The Physics of CMB Anisotropies

Erice, December 2000

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Sound Physics Seen

W. Hu – May 2000

- First: Nailed!
- Second: Constrained!
- Third: Hint...

\[ \Delta T (\mu K) \]

\[ l \text{ (multipole)} \]

\[ \theta \text{ (degrees)} \]
Projected Planck Errors

ΔT (μK) vs. θ (degrees) and l (multipole)
Outline

The Present
• Thermal History
• Initial Conditions
• Acoustic Oscillations
• Calculations
• First Peak
• Second Peak

The Future
• Higher Peaks
• Fish(er)ing for Parameters
• Polarization
• Testing Inflation
• Secondary Gravitational Effects
• Secondary Scattering
The Present
Thermal History

- $z > 1000; \ T_\gamma > 3000K$
  - Hydrogen ionized
  - Free electrons glue photons to baryons

\[
\gamma \leftrightarrow e^- \leftrightarrow p
\]
  - Compton scattering
  - Coulomb interactions

Photon–baryon fluid
Potential wells that later form structure

mean free path $\ll$ wavelength
Thermal History

- $z > 1000; \ T_\gamma > 3000K$
  - Hydrogen ionized
  - Free electrons glue photons to baryons
  - Photon–baryon fluid
  - Potential wells that later form structure

- $z \sim 1000; \ T_\gamma \sim 3000K$
  - Recombination
  - Fluid breakdown

- $z < 1000; \ T_\gamma < 3000K$
  - Gravitational redshifts & lensing
  - Reionization; rescattering

![Diagram](attachment:image.png)

- $\lambda \sim k^{-1}$
- $\theta \sim l^{-1}$
- Observer
Initial Conditions
Inflation and the Initial Conditions

- Inflation: (nearly) scale-invariant curvature (potential) perturbations
- Superluminal expansion $\rightarrow$ superhorizon scales $\rightarrow$ "initial conditions"
- Accompanying temperature perturbations due to cosmological redshift

Potential perturbation $\Psi = \text{time-time metric perturbation}$

$\delta t/t = \Psi \quad \rightarrow \quad \delta T/T = -\delta a/a = -2/3 \delta t/t = -2/3 \Psi$

Sachs & Wolfe (1967); White & Hu (1997)
Initial Conditions & the Sachs-Wolfe Effect

- **Initial** temperature perturbation
- **Observed** temperature perturbation
  Gravitational redshift: $\Psi$
  + Initial temperature: $+ \Theta$
- Potential = time-time $\Psi = \delta t/t$
- Metric perturbations

![Diagram showing time and space dimensions with a wave-like pattern representing Newtonian and comoving coordinates.](image-url)
Initial Conditions & the Sachs-Wolfe Effect

- **Initial** temperature perturbation
- **Observed** temperature perturbation
  - Gravitational redshift: \( \Psi \)
  - + Initial temperature: + \( \Theta \)
- Potential = time-time \( \Psi = \delta t/t \)
- Metric perturbations
- Matter–dominated expansion:
  \( a \propto t^{2/3}, \quad \delta a/a = 2/3 \quad \delta t/t \)
- Temperature falls as:
  \( T \propto a^{-1} \)
- Temperature fluctuation:
  \( \delta T/T = -\delta a/a \)
Initial Conditions & the Sachs-Wolfe Effect

- **Initial** temperature perturbation
- **Observed** temperature perturbation
  
  Gravitational redshift: $\Psi$
  
  + Initial temperature: $+ \Theta$
  
- Potential = time-time $\Psi = \delta t/t$

- Metric perturbations
  
- Matter-dominated expansion:
  
  $a \propto t^{2/3}$, $\delta a/a = 2/3 \delta t/t$

- Temperature falls as:
  
  $T \propto a^{-1}$

- Temperature fluctuation:
  
  $\delta T/T = -\delta a/a$

- Result
  
  Initial temperature perturbation: $\Theta \equiv \delta T/T = -\delta a/a = -2/3 \delta t/t = -2/3 \Psi$

  Observed temperature perturbation: $(\delta T/T)_{obs} = \Theta + \Psi = 1/3 \Psi$

Sachs & Wolfe (1967)  
White & Hu (1997)
Acoustic Oscillations
Acoustic Oscillations

- Photon pressure resists compression in potential wells
- Acoustic oscillations

Peebles & Yu (1970)

Acoustic Oscillations

- Photon pressure resists compression in potential wells
- Acoustic oscillations
- Gravity displaces zero point
  \[ \Theta \equiv \delta T/T = -\Psi \]

Oscillation amplitude = initial displacement from zero point
  \[ \Theta - (-\Psi) = 1/3 \Psi \]

References:
- Peebles & Yu (1970)
Acoustic Oscillations

- Photon **pressure** resists compression in **potential wells**
- Acoustic oscillations
- Gravity displaces zero point
  \[ \Theta \equiv \frac{\delta T}{T} = -\Psi \]

- Oscillation **amplitude** = initial displacement from zero pt.
  \[ \Theta - (-\Psi) = \frac{1}{3} \Psi \]
- Gravitational redshift: observed
  \[ (\frac{\delta T}{T})_{\text{obs}} = \Theta + \Psi \]
  oscillates around **zero**

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Peebles & Yu (1970)  
Acoustic Oscillations

- Photon pressure resists compression in potential wells
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  oscillates around zero

Second Extrema

Peebles & Yu (1970)

Harmonic Peaks

- Oscillations frozen at last scattering
- Wavenumbers at extrema = peaks
- Sound speed $c_s$

$\frac{\Delta T}{T} = -\frac{|\Psi|}{3}$

$\frac{\pi}{\text{sound horizon}}$

$\eta$

Harmonic Peaks

- Oscillations frozen at last scattering
- Wavenumbers at extrema = peaks
- Sound speed $c_s$

- Frequency $\omega = kc_s$; conformal time $\eta$
- Phase $\propto k$; $\phi = \int_0^{\text{last scattering}} d\eta \omega = k$ sound horizon
- Harmonic series in sound horizon $\phi_n = n\pi \rightarrow k_n = n\pi/\text{sound horizon}$

$\Delta T/T \quad \eta \quad \Delta T/T \quad \eta$

$-|\Psi|/3 \quad k_1 = \pi/\text{sound horizon} \quad k_2 = 2k_1 \quad -|\Psi|/3$

Hu & Sugyama (1995); Hu, Sugiyama & Silk (1997)
Projection into Angular Peaks

- Peaks in spatial power spectrum
- Projection on sphere
- Spherical harmonic decomposition

- Maximum power at $l = kd$
- Extended tail to $l \ll kd$
- Described by spherical bessel function $j_l(kd)$

$\text{Bond & Efstathiou (1987)} \quad \text{Hu & Sugiyama (1995); Hu & White (1997)}$
Projection into Angular Peaks

- Peaks in spatial power spectrum
- Projection on sphere
- Spherical harmonic decomposition

- Maximum power at $l = kd$
- Extended tail to $l \ll kd$
- 2D Transfer Function $T^2(k,l) \sim (2l+1)^2 \left[\Delta T/T\right]^2 j^2_l(kd)$

Transfer Function

Bessel Functions

Hu & Sugiyama (1995)
Doppler Effect

- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature
Doppler Effect

- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature
- Zero point not shifted by baryon drag
- Increased baryon inertia decreases effect
  \[ m_{\text{eff}} V^2 = \text{const.} \quad V \propto m_{\text{eff}}^{-1/2} = (1+R)^{-1/2} \]
Doppler Peaks?

- Doppler effect has lower amplitude and weak features from projection

\[ j_l(kd)Y_l^0 Y_0^0 \]

\[ j_l(kd)Y_l^0 Y_1^0 \]

Temperature peak

Doppler no peak

Hu & Sugiyama (1995)
Relative Contributions

$kd$

Hu & Sugiyama (1995); Hu & White (1997)
Relative Contributions

Hu & Sugiyama (1995); Hu & White (1997)
Mechanics of the Calculation

- Radiation distribution: \( f(x=\text{position}, t=\text{time}; n=\text{direction}, \nu=\text{frequency}) \)
- Expand in basis functions: (local angular dependence \( \otimes \) spatial dependence)
  \[ s Y_l^m \otimes e^{i\mathbf{k} \cdot \mathbf{x}} \]

Sources
(radiation & metric: monopole, dipole, quadrupole)

hierarchy
- plane wave ~ gradient
  \[ s Y_{l'}^m \otimes Y_l^0 \]
  \[ |l'\pm 1| \]

Addition of Angular Momentum
- plane wave \( \propto j_l Y_l^0 \)
  \[ s Y_l^m \otimes Y_{l''}^0 \]
  \[ |l\pm l'| \]
  \[ \sum_{l'\pm l''} (\text{clebsch-gordan}) j_l \]

Observables
-original codes
-Bond & Efstathiou (1984)
-Vittorio & Silk (1983)

- semi-analytic
-Seljak & Zaldarriaga (1996)
-Hu & White (1997)

-CMBFast
-Seljak (1994)
Semi-Analytic Calculation

- Treat Sources in the Tight Coupling Approximation
  - Expand the Boltzmann hierarchy equations in $1/(\text{optical depth per } \lambda)$
  - Also the trick to numerically integrating the stiff hierarchy equations
  - Closed Euler equation + Continuity equation = Oscillator equation

- Project Sources at Last Scattering
  - Integral equations
  - Visibility function of recombination
The First Peak:

Curvature
Age
Dark Energy
Shape of the First Peak

- Consistent with potential wells in place on superhorizon scale (inflation)
- Sharp fall from first peak indicates no continuous generation (defects)
Angular Diameter Distance

- A Classical Test
  Standard(ized) comoving ruler
  Measure angular extent
  Absolute scale drops out
  Infer curvature

- Upper limit 1st Peak Scale (Horizon)
  Upper limit on Curvature

- Calibrate 2 Physical Scales
  Sound horizon (peak spacing)
  Diffusion scale (damping tail)

Kamionkowski, Spergel & Sugiyama (1994)
Hu & White (1996)
Curvature and the Cosmological Constant

Gravitational Redshift

Shifted Acoustic Signature

$\Omega_K$

$\Omega_\Lambda$

Power

l

$\Omega W. Hu 2/98$
A Flat Universe!...

Riess et al. (1998)

Perlmutter et al. (1998)
First Peak Location

- BOOM's parabolic peak fit $185 < l_1 < 209 \ (2\sigma)$
- MAX's value $l_1 \sim 220$
Are They Consistent?

- Using $\Lambda$CDM models: $184 < l_1 < 216$ (2$\sigma$; BOOM)
- Joint analysis: $194 < l_1 < 218$ (2$\sigma$; BOOM+MAX)

Hu, Fukugita, Zaldarriaga, Tegmark (2000)
Is the Scale too Large?

- Fiducial flat $\Omega_m=0.35$, $h=0.65$ model: $l_1=221$ (excluded at $\sim 2.5\sigma$)
  (a) positive curvature (b) high Hubble constant / matter (c) dark energy

$\Omega_b h^2 > 0.019$

Hu, Fukugita, Zaldarriaga, Tegmark (2000)
Age and Dark Energy

- Flat solutions involve decreasing age of universe through $h$, $\Omega_m$
or dark energy equation of state $w=p/\rho$ (<13–13.5Gyr)

Hu, Fukugita, Zaldarriaga, Tegmark (2000)
Age and Dark Energy

- Region of consistency shrinking – headed to crisis?
- New physics? $w, m\nu, \alpha...$

Hu, Fukugita, Zaldarriaga, Tegmark (2000)
The Second Peak: Dark Baryons (and tilt)
Baryon Drag

- Baryons provide inertia
- Relative momentum density
  \[ R = \frac{\rho_b + p_b}{\rho_\gamma + p_\gamma} \frac{V_b}{V_\gamma} \propto \Omega_b h^2 \]
- Effective mass \( m_{\text{eff}} = (1 + R) \)

Hu & Sugiyama (1995) Low Baryon Content
Baryon Drag

- Baryons provide **inertia**
- Relative momentum density
  \[ R = \frac{(\rho_b + p_b)V_b}{(\rho_\gamma + p_\gamma)V_\gamma} \propto \Omega_b h^2 \]
- Effective **mass** \( m_{\text{eff}} = (1 + R) \)
- Baryons drag photons into potential wells \( \rightarrow \) **zero point** ↑
  - **Amplitude** ↑
  - **Frequency** ↓ \( (\omega \propto m_{\text{eff}}^{-1/2}) \)

- Constant \( R, \Psi \):
  \[ (1 + R)\ddot{\Theta} + (k^2/3)\Theta = -(1 + R)(k^2/3)\Psi \]
  \[ \Theta + \Psi = [\Theta(0) + (1 + R)\Psi(0)] \cos \left[ \frac{k\eta}{\sqrt{3}}(1 + R) \right] - R\Psi \]

High Baryon Content

Hu & Sugiyama (1995)
Baryon Drag

- Baryons provide inertia
- Relative momentum density
  \[ R = \left( \rho_b + p_b \right) V_b / \left( \rho_\gamma + p_\gamma \right) V_\gamma \propto \Omega_b h^2 \]
- Effective mass \( m_{\text{eff}} = (1 + R) \)
- Baryons drag photons into potential wells \( \rightarrow \) zero point
- Amplitude \( \uparrow \)
- Frequency \( \downarrow \) \( (\omega \propto m_{\text{eff}}^{-1/2}) \)

\[ \Theta + \Psi = \left[ \Theta(0) + (1 + R) \Psi(0) \right] \cos \left[ k_\eta / \sqrt{3} (1 + R) \right] - R \Psi \]

Alternating Peak Heights

Hu & Sugiyama (1995)
Baryons in the CMB

- High odd peaks

- Additional Effects
  - Time-varying potential
  - Dissipation/Fluid imperfections

\( \Omega_b h^2 \)

\( \text{Power} \)
• **BOOM** and **MAX** both show a low power at $l_2 = 2.4 \, l_1$

• $H_2 = \text{power at 1st/2nd} = (\Delta T_{l_2}/\Delta T_{l_1})^2$
Height of the Second Peak

- BOOM: \( H_2 = 0.37 \pm 0.044 \ (1\sigma) \)
- BOOM+MAX: \( H_2 = 0.38 \pm 0.04 \ (1\sigma) \)

\[
\Delta H_2 / H_2 = -0.65 \Delta \Omega_b h^2 / \Omega_b h^2 + 0.88 \Delta n / n + 0.14 \Delta \Omega_m h^2 / \Omega_m h^2
\]

75 < \( l \) < 400
400 < \( l \) < 600
(\( \Omega_\Lambda, \Omega_b h^2, n \))

\( \chi^2 \)

\( H_2 \)
• **Excludes fiducial LCDM** \((n=1, \Omega_b h^2=0.02, H_2=0.51)\) at \(\sim 3.3\sigma\)

• **Requires** \(\Omega_b h^2 > 0.022n\) (if \(\Omega_m h^2 > 0.16\), from \(l_1\), flat, \(h<0.8\))
Height of the Second Peak

- Excludes fiducial LCDM \((n=1, \Omega_b h^2=0.02, H_2=0.51)\) at \(~3.3\sigma\)
- Requires \(\Omega_b h^2 > 0.022n\) (if \(\Omega_m h^2>0.16\), from \(l_1\), flat, \(h<0.8\))

Tytler et al. \(~5\%\)

Conservative BBN upper limit

\(\Omega_m h^2=0.16\)

\(\Omega_b h^2\)

\(n\)
Summary

- We have entered a new era of precision cosmology
- First peak in the CMB power spectrum is due to acoustic waves at \( z \approx 1000 \) based on the precise shape measurement
- Consistent with simple inflationary models (scale-invariant superhorizon density perturbations, \( n > 0.85 \))

- First peak consistent with a flat universe but may indicate a young age (prefers \( \sim 10-11 \) Gyr, requires \( <13-14 \) Gyr from \( l_1 < 218 \) [95\%CL] possible causes: high Hubble constant, \( \Omega_m \), or dark energy \( p/\rho \))
- Requires dark baryons at least comparable to big-bang nucleosynthesis (prefers 25-50\% more: low second peak \( H_2 < 0.46 \) [95\%CL])
- Limits dark matter: lack of steep rise to third peak \( \Omega_m h^2 < 0.47 \)

- Still need to resolve the secondary peaks (stronger test of inflation)!
- Parameter degeneracies resolved with 3 peaks
The Future
Higher Peaks:
Matter–Radiation Ratio
and
Breaking the Degeneracies
Driving Effects and Matter/Radiation

- **Potential perturbation:** \[ k^2 \Psi = -4\pi G a^2 \delta \rho \] generated by radiation
- **Radiation → Potential:** inside sound horizon \( \delta \rho / \rho \) pressure supported \( \delta \rho \) hence \( \Psi \) decays with expansion

Hu & Sugiyama (1995)
Driving Effects and Matter/Radiation

- Potential perturbation: \( k^2 \Psi = -4\pi G a^2 \delta \rho \) generated by radiation

- **Radiation \( \rightarrow \) Potential:** inside sound horizon \( \delta \rho/\rho \) pressure supported \( \delta \rho \) hence \( \Psi \) decays with expansion

- **Potential \( \rightarrow \) Radiation:** \( \Psi \)–decay timed to drive oscillation
  \[ -2\Psi + (1/3)\Psi = -(5/3)\Psi \rightarrow 5x \text{ boost} \]

- Feedback stops at **matter domination**

Hu & Sugiyama (1995)
Driving Effects and Matter/Radiation

- Potential perturbation: \( k^2 \Psi = -4\pi Ga^2 \delta \rho \) generated by radiation
- Radiation → Potential: inside sound horizon \( \delta \rho/\rho \) pressure supported \( \delta \rho \) hence \( \Psi \) decays with expansion
- Potential → Radiation: \( \Psi \) decay timed to drive oscillation
  \[ -2\Psi + \frac{1}{3}\Psi = -\frac{5}{3}\Psi \rightarrow 5x \text{ boost} \]
- Feedback stops at matter domination

Hu & Sugiyama (1995)
Matter Density in the CMB

- Amplitude ramp across matter–radiation equality
- Radiation density fixed by CMB temperature & thermal history

- Measure $\Omega_m h^2$ from peak heights

$\Omega_m h^2$ vs. $l$
Dissipation / Diffusion Damping

- Imperfections in the coupled fluid → mean free path $\lambda_C$ in the baryons
- Random walk over diffusion scale: geometric mean of mfp & horizon
  $$\lambda_D \sim \lambda_C \sqrt{N} \sim \sqrt{\lambda_C \eta} \gg \lambda_C$$
- Overtake wavelength: $\lambda_D \sim \lambda$; second order in $\lambda_C/\lambda$
- Viscous damping for $R<1$; heat conduction damping for $R>1$

$N = \eta / \lambda_C$

Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)
Dissipation / Diffusion Damping

- Rapid increase at recombination as mfp $\uparrow$

- Independent of (robust to changes in) perturbation spectrum

- Robust physical scale for angular diameter distance test ($\Omega_K, \Omega_\Lambda$)

Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)
Parameter Estimation
Physical Decomposition & Information

- Combined
- Temperature
- Doppler

Power vs. $l$
Physical Decomposition & Information

- Fluid + Gravity
  → alternating peaks
  → photon-baryon ratio
  → $\Omega_b h^2$
Physical Decomposition & Information

- Fluid + Gravity
  - alternating peaks
  - photon-baryon ratio
  - $\Omega_b h^2$
  - driven oscillations
  - matter–radiation ratio
  - $\Omega_m h^2$
Physical Decomposition & Information

- Fluid + Gravity
  - alternating peaks
  - photon-baryon ratio
  - $\Omega_b h^2$
  - driven oscillations
  - matter–radiation ratio
  - $\Omega_m h^2$

- Fluid Rulers
  - sound horizon
  - damping scale
• Fluid + Gravity
  → alternating peaks
  → photon-baryon ratio
  → $\Omega_b h^2$
  → driven oscillations
  → matter–radiation ratio
  → $\Omega_m h^2$

• Fluid Rulers
  → sound horizon
  → damping scale

• Geometry
  → angular diameter
distance $f(\Omega_\Lambda, \Omega_K)$
  + flatness or no $\Omega_\Lambda$,
  → $\Omega_\Lambda$ or $\Omega_K$
Cosmological Parameters in the CMB

Baryon–Photon Ratio

Matter–Radiation Ratio

Curvature

Cosmological Constant

$\Omega_b h^2$

$\Omega_m h^2$

$\Omega_K$

$\Omega_\Lambda$

W.Hu 2/98
The Ubiquitous Fisher Matrix

- The Fisher matrix is defined in terms of the likelihood (or signal \( C_l \) and noise \( N_l \) power spectra) as

\[
F_{ij} = -\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \rangle = \frac{1}{2} \sum_{\ell} \frac{(2\ell + 1) f_{\text{sky}}}{(C_\ell + N_\ell)^2} \frac{\partial C_\ell}{\partial p_i} \frac{\partial C_\ell}{\partial p_j}
\]

- Its inverse, the curvature matrix, gives the optimal errors on \( p \), \( \sigma^2=(F^{-1})_{ii} \) including sampling and noise variance

- Useful for identifying degeneracies / constrained directions

- Problems: accuracy of derivatives and underlying parameterization can lead to widely diverging estimates:

  \[
  \sigma(h) \sim 1\% \quad \text{Jungman et al. (1996)}
  \]

  \[
  \sim 200\% \quad \text{Eisenstein, Hu, Tegmark (1999)}
  \]

**MAP:**

- \( \Omega_m h^2 \) 0.029
- \( \Omega_b h^2 \) 0.0026
- \( m_\nu \) 0.76
- \( \Omega_\Lambda \) 1.0
- \( \Omega_K \) 0.29
- \( \tau \) 0.64
- \( n_S \) 0.11
- \( T/S \) 0.45
- \( A \) 1.2
A Question of Degeneracies

- **Precision** of CMB parameter estimation *thwarted* by degeneracies
- Problem compounded at *large angular scales* by cosmic variance
- Example: tensor/scalar ratio and *reionization*
- Solution: go *beyond* primary temperature anisotropies
- **Polarization and Secondary Anisotropies**

![Graphs showing power spectra for (a) Gravity Waves and (b) Reionization]
Polarization
Why Measure the Polarization?

**Virtues** of polarization
- unlike temperature anisotropies, generated by scattering only
- tensor field on the sky; carries more info than scalar temperature

**Uses** of polarization
- **verify** the gravitational instability paradigm: fluctuations present during last scattering
- **probe** the reionization epoch: remove a leading source of ambiguity (degeneracy) in the temperature power spectrum
- get higher statistics on the acoustic peaks and their underlying parameters
- **reconstruct** the scalar, vector, tensor nature of the perturbations and hence the cosmology even if ab initio models are wrong
- **test** inflationary models by measuring the gravity wave amplitude: energy scale and shape of inflaton potential
Why Polarization is Difficult

- Source of polarization is the scattering of quadrupole anisotropies
- Rapid scattering destroys quadrupole anisotropies
- Polarization only from the optically thin period before full transparency

![Graph showing polarization variations with angular scale (l).](image)
Polarization from Thomson Scattering

- Thomson scattering of anisotropic radiation → linear polarization
- Polarization aligned with cold lobe of the quadrupole anisotropy
Perturbations & Their Quadrupoles

- Orientation of quadrupole relative to wave ($\mathbf{k}$) determines pattern
- Scalars (density) $m=0$
- Vectors (vorticity) $m=\pm 1$
- Tensors (gravity waves) $m=\pm 2$

Hu & White (1997)
• Polarization direction oriented with the cold lobe of the quadrupole
• A local observer will see a $\sin^2\theta$ pattern of $Q$-polarization
Polarization Patterns

Scalars

Vectors

Tensors

$E, B$

$l=2, m=0$

$l=2, m=1$

$l=2, m=2$
Electric & Magnetic Patterns

- **Global view:** behavior under **parity**
- **Local view:** alignment of **principle** vs. **polarization** axes

Kamionkowski, Kosowski, Stebbins (1997)
Zaldarriaga & Seljak (1997)
Hu & White (1997)
Local vs. Observable Polarization

• Thomson *scattering* generates a pure $E$-pattern locally
• Plane wave perturbation *modulates* the amplitude
• If modulation: in a 0° or 90° direction then $E$
in a 45° direction as polarization then $B$

Hu & White (1997)
Patterns and Perturbation Types

- Amplitude modulated by plane wave → Principle axis
- Direction determined by perturbation type → Polarization axis

Kamionkowski, Kosowski, Stebbins (1997); Zaldarriaga & Seljak (1997); Hu & White (1997)
Acoustic Peaks in the Polarization

- **Scalar** quadrupole follows the velocity perturbation
- **Acoustic velocity** out of phase with acoustic temperature
- **Correlation** oscillates at twice the frequency

\[
\frac{v}{2^3} \quad \text{cross} \quad \text{temp.} \\
\text{vel.-quad.}
\]
Scalar Power Spectra

Hu & White (1997)
Testing Inflation
Testing Inflation

- **Inflation** required to causally carry **density** fluctuations **outside horizon**
- **Naive test:** above 1°, we are looking above the horizon at last scattering hence **any power** indicates inflation
- **Problem:** temperature anisotropies can be generated **after last scattering**

- **Solution:** find effects **confined to last scattering**
  - **acoustic oscillations:** probes potentials just **before horizon crossing**
    - pros – **easy** to measure
    - cons – **indirect** (dynamical assumptions)

  **polarization:** causal scalar fluctuations **fall off** rapidly ($l^6$)
  - outside horizon (if first peak right then scalar dominated)
  - pros – **direct**
  - cons – **difficult** to measure

Hu & White (1997); Spergel & Zaldarriaga (1997)
Testing Inflation / Initial Conditions

- Superluminal expansion (inflation) required to generate superhorizon curvature (density) perturbations.
- Else perturbations are isocurvature initially with matter moving causally.
- Curvature (potential) perturbations drive acoustic oscillations.
- Ratio of peak locations.
- Harmonic series: curvature 1:2:3... isocurvature 1:3:5...

Hu & White (1996)
Testing Inflation / Initial Conditions

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Hu & White (1996)
Hu & White (1997)
Inflationary Dynamics

- **Tensor Amplitude** $\propto V$ (inflaton potential)
- Current upper limits: inflationary energy scale $< 2 \times 10^{16}$ GeV

- Tensor / Scalar **amplitude** $\propto (V'/V)^2$
  Scalar slope function of $(V'/V)^2$, $(V''/V)$

- Constrain shape, test **models** of inflation

  **Planck Errors:**
  - $T/S \pm 0.35$ (temp) $\pm 0.012$ (+pol.)
  - $n_S \pm 0.04$ (temp) $\pm 0.008$ (+pol.)

- **Consistency Relation** – test of slow-roll inflation
  Tensor **slope** $\propto (V'/V)^2$

- Meaningful test by Planck only possible if $T/S$ close to current limits

- **Next** next-generation satellite dedicated to polarization?
Secondary Anisotropies
Physics of Secondary Anisotropies

Primary Anisotropies

recombination
$z \sim 1000$

reionization
$z \sim 10$

$\Lambda$–domination
$z \sim 1$
Secondary Anisotropies: Power Spectra

- Gravitational Effects
  - ISW Effect (redshift from decaying potentials)
  - Weak Lensing (smooths peaks and generates power <1')

- Scattering Effects
  - Doppler Effect
  - Vishniac Effect (LSS kinetic SZ effect)
  - Patchy Reionization
    - SZ effect (LSS thermal)
Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00} = -(1 + \Psi)^2 \delta_{ij}$

Kofman & Starobinskii (1985)  
Hu & Sugiyama (1994)
Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00} = -(1 + \Psi)^2 \delta_{ij}$
- Perturbed cosmological redshift
  $$g_{ij} = a^2 (1 + \Psi)^2 \delta_{ij}$$
  $$\delta T/T = -\delta a/a = \Psi$$

Kofman & Starobinskii (1985)  
Hu & Sugiyama (1994)
**Integrated Sachs–Wolfe Effect**

- Potential redshift: \( g_{00} = -(1 + \Psi)^2 \delta_{ij} \)
- Perturbed cosmological redshift
  \[ g_{ij} = a^2 (1 + \Psi)^2 \delta_{ij} \]
  \[ \frac{\delta T}{T} = -\frac{\delta a}{a} = \Psi \]
- Time–varying potential
  Rapid compared with \( \lambda / c \)
  \[ \delta T / T = -2\Delta \Psi \]
  Slow compared with \( \lambda / c \)
  Redshift–blueshift cancel
- Imprint characteristic time scale of decay in angular spectrum

\[ l_{ISW} \sim d / \Delta \eta \]

Kofman & Starobinskii (1985)
Hu & Sugiyama (1994)
Calculation of Secondary Anisotropies

- Addition of angular momentum gives

\[
\text{multipole moment} = \int \left( \text{clebsch gordan} \right) \left( \text{bessel function} \right) \text{Source} \ d\left( \text{line of sight} \right)
\]

- Primary anisotropies: source sharply peaked at last scattering

  Tight Coupling Approximation:

\[
\text{multipole moment} \sim \left( \text{clebsch gordan} \right) \left( \text{bessel function} \right) \int \text{Source} \ d\left( \text{line of sight} \right)
\]

- Secondary anisotropies: source slowly–varying in time

  Weak Coupling Approximation:

\[
\text{multipole moment} \sim \text{Source} \left( \text{clebsch gordan} \right) \int \left( \text{bessel function} \right) d\left( \text{line of sight} \right)
\]

- Log power spectrum of \textbf{CMB} \sim (cg)*\text{Log power spectrum of source} / l

- Scalar source and scalar field on sky: weak coupling = limber approx.

Hu & White (1996); Hu (2000)
Cancellation of the Linear Effect

Last Scattering Surface

Cancellation

Observer

e−velocity

overdensity

redshifted γ

blueshifted γ
Modulated Doppler Effect

- e^− velocity
- unscattered γ
- blueshifted γ

Observer

Last Scattering Surface

overdensity,
ionization patch,
cluster...
Future Directions

• Precision measurements will allow sharp consistency tests of the inflationary CDM paradigm and determination of its underlying parameters

• Polarization can in principle enable probes of the early universe (inflationary dynamics)

  [challenge to extract from foregrounds and systematic effects]

• Secondary anisotropies probe the evolution of large scale structure in the universe

  [challenge to separate from primary anisotropies, foregrounds and each other] spectral signatures, non-gaussian signatures and cross correlation