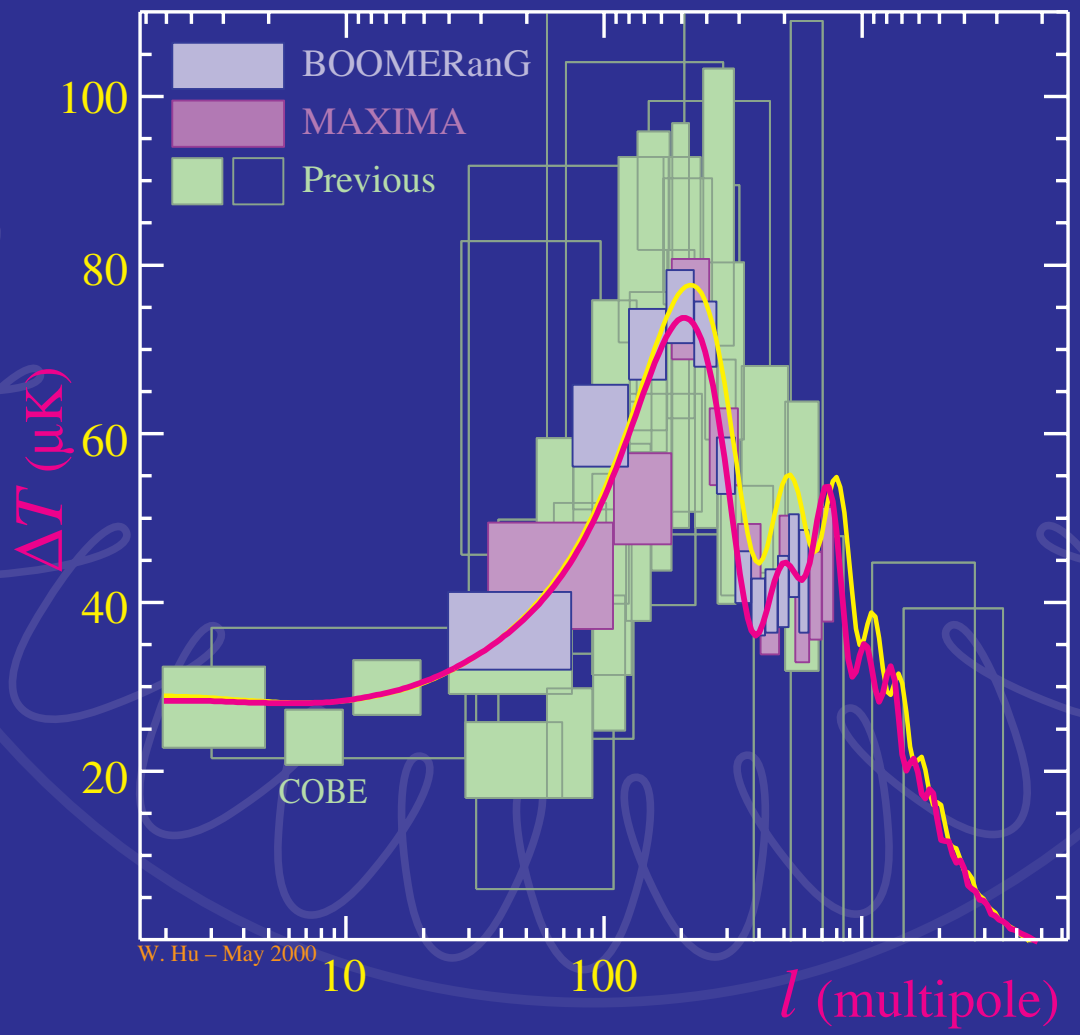
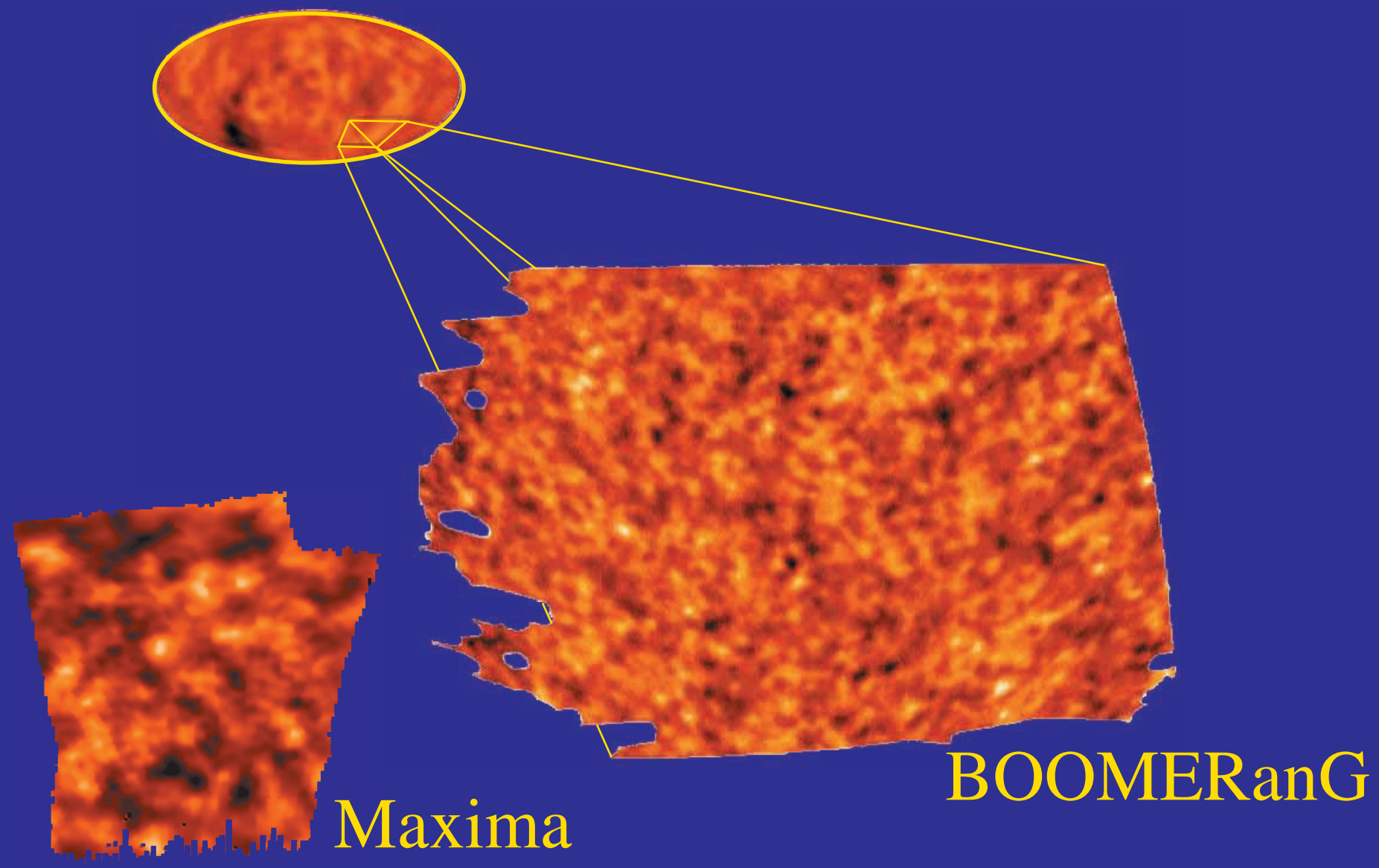
The Physics of CMB Anisotropies



Erice, December 2000

Wayne Hu

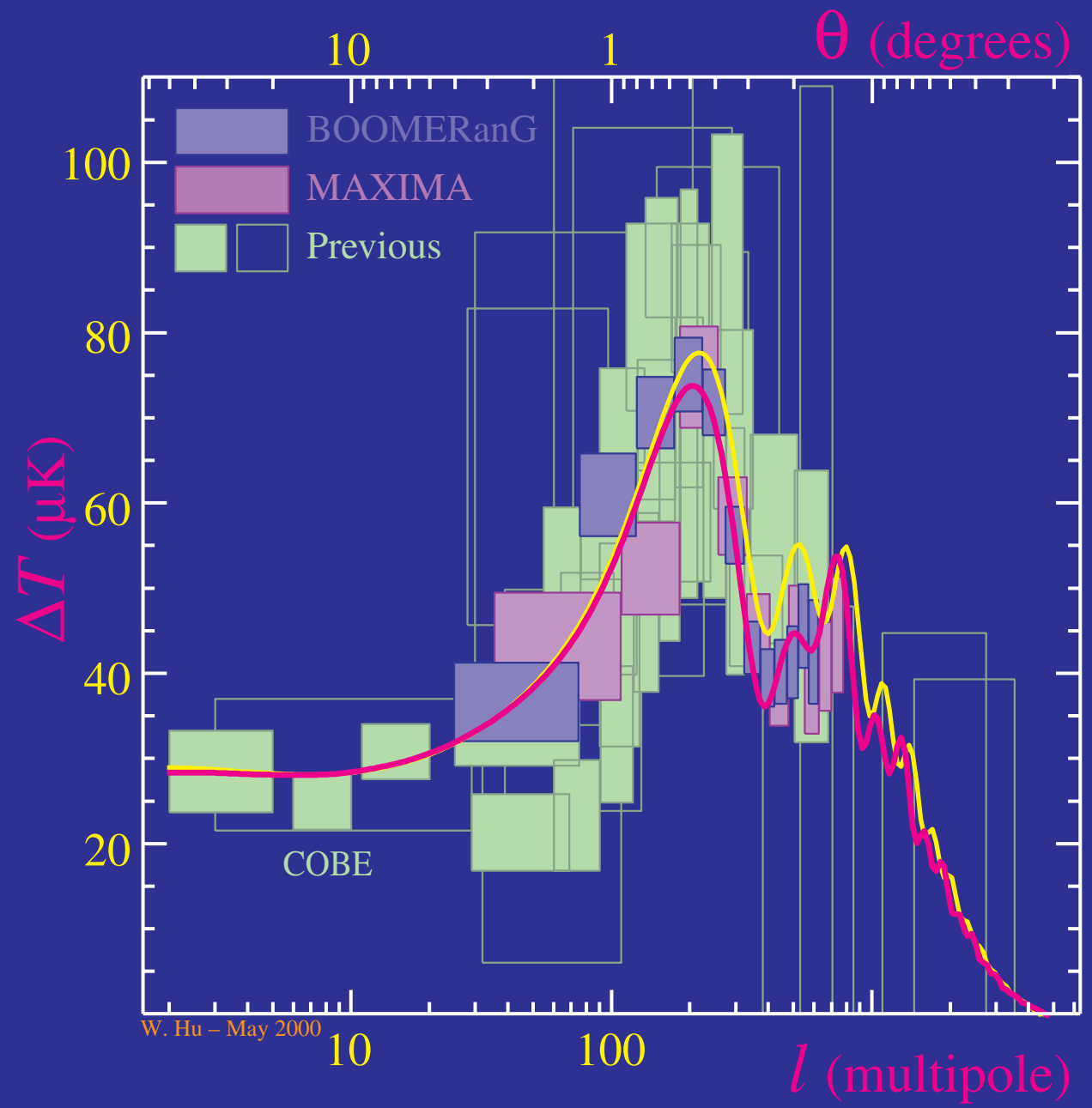
CMB Anisotropies



Hanany, *et al.* (2000)

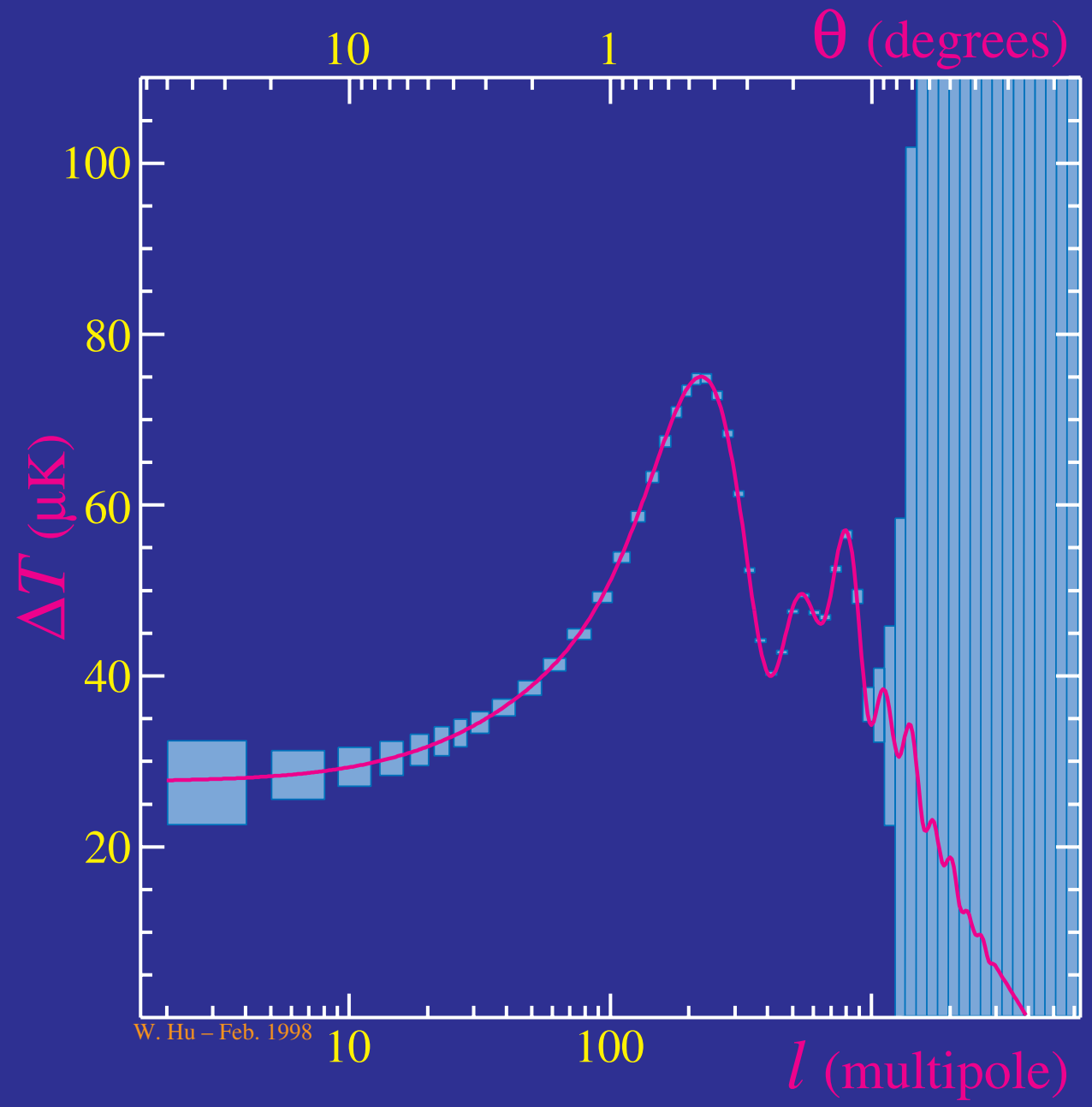
de Bernardis, *et al.* (2000)

Sound Physics Seen

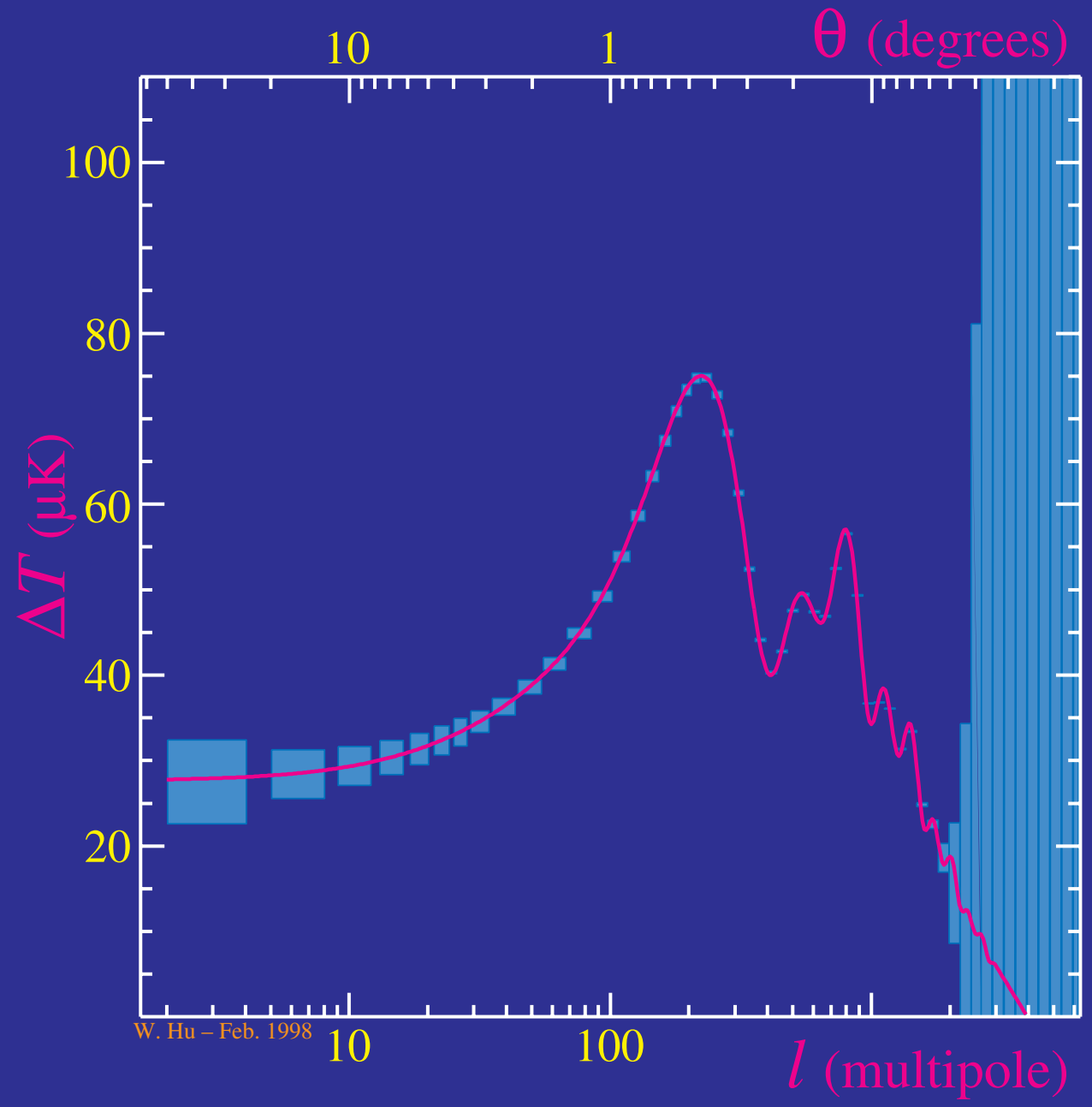


- First: Nailed!
- Second: Constrained!
- Third: Hint...

Projected MAP Errors



Projected Planck Errors



Outline

The Present

- Thermal History
- Initial Conditions
- Acoustic Oscillations
- Calculations
- First Peak
- Second Peak

The Future

- Higher Peaks
- Fish(er)ing for Parameters
- Polarization
- Testing Inflation
- Secondary Gravitational Effects
- Secondary Scattering

The Present

Thermal History

- $z > 1000$; $T_\gamma > 3000K$

  Hydrogen ionized

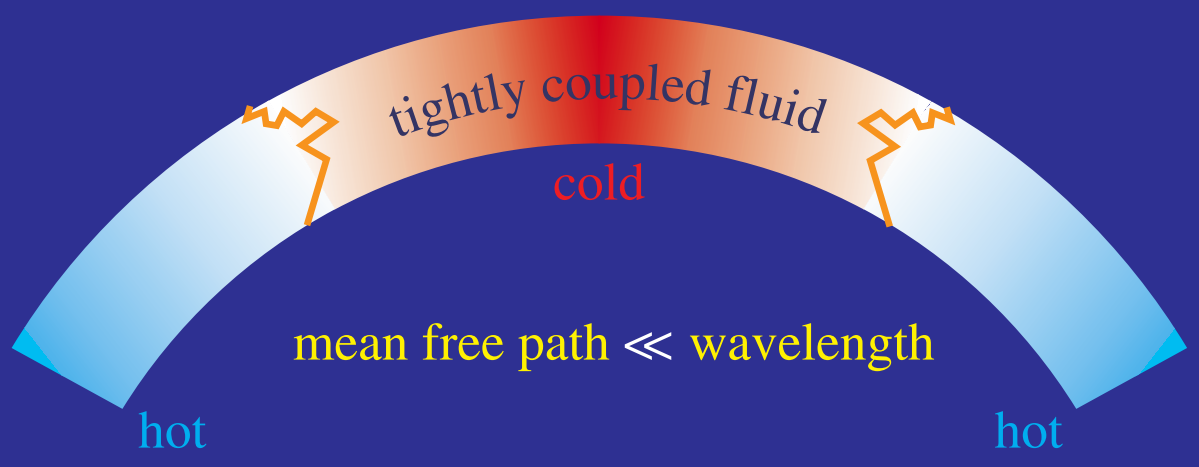
  Free electrons glue photons to baryons

  γ ~~~~~~~~ e^- ~~~~~~~~ p

  compton scattering coulomb interactions

  Photon–baryon fluid

  Potential wells that later form structure



Thermal History

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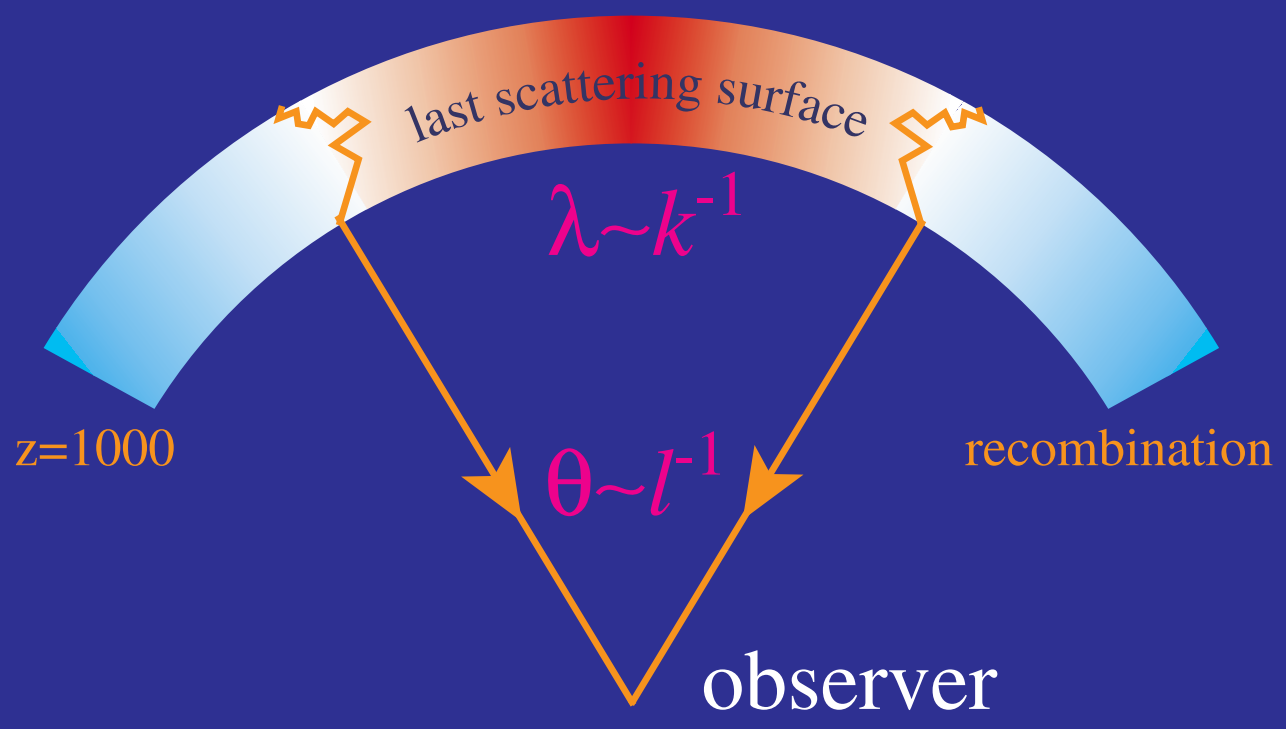
  Hydrogen ionized
  Free electrons glue photons to baryons

  γ ~~~~~~~~ e– ~~~~~~~~ p
  compton scattering coulomb interactions

  Photon–baryon fluid
  Potential wells that later form structure

- $z \sim 1000$; $T_\gamma \sim 3000K$
  Recombination
  Fluid breakdown

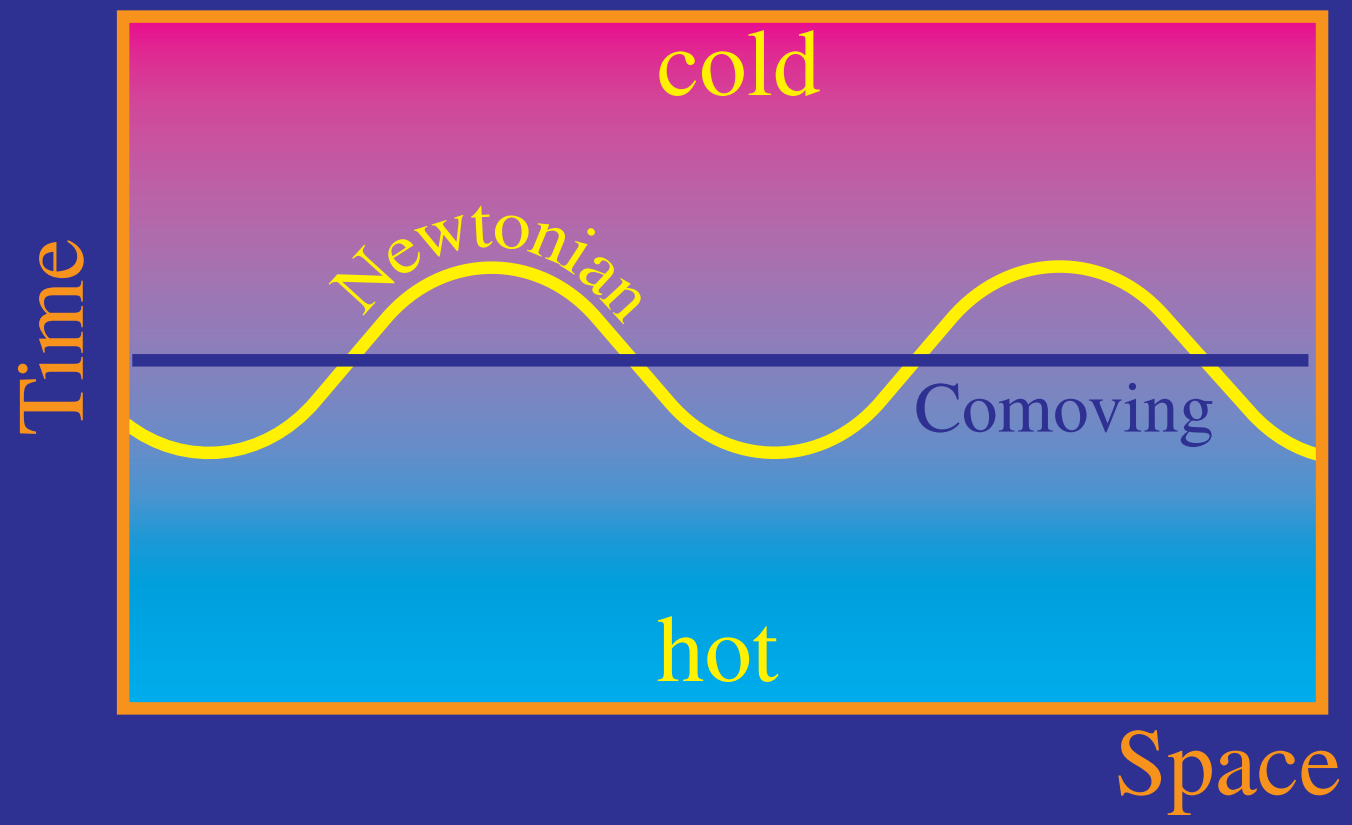
- $z < 1000$; $T_\gamma < 3000K$
  Gravitational redshifts & lensing
  Reionization; rescattering



Initial Conditions

Inflation and the Initial Conditions

- Inflation: (nearly) scale-invariant curvature (potential) perturbations
- Superluminal expansion → superhorizon scales → "initial conditions"
- Accompanying temperture perturbations due to cosmological redshift



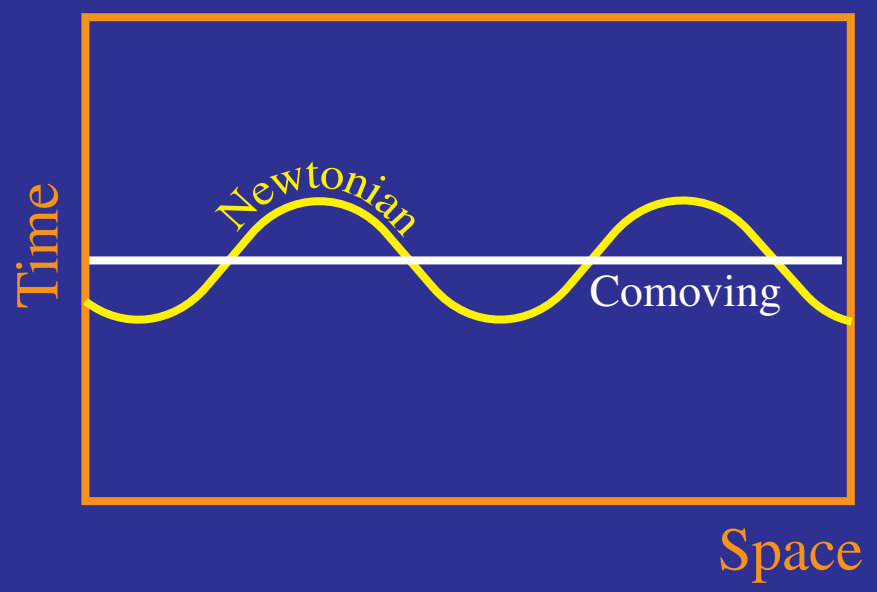
- Potential perturbation Ψ = time-time metric perturbation

$$\delta t/t = \Psi \quad \rightarrow \quad \delta T/T = -\delta a/a = -2/3\,\delta t/t = -2/3\,\Psi$$

Sachs & Wolfe (1967); White & Hu (1997)

Initial Conditions & the Sachs-Wolfe Effect

- Initial temperature perturbation
- Observed temperature perturbation
  Gravitational redshift: Ψ
  + Initial temperature: $+\Theta$
- Potential = time-time metric perturbations $\Psi = \delta t/t$



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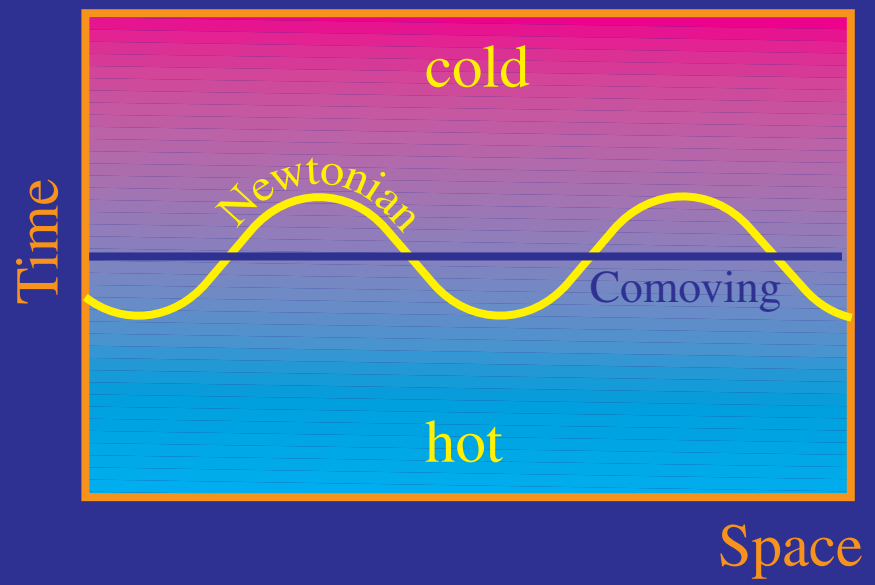

- Matter–dominated expansion: $a \propto t^{2/3}$, $\delta a/a = 2/3\ \delta t/t$
- Temperature falls as: $T \propto a^{-1}$
- Temperature fluctuation: $\delta T/T = -\delta a/a$

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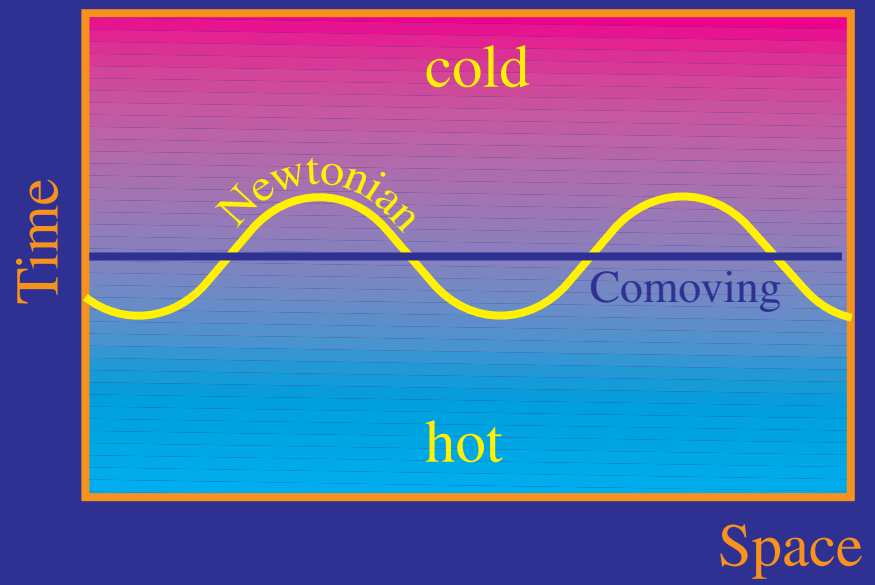

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- Temperature falls as: $T \propto a^{-1}$
- Temperature fluctuation: $\delta T/T = -\delta a/a$
- Result
  Initial temperature perturbation: $\Theta \equiv \delta T/T = -\delta a/a = -2/3\ \delta t/t$
  $= -2/3\ \Psi$

  Observed temperature perturbation: $(\delta T/T)_{obs} = \Theta + \Psi = 1/3\ \Psi$

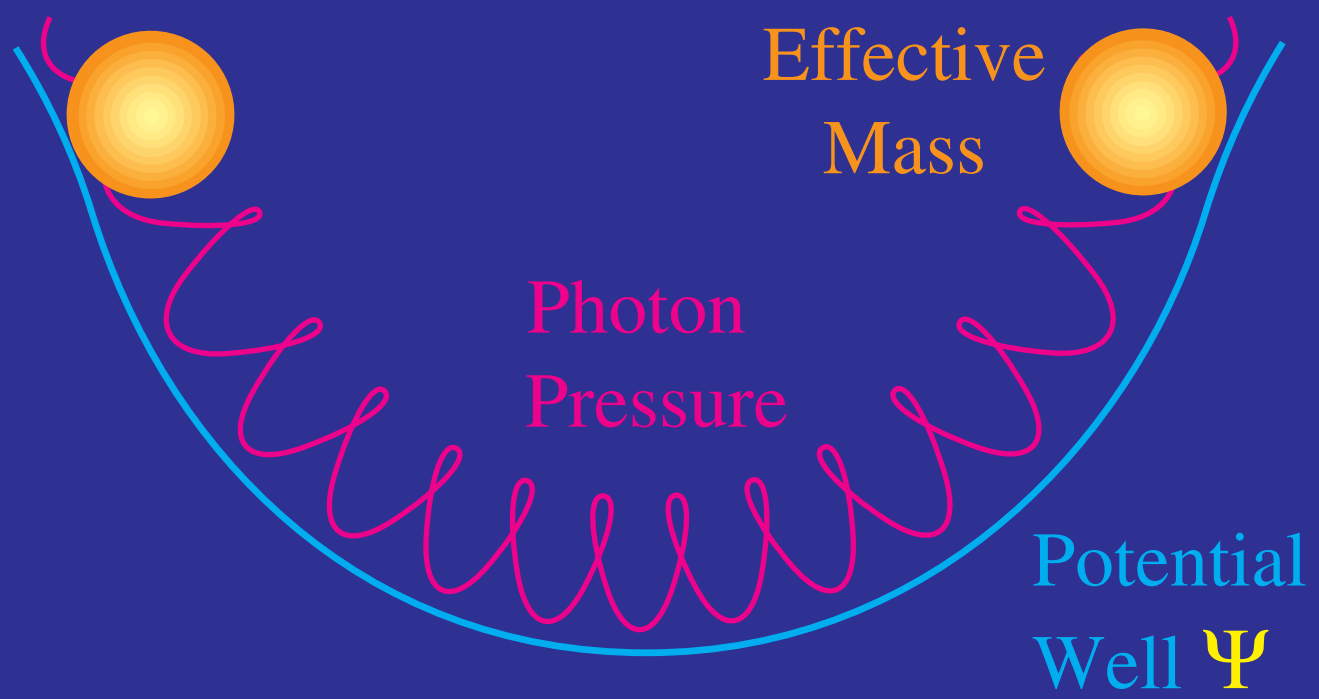
Sachs & Wolfe (1967)

White & Hu (1997)

Acoustic Oscillations

Acoustic Oscillations

- Photon pressure resists compression in potential wells
- Acoustic oscillations



Peebles & Yu (1970)

Hu & Sugyama (1995); Hu, Sugiyama & Silk (1997)

Acoustic Oscillations

- Photon pressure resists compression in potential wells
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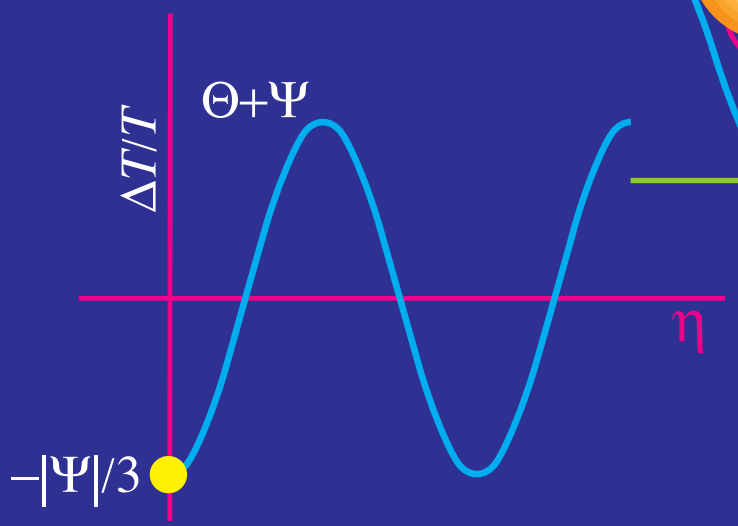
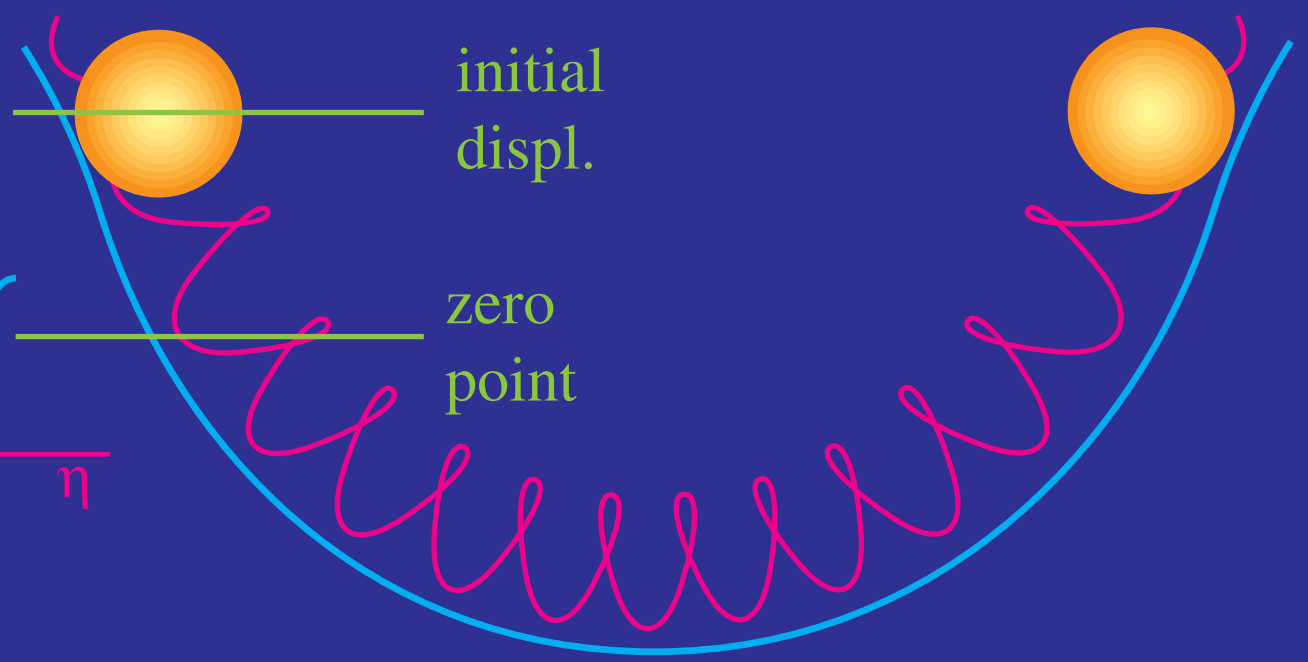
- Gravity displaces zero point
  $\Theta \equiv \delta T/T = -\Psi$

- Oscillation amplitude = initial displacement from zero pt.
  $\Theta - (-\Psi) = 1/3\Psi$





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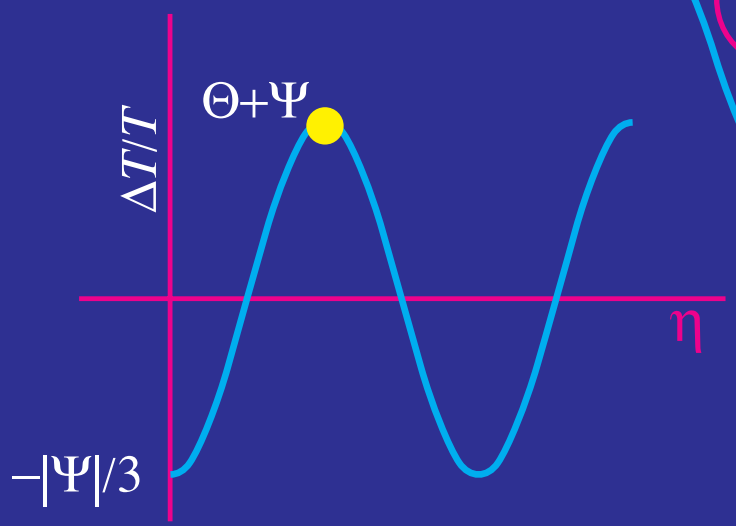

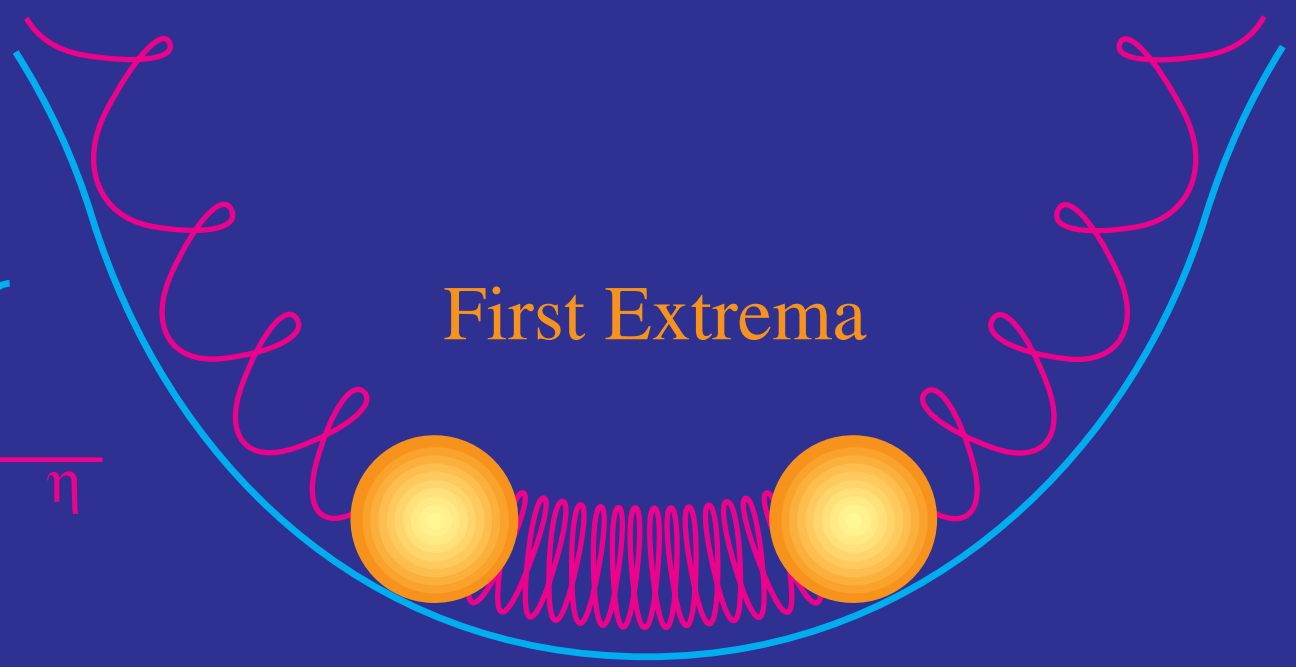

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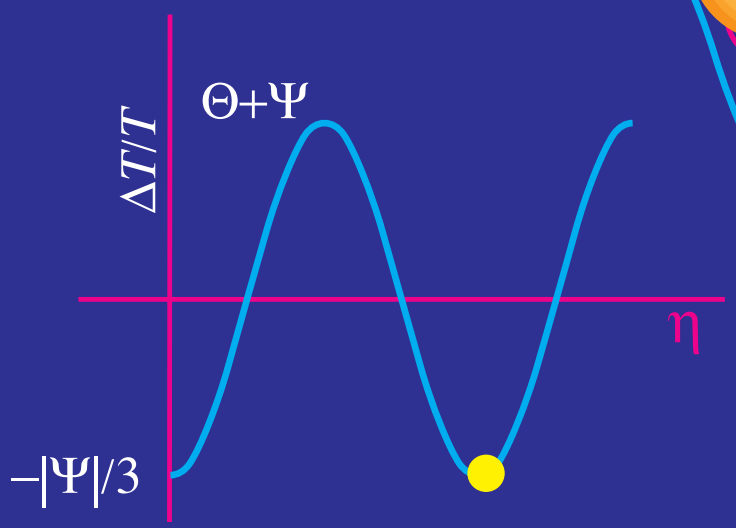
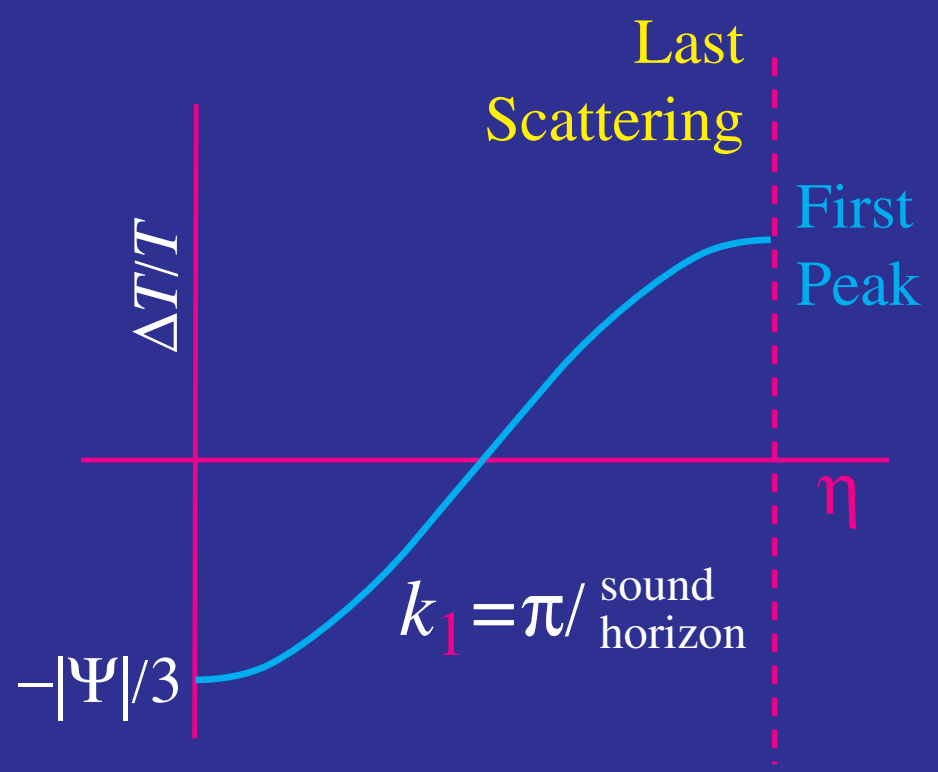


Peebles & Yu (1970)

Hu & Sugyama (1995); Hu, Sugiyama & Silk (1997)

Harmonic Peaks

- Oscillations frozen at last scattering
- Wavenumbers at extrema = peaks
- Sound speed c_s

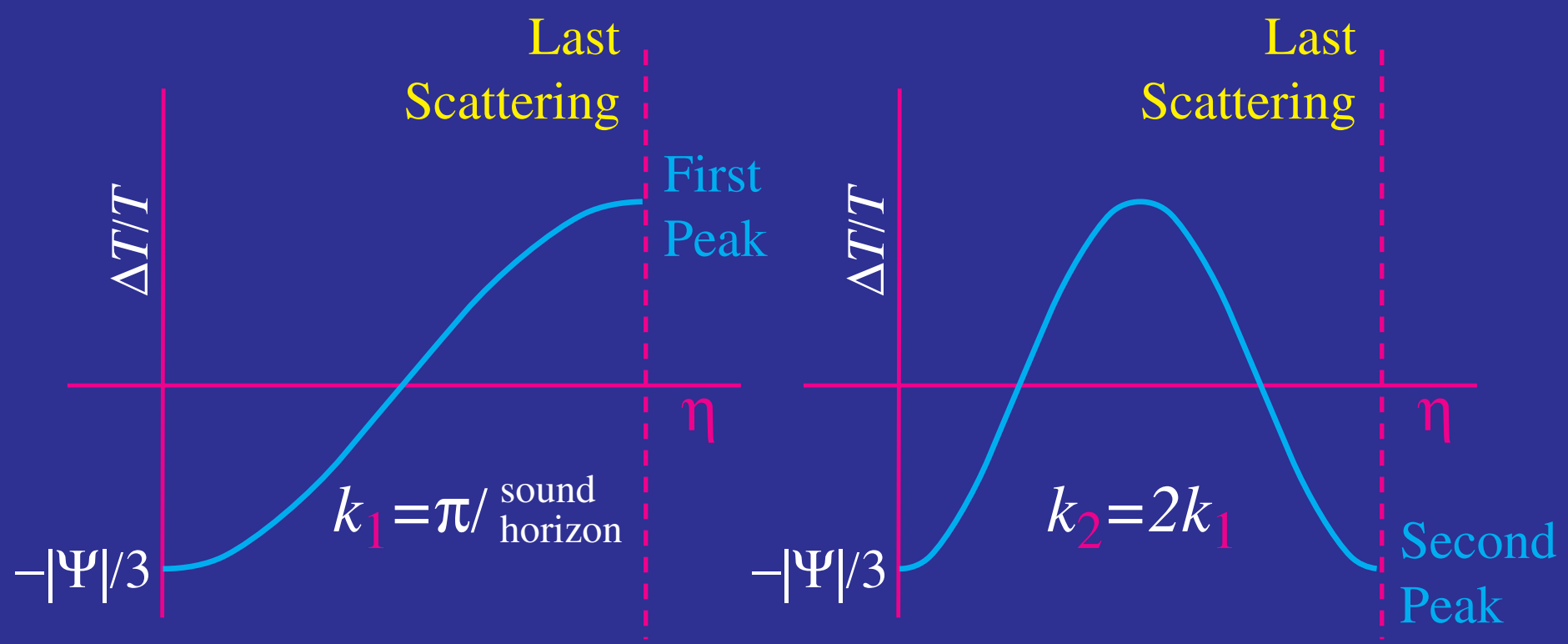


Hu & Sugyama (1995); Hu, Sugiyama & Silk (1997)

Harmonic Peaks

- Oscillations frozen at last scattering
- Wavenumbers at extrema = peaks
- Sound speed c_s

- Frequency $\omega = kc_s$; conformal time η
- Phase $\propto k$; $\phi = \int_0^{\text{last scattering}} d\eta\, \omega = k\, \text{sound horizon}$
- Harmonic series in sound horizon $\phi_n = n\pi \rightarrow k_n = n\pi/\text{sound horizon}$

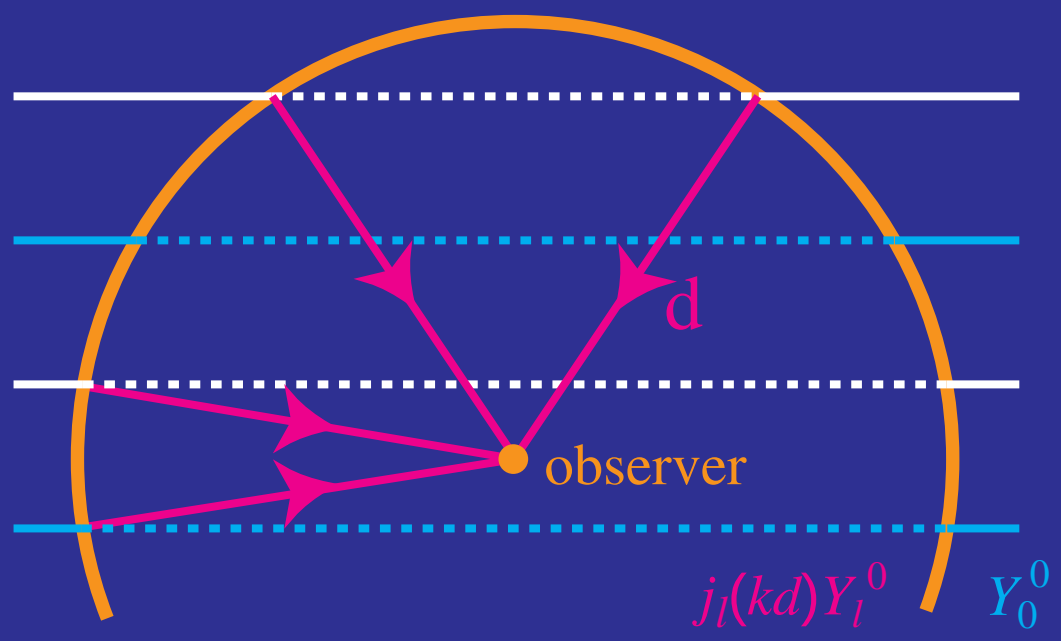


Hu & Sugyama (1995); Hu, Sugiyama & Silk (1997)

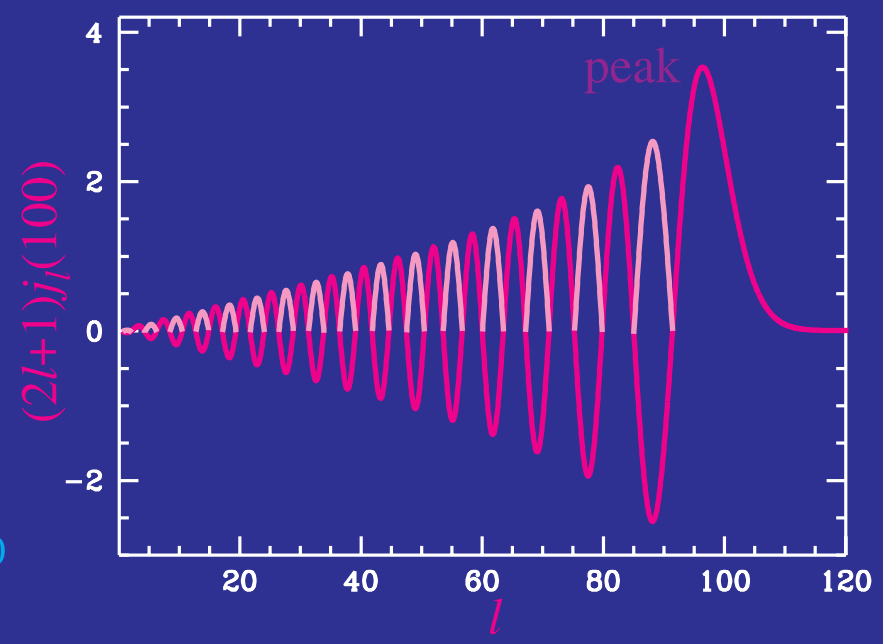
Projection into Angular Peaks

- Peaks in spatial power spectrum
- Projection on sphere
- Spherical harmonic decomposition

- Maximum power at $l = kd$
- Extended tail to $l \ll kd$
- Described by spherical bessel function $j_l(kd)$



Bond & Efstathiou (1987)

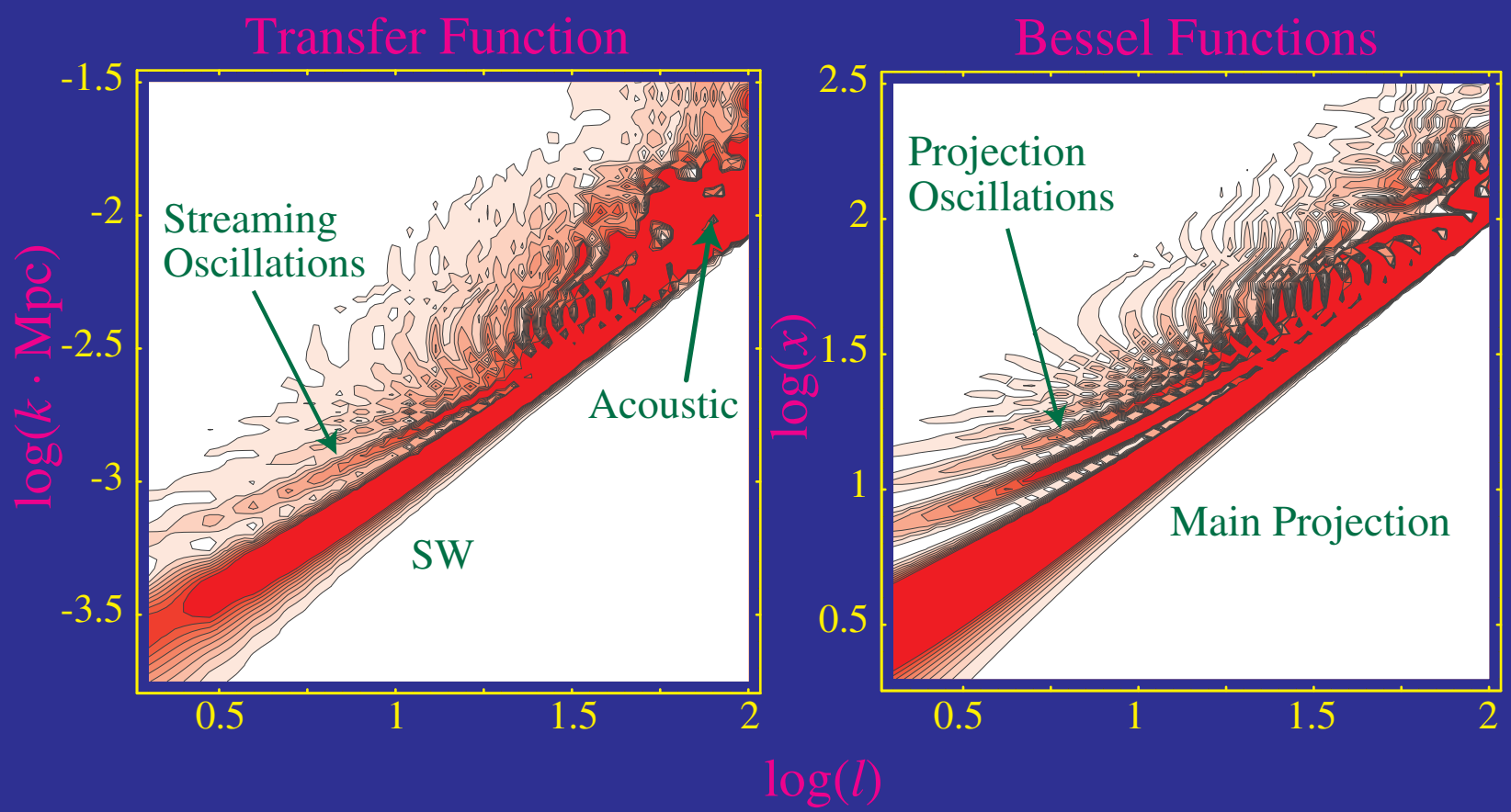


Hu & Sugiyama (1995); Hu & White (1997)

Projection into Angular Peaks

- Peaks in spatial power spectrum
- Projection on sphere
- Spherical harmonic decomposition

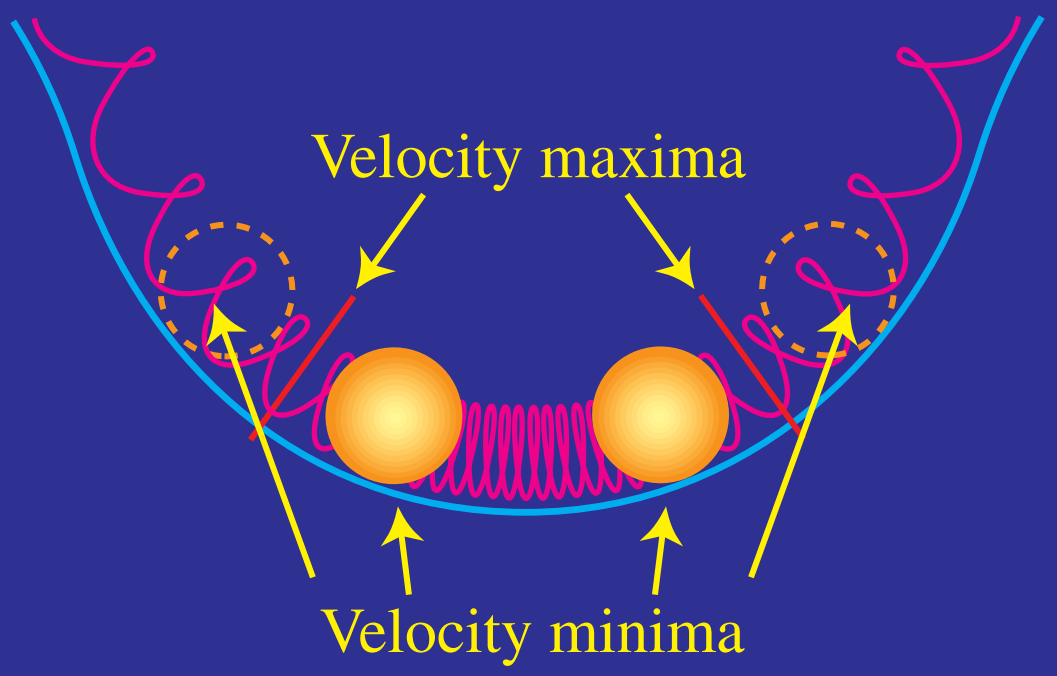
- Maximum power at $l = kd$
- Extended tail to $l \ll kd$
- 2D Transfer Function
  $T^2(k,l) \sim (2l+1)^2\,[\Delta T/T]^2\, j_l^2(kd)$



Hu & Sugiyama (1995)

Doppler Effect

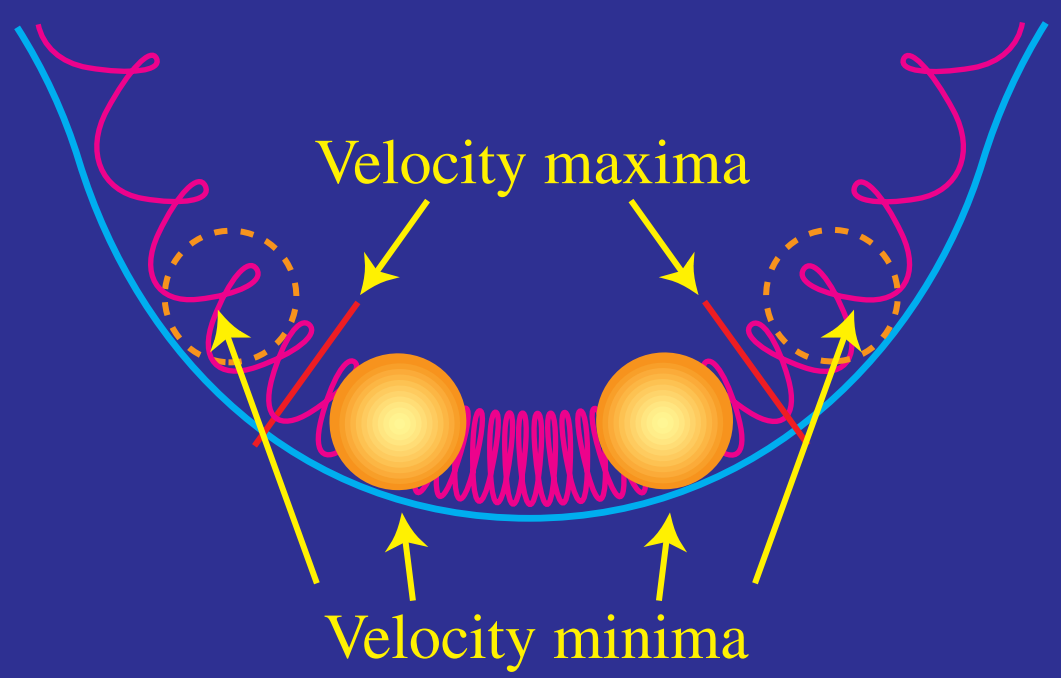
- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature

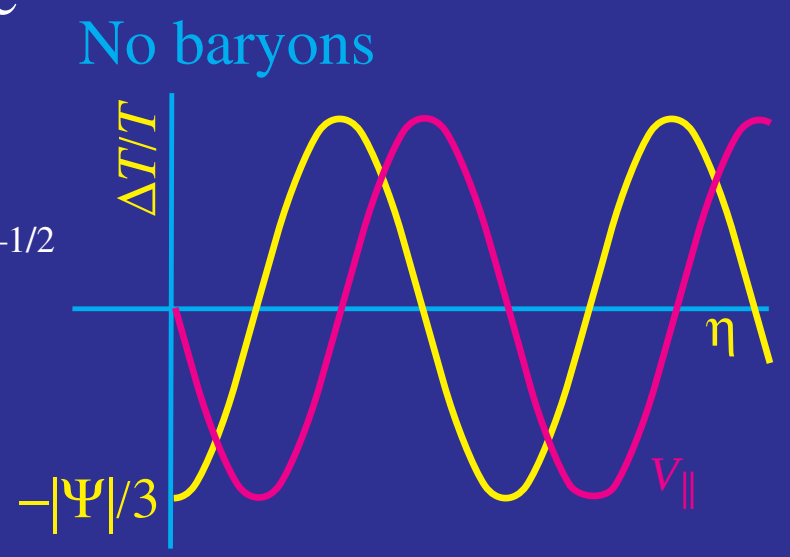


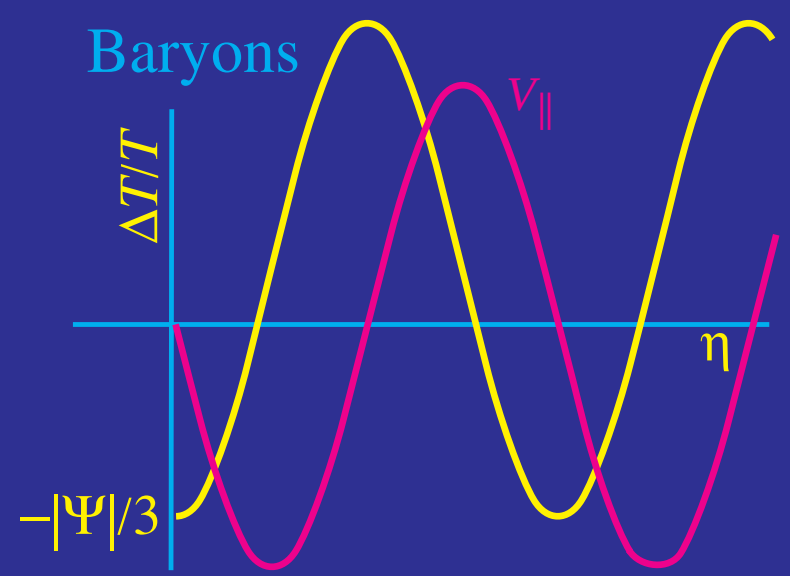
Doppler Effect

- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature
- Zero point not shifted by baryon drag
- Increased baryon inertia decreases effect

$$m_{\rm eff} V^2 = \text{const.} \qquad V \propto m_{\rm eff}^{-1/2} = (1+R)^{-1/2}$$

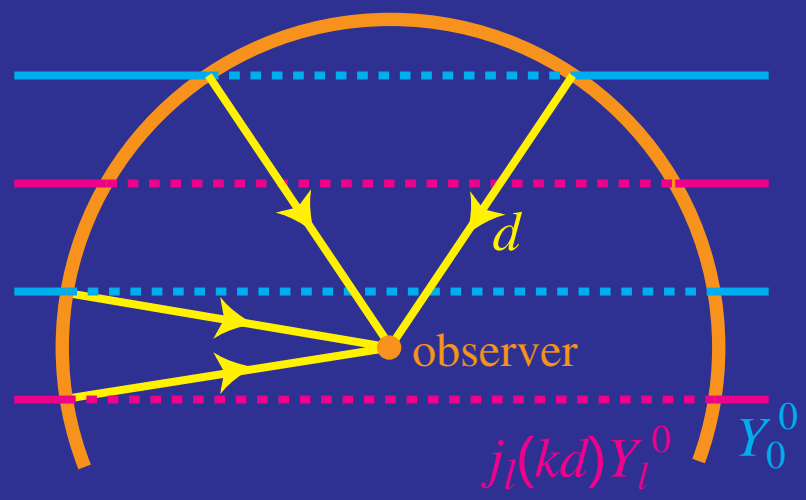


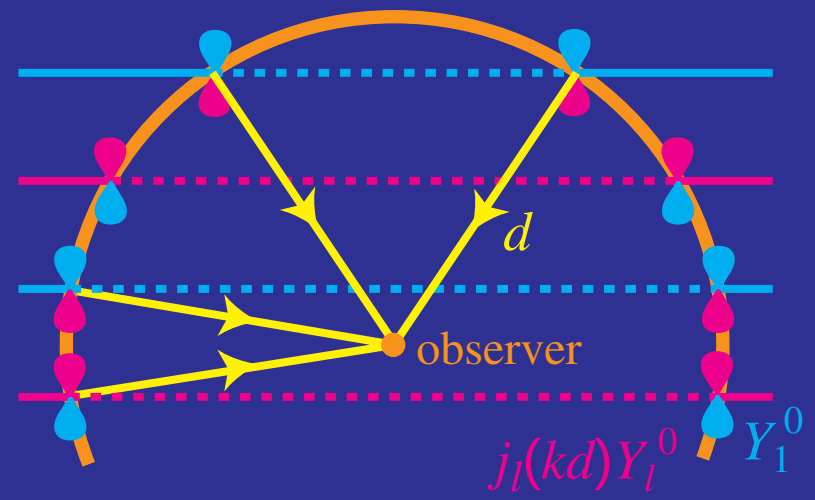


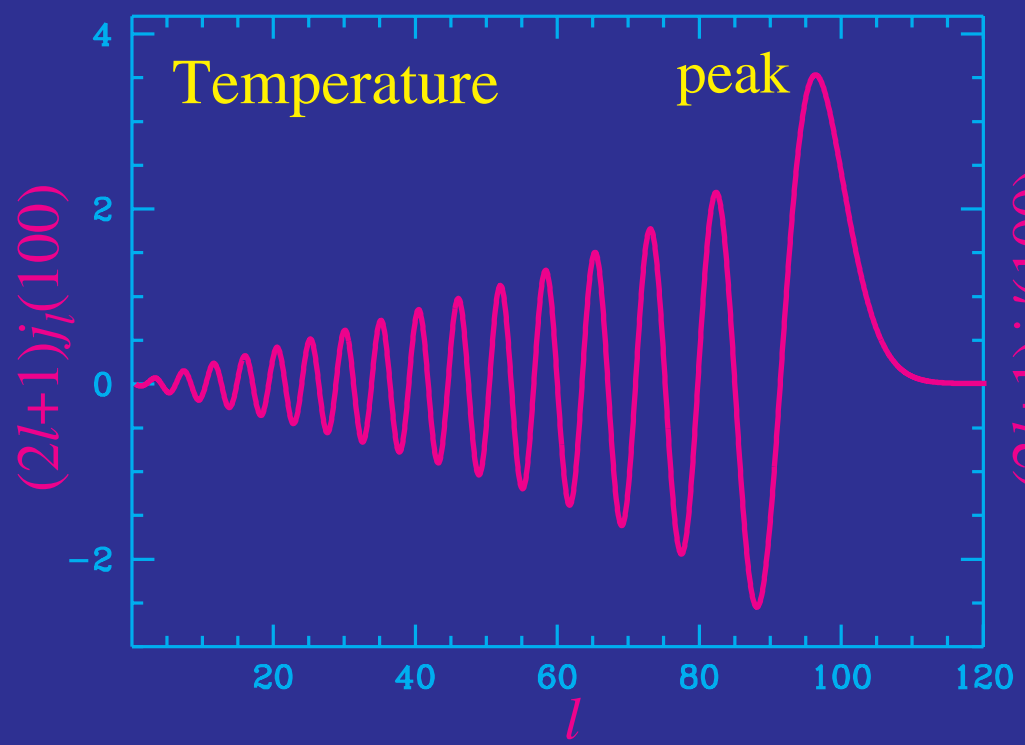


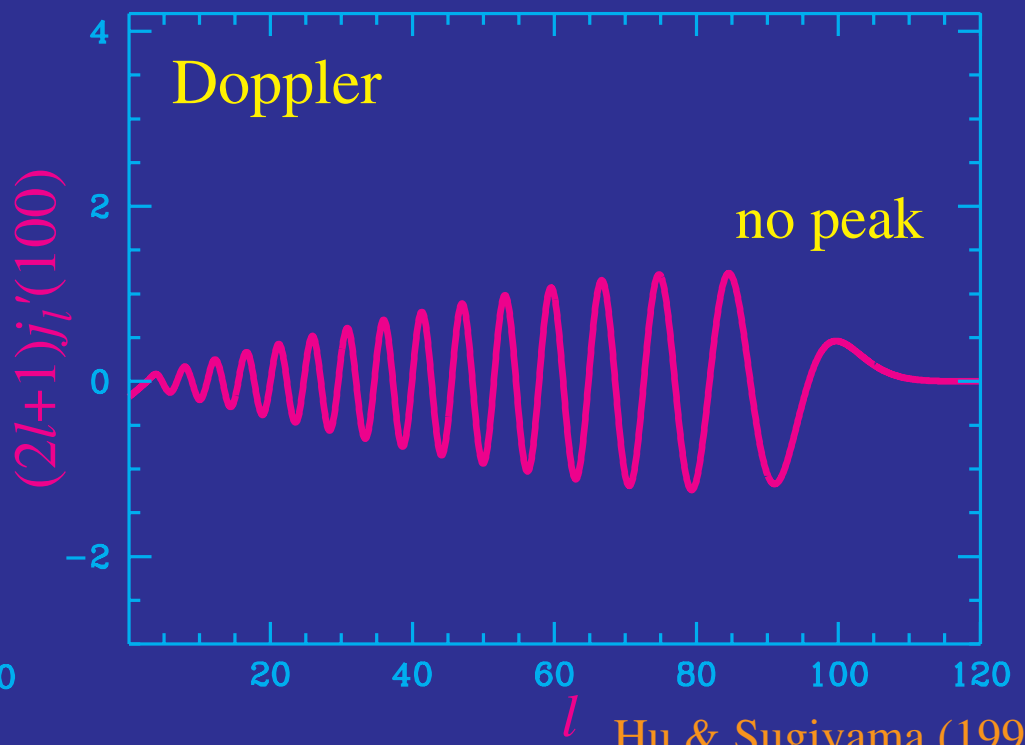
Doppler Peaks?

- Doppler effect has lower amplitude and weak features from projection



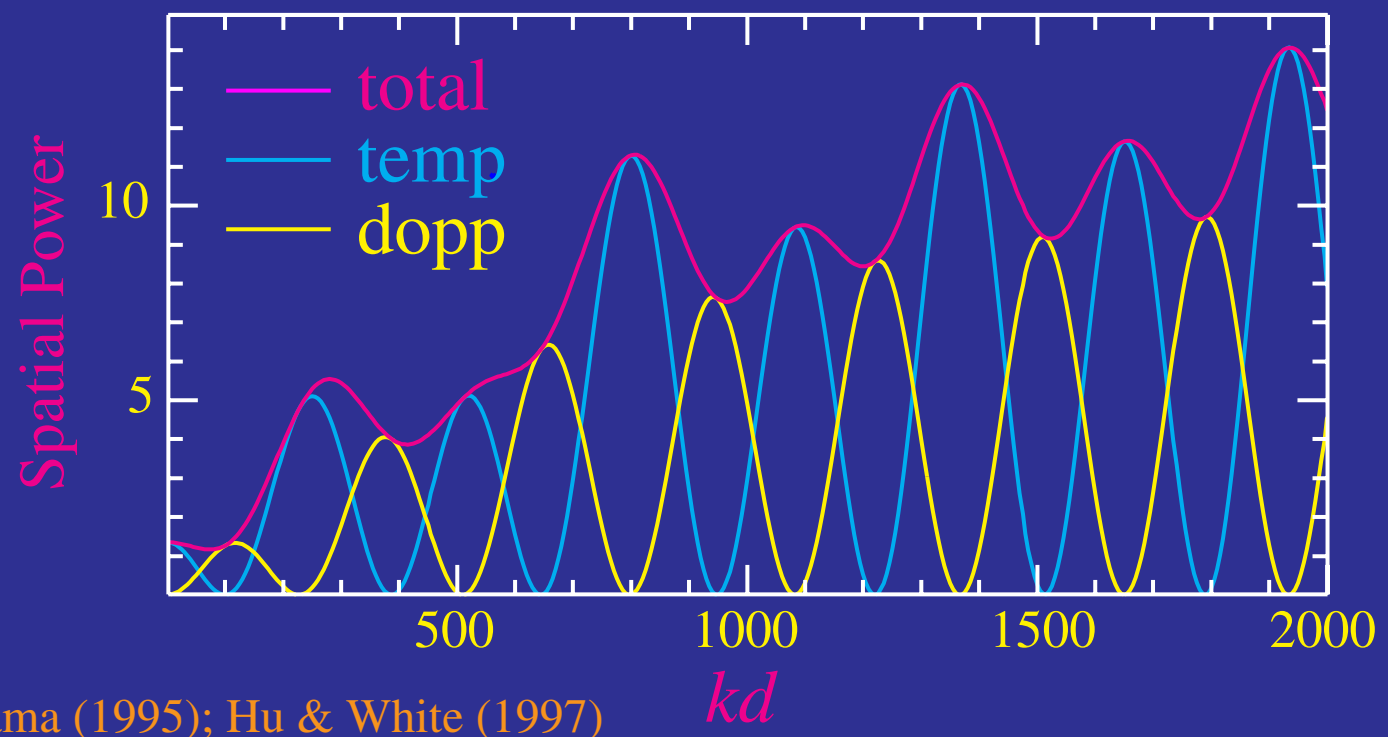






Hu & Sugiyama (1995)

Relative Contributions



Hu & Sugiyama (1995); Hu & White (1997)

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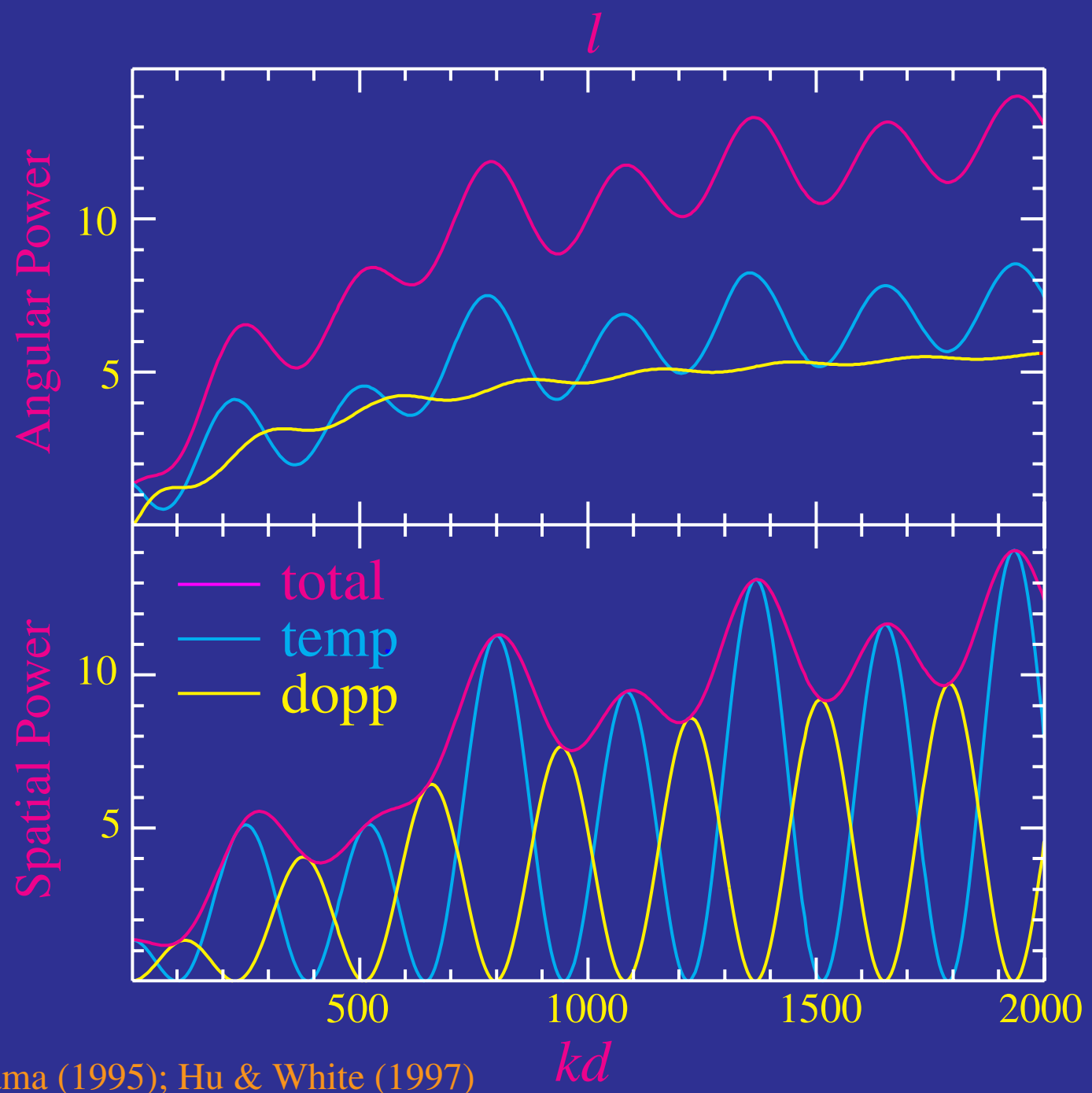


Hu & Sugiyama (1995); Hu & White (1997)

Mechanics of the Calculation

- Radiation distribution: $f(\mathbf{x}=\text{position}, t=\text{time}; \mathbf{n}=\text{direction}, \nu=\text{frequency})$
- Expand in basis functions: (local angular dependence ⊗ spatial dependence)

$$ {}_sY_l^m \otimes e^{i\mathbf{k}\cdot\mathbf{x}} $$

Sources

(radiation & metric: monopole, dipole, quadrupole)

hierarchy

plane wave ~ gradient

$$ {}_sY_{l'}^m \otimes Y_1^0 $$

$$ |l' \pm 1| $$

Addition of Angular Momentum

CMBFast

integral

plane wave $\propto j_l Y_l^0$

$$ {}_sY_{l'}^m \otimes Y_{l''}^0 $$

$$ |l \pm l'| $$

$$ \sum_{|l'-l''|}^{l'+l''} \text{(clebsch-gordan)}\, j_l $$

original codes

semi-analytic

Observables

Bond & Efstathiou (1984)
Vittorio & Silk (1983)

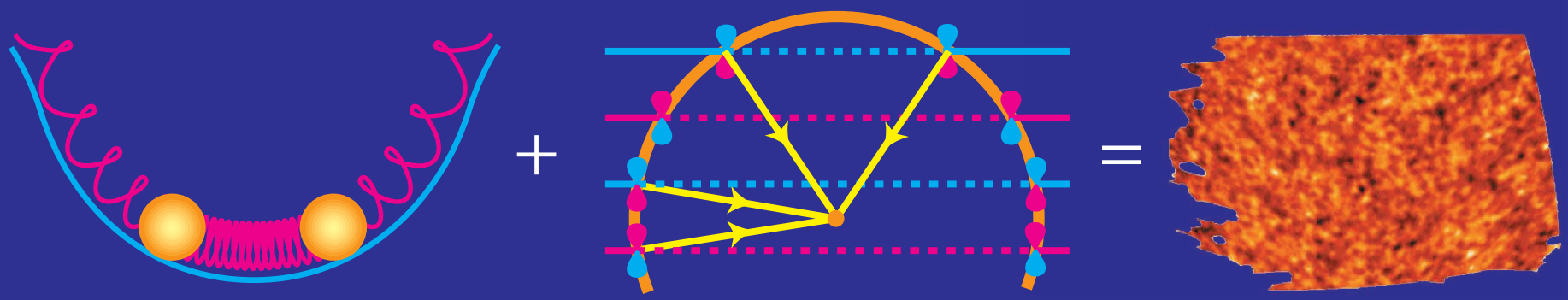
Seljak & Zaldarriaga (1996)
Hu & White (1997)

Hu & Sugiyama (1995)
Seljak (1994)

Semi-Analytic Calculation

- Treat Sources in the Tight Coupling Approximation
  - Expand the Boltzmann hierarchy equations in 1/(optical depth per λ)
  - Also the trick to numerically integrating the stiff hierarchy equations
  - Closed Euler equation + Continuity equation = Oscillator equation

- Project Sources at Last Scattering
  - Integral equations
  - Visibility function of recombination

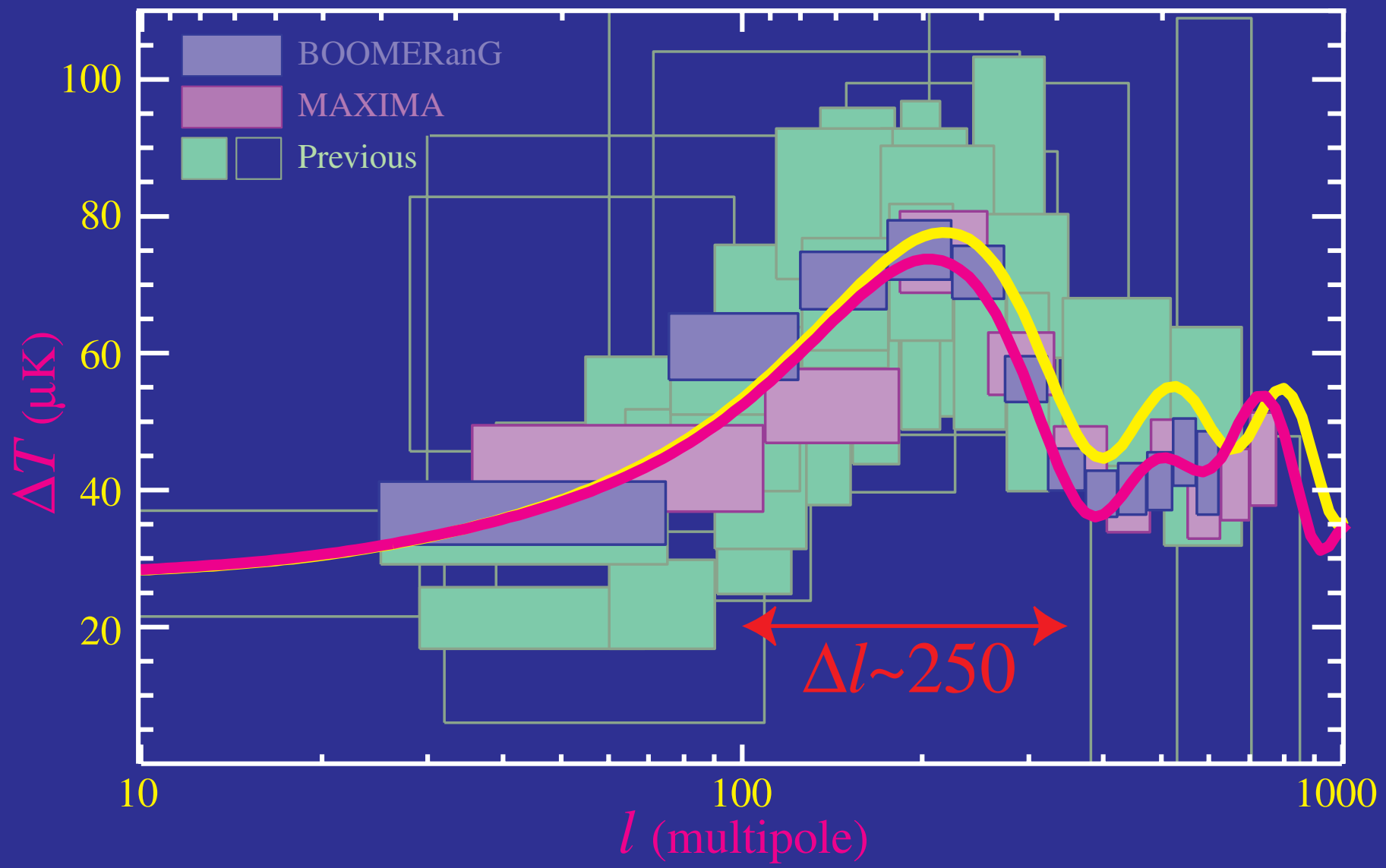


The First Peak:

Curvature
Age
Dark Energy

Shape of the First Peak

- Consistent with potential wells in place on superhorizon scale (inflation)
- Sharp fall from first peak indicates no continuous generation (defects)



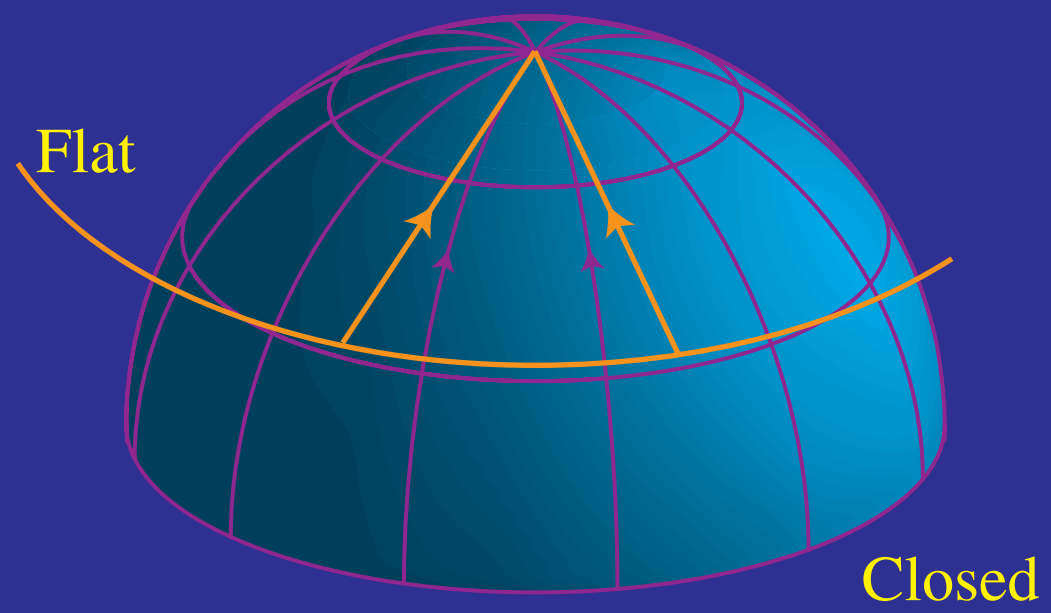
Angular Diameter Distance

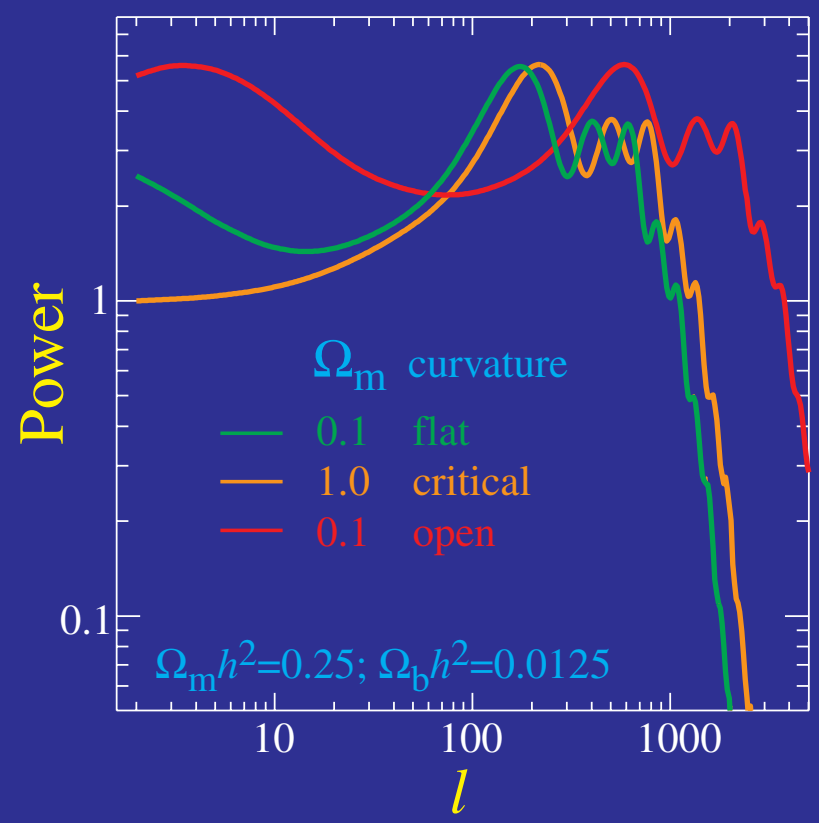
- A Classical Test
  - Standard(ized) comoving ruler
  - Measure angular extent
  - Absolute scale drops out

  Infer curvature

- Upper limit 1st Peak Scale (Horizon)
  Upper limit on Curvature
- Calibrate 2 Physical Scales
  - Sound horizon (peak spacing) $\Omega_m h^2$
  - Diffusion scale (damping tail) $\Omega_b h^2$

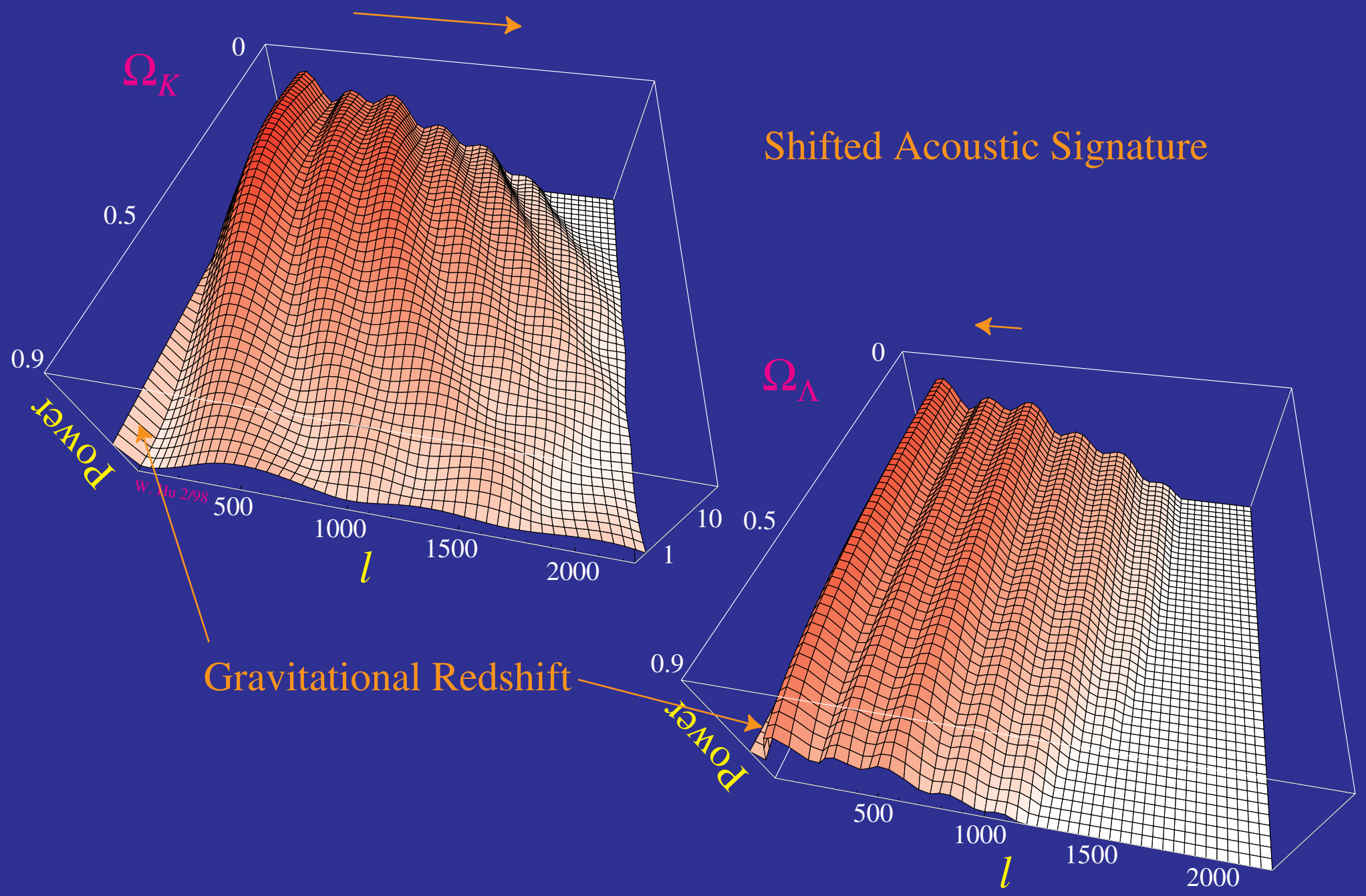
  IC's



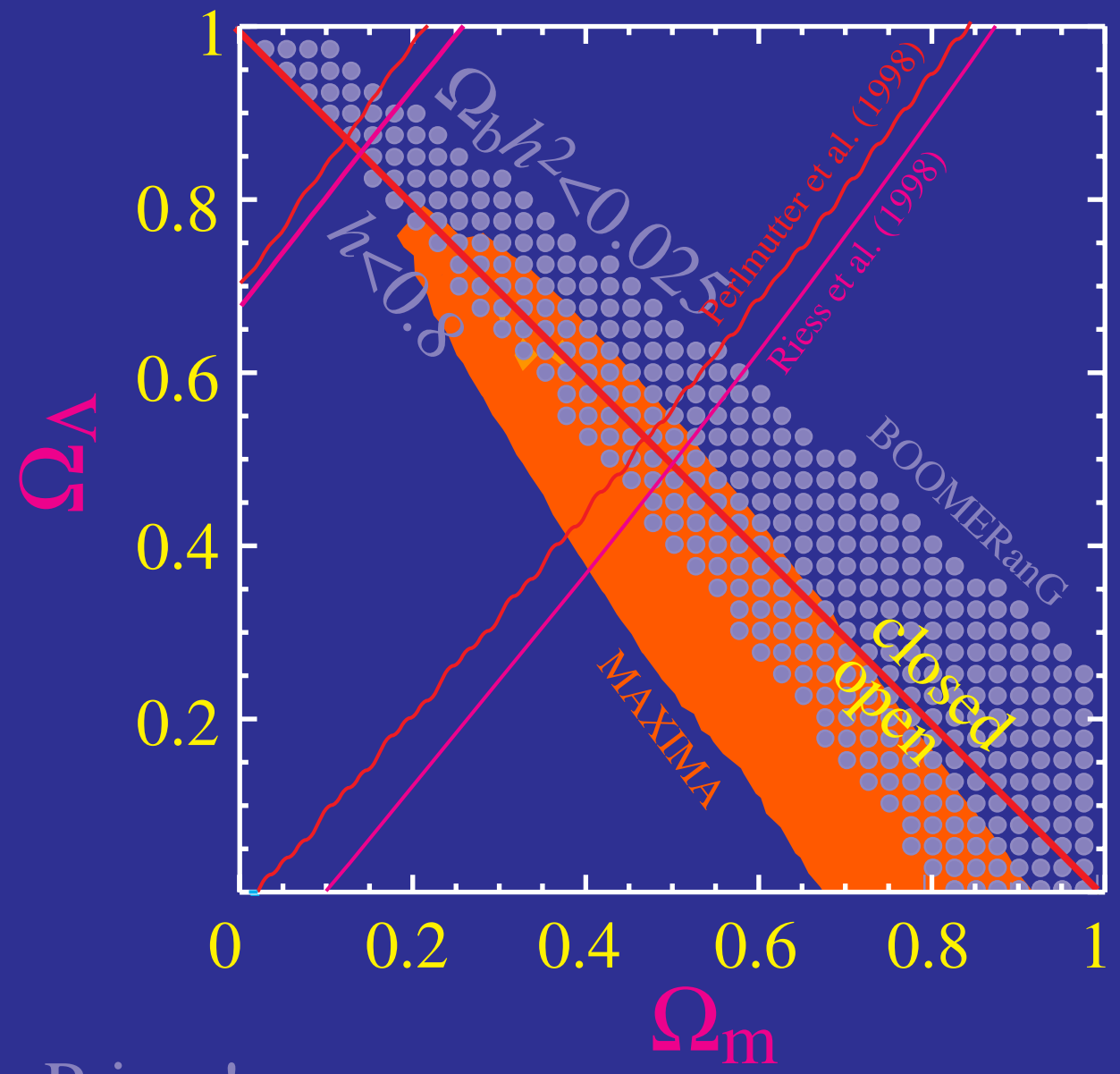


Kamionkowski, Spergel & Sugiyama (1994)
Hu & White (1996)

Curvature and the Cosmological Constant



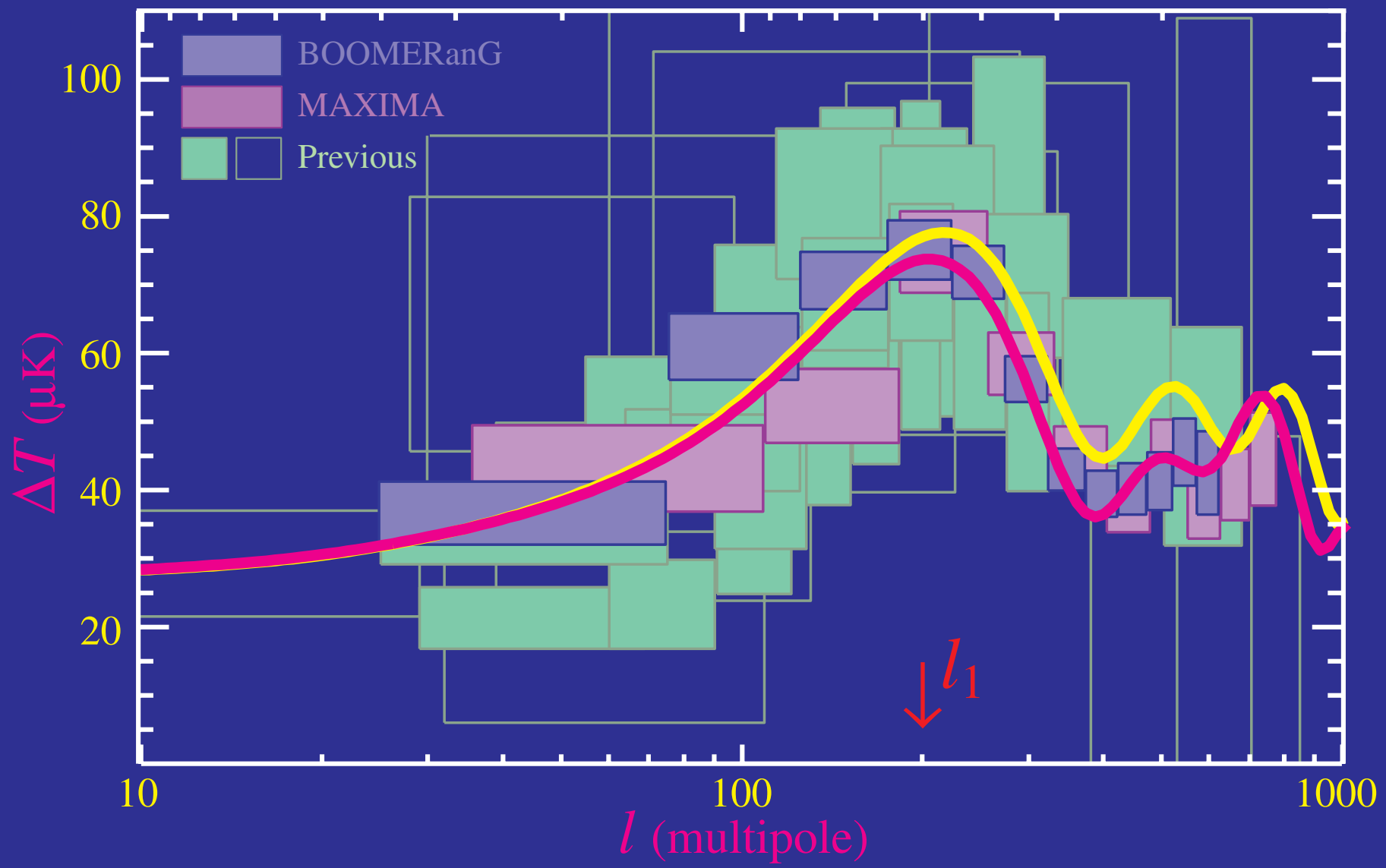
A Flat Universe!...?



Priors!

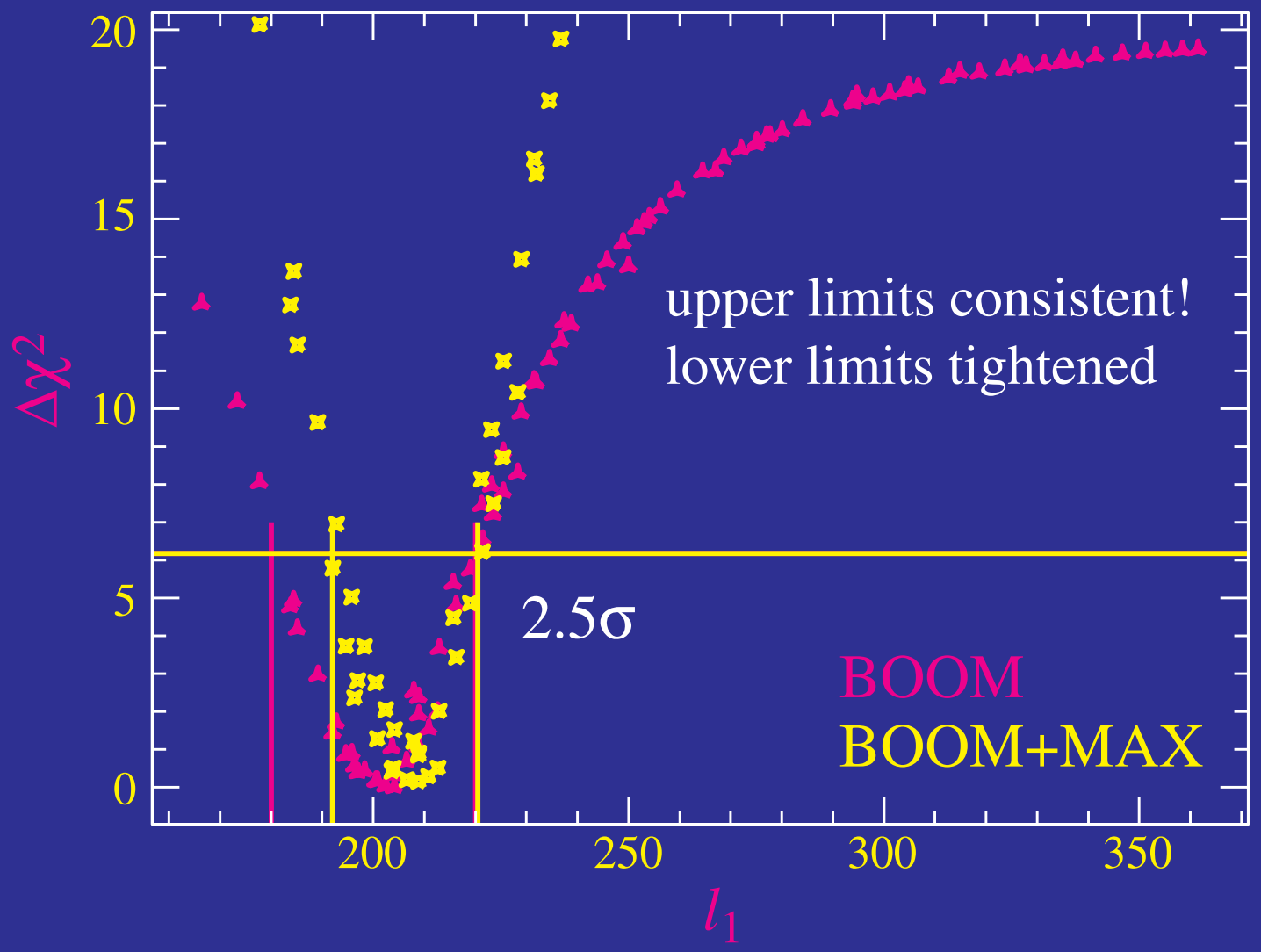
First Peak Location

- BOOM's parabolic peak fit $185<l_1<209$ (2σ)
- MAX's value l_1~220



Are They Consistent?

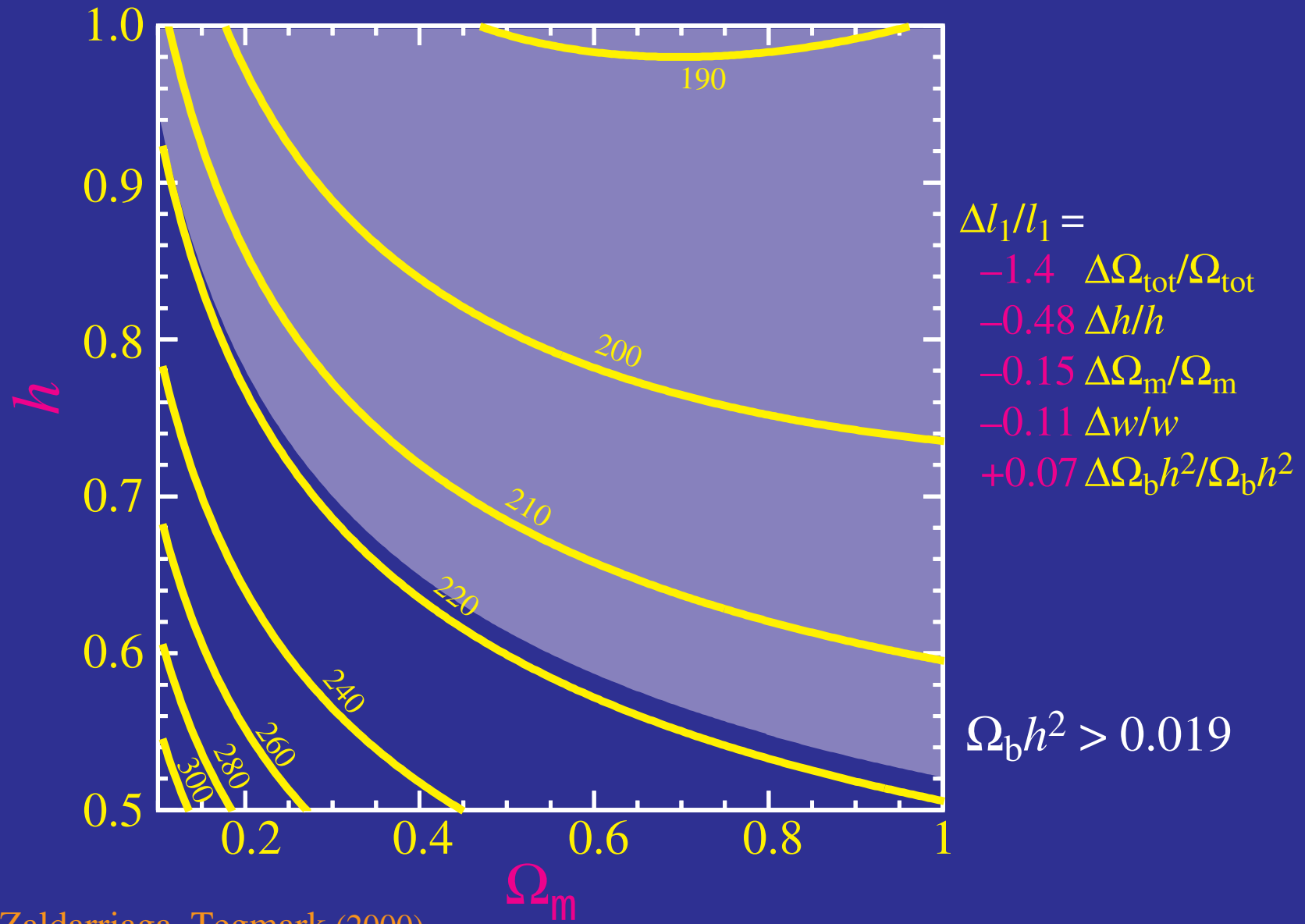
- Using ΛCDM models: $184<l_1<216$ (2σ; BOOM)
- Joint analysis: $194<l_1<218$ (2σ; BOOM+MAX)



Hu, Fukugita, Zaldarriaga, Tegmark (2000)

Is the Scale too Large?

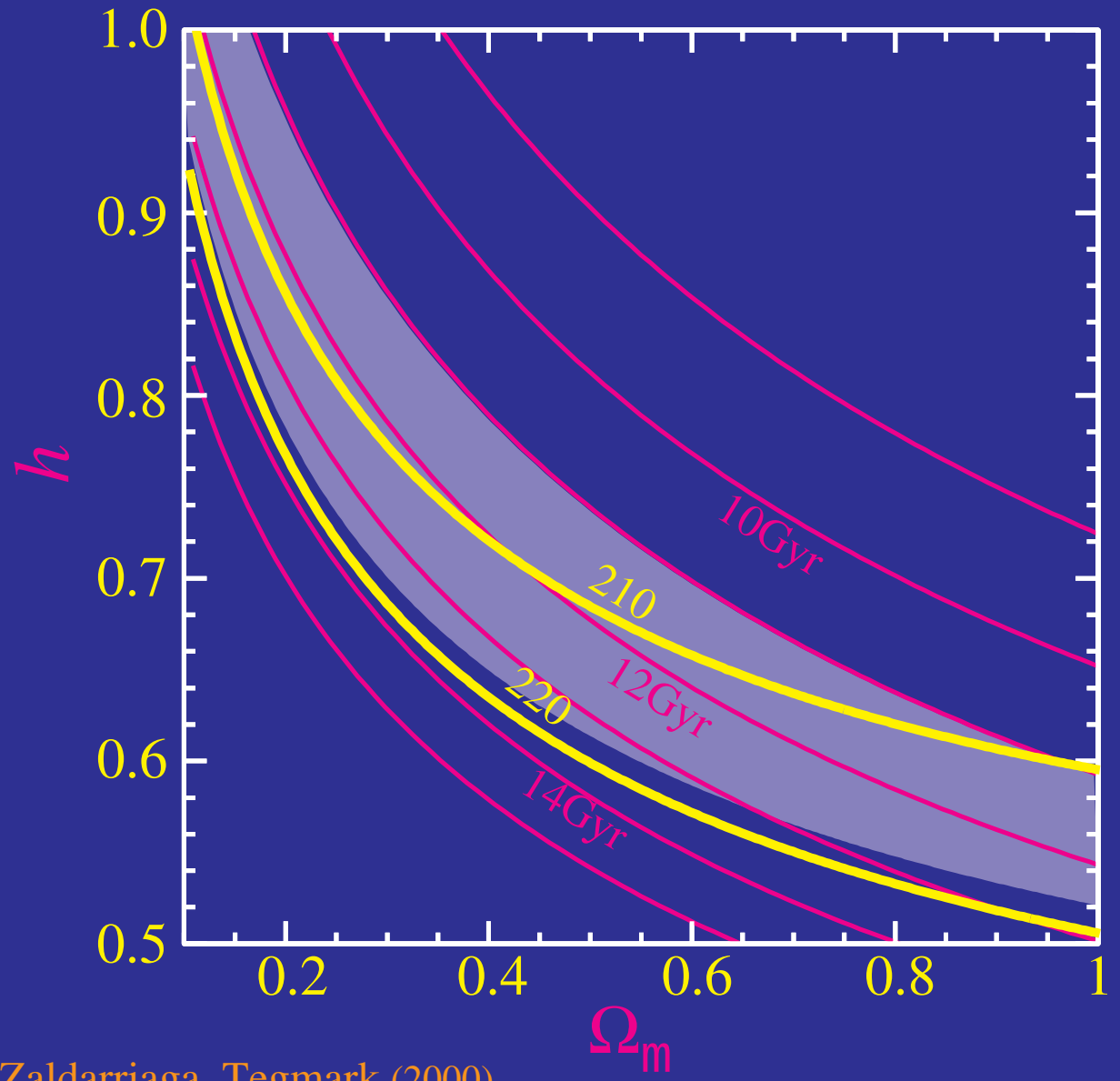
- Fiducial flat Ω_m=0.35, h=0.65 model: l_1=221 (excluded at ~2.5σ)
  (a) positive curvature (b) high Hubble constant / matter (c) dark energy



Hu, Fukugita, Zaldarriaga, Tegmark (2000)

Age and Dark Energy

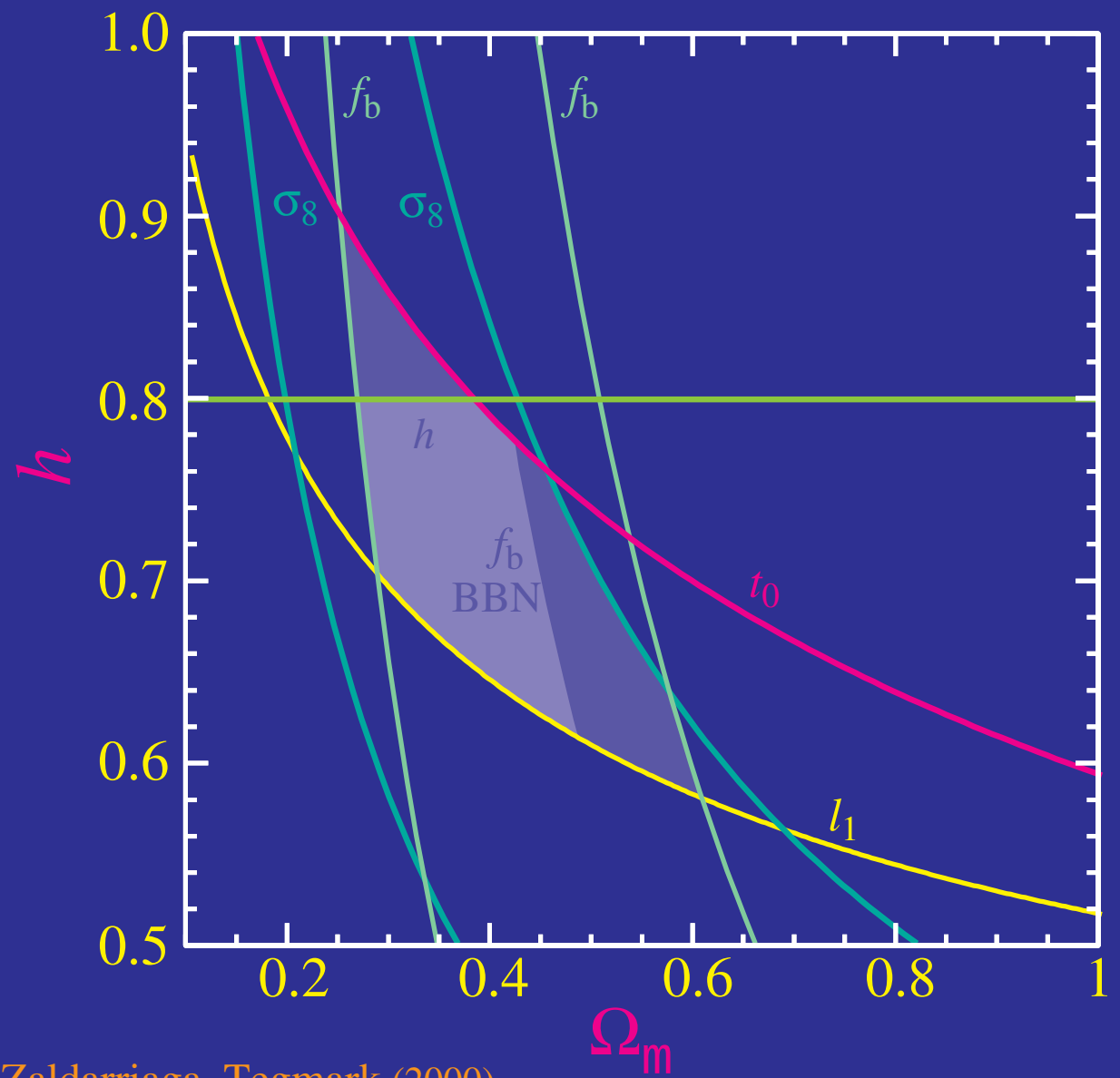
- Flat solutions involve decreasing age of universe through h, Ω_m or dark energy equation of state $w=p/\rho$ (<13–13.5Gyr)



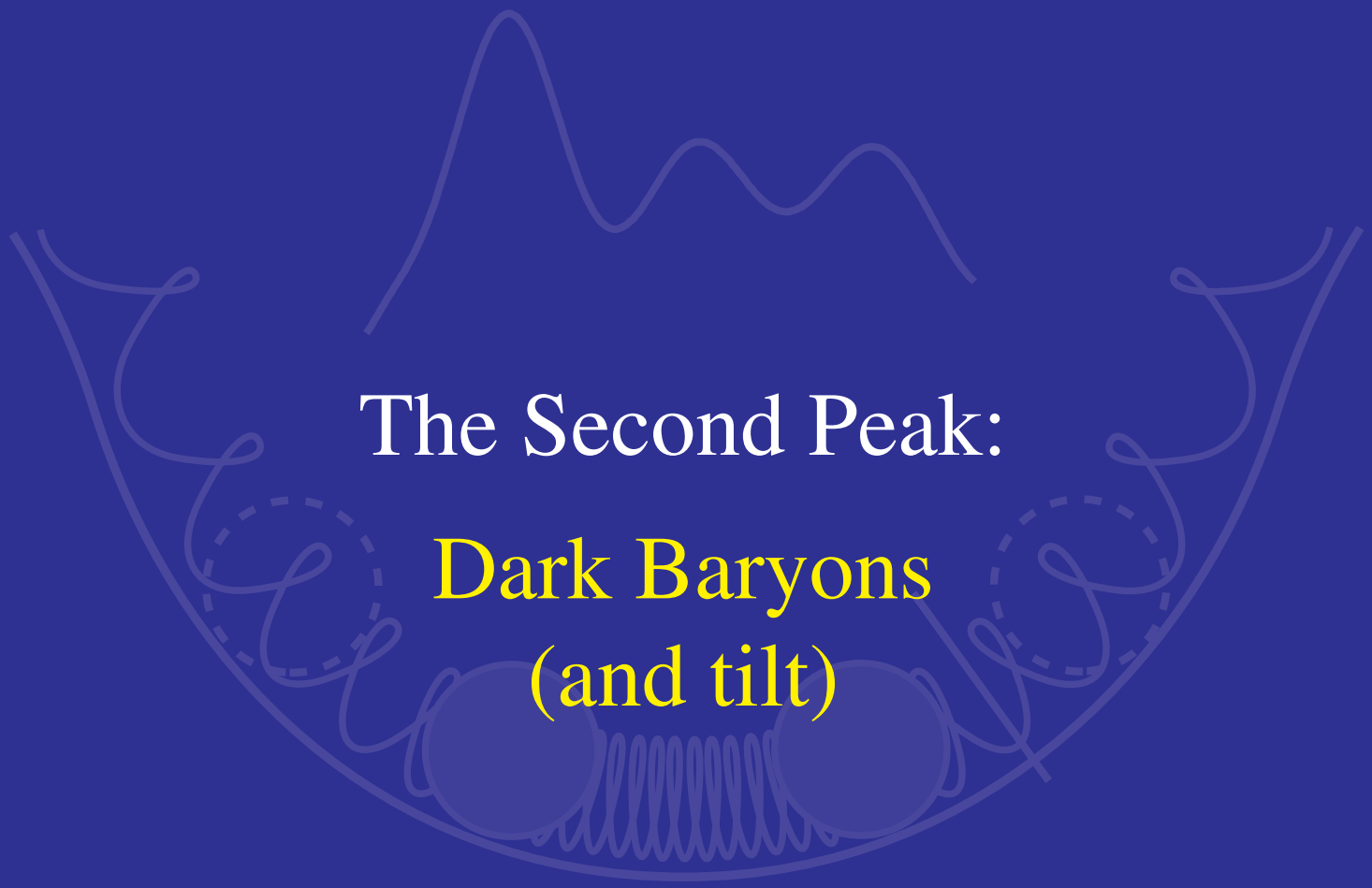
Hu, Fukugita, Zaldarriaga, Tegmark (2000)

Age and Dark Energy

- Region of consistency shrinking – headed to crisis?
- New physics? w, m_ν, α...



Hu, Fukugita, Zaldarriaga, Tegmark (2000)



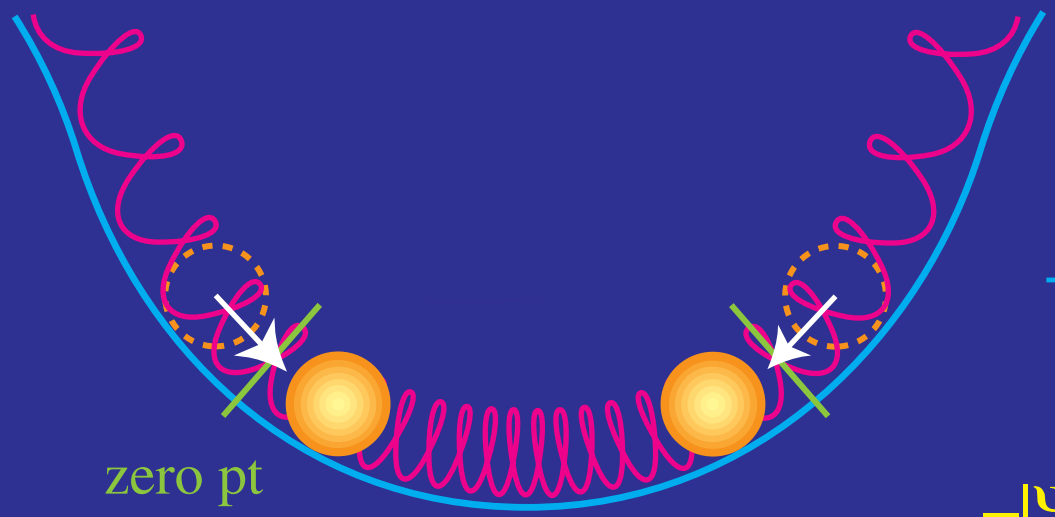
The Second Peak:

Dark Baryons
(and tilt)

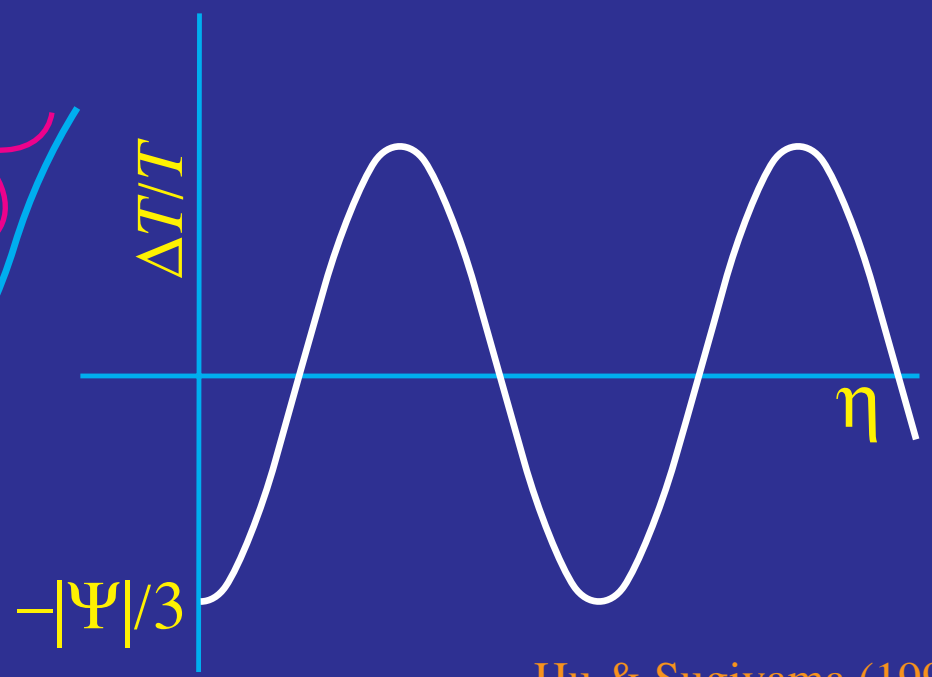
Baryon Drag

- Baryons provide inertia
- Relative momentum density

  $R = (\rho_b + p_b)V_b / (\rho_\gamma + p_\gamma)V_\gamma \propto \Omega_b h^2$
- Effective mass $m_{eff} = (1 + R)$



Low Baryon Content



Hu & Sugiyama (1995)

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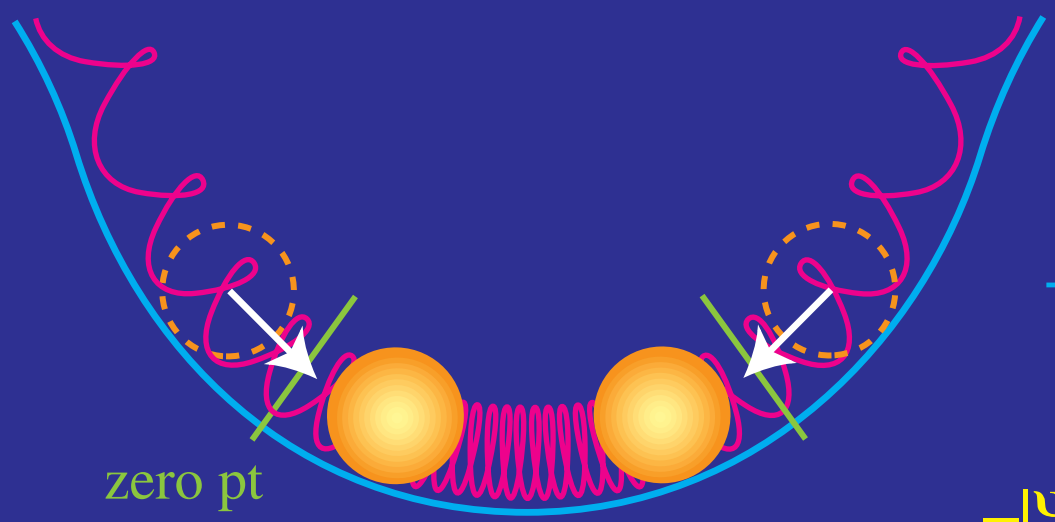
- Effective mass $m_{eff} = (1 + R)$

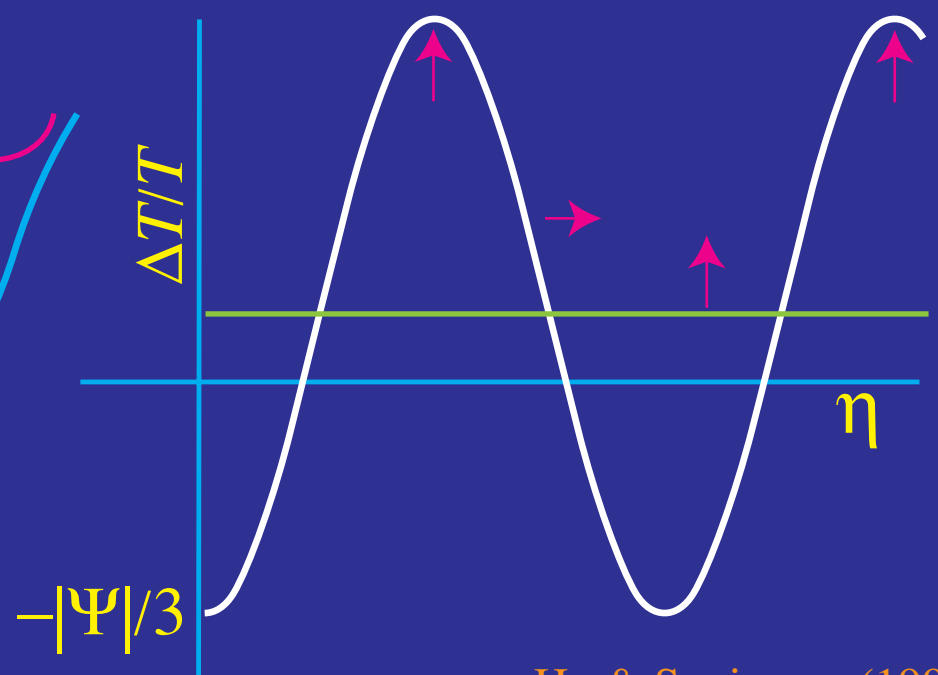
- Baryons drag photons into potential wells → zero point ↑
- Amplitude ↑
- Frequency ↓ $(\omega \propto m_{eff}^{-1/2})$

- Constant R, Ψ: $(1+R)\ddot{\Theta} + (k^2/3)\Theta = -(1+R)(k^2/3)\Psi$

$$\Theta + \Psi = [\Theta(0) + (1+R)\Psi(0)] \cos[k\eta/\sqrt{3}(1+R)] - R\Psi$$



High Baryon Content



Hu & Sugiyama (1995)

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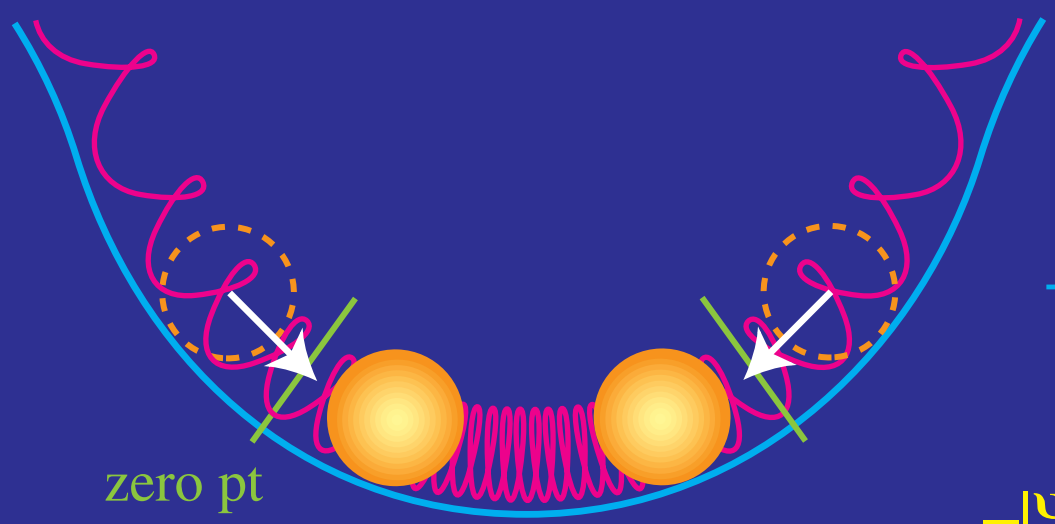
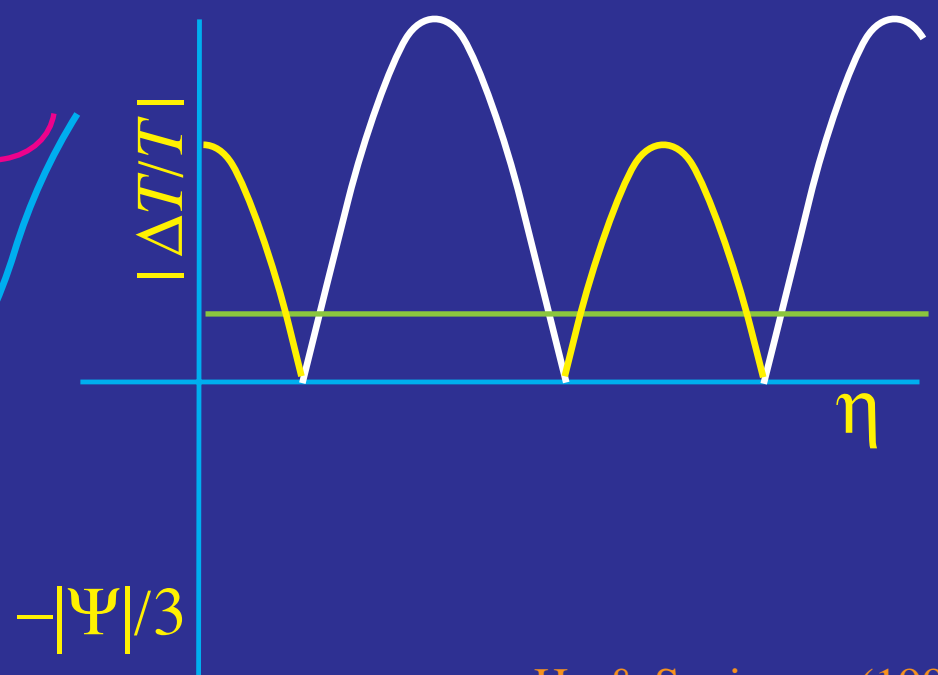
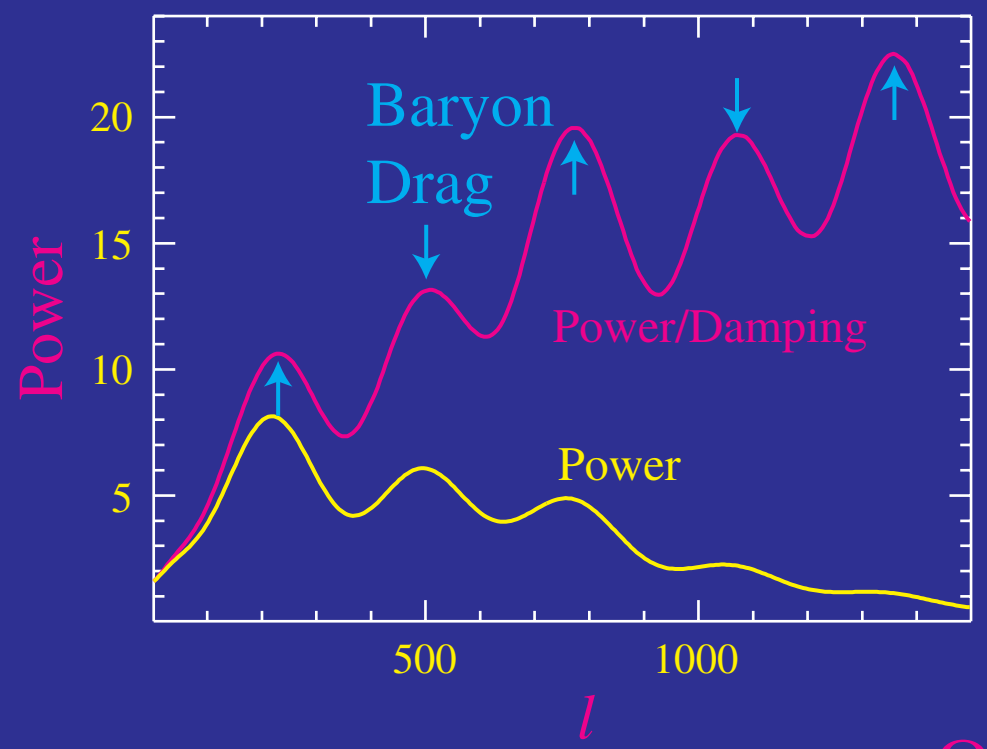


Alternating Peak Heights



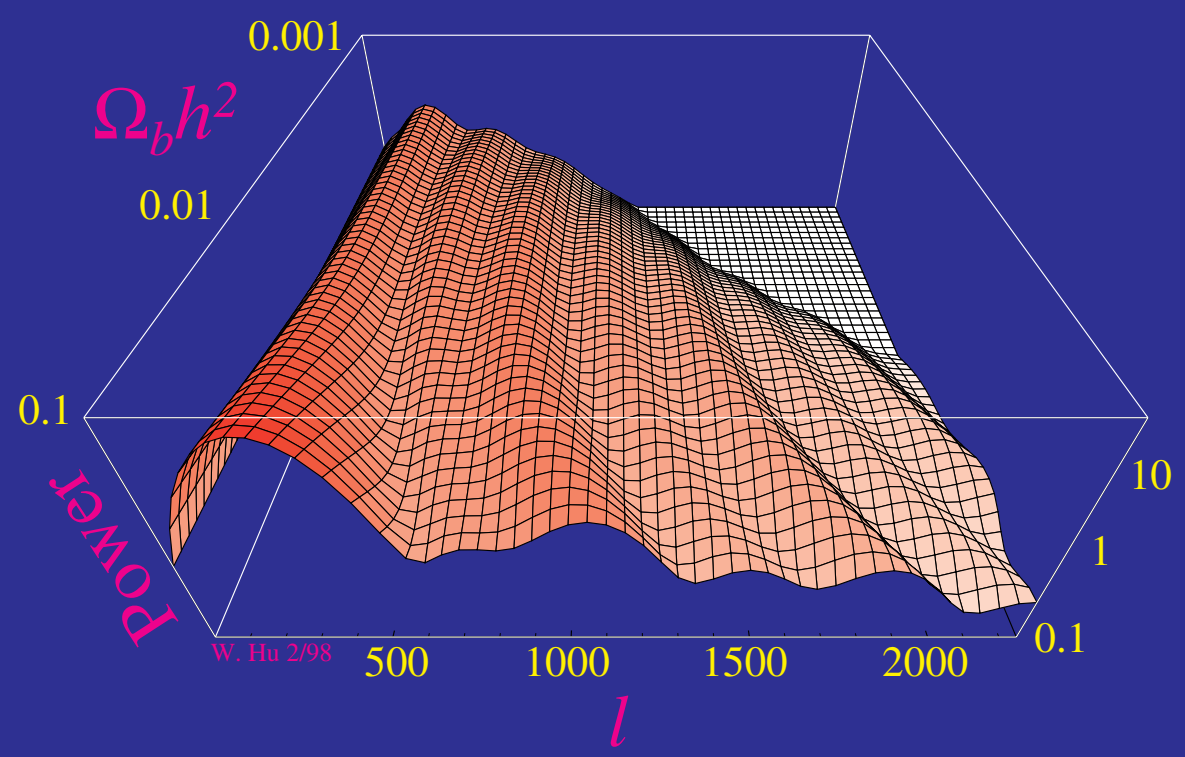
Hu & Sugiyama (1995)

Baryons in the CMB



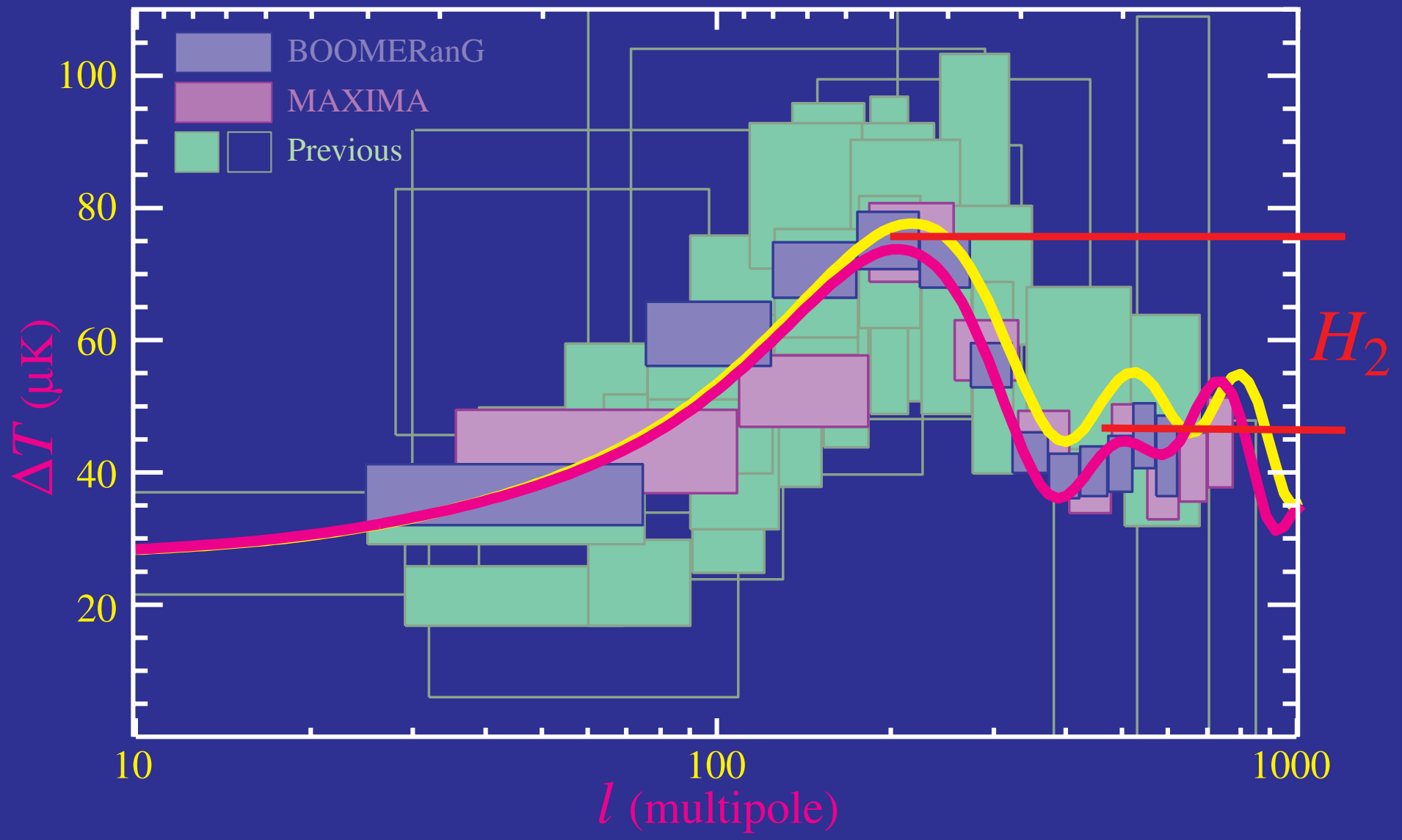
- High odd peaks

- Additional Effects
  - Time–varying potential
  - Dissipation/Fluid imperfections



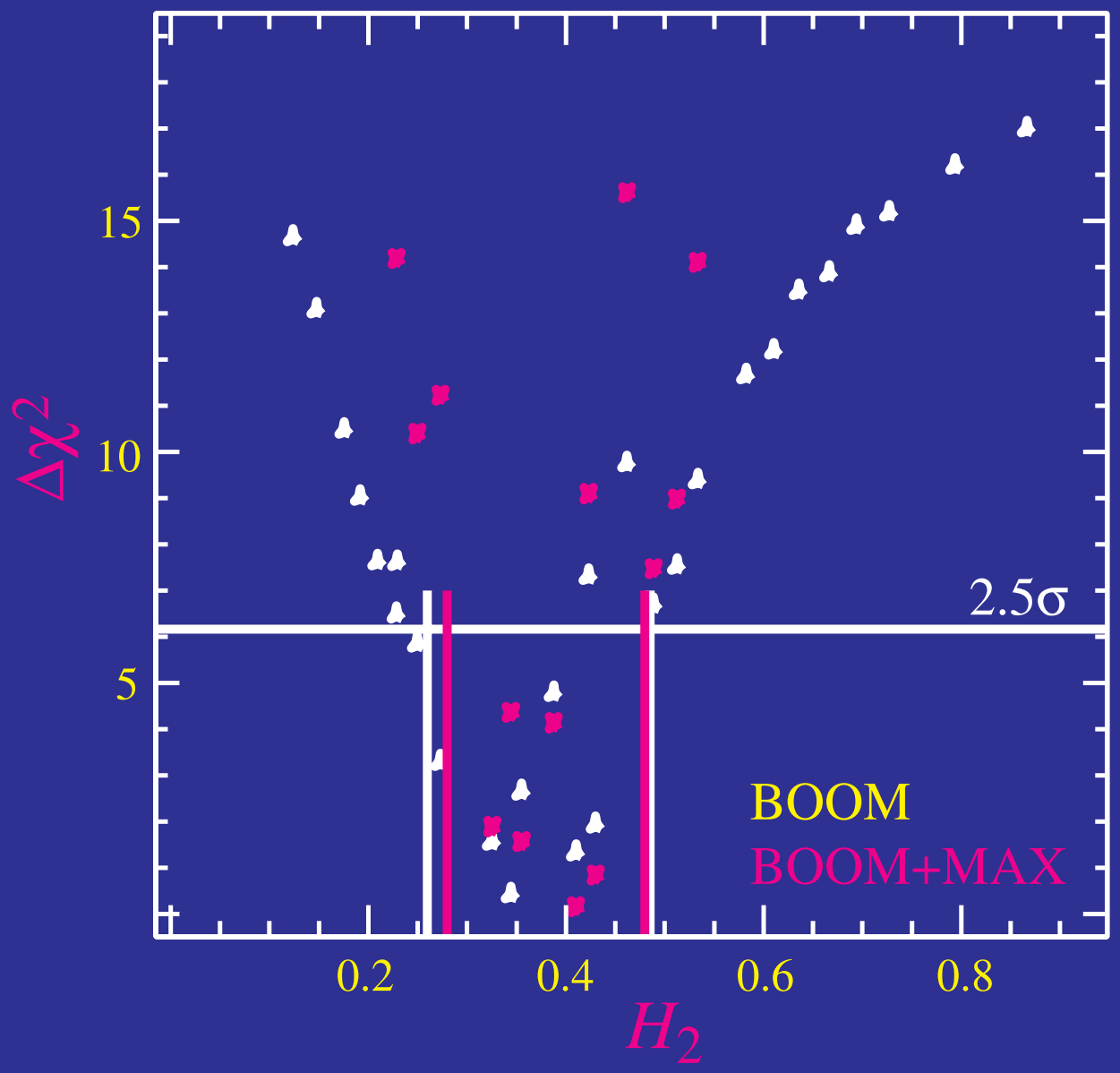
Height of the Second Peak

- BOOM and MAX both show a low power at $l_2 = 2.4\ l_1$
- H_2 = power at 1st/2nd = $(\Delta T_{l_2}/\Delta T_{l_1})^2$



Height of the Second Peak

- BOOM : H_2=0.37 ±0.044 (1σ)
- BOOM+MAX: H_2=0.38 ±0.04 (1σ)



$75 < l < 400$

+

$400 < l < 600$

$(\Omega_\Lambda, \Omega_b h^2, n)$

$\Delta H_2/H_2 =$

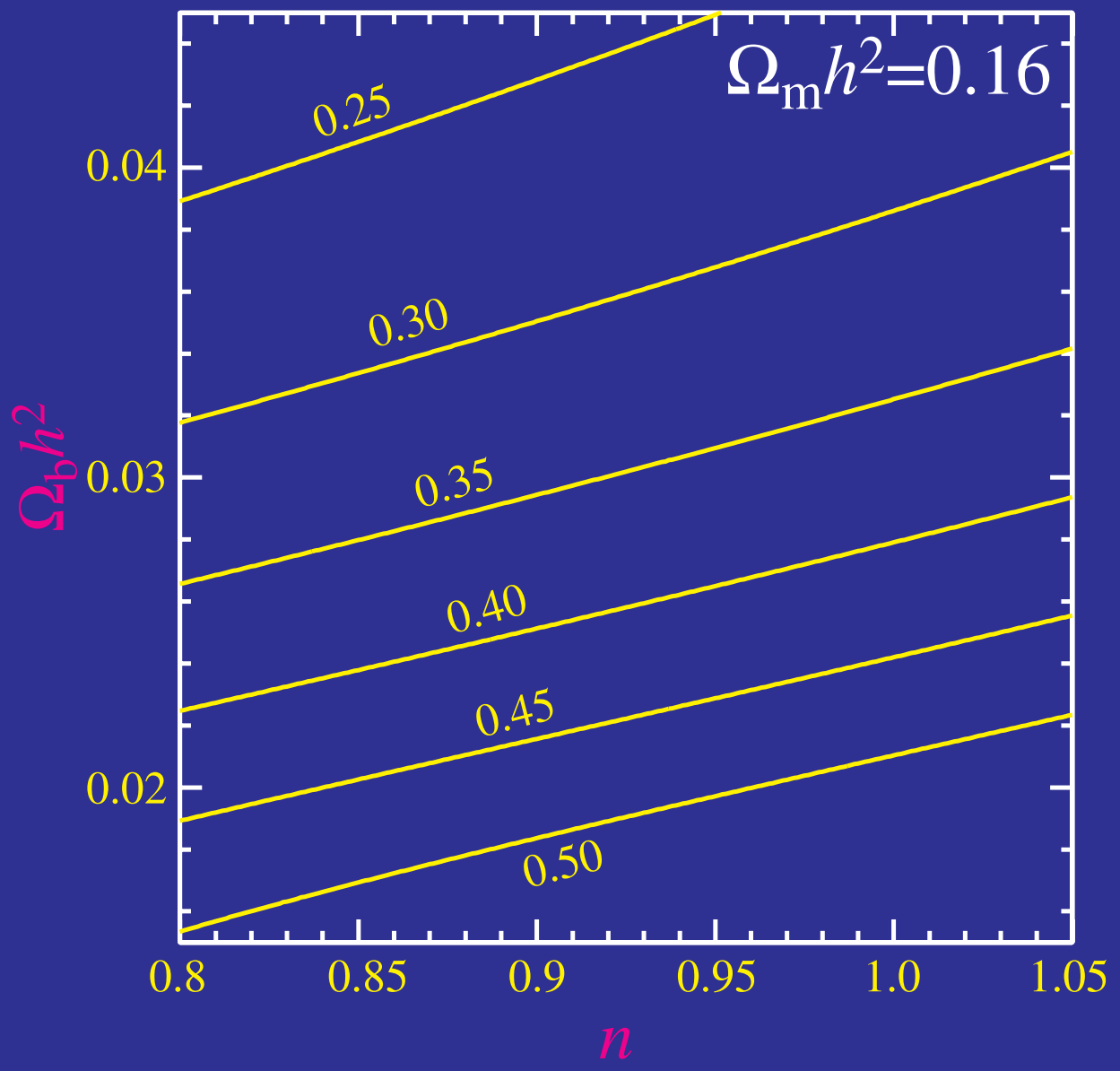
$-0.65\ \Delta\Omega_b h^2/\Omega_b h^2$

$+0.88\ \Delta n/n$

$+0.14 \Delta\Omega_m h^2/\Omega_m h^2$

Height of the Second Peak

- Excludes fiducial LCDM (n=1, $\Omega_b h^2$=0.02, H_2=0.51) at ~3.3σ
- Requires $\Omega_b h^2 > 0.022n$ (if $\Omega_m h^2$>0.16, from l_1, flat, h<0.8)



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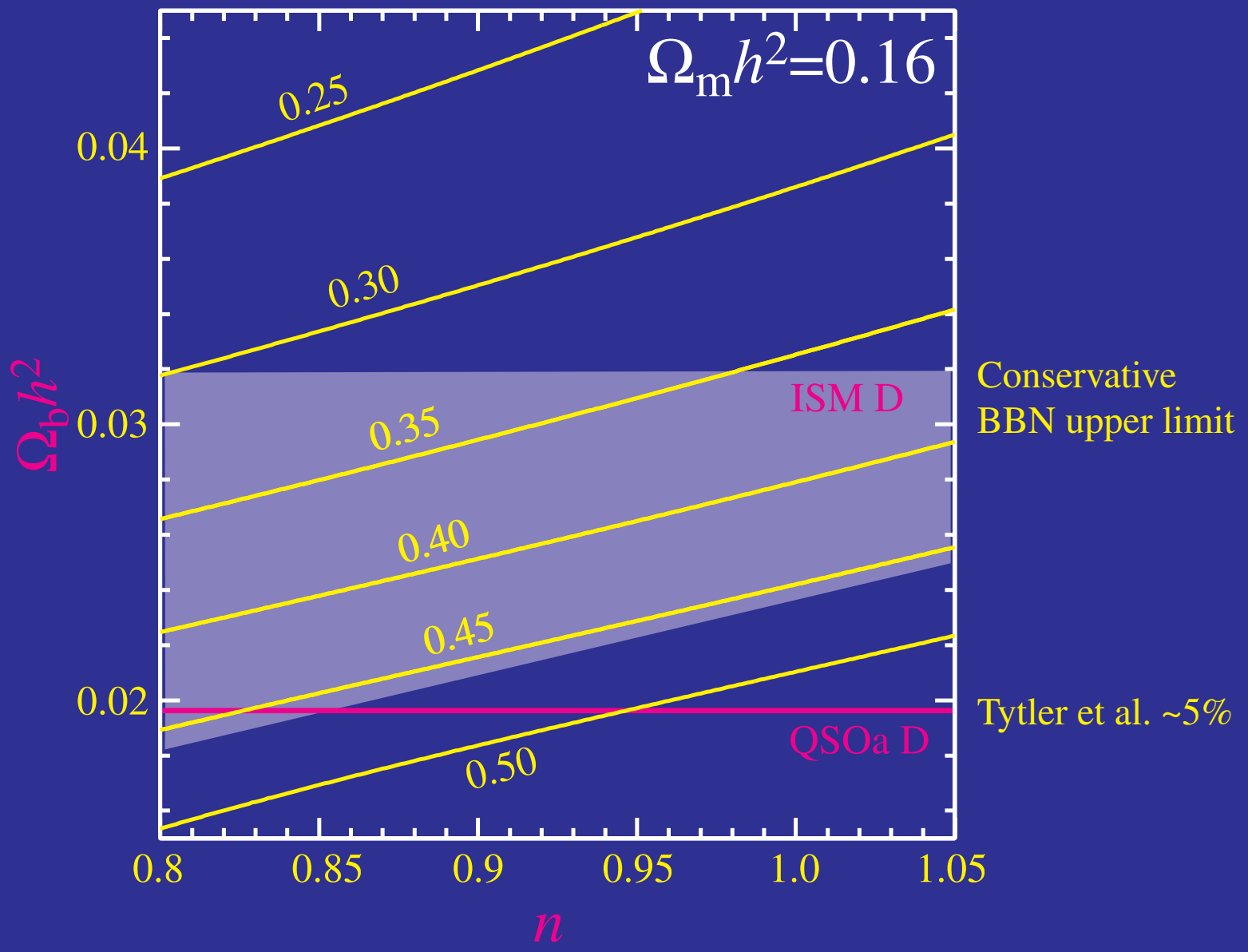


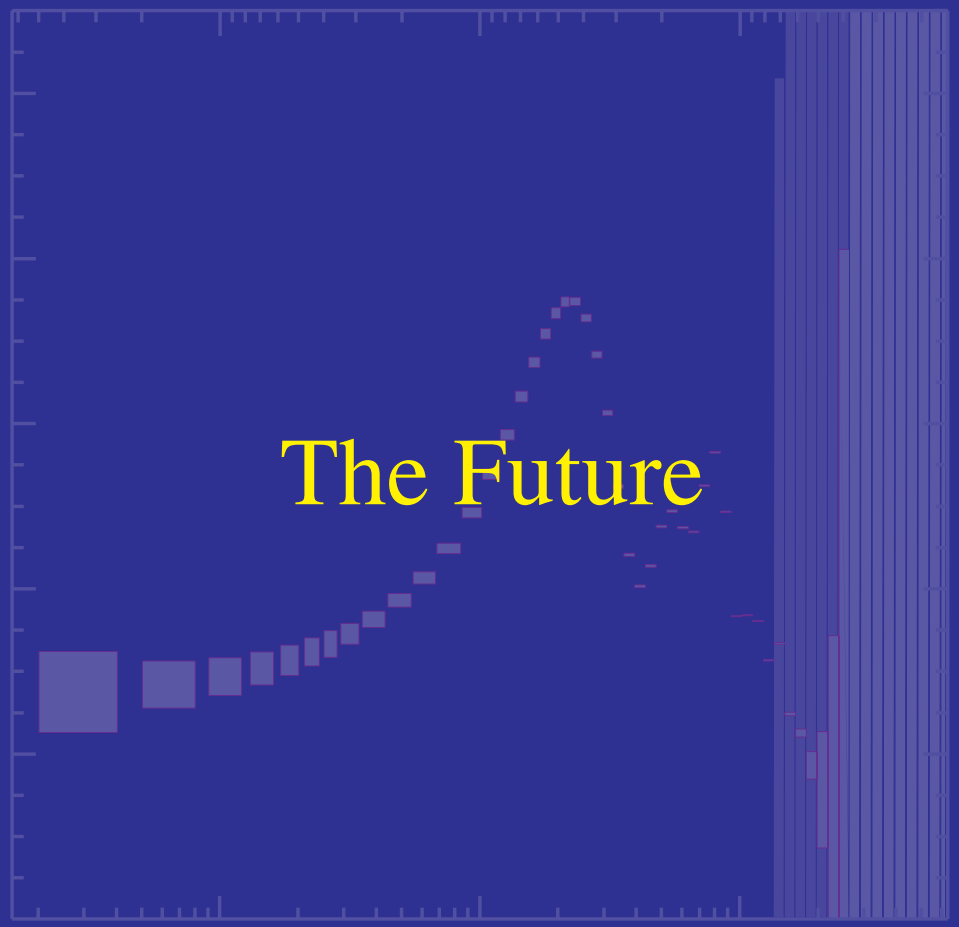
Summary

- We have entered a new era of precision cosmology
- First peak in the CMB power spectrum is due to acoustic waves at z~1000 based on the precice shape measurement
- Consistent with simple inflationary models (scale-invariant superhorizon density perturbations, n>0.85)

- First peak consistent with a flat universe but may indicate a young age (prefers ~10-11Gyrs, requires <13-14Gyrs from l_1<218 [95%CL] possible causes: high Hubble constant, Ω_m, or dark energy p/ρ)
- Requires dark baryons at least comparable to big-bang nucleosynthesis (prefers 25-50% more: low second peak H_2<0.46 [95%CL])
- Limits dark matter: lack of steep rise to third peak $\Omega_m h^2 < 0.47$)

- Still need to resolve the secondary peaks (stronger test of inflation)!
- Parameter degeneracies resolved with 3 peaks

The Future

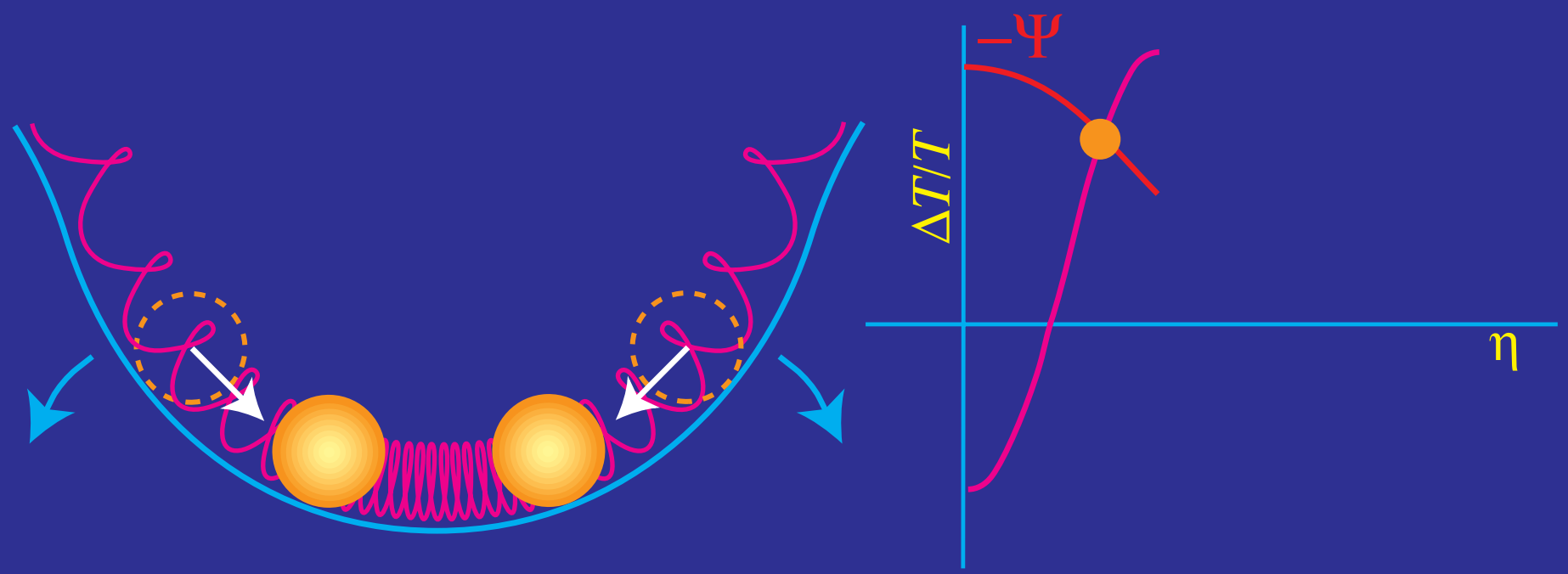


Higher Peaks:

Matter–Radiation Ratio
and
Breaking the Degeneracies

Driving Effects and Matter/Radiation

- Potential perturbation: $k^2\Psi = -4\pi G a^2 \delta\rho$ generated by radiation
- Radiation $\rightarrow$ Potential: inside sound horizon $\delta\rho/\rho$ pressure supported $\delta\rho$ hence Ψ decays with expansion



Hu & Sugiyama (1995)

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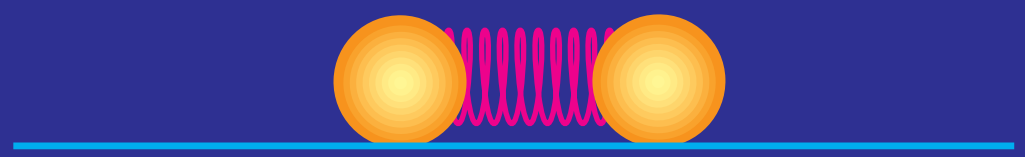
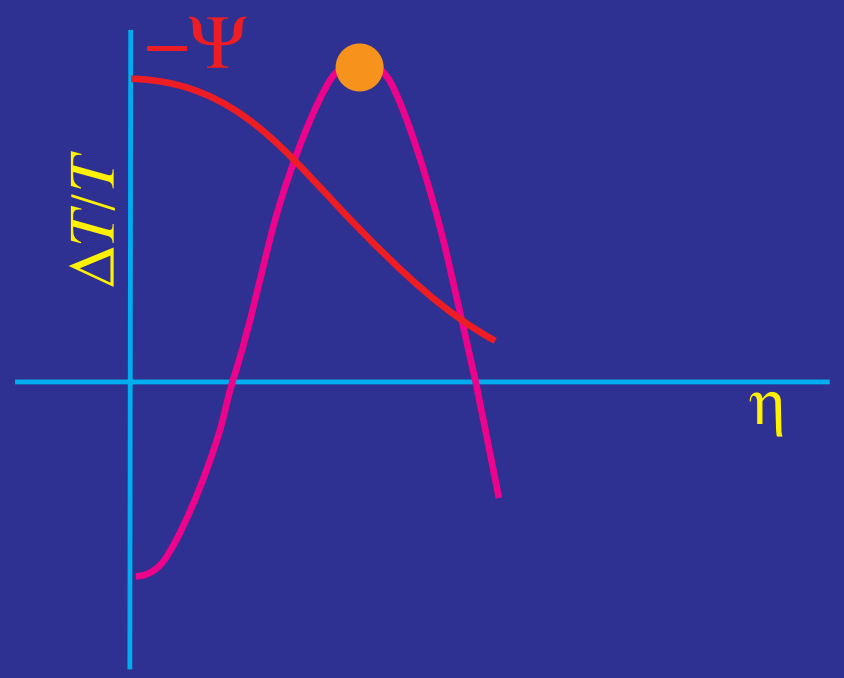
- Feedback stops at matter domination





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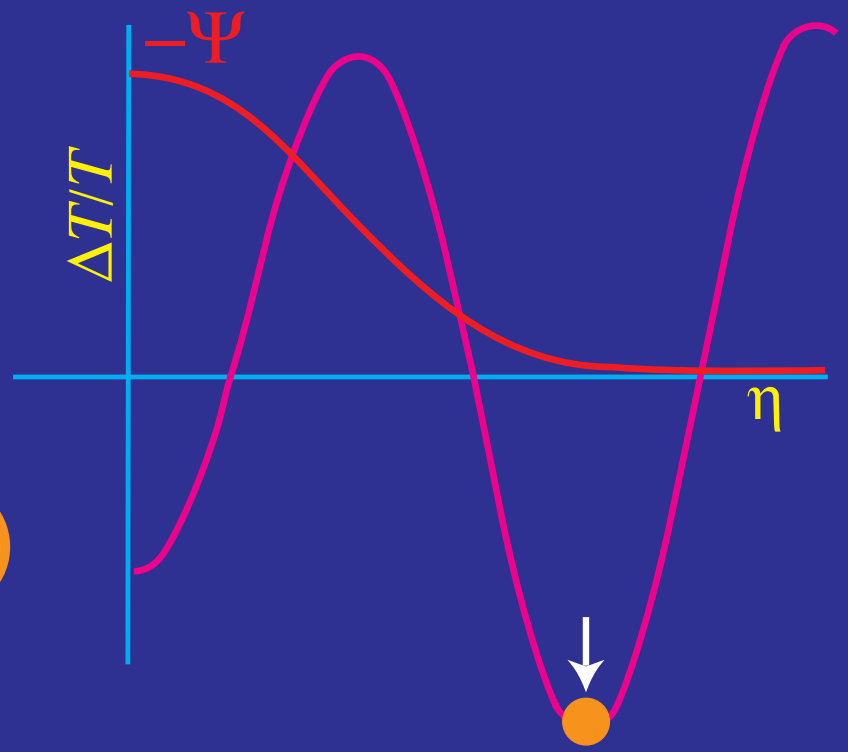
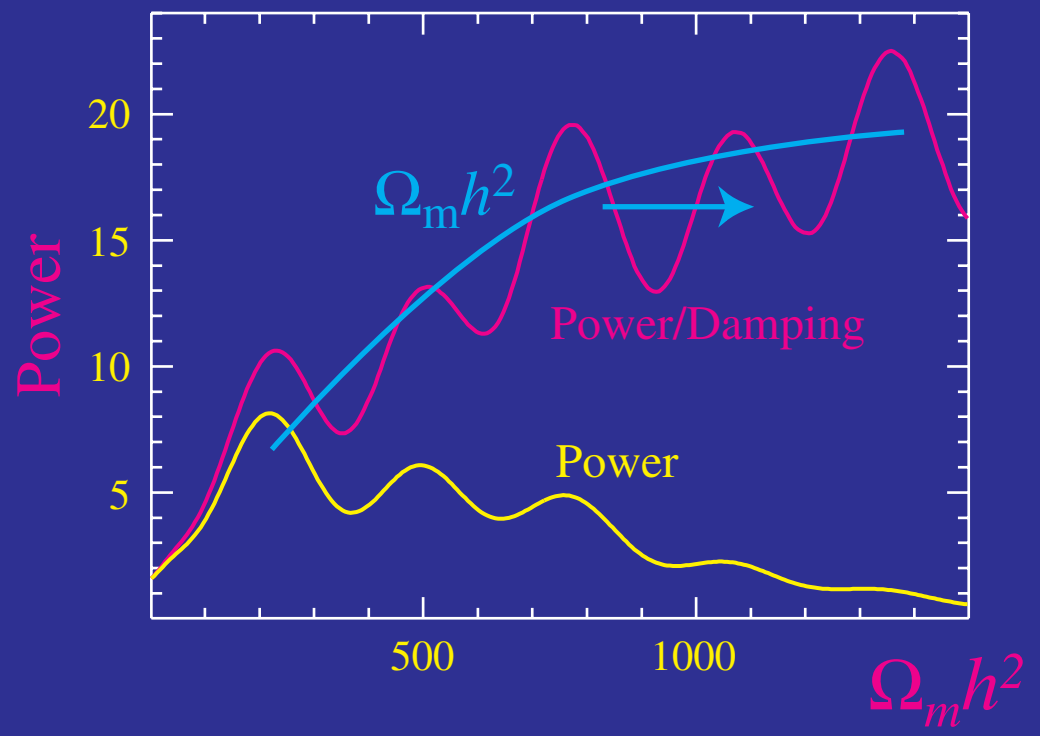
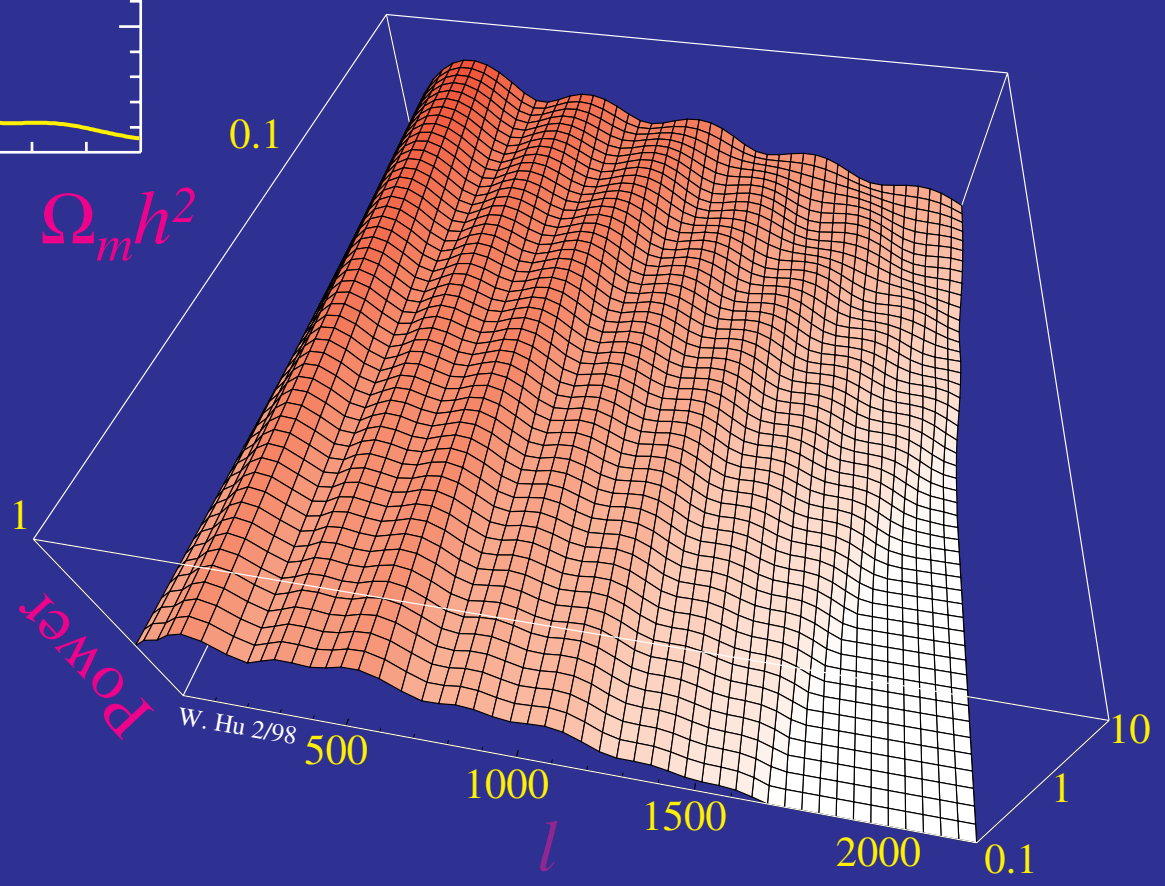


Hu & Sugiyama (1995)

Matter Density in the CMB



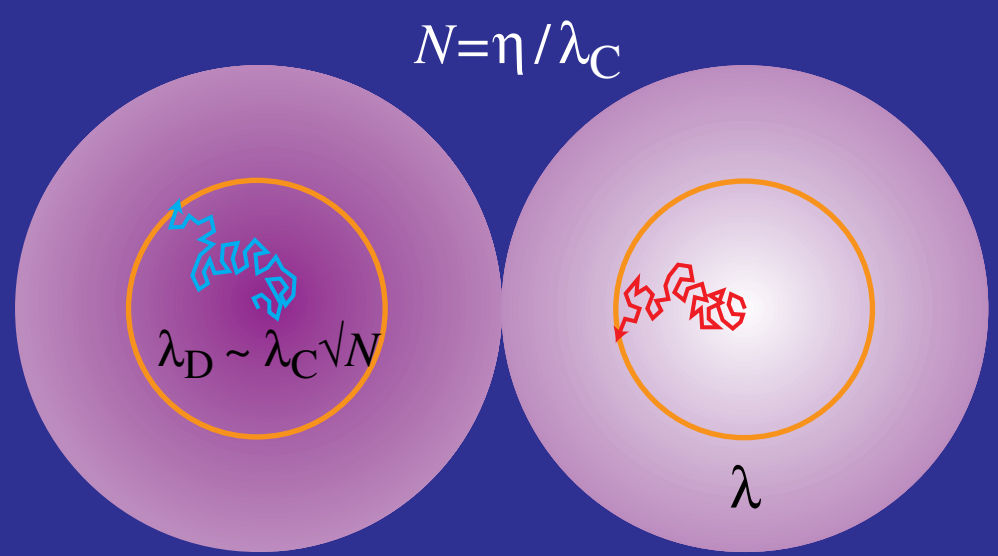
- Amplitude ramp across matter–radiation equality
- Radiation density fixed by CMB temperature & thermal history

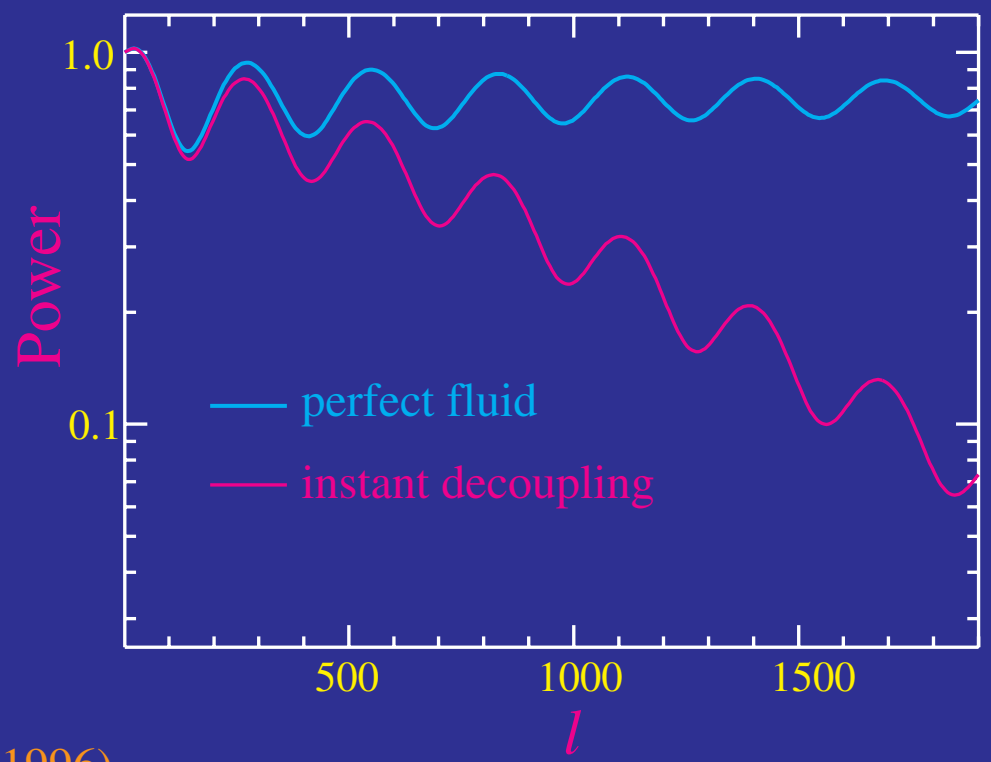


- Measure $\Omega_m h^2$ from peak heights

Dissipation / Diffusion Damping

- Imperfections in the coupled fluid → mean free path λ_C in the baryons
- Random walk over diffusion scale: geometric mean of mfp & horizon
  $\lambda_D \sim \lambda_C\sqrt{N} \sim \sqrt{\lambda_C \eta} \gg \lambda_C$
- Overtake wavelength: $\lambda_D \sim \lambda$; second order in λ_C/λ
- Viscous damping for $R<1$; heat conduction damping for $R>1$

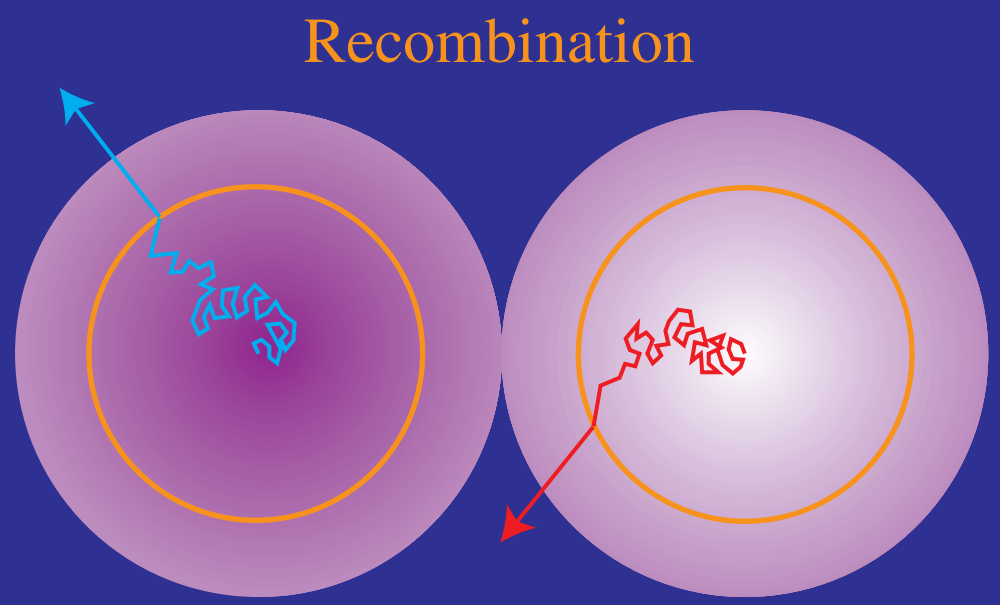


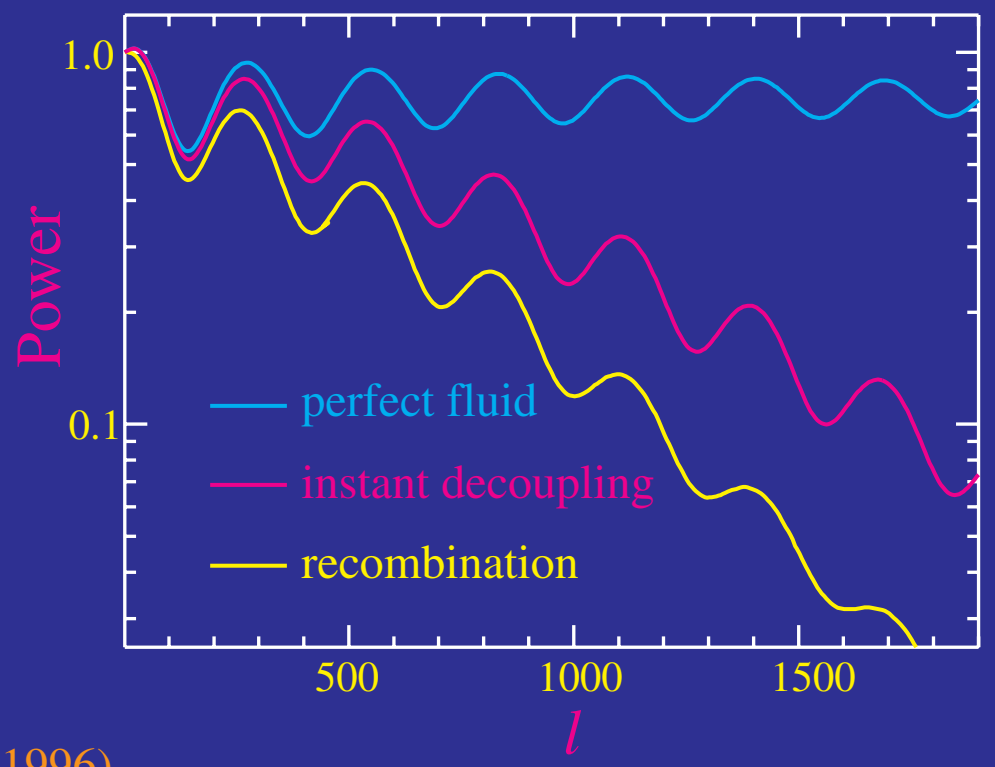


Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)

Dissipation / Diffusion Damping

- Rapid increase at recombination as mfp ↑
- Independent of (robust to changes in) perturbation spectrum
- Robust physical scale for angular diameter distance test (Ω_K, Ω_Λ)





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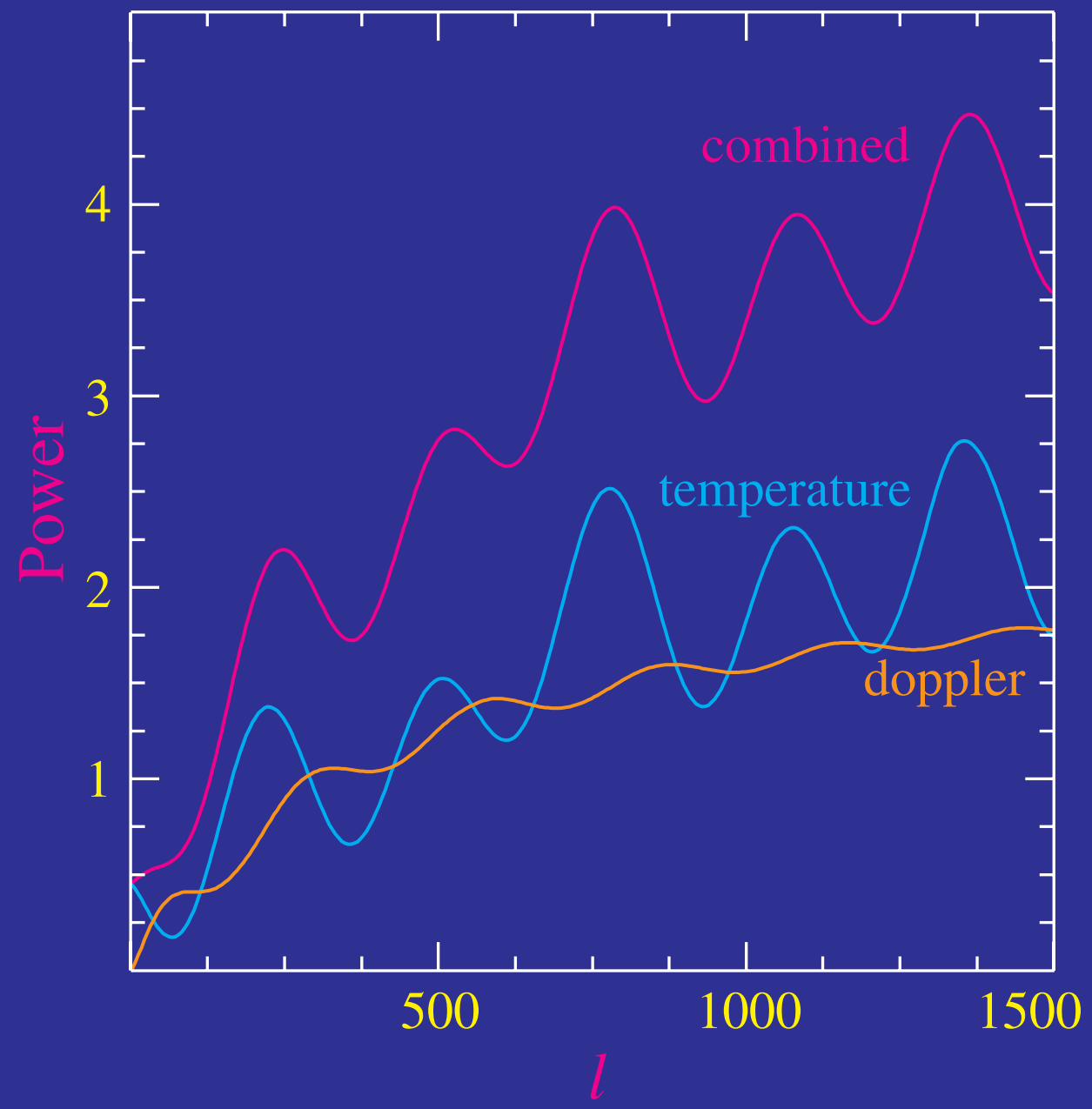
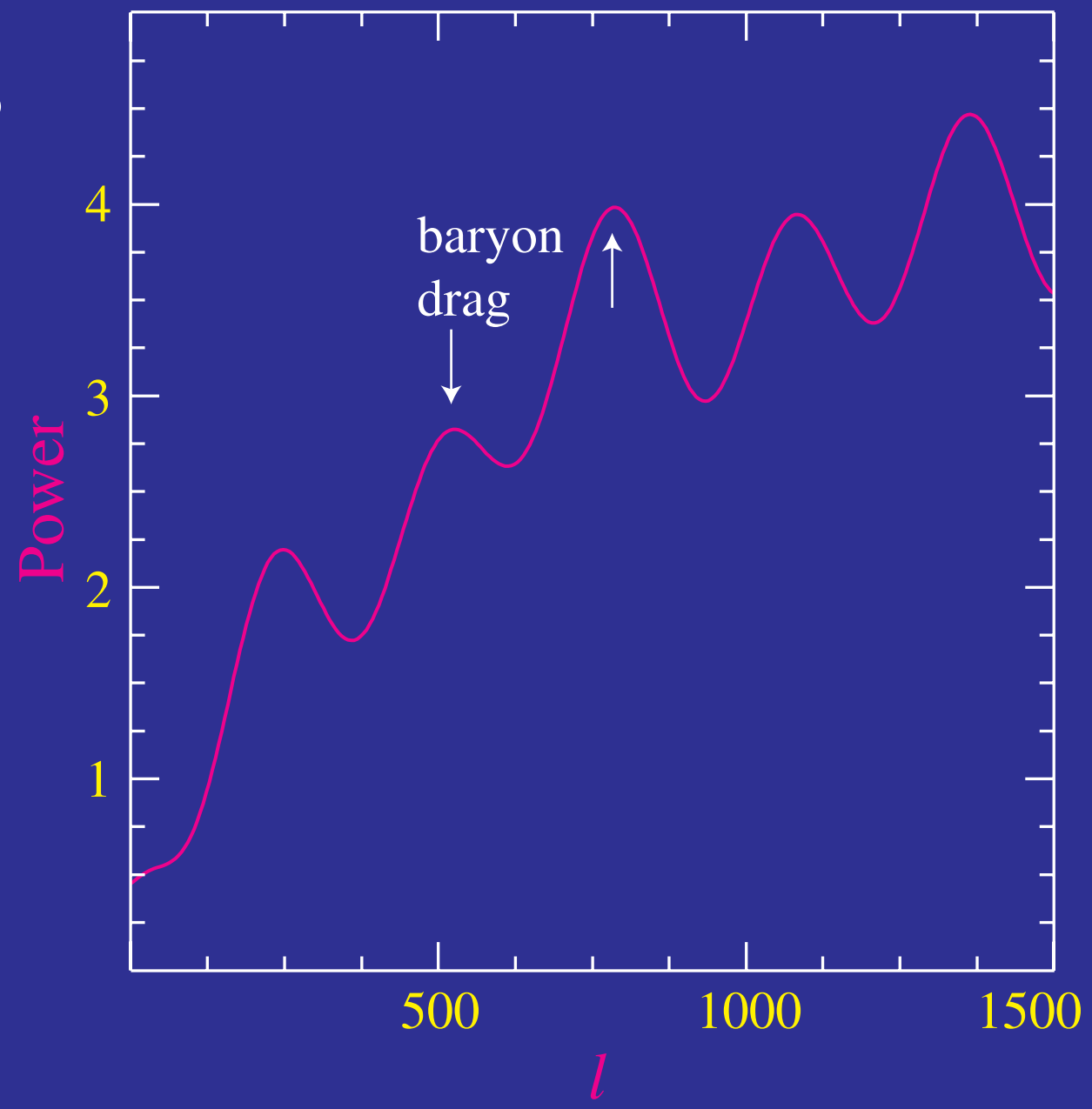
Parameter Estimation

Physical Decomposition & Information



Physical Decomposition & Information

- Fluid + Gravity
  - → alternating peaks
  - → photon-baryon ratio
  - → $\Omega_b h^2$



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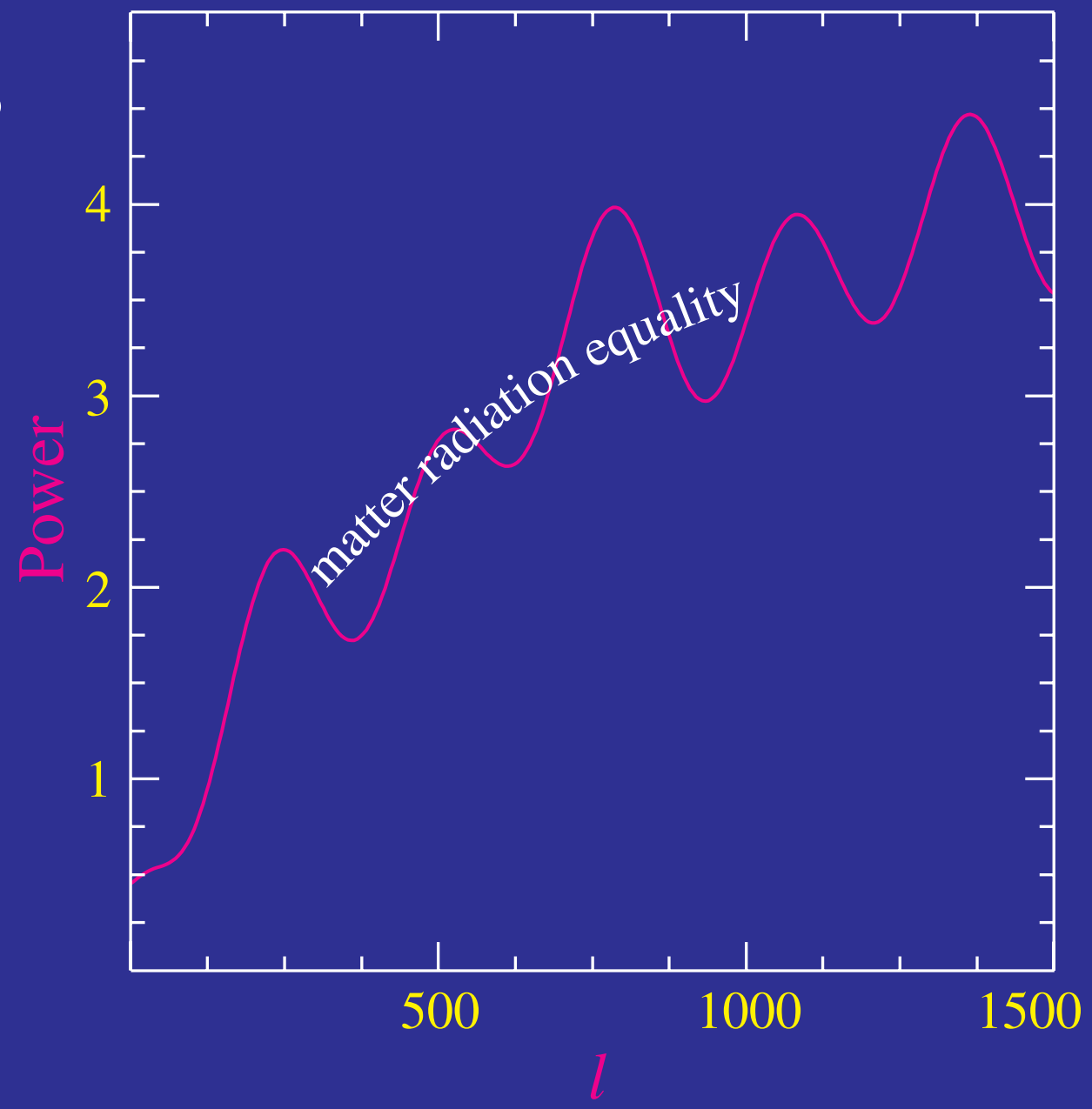
  - → $\Omega_m h^2$



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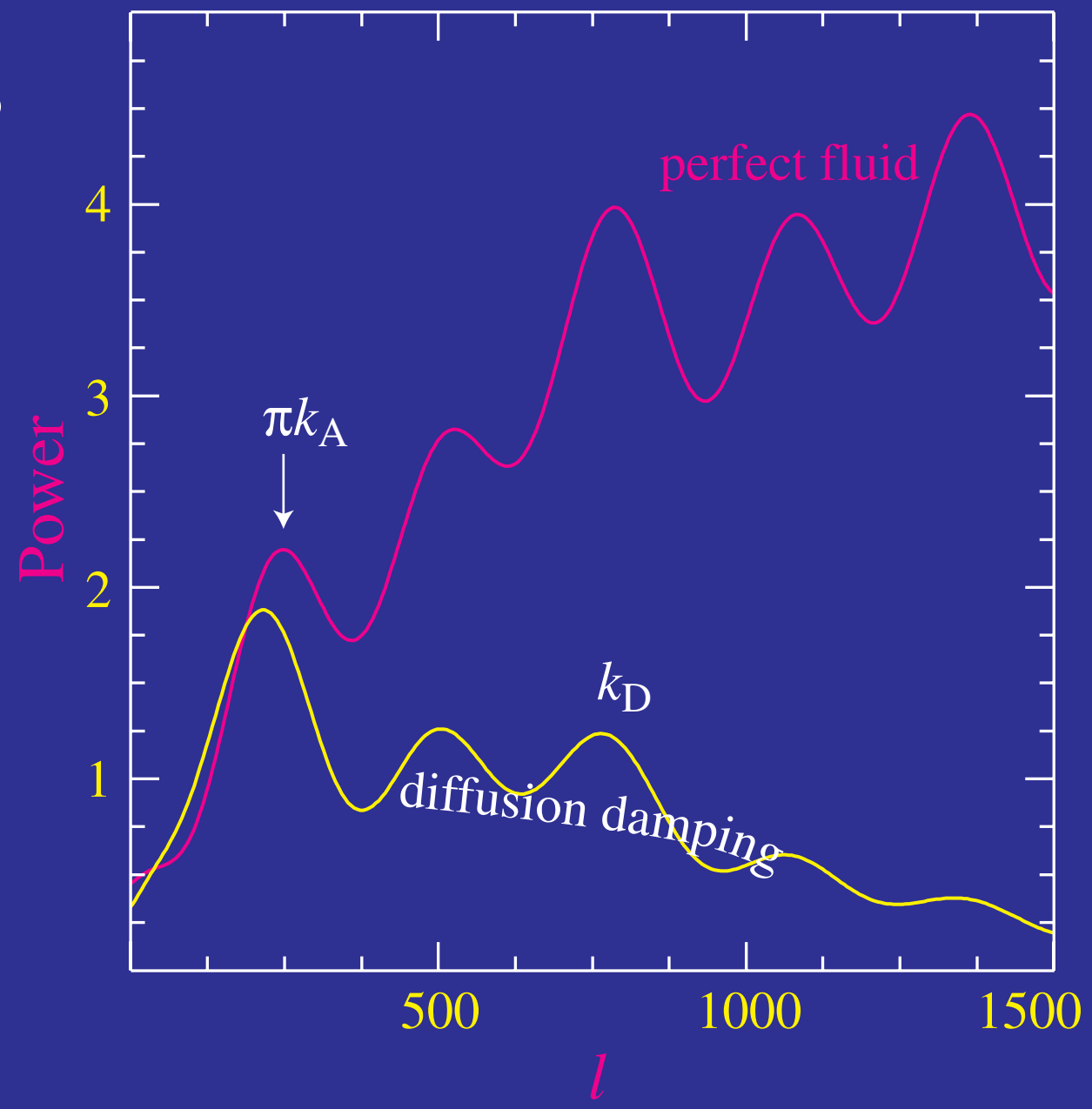
- Fluid Rulers
  - → sound horizon
  - → damping scale



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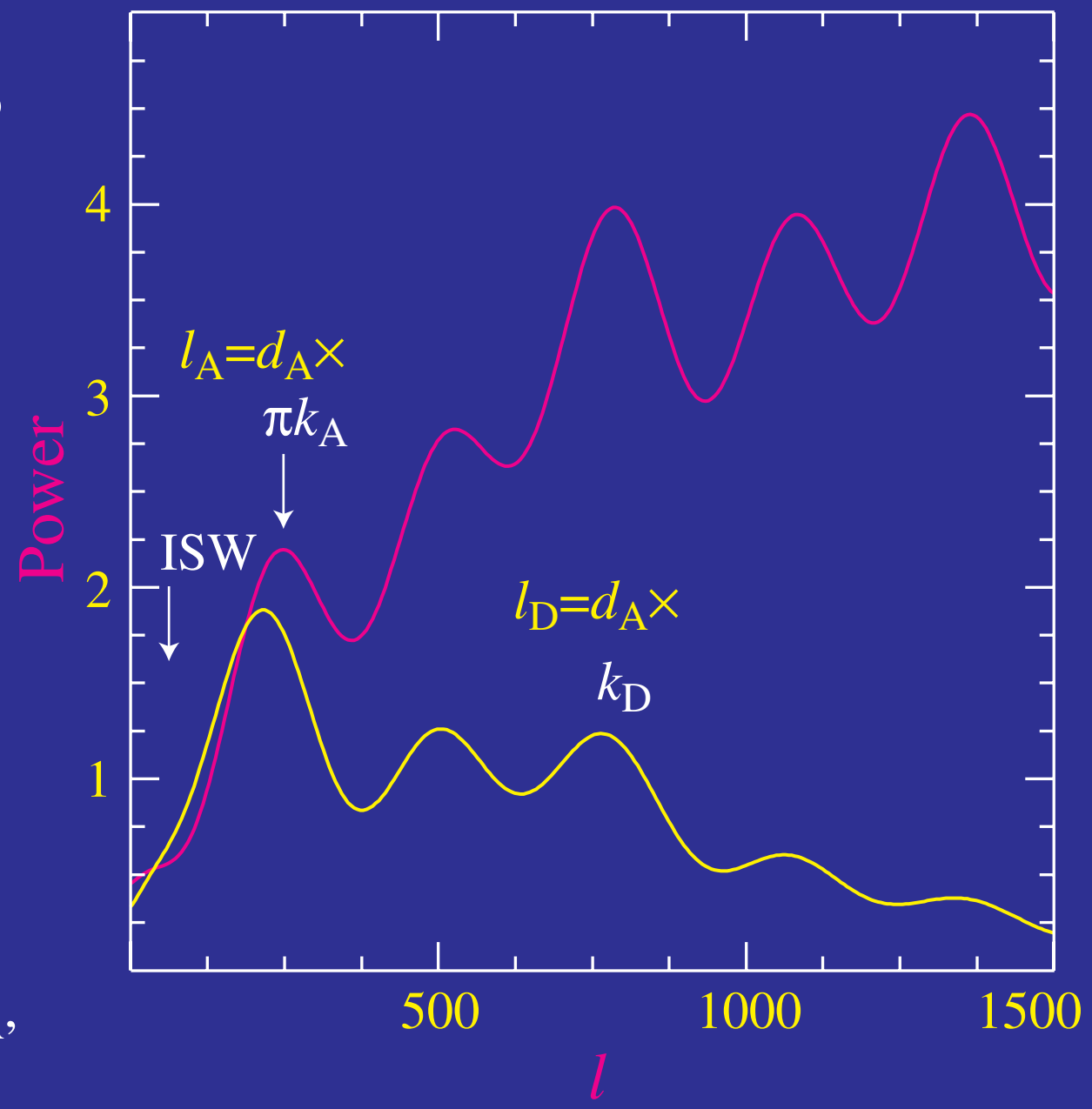
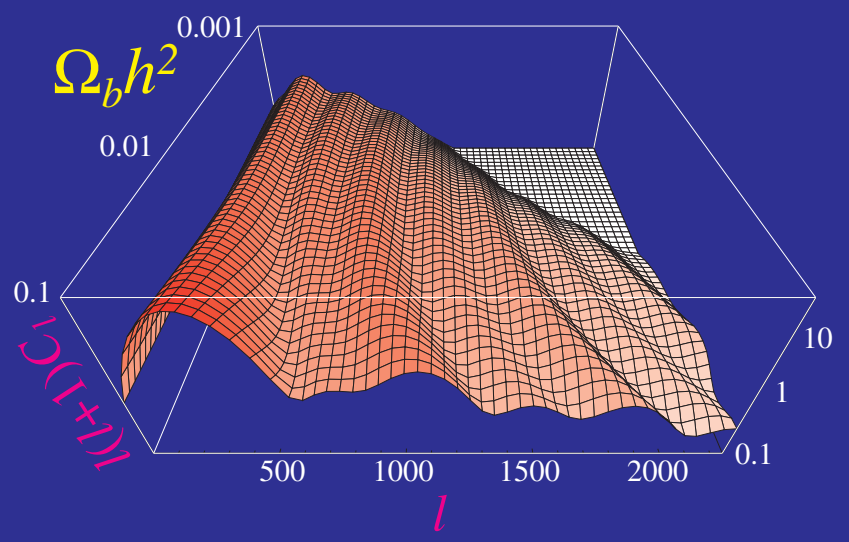
  - → $\Omega_m h^2$
- Fluid Rulers
  - → sound horizon
  - → damping scale
- Geometry
  - → angular diameter distance $f(\Omega_\Lambda,\Omega_K)$ + flatness or no Ω_Λ,
  - → Ω_Λ or Ω_K

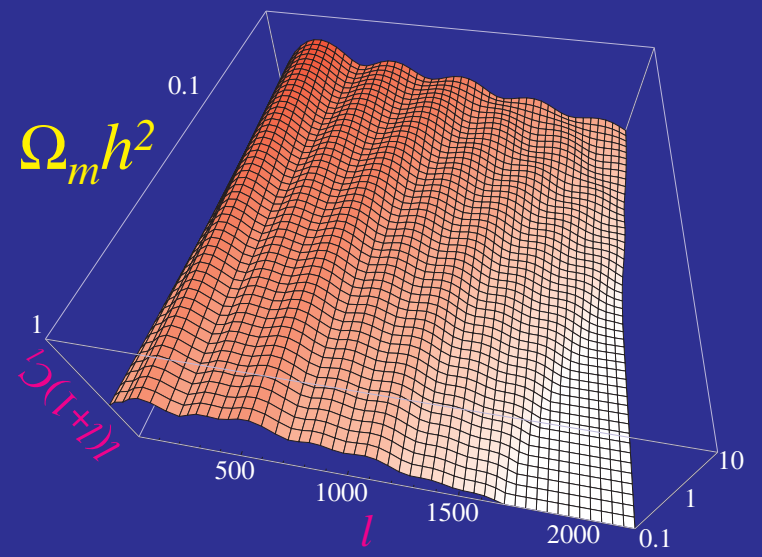


Cosmological Parameters in the CMB

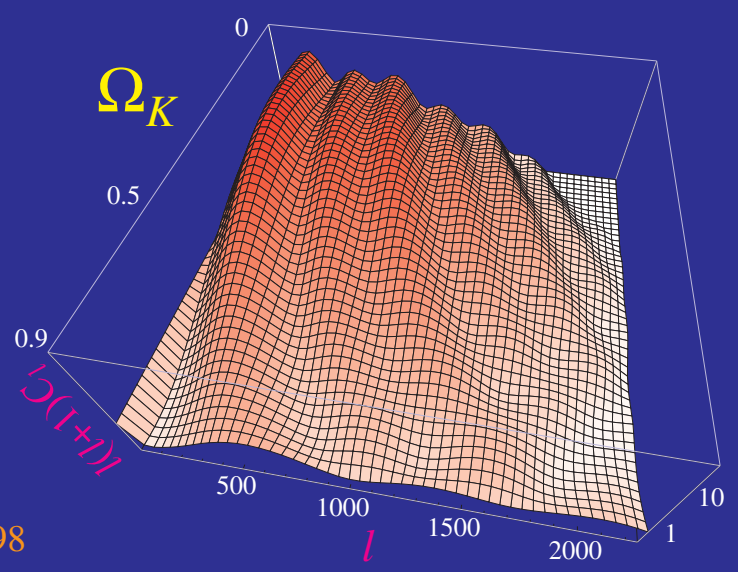
Baryon–Photon Ratio



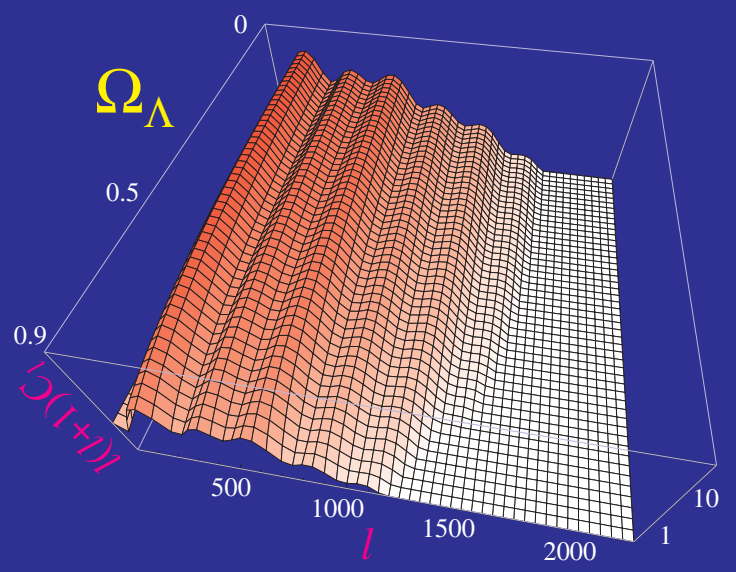
Matter–Radiation Ratio



Curvature



Cosmological Constant



W.Hu 2/98

The Ubiquitous Fisher Matrix

- The Fisher matrix is defined in terms of the likelihood (or signal C_l and noise N_l power spectra) as

$$F_{ij} = -\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \rangle = \frac{1}{2} \sum_\ell \frac{(2\ell+1) f_{\rm sky}}{(C_\ell + N_\ell)^2} \frac{\partial C_\ell}{\partial p_i} \frac{\partial C_\ell}{\partial p_j}$$

- Its inverse, the curvature matrix, gives the optimal errors on p, $\sigma^2 = (F^{-1})_{ii}$ including sampling and noise variance
- Useful for identifying degeneracies / constrained directions
- Problems: accuracy of derivatives and underlying parameterization can lead to widely diverging estimates:

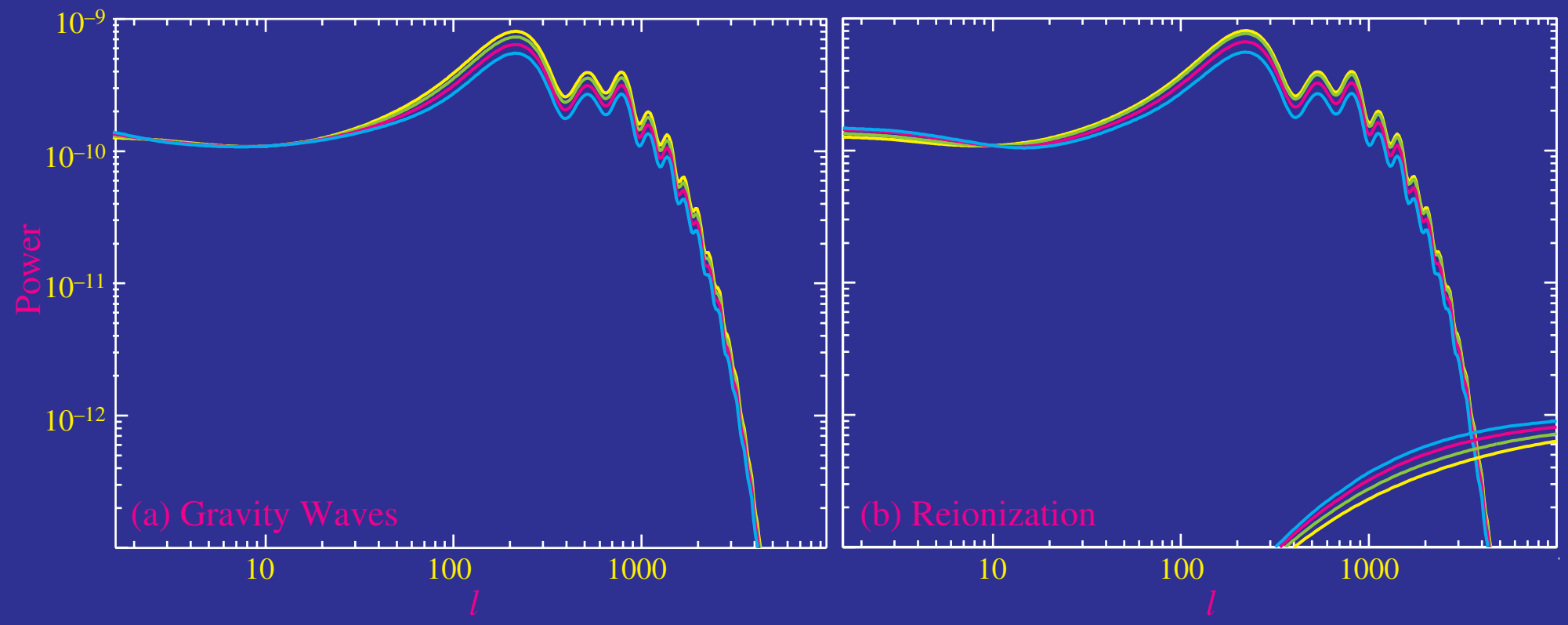
$\sigma(h)$	~1%	Jungman et al. (1996)
	~200%	Eisenstein, Hu, Tegmark (1999)

MAP:

$\Omega_m h^2$	$\Omega_b h^2$	m_ν	Ω_Λ	Ω_K	τ	n_S	T/S	A
0.029	0.0026	0.76	1.0	0.29	0.64	0.11	0.45	1.2

A Question of Degeneracies

- Precision of CMB parameter estimation thwarted by degeneracies
- Problem compounded at large angular scales by cosmic variance
- Example: tensor/scalar ratio and reionization
- Solution: go beyond primary temperature anisotropies
- Polarization and Secondary Anisotropies



Polarization

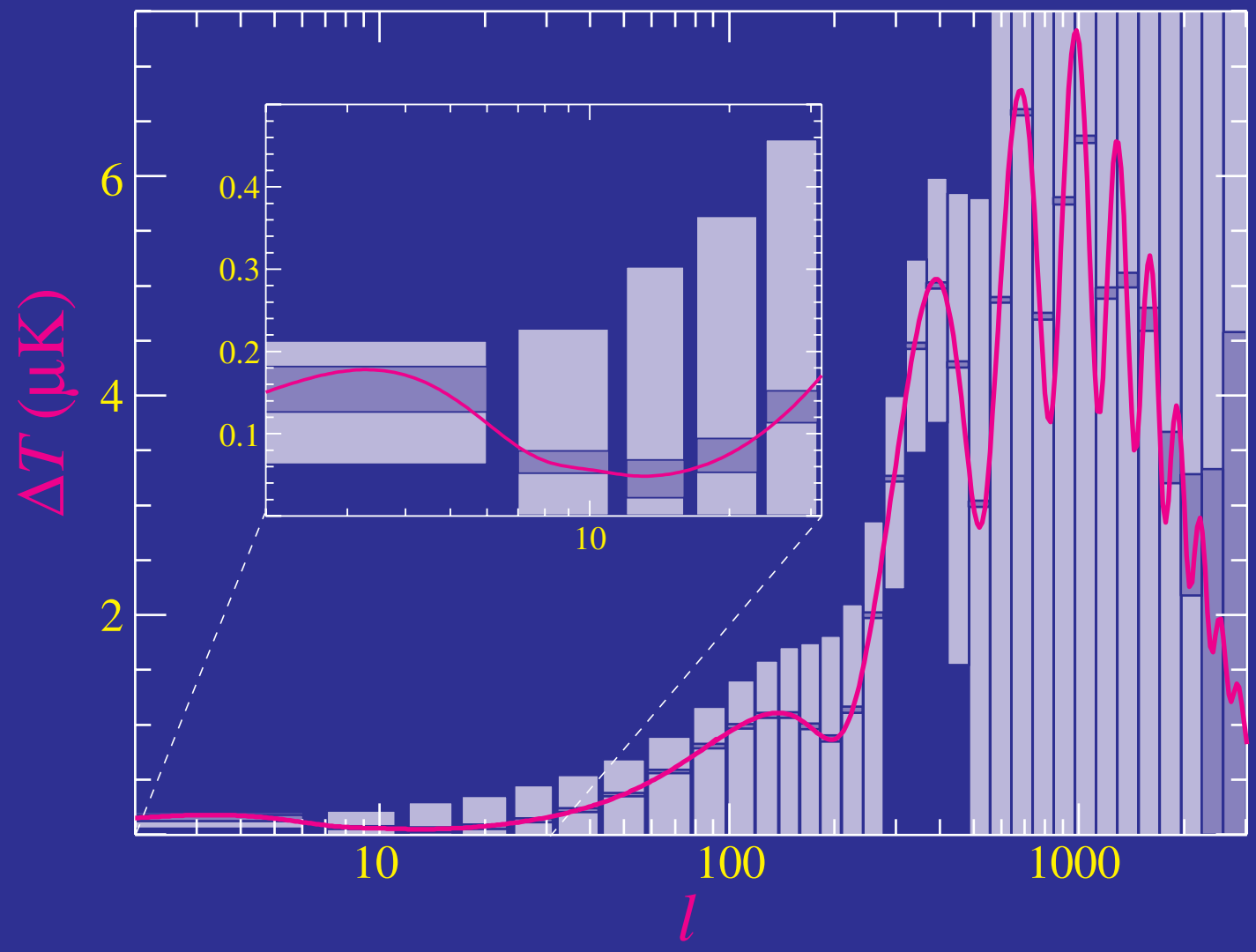
Why Measure the Polarization?

- Virtues of polarization
  - unlike temperature anisotropies, generated by scattering only
  - tensor field on the sky; carries more info than scalar temperature

- Uses of polarization
  - verify the gravitational instability paradigm: fluctuations present during last scattering
  - probe the reionization epoch: remove a leading source of ambiguity (degeneracy) in the temperature power spectrum
  - get higher statistics on the acoustic peaks and their underlying parameters
  - reconstruct the scalar, vector, tensor nature of the perturbations and hence the cosmology even if ab initio models are wrong
  - test inflationary models by measuring the gravity wave amplitude: energy scale and shape of inflaton potential

Why Polarization is Difficult

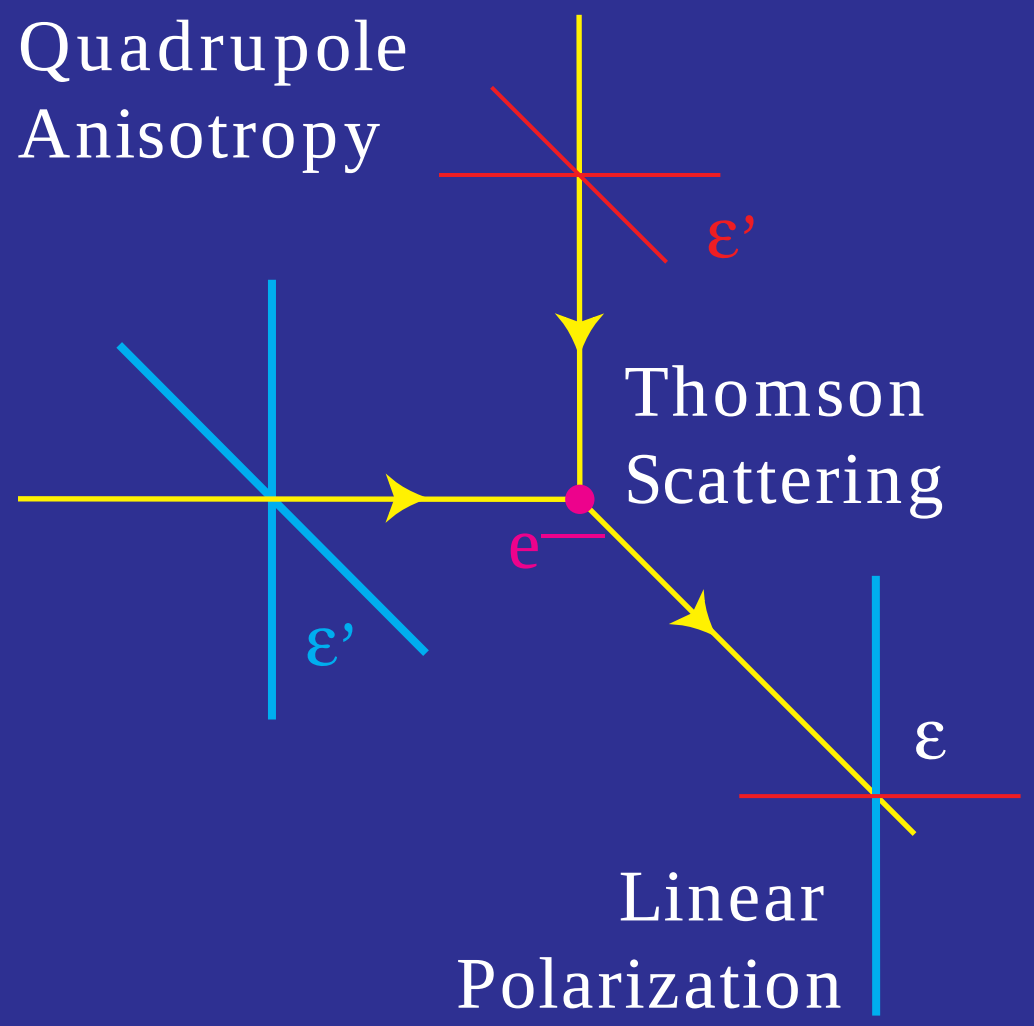
- Source of polarization is the scattering of quadrupole anisotropies
- Rapid scattering destroys quadrupole anisotropies
- Polarization only from the optically thin period before full transparency



< 10%

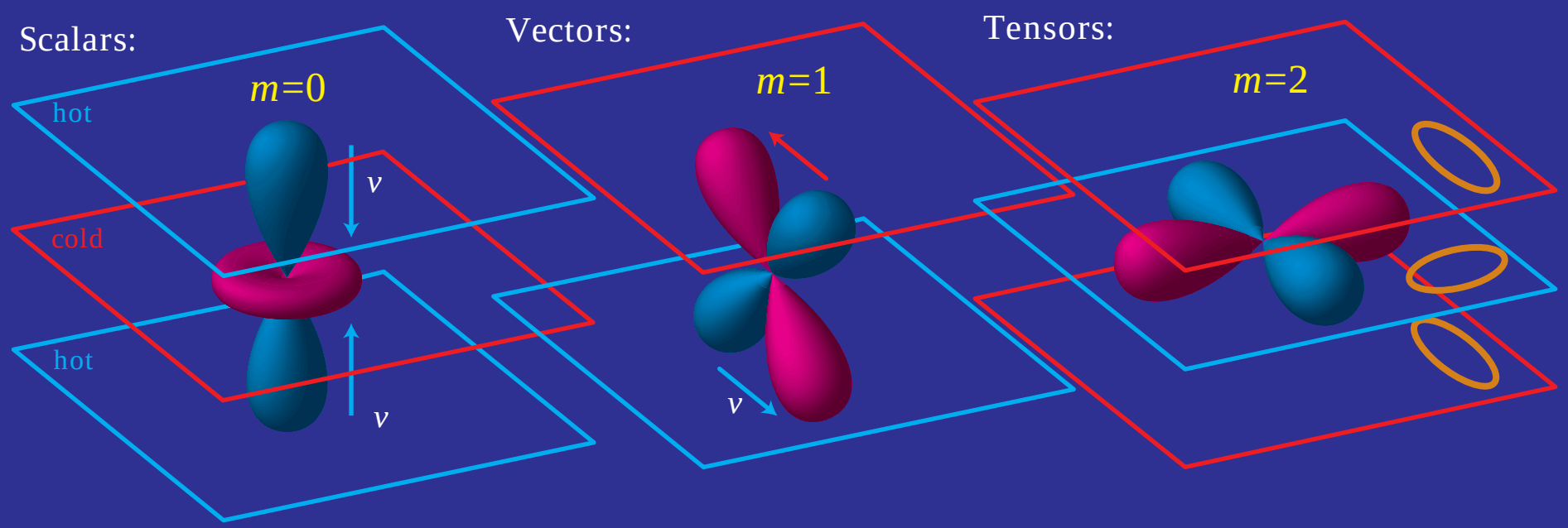
Polarization from Thomson Scattering

- Thomson scattering of anisotropic radiation → linear polarization
- Polarization aligned with cold lobe of the quadrupole anisotropy



Perturbations & Their Quadrupoles

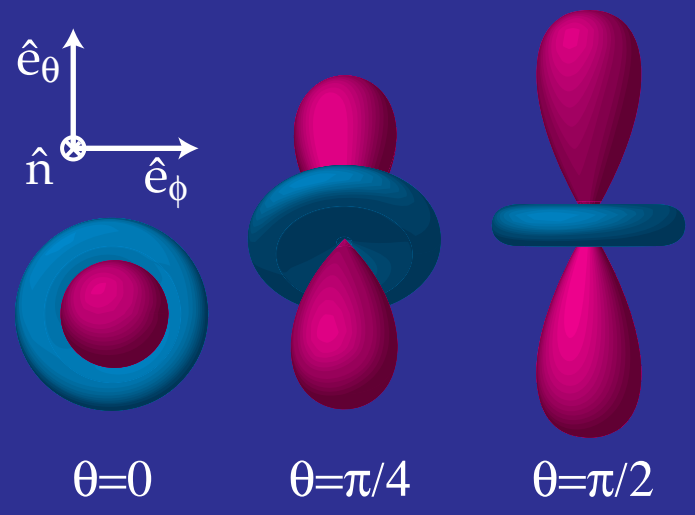
- Orientation of quadrupole relative to wave (**k**) determines pattern
- Scalars (density) m=0
- Vectors (vorticity) m=±1
- Tensors (gravity waves) m=±2

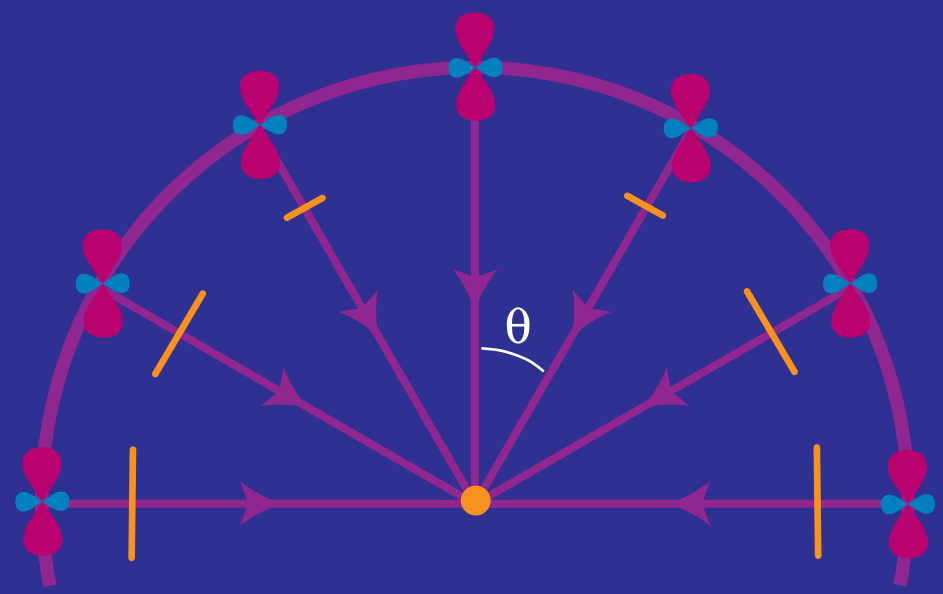


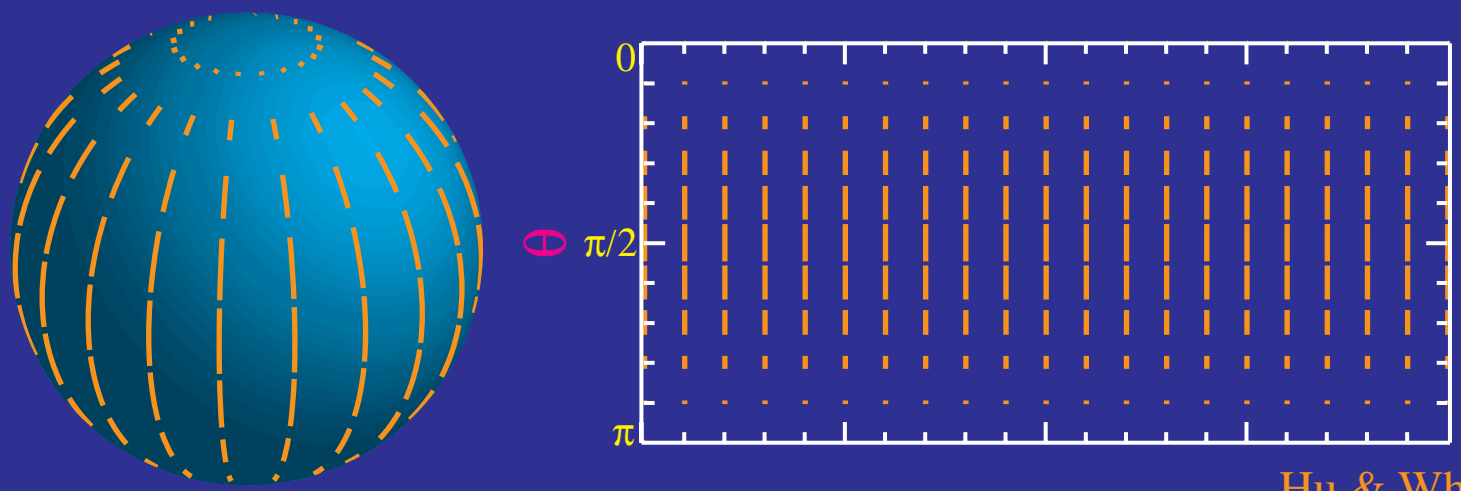
Hu & White (1997)

Polarization on the Sphere

- Polarization direction oriented with the cold lobe of the quadrupole
- A local observer will see a $\sin^2\theta$ pattern of Q-polarization



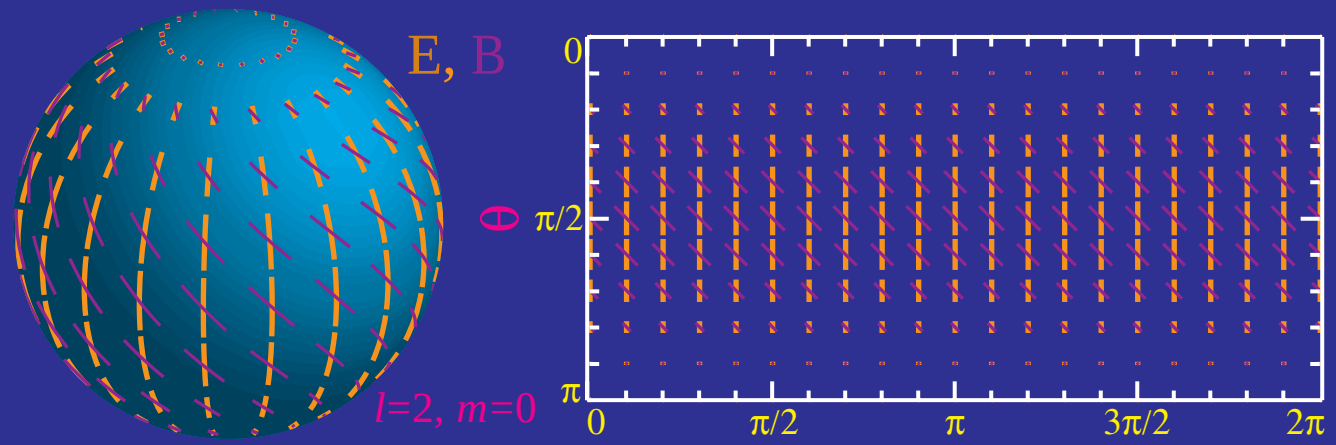




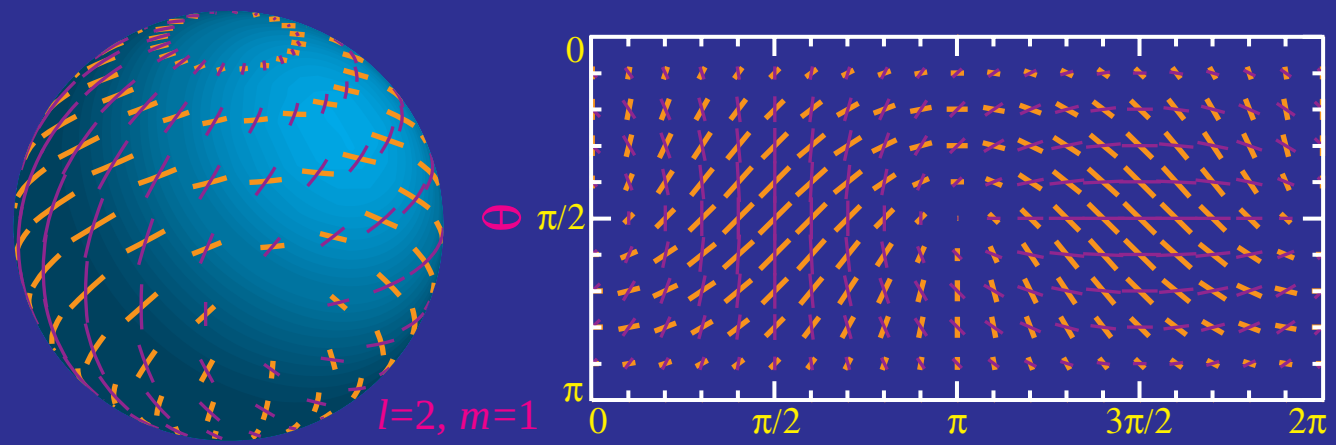
Hu & White (1997)

Polarization Patterns

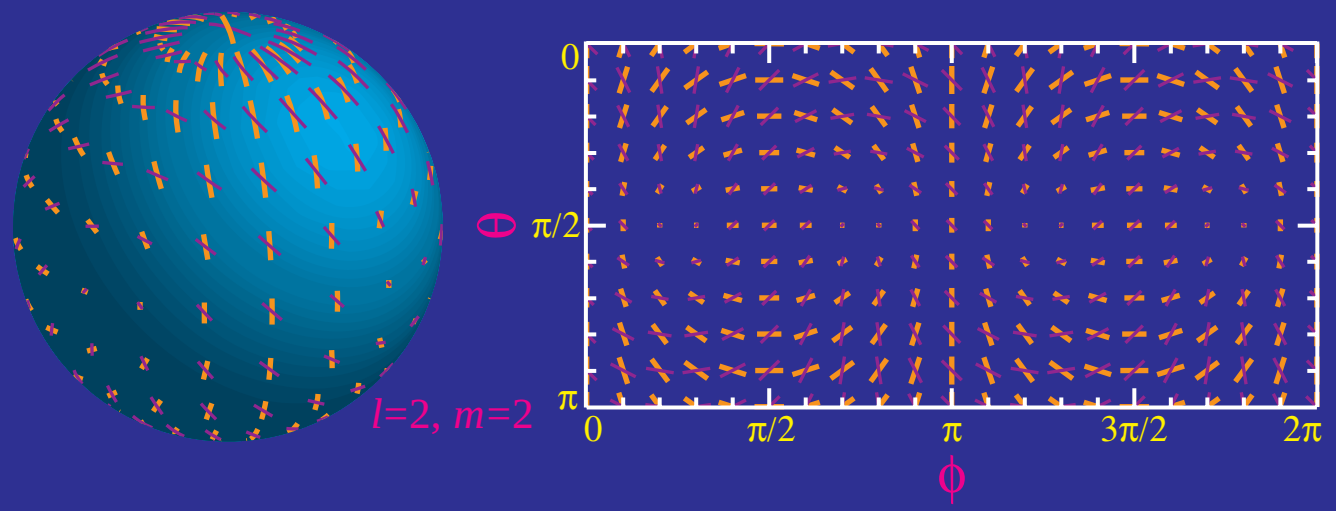
Scalars



Vectors

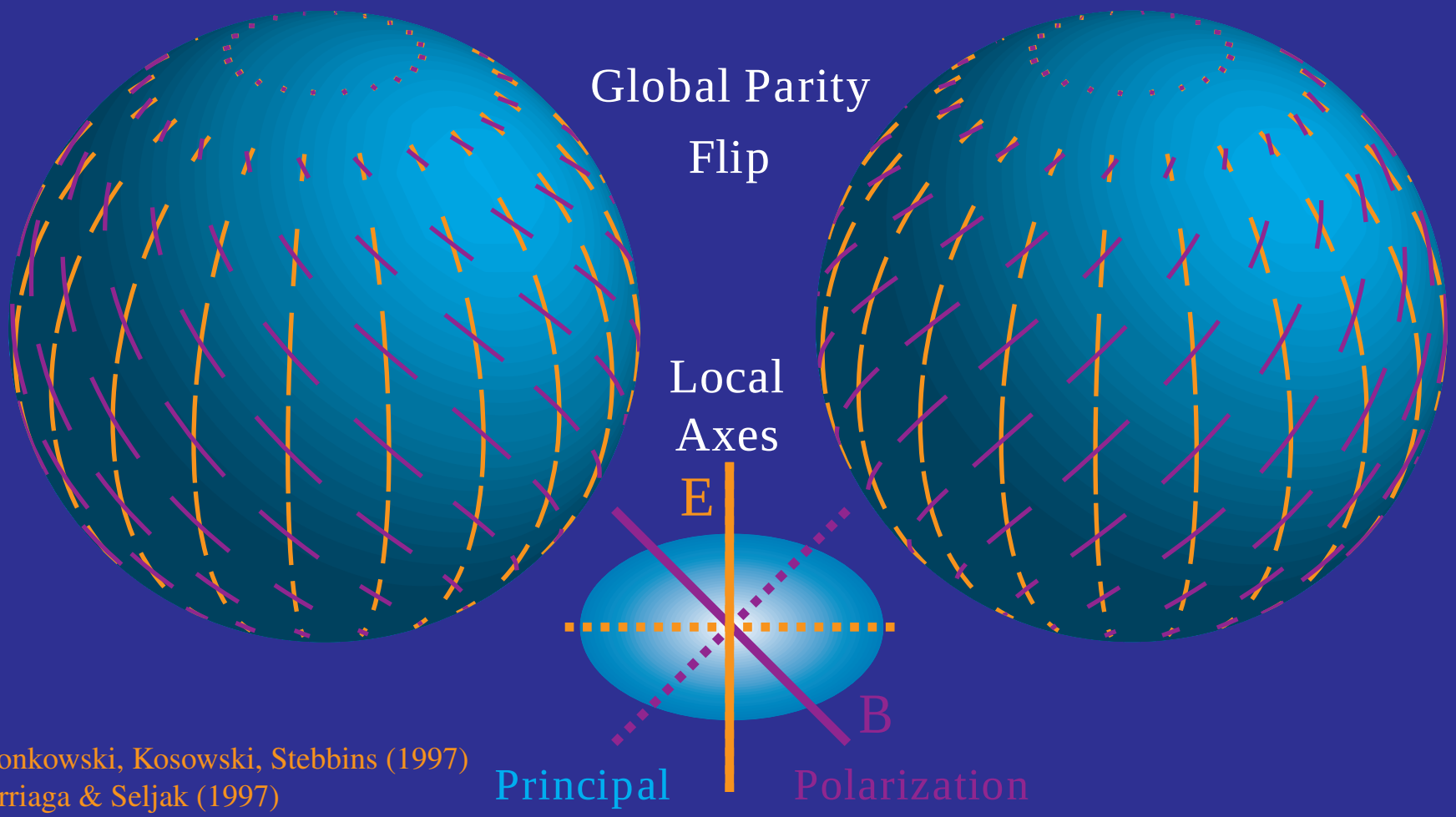


Tensors



Electric & Magnetic Patterns

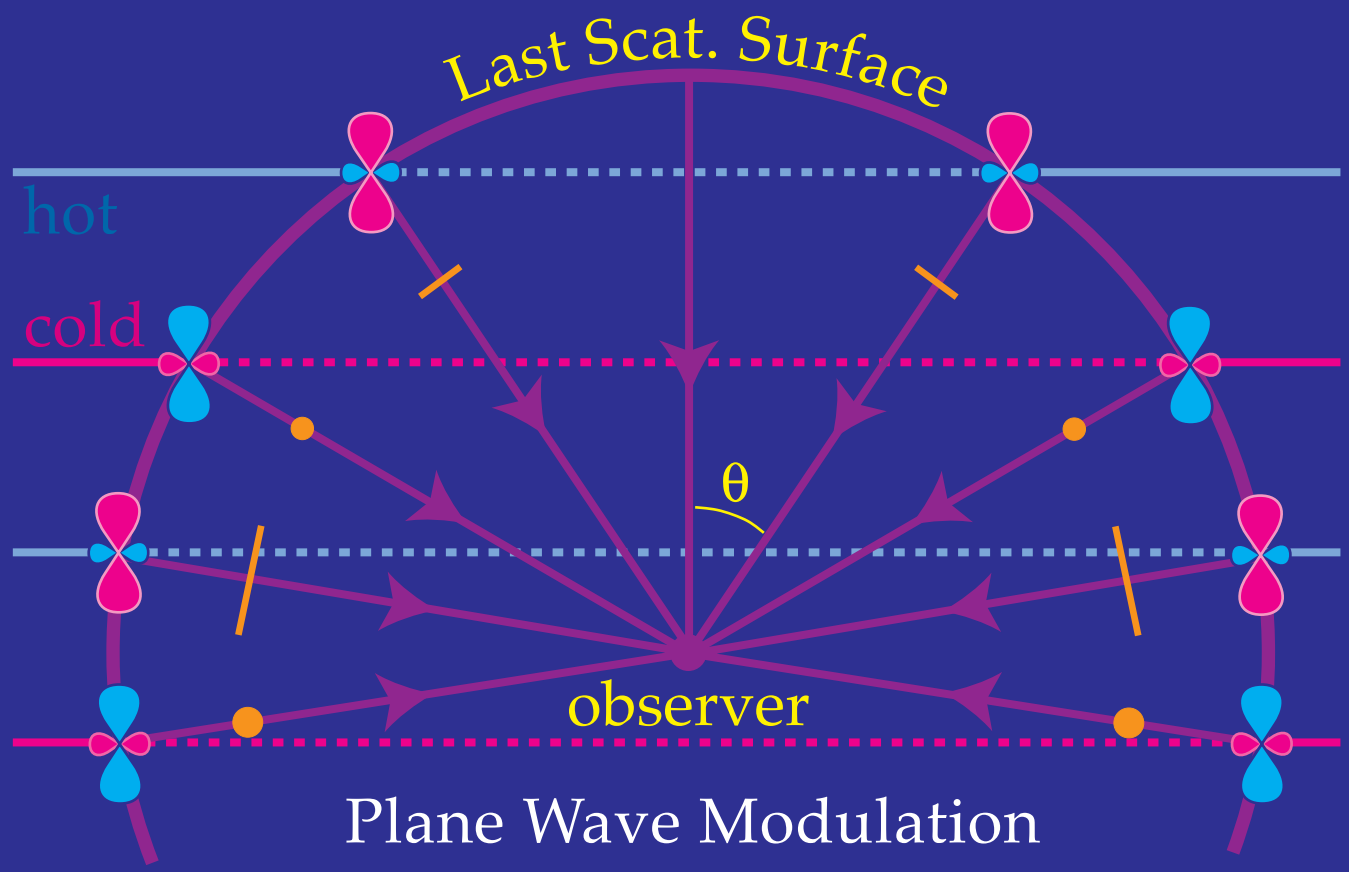
- Global view: behavior under parity
- Local view: alignment of principle vs. polarization axes



Kamionkowski, Kosowski, Stebbins (1997)
Zaldarriaga & Seljak (1997)
Hu & White (1997)

Local vs. Observable Polarization

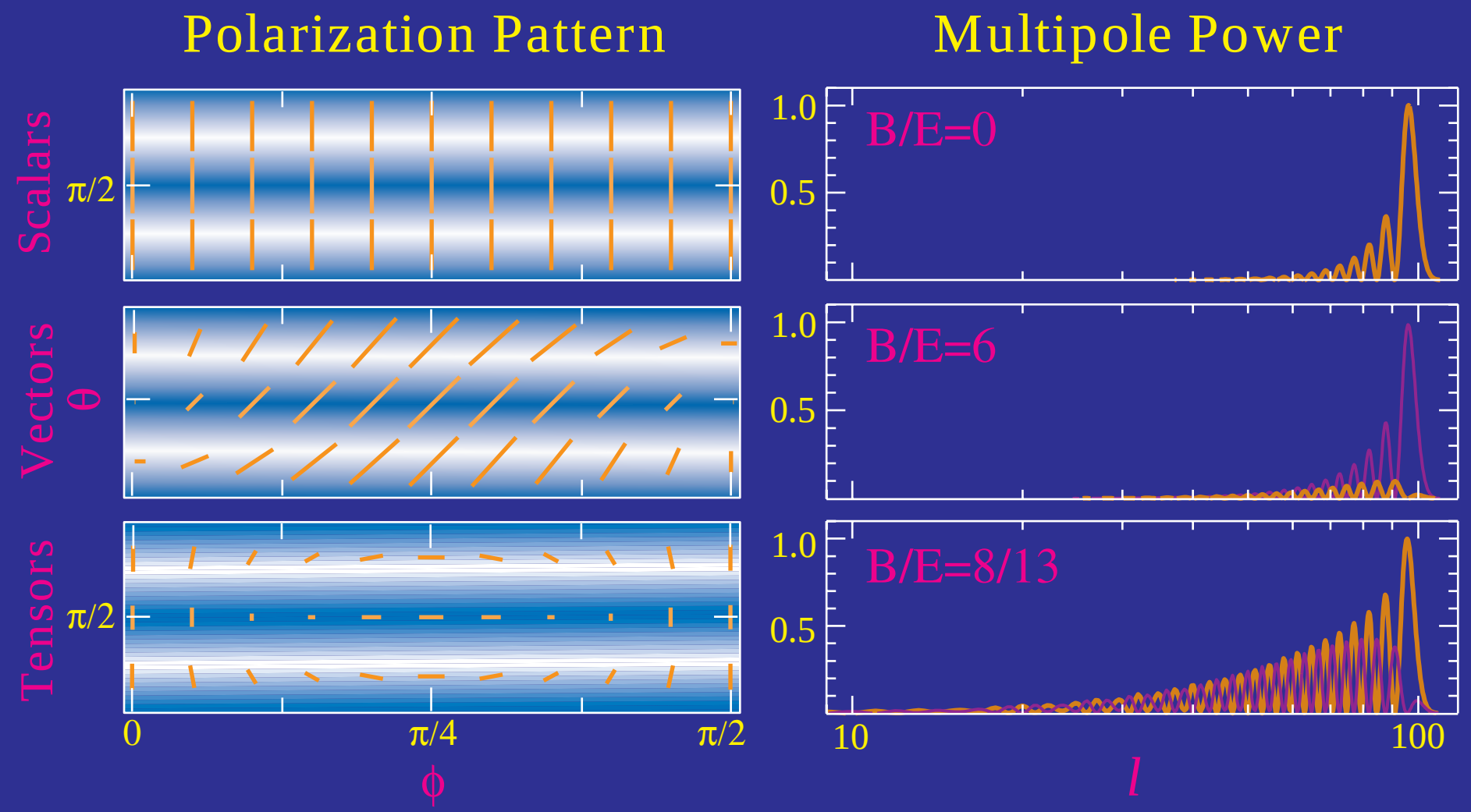
- Thomson scattering generates a pure *E*-pattern locally
- Plane wave perturbation modulates the amplitude
- If modulation: in a 0º or 90º direction then *E*
  in a 45º direction as polarization then *B*



Hu & White (1997)

Patterns and Perturbation Types

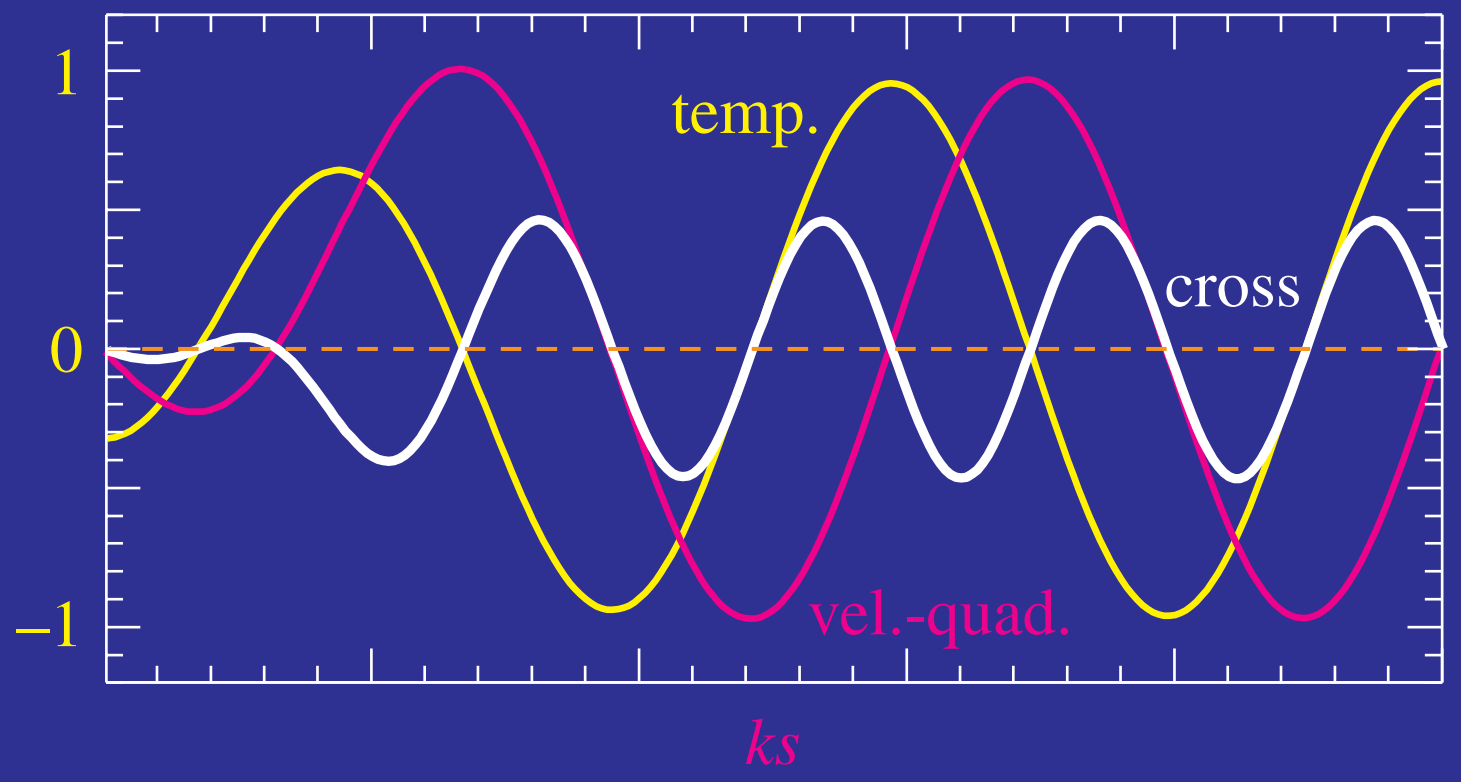
- Amplitude modulated by plane wave → Principle axis
- Direction detemined by perturbation type → Polarization axis



Kamionkowski, Kosowski, Stebbins (1997); Zaldarriaga & Seljak (1997); Hu & White (1997)

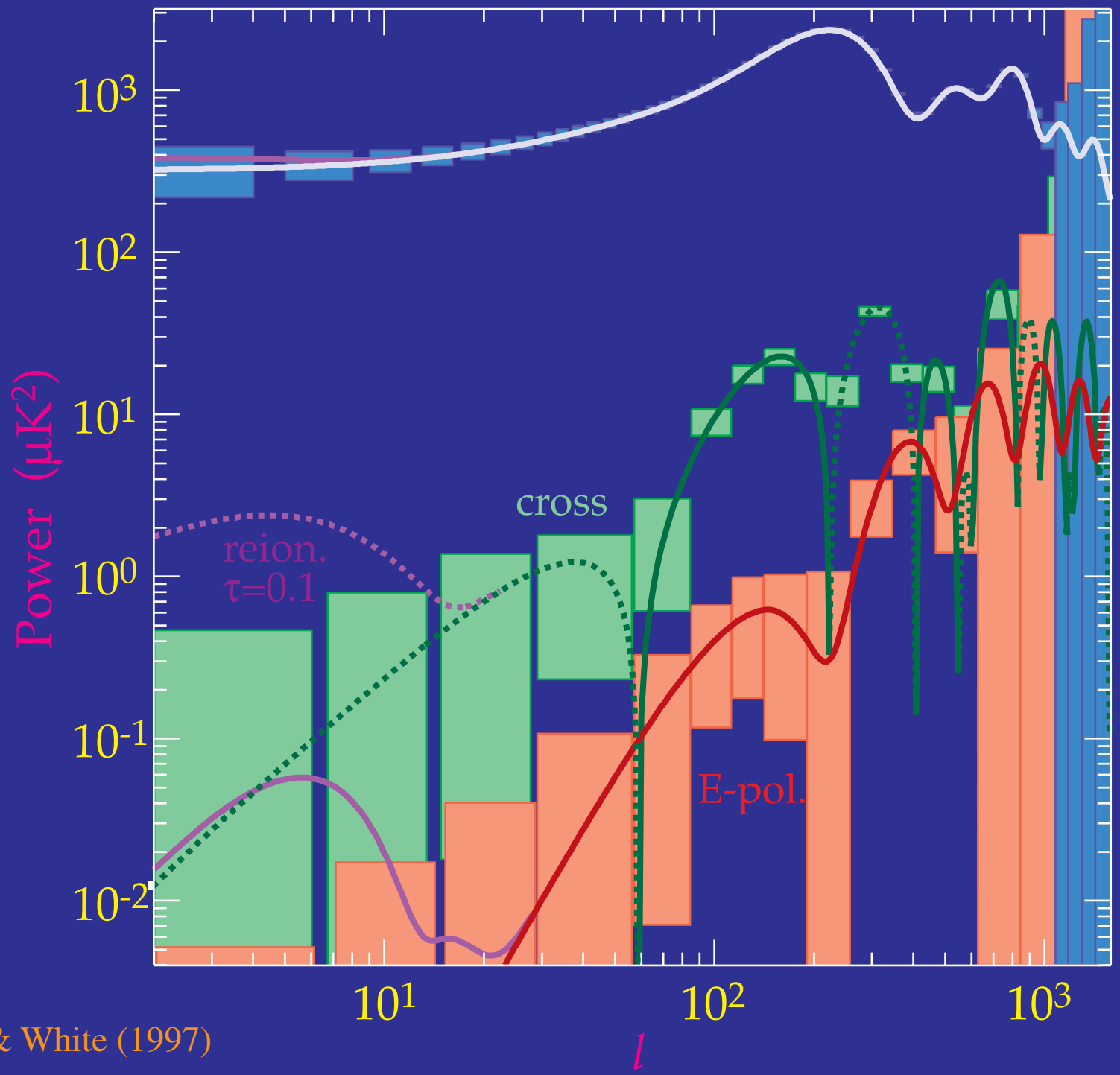
Acoustic Peaks in the Polarization

- Scalar quadrupole follows the velocity perturbation
- Acoustic velocity out of phase with acoustic temperature
- Correlation oscillates at twice the frequency



Hu & White (1997)

Scalar Power Spectra



Hu & White (1997)

Testing Inflation

Testing Inflation

- Inflation required to causally carry density fluctuations outside horizon
- Naive test: above 1º, we are looking above the horizon at last scattering
  hence any power indicates inflation
- Problem: temperature anisotropies can be generated after last scattering

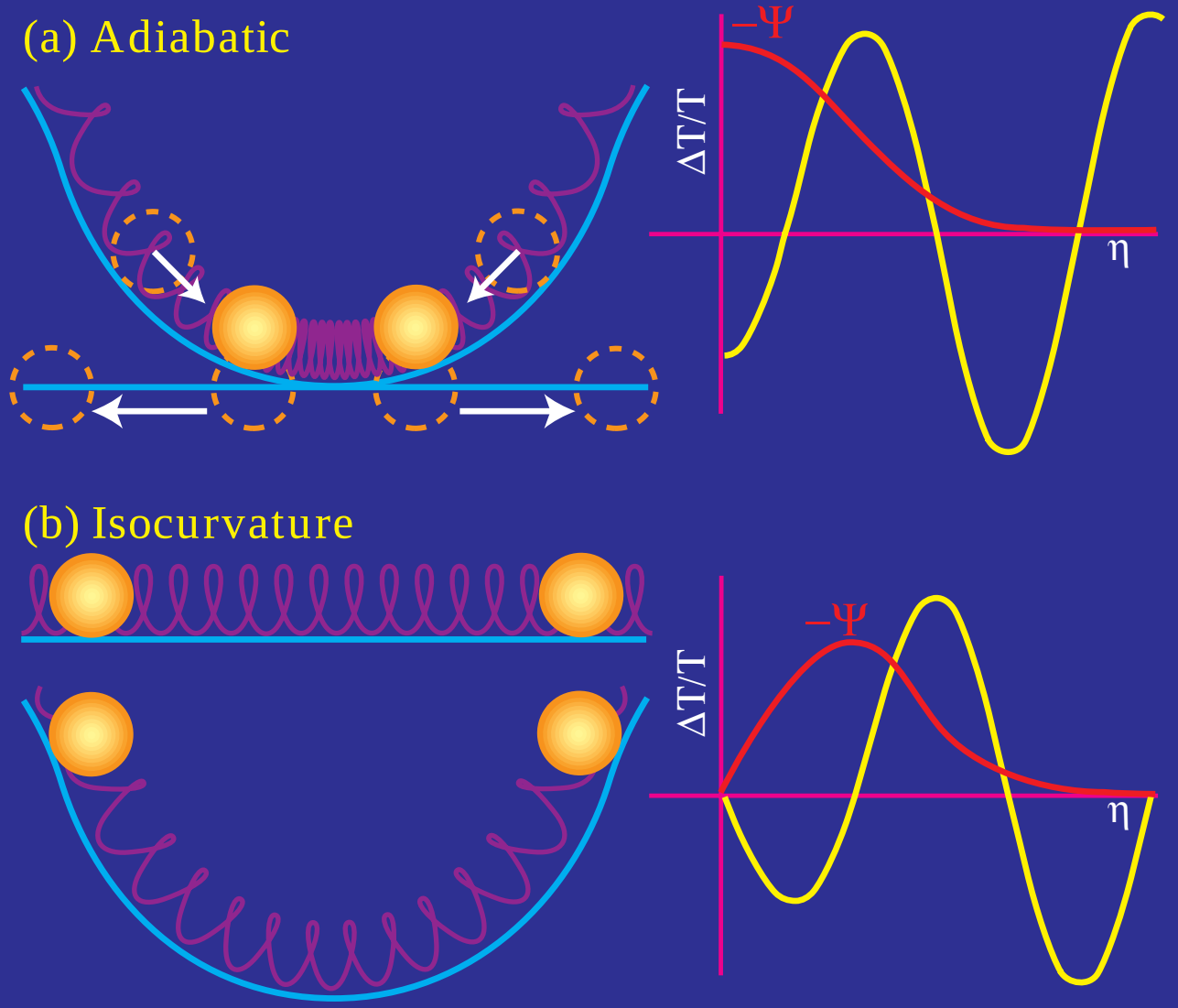
- Solution: find effects confined to last scattering
  acoustic oscillations: probes potentials just before horizon crossing
  pros – easy to measure
  cons – indirect (dynamical assumptions)

  polarization: causal scalar fluctuations fall off rapidly (l^6)
  outside horizon (if first peak right then scalar dominated)
  pros – direct
  cons – difficult to measure

Hu & White (1997); Spergel & Zaldarriaga (1997)

Testing Inflation / Initial Conditions

- Superluminal expansion (inflation) required to generate superhorizon curvature (density) perturbations
- Else perturbations are isocurvature initially with matter moving causally
- Curvature (potential) perturbations drive acoustic oscillations
- Ratio of peak locations
- Harmonic series:
  curvature 1:2:3...
  isocurvature 1:3:5...



Hu & White (1996)

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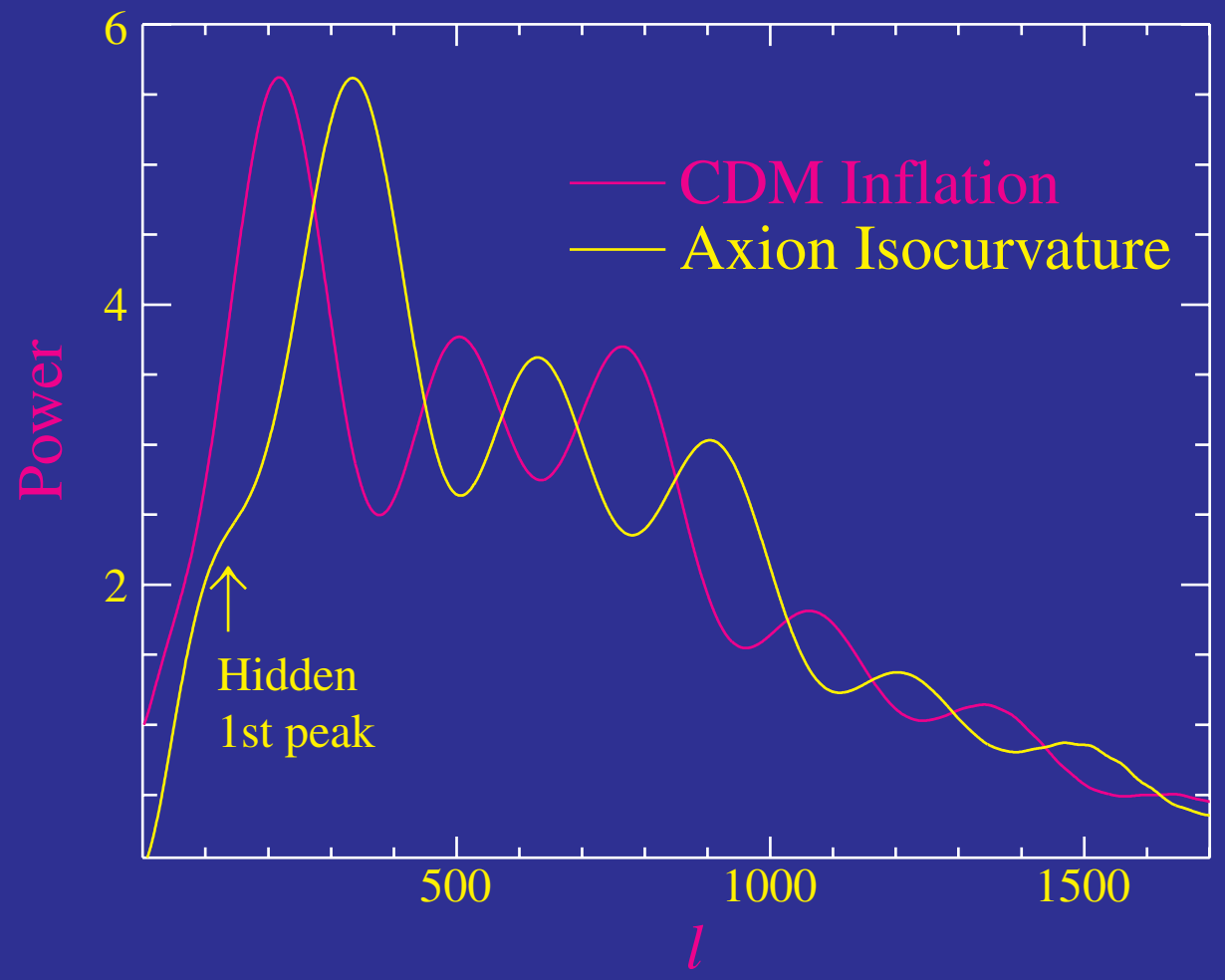
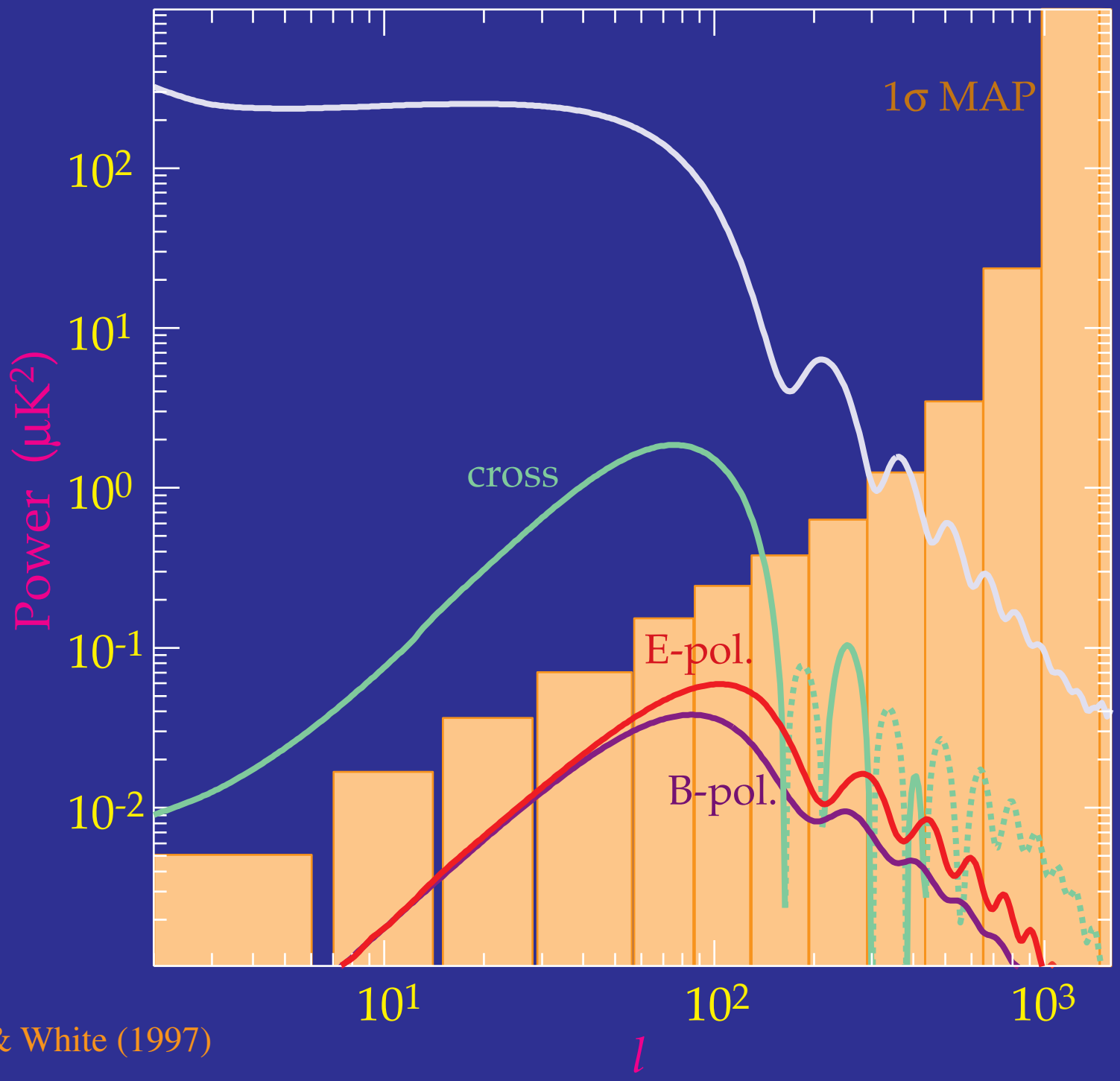


Hu & White (1996)

Tensor Power Spectra



Hu & White (1997)

Inflationary Dynamics

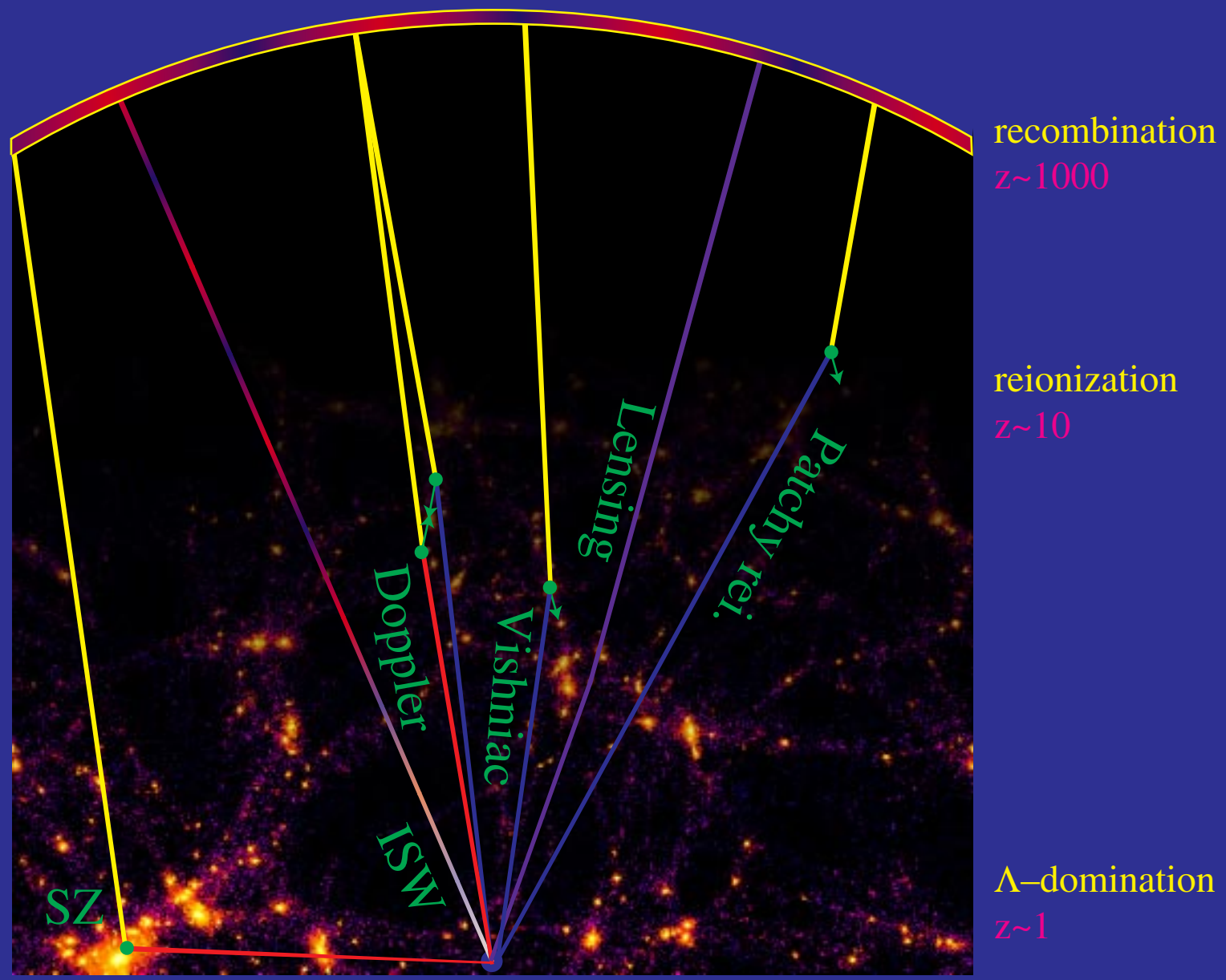
- Tensor Amplitude $\propto$ V (inflaton potential)
- Current upper limits: inflationary energy scale $< 2 \times 10^{16}$ GeV

- Tensor / Scalar amplitude $\propto (V'/V)^2$
  Scalar slope function of $(V'/V)^2$, (V''/V)
- Constrain shape, test models of inflation
  Planck Errors: T/S ± 0.35 (temp) ± 0.012 (+pol.)
  n_s ± 0.04 (temp) ± 0.008 (+pol.)

- Consistency Relation – test of slow-roll inflation
  Tensor slope $\propto (V'/V)^2$
- Meaningful test by Planck only possible if T/S close to current limits
- Next next-generation satellite dedicated to polarization?

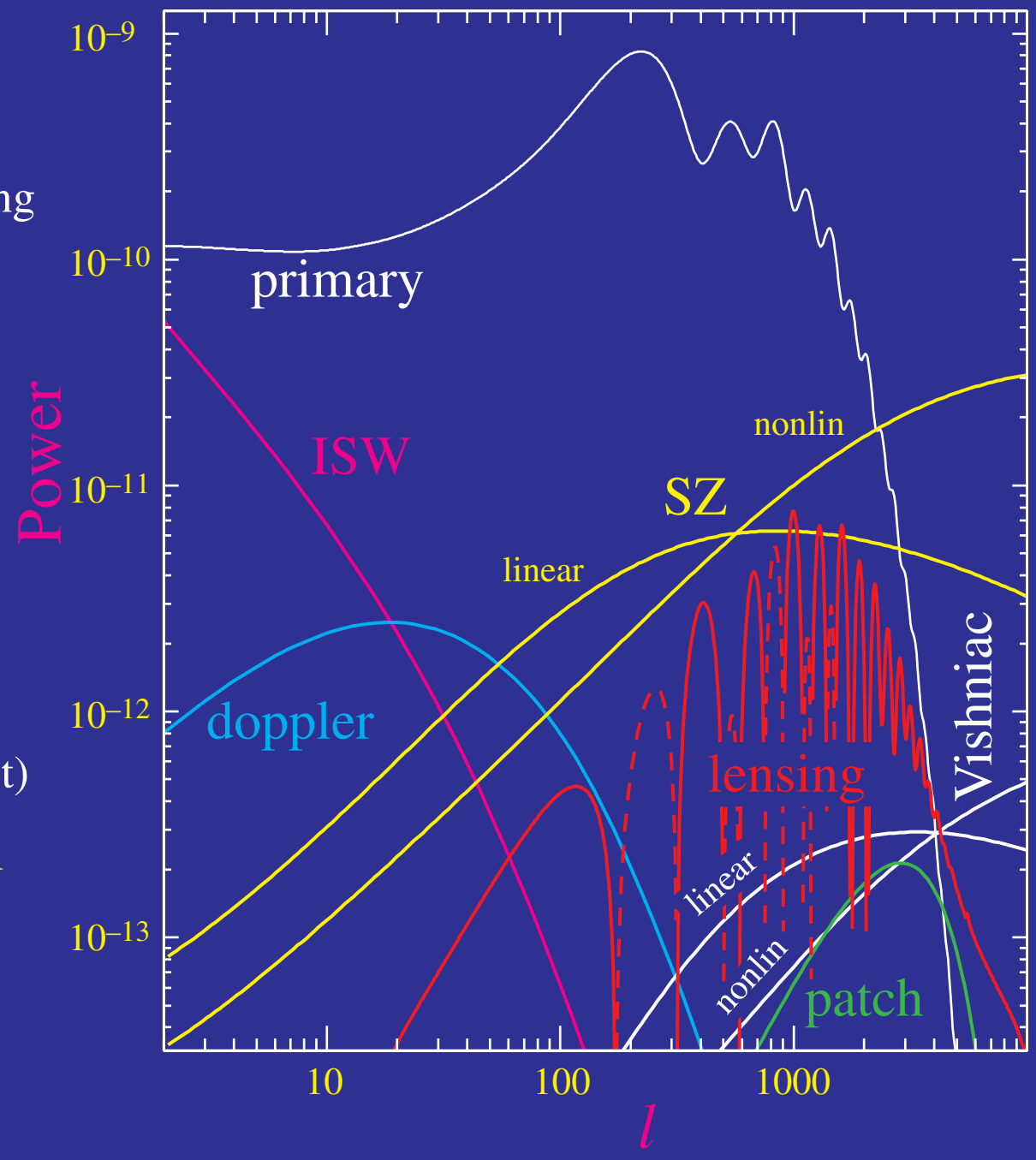
Secondary Anisotropies

Physics of Secondary Anisotropies



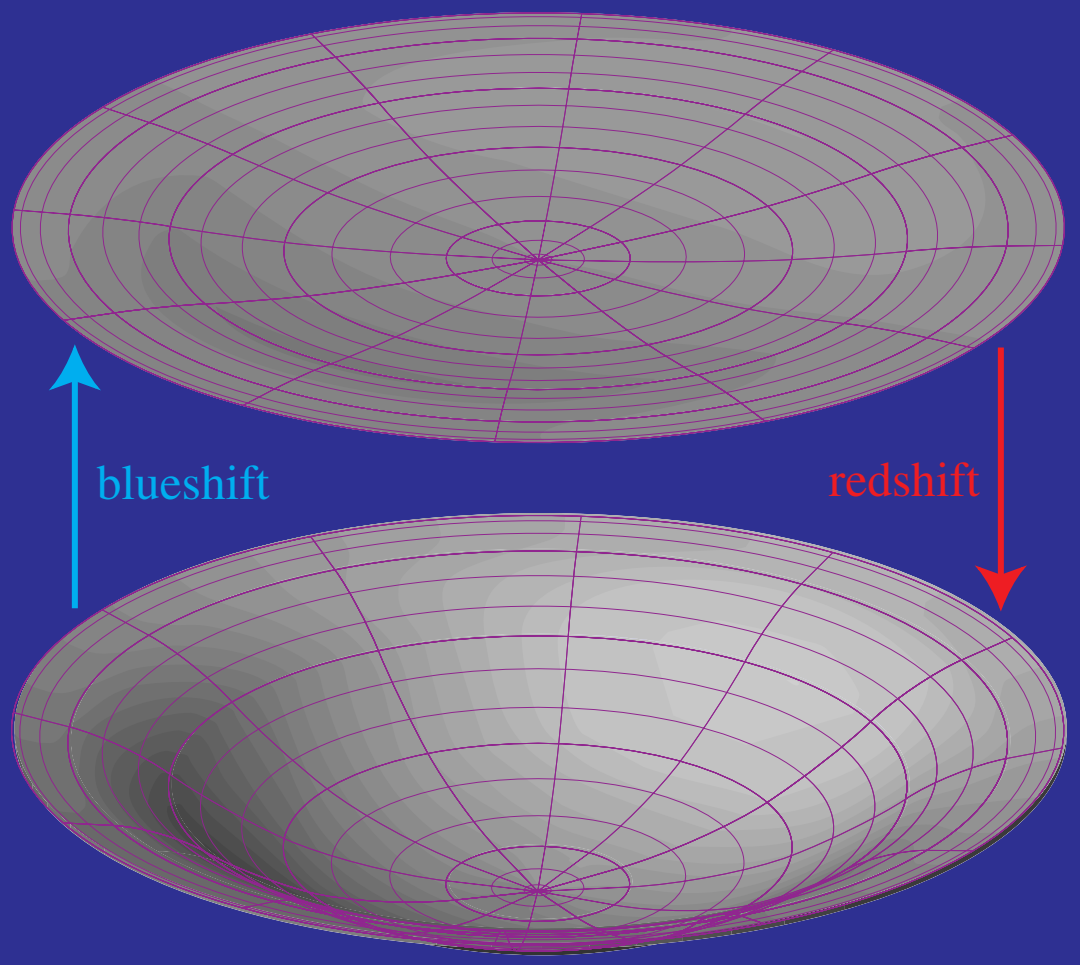
Secondary Anisotropies: Power Spectra

- Gravitational Effects
  - ISW Effect
    (redshift from decaying potentials)
  - Weak Lensing
    (smooths peaks and generates power <1')
- Scattering Effects
  - Doppler Effect
  - Vishniac Effect
    (LSS kinetic SZ effect)
  - Patchy Reionization
  - SZ effect
    (LSS thermal)



Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00}=-(1+\Psi)^2\ \delta_{ij}$



Kofman & Starobinskii (1985)

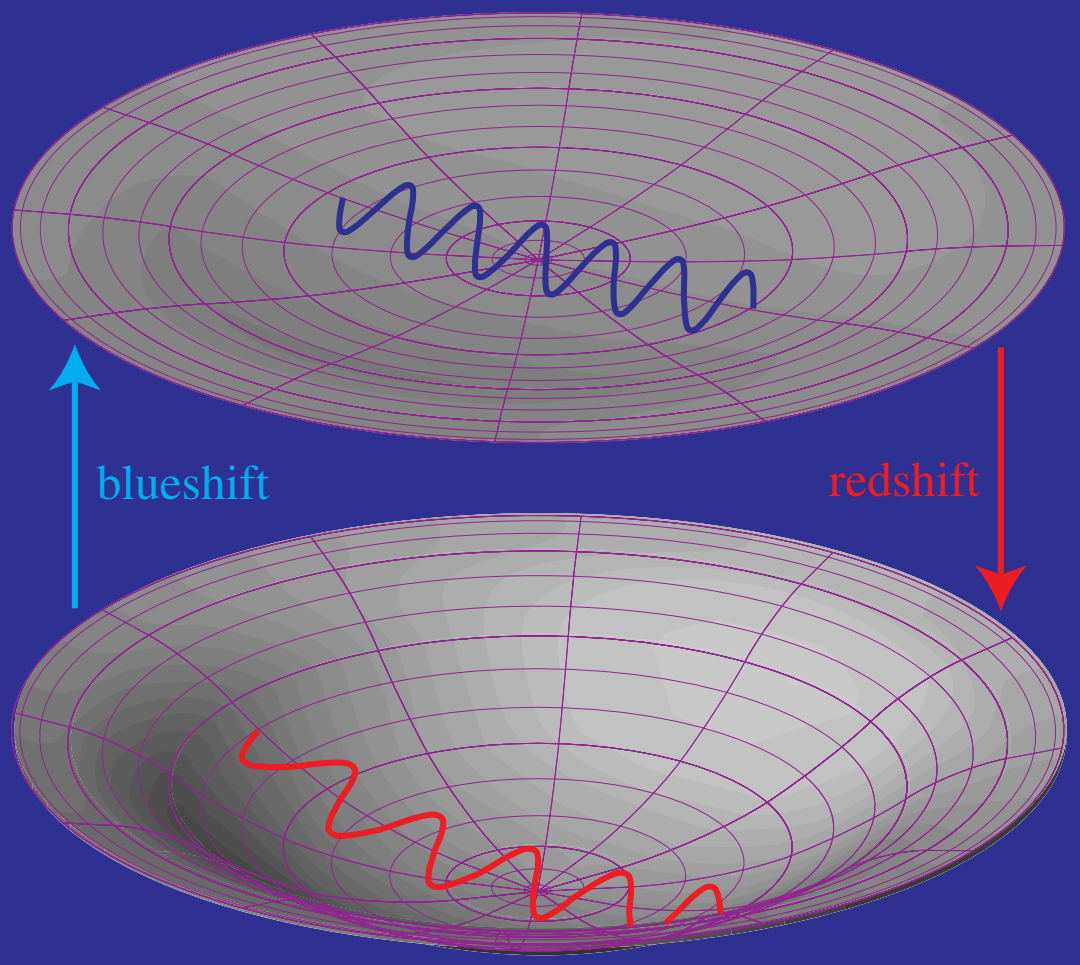
Hu & Sugiyama (1994)

Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00}=-(1+\Psi)^2\,\delta_{ij}$
- Perturbed cosmological redshift

  $g_{ij}=a^2(1+\Psi)^2\,\delta_{ij}$

  $\delta T/T = -\delta a/a = \Psi$



Kofman & Starobinskii (1985)

Hu & Sugiyama (1994)

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- Perturbed cosmological redshift

  $g_{ij}=a^2(1+\Psi)^2\,\delta_{ij}$

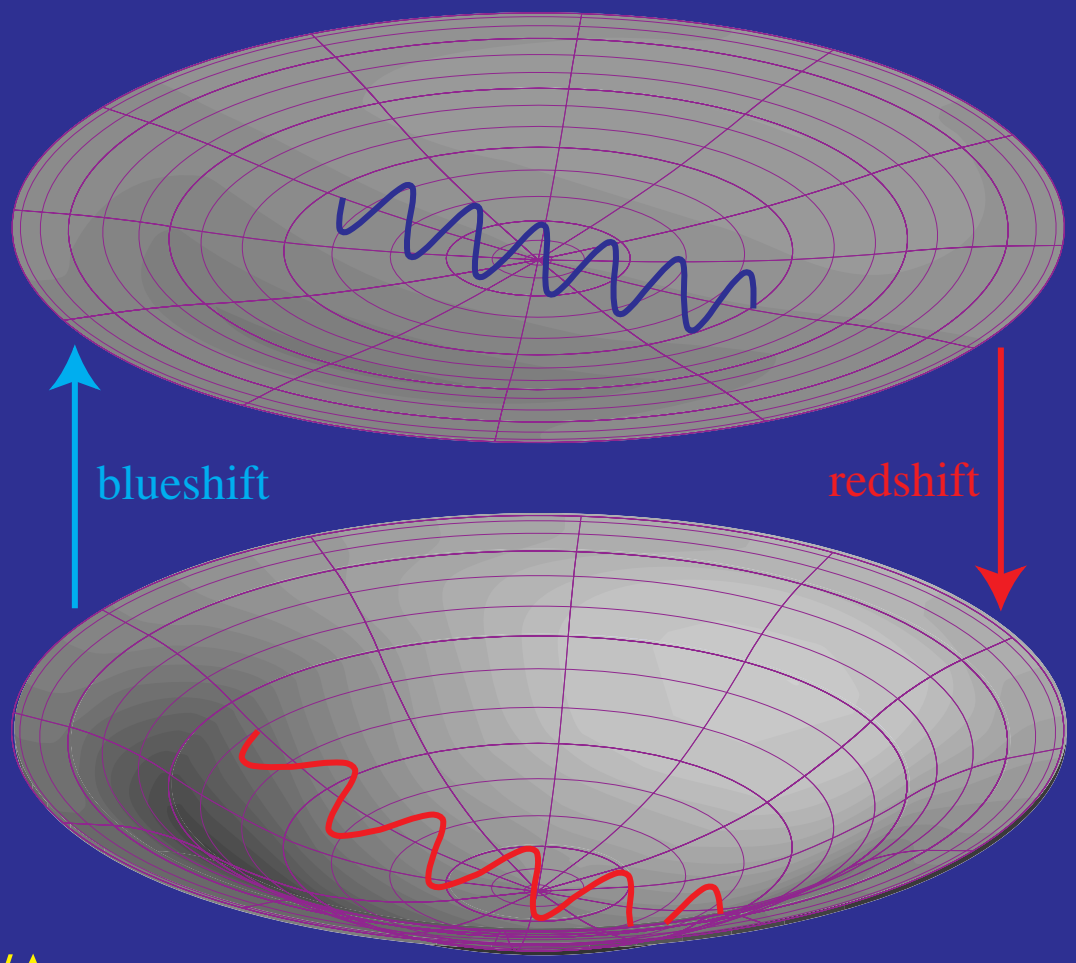
  $\delta T/T = -\delta a/a = \Psi$
- Time–varying potential

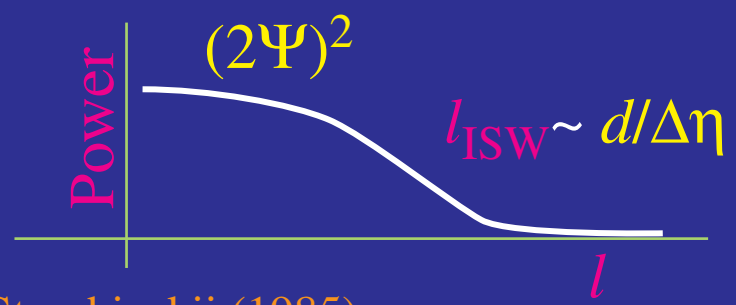
  Rapid compared with λ/c

  $\delta T/T = -2\Delta\Psi$

  Slow compared with λ/c

  redshift–blueshift cancel
- Imprint characteristic time scale of decay in angular spectrum





Kofman & Starobinskii (1985)

Hu & Sugiyama (1994)

Calculation of Secondary Anisotropies

- Addition of angular momentum gives

$$\begin{matrix}\text{multipole}\\ \text{moment}\end{matrix} = \int \begin{pmatrix}\text{clebsch}\\ \text{gordan}\end{pmatrix}\begin{pmatrix}\text{bessel}\\ \text{function}\end{pmatrix} \text{Source} \; d\begin{pmatrix}\text{line of}\\ \text{sight}\end{pmatrix}$$

- Primary anisotropies: source sharply peaked at last scattering

  Tight Coupling Approximation:

$$\begin{matrix}\text{multipole}\\ \text{moment}\end{matrix} \sim \begin{pmatrix}\text{clebsch}\\ \text{gordan}\end{pmatrix}\begin{pmatrix}\text{bessel}\\ \text{function}\end{pmatrix} \int \text{Source} \; d\begin{pmatrix}\text{line of}\\ \text{sight}\end{pmatrix}$$

- Secondary anisotropies: source slowly–varying in time

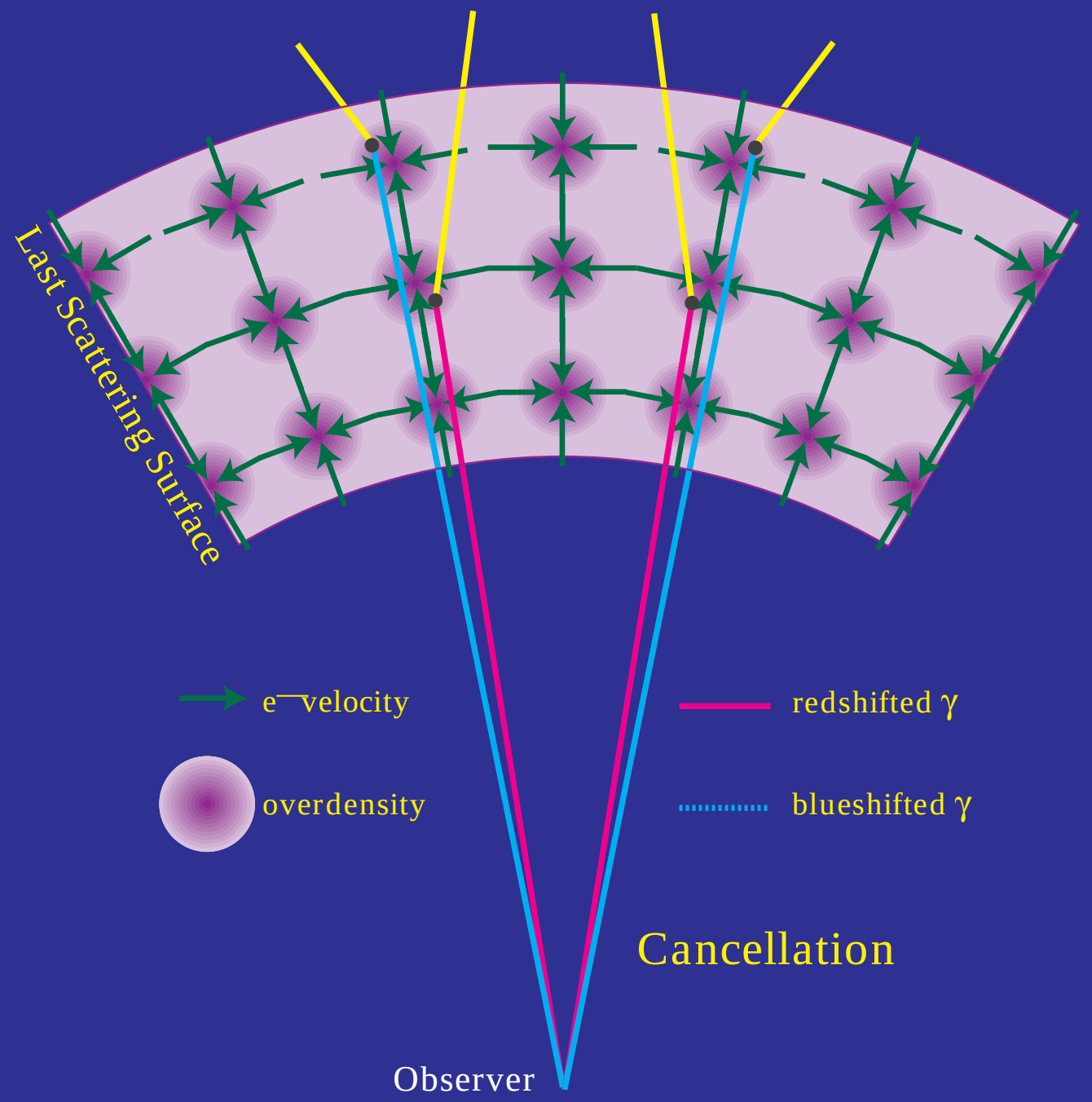
  Weak Coupling Approximation:

$$\begin{matrix}\text{multipole}\\ \text{moment}\end{matrix} \sim \text{Source} \begin{pmatrix}\text{clebsch}\\ \text{gordan}\end{pmatrix} \int \begin{pmatrix}\text{bessel}\\ \text{function}\end{pmatrix} d\begin{pmatrix}\text{line of}\\ \text{sight}\end{pmatrix}$$

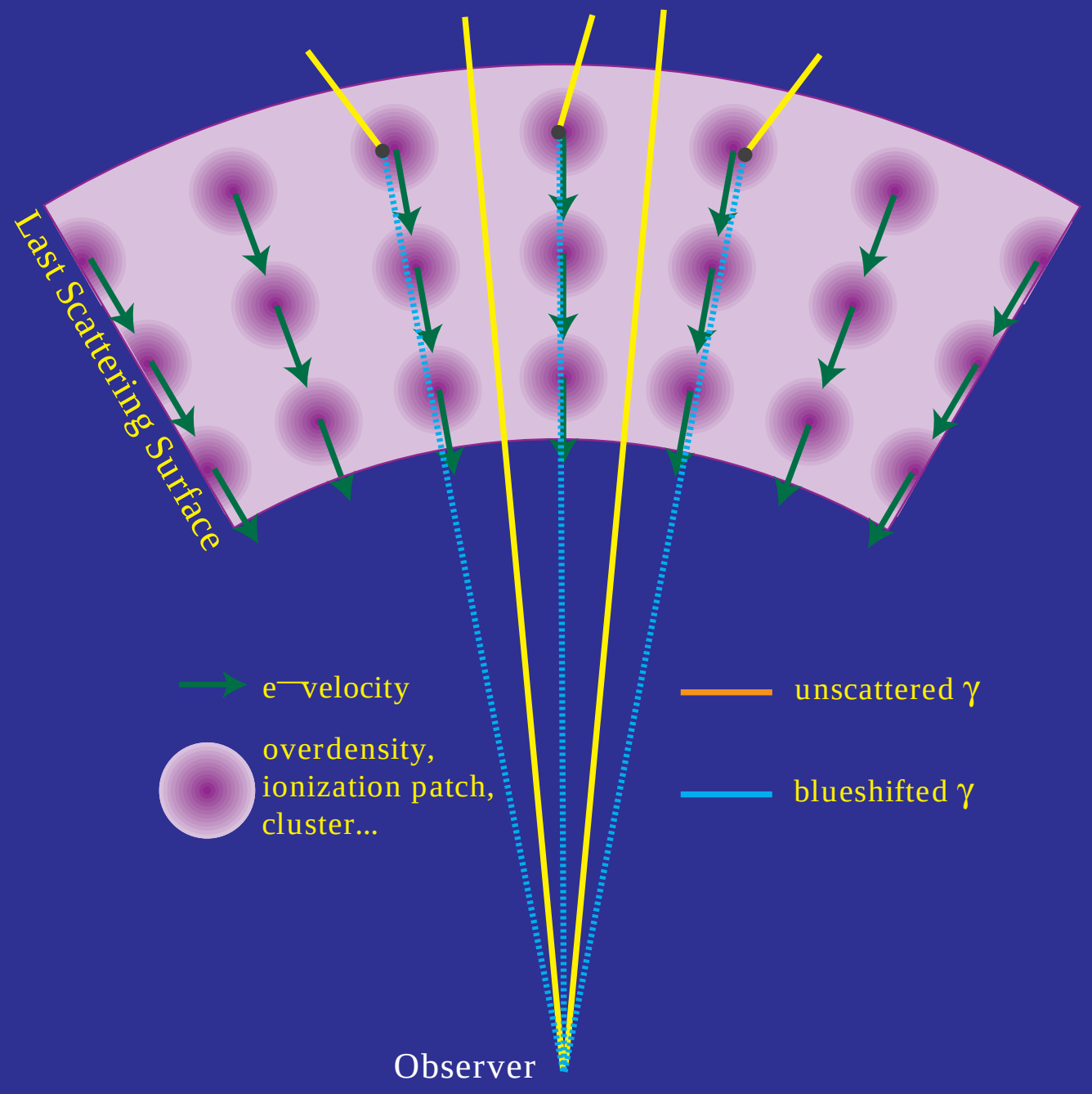
- Log power spectrum of CMB ~ (cg)*Log power spectrum of source / l
- Scalar source and scalar field on sky: weak coupling = limber approx.

Hu & White (1996); Hu (2000)

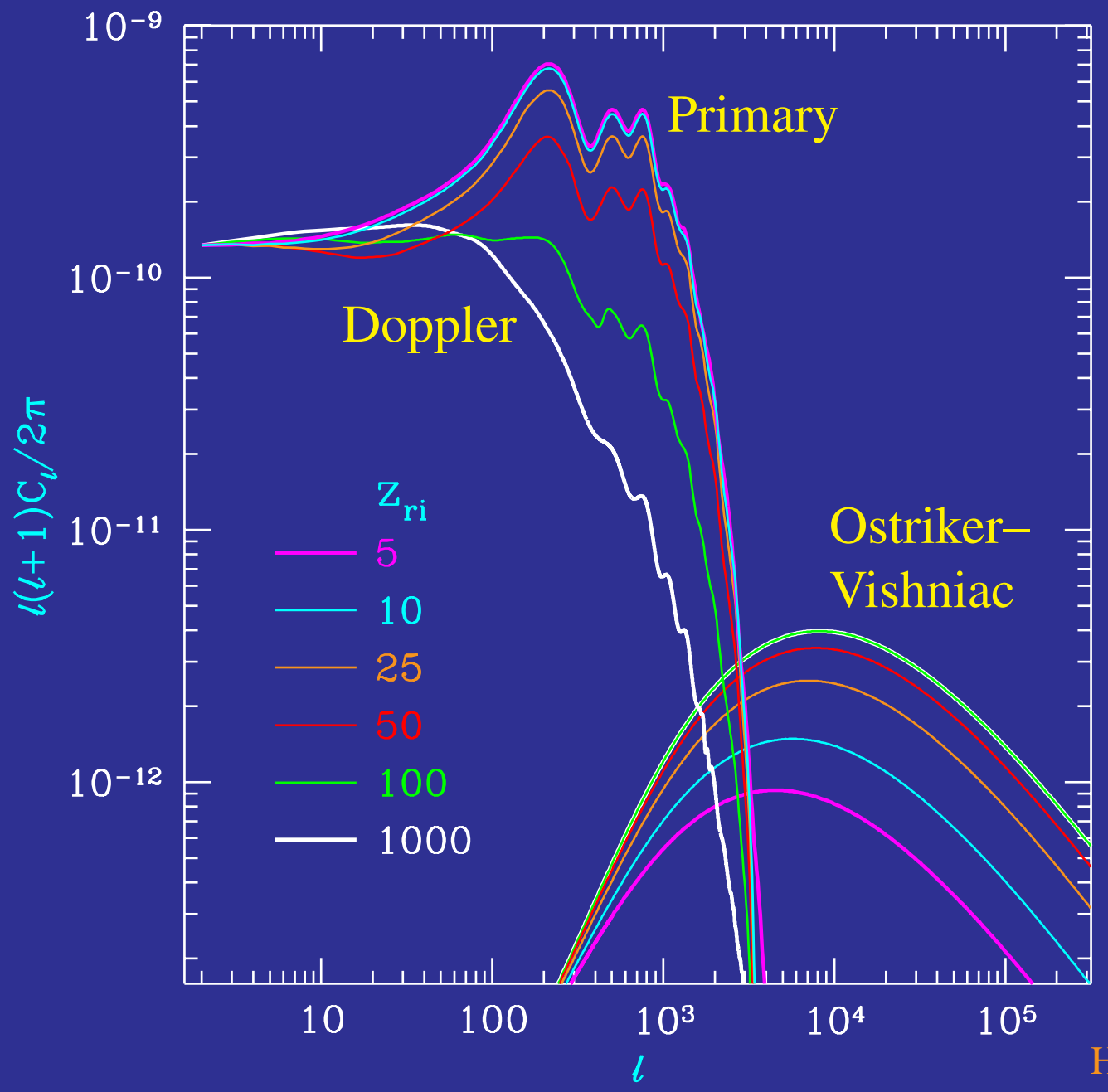
Cancellation of the Linear Effect



Modulated Doppler Effect



Ostriker–Vishniac Effect



Hu & White (1996)

Future Directions

- Precision measurements will allow sharp consistency tests of the inflationary CDM paradigm and determination of its underlying parameters

- Polarization can in principle enable probes of the early universe (inflationary dynamics)

  [challenge to extract from foregrounds and systematic effects]

- Secondary anisotropies probe the evolution of large scale structure in the universe

  [challenge to separate from primary anisotropies, foregrounds and each other]
  spectral signatures, non-gaussian signatures and cross correlation