Cosmological Chameleon

Wayne Hu
Copenhagen, August 2011
Outline

- Falsifying $\Lambda$ in favor of Modified Gravity?
- Cosmological Statistics in Non-Linear Regime
- Parameterized Post Friedmann Chameleon Description
- Collaborators

  Wenjuan Fang  Dragan Huterer
  Simone Ferarro  Yin Li
  Marcos Lima  Lucas Lombriser
  Michael Mortonson  Hiro Oyaizu
  Fabian Schmidt  Hiranya Peiris
  Iggy Sawicki  Uros Seljak
  Yong-Seon Song  Anze Slosar
  Amol Upadhye  Sheng Wang
  Ali Vanderveld  Alexey Vikhlinin
Cosmic Acceleration

- Geometric measures of distance redshift from SN, CMB, BAO

![Graph showing relation between supernovae luminosity and flux, with redshift plotted on the x-axis and a measure related to the speed of light, $\Delta \mu$, on the y-axis. The graph includes error bars and a fitted line.](image)
• General relativity says *Gravity = Geometry*

• And *Geometry = Matter-Energy*

• Could the *missing energy* required by *acceleration* be an *incomplete* description of how *matter determines geometry*?
Geometry Predicts Growth
Falsifying $\Lambda$CDM

- $\Lambda$ slows growth of structure in highly predictive way
Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way

Cosmological Constant
- Deviation significantly >2% rules out $\Lambda$ with or without curvature

Quintessence
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in $H_0$]
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)
Elephantine Predictions
Elephantine Predictions

- Geometric constraints on the cosmological parameters of $\Lambda$CDM
- Convert to distributions for the predicted average number of clusters above a given mass and redshift
ΛCDM Falsified?

- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the $M(z)$ curve
- No currently known high mass, high redshift cluster violates this bound

Mortonson, Hu, Huterer (2010)
Pink Elephant Parade

- SPT catalogue on 2500 sq degrees

Williamson et al (2011)
Predictions for Cosmic Shear

- $\Lambda$CDM statistical prediction for cosmic shear and sources at $z=0.5, 2, 3.5$

\[ \frac{l(l+1)C_l}{2\pi} \]

Vanderveld et al (2011)
Testing Gravity:
$f(R)$ Example
Dynamical vs Lensing Mass

- Newtonian potential: $\Psi = \delta g_{00}/2g_{00}$ which non-relativistic particles feel

- Space curvature: $\Phi = \delta g_{ii}/2g_{ii}$ which also deflects photons

- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass
Testing Gravity with Cosmology

- Currently most **incisive tests** are in the **non-linear regime**
- With **smooth dark energy**, the **linear growth** function predicts **non-linear statistics**
  - Mass function
  - Power spectrum (halo model, Halofit), \( n \)-pt function
- With **modified gravity**, how does the **linear growth** function relate to **non-linear statistics**?
- Gravity well tested **locally** and must return to **General Relativity**
- Smooth dark energy **linear \( \leftrightarrow \) non-linear** must break
- How do we describe this? **Parameterized Post Friedmann** description
- Where does **cosmology** best complement **local** tests?
Modified Action $f(R)$ Model

- $R$: Ricci scalar or “curvature”
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]$$

- $f_R \equiv df / dR$: additional propagating scalar degree of freedom  
  (metric variation)
- $f_{RR} \equiv d^2f / dR^2$: Compton wavelength of $f_R$ squared, inverse mass squared
- $B$: Compton wavelength of $f_R$ squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}$$

- $' \equiv d/d \ln a$: scale factor as time coordinate
Three Regimes

- Fully worked $f(R)$ and DGP examples show 3 regimes
- Superhorizon regime: $\zeta = \text{const.}, \ g(a)$
- Linear regime - closure condition - analogue of “smooth” dark energy density:

\[
\frac{\nabla^2(\Phi - \Psi)}{2} = -4\pi G a^2 \Delta \rho
\]

\[
g(a, x) \leftrightarrow g(a, k)
\]

$G$ can be promoted to $G(a)$ but conformal invariance relates fluctuations to field fluctuation that is small

- Non-linear regime:

\[
\frac{\nabla^2(\Phi - \Psi)}{2} = -4\pi G a^2 \Delta \rho
\]

\[
\nabla^2 \Psi = 4\pi G a^2 \Delta \rho - \frac{1}{2} \nabla^2 \phi
\]
Nonlinear Interaction

Nonlinearity in field equation recovers linear theory if $N[\phi] \rightarrow 0$

$$\nabla^2 \phi = g_{\text{lin}}(a)a^2 \left( 8\pi G \Delta \rho - N[\phi] \right)$$

- For $f(R)$, $\phi = f_R$ and
  $$N[\phi] = \delta R(\phi)$$
  a nonlinear function of the field
  Linked to gravitational potential

- For DGP, $\phi$ is the brane-bending mode and
  $$N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$
  a nonlinear function of second derivatives of the field
  Linked to density fluctuation - Galileon invariance - no self-shielding of external forces
Non-Linear Chameleon

- For $f(R)$ the field equation

$$\nabla^2 f_R \approx \frac{1}{3} (\delta R(f_R) - 8\pi G \delta \rho)$$

is the non-linear equation that returns general relativity

- High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$

- Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if

$$\Delta f_R \leq \frac{2}{3} \Phi,$$

else required field gradients too large despite $\delta R = 8\pi G \delta \rho$ being the local minimum of effective potential
Cosmological Simulations

- Solve for the **extra field** contribution to the **gravitational potential**–nonlinear Poisson-like equation – via **relaxation**

- Add this source to the **usual Poisson equation**

\[ \nabla^2 \Psi = 4\pi G a^2 \delta \rho - \frac{1}{2} \nabla^2 \phi \]

- Matter evolution given metric unchanged: usual **motion of matter** in a gravitational potential \( \Psi \)

- Prescription for **N-body** code

- **Particle Mesh (PM)** for the Poisson equation
  [adaptive mesh for high resolution: Zhao, Li, Koyama (2010,2011)]

- Field equation is a non-linear Poisson equation: **relaxation** method for \( f_R \)

- **Initial conditions** set to GR at high redshift
Environment Dependent Force

- Chameleon suppresses extra force (scalar field) in high density, deep potential regions

\[ f_R = 10^{-6} \]

- density: \( \max[\ln(1+\delta)] \)
- potential: \( \min[\Psi] \)
- field: \( \min[f_R/f_{R0}] \)
Environment Dependent Force

- For large background field, gradients in the scalar prevent the chameleon from appearing

\[
\begin{array}{ccc}
\text{density: max[ln(1+\delta)]} & \text{potential: min[}\Psi]\text{]} & \text{field: min[f_R/f_{R0}]} \\
\hline
f_{R0}=10^{-6} & \text{image} & \text{image} \\
\hline
f_{R0}=10^{-4} & \text{image} & \text{image} \\
\end{array}
\]

Enhanced abundance of rare dark matter halos (clusters) with extra force
Cluster $f(R)$ Constraints

- Clusters provide best current cosmological constraints on $f(R)$ models.
- Spherical collapse rescaling to place constraints on full range of inverse power law models of index $n$.

Cluster $f(R)$ Constraints

- Approaching competitiveness with solar system + Galaxy constraints of few $10^{-6}$ at low $n$
- Vastly different scale

Chameleon Mass Function

- **Chameleon effect suppresses the enhancement at high masses**
- **Pile up of abundance at intermediate group scale**

Lima, Schmidt, Oyaizu, Hu (2008)
Chameleon Pile-Up

- Chameleon threshold at intermediate masses \( (10^{13} \, h^{-1} \, M_\odot) \)
- Mergers from smaller masses continues, to higher masses stops
- Pile up of halos at threshold
PPF Parameterization

- Interpolate between linear $f(R)$ enhanced $\sigma(M)$ and ordinary gravity

Li & Hu (2011)
Chameleon Mass Function

- Conservation of mass in halos leads to pile up in model

Li & Hu (2011)
Universal Chameleon Mass Function

- Two parameters describe a wide range in $f(R)$ models

Li & Hu (2011)
Halo Model

- Suppression of 1 halo term on cluster scales

Li & Hu (2011)
• Connect to **linear regime** with interpolation of HaloFit

Li & Hu (2011)
Motion: Environment & Object

- Self-field of a “test mass” can saturate an external field (for $f(R)$ in the gradient, for DGP in the second derivatives)

\[ \phi_{\text{ext}} \]
\[ \phi_{\text{int}} \]
\[ \phi_{\text{tot}} \]

Hui, Nicolis, Stubbs (2009)
Jain & Vanderplas (2011)
Zhao, Li, Koyama (2011)
Summary

- Smooth dark energy and General Relativity, distance $\rightarrow$ growth
- $\Lambda$CDM places firm upper bound on growth of structure for all quintessence models, e.g. for cluster abundance or cosmic shear
- Strongest tests of predictions currently in non-linear regime
- Viable modified gravity scenarios must satisfy local tests of gravity, also in non-linear regime:
  - generally as function of environment not length
- $f(R)$ models employ chameleon mechanism - hide changes in deep potential wells
- Deep potential wells associated with dark matter halos
- Abundance exhibits double enhancement at chameleon threshold
- PPF Halo model parameterizes the chameleon mechanism for cosmological statistics