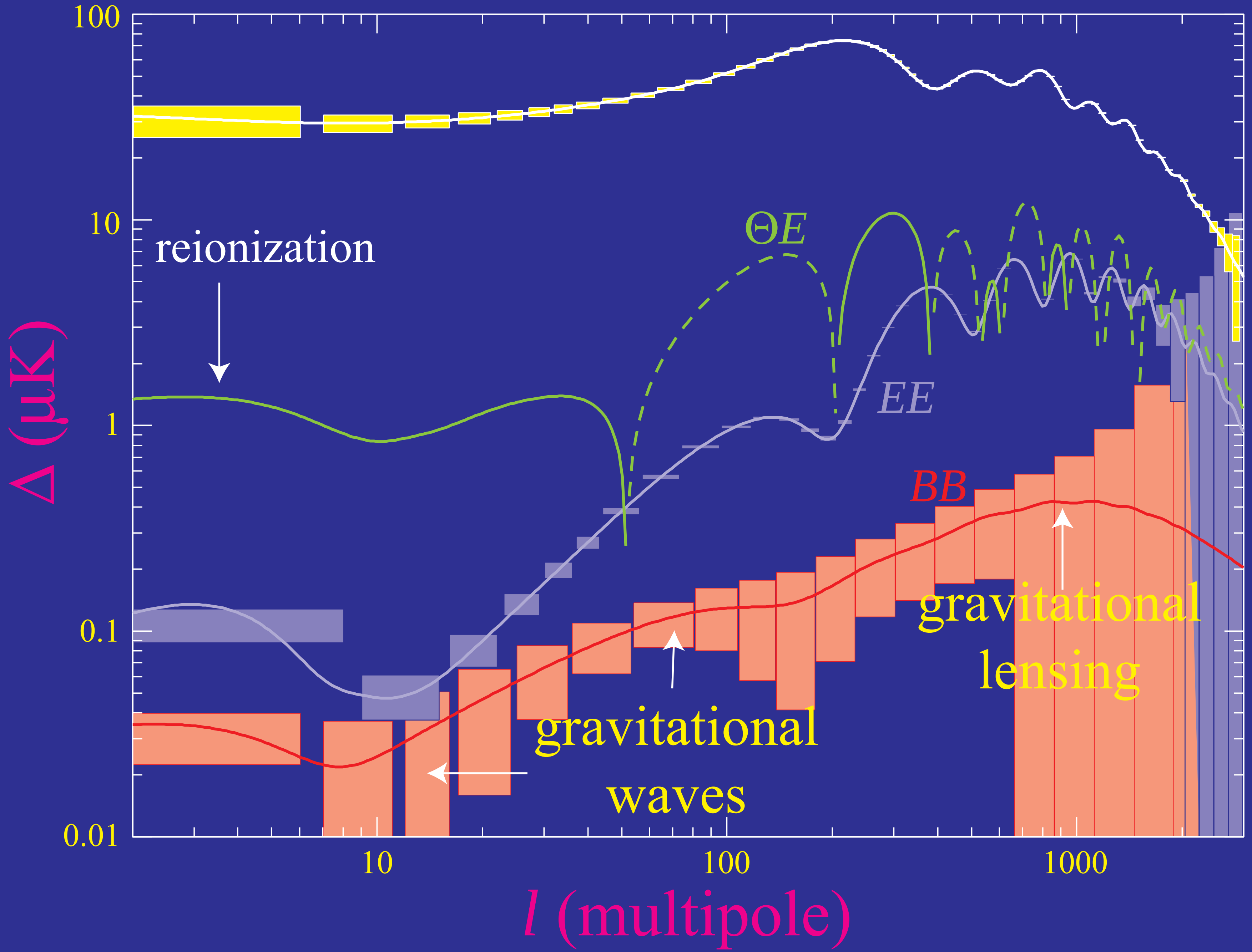
Cross Correlation

- Cross correlation of temperature and polarization

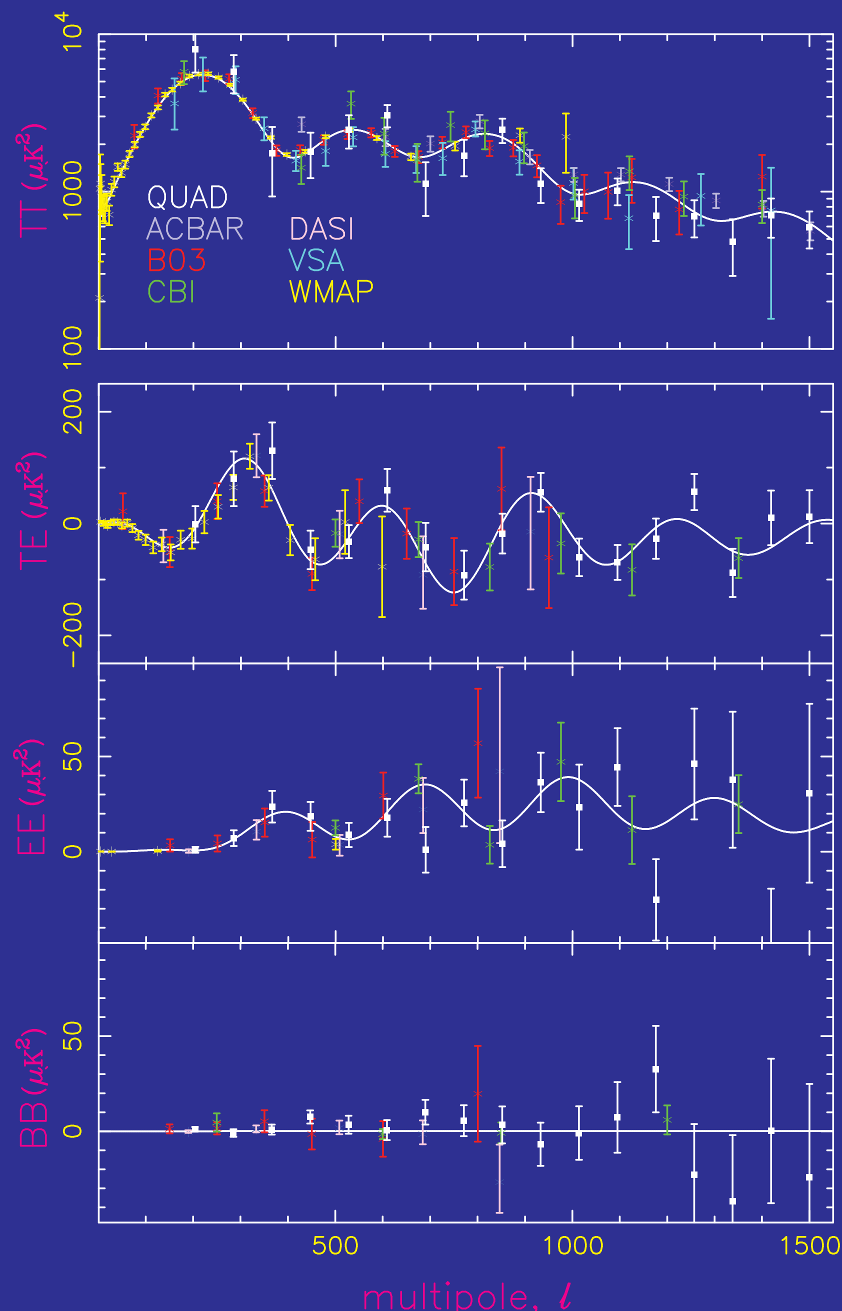
$$(\Theta + \Psi)(v_\gamma) \propto \cos(ks)\sin(ks) \propto \sin(2ks)$$

- Oscillation at twice the frequency
- Correlation: radial or tangential around hot spots
- Partial correlation: easier to measure if polarization data is noisy, harder to measure if polarization data is high S/N or if bands do not resolve oscillations
- Good check for systematics and foregrounds
- Comparison of temperature and polarization is proof against features in initial conditions mimicking acoustic features

Temperature and Polarization Spectra



Recent Data



Ade et al (QUAD, 2007)

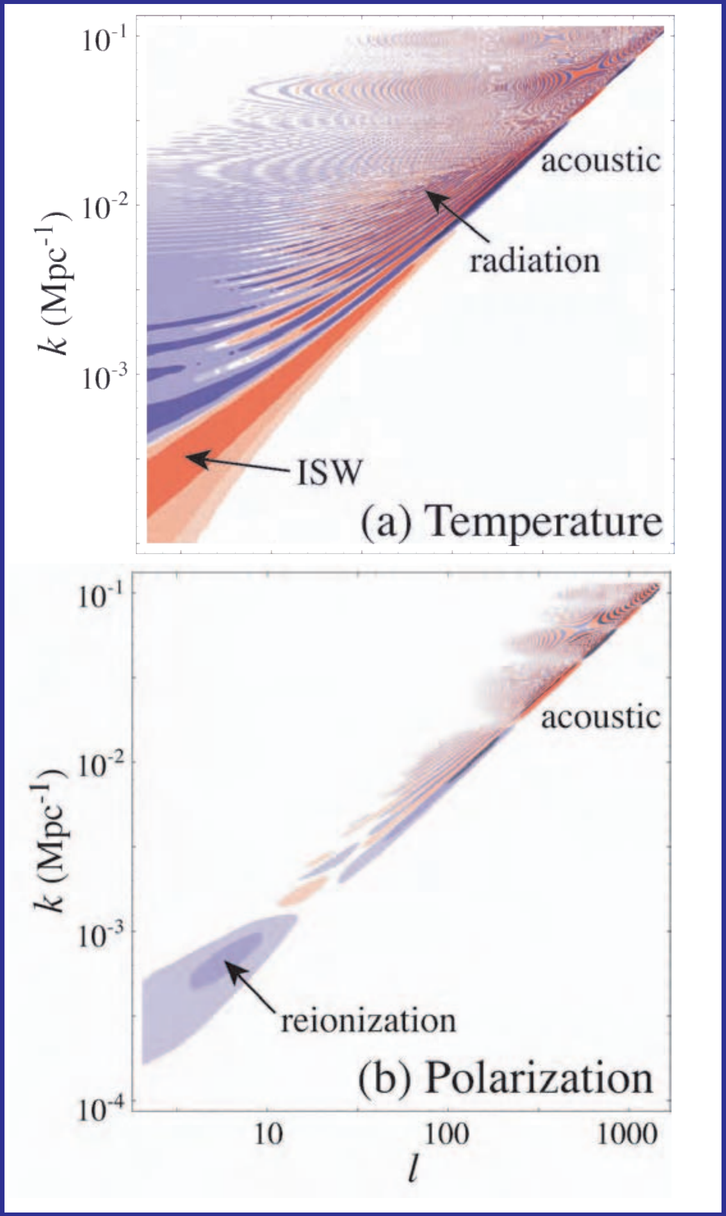
Why Care?

- In the standard model, acoustic polarization spectra uniquely predicted by same parameters that control temperature spectra
- Validation of standard model
- Improved statistics on cosmological parameters controlling peaks

- Polarization is a complementary and intrinsically more incisive probe of the initial power spectrum and hence inflationary (or alternate) models

- Acoustic polarization is lensed by the large scale structure into B-modes
- Lensing B-modes sensitive to the growth of structure and hence neutrino mass and dark energy
- Contaminate the gravitational wave B-mode signature

Transfer of Initial Power

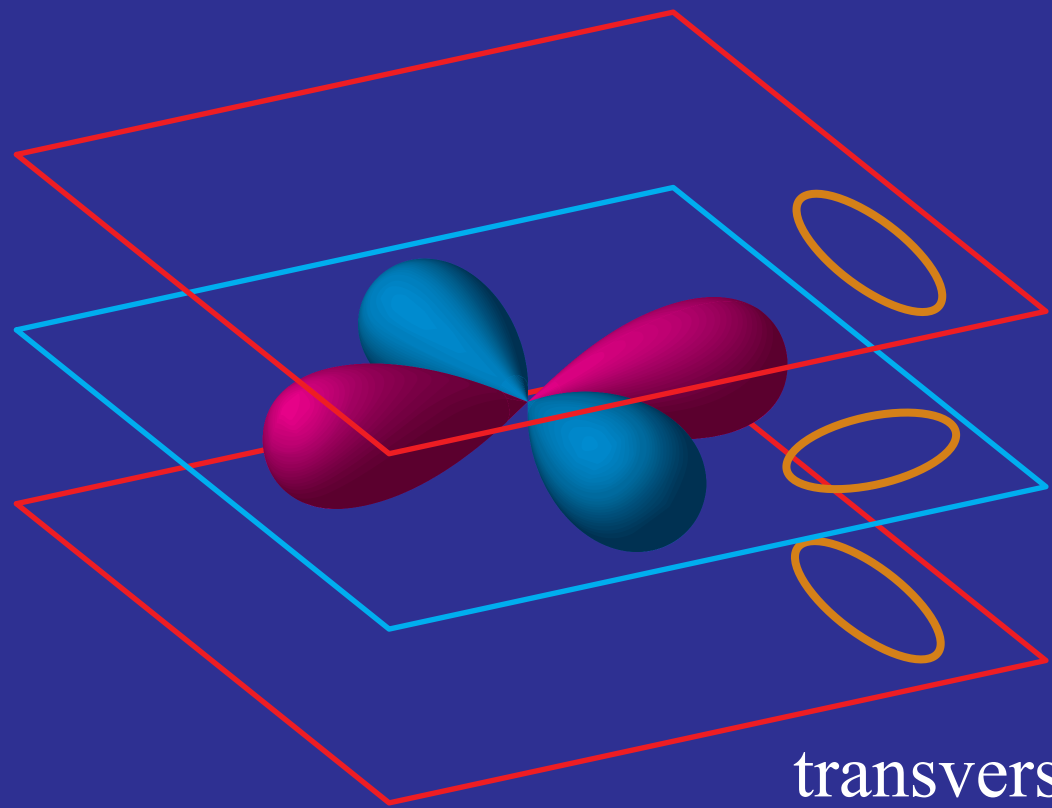


Hu & Okamoto (2003)

Gravitational Waves

Gravitational Waves

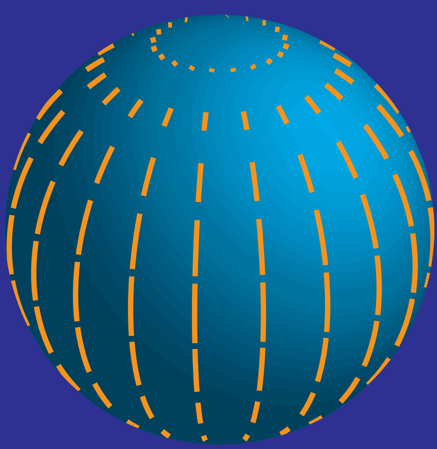
- Inflation predicts near scale invariant spectrum of gravitational waves
- Amplitude proportional to the square of the $E_i = V^{1/4}$ energy scale
- If inflation is associated with the grand unification $E_i \sim 10^{16}$ GeV and potentially observable

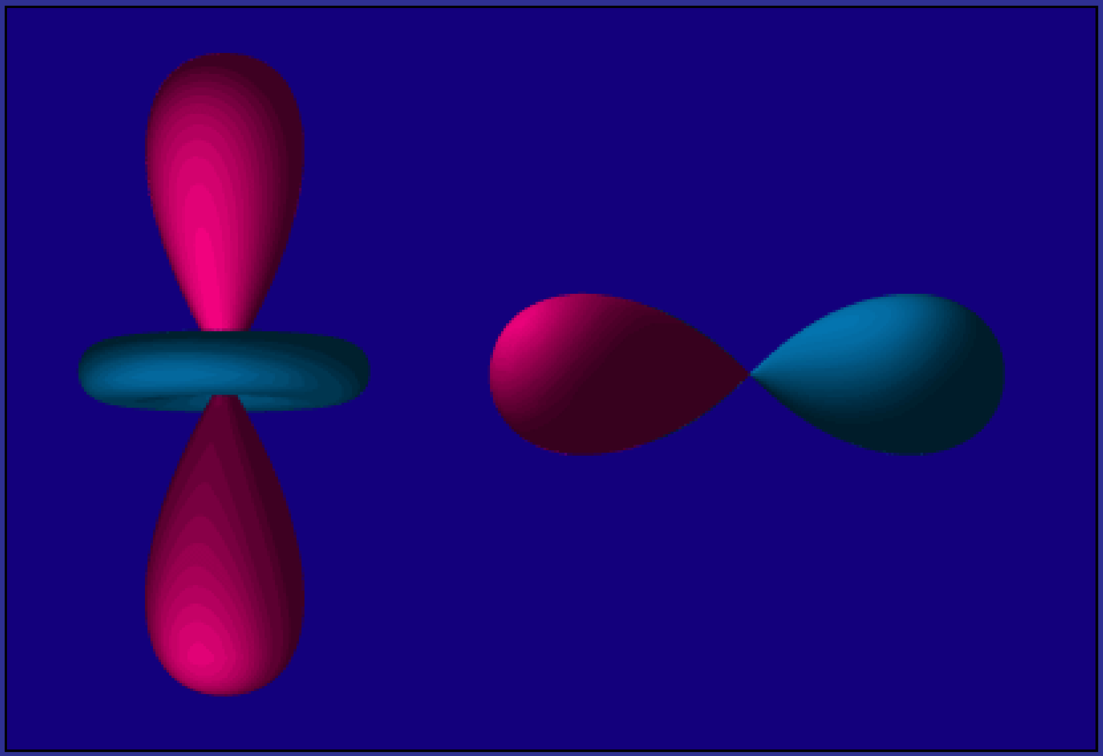


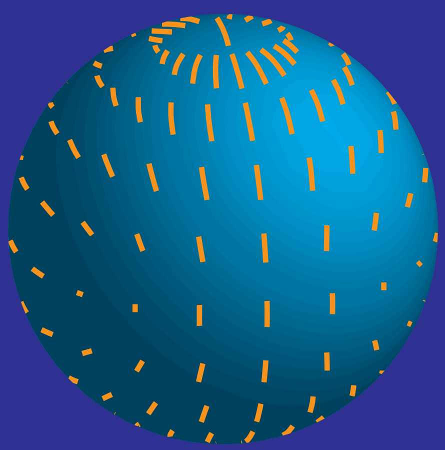
transverse-traceless distortion

Gravitational Wave Pattern

- Projection of the quadrupole anisotropy gives polarization pattern
- Transverse polarization of gravitational waves breaks azimuthal symmetry







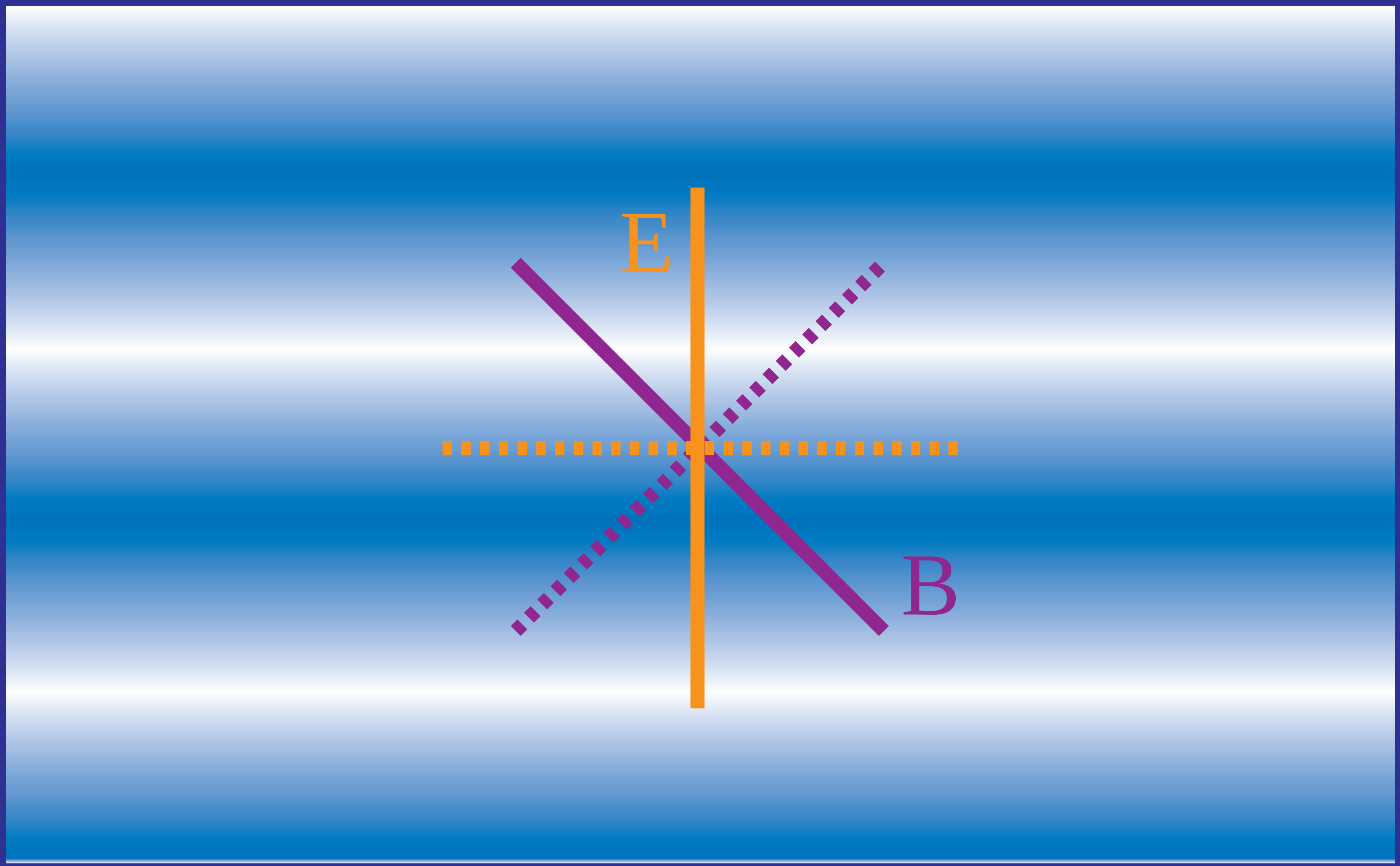
density perturbation

gravitational wave

Electric & Magnetic Polarization

(a.k.a. gradient & curl)

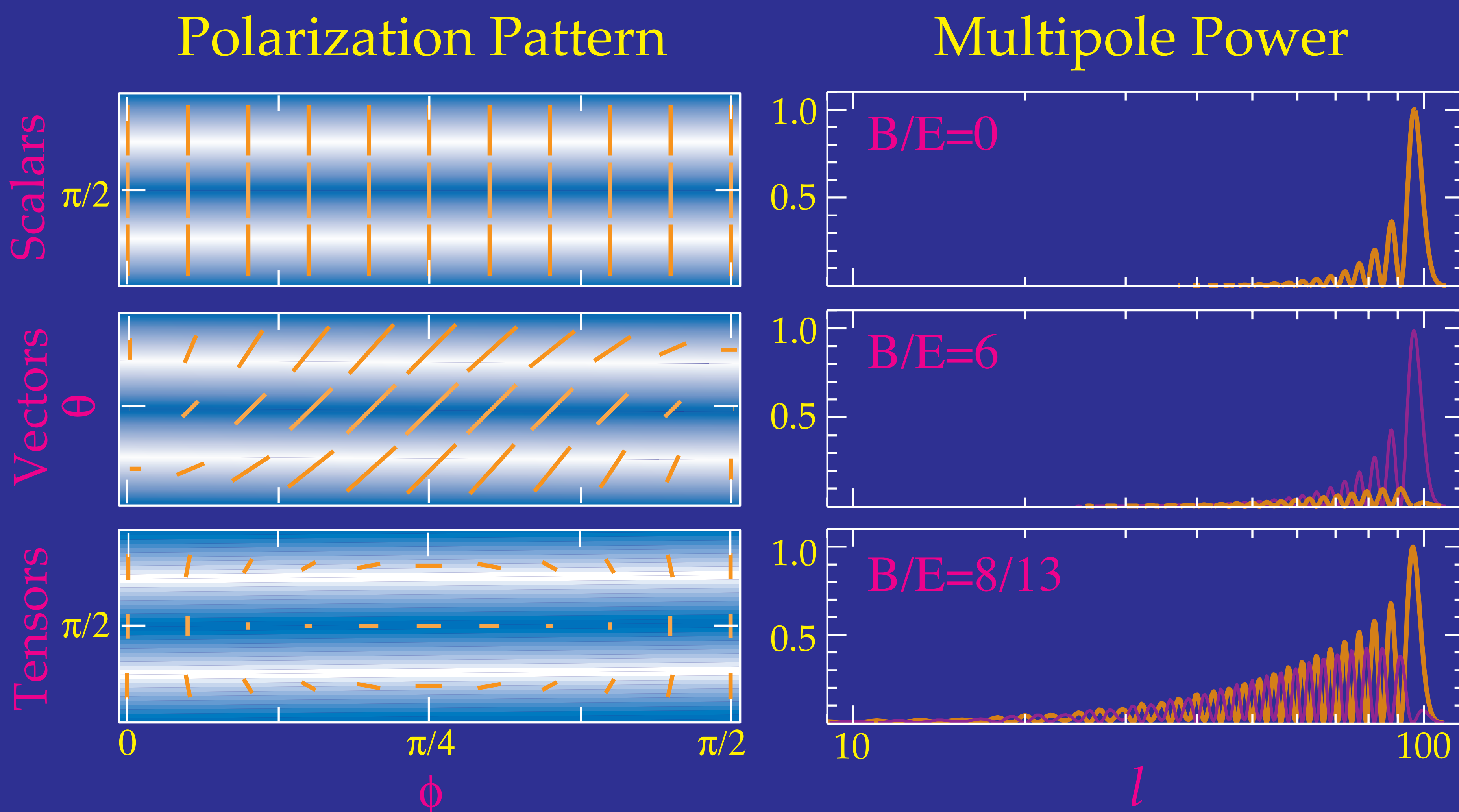
- Alignment of principal vs polarization axes
  (curvature matrix vs polarization direction)



Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)

Patterns and Perturbation Types

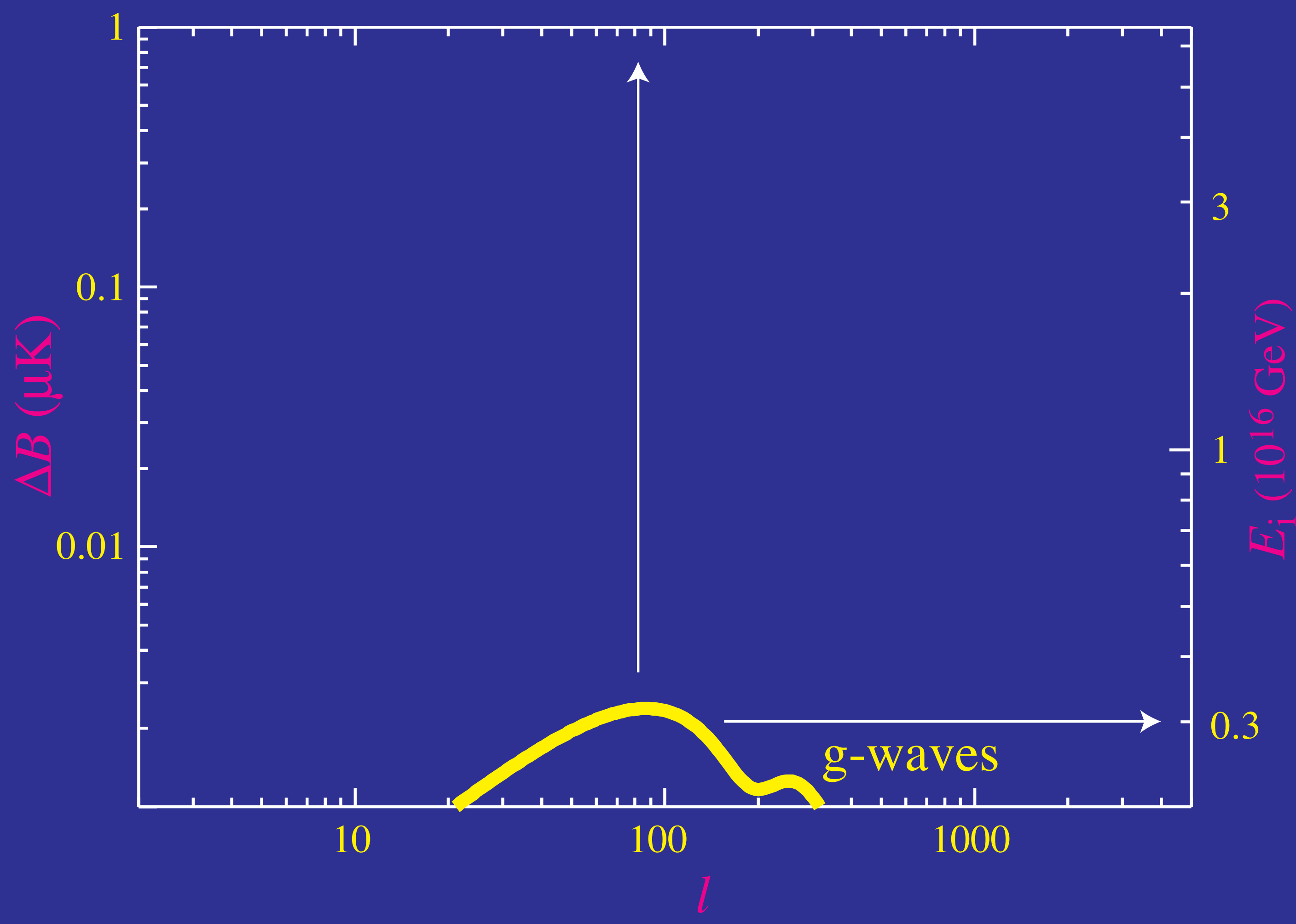
- Amplitude modulated by plane wave → Principal axis
- Direction detemined by perturbation type → Polarization axis



Kamionkowski, Kosowski, Stebbins (1997); Zaldarriaga & Seljak (1997); Hu & White (1997)

Scaling with Inflationary Energy Scale

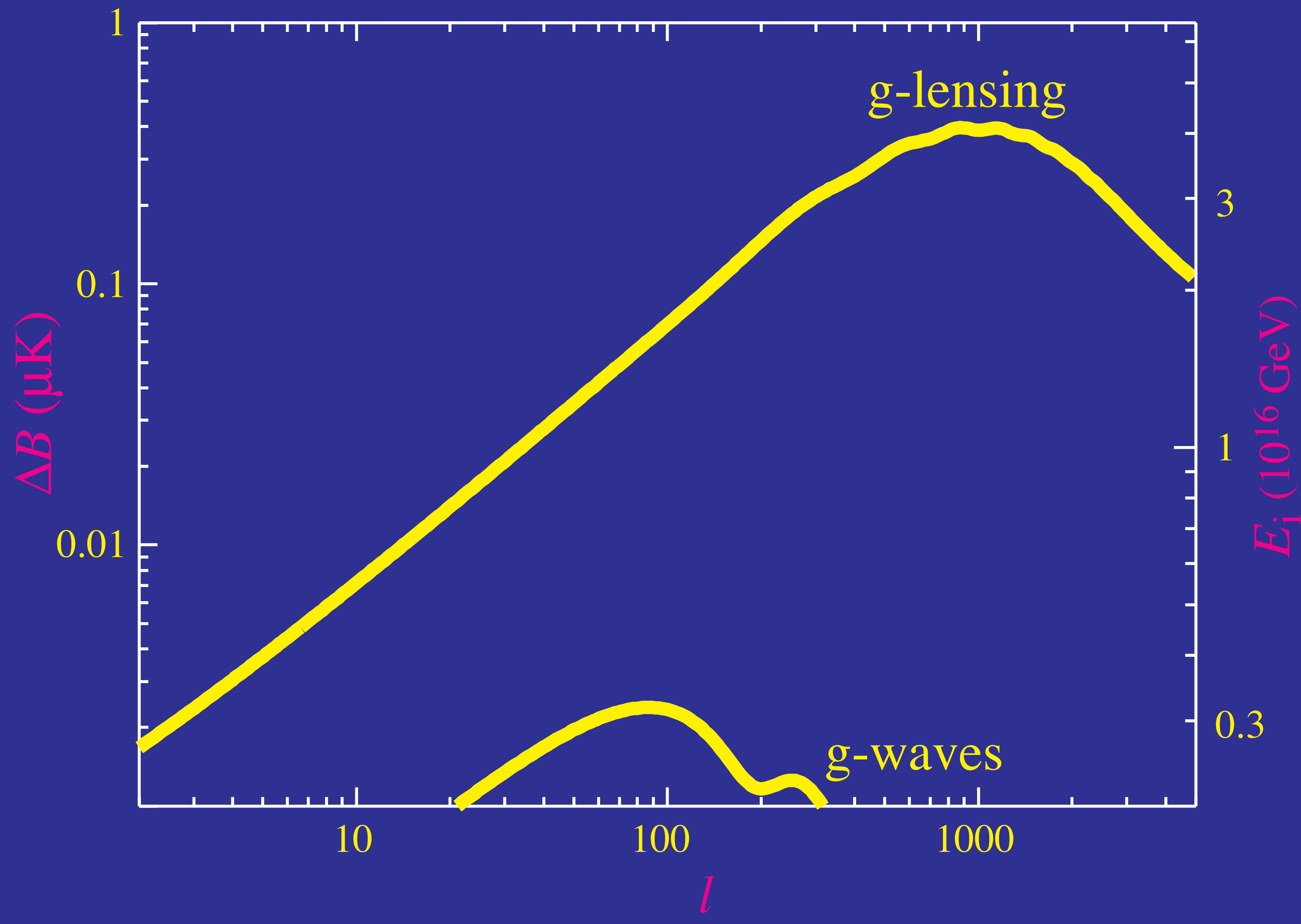
- RMS B-mode signal scales with inflationary energy scale squared E_i^2



Contamination for Gravitational Waves

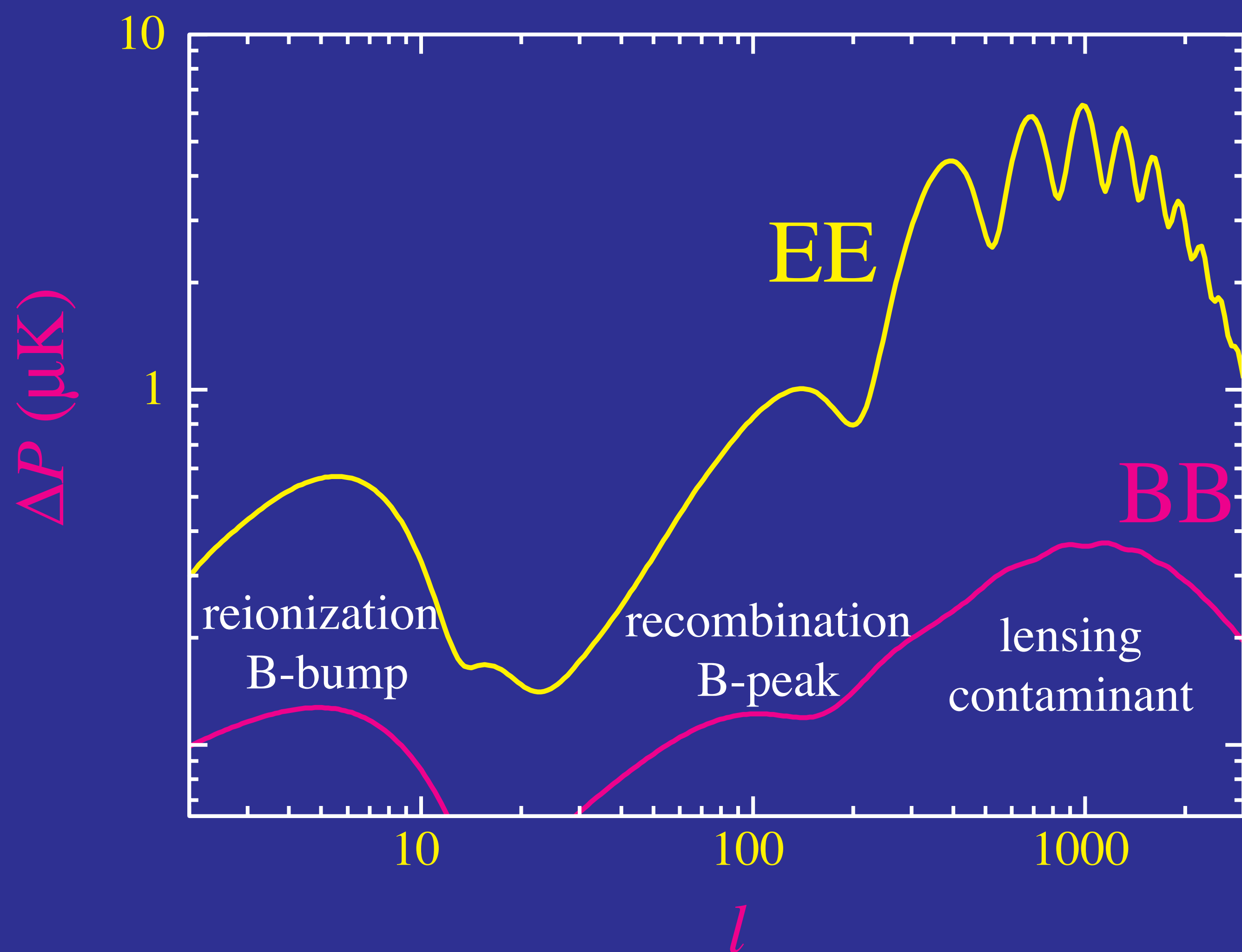
- Gravitational lensing contamination of B-modes from gravitational waves cleaned to E_i~0.3 x 10^{16} GeV

Hu & Okamoto (2002) limits by Knox & Song (2002); Cooray, Kedsen, Kamionkowski (2002)



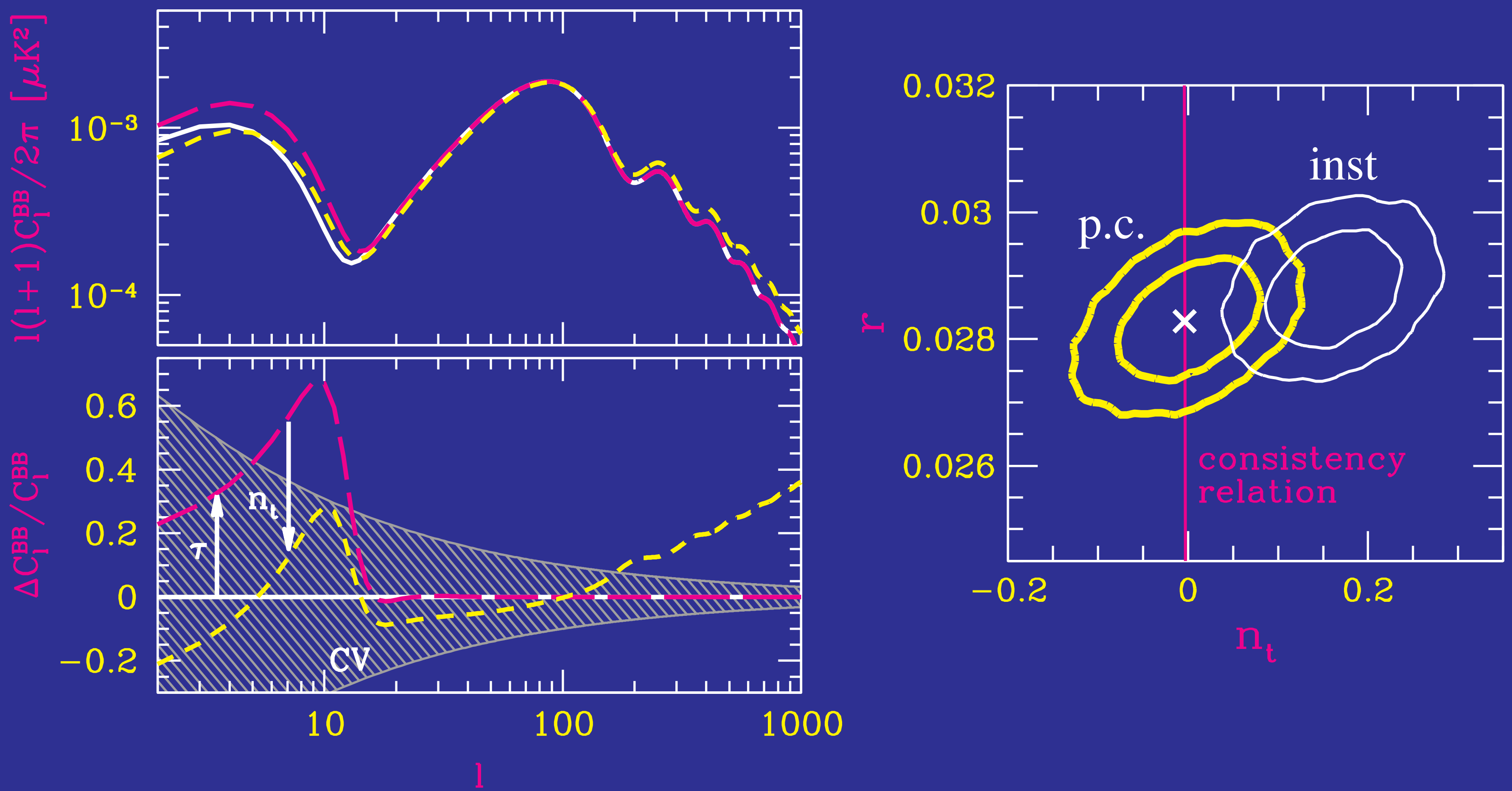
The B-Bump

- Rescattering of gravitational wave anisotropy generates the B-bump
- Potentially the most sensitive probe of inflationary energy scale
- Potentially enables test of consistency relation (slow roll)



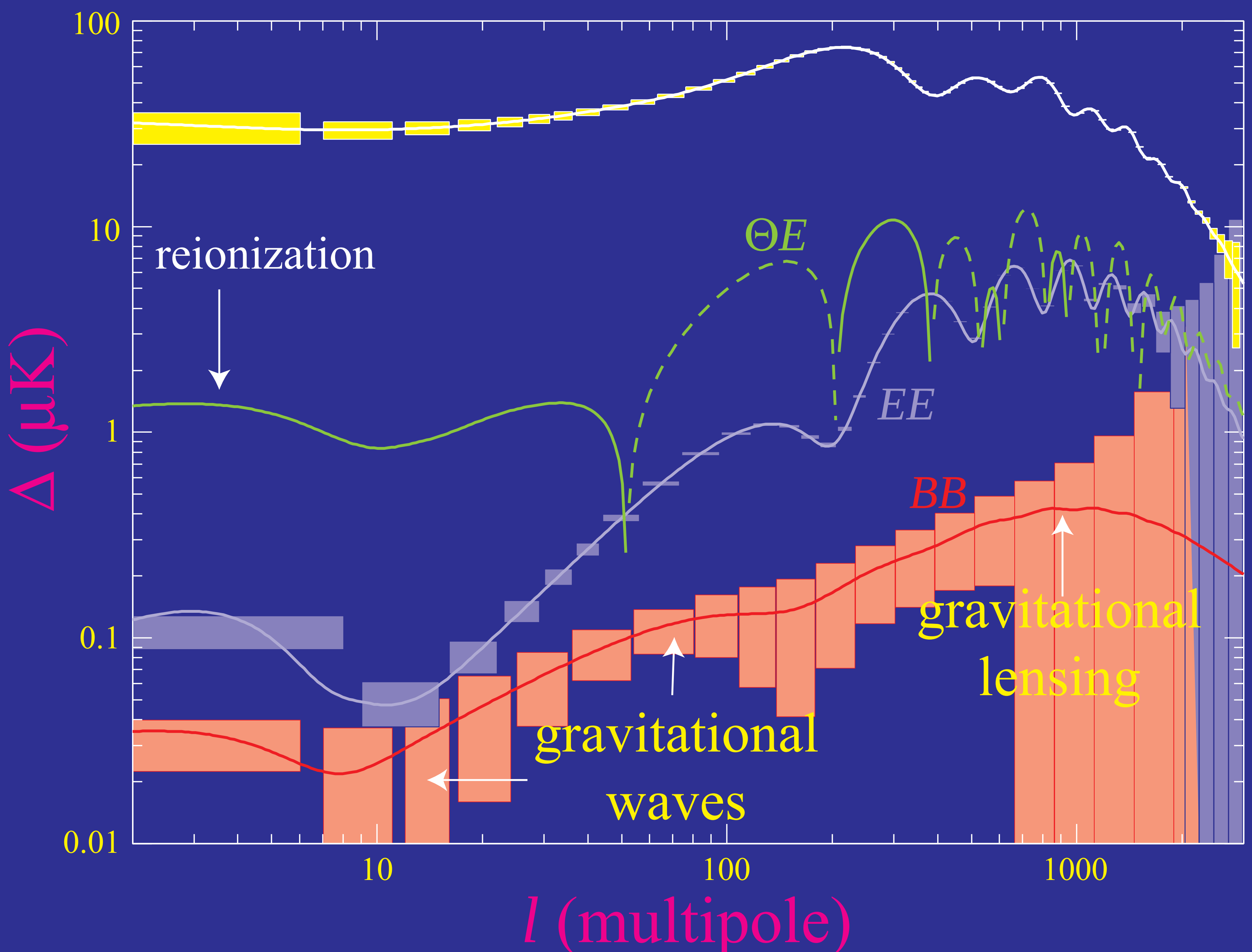
Slow Roll Consistency Relation

- Consistency relation between tensor-scalar ratio and tensor tilt $r = -8n_t$ tested by reionization
- Reionization uncertainties controlled by a complete p.c. analysis



Mortonson & Hu (2007)

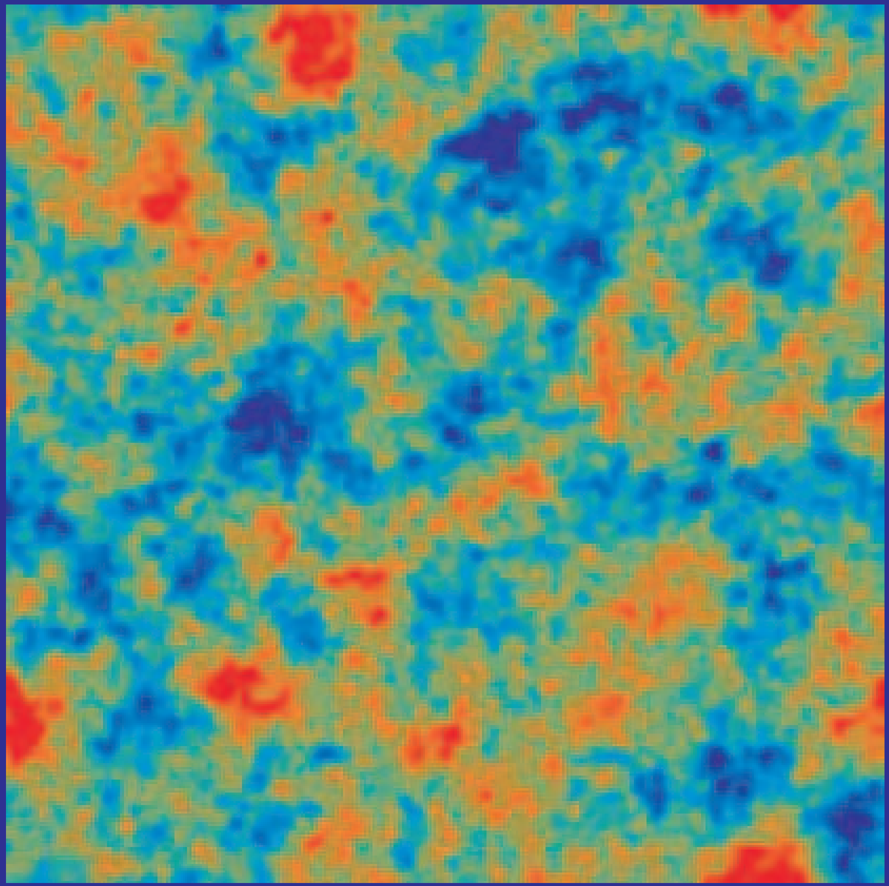
Temperature and Polarization Spectra



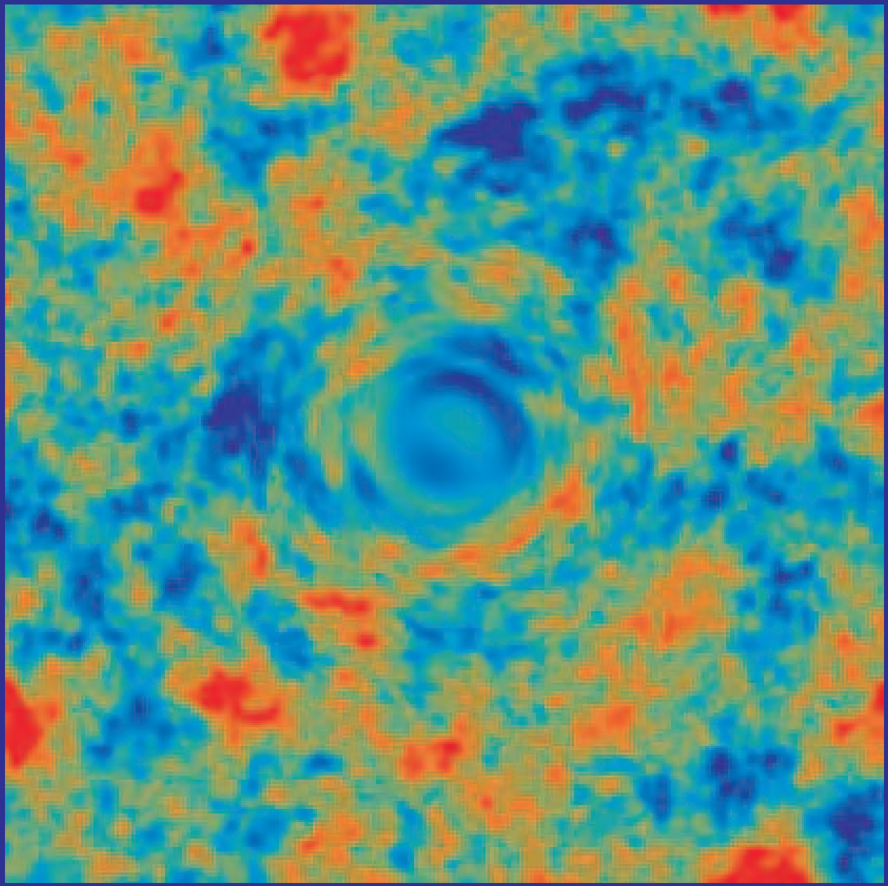
Gravitational Lensing

Gravitational Lensing

- Gravitational lensing by large scale structure distorts the observed temperature and polarization fields
- Exaggerated example for the temperature

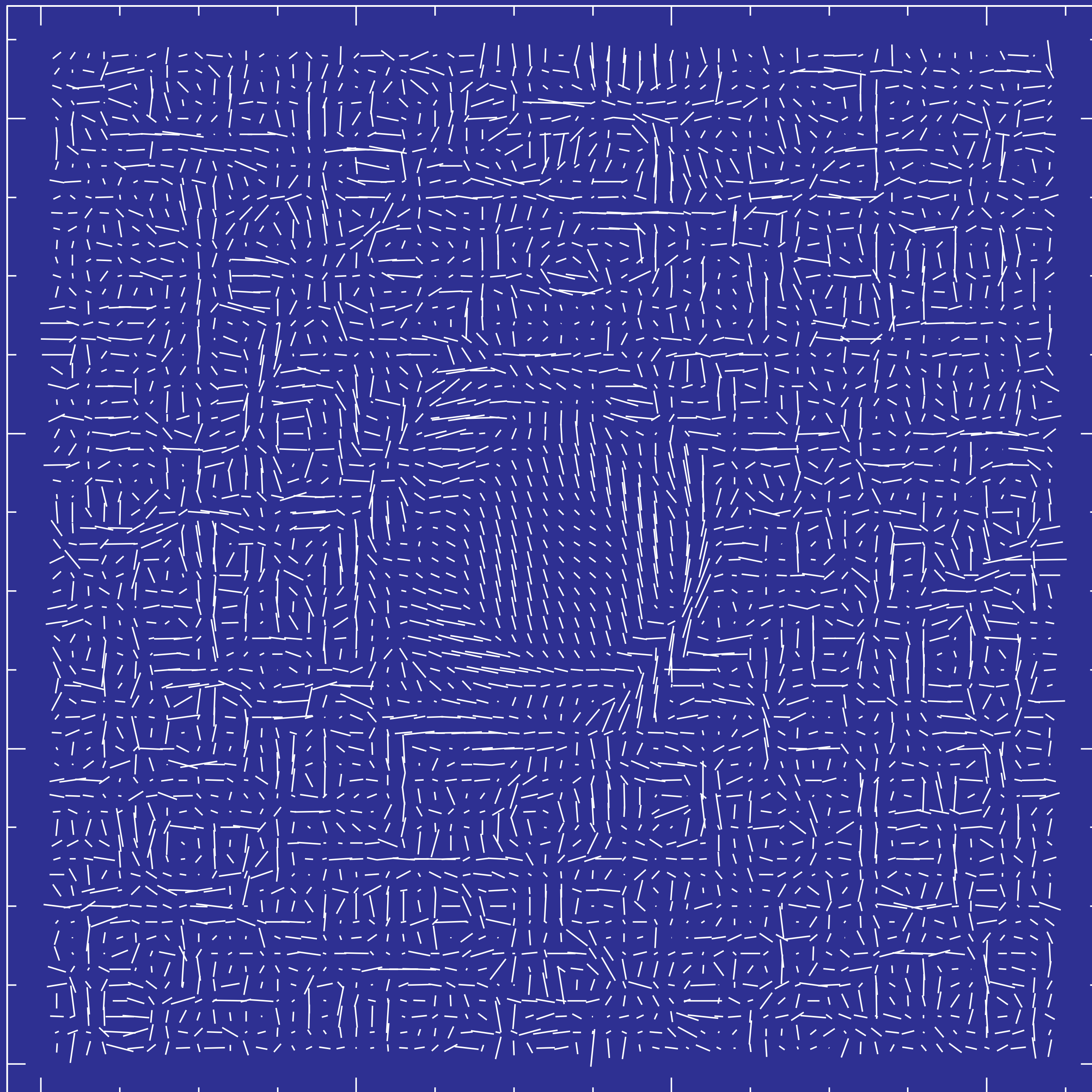


Original



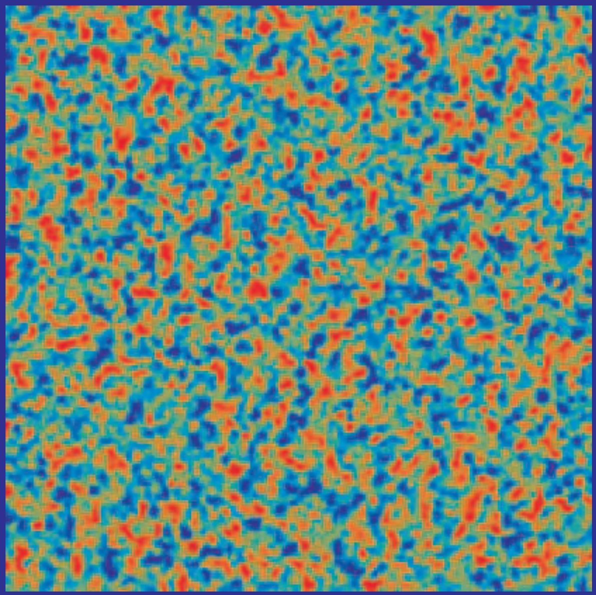
Lensed

Polarization Lensing

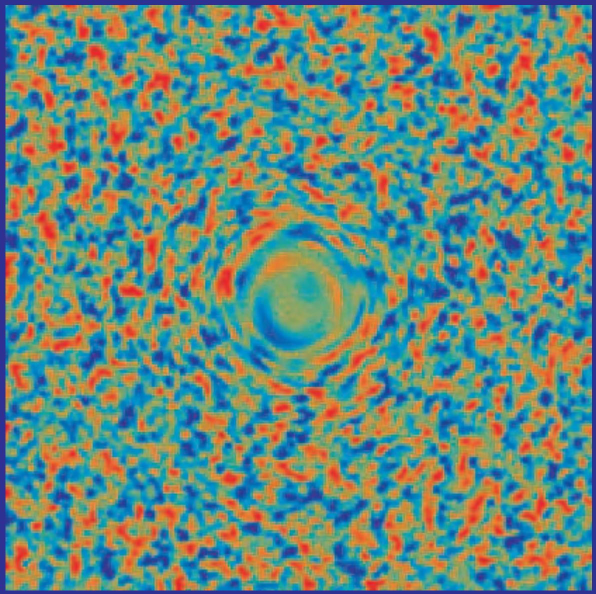


Polarization Lensing

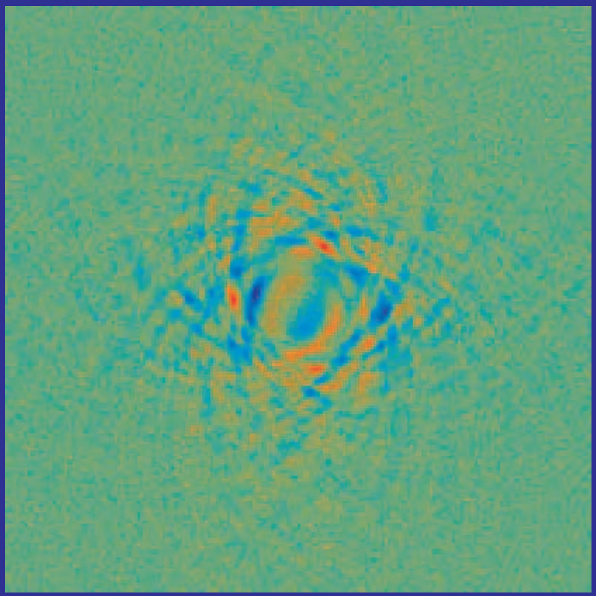
- Since E and B denote the relationship between the polarization amplitude and direction, warping due to lensing creates B-modes



Original



Lensed E

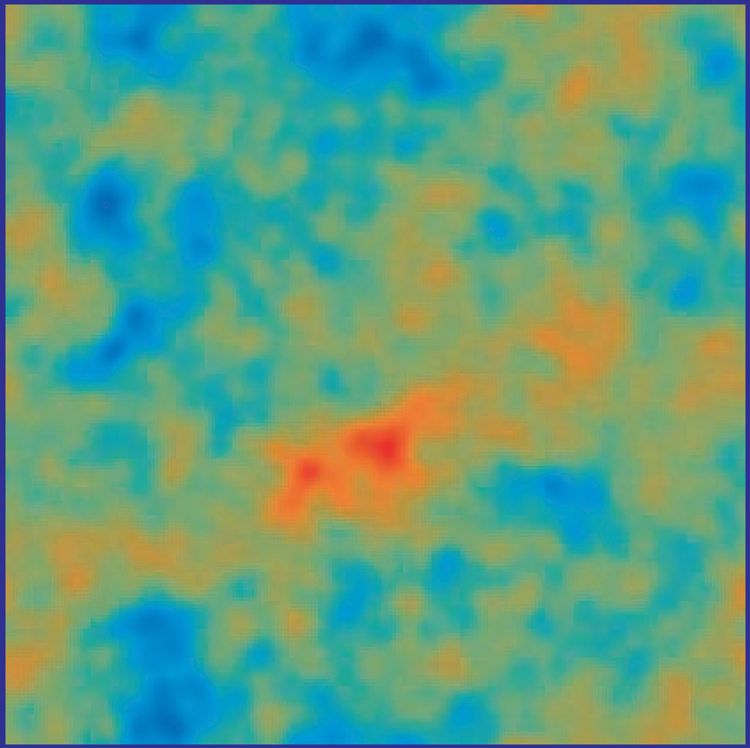


Lensed B

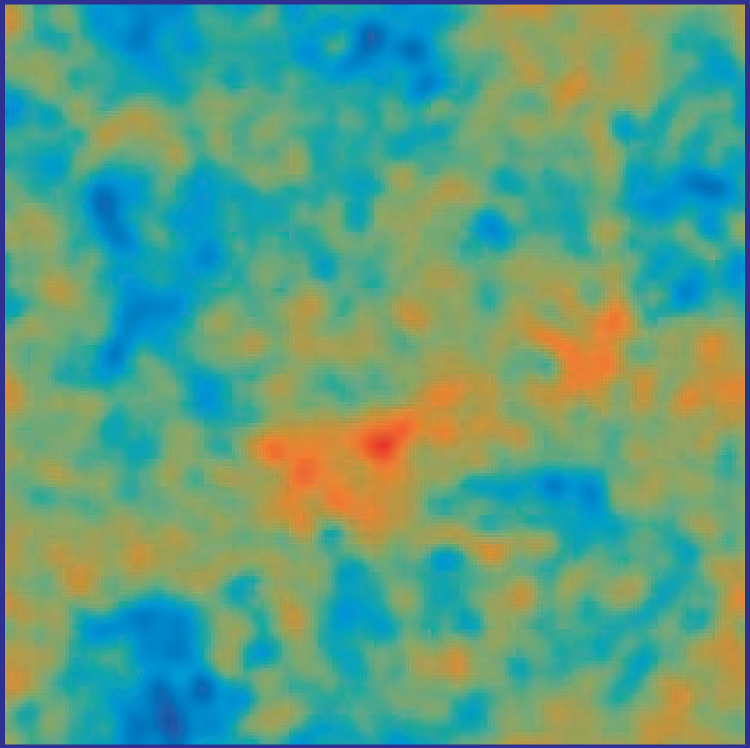
Zaldarriaga & Seljak (1998) [figure: Hu & Okamoto (2001)]

Reconstruction from Polarization

- Lensing B-modes correlated to the orignal E-modes in a specific way
- Correlation of E and B allows for a reconstruction of the lens
- Reference experiment of 4' beam, 1μK' noise and 100 deg^2



Original Mass Map



Reconstructed Mass Map

Hu & Okamoto (2001) [iterative improvement Hirata & Seljak (2003)]

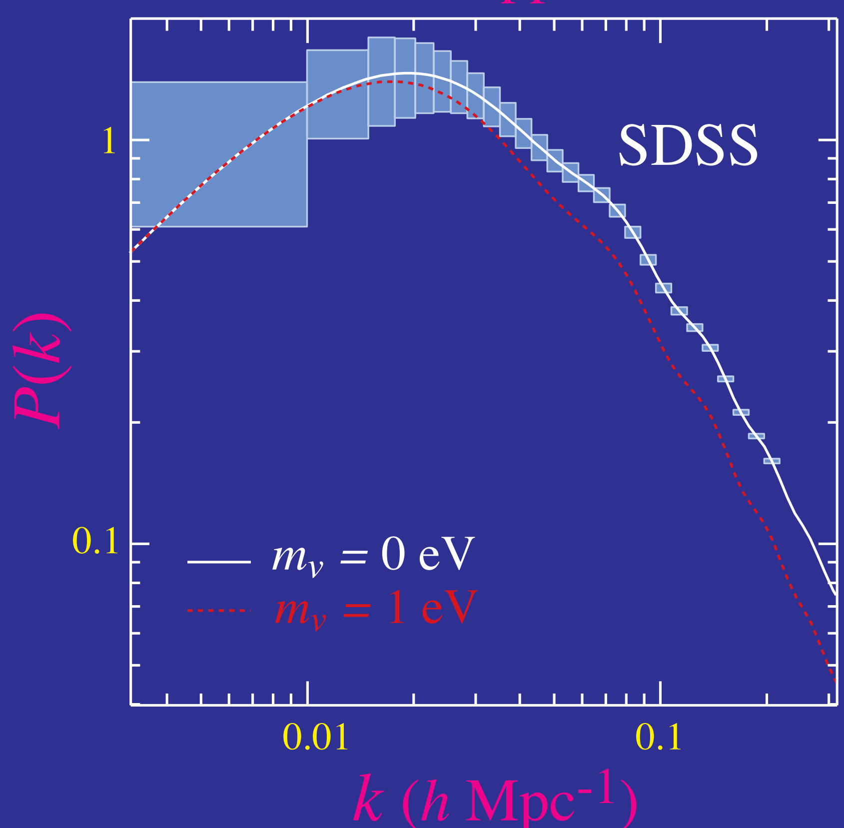
Why Care

- Gravitational lensing sensitive to amount and hence growth of structure
- Examples: massive neutrinos - $d \ln C_\ell^{BB}/dm_\nu \approx -1/3$eV, dark energy - $d \ln C_\ell^{BB}/dw \approx -1/8$
- Mass reconstruction measures the large scale structure on large scales and the mass profile of objects on small scales
- Examples: large scale decontamination of the gravitational wave B modes; lensing by SZ clusters combined with optical weak lensing can make a distance ratio test of the acceleration

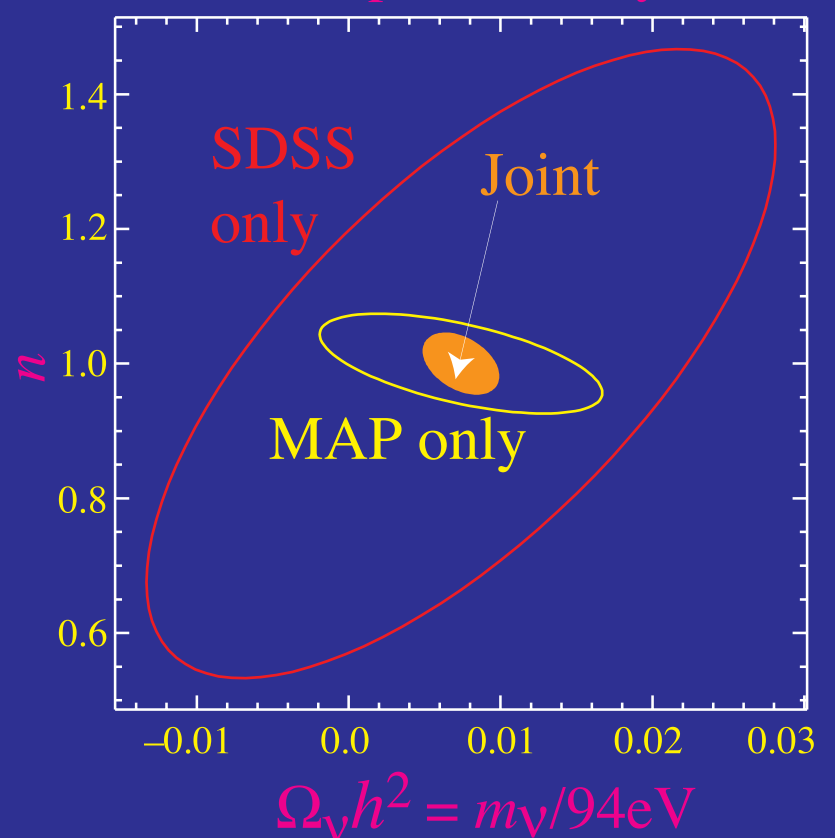
Weighing Neutrinos

- Massive neutrinos suppress power strongly on small scales [$\Delta P/P \approx -8\Omega_\nu/\Omega_m$]: well modeled by [$c_{\rm eff}^2 = w_g$, $c_{\rm vis}^2 = w_g$, w_g: $1/3 \to 1$]
- Degenerate with other effects [tilt n, $\Omega_m h^2$...]
- CMB signal small but breaks degeneracies
- 2σ Detection: 0.3eV [Map (pol) + SDSS]

Power Suppression



Complementarity



Hu, Eisenstein, & Tegmark (1998); Eisenstein, Hu & Tegmark (1998)

Lecture III: Summary

- Polarization by Thomson scattering of quadrupole anisotropy
- Quadrupole anisotropy only sustained in optically thin conditions of reionization and the end of recombination
- Reionization generates E-modes at low multipoles from and correlated to the Sachs-Wolfe anisotropy
- Reionization polarization enables study of ionization history, low multipole anomalies, gravitational waves
- Dissipation of acoustic waves during recombination generates quadrupoles and correlated polarization peaks
- Recombination polarization provides consistency checks, features in power spectrum, source of graviational lensing B modes
- Gravitational waves B-mode polarization sensitive to inflation energy scale and tests slow roll consistency relation