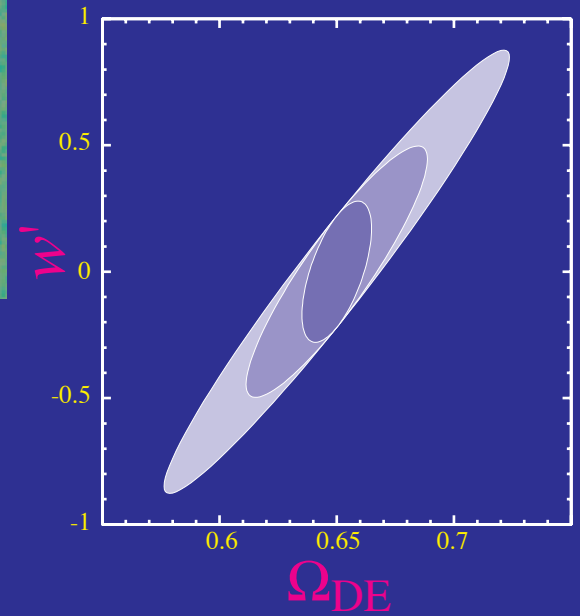
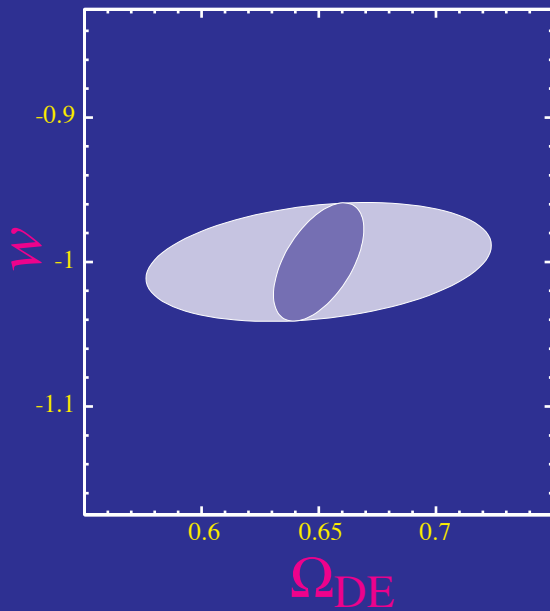
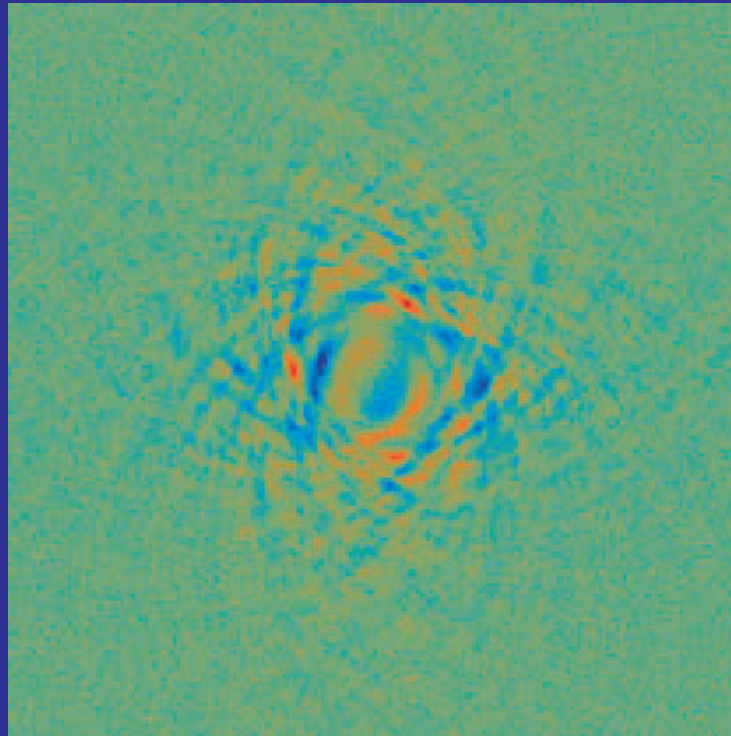


Gravitational Lensing and the Dark Energy



Future Prospects

Wayne Hu

SLAC, August 2002

Outline

- Standard model of cosmology
- Gravitational lensing
- Lensing probes of dark energy

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- Takemi Okamoto
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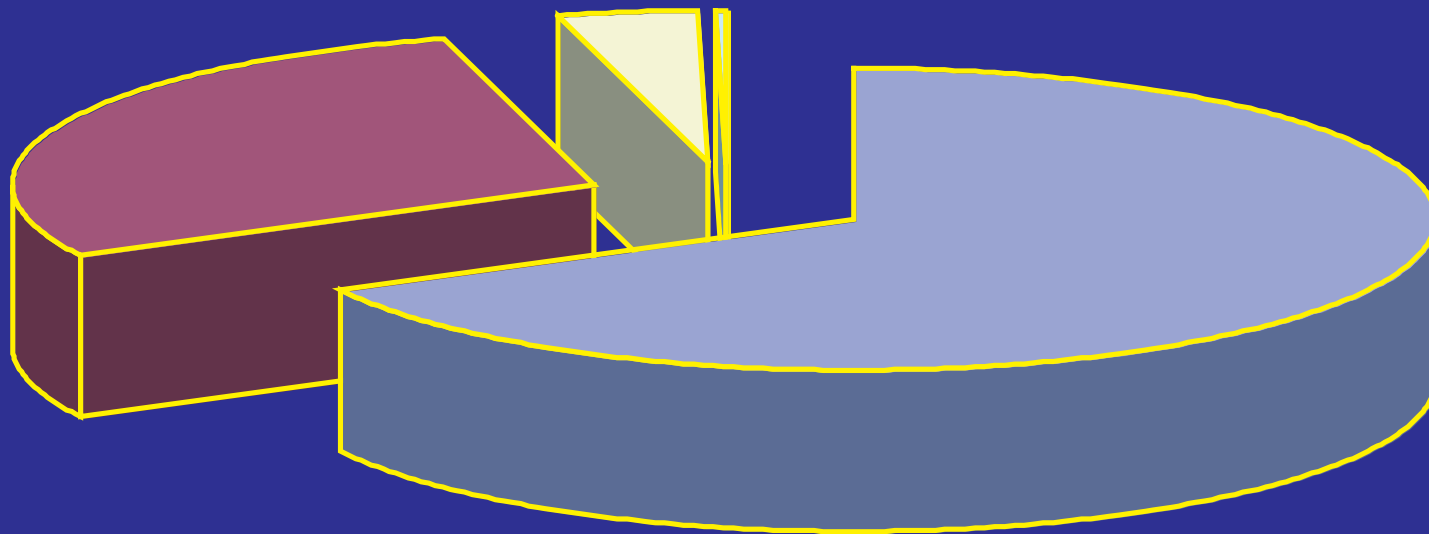


<http://background.uchicago.edu>
("Presentations" in PDF)

Dark Energy
and the
Standard Model of Cosmology

If its not dark, it doesn't matter!

- Cosmic matter-energy budget:

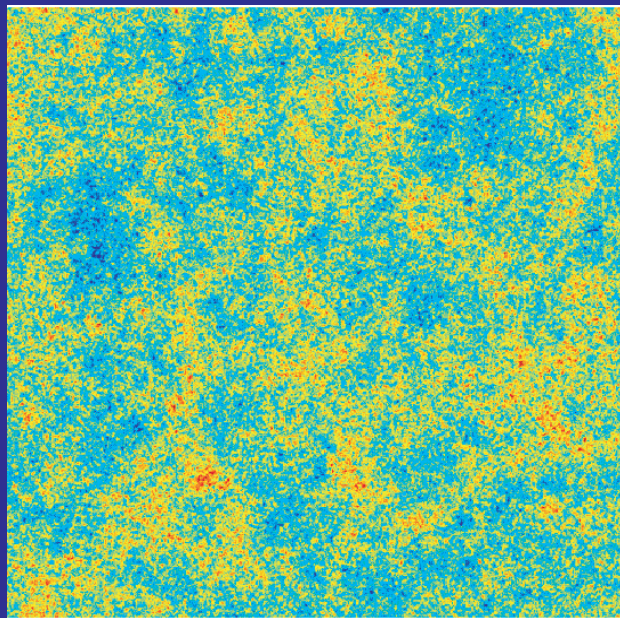


■ Dark Energy
■ Dark Matter
■ Dark Baryons

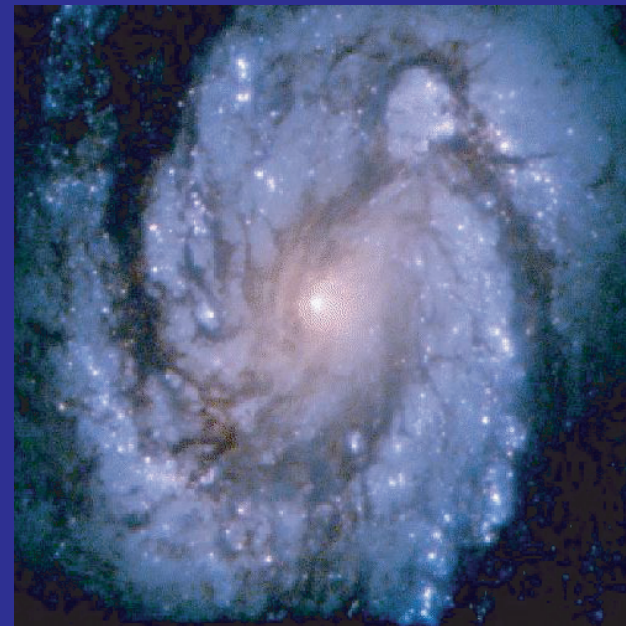
■ Visible Matter
■ Dark Neutrinos

Making Light of the Dark Side

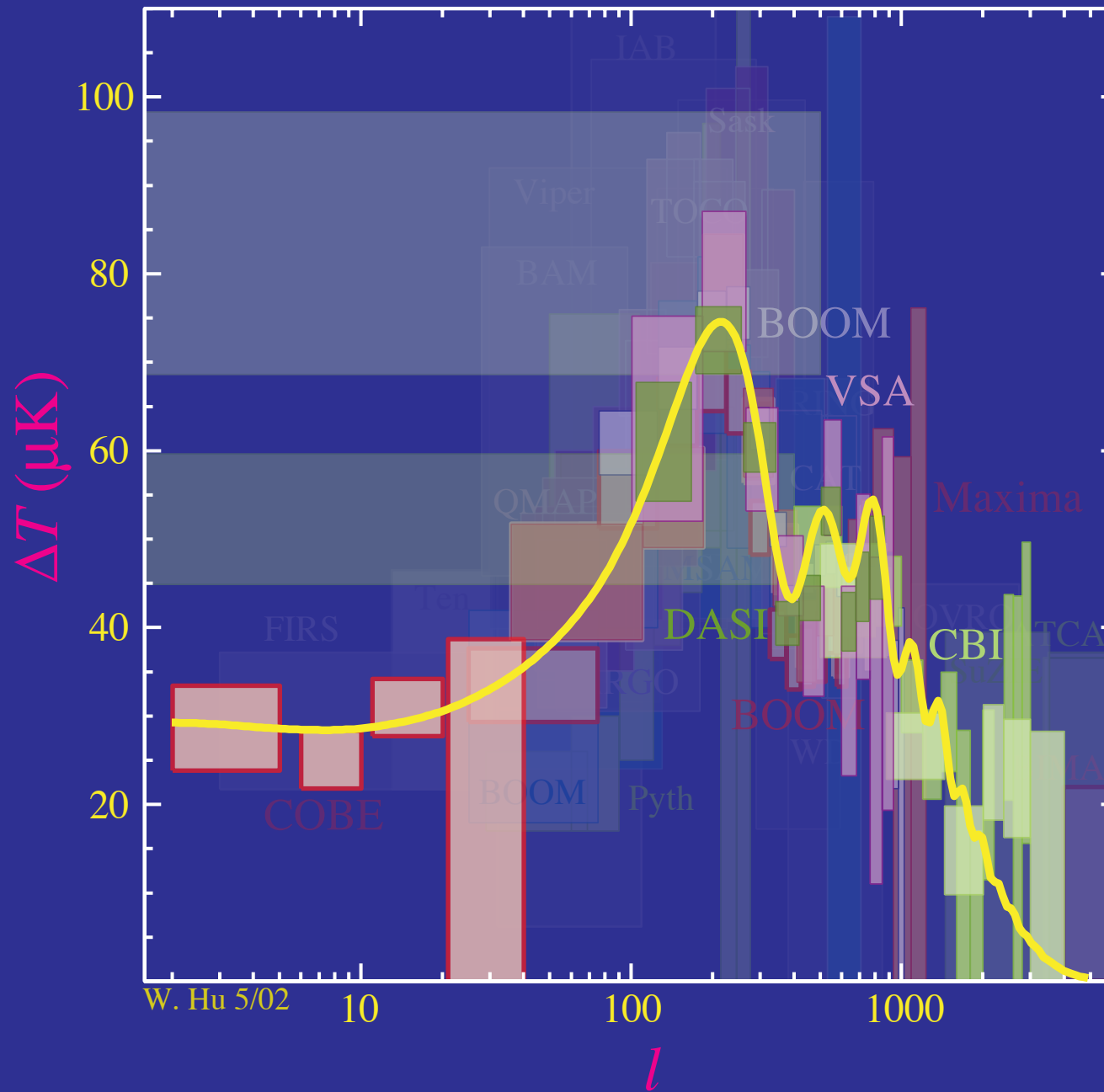
- Visible **structures** and the processes that form them are our only cosmological probe of the **dark components**
- In the standard, well-verified, cosmological model, structures grow through **gravitational instability** from small-fluctuations (perhaps formed during **inflation**)



15 Gyr



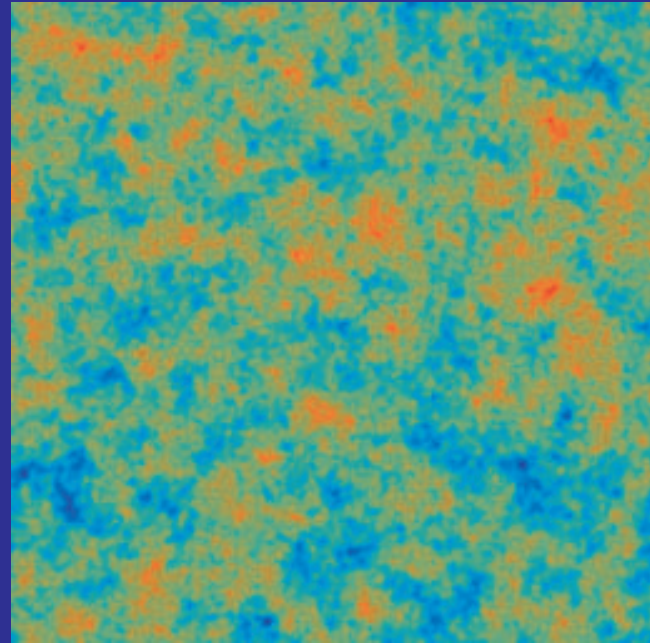
CMB: high redshift anchor



Power Spectra of Maps

- Original

↑
64°
↓



- Band Filtered

Photon-Baryon Plasma

- Before $z \sim 1000$ when the CMB was $T > 3000\text{K}$, hydrogen ionized
- Free electrons act as "glue" between photons and baryons by Compton scattering and Coulomb interactions
- Nearly perfect fluid

Peak Location

- Fundamental **physical scale**, the distance sound travels, becomes an **angular scale** by simple projection according to the angular diameter distance D_A

$$\theta_A = \lambda_A / D_A$$

$$\ell_A = k_A D_A$$

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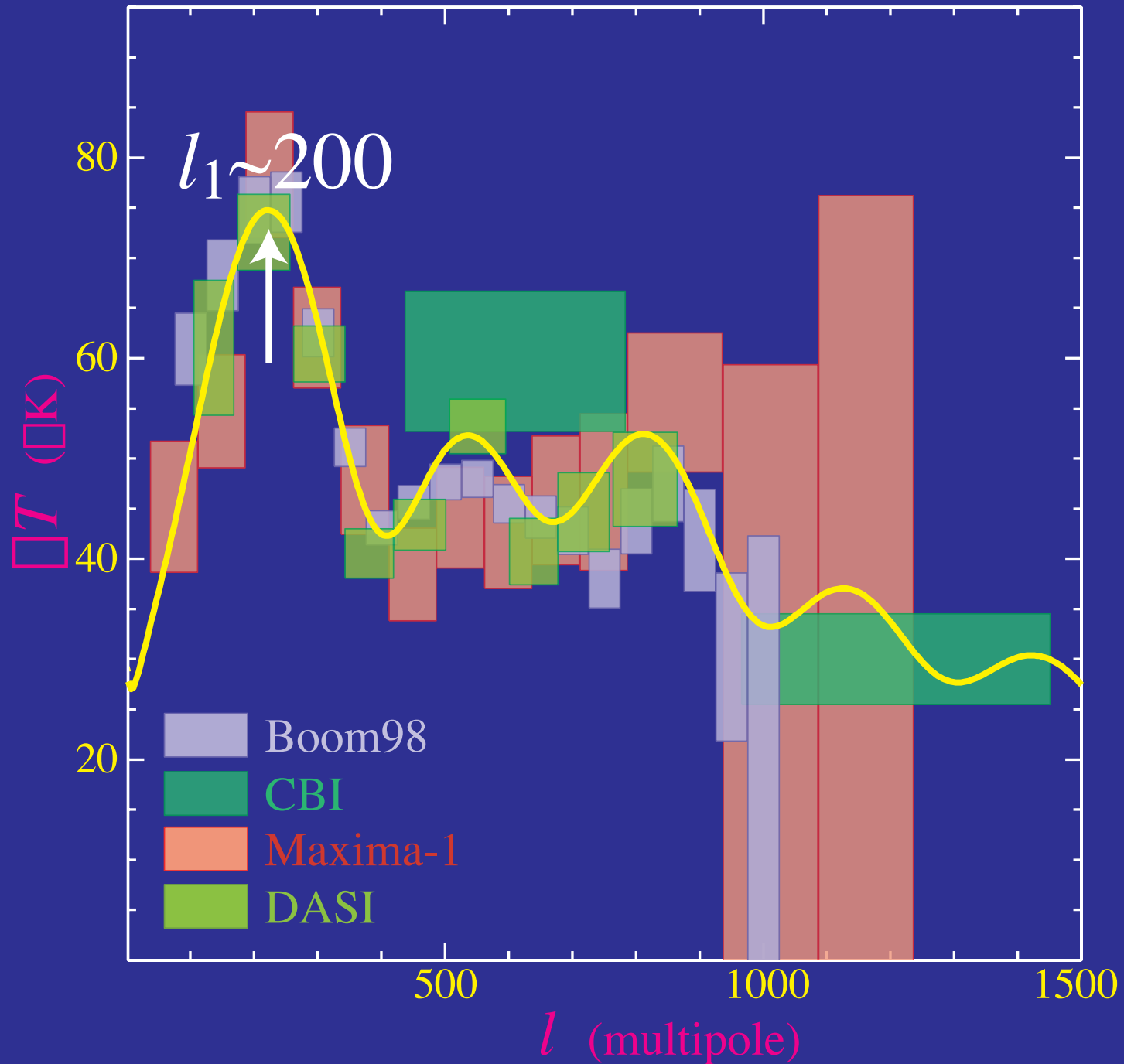
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- In a **matter-dominated** universe $\eta \propto a^{1/2}$ so $\theta_A \approx 1/30 \approx 2^\circ$ or

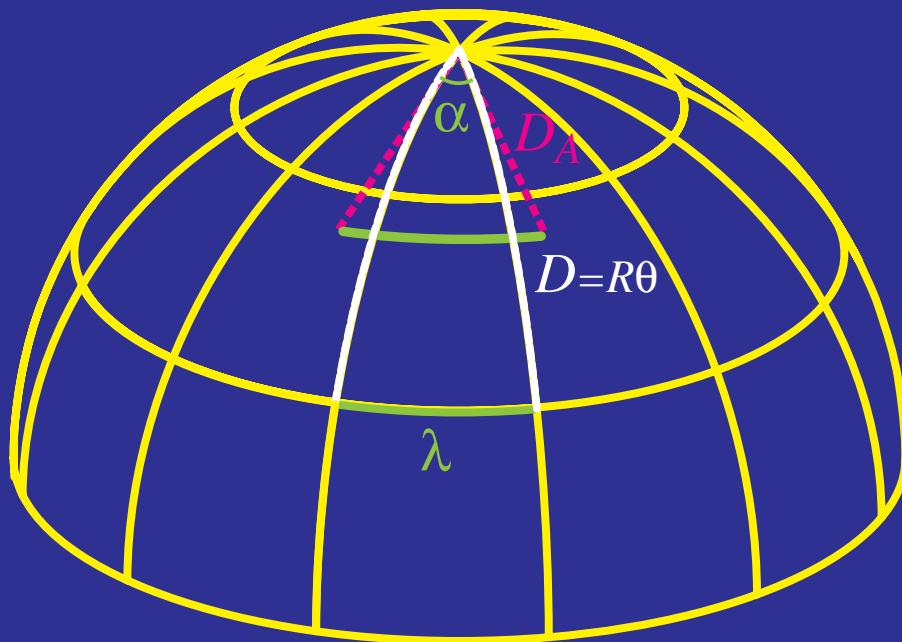
$$\ell_A \approx 200$$

Angular Diameter Distance Test



Curvature

- In a **curved universe**, the apparent or **angular diameter distance** is no longer the conformal distance $D_A = R \sin(D/R) \neq D$



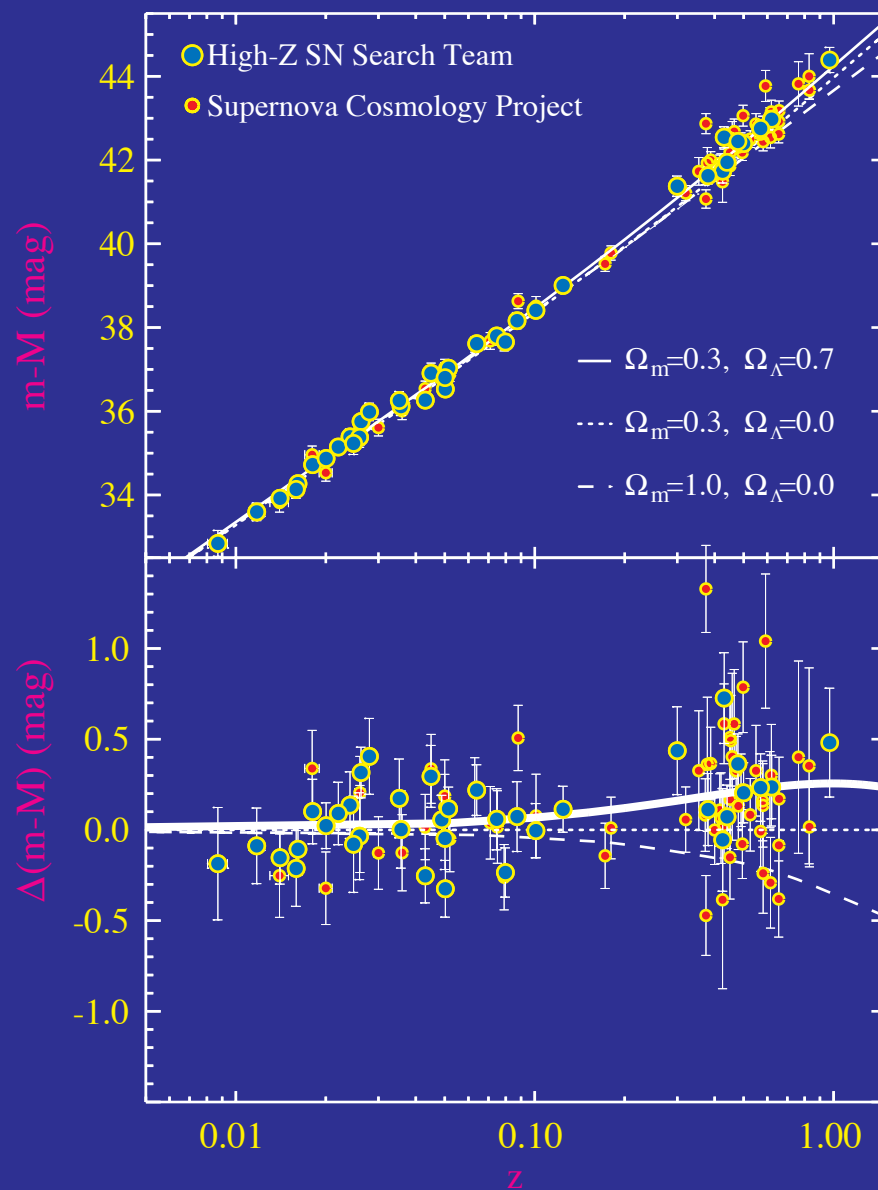
- Objects in a **closed universe** are **further** than they appear!
gravitational **lensing** of the background...

Curvature in the Power Spectrum

- Angular location of harmonic peaks
- Flat = critical density = missing dark energy

Accelerated Expansion from SNe

- Missing energy must also accelerate the expansion at low redshift



compilation from High-z team

Acceleration Implies Negative Pressure

- Role of **pressure** in the background cosmology
- Homogeneous **Einstein equations** $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ imply the two **Friedman equations** (flat universe, or associating curvature $\rho_K = -3K/8\pi G a^2$)

$$\left(\frac{1}{a} \frac{da}{dt}\right)^2 = \frac{8\pi G}{3} \rho$$
$$\frac{1}{a} \frac{d^2 a}{dt^2} = -\frac{4\pi G}{3} (\rho + 3p)$$

so that the total equation of state $w \equiv p/\rho < -1/3$ for acceleration

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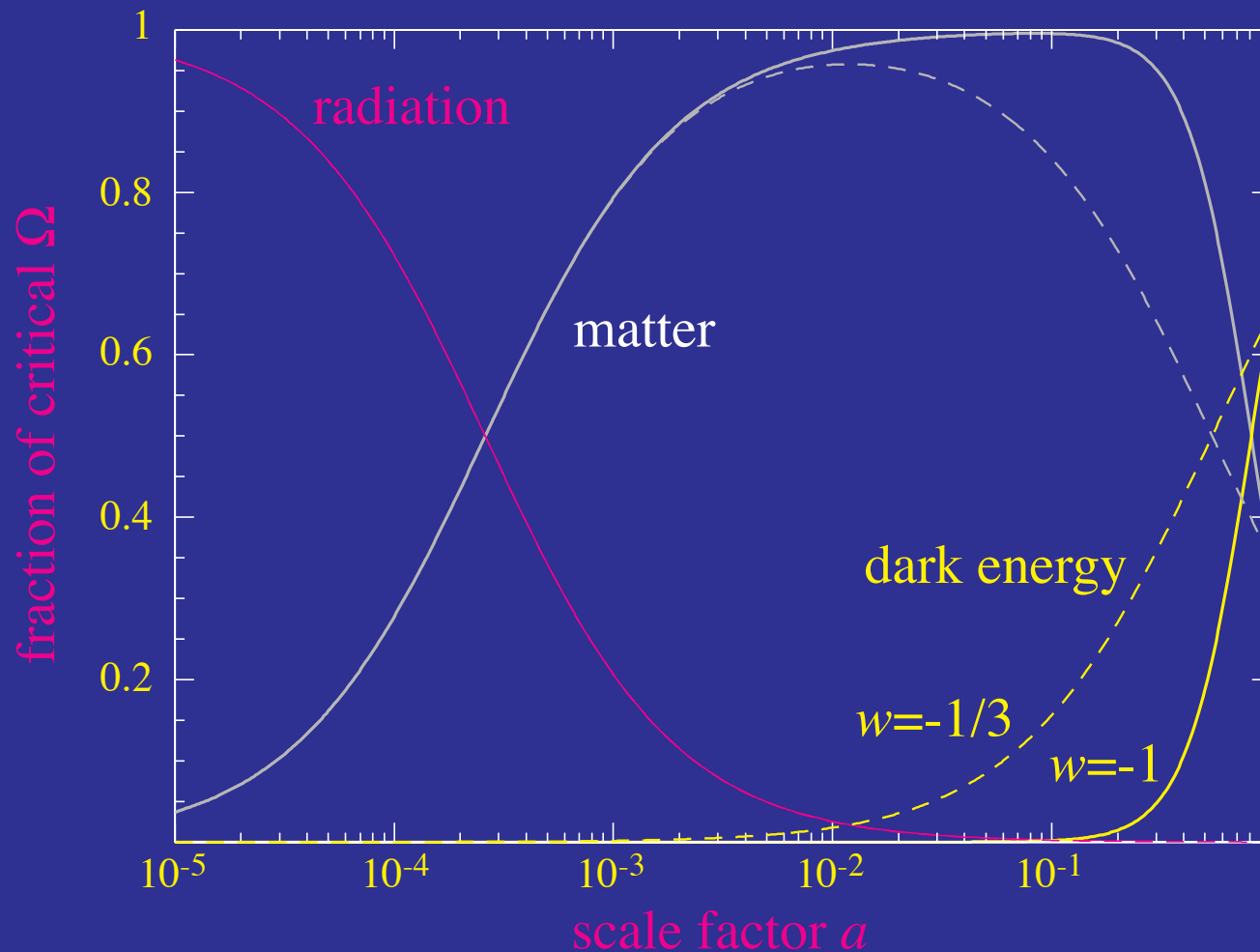
- **Conservation equation** $\nabla^\mu T_{\mu\nu} = 0$ implies

$$\frac{\dot{\rho}}{\rho} = -3(1 + w) \frac{\dot{a}}{a}$$

- so that ρ must scale more slowly than a^{-2}

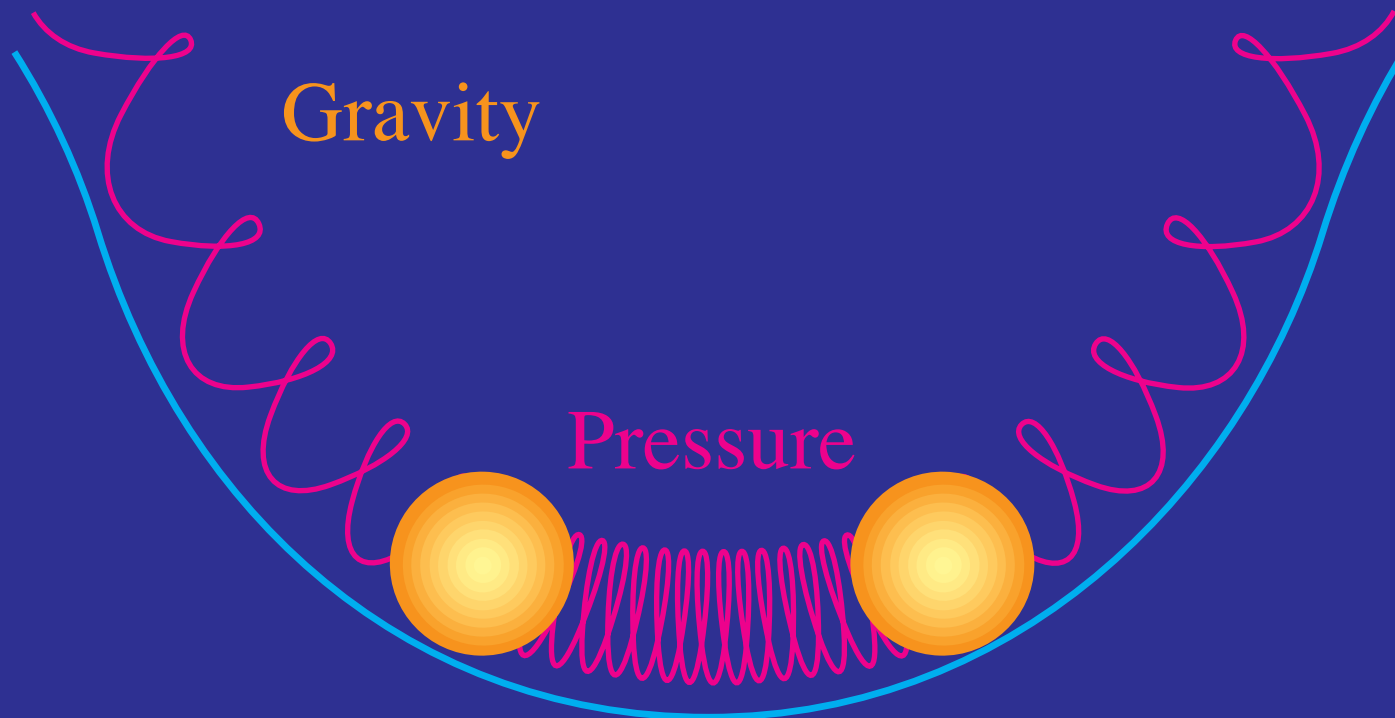
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Dark Mystery?

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pressure gradients, not pressure, establish stability

- **Candidates:**

Cosmological constant $w=-1$, constant in space and time, but >60 orders of magnitude off vacuum energy prediction

Ultralight **scalar field**, slowly rolling in a potential,
Klein-Gordon equation: sound speed $c_s^2 = \delta p / \delta \rho = 1$

Tangled defects $w=-1/3, -2/3$ but relativistic sound speed
("solid" dark matter)

Dark Energy Probes

- (Comoving) distance-redshift relation: $a = (1 + z)^{-1}$

$$D = \int_a^1 da \frac{1}{a^2 H(a)} = \int_0^z dz \frac{1}{H(a)}$$

$$H^2(a) = \frac{8\pi G}{3}(\rho_m + \rho_{DE})$$

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- Pressure growth suppression: $\delta \equiv \delta\rho_m/\rho_m \propto a\phi$

$$\frac{d^2\phi}{d\ln a^2} + \left[\frac{5}{2} - \frac{3}{2}w(z)\Omega_{DE}(z) \right] \frac{d\phi}{d\ln a} + \frac{3}{2}[1 - w(z)]\Omega_{DE}(z)\phi = 0,$$

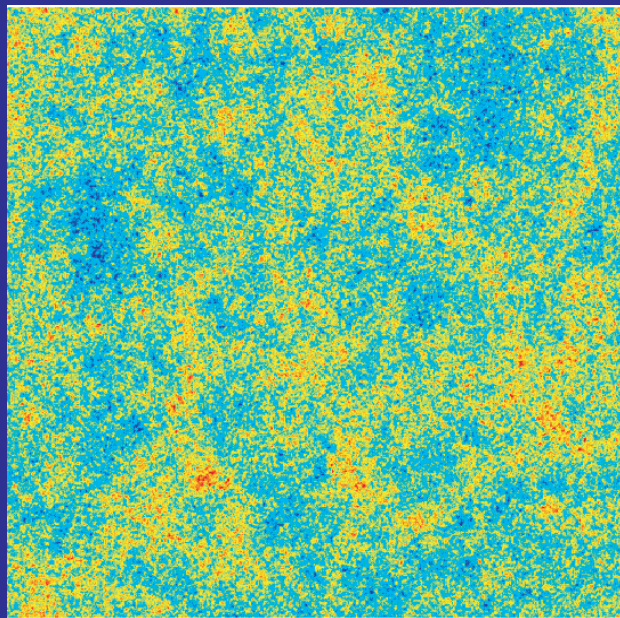
where $w \equiv p_{DE}/\rho_{DE}$ and $\Omega_{DE} \equiv \rho_{DE}/(\rho_m + \rho_{DE})$

e.g. galaxy cluster abundance, gravitational lensing...

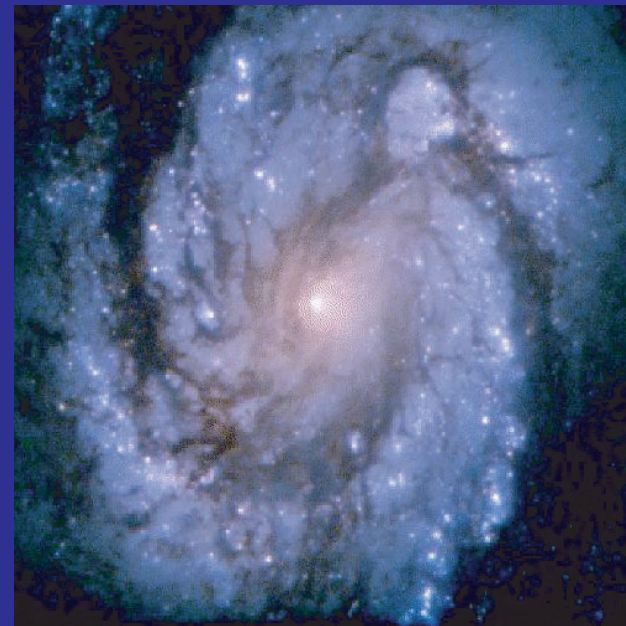
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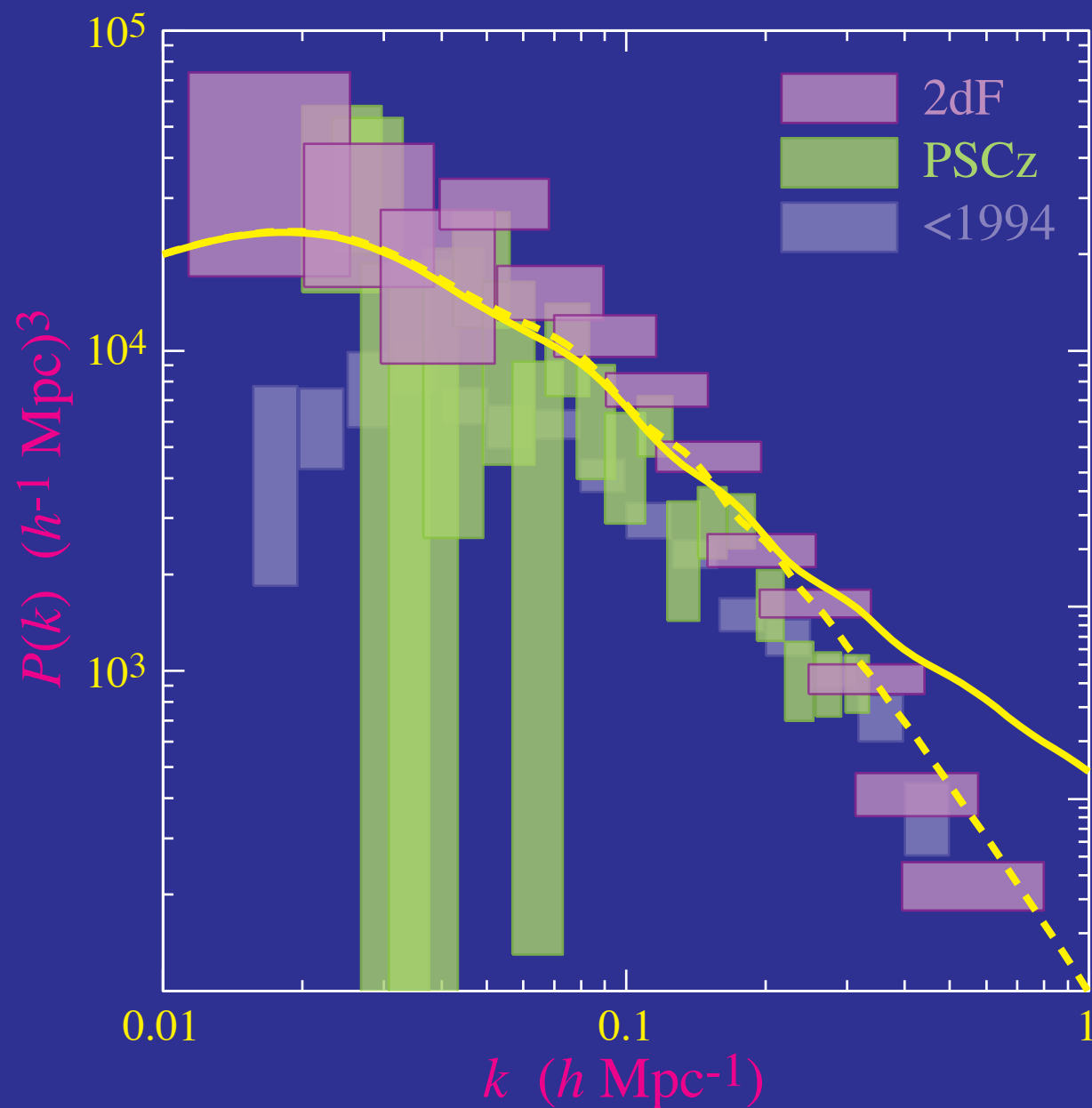


Structure Formation Simulation

- Simulation (by A. Kravstov)

Galaxy Power Spectrum Data

- Galaxy clustering tracks the dark matter – but bias depends on type



A Fundamental Problem

- All cosmological observables relate to the **luminous matter**: photon-baryon plasma, galaxies, clusters of galaxies, supernovae
- Implications for the dark energy or cosmology in general depend on modelling the **formation** and **evolution** of luminous objects
- Success of CMB anisotropy is in large part based on the **solid theoretical grounding** of its formation and evolution – well understood linear gravitational physics

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- Success of CMB anisotropy is in large part based on the **solid theoretical grounding** of its formation and evolution – well understood linear gravitational physics
- Distortion of the images of luminous objects by **gravitational lensing is equally well understood**
- Problem: image distortion is typically at %-level – very demanding for the control of **systematic errors** – but recall CMB is 10^{-5} level! (Tyson, Wenk & Valdes 1990)

Example of Weak Lensing

- Toy example of lensing of the CMB primary anisotropies
- Shearing of the image

Lensing Observables

- Image distortion described by Jacobian matrix of the remapping

$$\mathbf{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix},$$

where κ is the convergence, γ_1, γ_2 are the shear components

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- related to the gravitational potential Φ by spatial derivatives

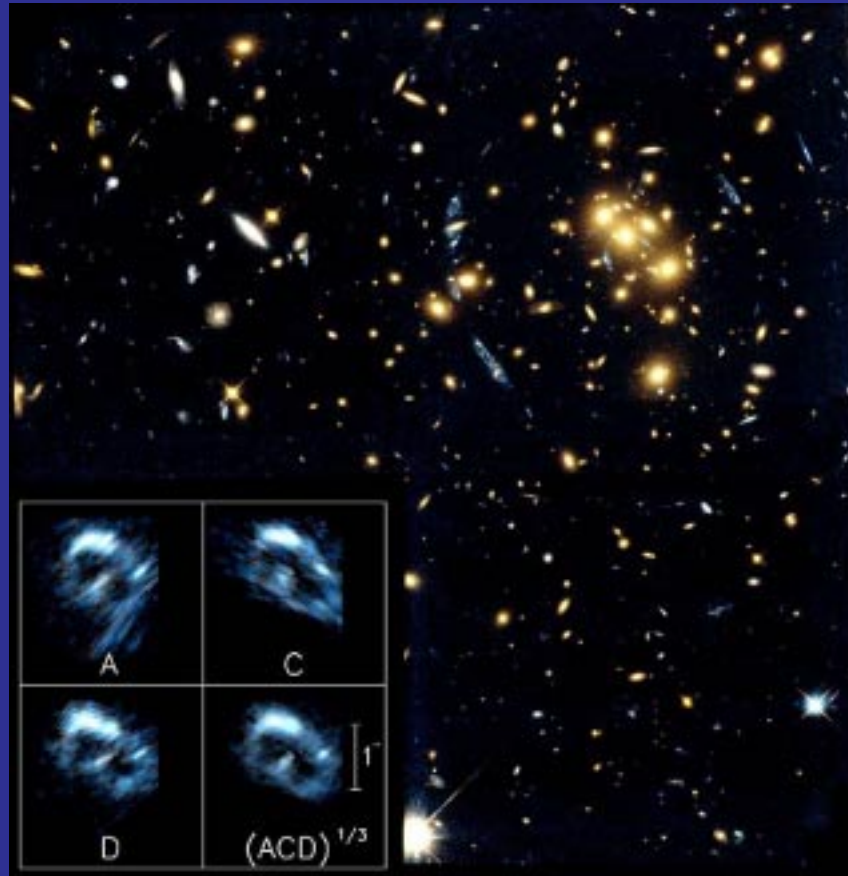
$$\psi_{ij}(z_s) = 2 \int_0^{z_s} dz \frac{dD}{dz} \frac{D(D_s - D)}{D_s} \Phi_{,ij},$$

$\psi_{ij} = \delta_{ij} - A_{ij}$, i.e. via Poisson equation

$$\kappa(z_s) = \frac{3}{2} H_0^2 \Omega_m \int_0^{z_s} dz \frac{dD}{dz} \frac{D(D_s - D)}{D_s} \delta/a,$$

Gravitational Lensing by LSS

- Shearing of galaxy images reliably detected in clusters
- Main systematic effects are instrumental rather than astrophysical

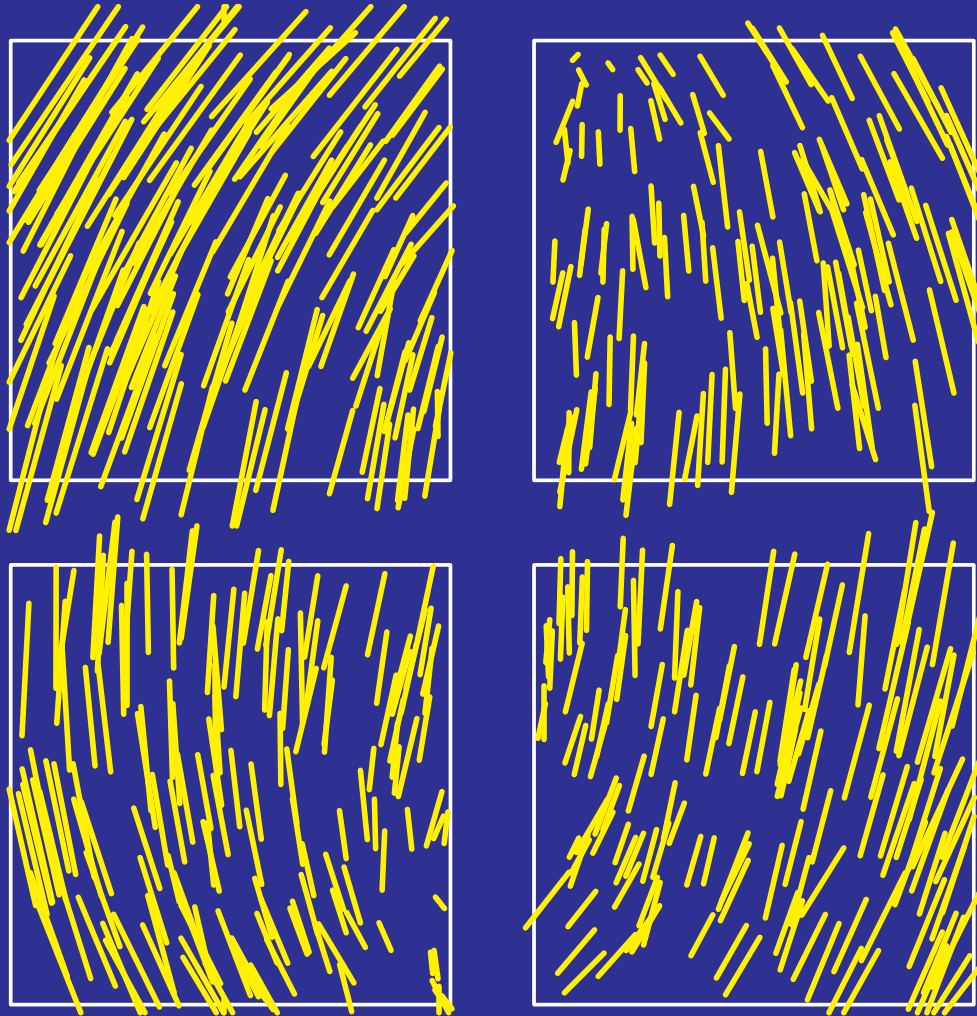


Cluster (Strong) Lensing: 0024+1654

Colley, Turner, & Tyson (1996)

Instrumental Systematics

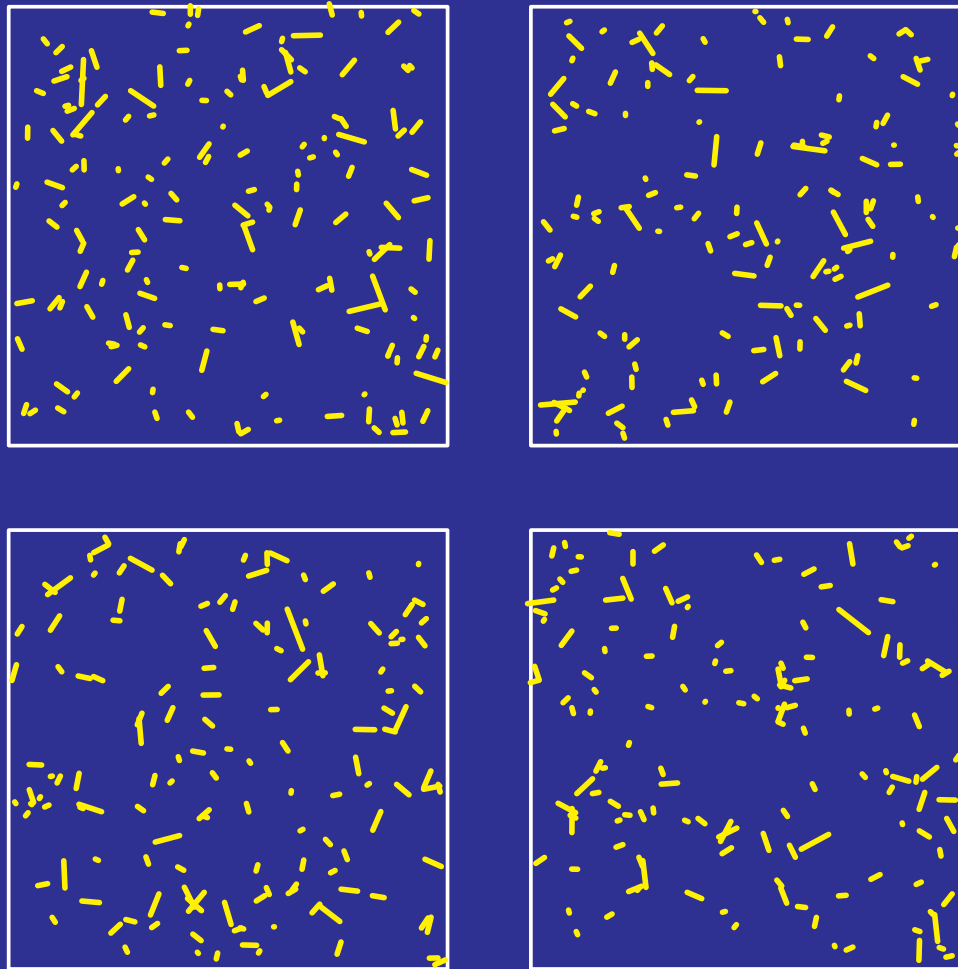
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○ 1% ellipticity

Instrumental Systematics

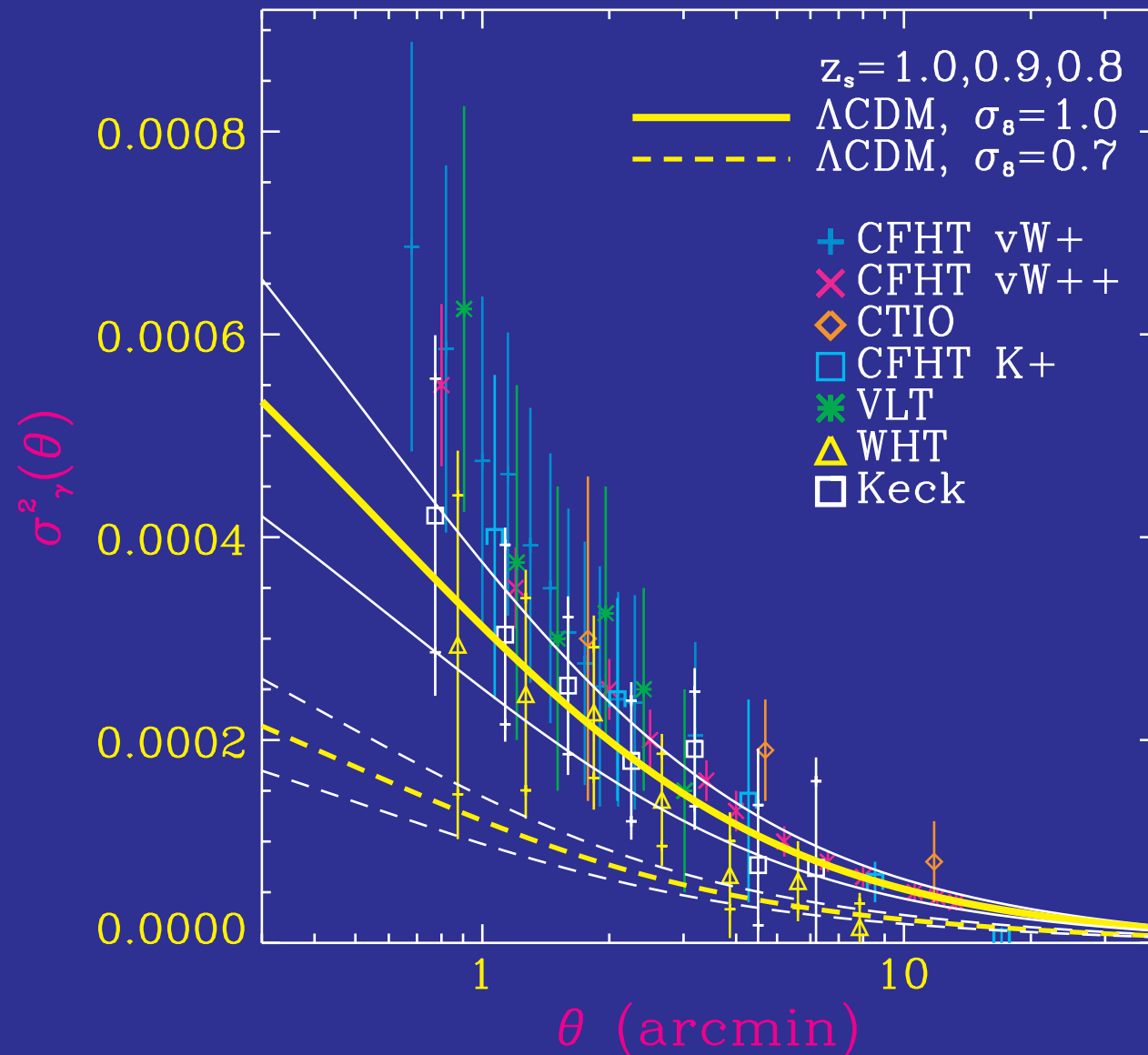
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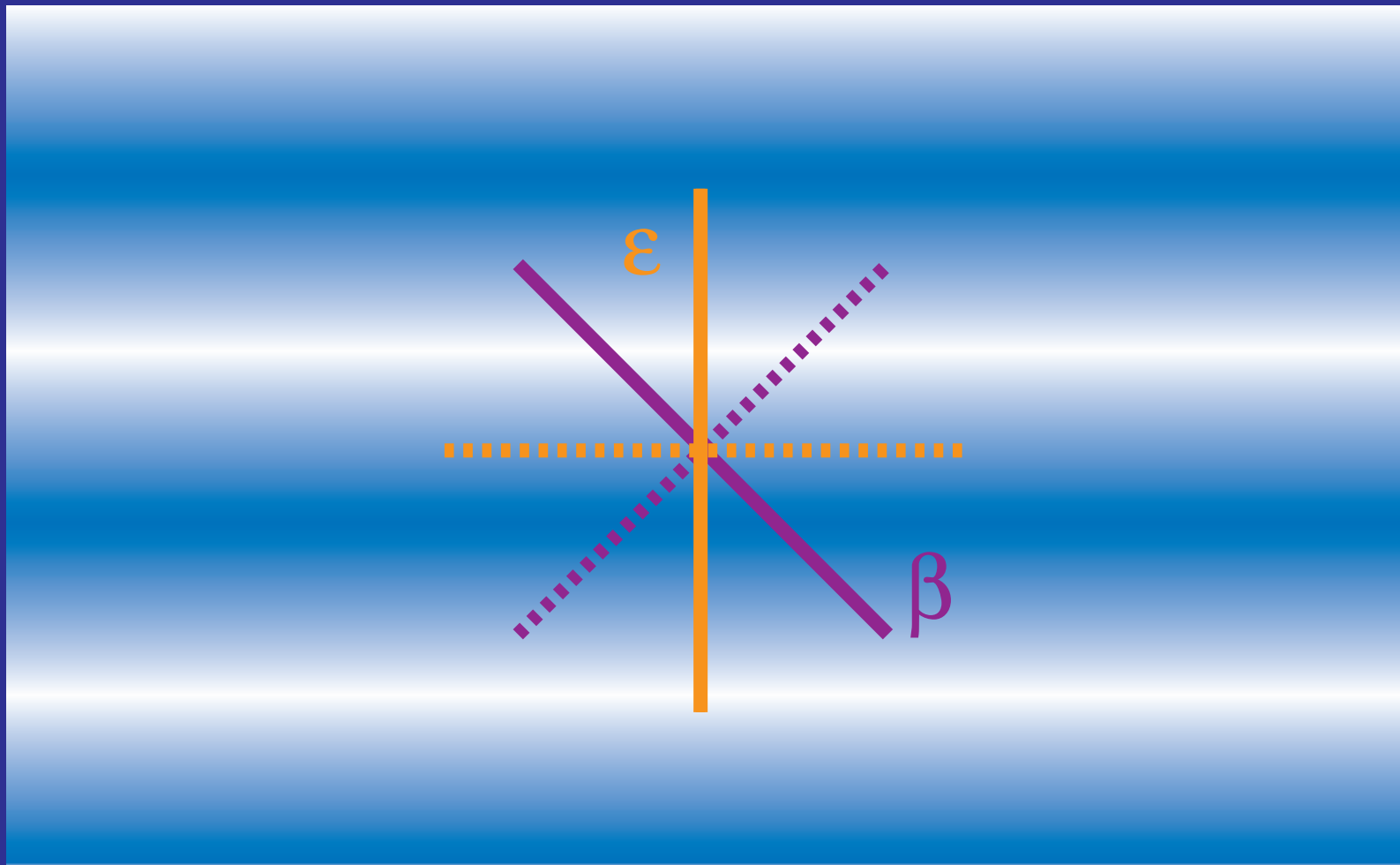
Cosmic Shear Data

- Shear variance as a function of smoothing scale



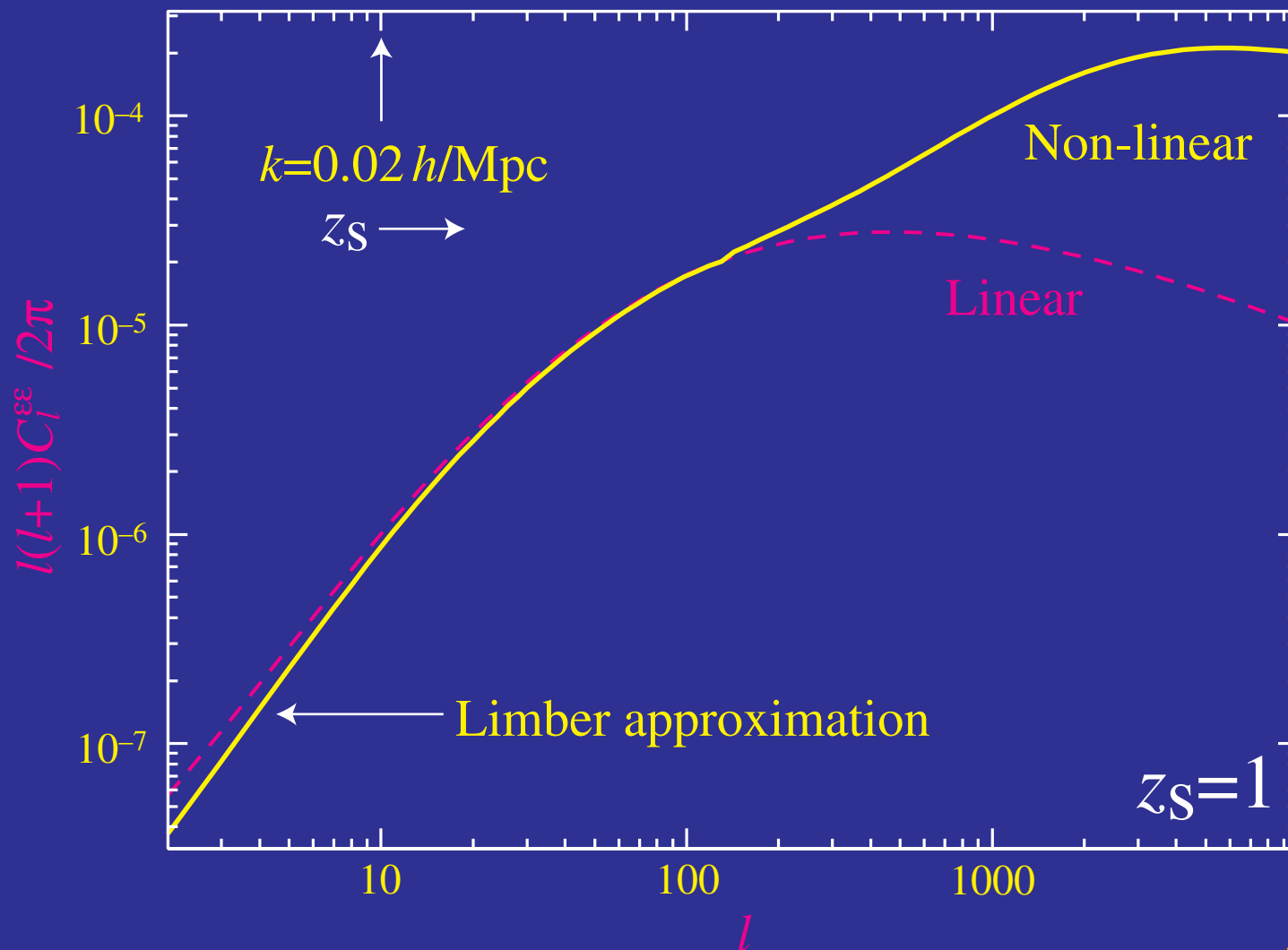
Shear Power Modes

- Alignment of **shear** and **wavevector** defines modes



Shear Power Spectrum

- Lensing weighted Limber projection of density power spectrum
- ϵ -shear power = κ power

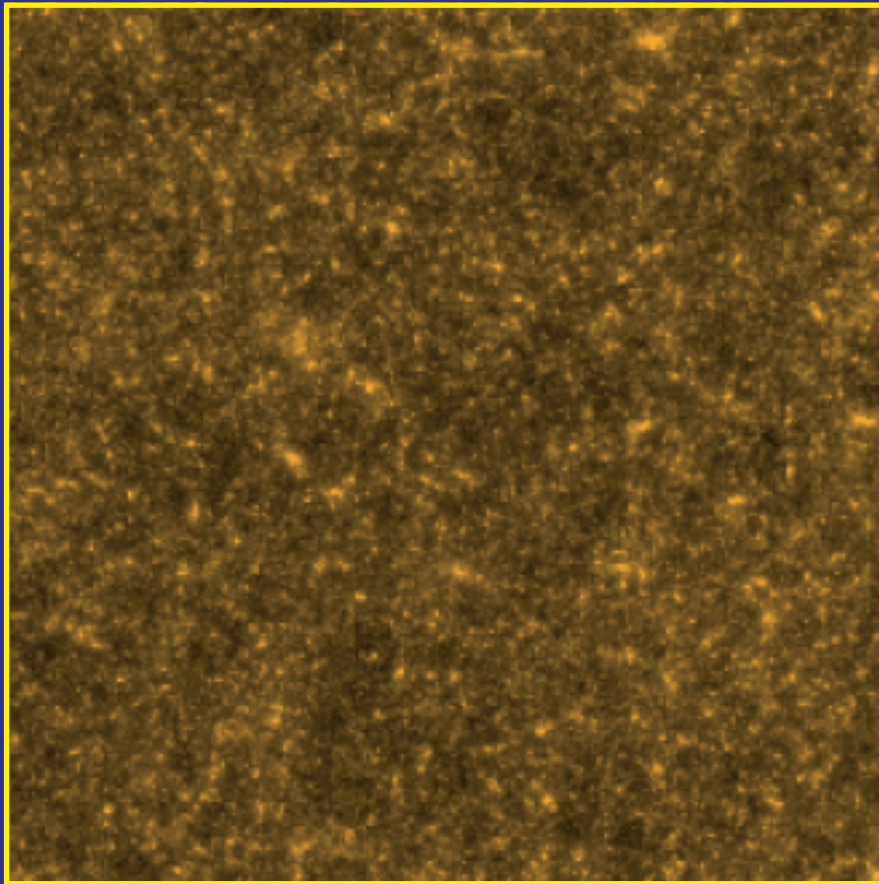


Kaiser (1992)
Jain & Seljak (1997)
Hu (2000)

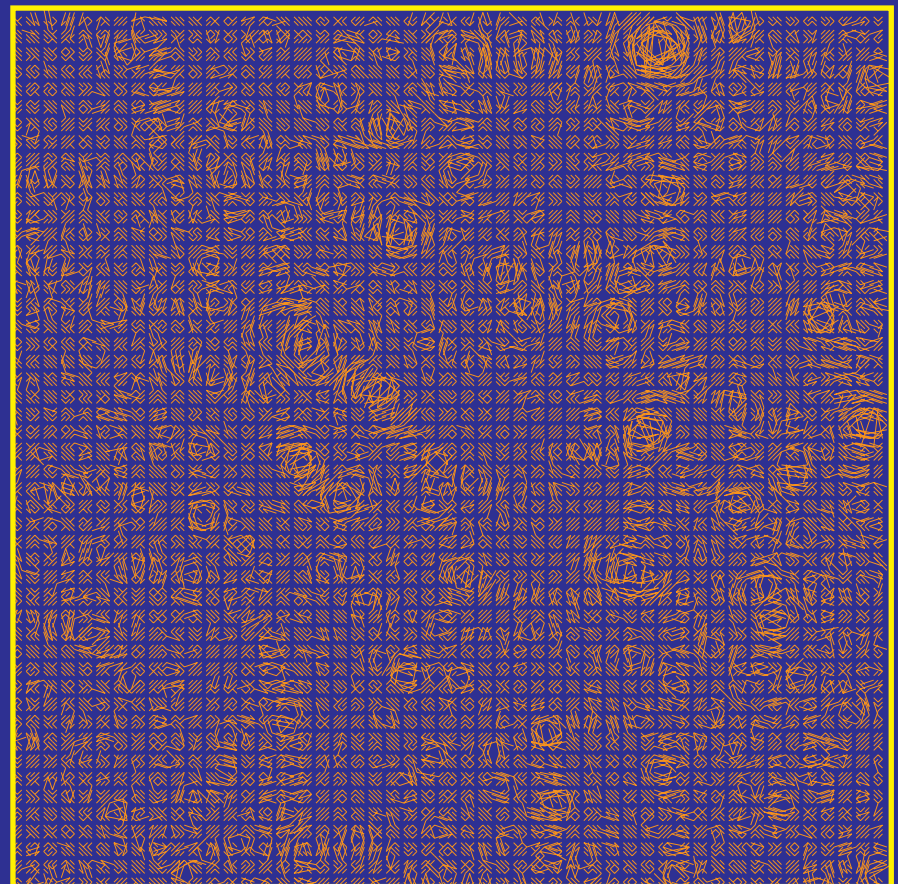
PM Simulations

- Simulating mass distribution is a routine exercise

Convergence



Shear

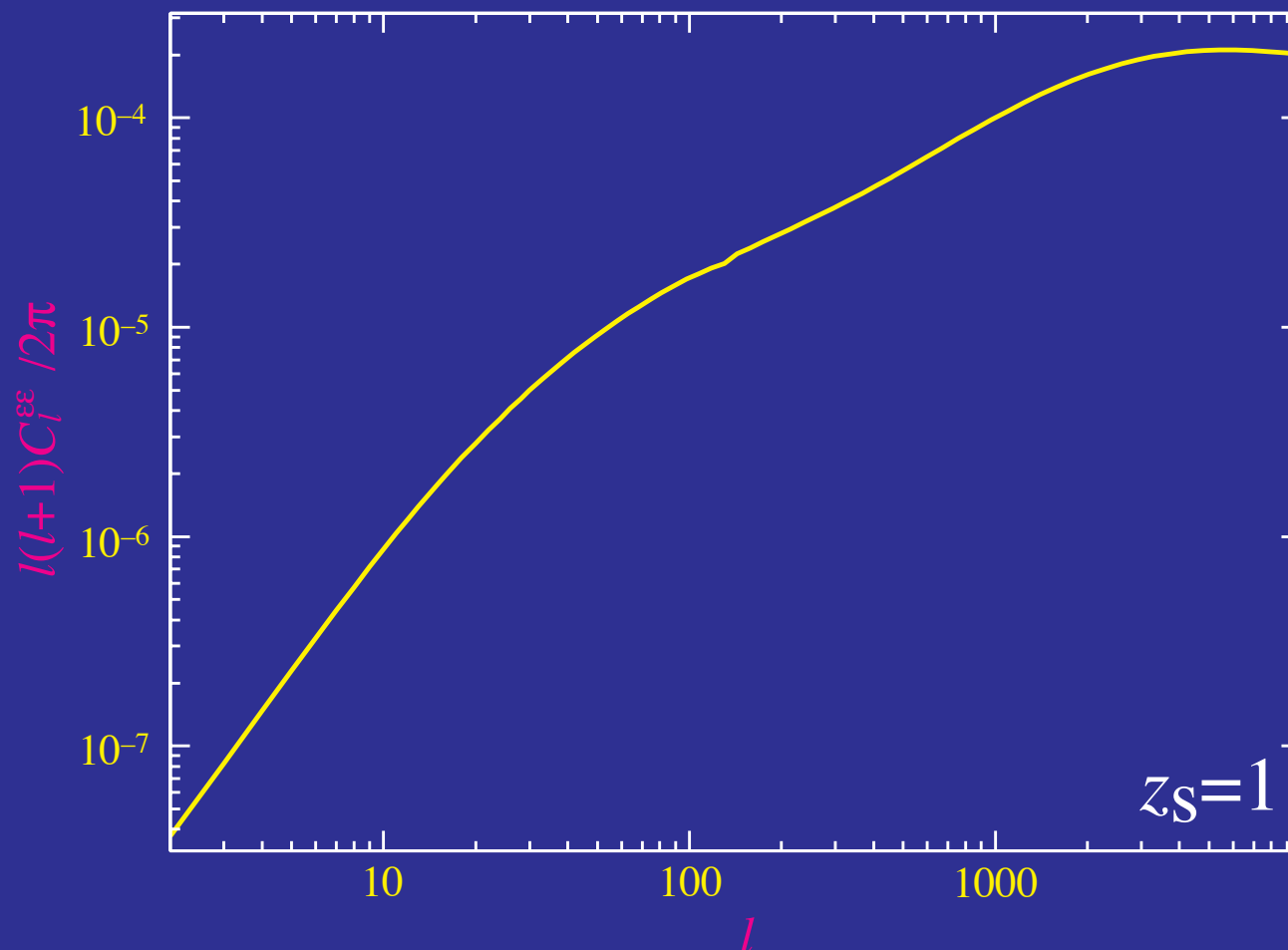


$6^\circ \times 6^\circ$ FOV; 2' Res.; 245–75 h^{-1} Mpc box; 480–145 h^{-1} kpc mesh; 2–70 $10^9 M_\odot$

Dark Energy and Gravitational Lensing

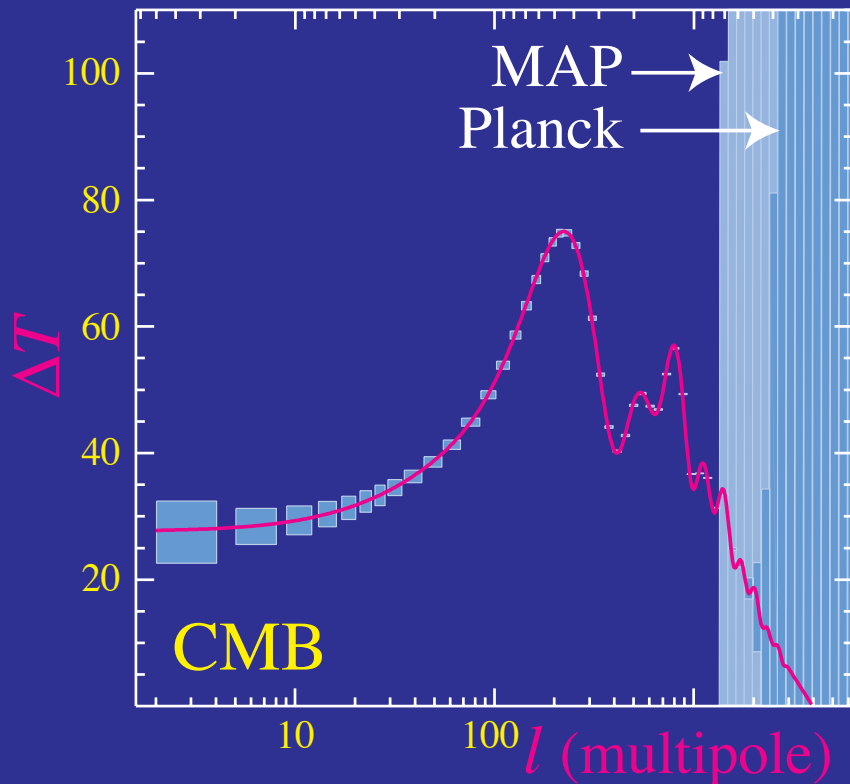
Degeneracies

- All parameters of **initial condition**, **growth** and **distance redshift** relation $D(z)$ enter
- Nearly **featureless** power spectrum results in **degeneracies**



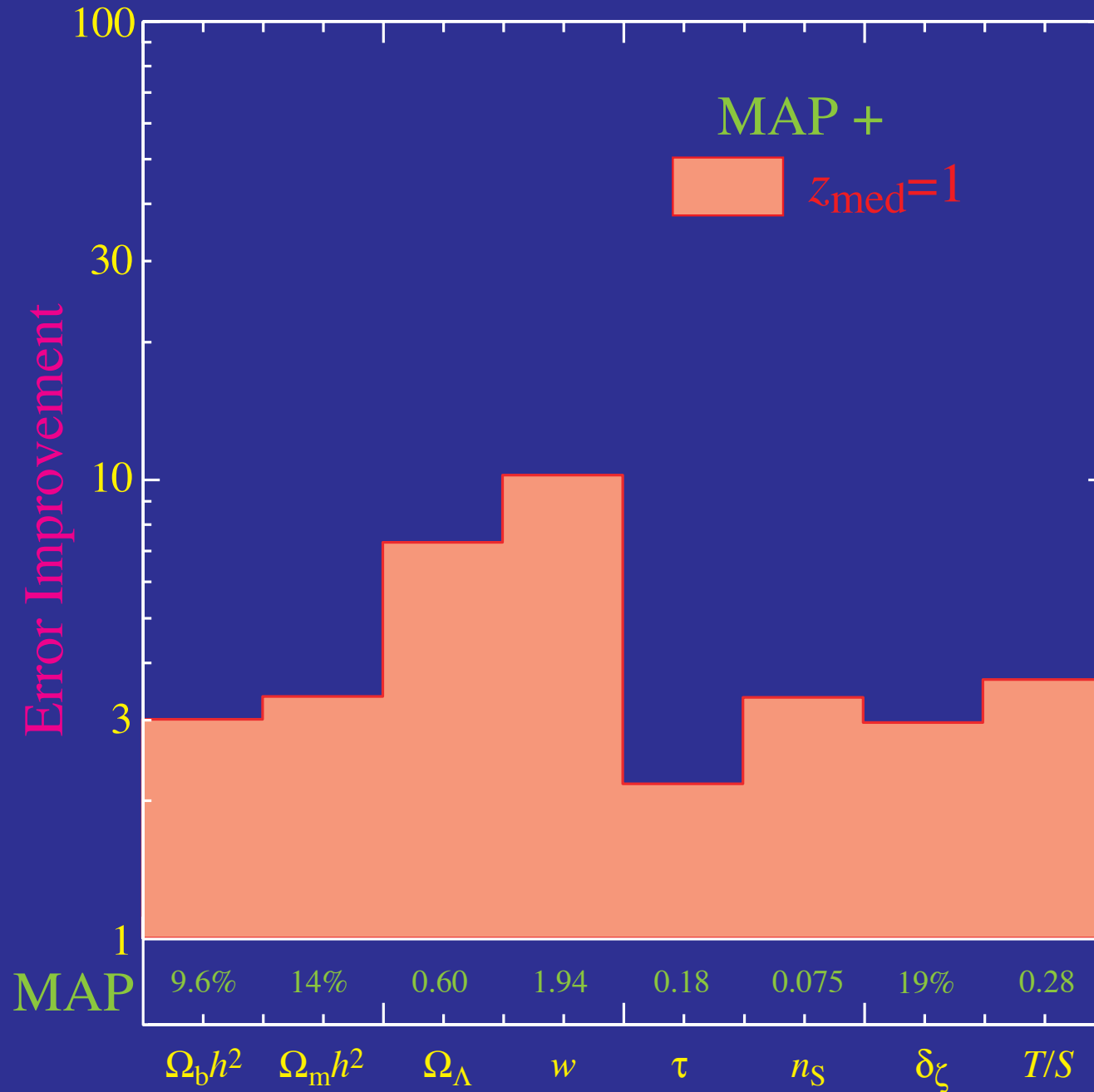
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- Combine with information from the **CMB: complementarity** (Hu & Tegmark 1999)
- **Crude tomography** with source divisions (Hu 1999; Hu 2001)
- **Fine tomography** with source redshifts (Hu & Keeton 2002; Hu 2002)

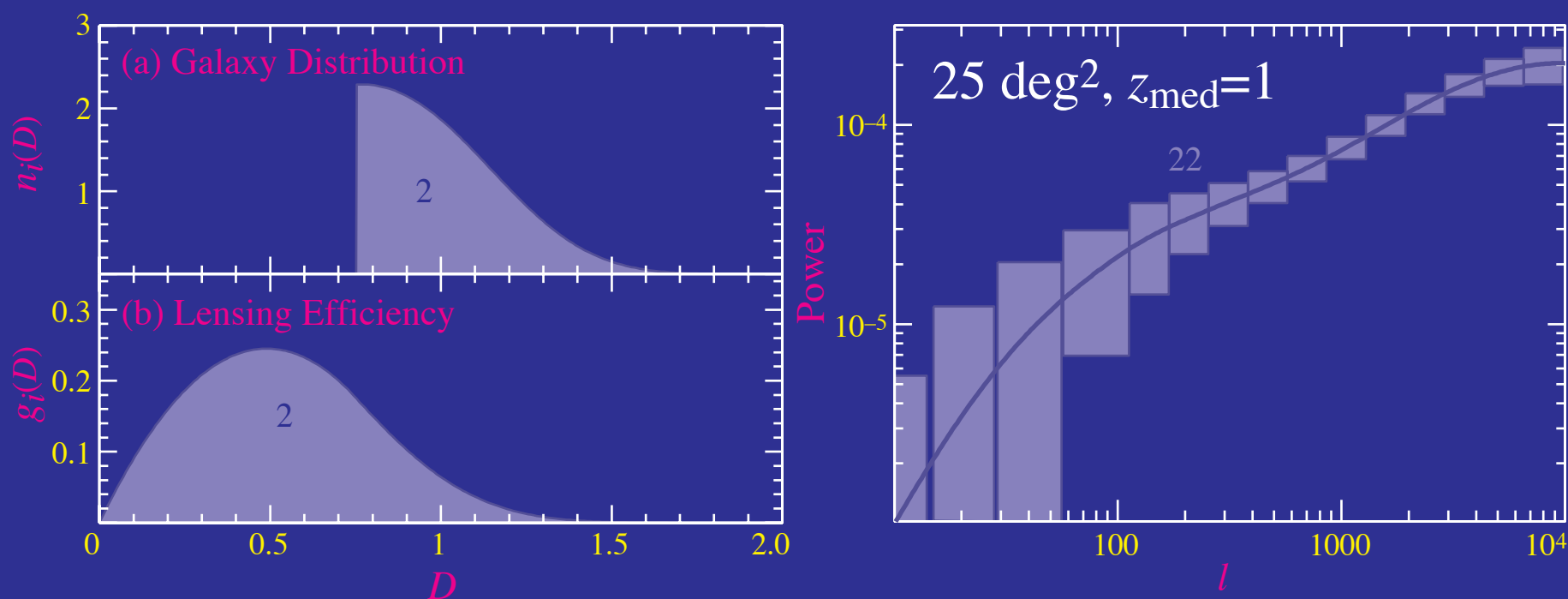
Error Improvement: 1000deg²



Cosmological Parameters Hu & Tegmark (1999) Hu (2001)

Crude Tomography

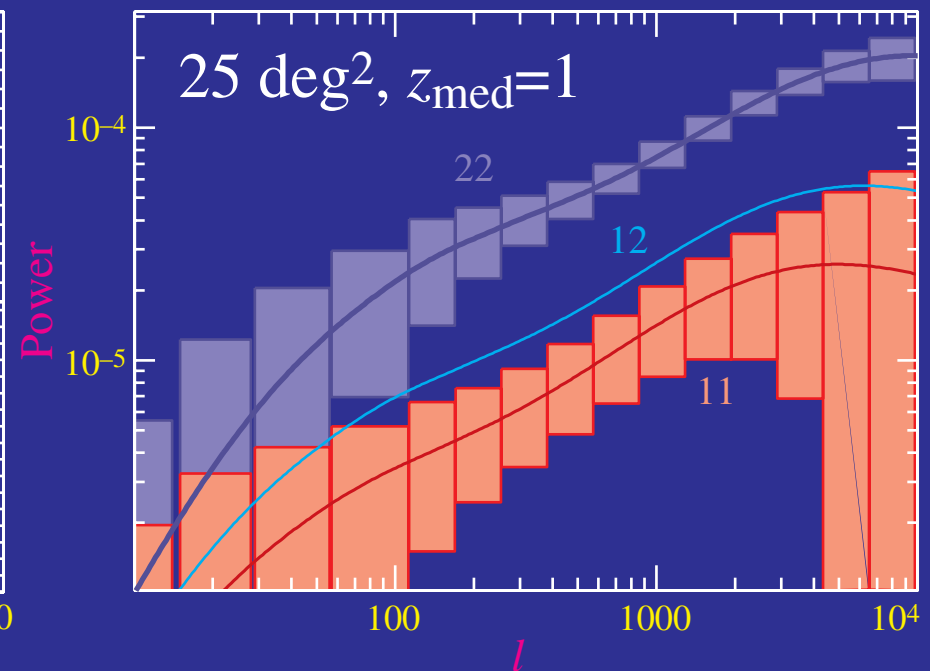
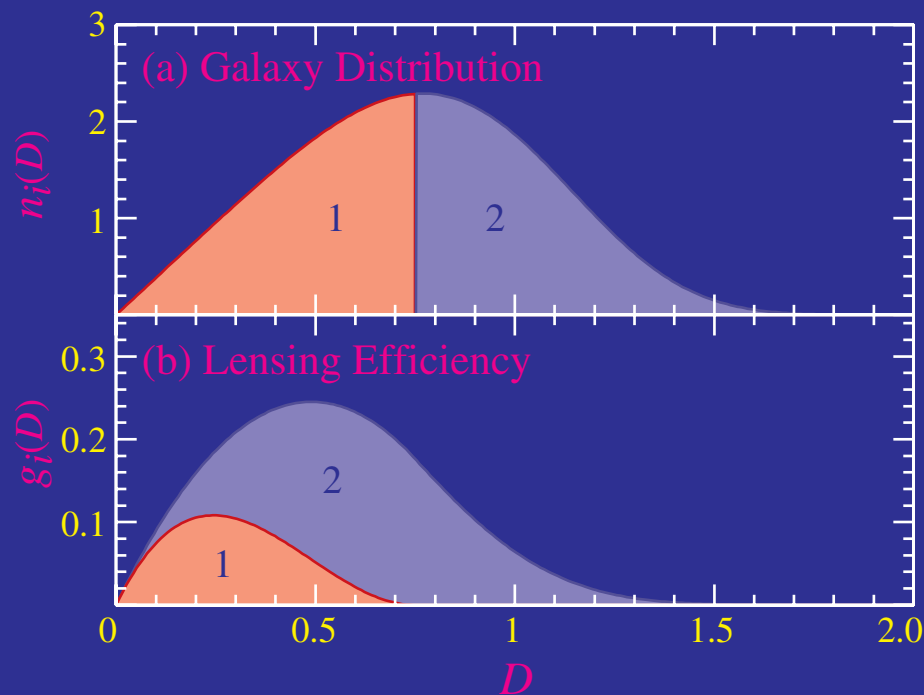
- Divide sample by **photometric redshifts**



Hu (1999)

Crude Tomography

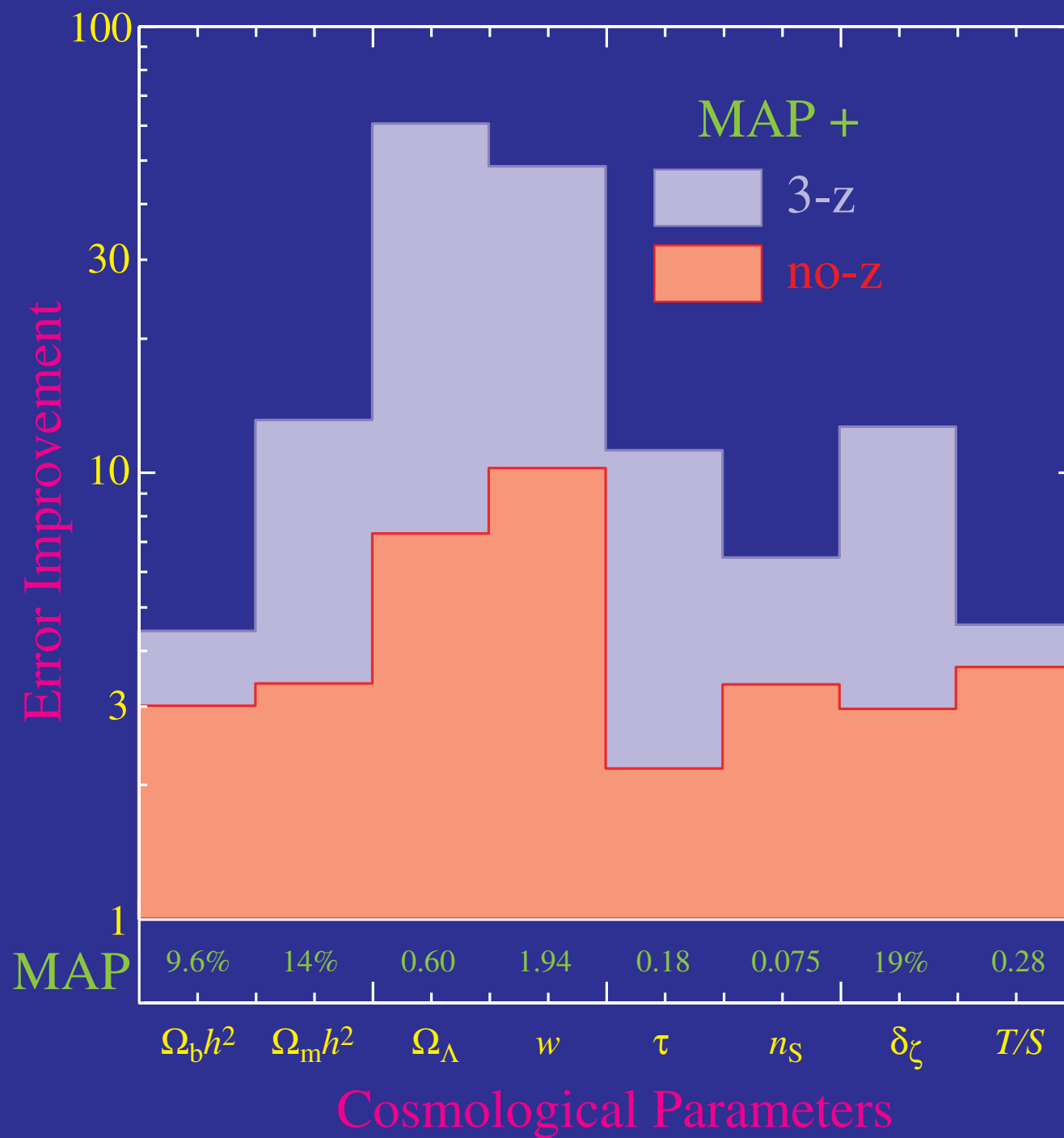
- Divide sample by **photometric redshifts**
- Cross **correlate** samples



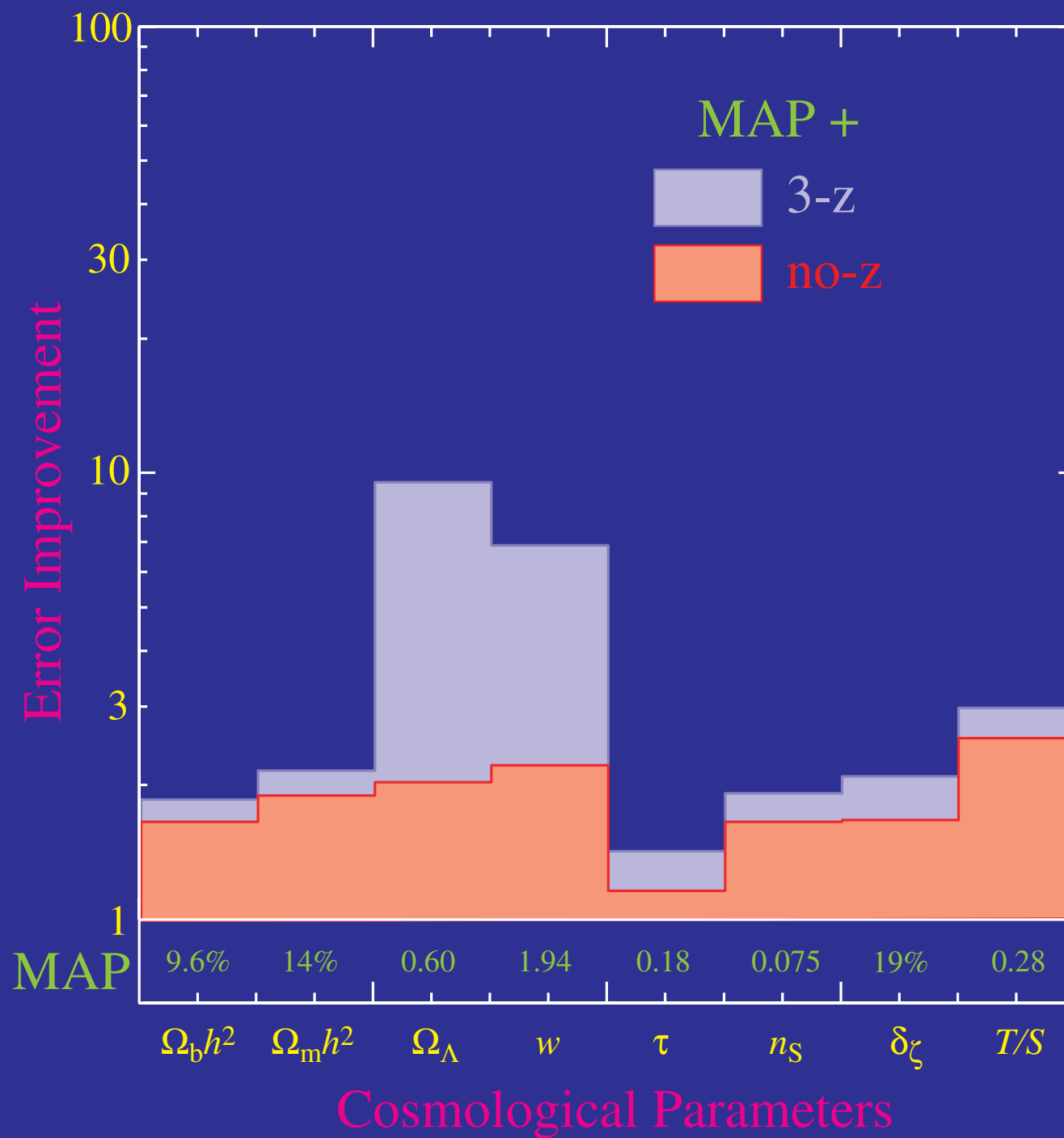
- **Order of magnitude** increase in precision **even after** CMB breaks degeneracies

Hu (1999)

Error Improvement: 1000deg²

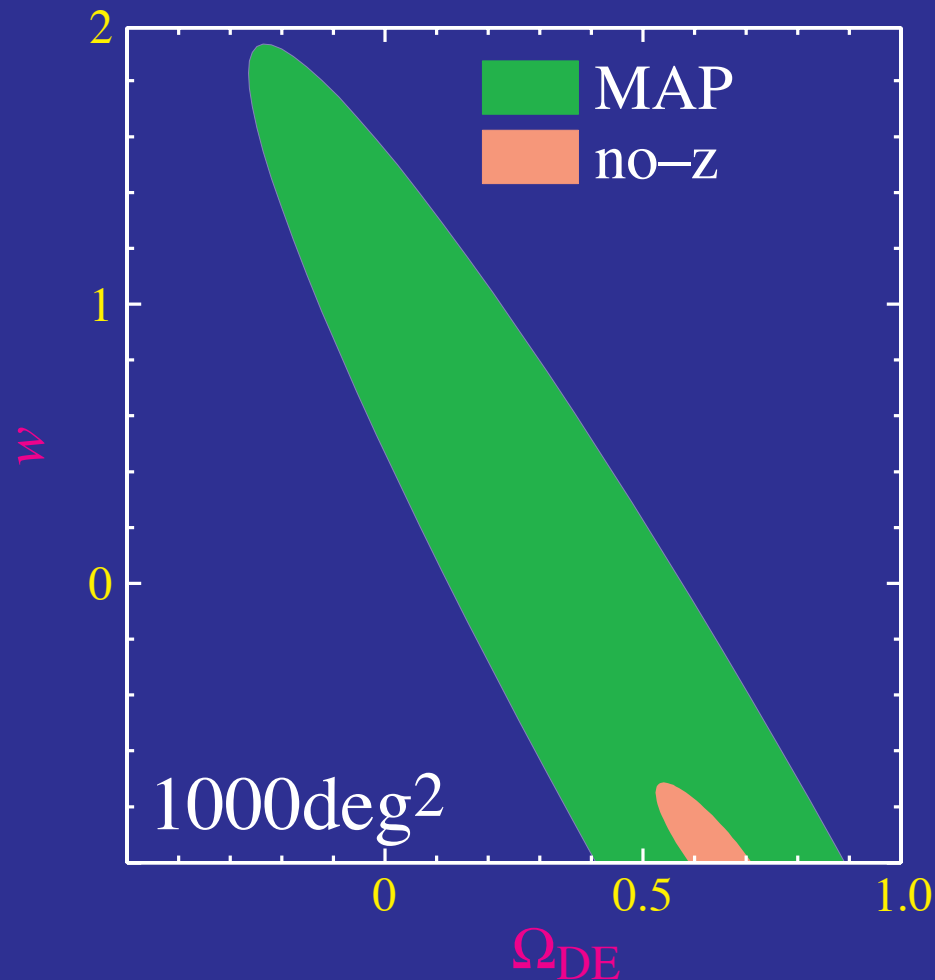


Error Improvement: 25deg²



Dark Energy & Tomography

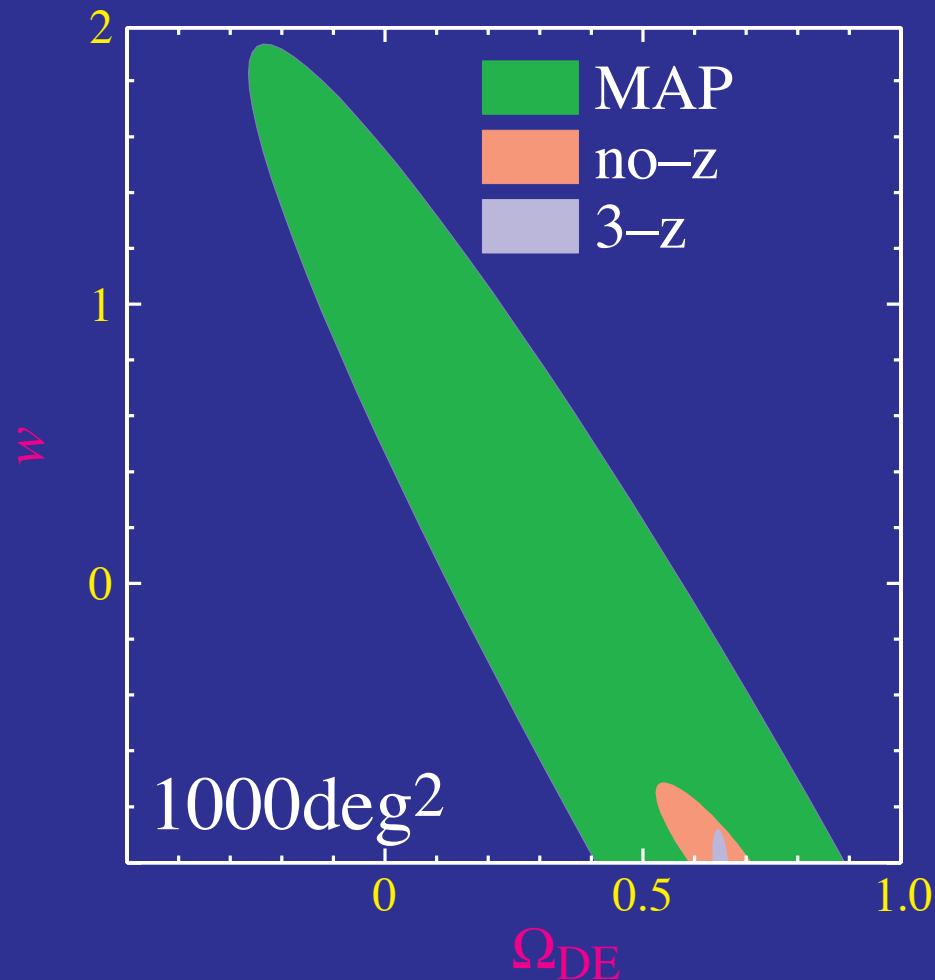
- Both CMB and **tomography** help lensing provide interesting constraints on **dark energy**



$l < 3000$; 56 gal/deg²

Dark Energy & Tomography

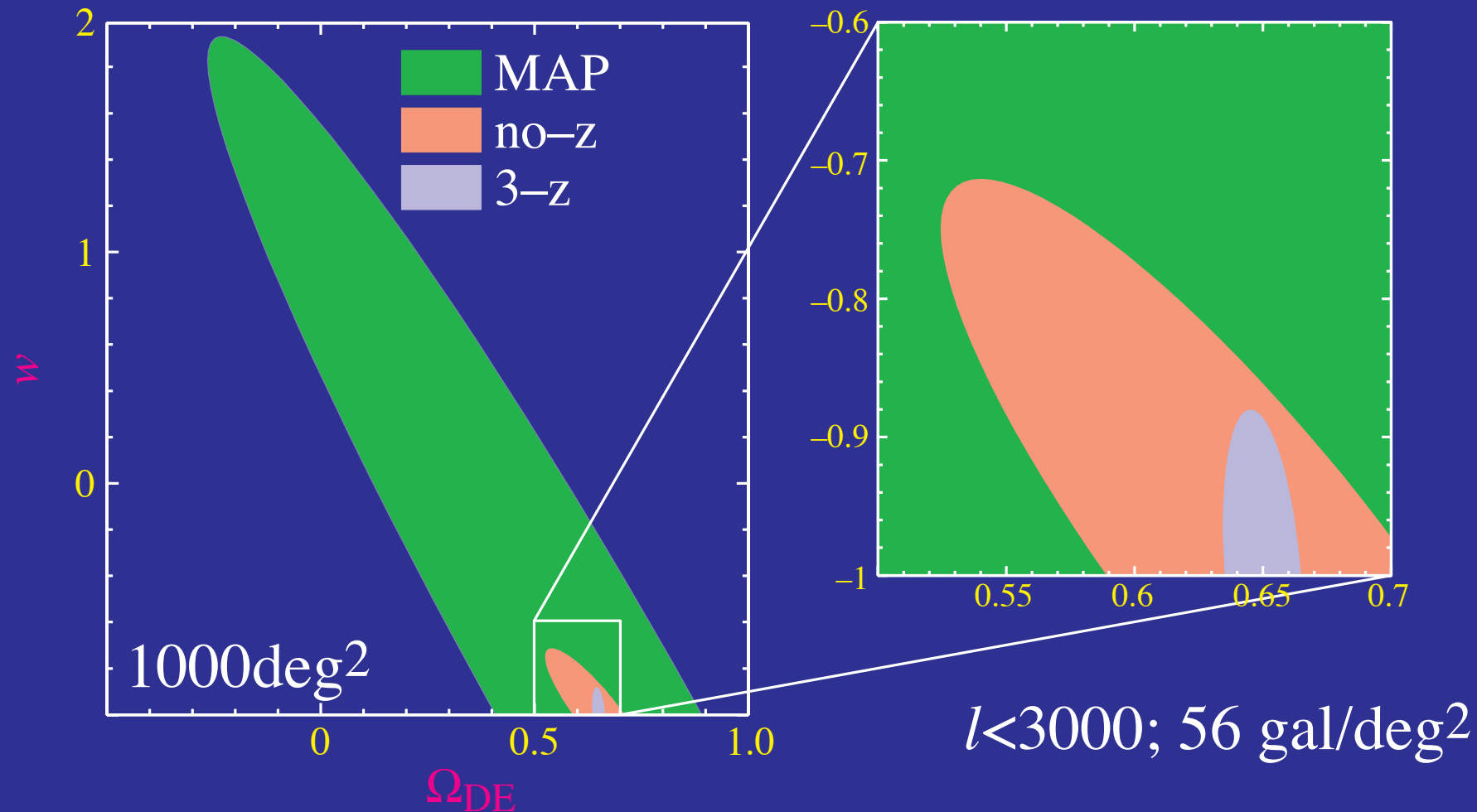
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Hidden Dark Energy Information

- Most of the information on the dark energy is hidden in the temporal or radial dimension
- Evolution of growth rate (dark energy pressure slows growth)
- Evolution of distance-redshift relation

Hidden Dark Energy Information

- Most of the information on the **dark energy** is hidden in the **temporal** or **radial** dimension
- Evolution of **growth rate** (dark energy pressure slows growth)
- Evolution of **distance-redshift** relation
- Lensing is inherently **two dimensional**: all mass along the line of sight lenses
- Tomography implicitly or explicitly **reconstructs radial dimension** with source redshifts
- Photometric redshift errors currently $\Delta z < 0.1$ out to $z \sim 1$ and allow for **”fine” tomography**

Fine Tomography

- Convergence – projection of $\Delta = \delta/a$ for each z_s

$$\kappa(z_s) = \frac{3}{2} H_0^2 \Omega_m \int_0^{z_s} dz \frac{dD}{dz} \frac{D(D_s - D)}{D_s} \Delta,$$

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$$\mathbf{d}_\kappa = \mathbf{P}_{\kappa\Delta} \mathbf{s}_\Delta + \mathbf{n}_\kappa ,$$

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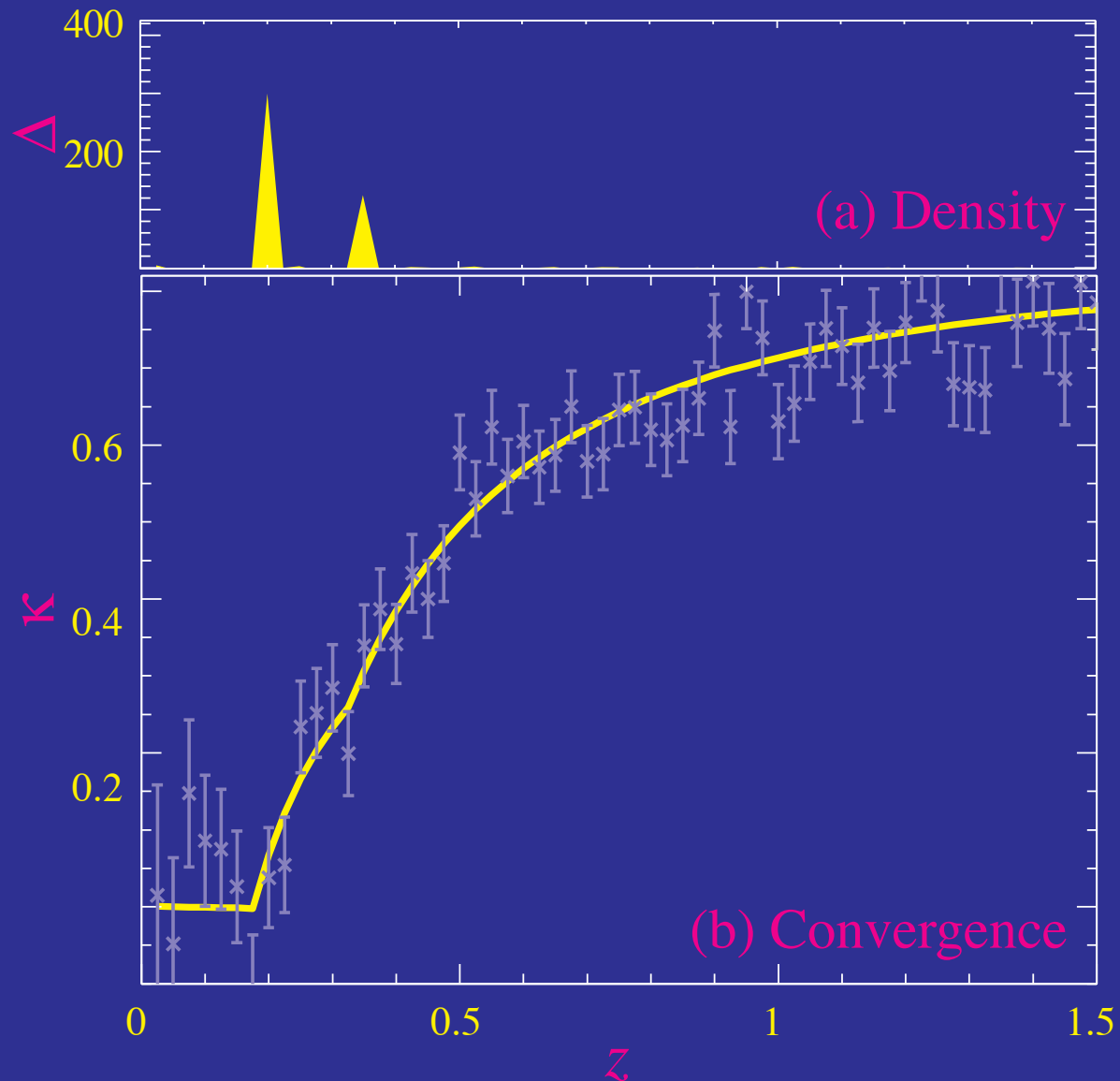
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- Well-posed (Taylor 2002) but noisy inversion (Hu & Keeton 2002)
- Noise properties differ from signal properties \rightarrow **optimal filters**

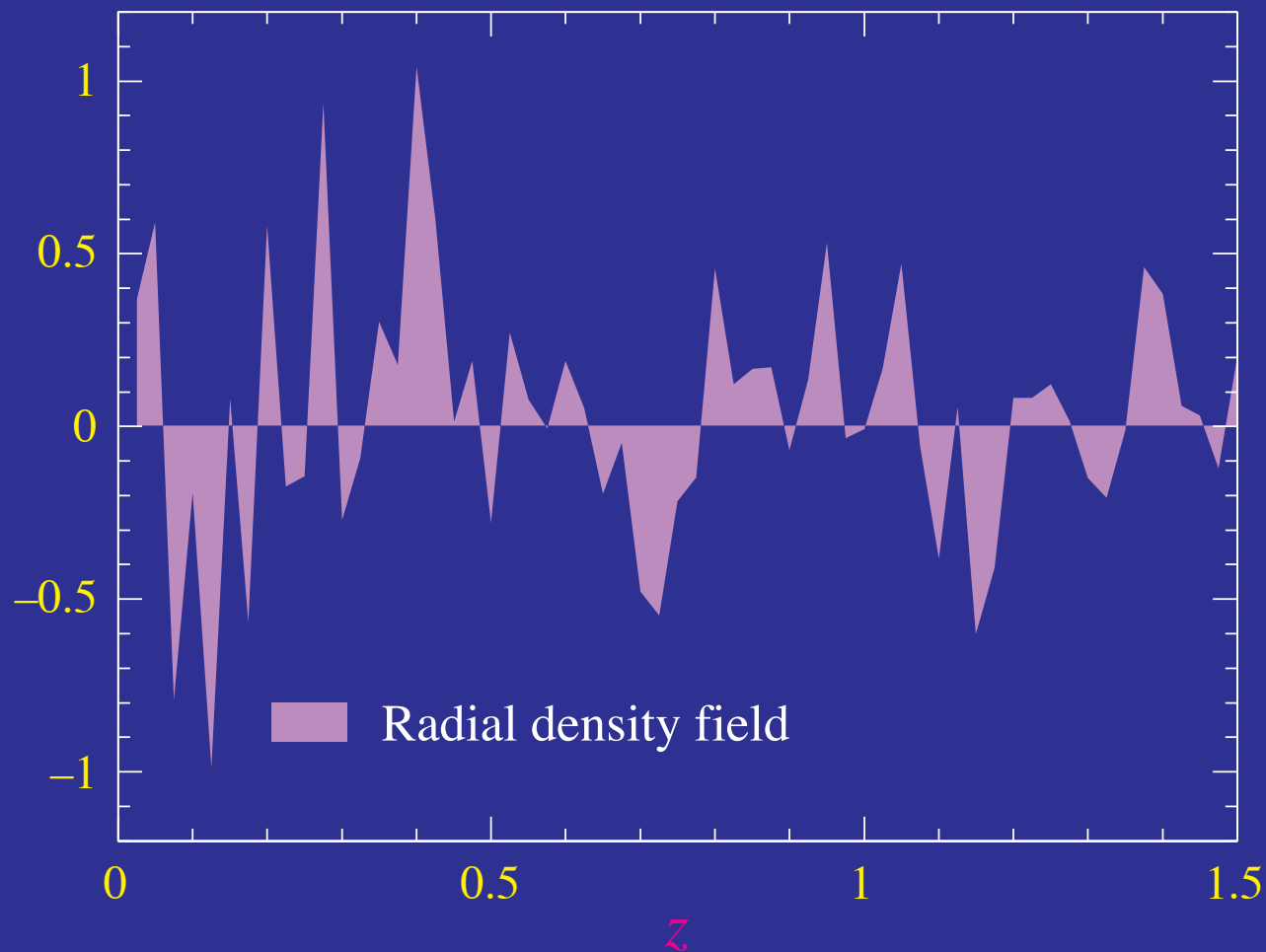
Hidden in Noise

- Derivatives of noisy convergence isolate radial structures



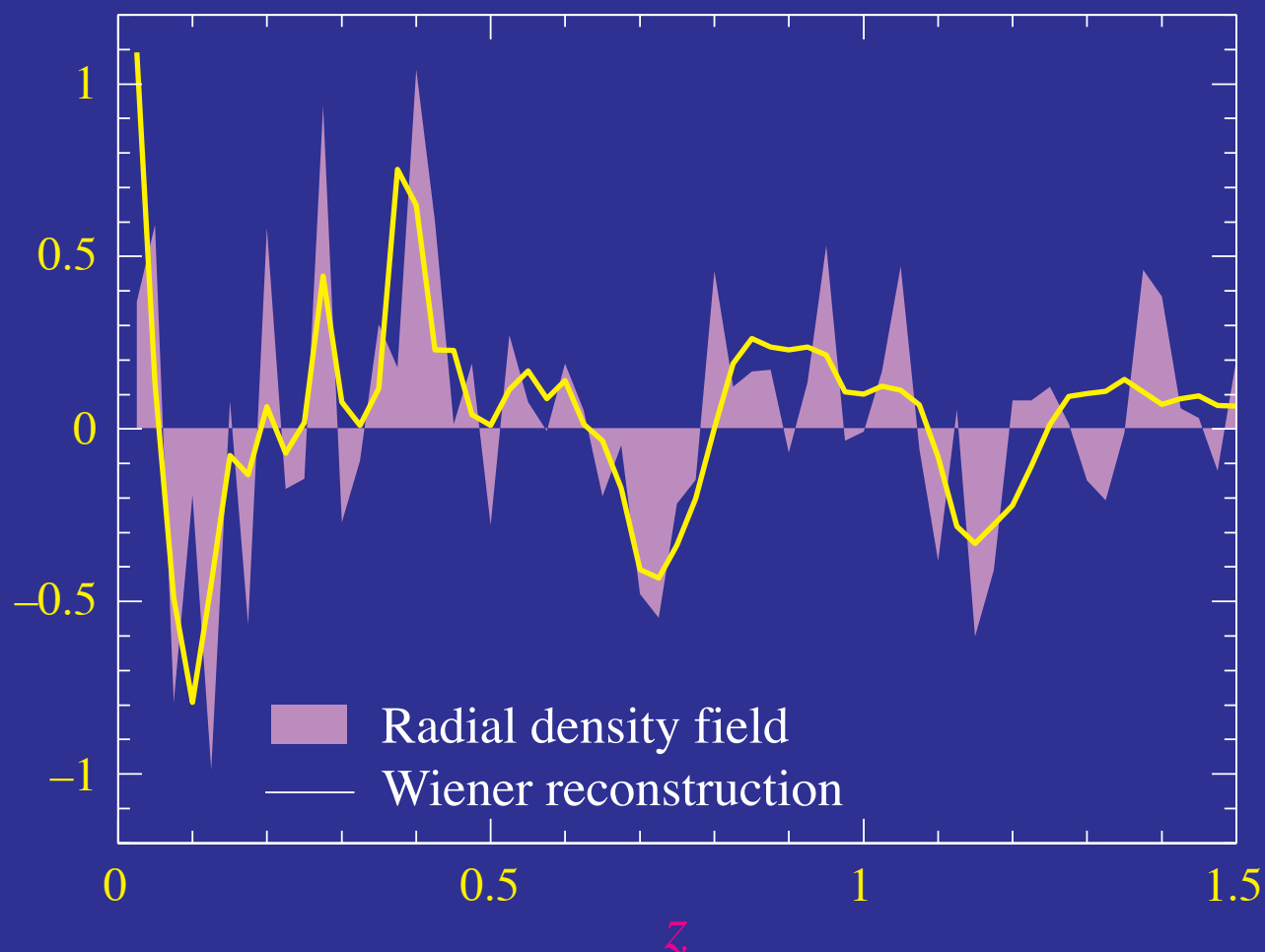
Fine Tomography

- Tomography can produce direct 3D dark matter maps, but realistically only broad features (Hu & Keeton 2002)



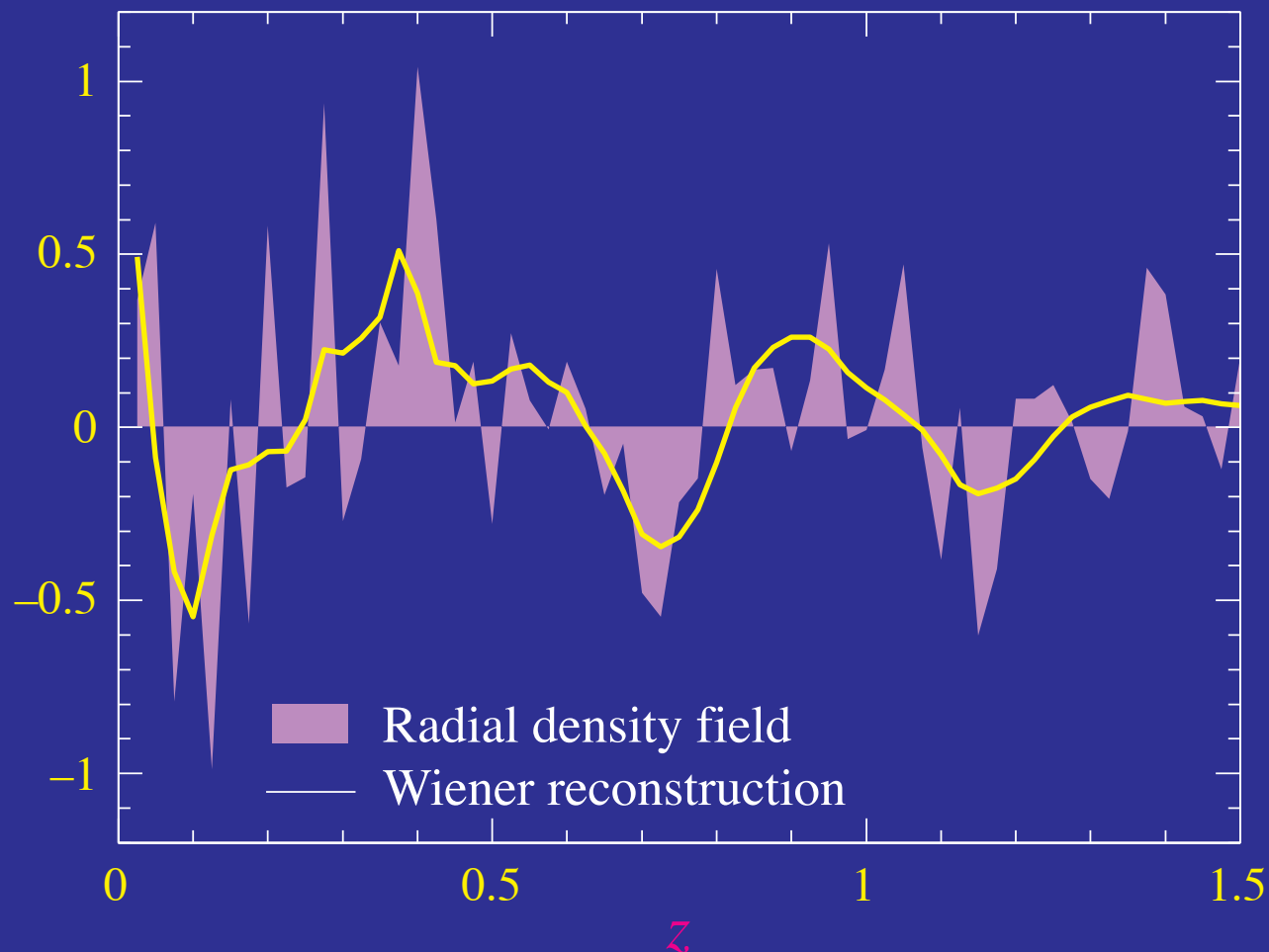
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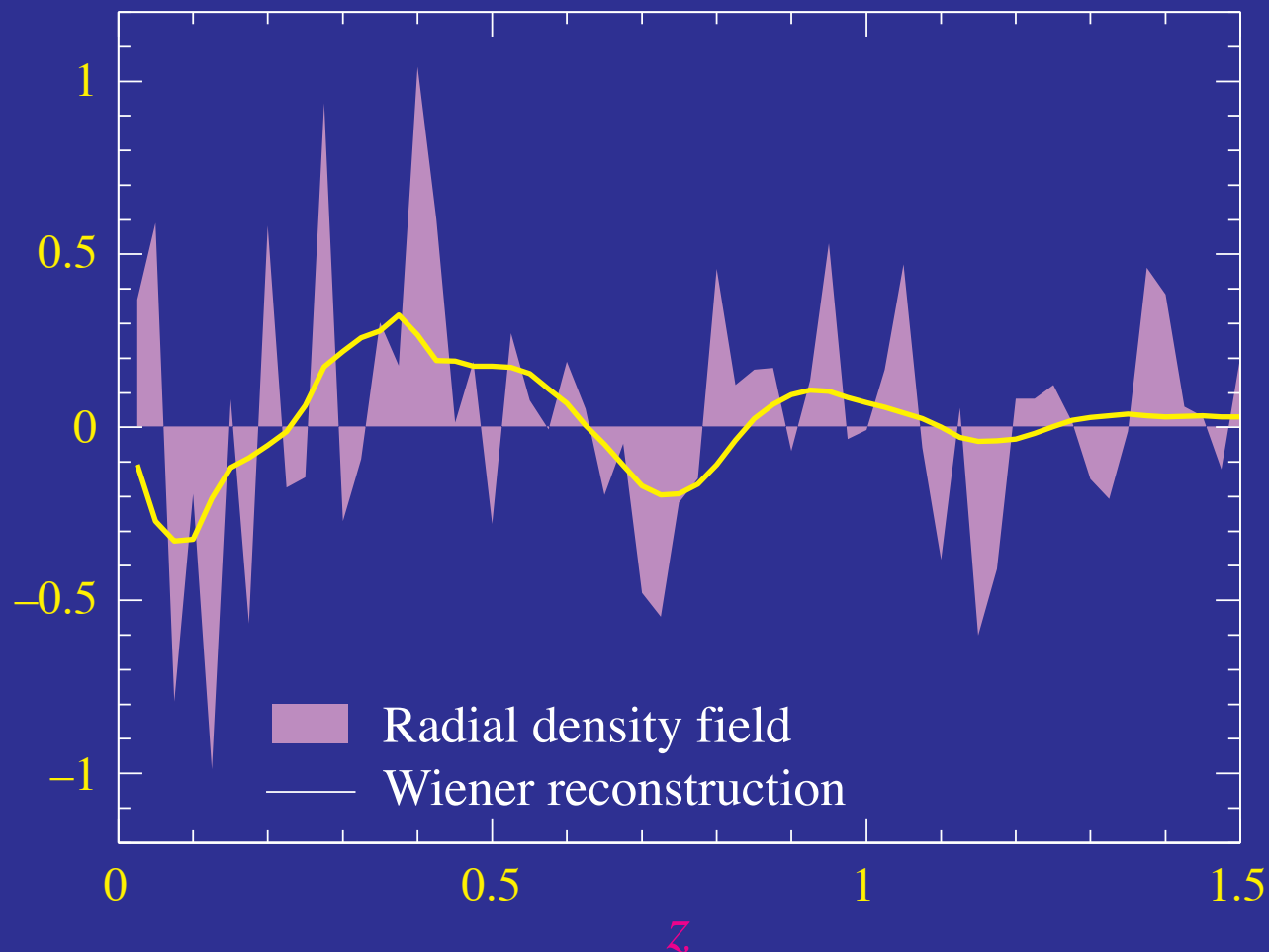
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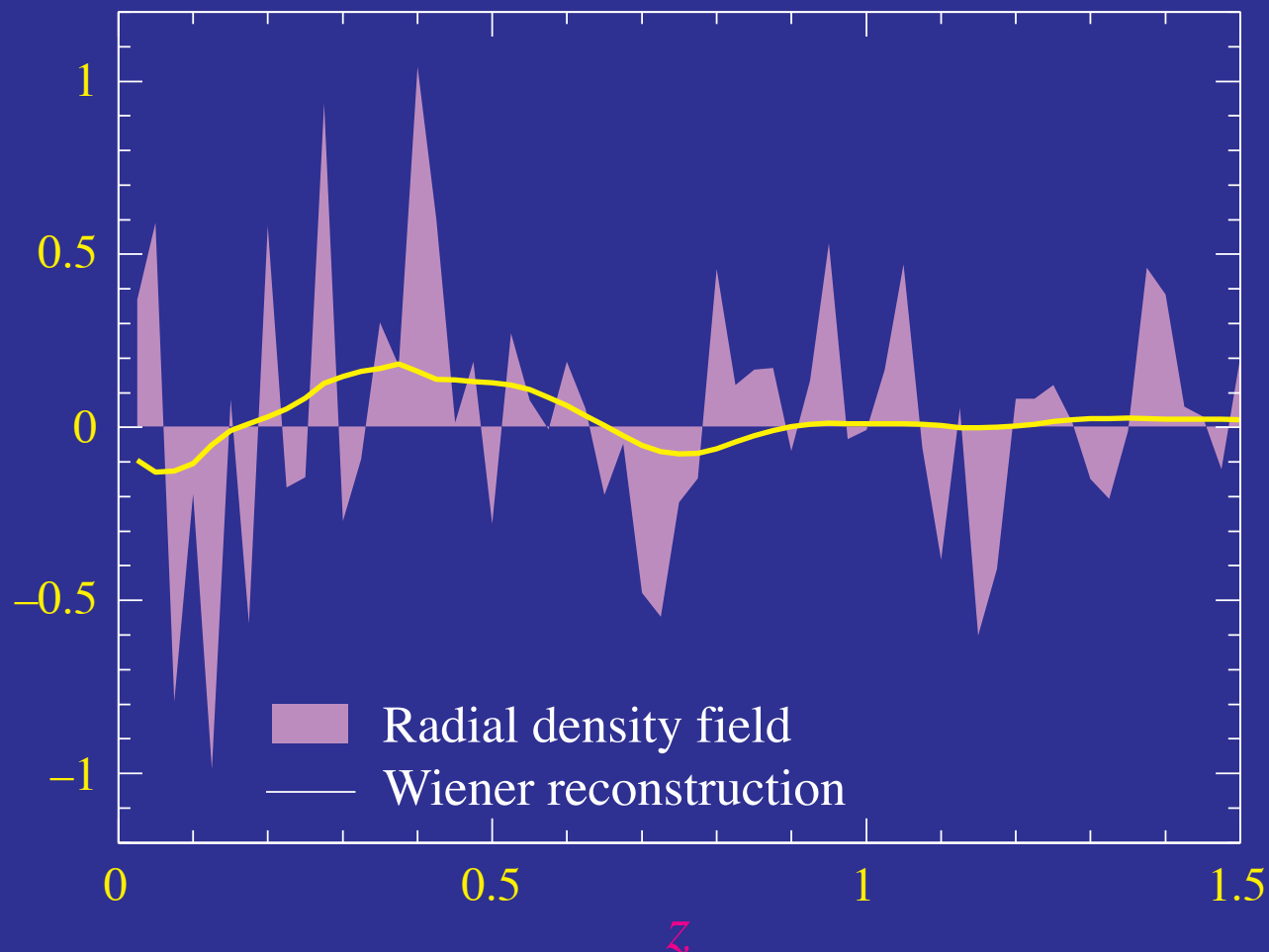
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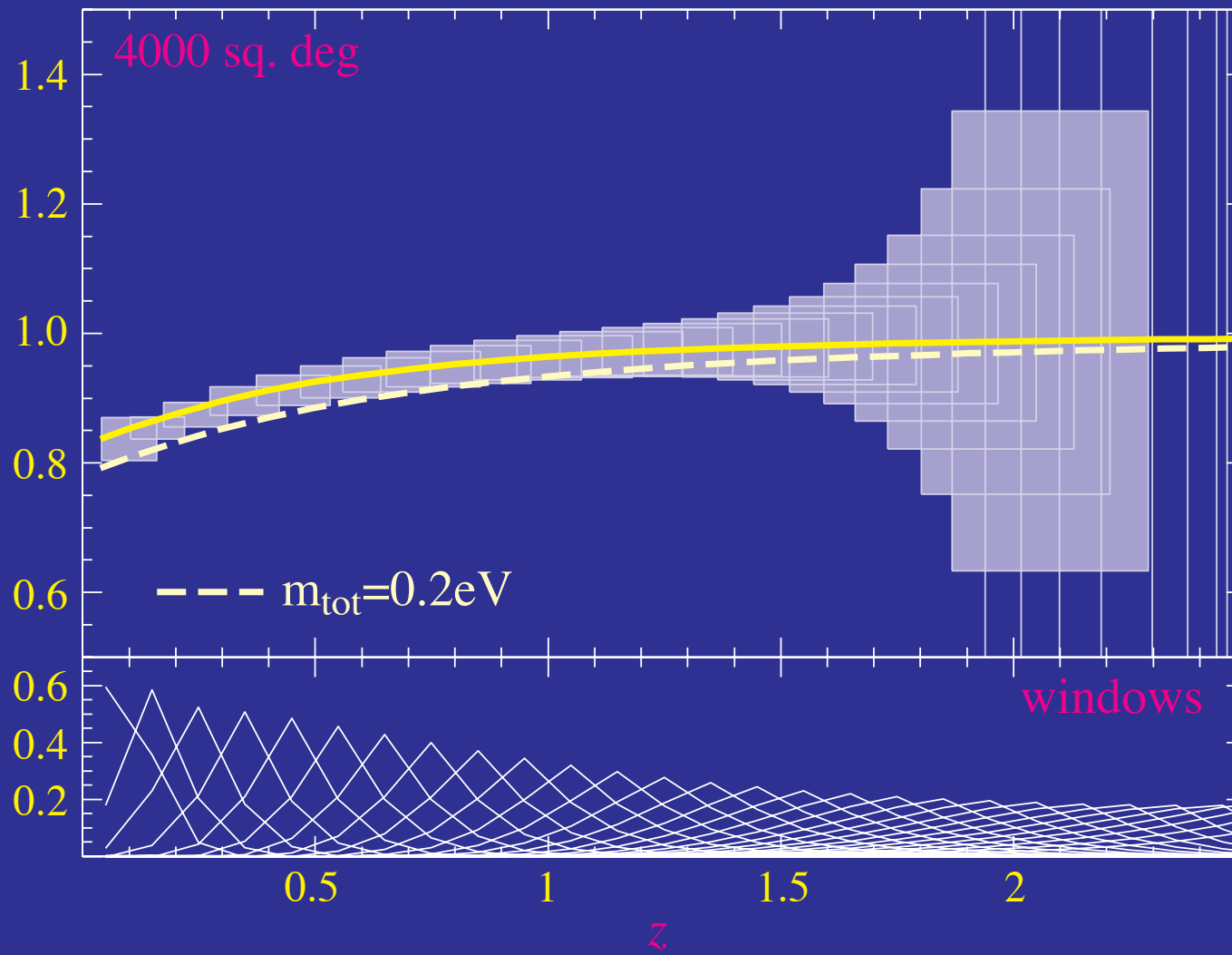
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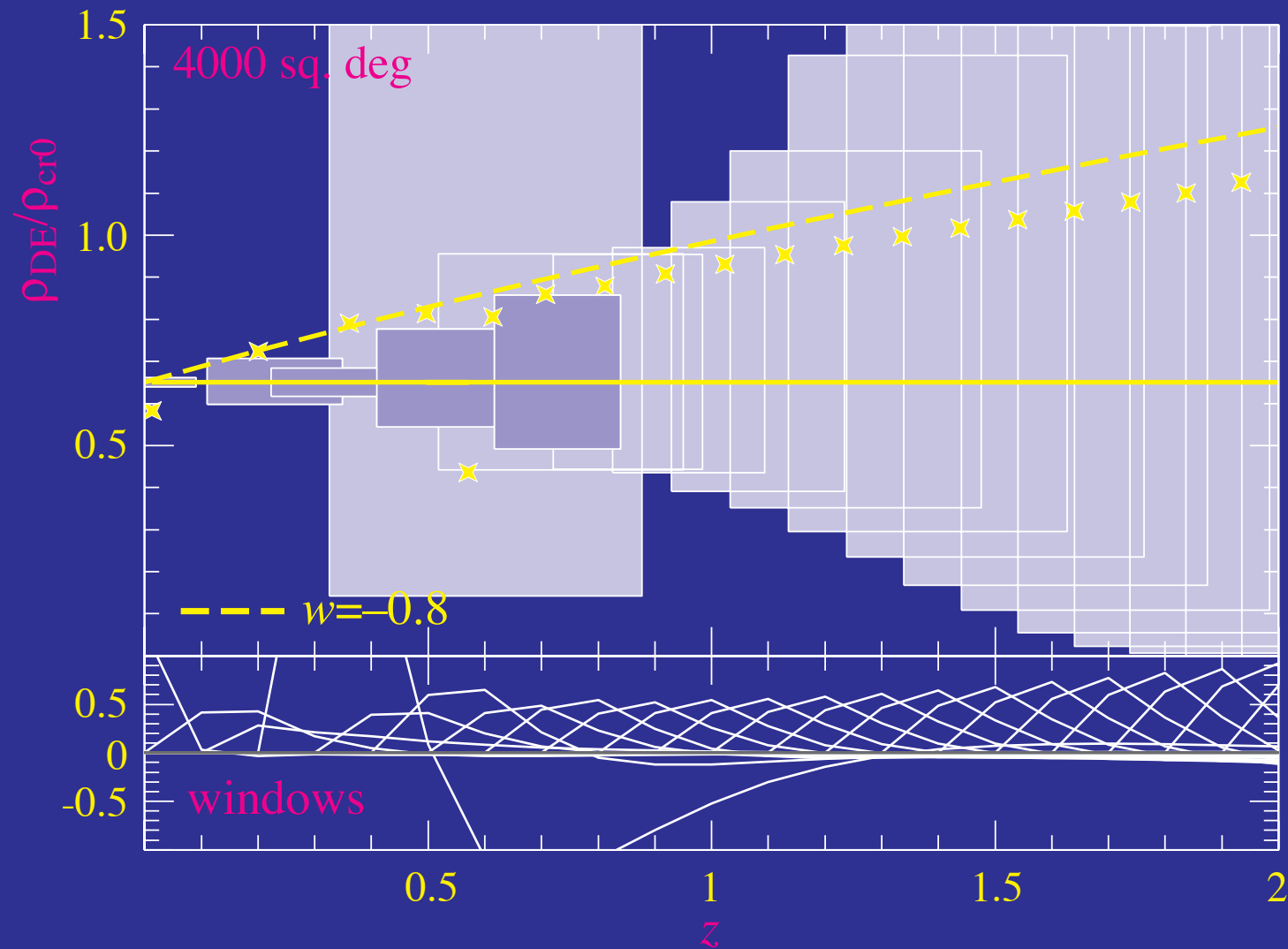
Growth Function

- Localized constraints (fixed distance-redshift relation)



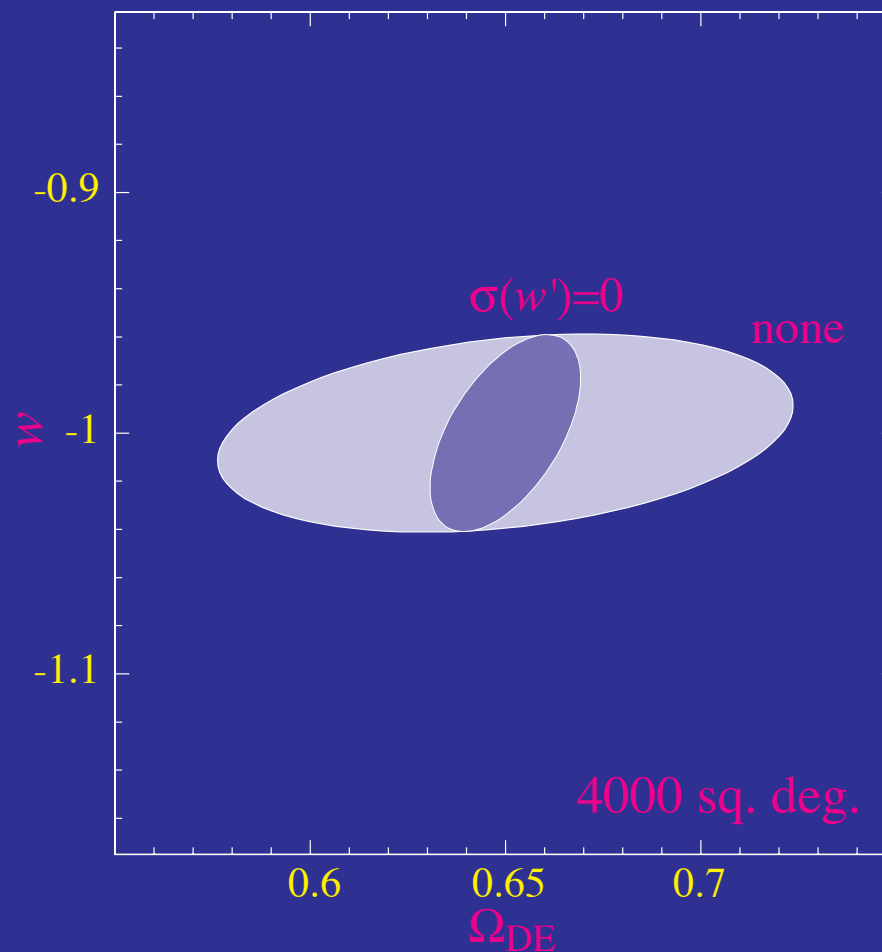
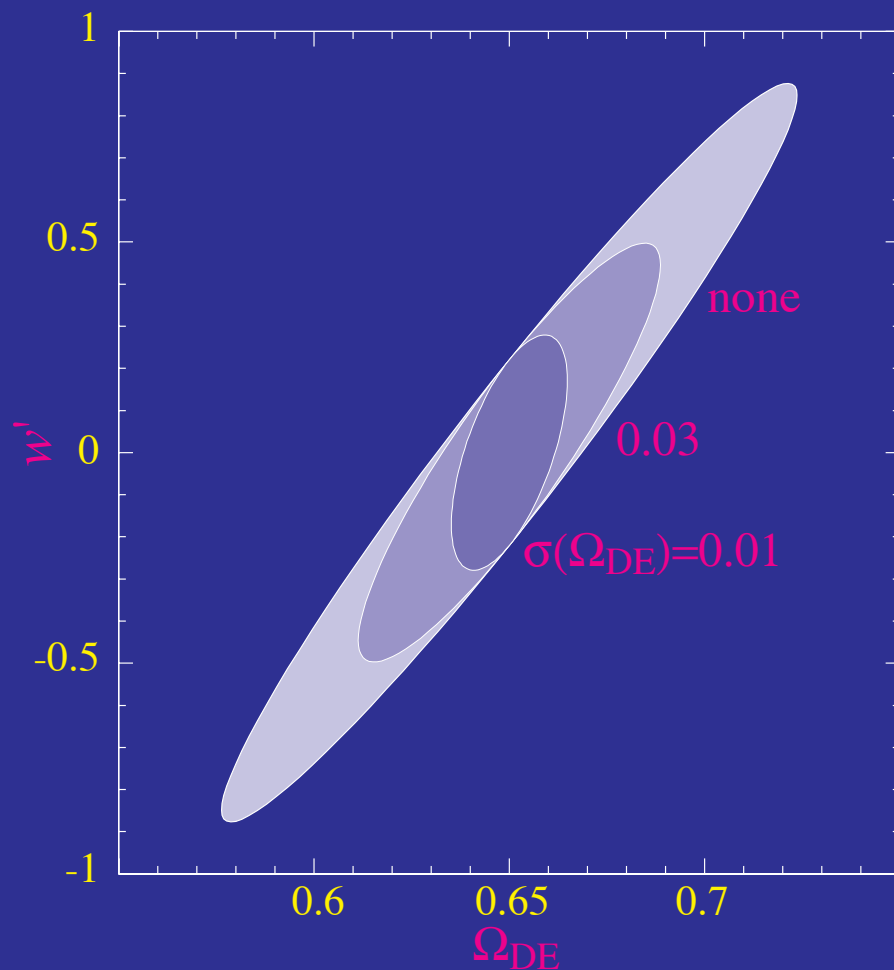
Dark Energy Density

- Localized constraints (with cold dark matter)



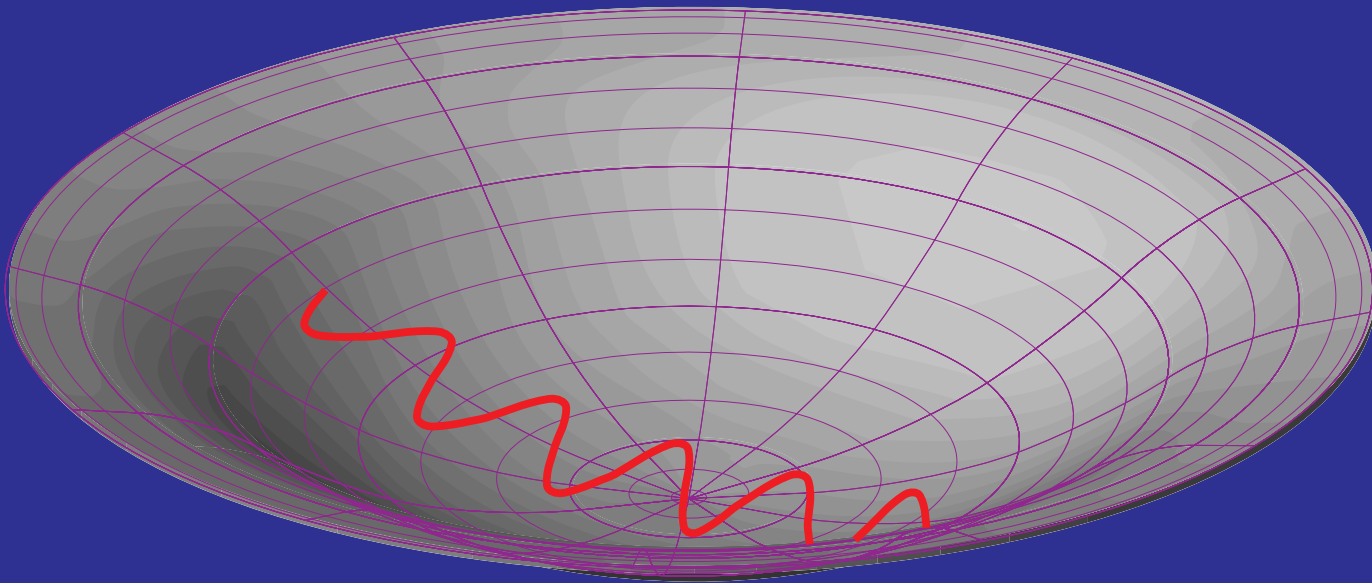
Dark Energy Parameters

- Three parameter dark energy model (Ω_{DE} , w , $dw/dz=w'$)



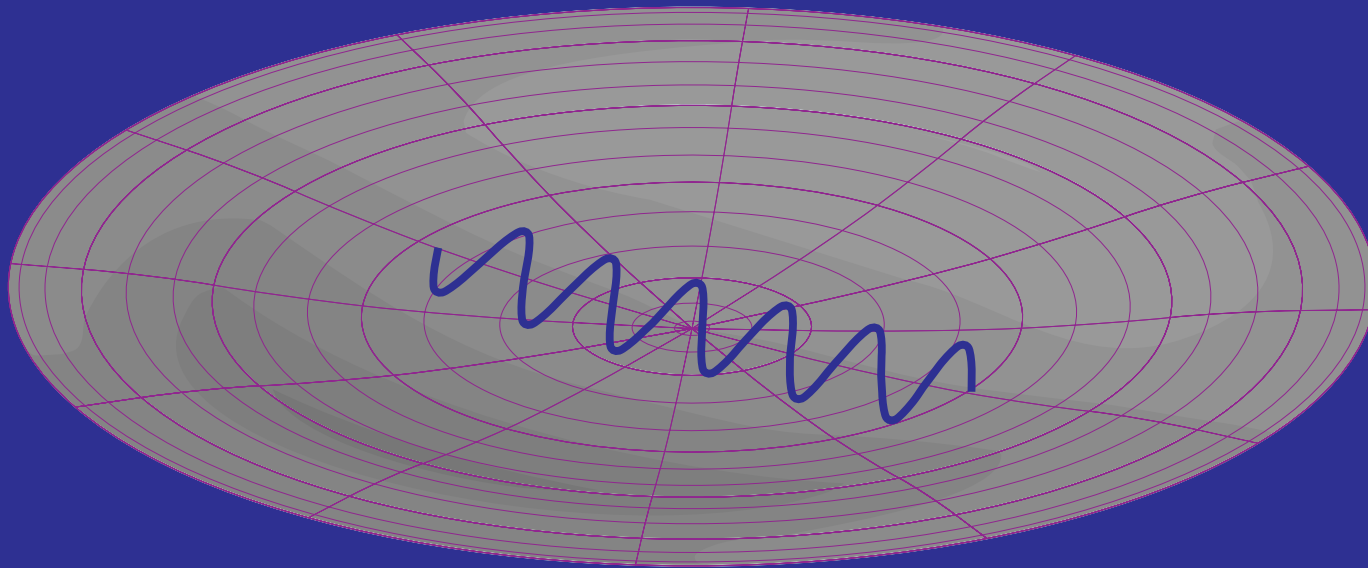
ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: $\Delta T/T = 2\Delta\Phi$
- Effect from potential hills and wells cancel on small scales



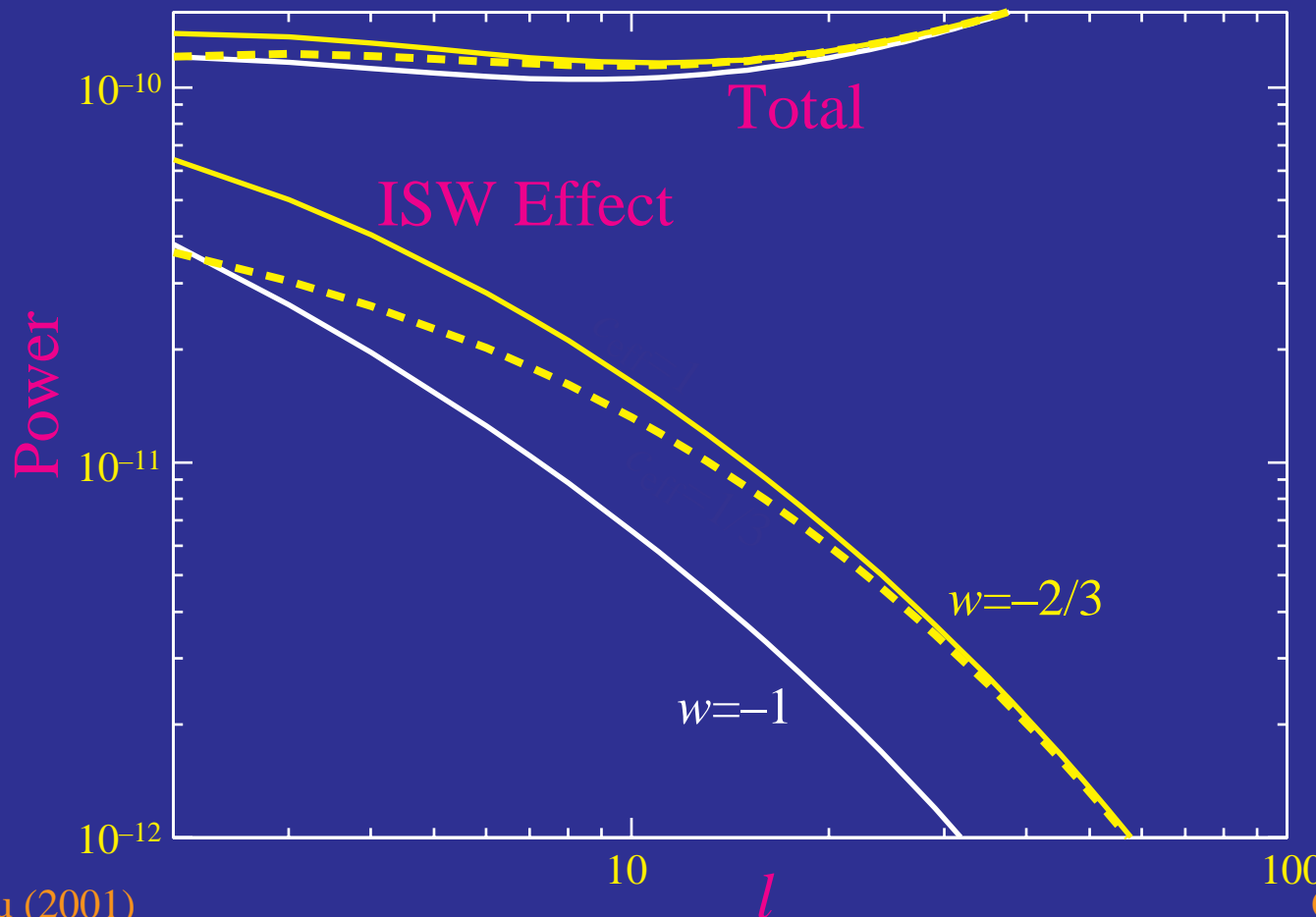
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ISW Effect and Dark Energy

- Raising **equation of state** increases redshift of dark energy domination and **raises** the **ISW effect**
- Lowering the **sound speed** increases clustering and **reduces** ISW effect at large angles

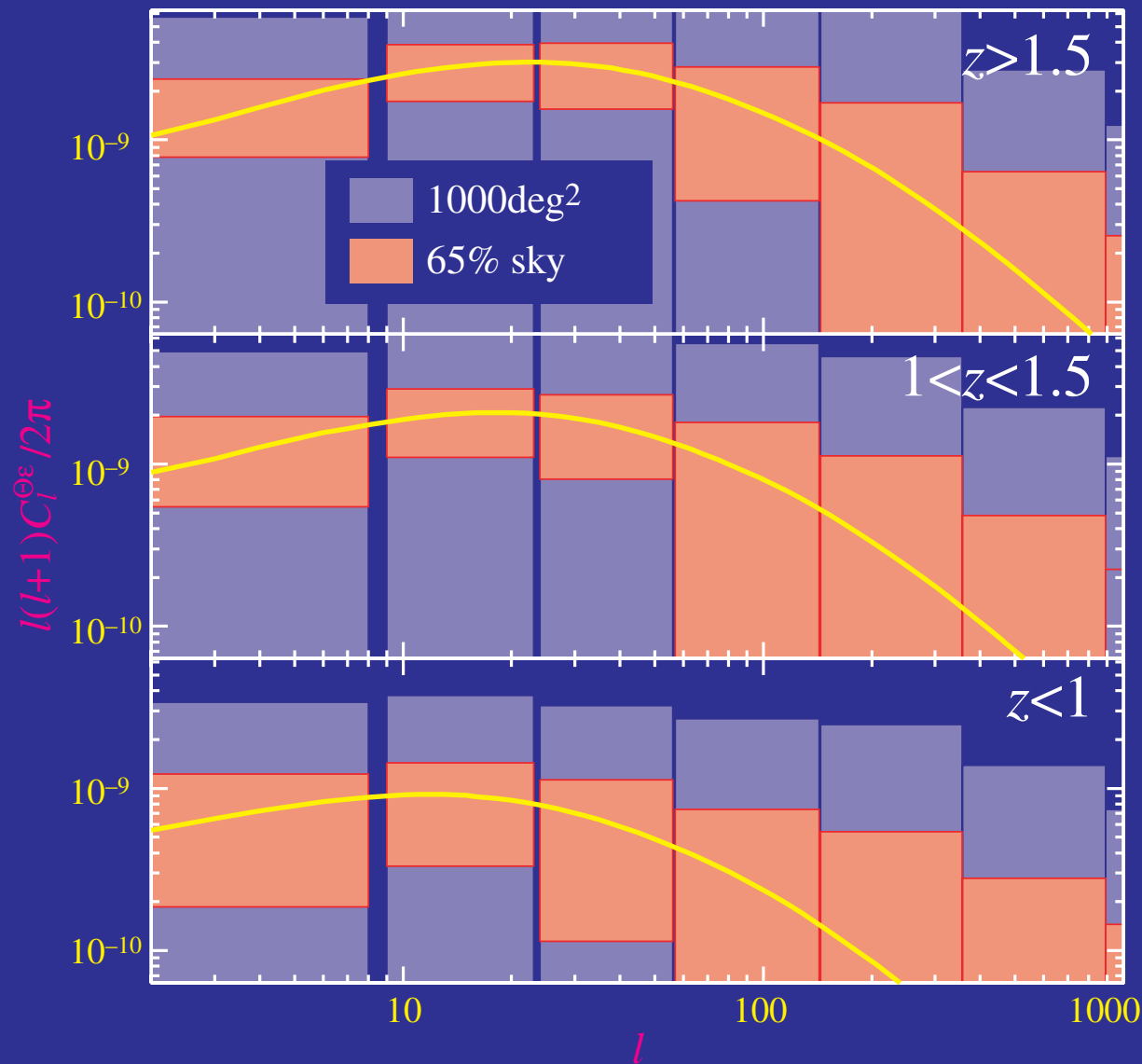


Hu (1998); Hu (2001)

Coble et al. (1997)
Caldwell et al. (1998)

Direct Detection of Dark Energy?

- In the presence of dark energy, shear is correlated with CMB temperature via ISW effect

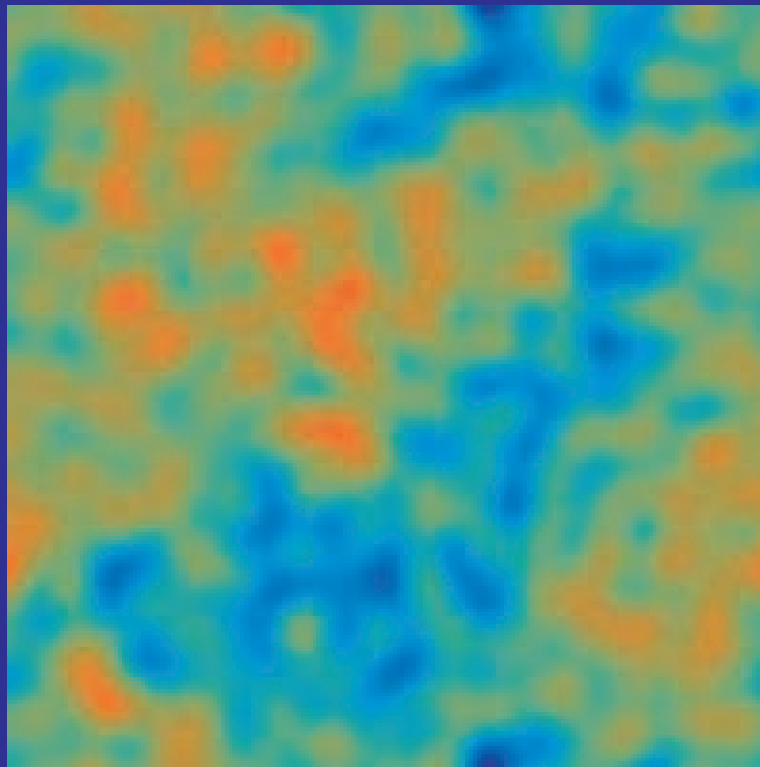


Lensing of a Gaussian Random Field

- CMB temperature and polarization anisotropies are Gaussian random fields – unlike galaxy weak lensing
- Average over many noisy images – like galaxy weak lensing

Lensing by a Gaussian Random Field

- Mass distribution at large angles and high redshift in the linear regime
- Projected mass distribution (low pass filtered reflecting deflection angles): 1000 sq. deg



rms deflection

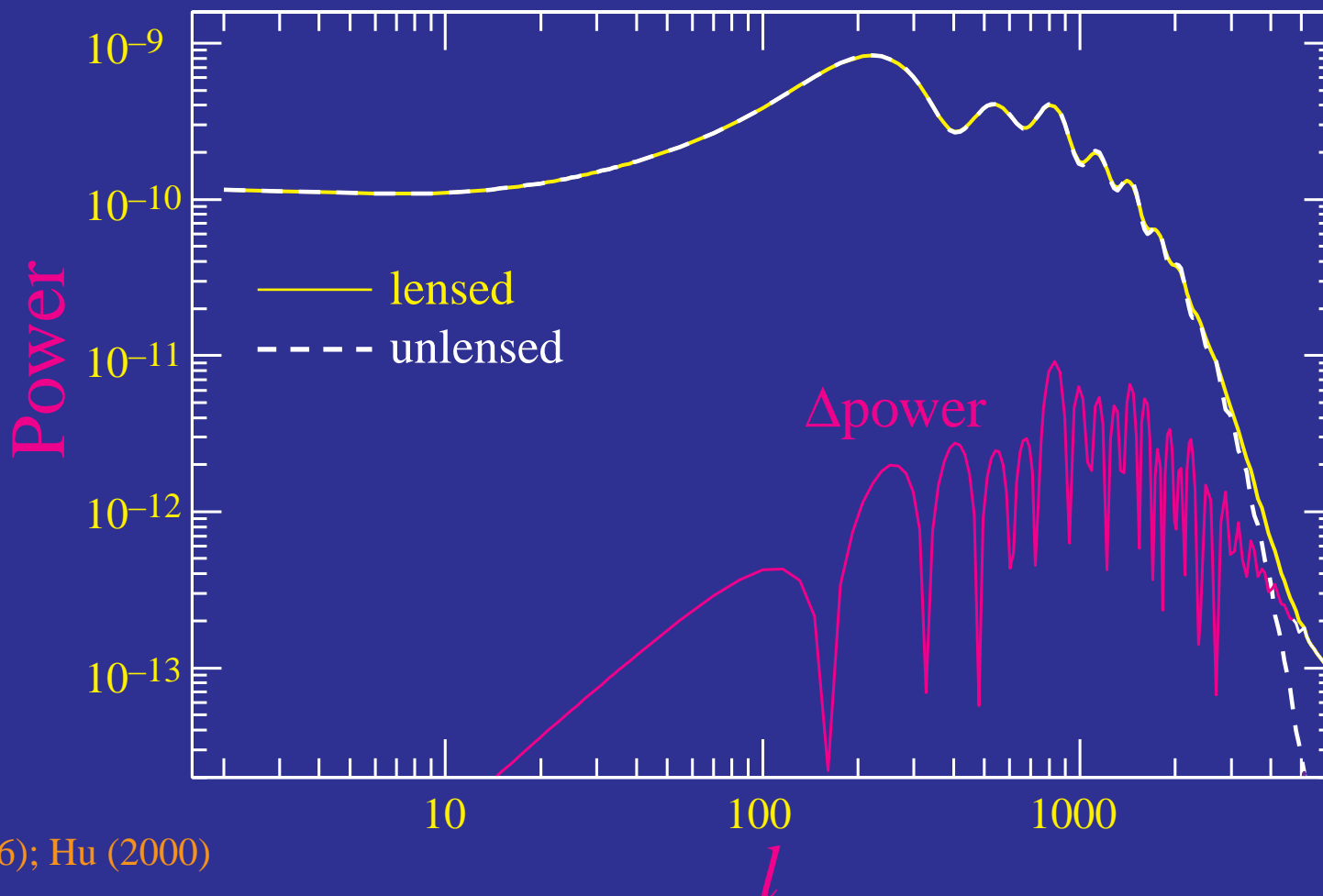
2.6'

deflection coherence

10°

Lensing in the Power Spectrum

- Lensing **smooths** the power spectrum with a width $\Delta l \sim 60$
- Convolution with specific kernel: higher order **correlations** between **multipole moments** – not apparent in **power**



Reconstruction from the CMB

- Correlation between **Fourier moments** reflect **lensing potential**

$$\kappa = \nabla^2 \phi$$

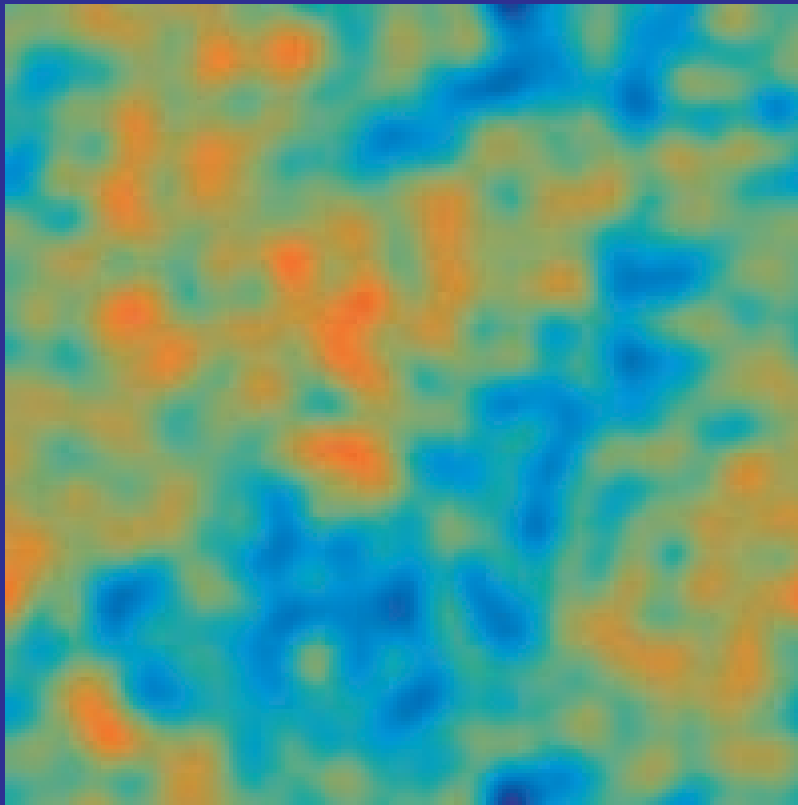
$$\langle x(\mathbf{l})x'(\mathbf{l}') \rangle_{\text{CMB}} = f_\alpha(\mathbf{l}, \mathbf{l}') \phi(\mathbf{l} + \mathbf{l}') ,$$

where $x \in$ **temperature, polarization fields** and f_α is a fixed weight that reflects geometry

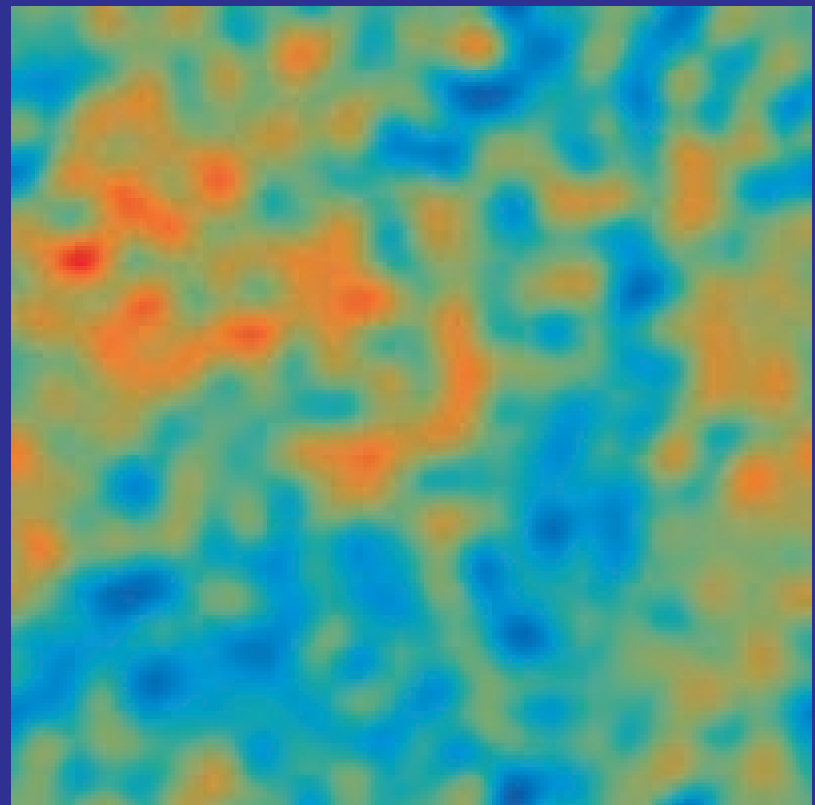
- Each pair forms a **noisy estimate** of the potential or projected mass
- just like a pair of galaxy shears
- **Minimum variance weight** all pairs to form an estimator of the lensing mass

Quadratic Reconstruction

- Matched filter (minimum variance) averaging over pairs of multipole moments
- Real space: divergence of a temperature-weighted gradient



original



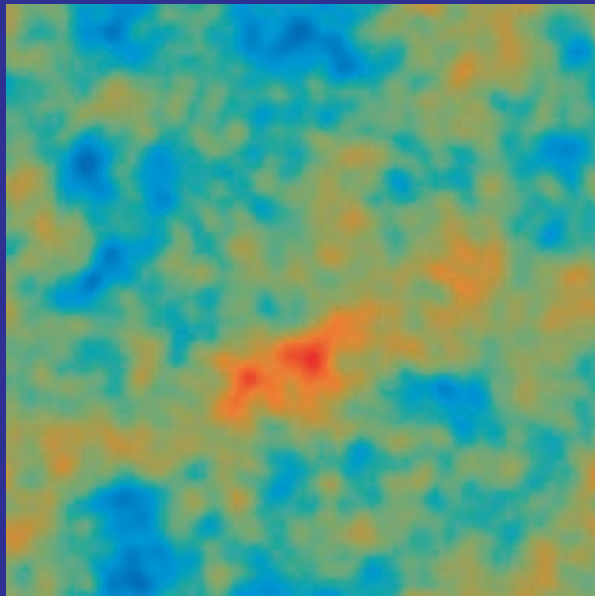
reconstructed

Hu (2001) potential map (1000sq. deg)

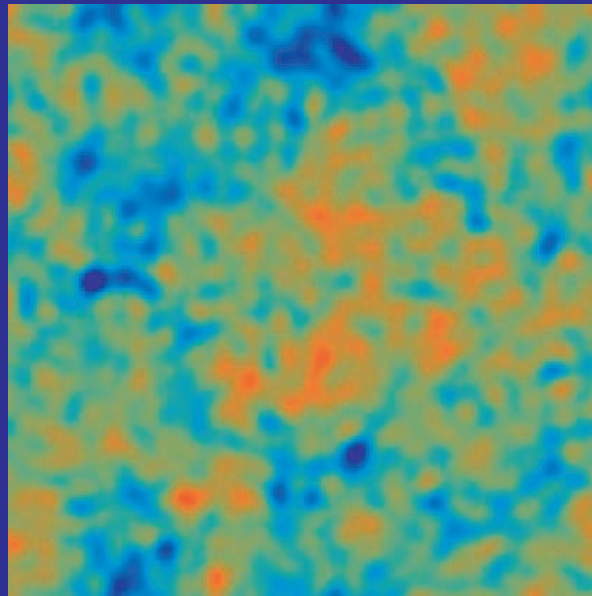
1.5' beam; $27\mu\text{K-arcmin}$ noise

Ultimate (Cosmic Variance) Limit

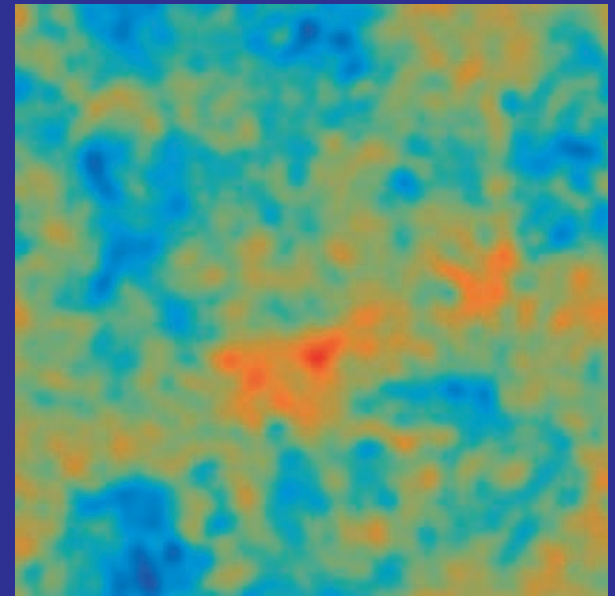
- Cosmic variance of CMB fields sets ultimate limit
- Polarization allows mapping to finer scales ($\sim 10'$)



mass



temp. reconstruction

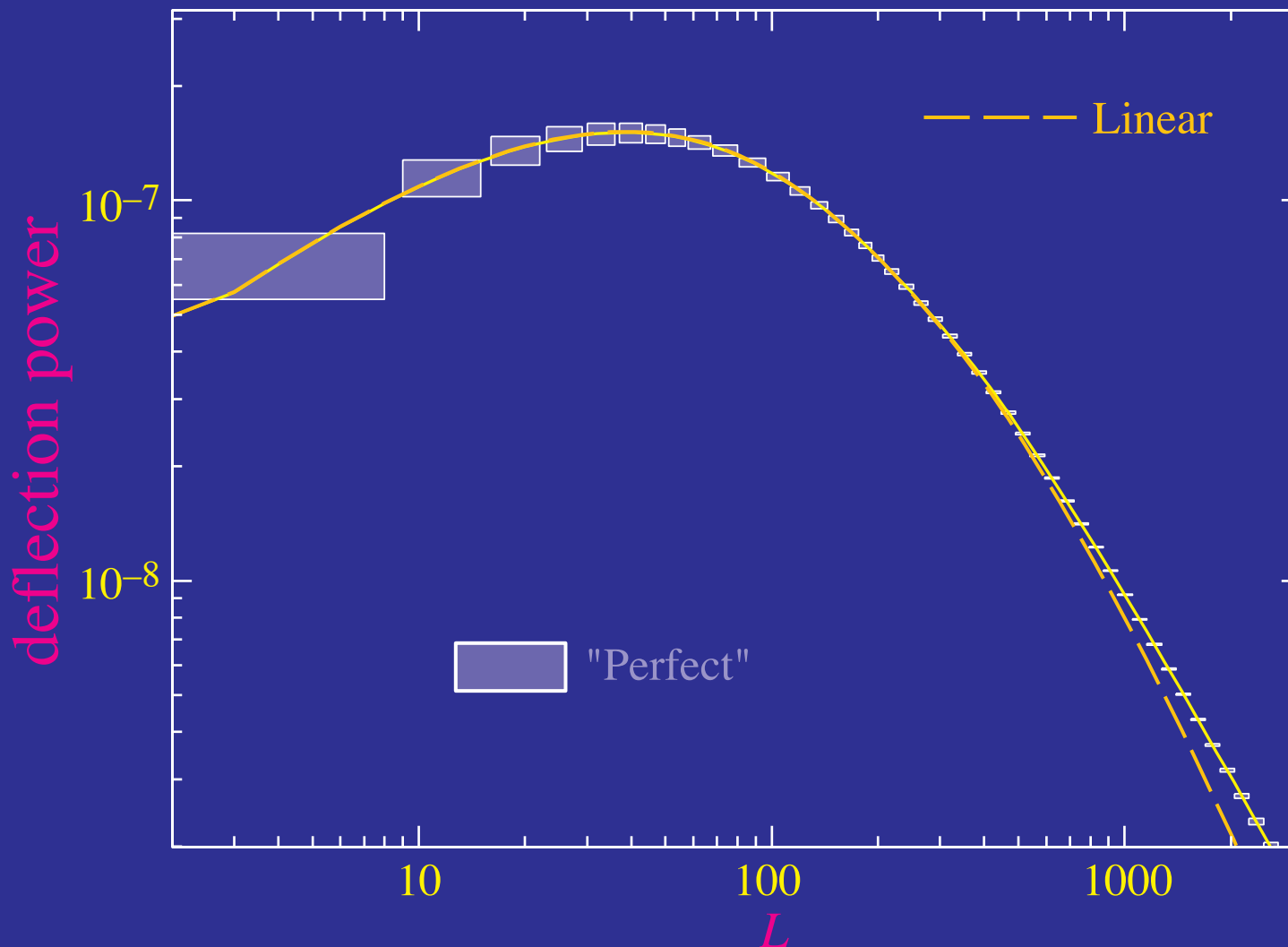


EB pol. reconstruction

100 sq. deg; 4' beam; $1\mu\text{K}$ -arcmin

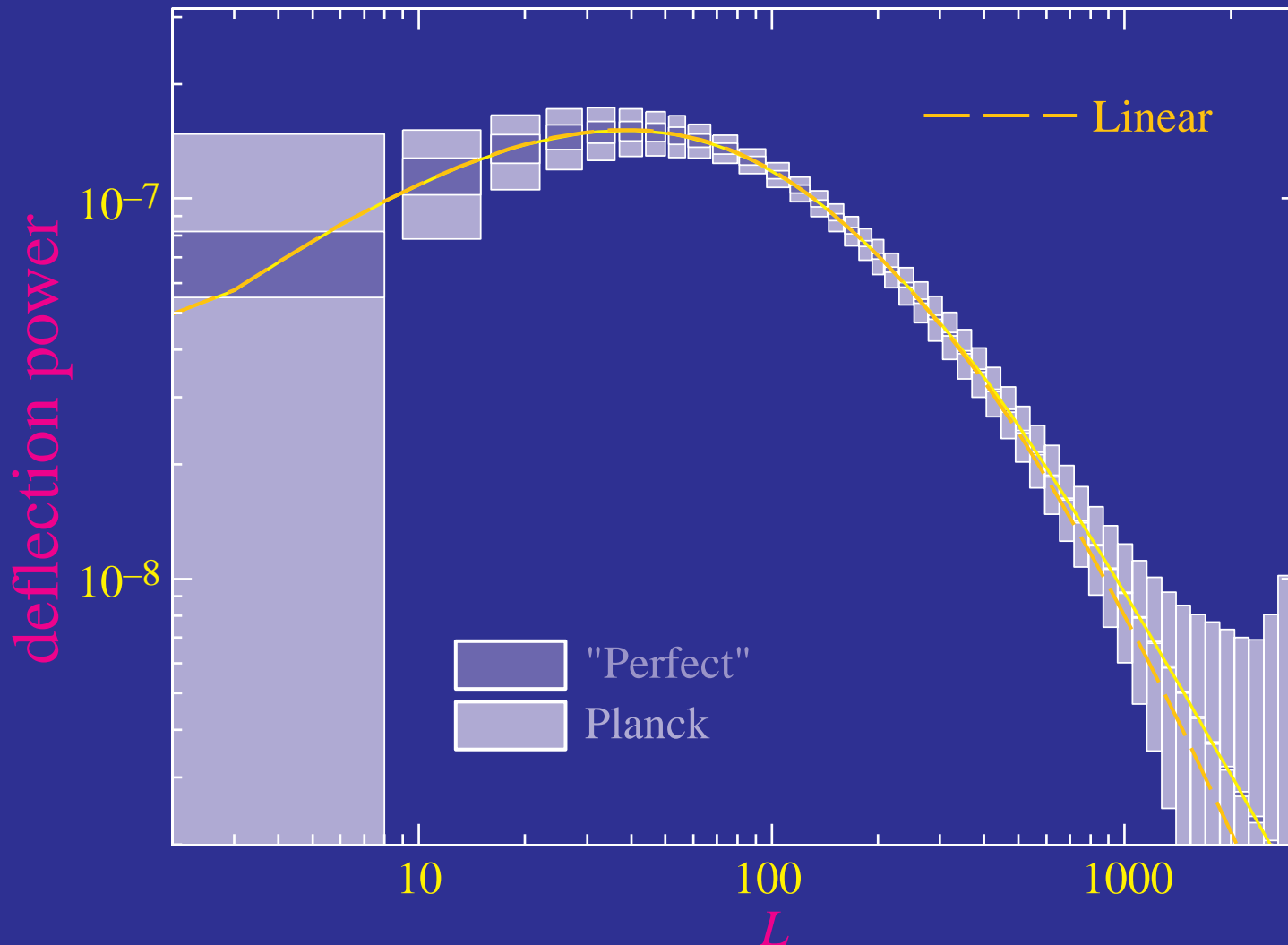
Matter Power Spectrum

- Measuring projected **matter power** spectrum to cosmic variance limit across whole **linear regime** $0.002 < k < 0.2 \ h/\text{Mpc}$



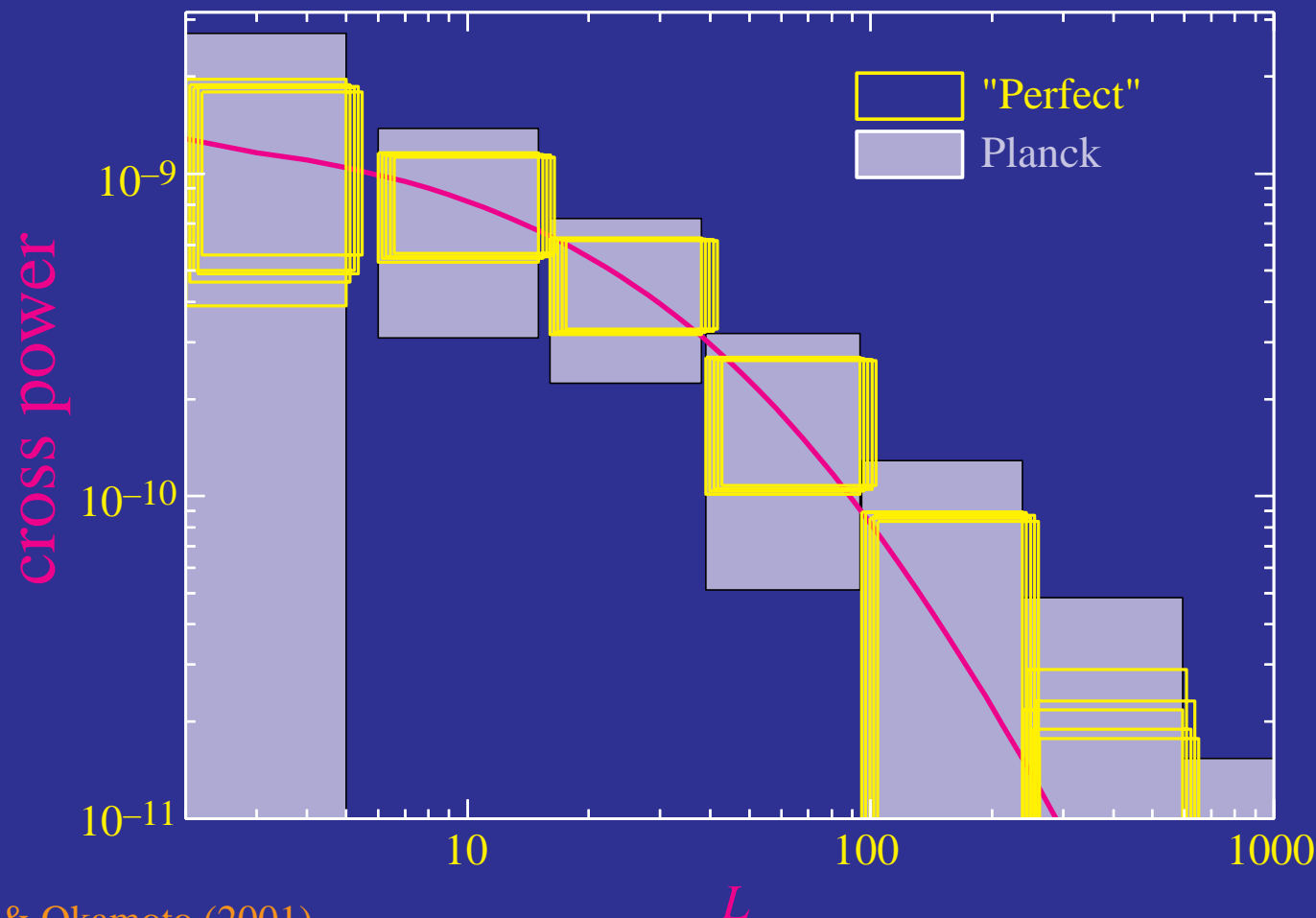
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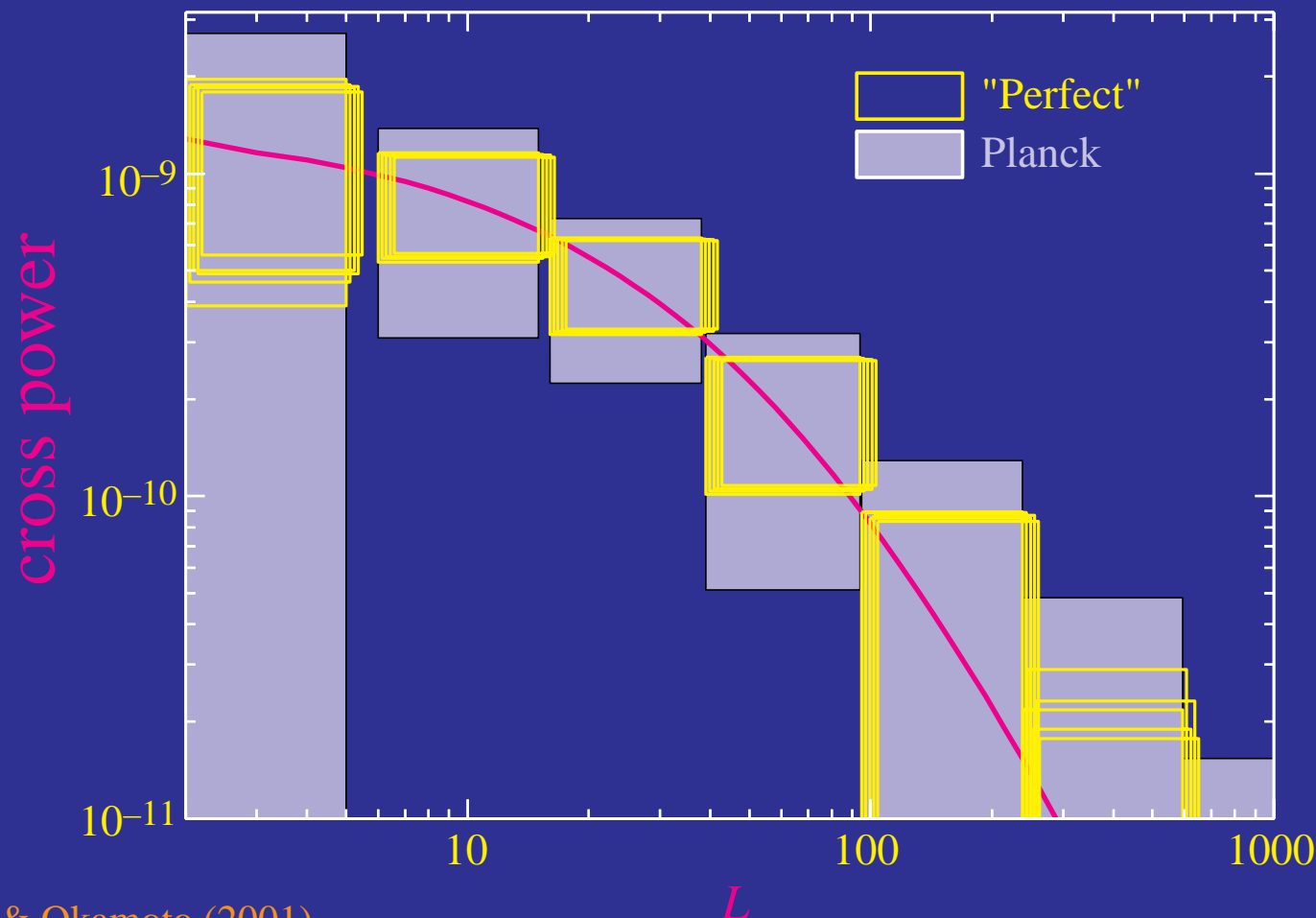
Cross Correlation with Temperature

- Any correlation is a **direct detection** of a **smooth energy density** component through the **ISW** effect
- **5** nearly independent measures in **temperature** & **polarization**



Cross Correlation with Temperature

- Any correlation is a **direct detection** of a **smooth energy density** component through the **ISW effect**
- Show dark energy smooth **>5-6 Gpc** scale, **test quintessence**



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- Standard model of cosmology well-established
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- Dark matter distribution and its dependence on dark energy well-understood
- Luminous tracers (supernovae/galaxies/clusters) require modelling of formation/evolution
- Gravitational lensing avoids ambiguity, utilizes luminous objects only as background image
- Evolution of dark energy can be extracted tomographically
- Clustering of dark energy (test of scalar field paradigm) extractable from wide-field CMB lensing