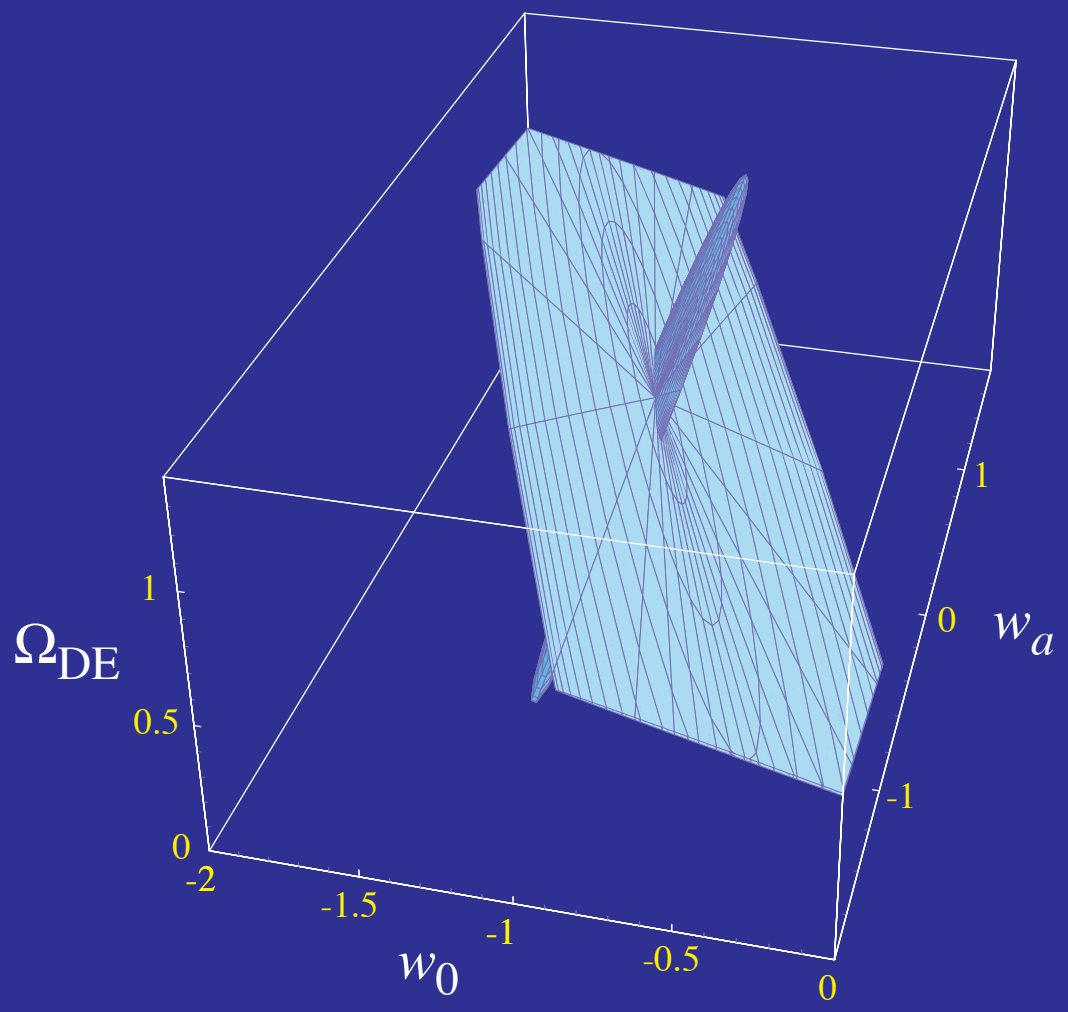
Gravitational Lensing of the CMB



Wayne Hu

Leiden, August 2006

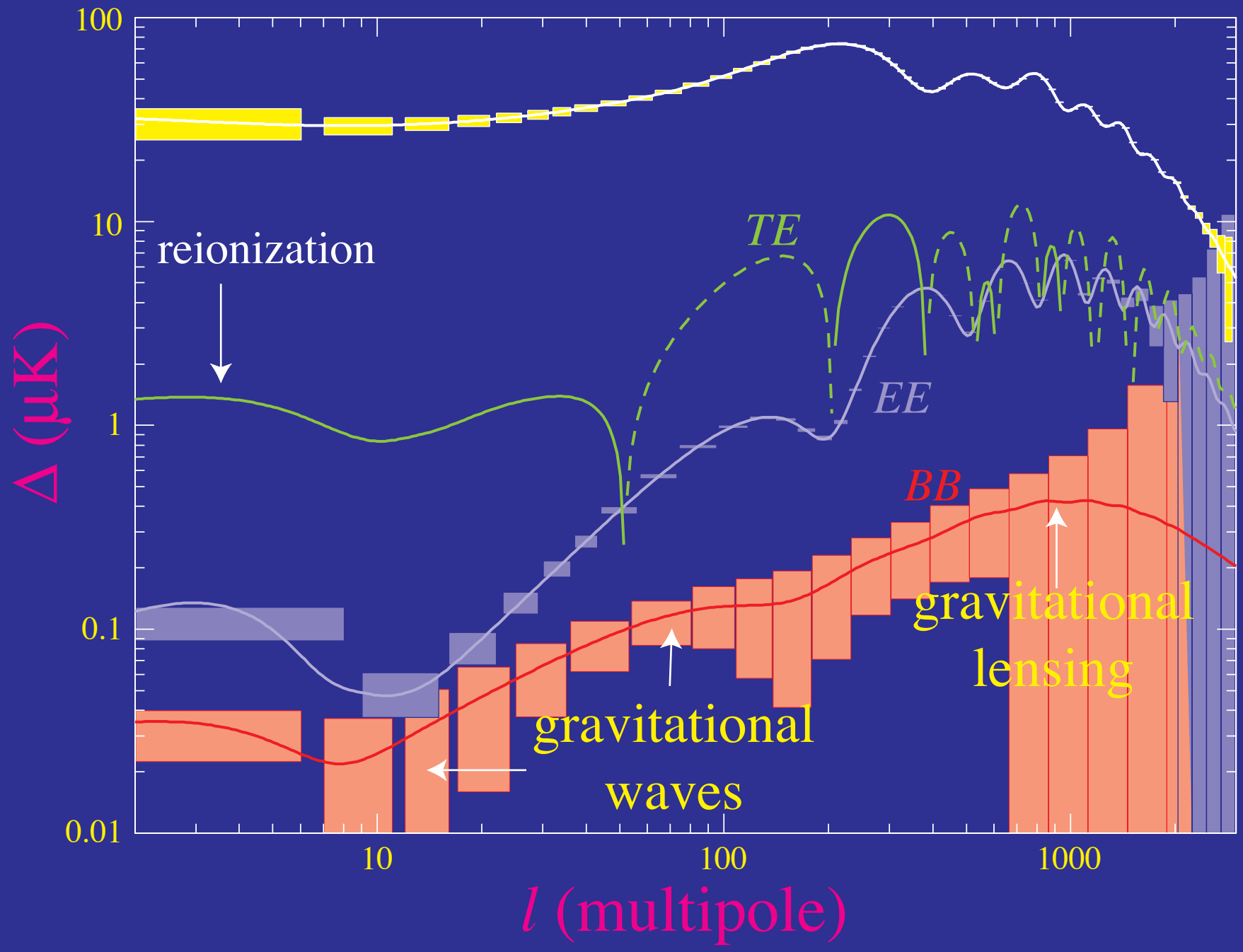
Outline

- Gravitational Lensing of Temperature and Polarization Fields
- Cosmological Observables from Lensed Power Spectra
- Complementarity with Dark Energy Probes
- Direct Mass Reconstruction

Collaborators:

- Dragan Huterer
- Manoj Kaplinghat
- Takemi Okamoto
- Kendrick Smith

Temperature and Polarization Spectra



Lensing of CMB Fields

Gravitational Lensing

- Lensing is a surface brightness conserving remapping of source to image planes by the gradient of the projected potential

$$\phi(\hat{\mathbf{n}}) = 2 \int \frac{dz}{H(z)} \frac{D_A(D_s - D)}{D_A(D)\, D_A(D_s)} \Phi(D_A \hat{\mathbf{n}}, D)\,,$$

  such that the fields are remapped as

$$x(\hat{\mathbf{n}}) \to x(\hat{\mathbf{n}} + \nabla\phi)\,,$$

  where $x \in \{T, Q, U\}$ temperature and polarization.

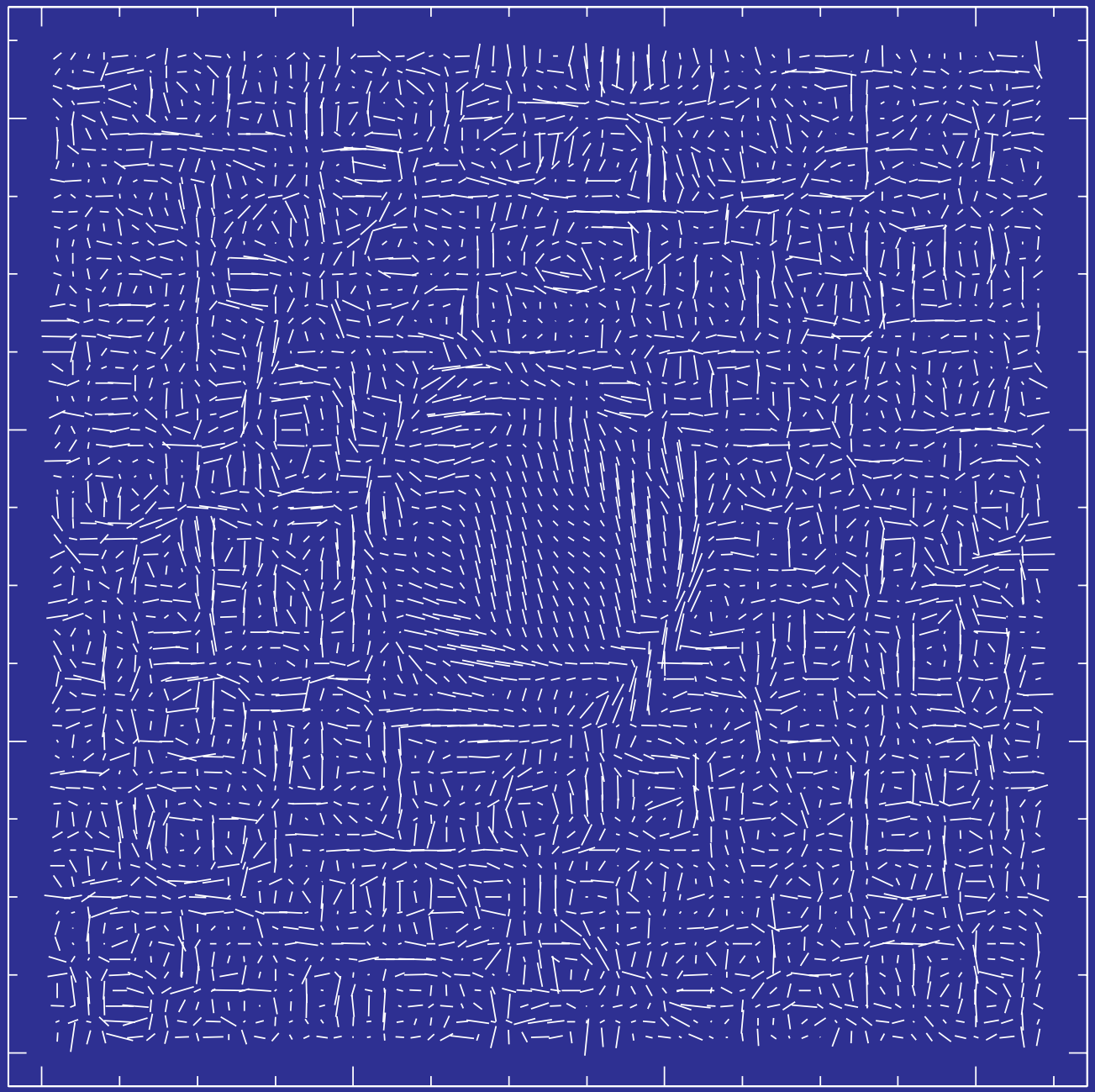
- Taylor expansion leads to product of fields and Fourier mode-coupling

- Appears in the power spectrum as a convolution kernel for T and E and an $E \to B$.

Lensing of a Gaussian Random Field

- CMB temperature and polarization anisotropies are Gaussian random fields – unlike galaxy weak lensing
- Average over many noisy images – like galaxy weak lensing

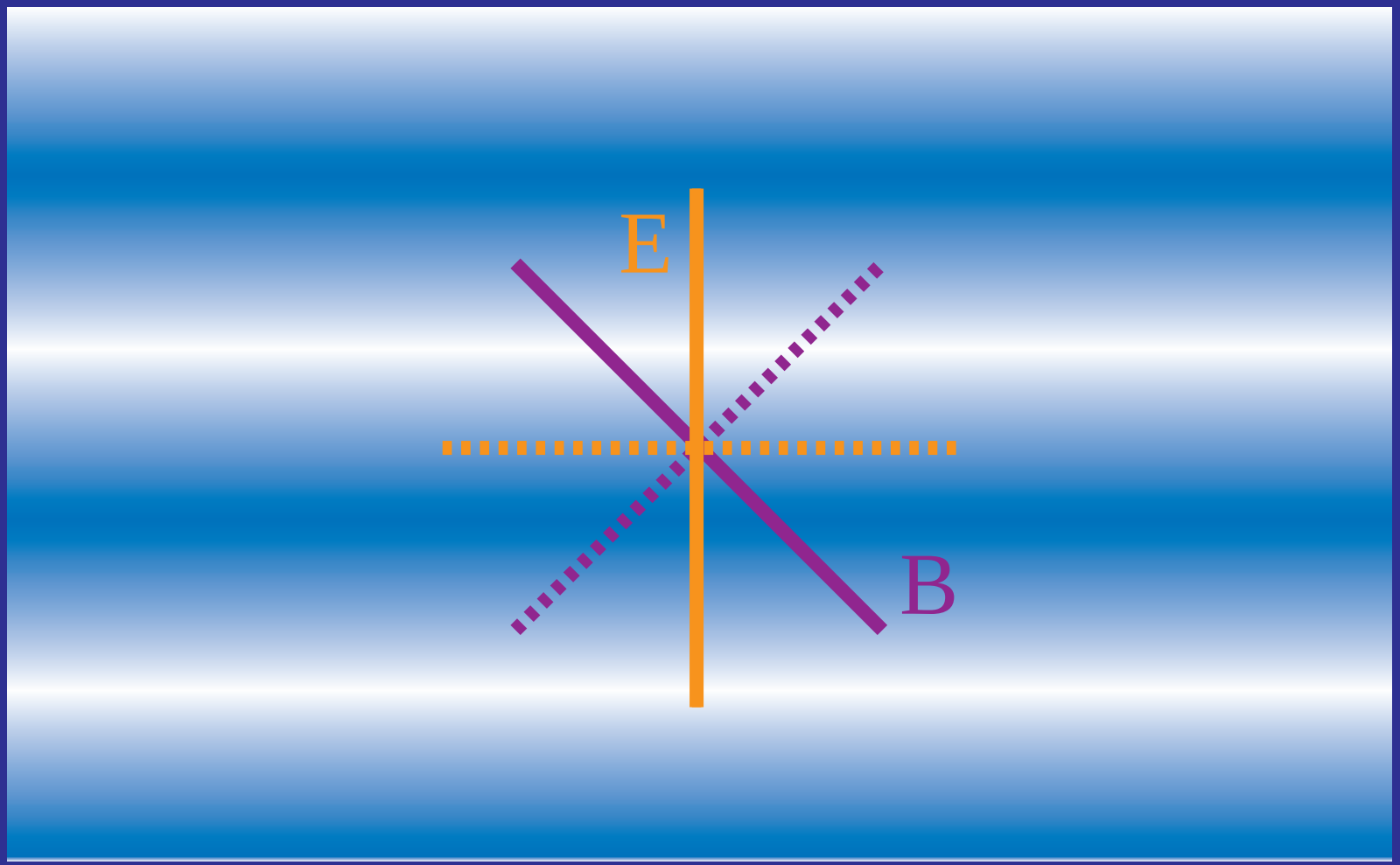
Polarization Lensing



Electric & Magnetic Polarization

(a.k.a. gradient & curl)

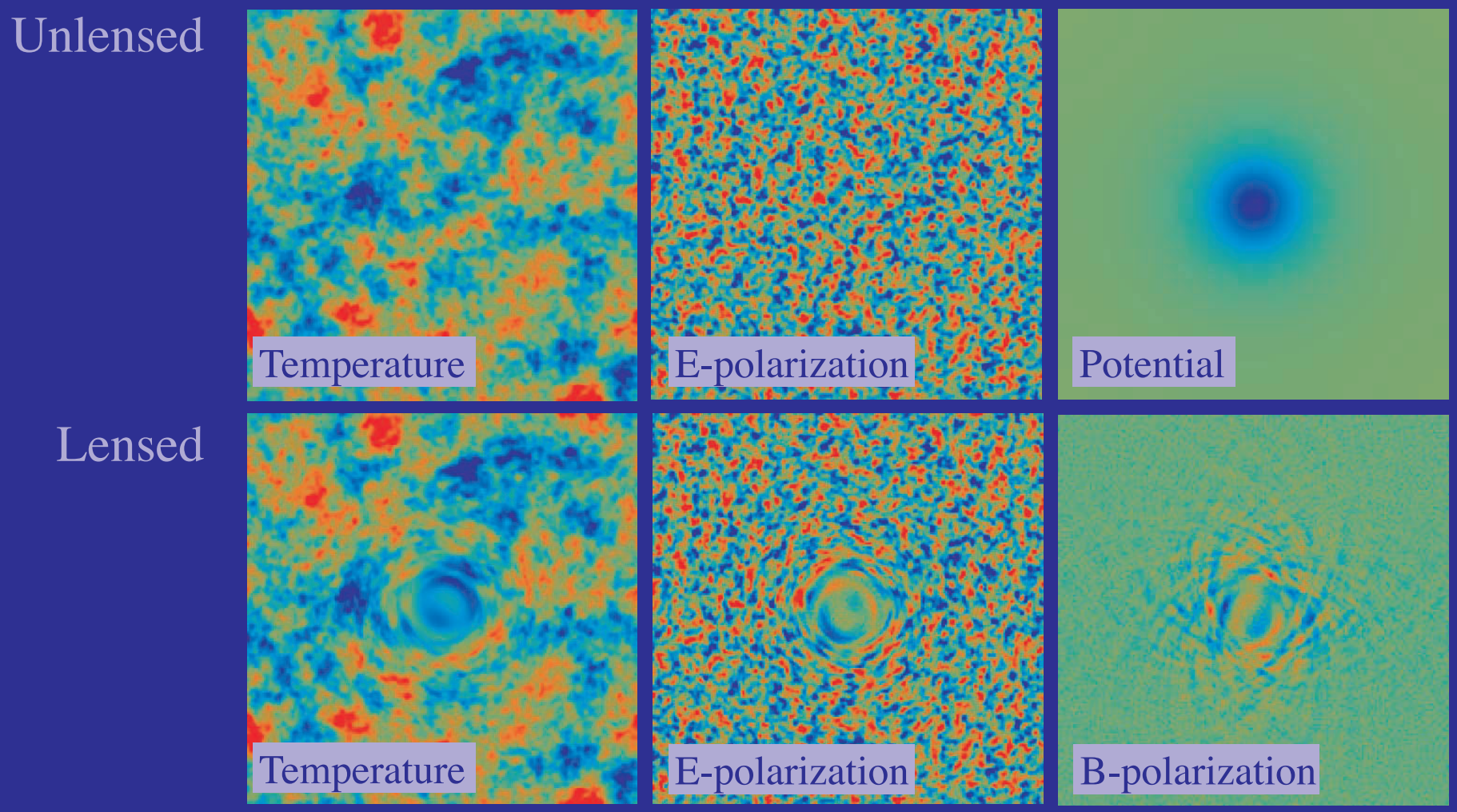
- Alignment of principal vs polarization axes (curvature matrix vs polarization direction)



Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)

Temperature & Polarization

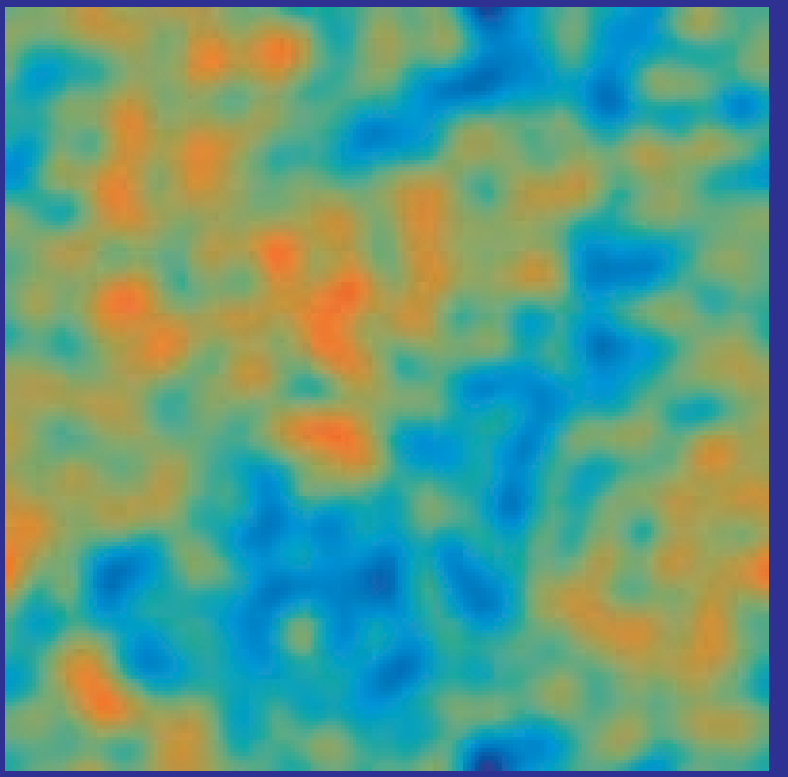
- Warping of the polarization field generates B-modes from E-modes at recombination (100 sq deg.)



Zaldarriaga & Seljak (1999) [figure from Hu & Okamoto (2001)]

Lensing by a Gaussian Random Field

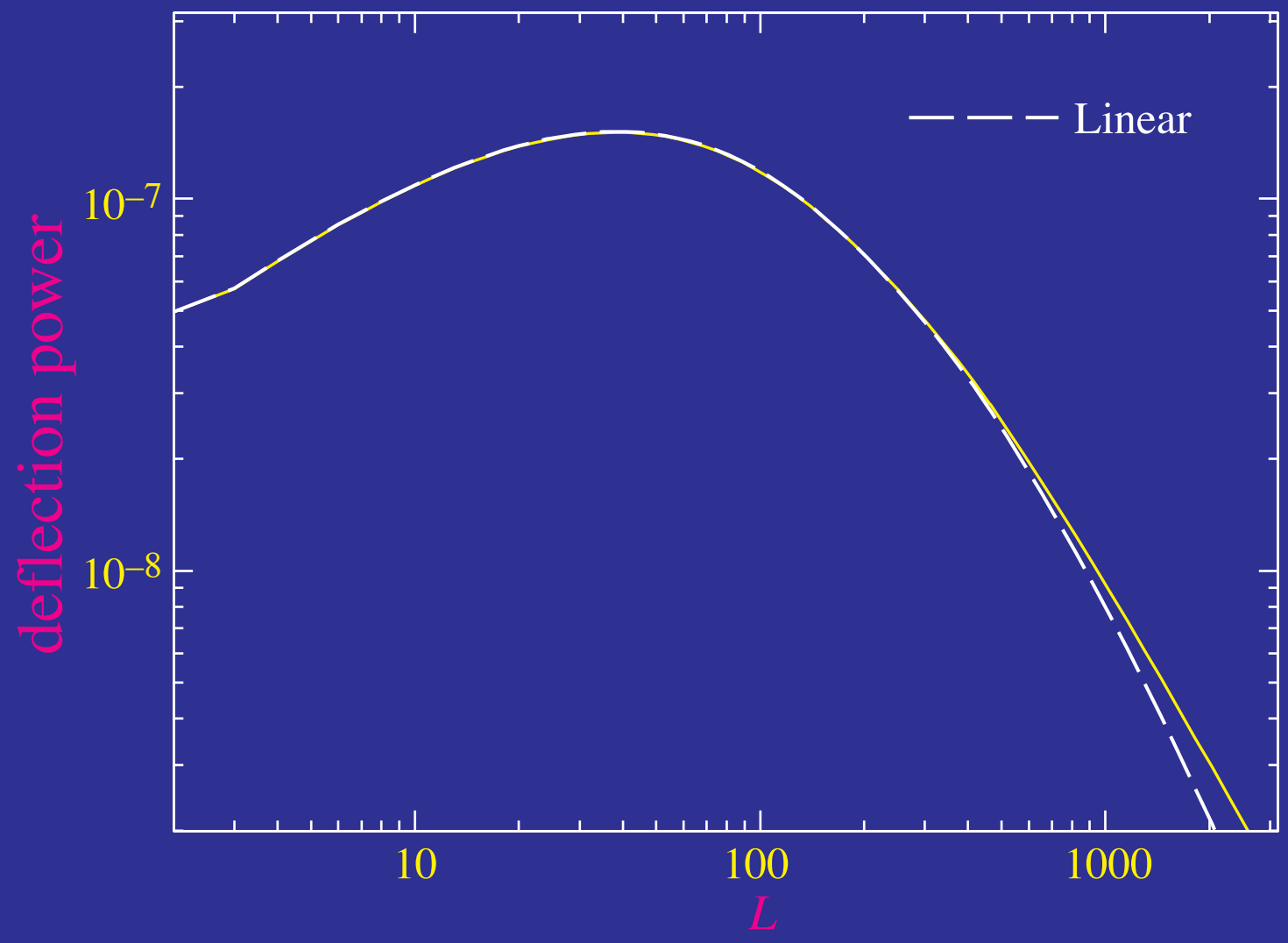
- Mass distribution at large angles and high redshift in in the linear regime
- Projected mass distribution (low pass filtered reflecting deflection angles): 1000 sq. deg



rms deflection
2.6'
deflection coherence
10°

Deflection Power Spectrum

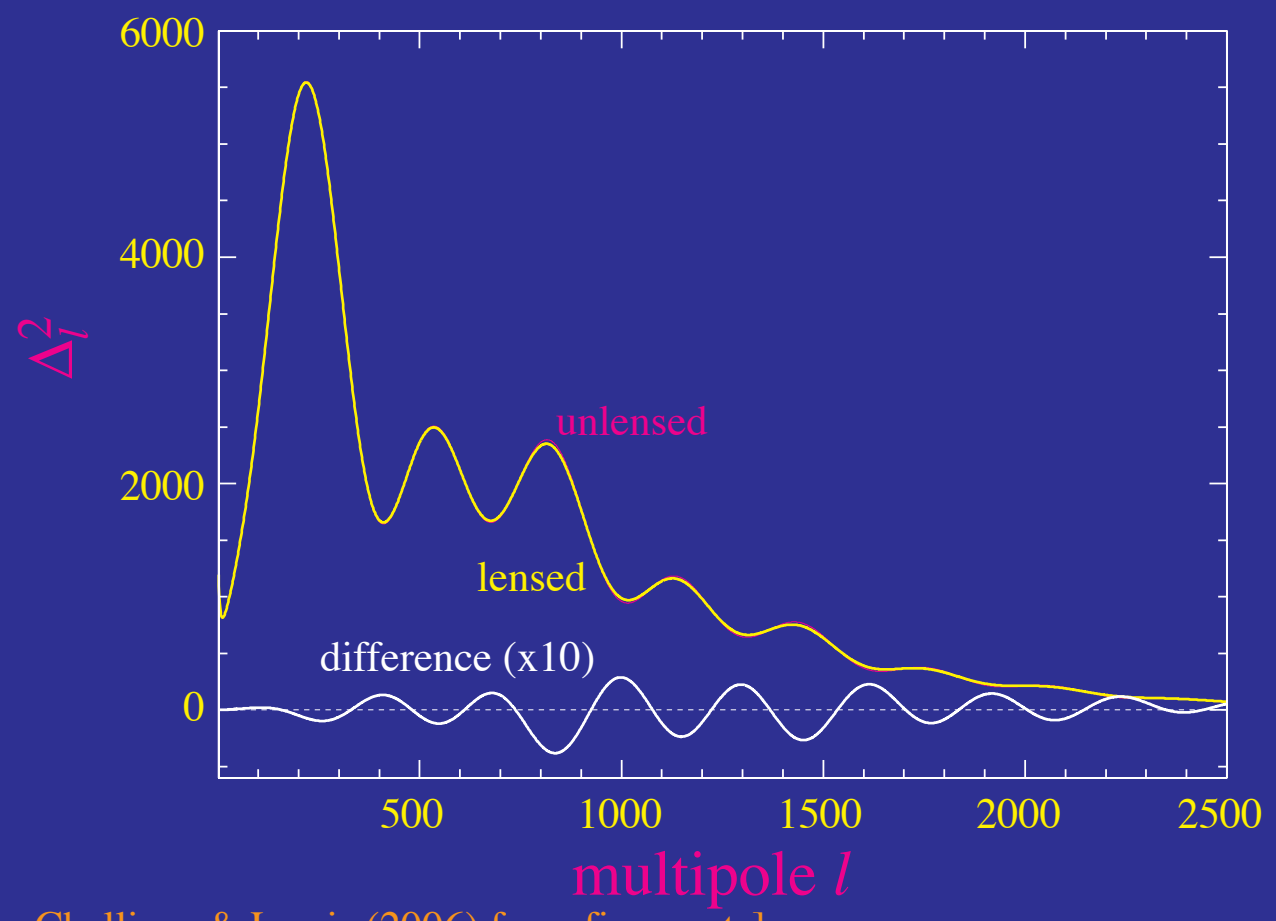
- Fundamental observable is deflection power spectrum (or convergence / l^2)
- Nearly entirely in linear regime



Power Spectrum Observables

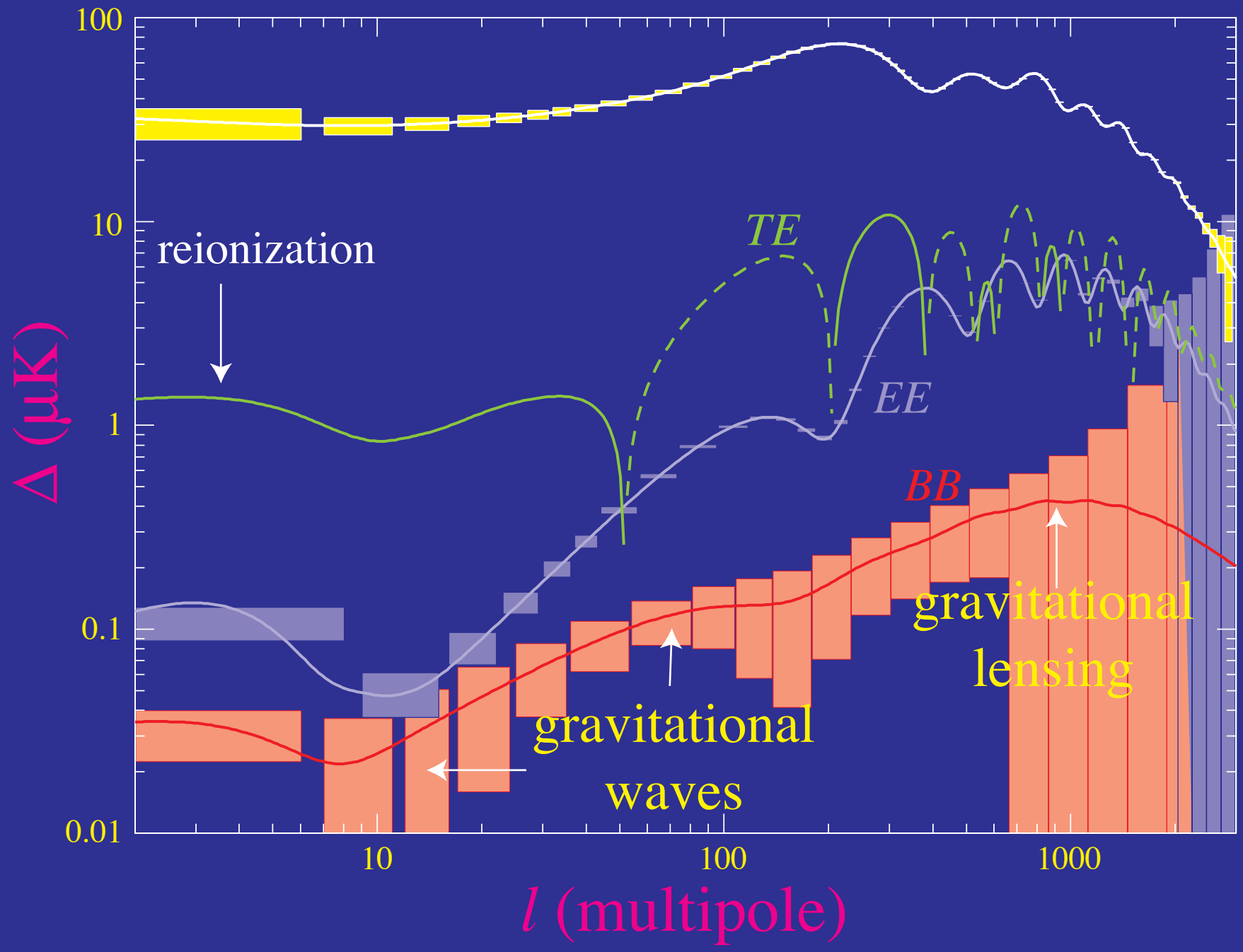
Temperature Power Spectrum

- Lensing acts to smooth the temperature (and E polarization peaks)
- Subtle effect reaches 10% deep in the damping tail
- Statistically detectable at high significance with Planck in the absence of other secondaries and foregrounds



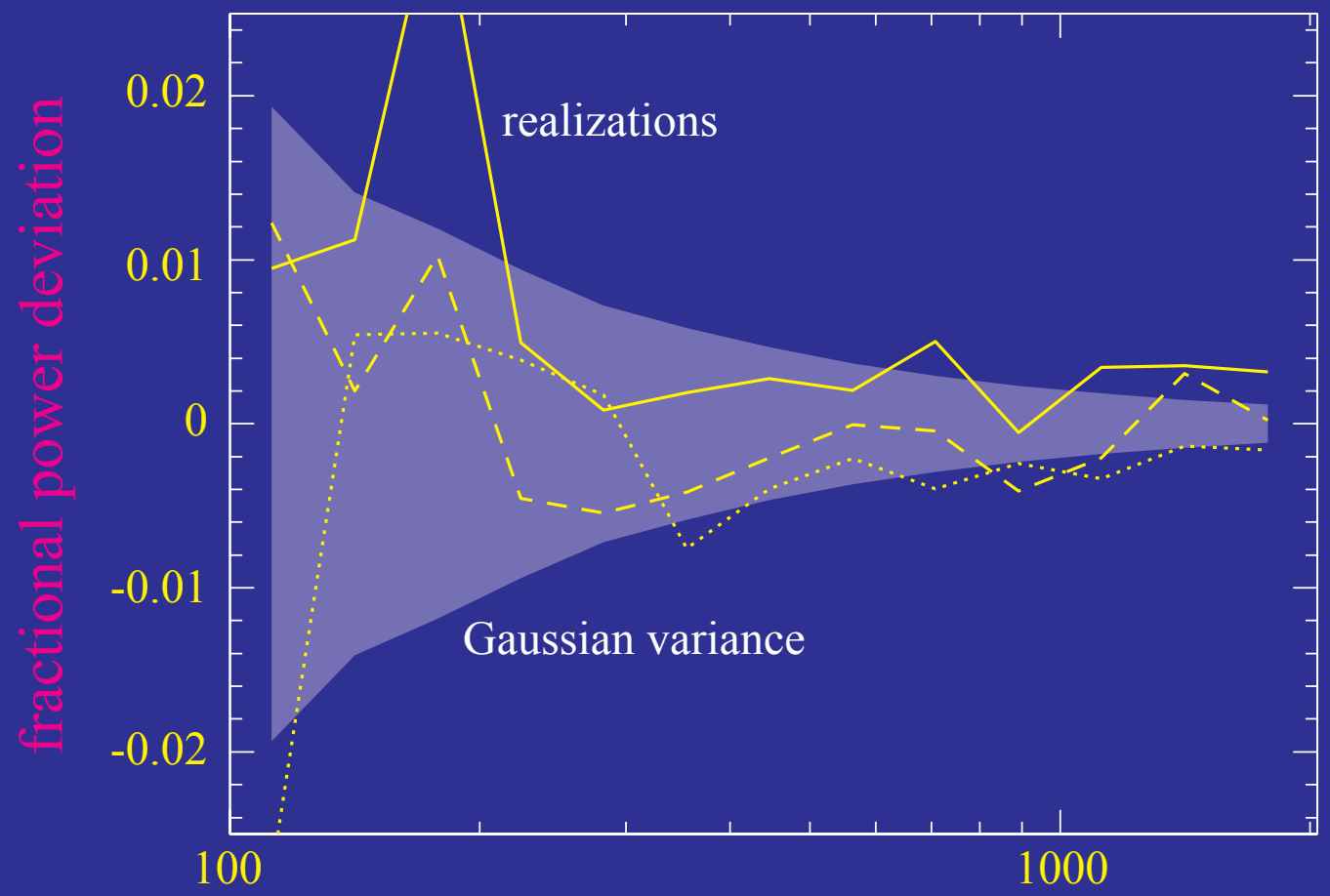
Seljak (1996) [see Challinor & Lewis (2006) for refinements]

Temperature and Polarization Spectra



Power Spectrum Measurements

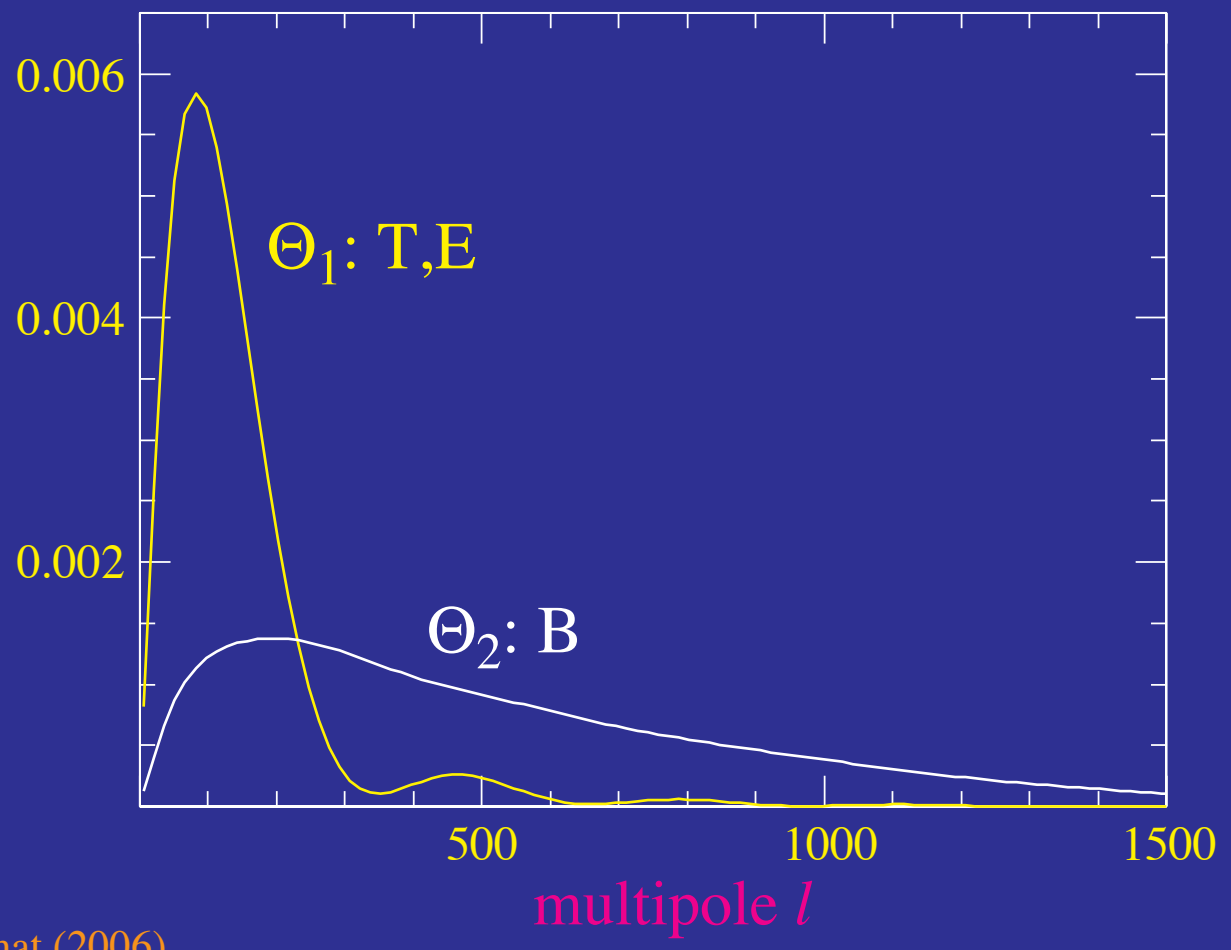
- Lensed field is non-Gaussian in that a single degree scale lens controls the polarization at arcminutes
- Increased variance and covariance implies that 10x as much sky needed compared with Gaussian fields



Smith, Hu & Kaplinghat (2004)

Lensed Power Spectrum Observables

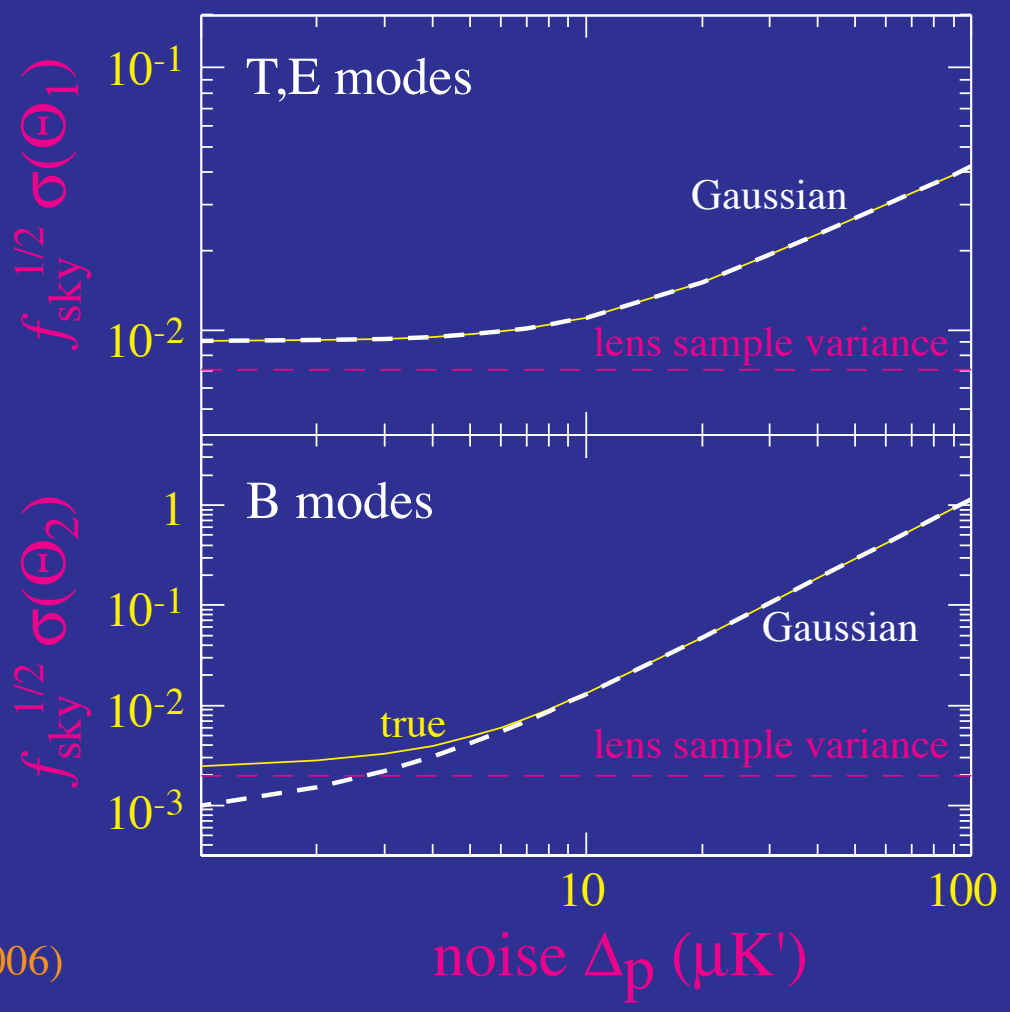
- Principal components show two observables in lensed power spectra
- Temperature and E-polarization: deflection power at l~100
  B-polarization: deflection power at l~500
- Normalized so that observables error = fractional lens power error



Smith, Hu & Kaplinghat (2006)

Constraints on Lensing Observables

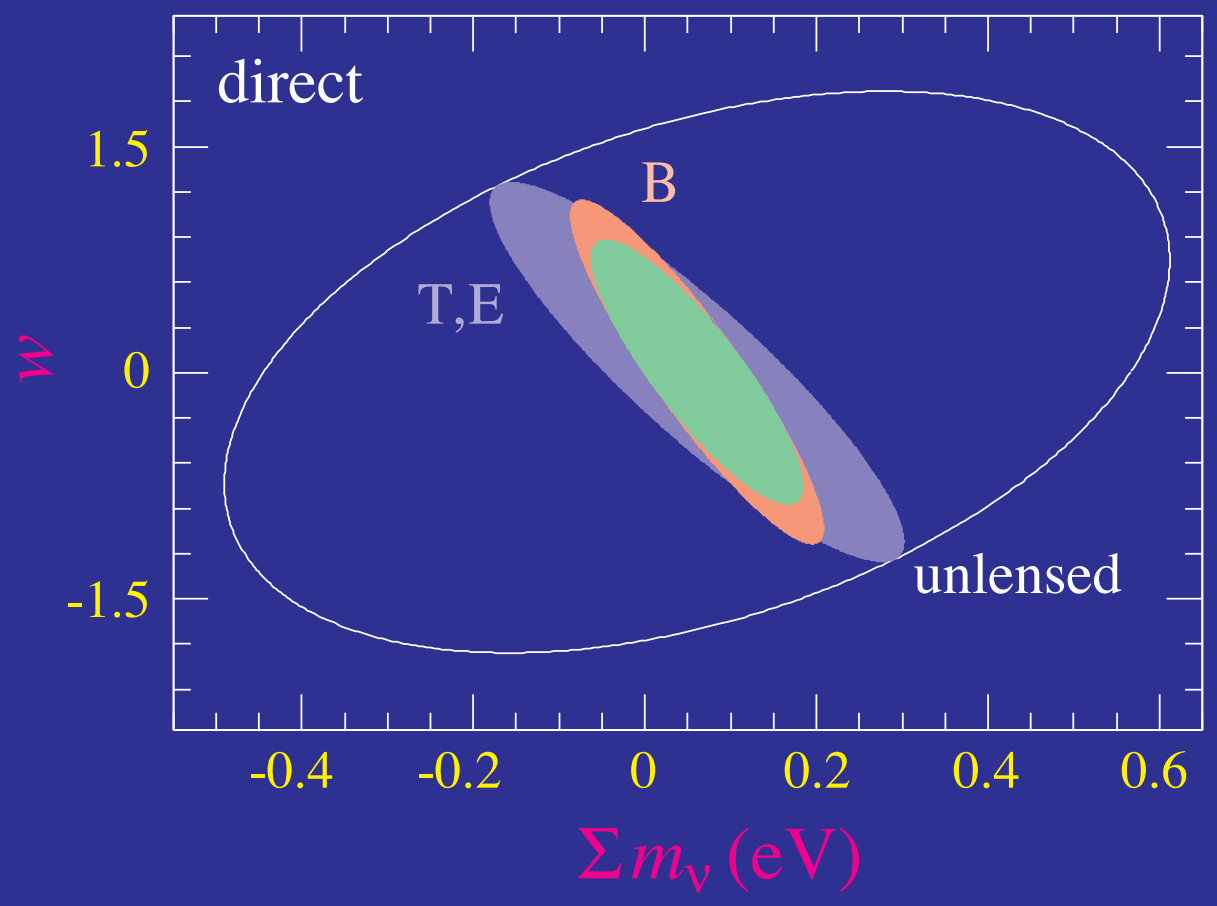
- Lensing observables in T,E are limited by CMB sample variance
- Lensing observables in B are limited by lens sample variance
- B-modes require 10x as much sky at high signal-to-noise or 3x as much sky at the optimal signal-to-noise with Δ_P=4.7uK'



Smith, Hu & Kaplinghat (2006)

Lensing Observables

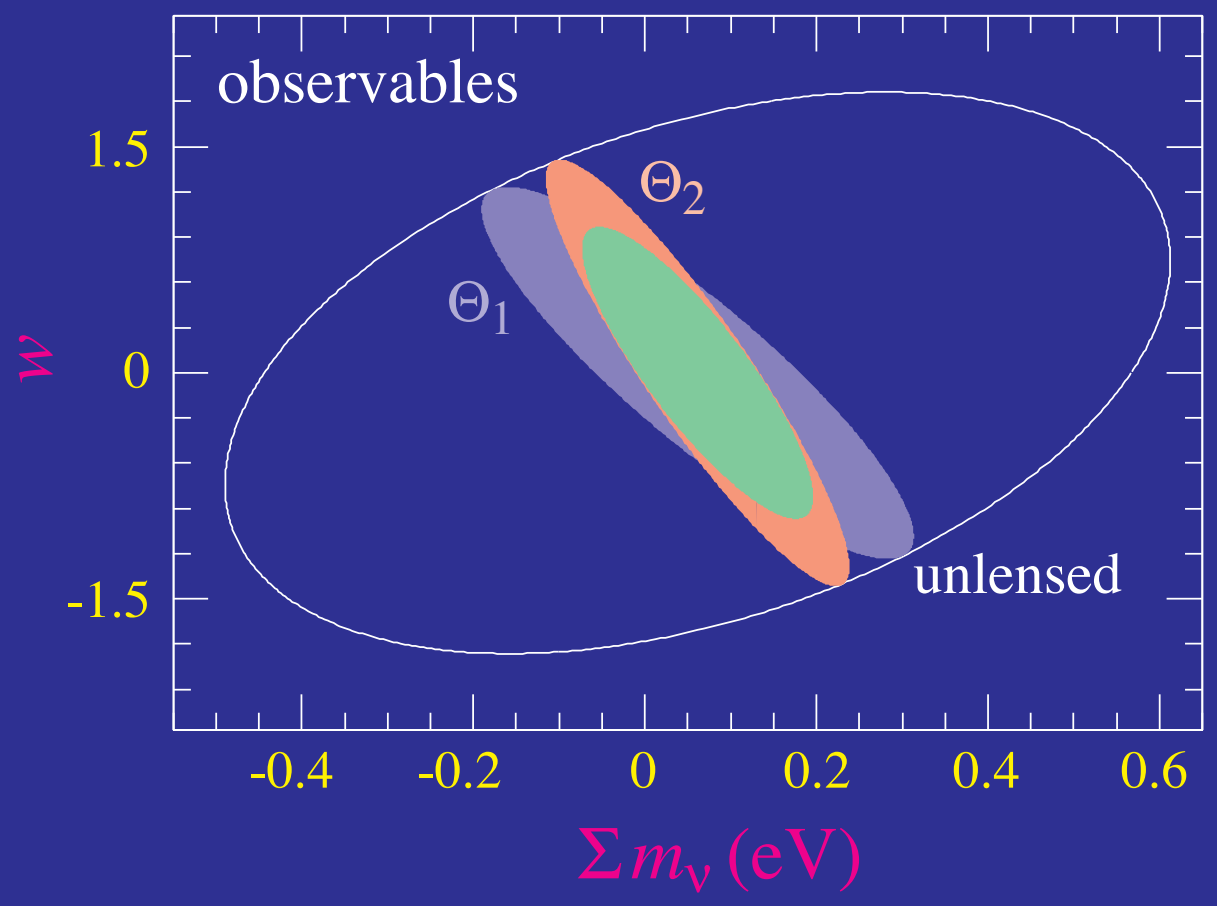
- Lensing observables provide a simple way of accounting for non-Gaussianity and parameter degeneracies
- Direct forecasts for Planck + 10% sky with noise Δ_P=1.4uK'



Smith, Hu, Kaplinghat (2006) [see also: Kaplinghat et. al 2003, Acquaviva & Baccigalupi 2005, Smith et al 2005, Li et al 2006]

Lensing Observables

- Lensing observables provide a simple way of accounting for non-Gaussianity and parameter degeneracies
- Observables forecasts for Planck + 10% sky with noise Δ_P=1.4uK'



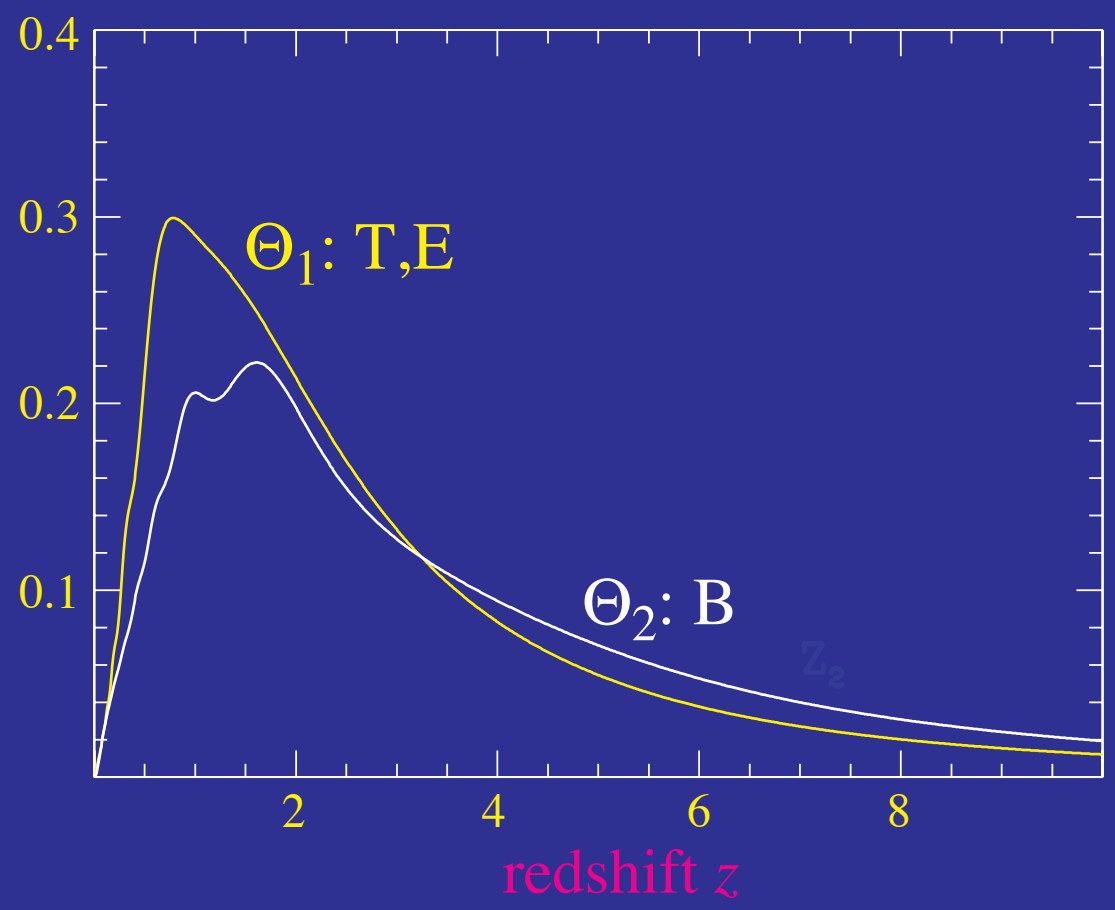
Smith, Hu, Kaplinghat (2006)

Complementarity

Redshift Sensitivity

- Lensing observables probe distance and structure at high redshift

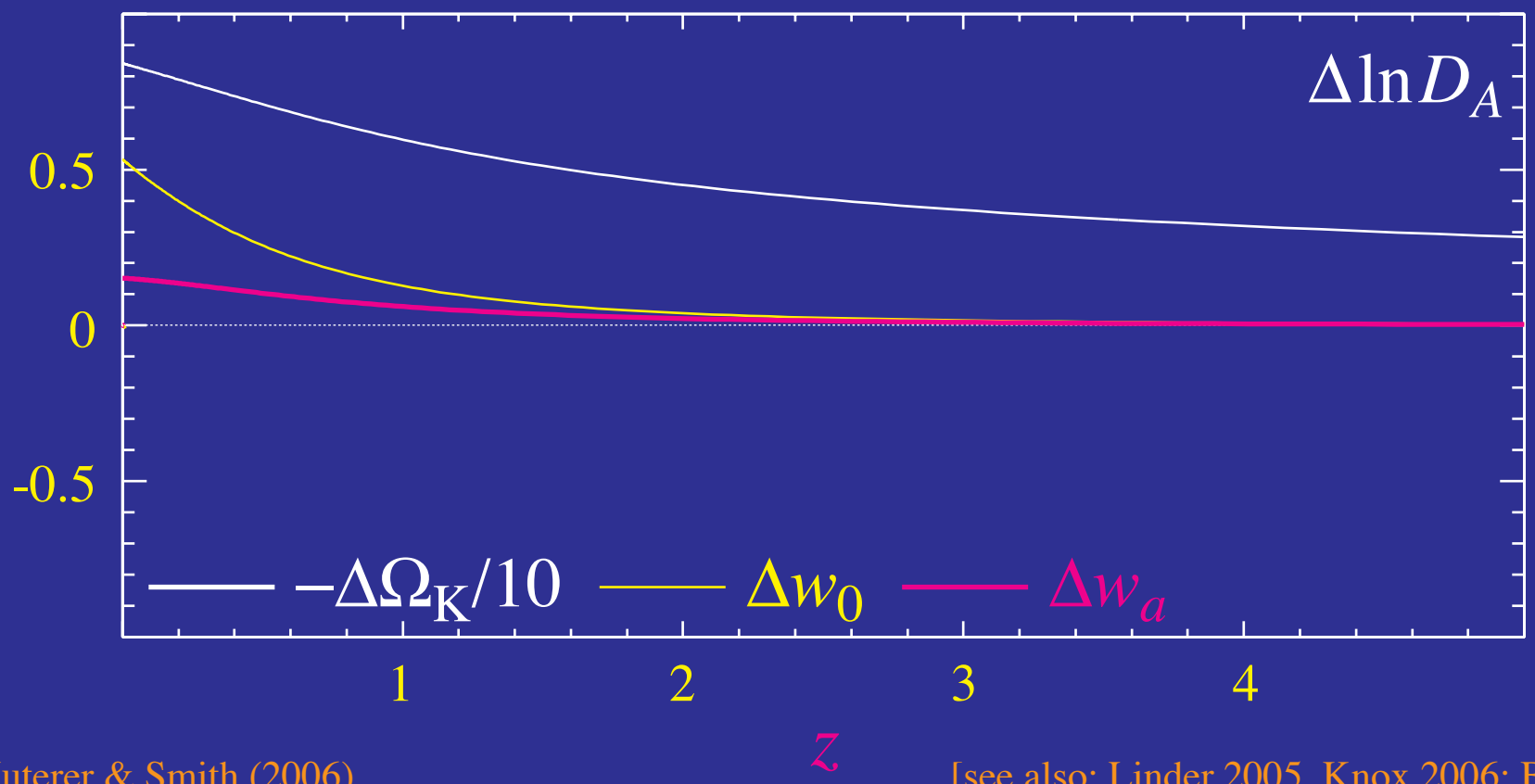
$$\frac{\delta\Theta_i}{\Theta_i} = \left[\left(3 - \frac{d\ln\Delta_m^2}{d\ln k}\right)\frac{\delta D_A}{D_A} - \frac{\delta H}{H} + 2\frac{\delta G}{G} + 2\frac{\delta D_A(D_s - D)}{D_A(D_s - D)}\right]$$



Smith, Hu & Kaplinghat (2006)

Curvature, H_0 and Dark Energy

- CMB peaks fix distance to recombination and $\Omega_m h^2$
- Deviations from ΛCDM distance at low z indicate either spatial curvature or dark energy equation of state $w=w_0-(1-a)w_a\neq-1$
- CMB lensing measures intermediate z and can eliminate curvature degeneracy

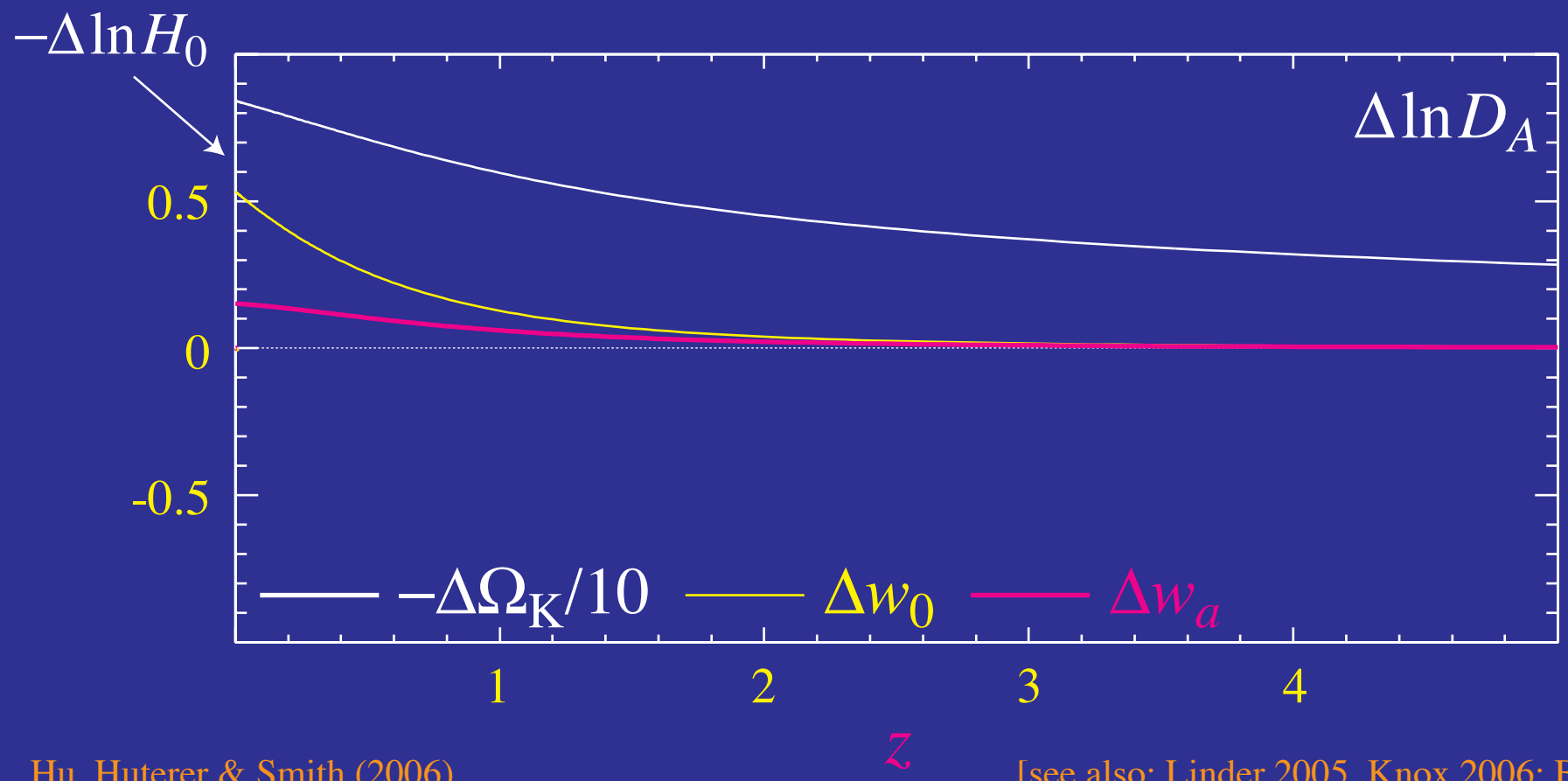


Hu, Huterer & Smith (2006)

[see also: Linder 2005, Knox 2006; Bernstein 2006]

Curvature, H_0 and Dark Energy

- CMB peaks fix distance to recombination and $\Omega_m h^2$
- Deviations from ΛCDM distance at low z indicate either spatial curvature or dark energy equation of state $w=w_0-(1-a)w_a\neq-1$
- Allowing H_0 to measure the dark energy or

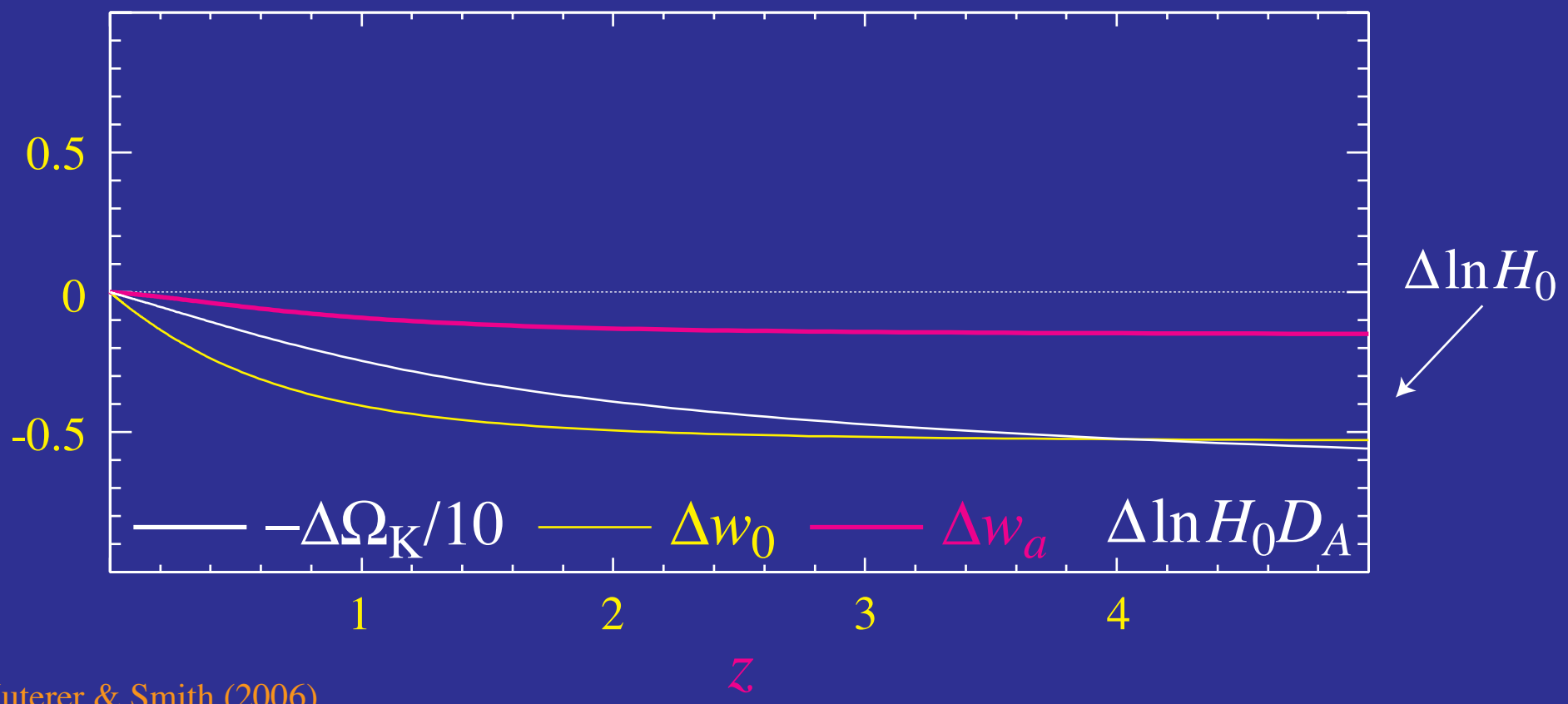


Hu, Huterer & Smith (2006)

[see also: Linder 2005, Knox 2006; Bernstein 2006]

Curvature, H_0 and Dark Energy

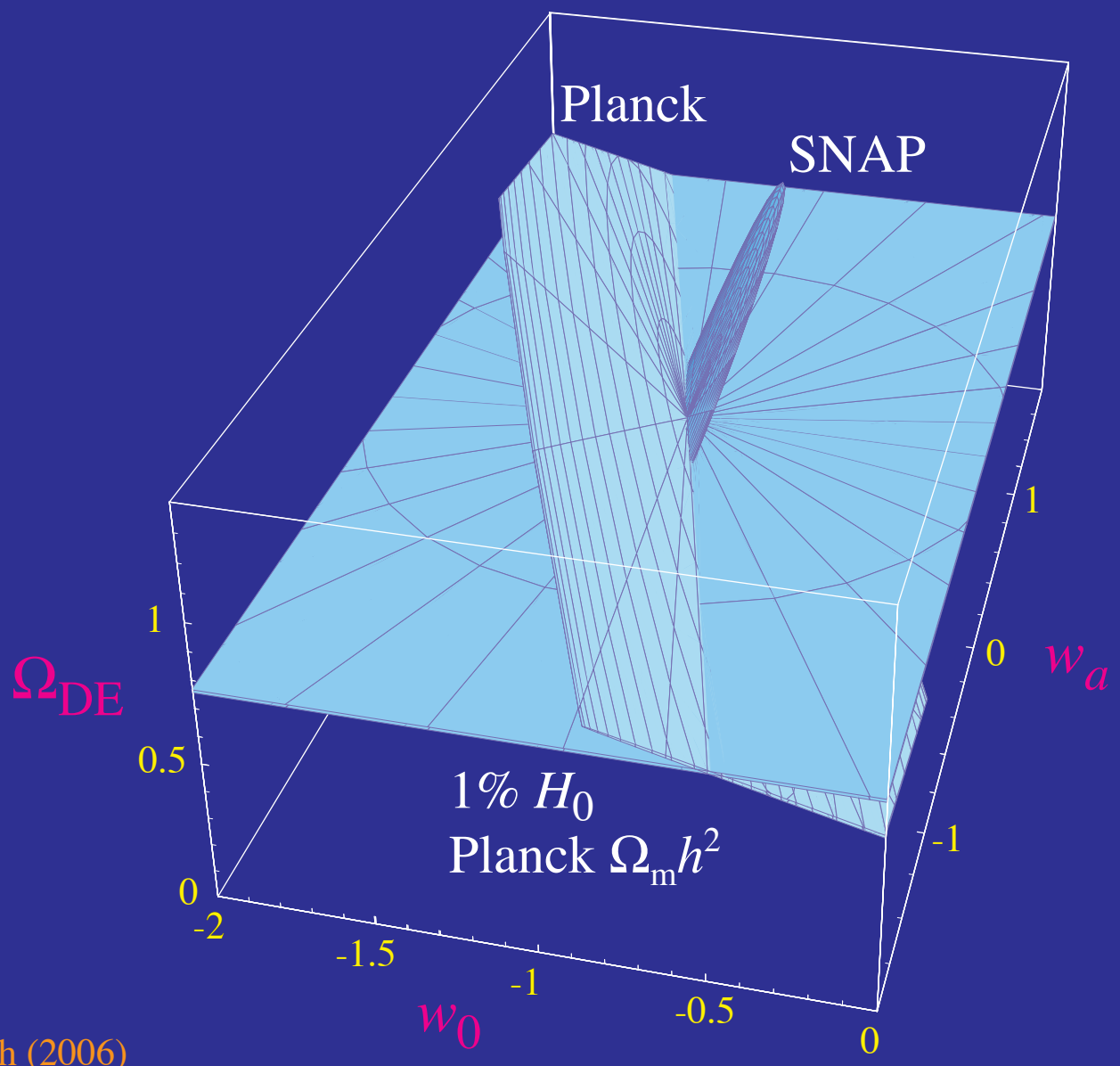
- CMB peaks fix distance to recombination and $\Omega_m h^2$
- Deviations from ΛCDM distance at low z indicate either spatial curvature or dark energy equation of state $w=w_0-(1-a)w_a\neq-1$
- SNIa relative distance measurements to measure (w_0, w_a)
- (Alternately H_0 can remove the curvature degeneracy)



Hu, Huterer & Smith (2006)

Flat Universe Precision

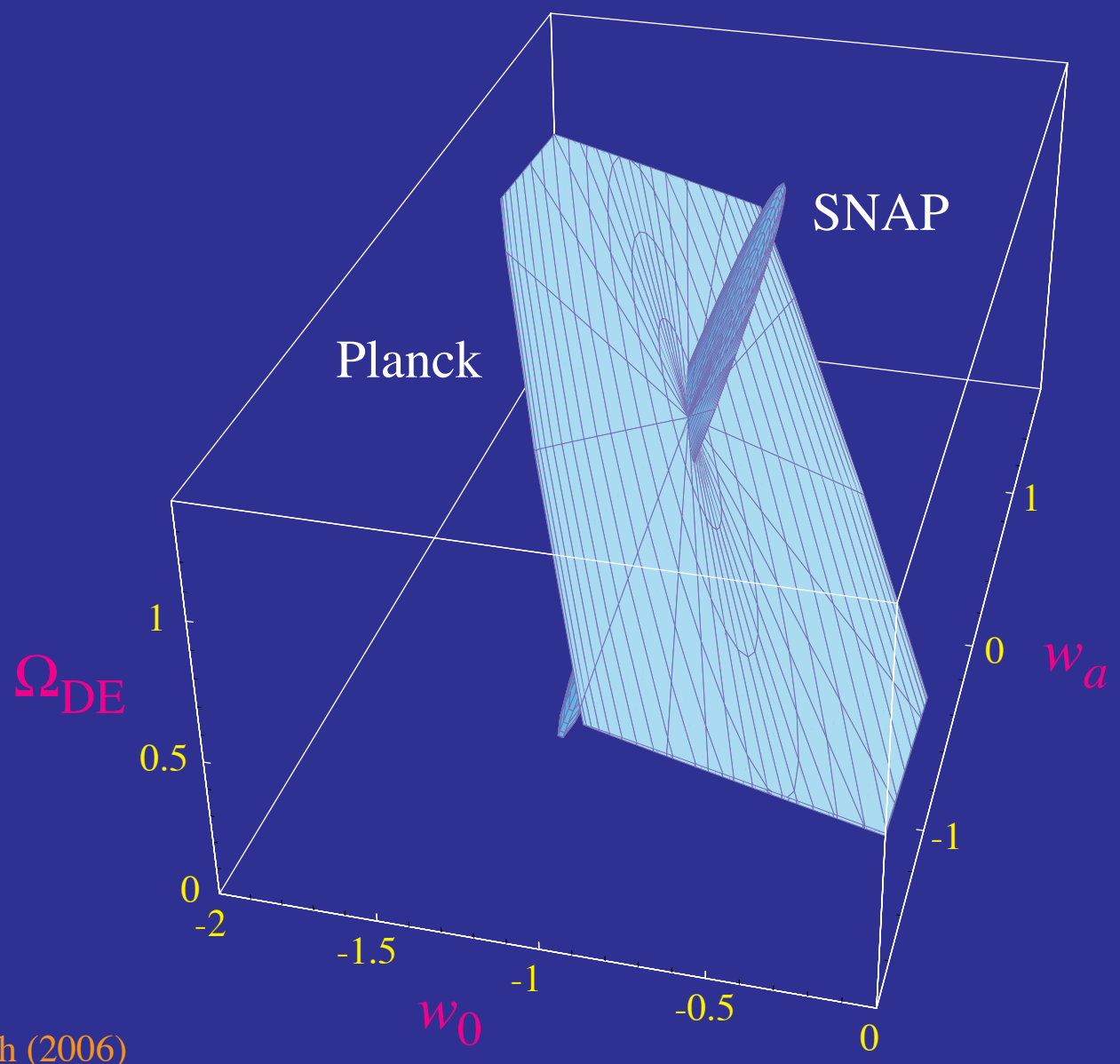
- Planck acoustic peaks, 1% H_0, SNAP SNe to z=1.7 in a flat universe



Hu, Huterer & Smith (2006)

Flat Universe Precision

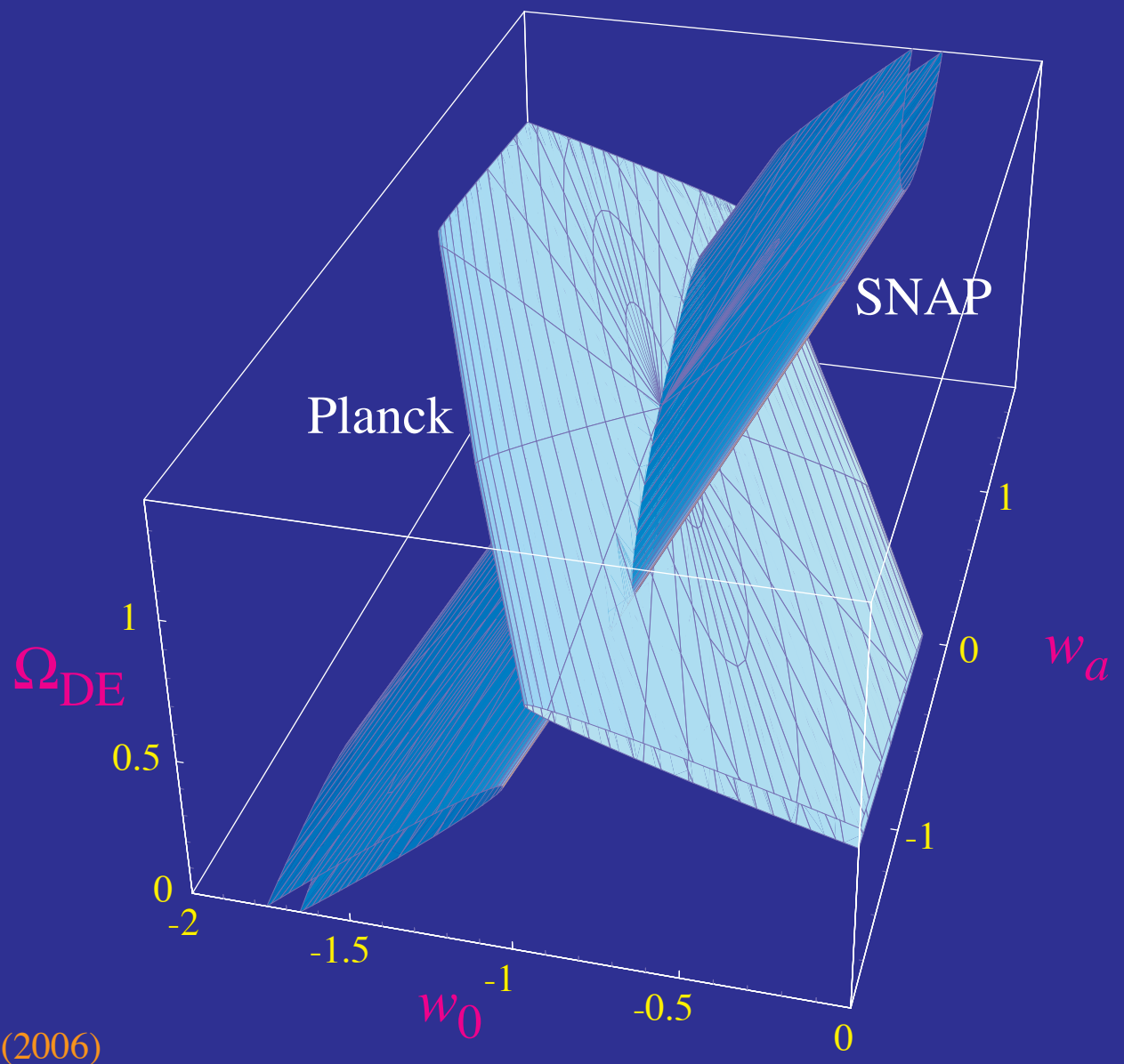
- Planck acoustic peaks, SNAP SNe to z=1.7 in a flat universe



Hu, Huterer & Smith (2006)

Marginalizing Curvature w/o Lensing

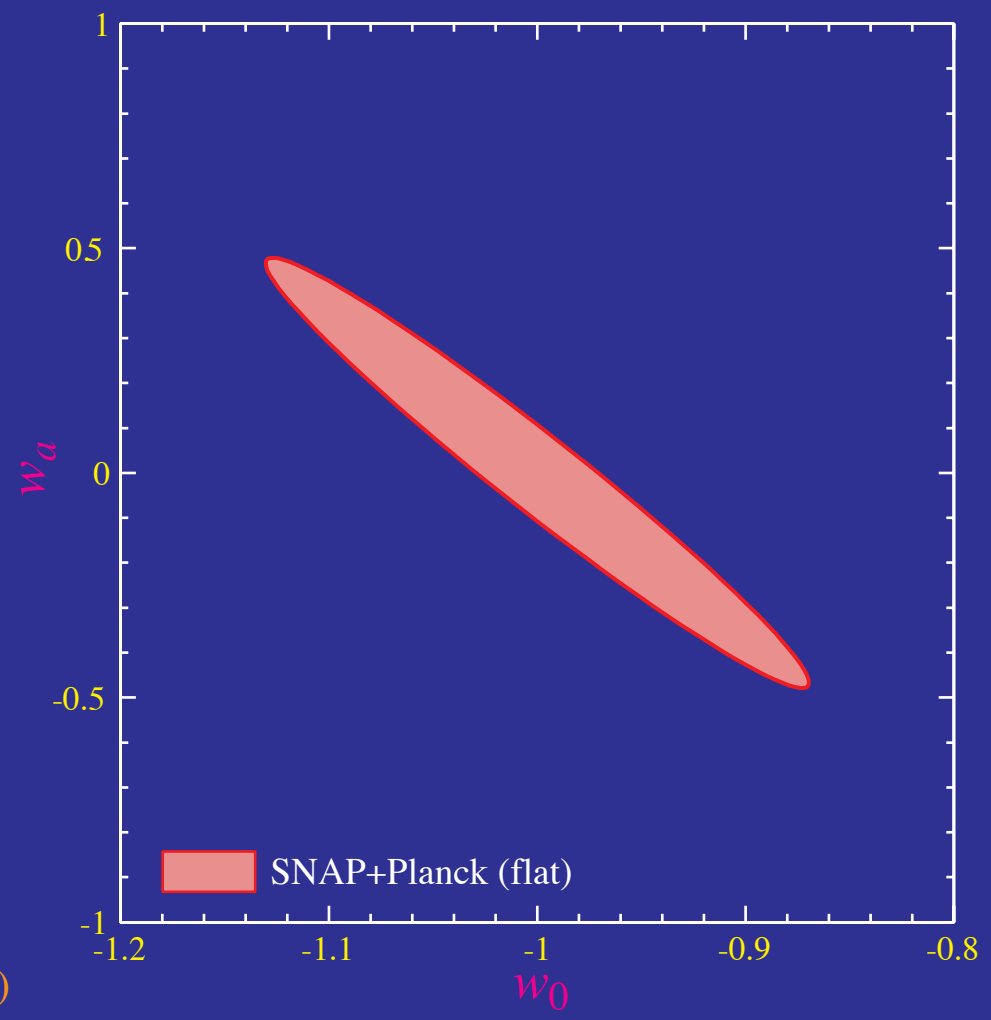
- Marginalizing curvature acts as a superposition of error ellipses



Hu, Huterer & Smith (2006)

Dark Energy Equation of State

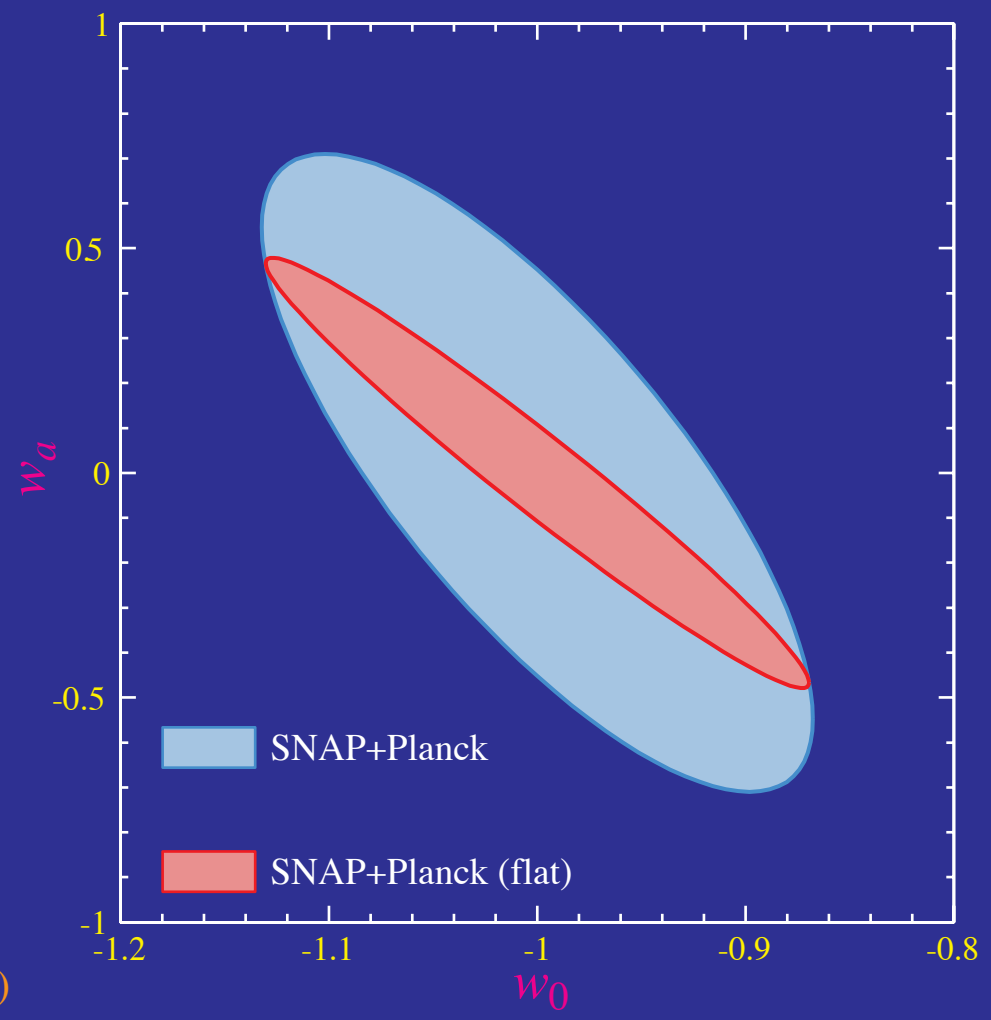
- Marginalizing curvature degrades 68% CL area by 4.8
- CMB lensing information from SPTpol (~3% B-mode power) fully restores constraints



Hu, Huterer & Smith (2006)

Dark Energy Equation of State

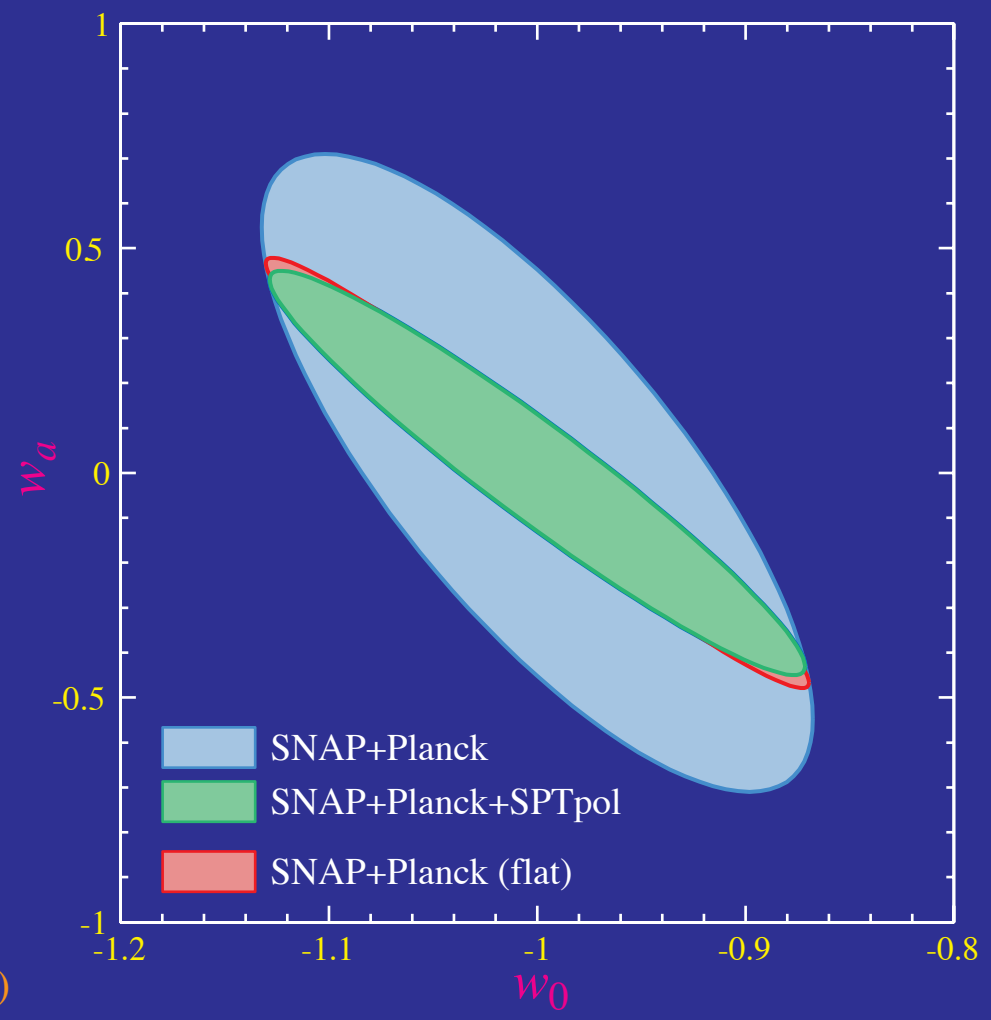
- Marginalizing curvature degrades 68% CL area by 4.8
- CMB lensing information from SPTpol (~3% B-mode power) fully restores constraints



Hu, Huterer & Smith (2006)

Dark Energy Equation of State

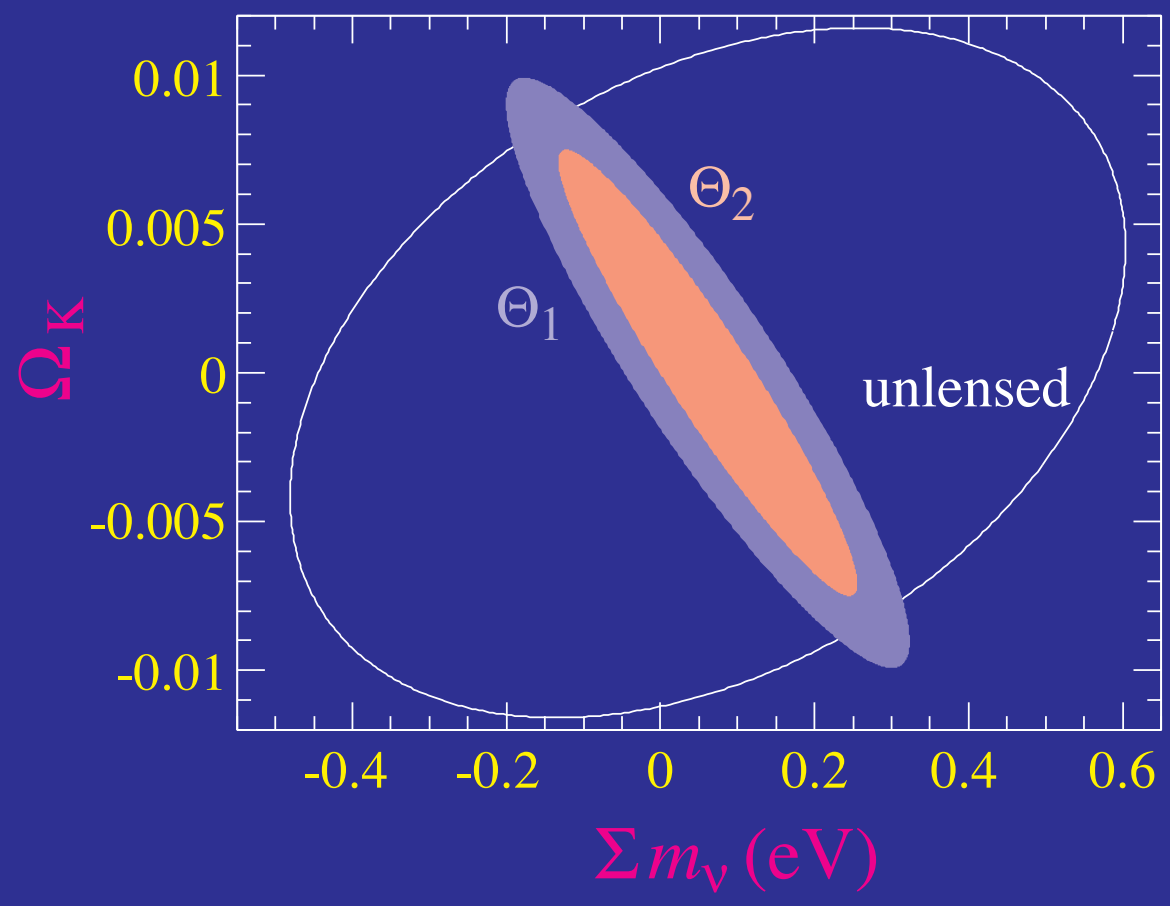
- Marginalizing curvature degrades 68% CL area by 4.8
- CMB lensing information from SPTpol (~3% B-mode power) fully restores constraints



Hu, Huterer & Smith (2006)

Degeneracy with Massive Neutrinos

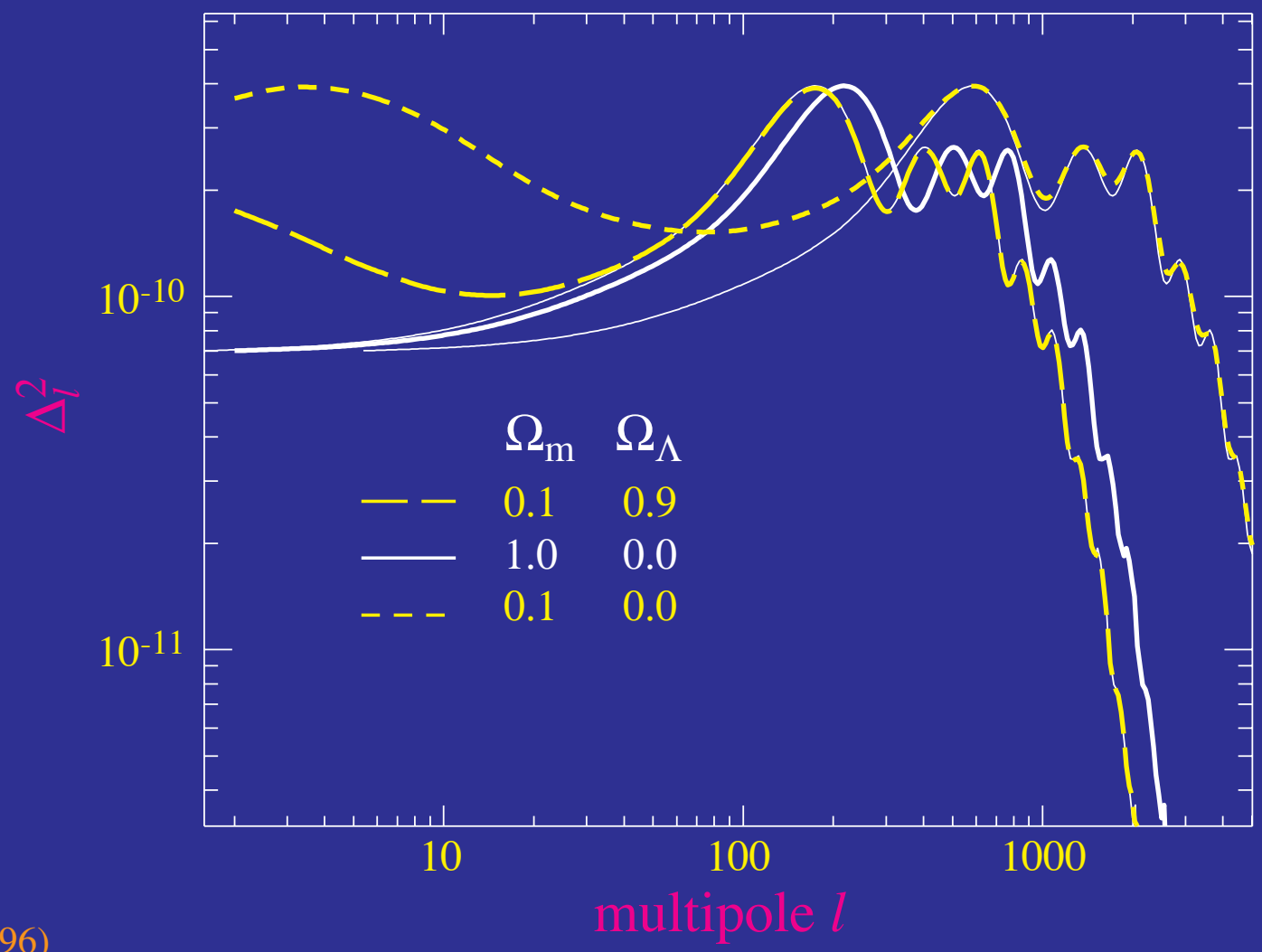
- Lensing observables are nearly degenerate in neutrinos and curvature
- Planck + 10% sky with noise Δ_P=1.4uK'



Smith, Hu, Kaplinghat (2006)

Degeneracy with Massive Neutrinos

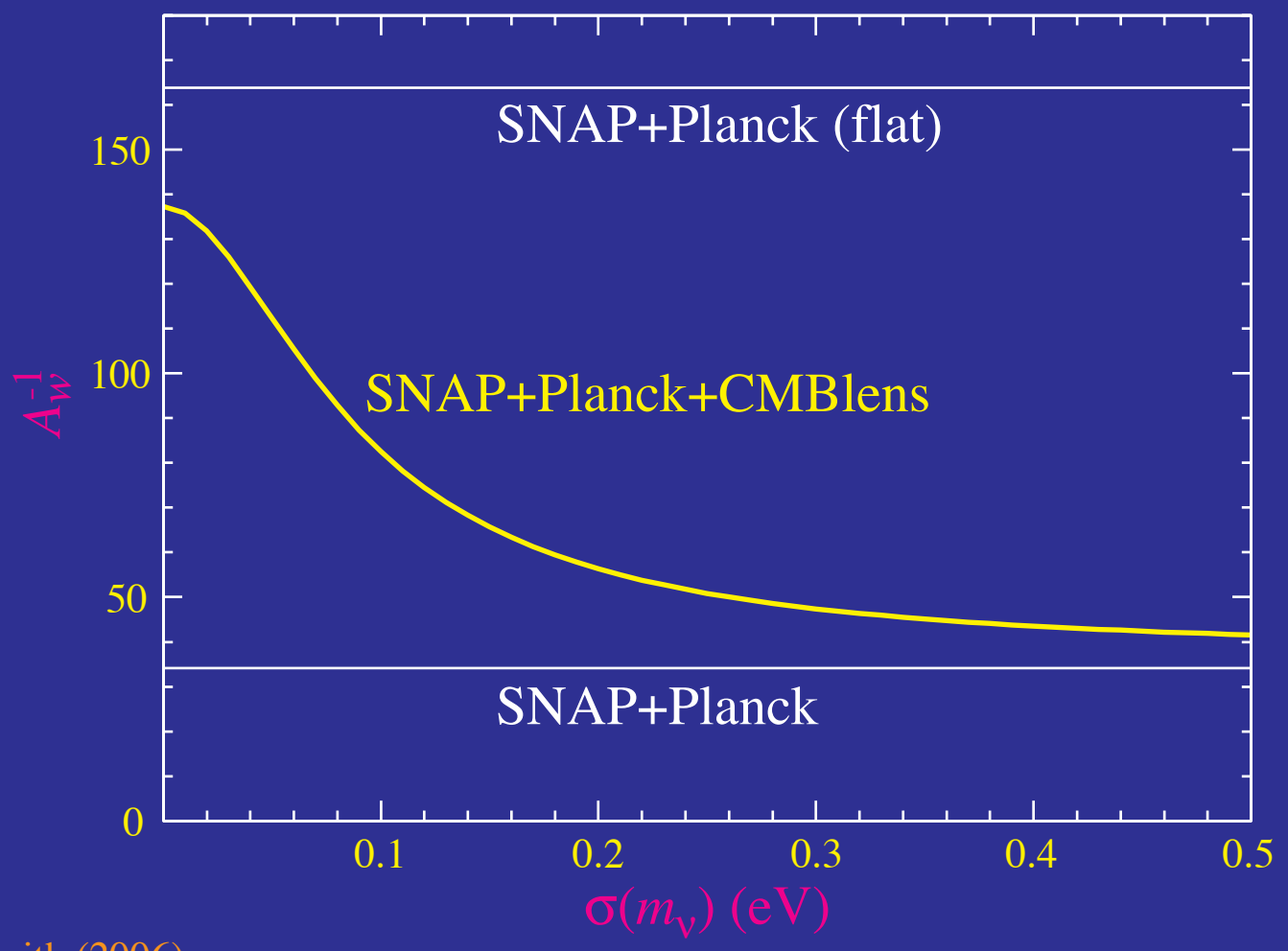
- Degeneracy is partially an accidental cancellation of intrinsic information from curvature across last scattering surface
- Degeneracy potentially removeable beyond power spectra



Hu & White (1996)

Priors on Massive Neutrinos

- In a normal hierarchy and with the lightest neutrino <0.01eV, masses are well enough determined from oscillation experiments already
- More generally, priors on the sum of masses <0.1eV required



inverse area in w_0xw_a

Hu, Huterer & Smith (2006)

Mass Reconstruction

Quadratic Estimator

- Taylor expand mapping

$$T(\hat{\mathbf{n}}) = \tilde{T}(\hat{\mathbf{n}} + \nabla\phi)$$
$$= \tilde{T}(\hat{\mathbf{n}}) + \nabla_i\phi(\hat{\mathbf{n}})\nabla^i\tilde{T}(\hat{\mathbf{n}}) + \ldots$$

- Fourier decomposition $\rightarrow$ mode coupling of harmonics

$$T(\mathbf{l}) = \int d\hat{\mathbf{n}}\, T(\hat{\mathbf{n}}) e^{-i\mathbf{l}\cdot\hat{\mathbf{n}}}$$
$$= \tilde{T}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} (\mathbf{l} - \mathbf{l}_1)\cdot\mathbf{l}_1\, \tilde{T}(\mathbf{l}_1)\phi(\mathbf{l} - \mathbf{l}_1)$$

- Consider fixed lens and Gaussian random CMB realizations: each pair is an estimator of the lens at $\mathbf{L} = \mathbf{l}_1 + \mathbf{l}_2$ (Hu 2001):

$$\langle T(\mathbf{l})T'(\mathbf{l}')\rangle_{\rm CMB} \approx \left[\tilde{C}^{TT}_{l_1}(\mathbf{L}\cdot\mathbf{l}_1) + \tilde{C}^{TT}_{l_2}(\mathbf{L}\cdot\mathbf{l}_2)\right]\phi(\mathbf{L}) \quad (\mathbf{l} \neq -\mathbf{l}')$$

Reconstruction from the CMB

- Generalize to polarization: each quadratic pair of fields estimates the lensing potential (Hu & Okamoto 2002)

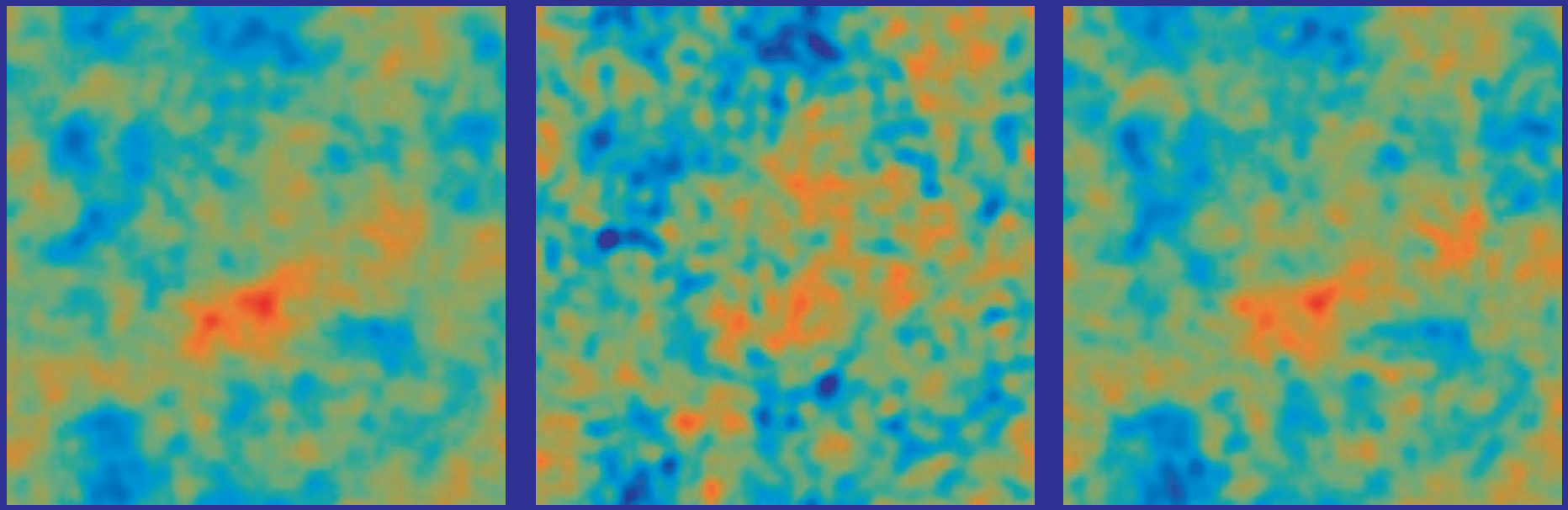
$$\langle x(\mathbf{l})x'(\mathbf{l}')\rangle_{\rm CMB} = f_\alpha(\mathbf{l},\mathbf{l}')\phi(\mathbf{l}+\mathbf{l}')\,,$$

  where $x \in$ temperature, polarization fields and f_α is a fixed weight that reflects geometry

- Each pair forms a noisy estimate of the potential or projected mass - just like a pair of galaxy shears
- Minimum variance weight all pairs to form an estimator of the lensing mass
- Generalize to inhomogeneous noise, cut sky and maximum likelihood by iterating the quadratic estimator (Seljak & Hirata 2002)

Ultimate (Cosmic Variance) Limit

- Cosmic variance of CMB fields sets ultimate limit
- Polarization allows mapping to finer scales (~10')



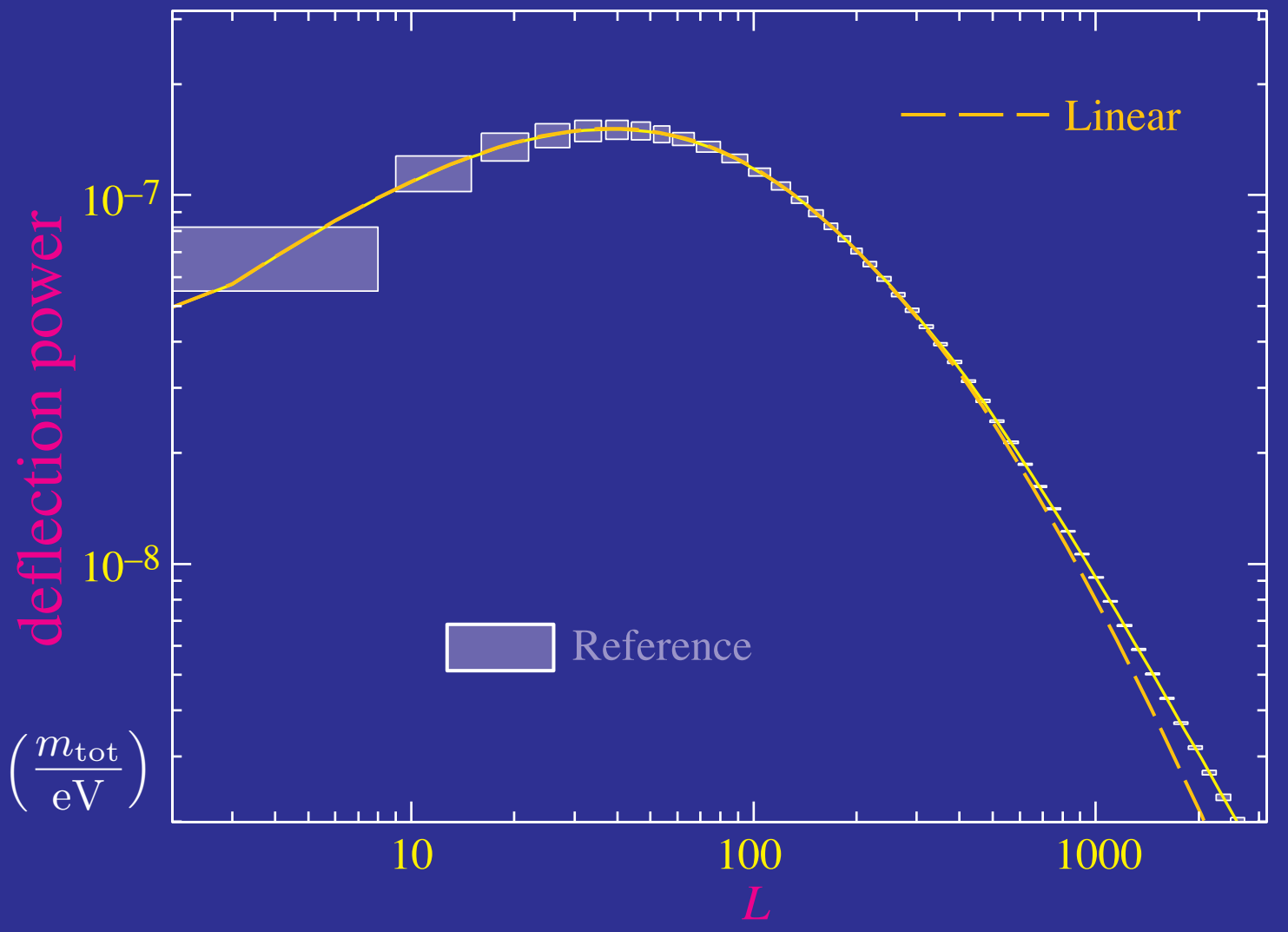
mass | temp. reconstruction | EB pol. reconstruction

100 sq. deg; 4' beam; 1μK-arcmin

Hu & Okamoto (2001)

Matter Power Spectrum

- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime 0.002< k < 0.2 *h*/Mpc

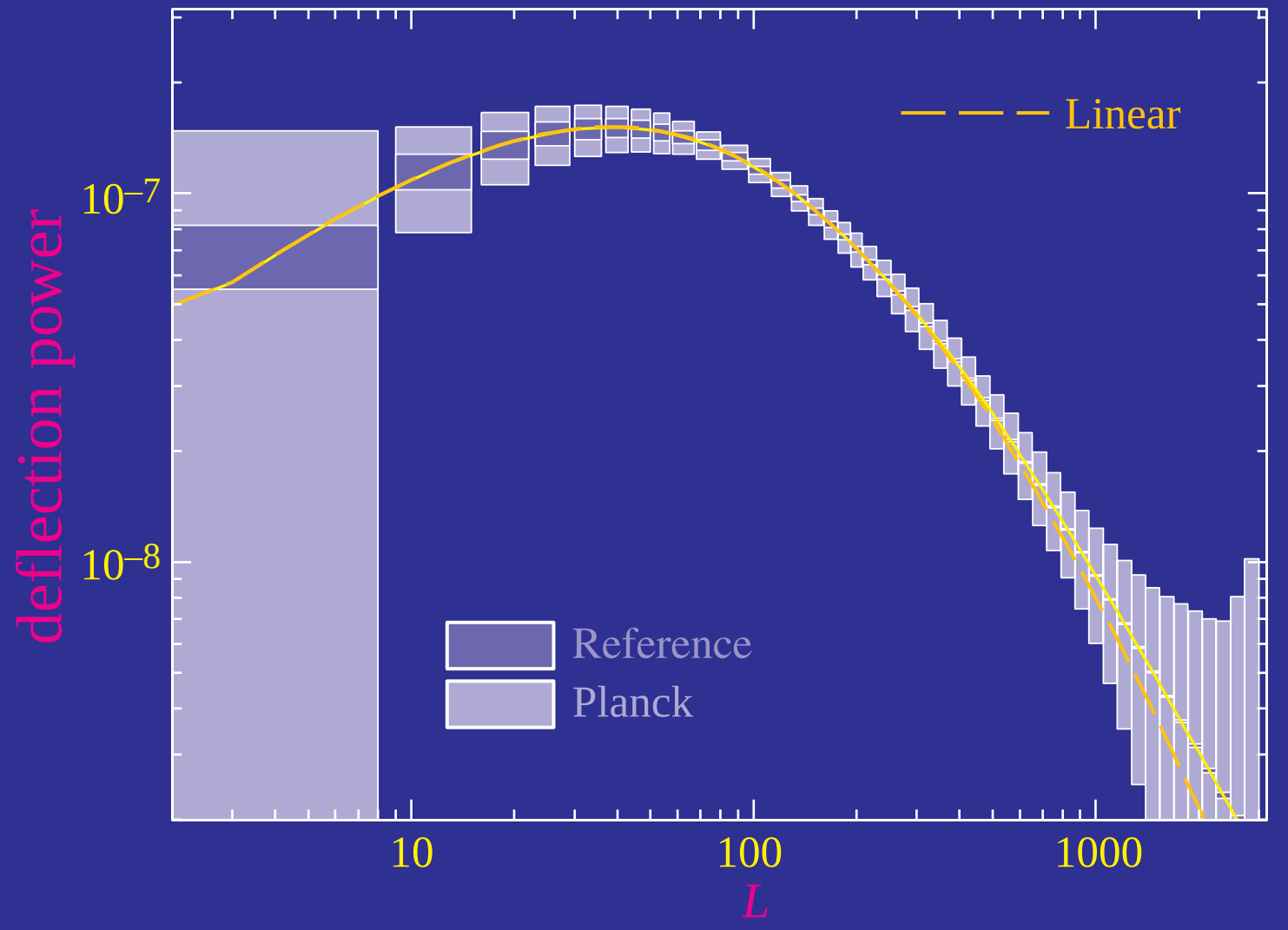


$$\frac{\Delta P}{P} \approx -0.6 \left(\frac{m_{\rm tot}}{\rm eV}\right)$$

Hu & Okamoto (2001)

Matter Power Spectrum

- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime 0.002< k < 0.2 *h*/Mpc



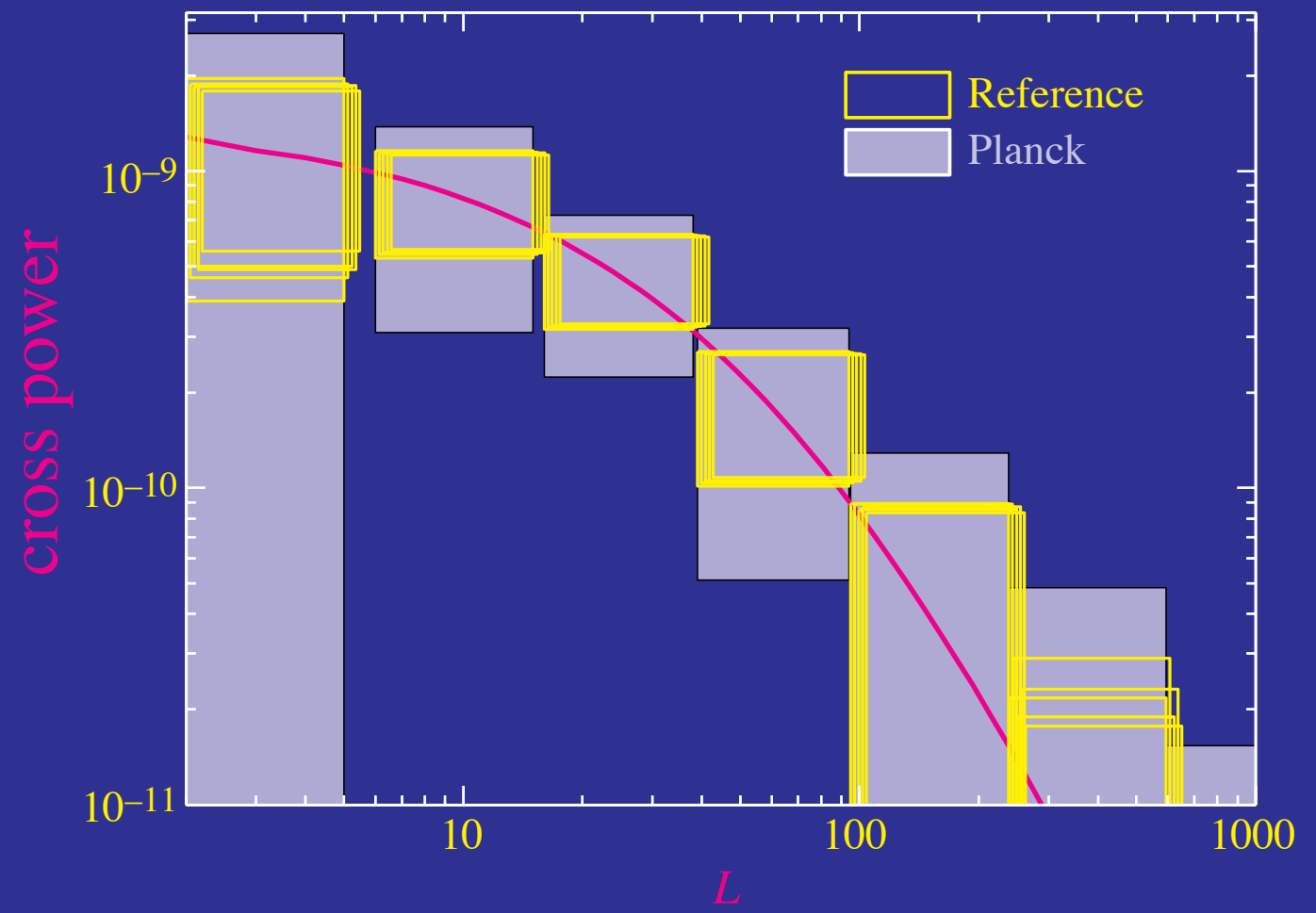
Hu & Okamoto (2001)

Breaking Degeneracies

- Reconstructed power spectrum comes from the non-Gaussian part of the CMB: 4pt and higher correlations
- Contains more information than the two lensing observables which probe the amplitude of the power spectrum at around only two multipoles
- Degeneracies between neutrinos, curvature and the dark energy can potentially be broken (Hu 2002, Kaplinghat, Knox & Song 2003)
- Small biases must be removed from higher order Taylor terms and other non-Gaussian secondaries and foregrounds
- Further study of techniques needed to insure accuracy of measurements

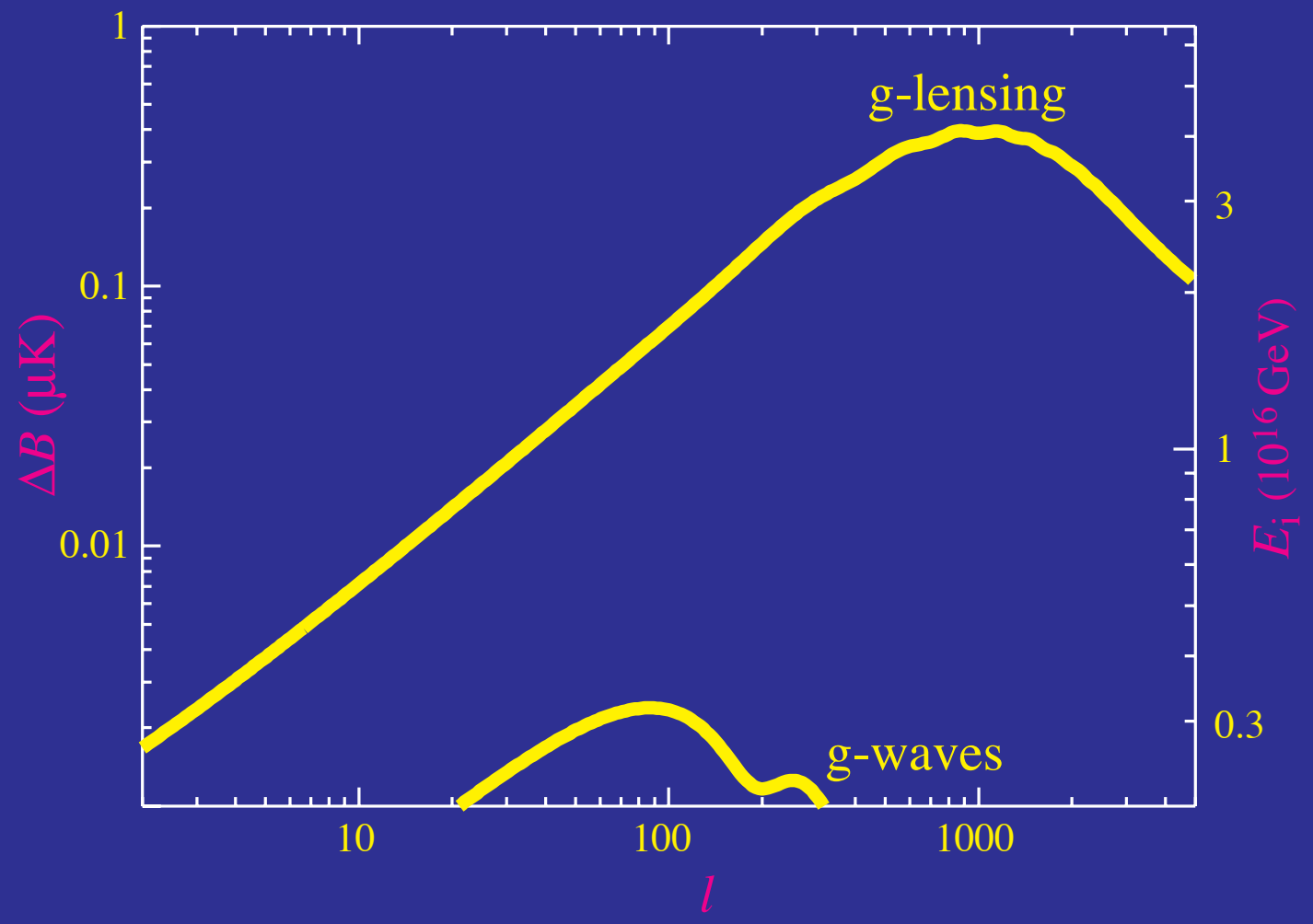
Cross Correlation with Temperature

- Correlation with ISW effect tests the nature of acceleration
- Tests smoothness of dark energy (scalar field) Hu & Okamoto (2002)
- Tests modified gravity (e.g. DGP braneworld) Zhang (2006)



De-Lensing the Polarization

- Gravitational lensing contamination of B-modes from gravitational waves cleaned to E_i~0.3 x 10^{16} GeV
  Hu & Okamoto (2002); Knox & Song (2002); Cooray, Kedsen, Kamionkowski (2002)
- Potentially further with maximum likelihood Hirata & Seljak (2004)



Summary

- Gravitational lensing of the CMB should be detectable in next generation temperature and polarization measurements
- CMB power spectra measure two lensing observables, associated with convergence at $\ell \sim 100, 500$ and $z \sim 1 - 4$
- Observable measurements limited by sample variance of lenses regardless of how well arcminute CMB anisotropy measured
- Lensing observables is a useful framework for studying parameter degeneracies and complementarity with other cosmological probes
- SPTpol and other experiments can fix spatial curvature well enough for SNAP and Planck dark energy measurements
- Mass reconstruction can help break degeneracies, test fundamental principles in acceleration, and clean the polarization field for gravitational wave studies