

Cosmic Acceleration from Modified Gravity:

$f(R)$

A Worked Example

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Why Study $f(R)$?

- Cosmic acceleration, like the cosmological constant, can either be viewed as arising from

Missing, or dark energy, with $w \equiv \bar{p}/\bar{\rho} < -1/3$

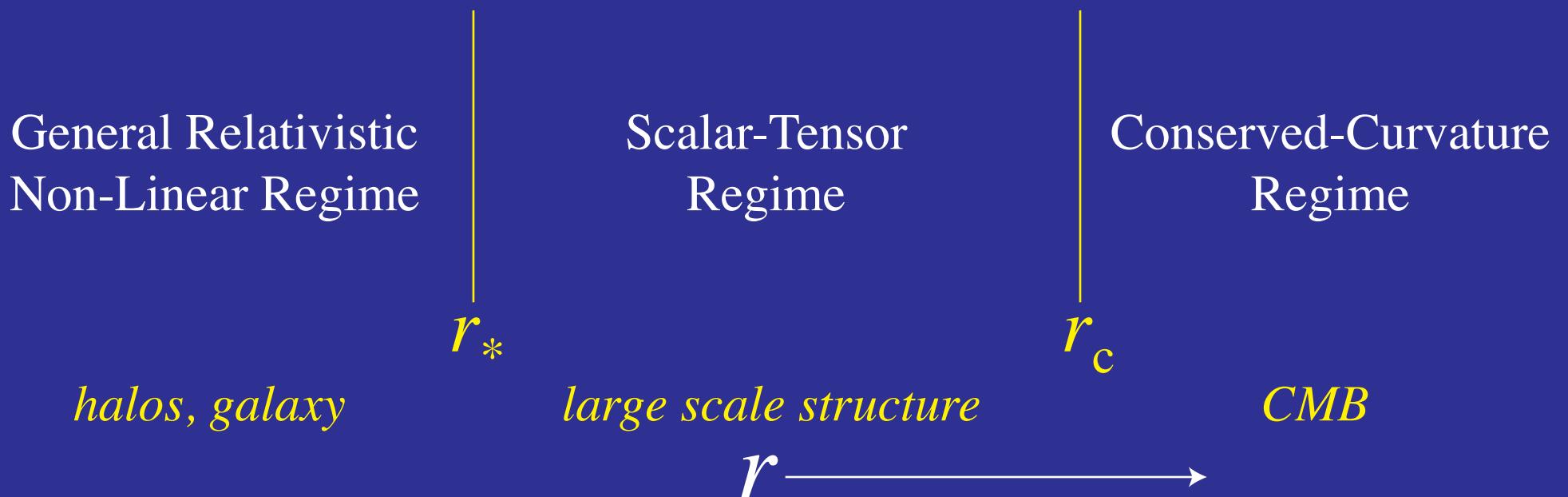
Modification of gravity on large scales

$$\begin{aligned} G_{\mu\nu} &= 8\pi G \left(T_{\mu\nu}^M + T_{\mu\nu}^{DE} \right) \\ F(g_{\mu\nu}) + G_{\mu\nu} &= 8\pi G T_{\mu\nu}^M \end{aligned}$$

- Proof of principle models for both exist: quintessence, k-essence; DGP braneworld acceleration, $f(R)$ modified action
- Compelling models for either explanation lacking
- Study models as illustrative toy models whose features can be generalized

Three Regimes

- Three regimes defined by $\gamma = -\Phi/\Psi$ BUT with different dynamics
- Examples $f(R)$ and DGP braneworld acceleration
- Parameterized Post-Friedmann description
- Non-linear regime follows a halo paradigm but a full parameterization still lacking and theoretical, examples few: $f(R)$ now fully worked



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Outline

- $f(R)$ Basics and Background
- Linear Theory Predictions
- N-body Simulations and the Chameleon
- Collaborators:
 - Marcos Lima
 - Hiro Oyaizu
 - Hiranya Peiris
 - Iggy Sawicki
 - Fabian Schmidt
 - Yong-Seon Song

Apologies to...

- [1] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. **D70**, 043528 (2004), astro-ph/0306438.
- [2] S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007), hep-th/0601213.
- [3] S. Capozziello, Int. J. Mod. Phys. **D11**, 483 (2002), gr-qc/0201033.
- [4] S. Capozziello, S. Carloni, and A. Troisi (2003), astro-ph/0303041.
- [5] S. Nojiri and S. D. Odintsov, Phys. Rev. **D68**, 123512 (2003), hep-th/0307288.
- [6] S. Nojiri and S. D. Odintsov, Phys. Lett. **B576**, 5 (2003), hep-th/0307071.
- [7] V. Faraoni, Phys. Rev. **D72**, 124005 (2005), gr-qc/0511094.
- [8] A. de la Cruz-Dombriz and A. Dobado, Phys. Rev. **D74**, 087501 (2006), gr-qc/0607118.
- [9] N. J. Poplawski, Phys. Rev. **D74**, 084032 (2006), gr-qc/0607124.
- [10] A. W. Brookfield, C. van de Bruck, and L. M. H. Hall, Phys. Rev. **D74**, 064028 (2006), hep-th/0608015.
- [11] B. Li, K. C. Chan, and M. C. Chu (2006), astro-ph/0610794.
- [12] T. P. Sotiriou and S. Liberati, Annals Phys. **322**, 935 (2007), gr-qc/0604006.
- [13] T. P. Sotiriou, Phys. Lett. **B645**, 389 (2007), gr-qc/0611107.
- [14] T. P. Sotiriou, Class. Quant. Grav. **23**, 5117 (2006), gr-qc/0604028.
- [15] R. Bean, D. Bernat, L. Pogosian, A. Silvestri, and M. Trodden, Phys. Rev. **D75**, 064020 (2007), astro-ph/0611321.
- [16] S. Baghram, M. Farhang, and S. Rahvar, Phys. Rev. **D75**, 044024 (2007), astro-ph/0701013.
- [17] D. Bazeia, B. Carneiro da Cunha, R. Menezes, and A. Y. Petrov (2007), hep-th/0701106.
- [18] B. Li and J. D. Barrow, Phys. Rev. **D75**, 084010 (2007), gr-qc/0701111.
- [19] S. Bludman (2007), astro-ph/0702085.
- [20] T. Rador (2007), hep-th/0702081.
- [21] T. Rador, Phys. Rev. **D75**, 064033 (2007), hep-th/0701267.
- [22] L. M. Sokolowski (2007), gr-qc/0702097.
- [23] V. Faraoni, Phys. Rev. **D75**, 067302 (2007), gr-qc/0703044.
- [24] V. Faraoni, Phys. Rev. **D74**, 104017 (2006), astro-ph/0610734.
- [25] S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. **36**, 1765 (2004), hep-th/0308176.
- [26] P. Wang and X.-H. Meng, TSPU Vestnik **44N7**, 40 (2004), astro-ph/0406455.
- [27] X.-H. Meng and P. Wang, Gen. Rel. Grav. **36**, 1947 (2004), gr-qc/0311019.
- [28] M. C. B. Abdalla, S. Nojiri, and S. D. Odintsov, Class. Quant. Grav. **22**, L35 (2005), hep-th/0409177.
- [29] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, and S. Zerbini, JCAP **0502**, 010 (2005), hep-th/0501096.
- [30] S. Capozziello, V. F. Cardone, and A. Troisi, Phys. Rev. **D71**, 043503 (2005), astro-ph/0501426.
- [31] G. Allemandi, A. Borowiec, M. Francaviglia, and S. D. Odintsov, Phys. Rev. **D72**, 063505 (2005), gr-qc/0504057.
- [32] T. Koivisto and H. Kurki-Suonio, Class. Quant. Grav. **23**, 2355 (2006), astro-ph/0509422.
- [33] T. Clifton and J. D. Barrow, Phys. Rev. **D72**, 103005 (2005), gr-qc/0509059.
- [34] O. Mena, J. Santiago, and J. Weller, Phys. Rev. Lett. **96**, 041103 (2006), astro-ph/0510453.
- [35] M. Amarzguioui, O. Elgaroy, D. F. Mota, and T. Multamaki, Astron. Astrophys. **454**, 707 (2006), astro-ph/0510519.
- [36] I. Brevik, Int. J. Mod. Phys. **D15**, 767 (2006), gr-qc/0601100.
- [37] T. Koivisto, Phys. Rev. **D73**, 083517 (2006), astro-ph/0602031.
- [38] S. E. Perez Bergliaffa, Phys. Lett. **B642**, 311 (2006), gr-qc/0608072.
- [39] G. Cognola, M. Gastaldi, and S. Zerbini (2007), gr-qc/0701138.
- [40] S. Capozziello and R. Garattini, Class. Quant. Grav. **24**, 1627 (2007), gr-qc/0702075.
- [41] S. Nojiri and S. D. Odintsov, Phys. Rev. **D74**, 086005 (2006), hep-th/0608008.
- [42] S. Nojiri and S. D. Odintsov (2006), hep-th/0610164.
- [43] S. Capozziello, S. Nojiri, S. D. Odintsov, and A. Troisi, Phys. Lett. **B639**, 135 (2006), astro-ph/0604431.
- [44] S. Fay, S. Nesseris, and L. Perivolaropoulos (2007), gr-qc/0703006.
- [45] S. Fay, R. Tavakol, and S. Tsujikawa, Phys. Rev. **D75**, 063509 (2007), astro-ph/0701479.
- [46] S. Nojiri, S. D. Odintsov, and M. Sasaki, Phys. Rev. **D71**, 123509 (2005), hep-th/0504052.
- [47] M. Sami, A. Toporensky, P. V. Tretyakov, and S. Tsujikawa, Phys. Lett. **B619**, 193 (2005), hep-th/0504154.
- [48] G. Calcagni, S. Tsujikawa, and M. Sami, Class. Quant. Grav. **22**, 3977 (2005), hep-th/0505193.
- [49] S. Tsujikawa and M. Sami, JCAP **0701**, 006 (2007), hep-th/0608178.
- [50] Z.-K. Guo, N. Ohta, and S. Tsujikawa, Phys. Rev. **D75**, 023520 (2007), hep-th/0610336.
- [51] A. K. Sanyal, Phys. Lett. **B645**, 1 (2007), astro-ph/0608104.
- [52] B. M. Leith and I. P. Neupane (2007), hep-th/0702002.
- [53] B. M. Carter and I. P. Neupane, JCAP **0606**, 004 (2006), hep-th/0512262.
- [54] T. Koivisto and D. F. Mota, Phys. Rev. **D75**, 023518 (2007), hep-th/0609155.
- [55] T. Koivisto and D. F. Mota, Phys. Lett. **B644**, 104 (2007), astro-ph/0606078.
- [56] S. Nojiri, S. D. Odintsov, and M. Sami, Phys. Rev. **D74**, 046004 (2006), hep-th/0605039.
- [57] S. Nojiri and S. D. Odintsov (2006), hep-th/0611071.
- [58] G. Cognola, E. Elizalde, S. Nojiri, S. Odintsov, and S. Zerbini, Phys. Rev. **D75**, 086003 (2007), hep-th/0611198.
- [59] S. Nojiri and S. D. Odintsov, Phys. Lett. **B631**, 1 (2005), hep-th/0508049.
- [60] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, and S. Zerbini, Phys. Rev. **D73**, 084007 (2006), hep-th/0601008.
- [61] Y.-S. Song, W. Hu, and I. Sawicki, Phys. Rev. **D75**, 044004 (2007), astro-ph/0610532.
- [62] S. Nojiri, S. D. Odintsov, and P. V. Tretyakov (2007), arXiv:0704.2520 [hep-th].
- [63] T. Multamaki and I. Vilja, Phys. Rev. **D73**, 024018 (2006), astro-ph/0506692.
- [64] R. P. Woodard (2006), astro-ph/0601672.
- [65] T. P. Sotiriou, Gen. Rel. Grav. **38**, 1407 (2006), gr-qc/0507027.
- [66] J. A. R. Cembranos, Phys. Rev. **D73**, 064029 (2006), gr-qc/0507039.
- [67] G. Allemandi, M. Francaviglia, M. L. Ruggiero, and A. Tartaglia, Gen. Rel. Grav. **37**, 1891 (2005), gr-qc/0506123.
- [68] M. L. Ruggiero and L. Iorio, JCAP **0701**, 010 (2007), gr-qc/0607093.
- [69] T. P. Sotiriou and E. Barausse, Phys. Rev. **D75**, 084007 (2007), gr-qc/0612065.
- [70] C.-G. Shao, R.-G. Cai, B. Wang, and R.-K. Su, Phys. Lett. **B633**, 164 (2006), gr-qc/0511034.
- [71] A. J. Bustelo and D. E. Barraco, Class. Quant. Grav. **24**, 2333 (2007), gr-qc/0611149.
- [72] G. J. Olmo, Phys. Rev. **D75**, 023511 (2007), gr-qc/0612047.
- [73] P.-J. Zhang (2007), astro-ph/0701662.
- [74] K. Kainulainen, J. Piilonen, V. Reijonen, and D. Sunhede (2007), arXiv:0704.2729 [gr-qc].
- [75] T. Chiba, Phys. Lett. **B575**, 1 (2003), astro-ph/0307338.
- [76] A. L. Erickcek, T. L. Smith, and M. Kamionkowski, Phys. Rev. **D74**, 121501 (2006), astro-ph/0610483.
- [77] T. Chiba, T. L. Smith, and A. L. Erickcek (2006), astro-ph/0611867.
- [78] J. Khoury and A. Weltman, Phys. Rev. **D69**, 044026 (2004), astro-ph/0309411.
- [79] I. Navarro and K. Van Acoleyen (2006), gr-qc/0611127.
- [80] T. Faulkner, M. Tegmark, E. F. Bunn, and Y. Mao (2006), astro-ph/0612569.
- [81] D. F. Mota and J. D. Barrow, Phys. Lett. **B581**, 141 (2004), astro-ph/0306047.
- [82] D. F. Mota and J. D. Barrow, Mon. Not. Roy. Astron. Soc. **349**, 291 (2004), astro-ph/0309273.
- [83] L. Amendola, D. Polarski, and S. Tsujikawa (2006), astro-ph/0603703.
- [84] I. Sawicki and W. Hu, Phys. Rev. **D75**, 127502 (2007), astro-ph/0702278.
- [85] P. Zhang, Phys. Rev. **D73**, 123504 (2006), astro-ph/0511218.
- [86] A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980).
- [87] T. Faulkner, M. Tegmark, E. F. Bunn, and Y. Mao (2006), astro-ph/0612569.
- [88] A. Vikman, Phys. Rev. **D71**, 023515 (2005), astro-ph/0407107.
- [89] W. Hu, Phys. Rev. D **71**, 047025 (2005), astro-ph/0410680.
- [90] Z.-K. Guo, Y.-S. Piao, X.-M. Zhang, and Y.-Z. Zhang, Phys. Lett. **B608**, 177 (2005), astro-ph/0410654.
- [91] L. Amendola and S. Tsujikawa (2007), arXiv:0705.0396 [astro-ph].
- [92] A. D. Dolgov and M. Kawasaki, Phys. Lett. **B573**, 1 (2003), astro-ph/0307285.
- [93] M. D. Seifert (2007), gr-qc/0703060.
- [94] J. A. R. Cembranos, Phys. Rev. **D73**, 064029 (2006), gr-qc/0507039.
- [95] J. N. Bahcall, A. M. Serenelli, and S. Basu, Astrophys. J. Lett. **621**, L85 (2005), arXiv:astro-ph/0412440.
- [96] J. E. Vernazza, E. H. Avrett, and R. Loeser, Astrophys. J. Suppl. **45**, 635 (1981).
- [97] E. C. Sittler, Jr. and M. Guhathakurta, Astrophys. J. **523**, 812 (1999).
- [98] C. M. Will, Living Reviews in Relativity **9** (2006), URL <http://www.livingreviews.org/lrr-2006-3>.
- [99] A. Klypin, H. Zhao, and R. S. Somerville, Astrophys. J. **573**, 597 (2002), astro-ph/0110390.

$f(R)$ Basics

Cast of $f(R)$ Characters

- R : Ricci scalar or “curvature”
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]$$

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$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B : Compton wavelength of f_R squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}$$

- $' \equiv d/d \ln a$: scale factor as time coordinate

Modified Einstein Equation

- In the **Jordan frame**, gravity becomes 4th order but matter remains minimally coupled and separately conserved

$$G_{\alpha\beta} + f_R R_{\alpha\beta} - \left(\frac{f}{2} - \square f_R \right) g_{\alpha\beta} - \nabla_\alpha \nabla_\beta f_R = 8\pi G T_{\alpha\beta}$$

- Trace can be interpreted as a **scalar field equation** for f_R with a density-dependent effective potential ($p = 0$)

$$3\square f_R + f_R R - 2f = R - 8\pi G\rho$$

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- For small deviations, $|f_R| \ll 1$ and $|f/R| \ll 1$,

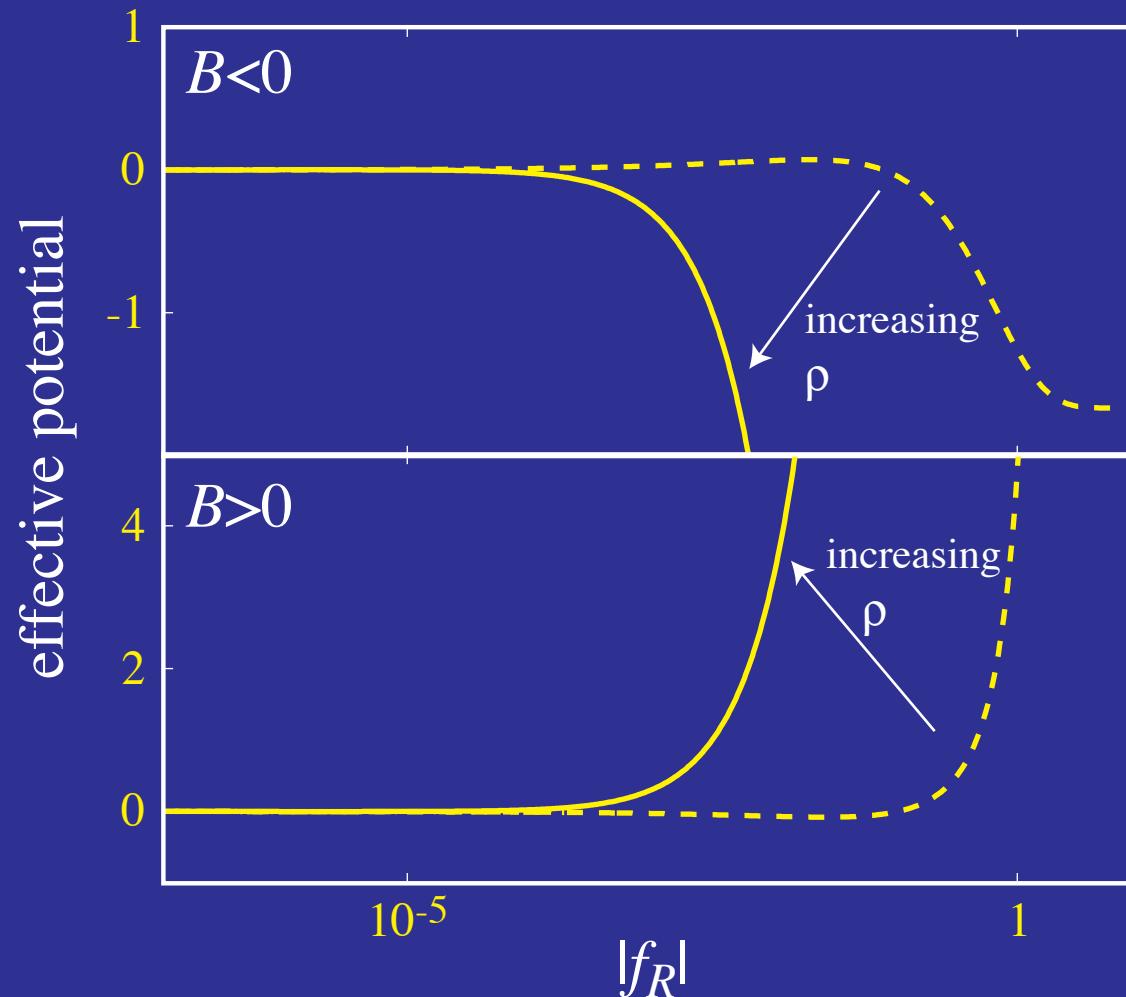
$$\square f_R \approx \frac{1}{3} (R - 8\pi G \rho)$$

the field is sourced by the deviation from GR relation between curvature and density and has a mass

$$m_{f_R}^2 \approx \frac{1}{3} \frac{\partial R}{\partial f_R} = \frac{1}{3 f_{RR}}$$

Effective Potential

- Scalar f_R rolls in an effective potential that depends on density
- At high density, extrema is at GR $R=8\pi G\rho$
- Minimum for $B>0$, pinning field to $|f_R| \ll 1$, maximum for $B<0$



$f(R)$ Expansion History

Modified Friedmann Equation

- Expansion history parameterization: Friedmann equation becomes

$$H^2 - f_R(HH' + H^2) + \frac{1}{6}f + H^2 f_{RR} R' = \frac{8\pi G \rho}{3}$$

- Reverse engineering $f(R)$ from the expansion history: for any desired H , solve a 2nd order diffeq to find $f(R)$
- Allows a family of $f(R)$ models, parameterized in terms of the Compton wavelength parameter B

Modified Friedmann Equation

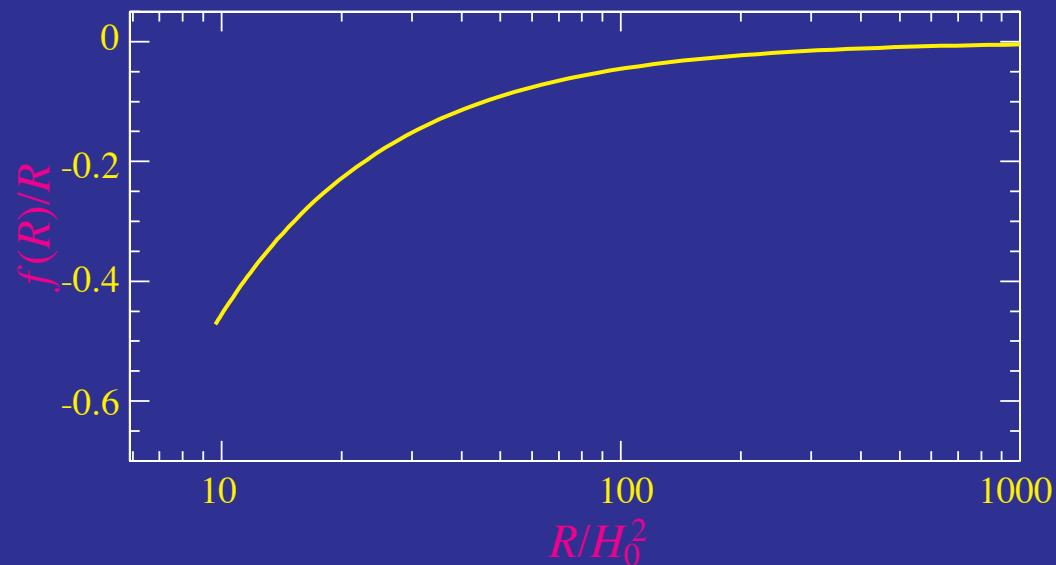
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- Reverse engineering $f(R)$ from the expansion history: for any desired H , solve a 2nd order diffeq to find $f(R)$
- Allows a family of $f(R)$ models, parameterized in terms of the Compton wavelength parameter B
- Formally includes models where $B < 0$, such as $f(R) = -\mu^4/R$, leading to confusion as to whether such models provide viable expansion histories
- Answer: no these have short-time scale tachyonic instabilities at high curvature and limit as $B \rightarrow 0$ from below is not GR
- $B > 0$ family has very different implications for structure formation but with identical distance-redshift relations

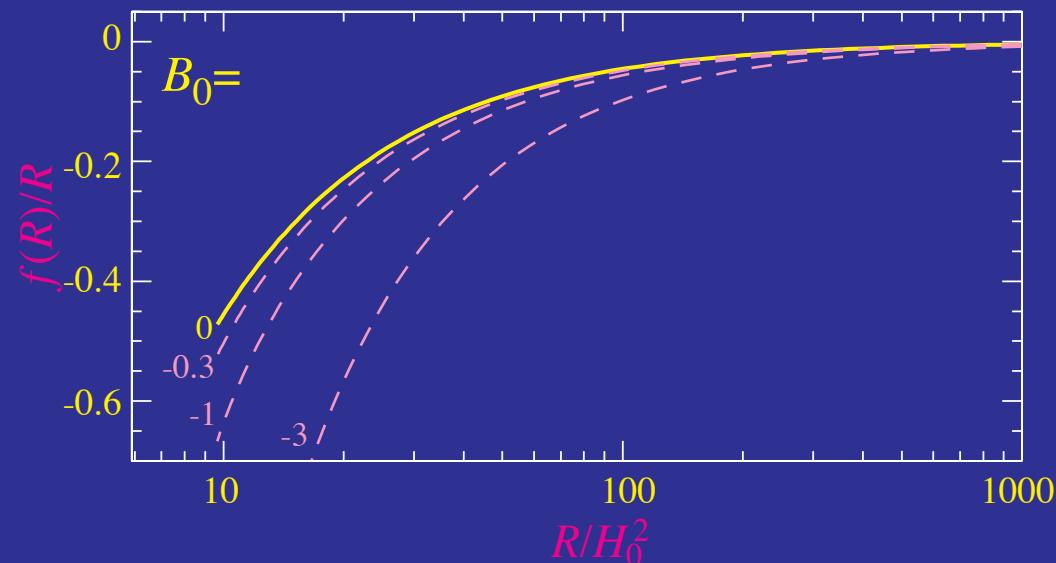
Expansion History Family of $f(R)$

- Each expansion history, matched by dark energy model [$w(z), \Omega_{\text{DE}}, H_0$] corresponds to a family of $f(R)$ models due to its 4th order nature
- Parameterized by $B \propto f_{RR} = d^2f/dR^2$ evaluated at $z=0$



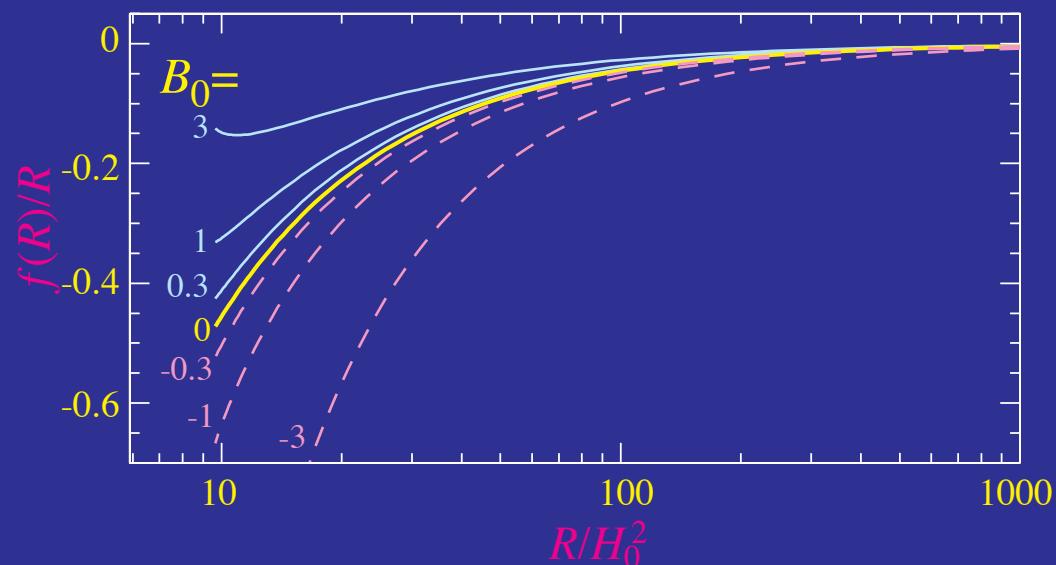
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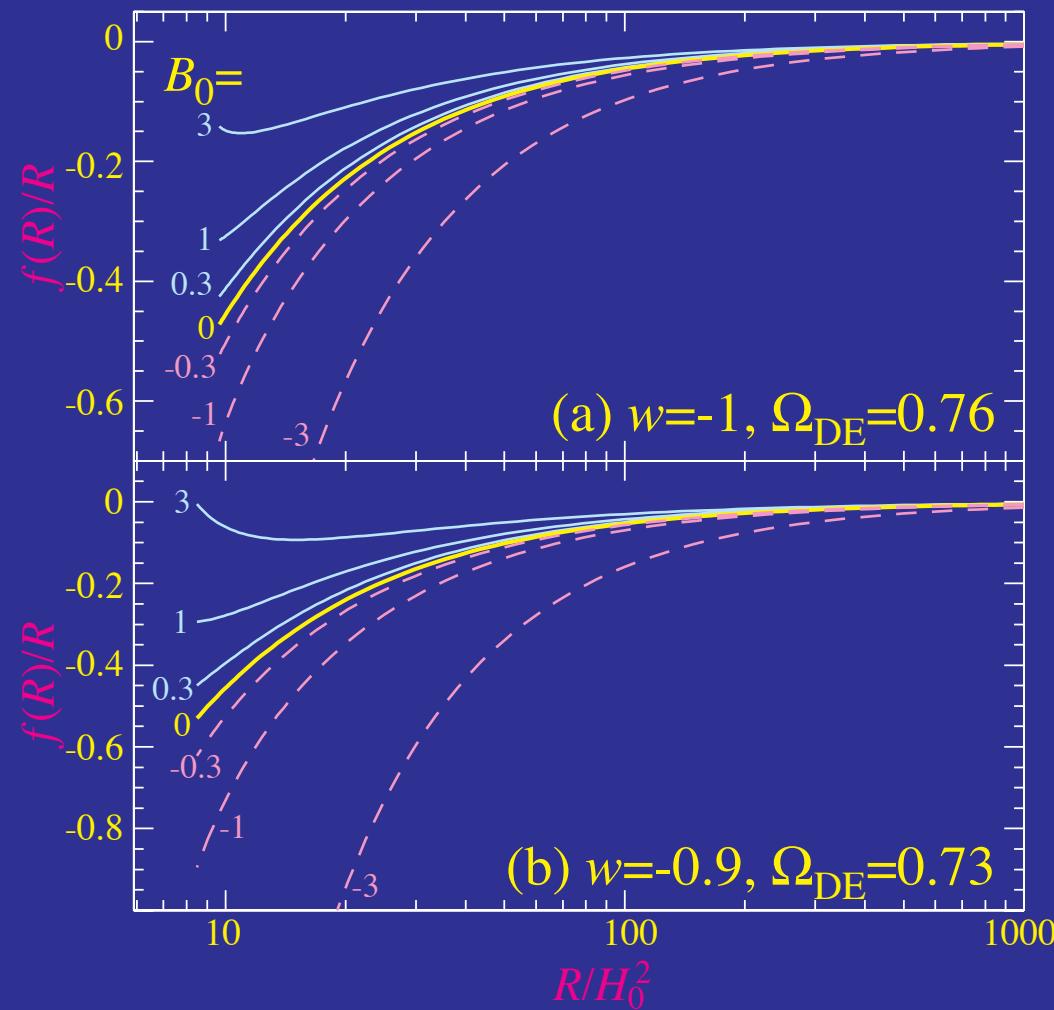
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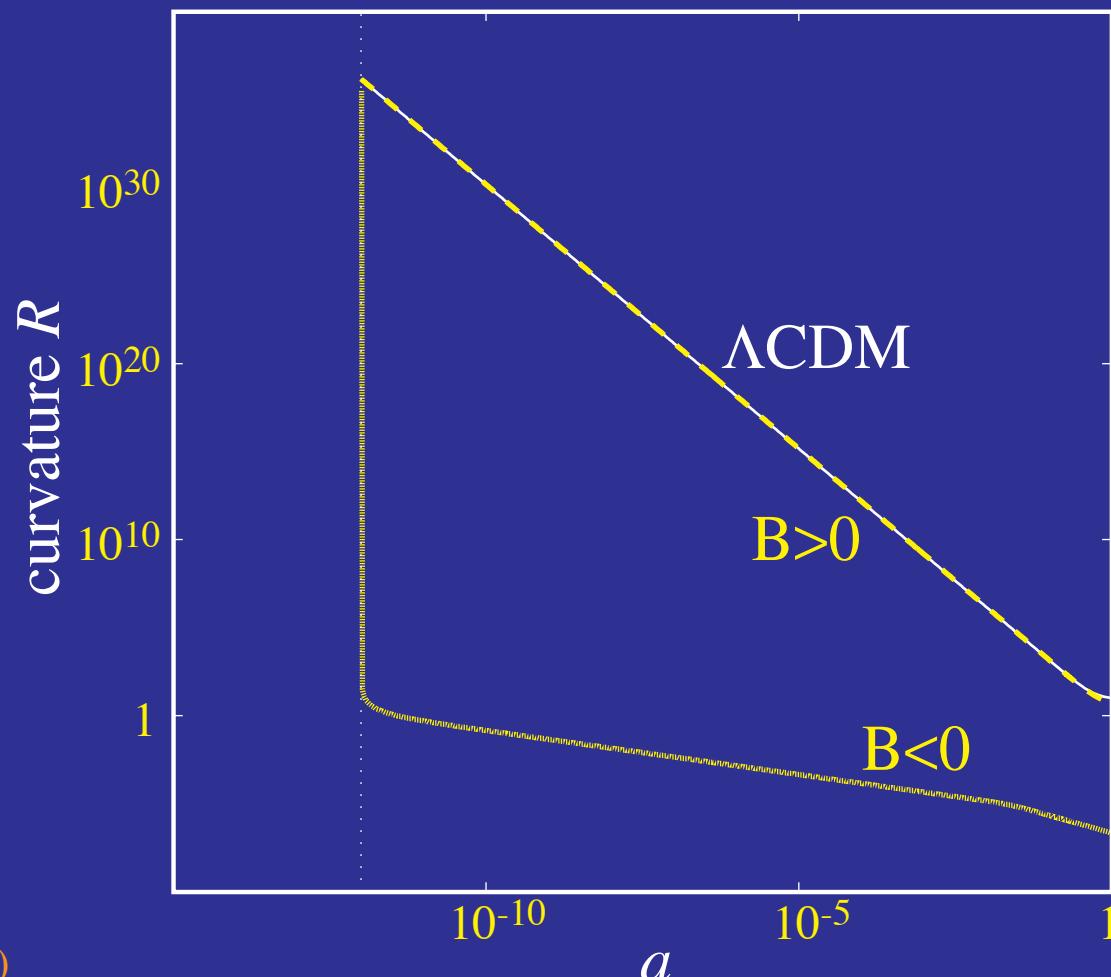
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Instability at High Curvature

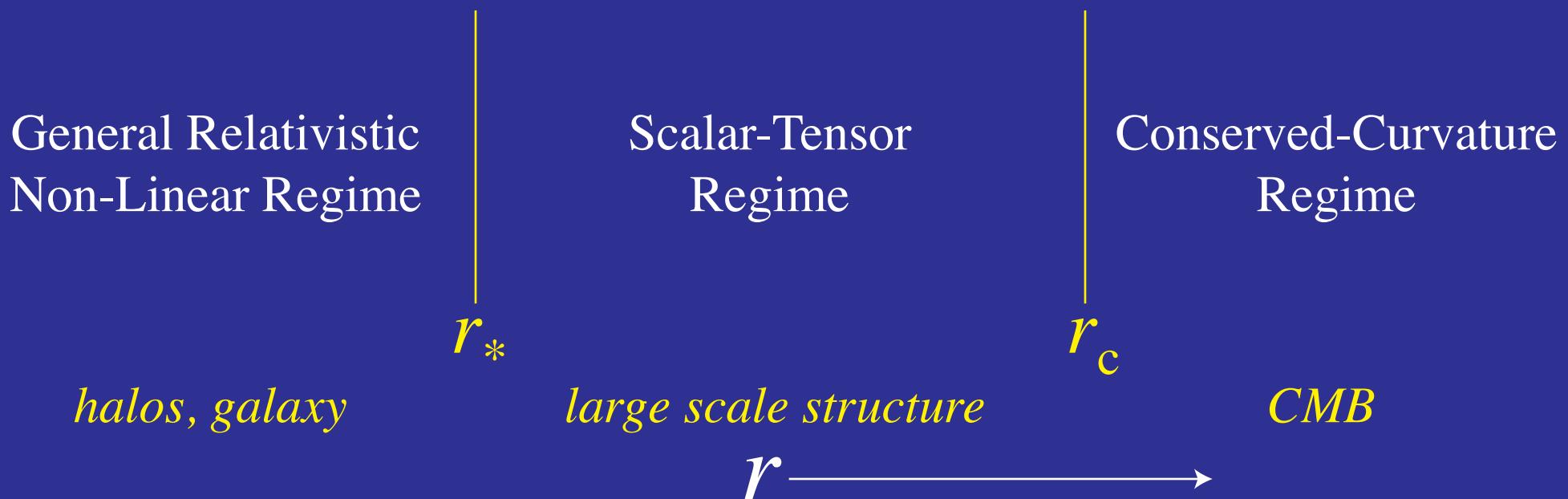
- Tachyonic instability for negative mass squared $B < 0$ makes high curvature regime increasingly unstable: high density \neq high curvature
- Linear metric perturbations immediately drop the expansion history to low curvature solution



$f(R)$ Linear Theory

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Curvature Conservation

- On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)
- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d \ln a$ (Bardeen 1980)
- Gauge transformation to Newtonian gauge

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2$$

yields (Hu & Eisenstein 1999)

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = 0$$

- Modified gravity theory supplies the closure relationship $\Phi = -\gamma(\ln a)\Psi$ between and expansion history $H = \dot{a}/a$ supplies rest.

Linear Theory for $f(R)$

- In $f(R)$ model, “superhorizon” behavior persists until Compton wavelength smaller than fluctuation wavelength $B^{1/2}(k/aH) < 1$
- Once Compton wavelength becomes larger than fluctuation

$$B^{1/2}(k/aH) > 1$$

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- Small scale density growth enhanced and

$$8\pi G \rho > R$$

low curvature regime with order unity deviations from GR

- Transitions in the non-linear regime where the Compton wavelength can shrink via chameleon mechanism
- Given $k_{\text{NL}}/aH \gg 1$, even very small f_R have scalar-tensor regime

Deviation Parameter

- Express the 4th order nature of equations as a deviation parameter

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = \left(\frac{k}{aH}\right)^2 B\epsilon$$

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- Einstein equation become a second order equation for ϵ
- In high redshift, high curvature R limit this is

$$\epsilon'' + \left(\frac{7}{2} + 4\frac{B'}{B}\right)\epsilon' + \frac{2}{B}\epsilon = \frac{1}{B} \times \text{metric sources}$$

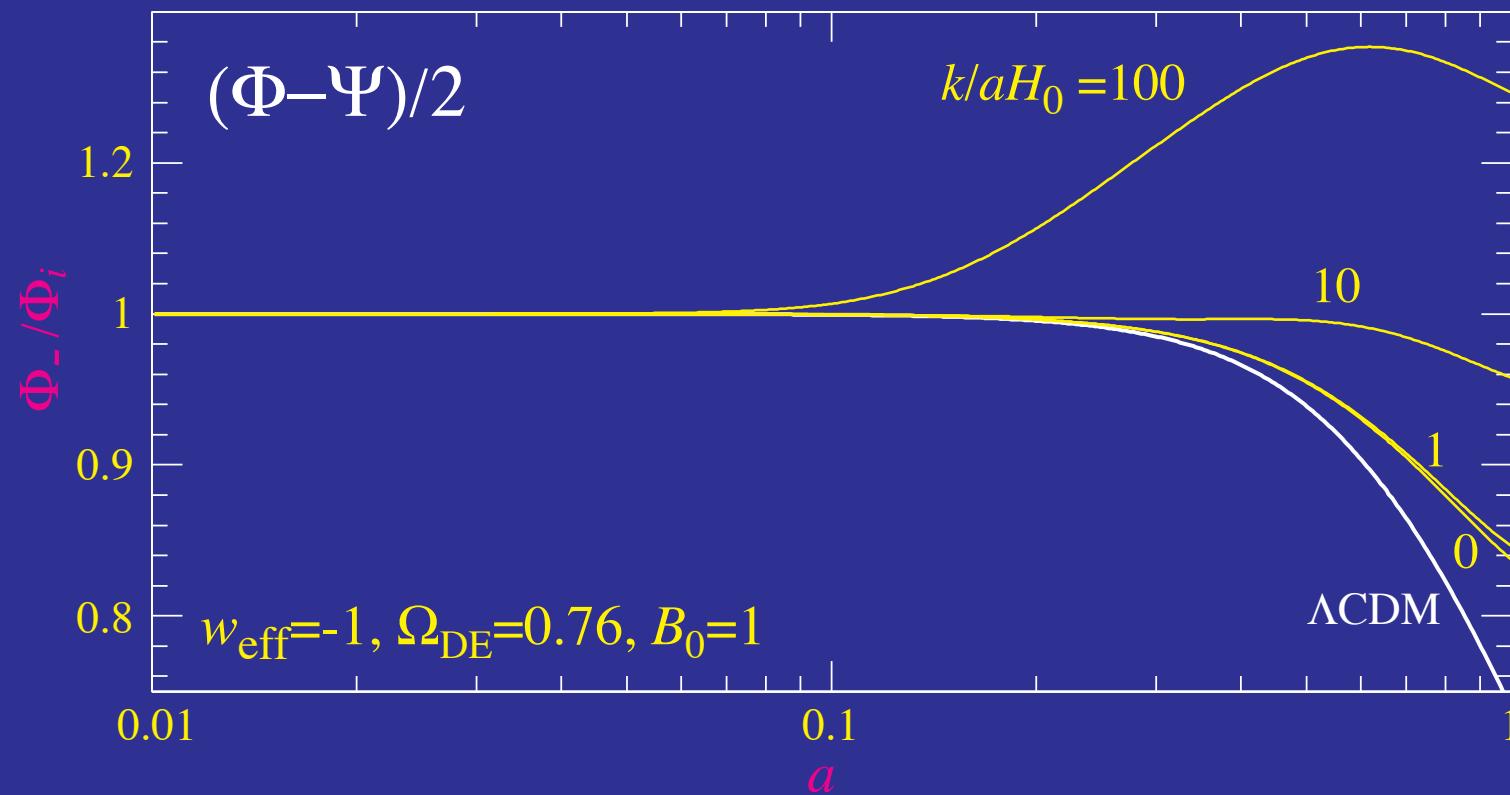
$$B = \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}$$

- $R \rightarrow \infty, B \rightarrow 0$ and for $B < 0$ short time-scale tachyonic instability appears making previous models not cosmologically viable

$$f(R) = -M^{2+2n}/R^n$$

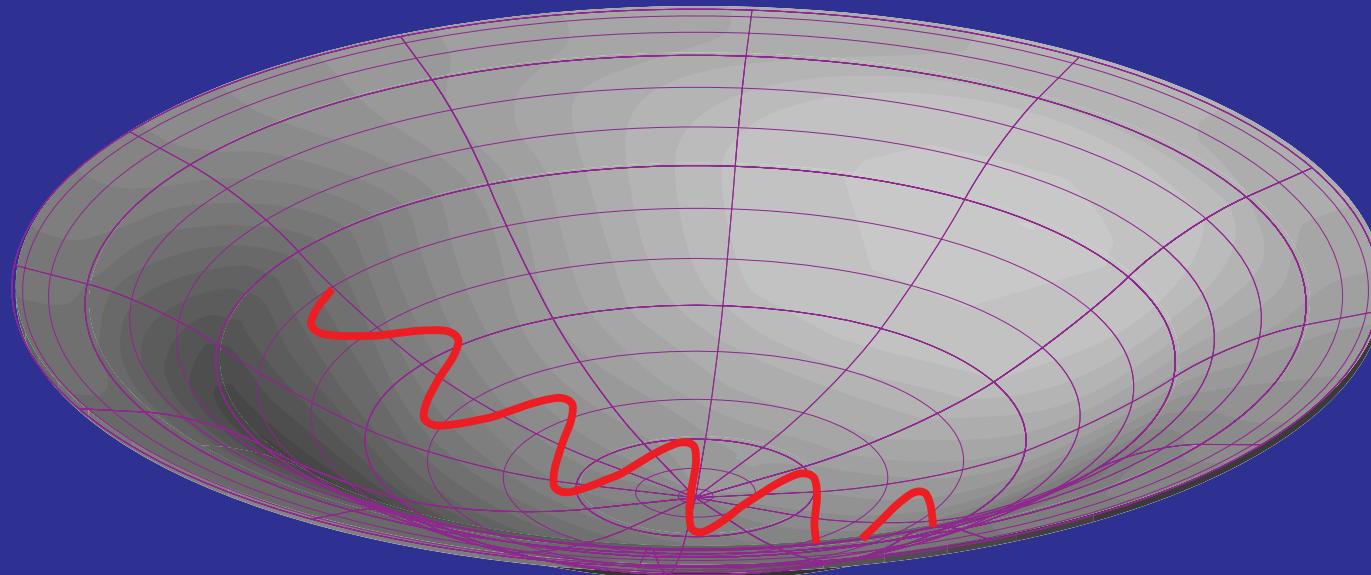
Potential Growth

- On the stable $B>0$ branch, potential evolution **reverses** from decay to **growth** as wavelength becomes smaller than Compton scale
- Quasistatic equilibrium reached in linear theory with $\gamma=-\Phi/\Psi=1/2$ until non-linear effects restore $\gamma=1$



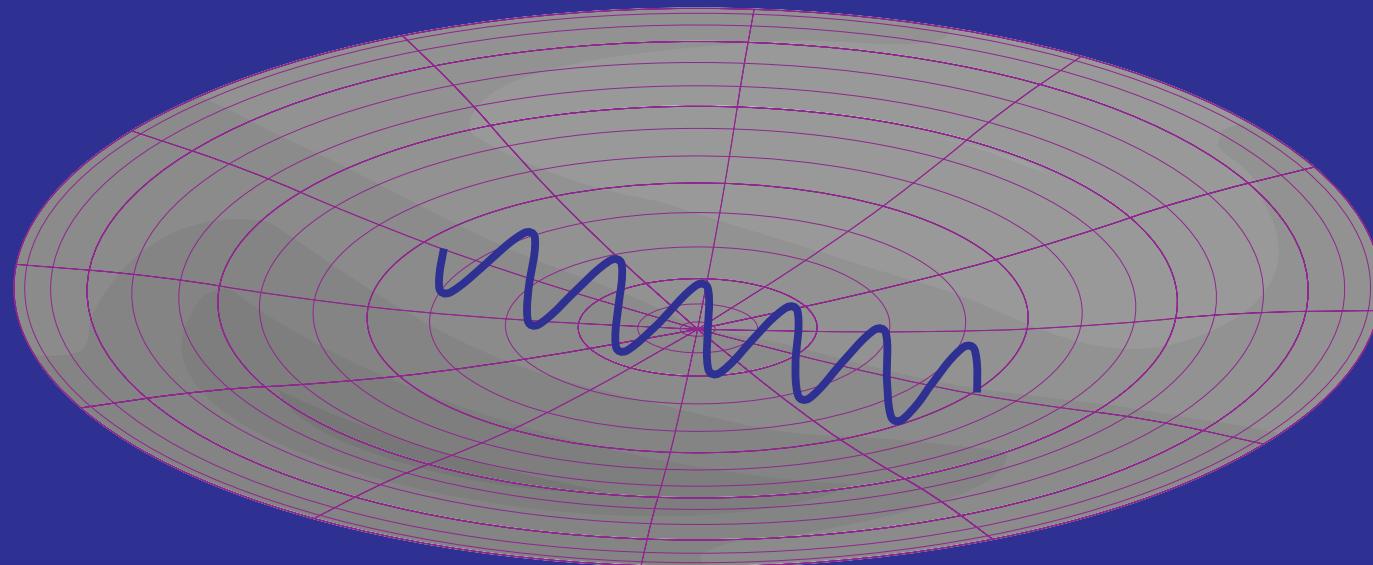
Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi - \Psi)$



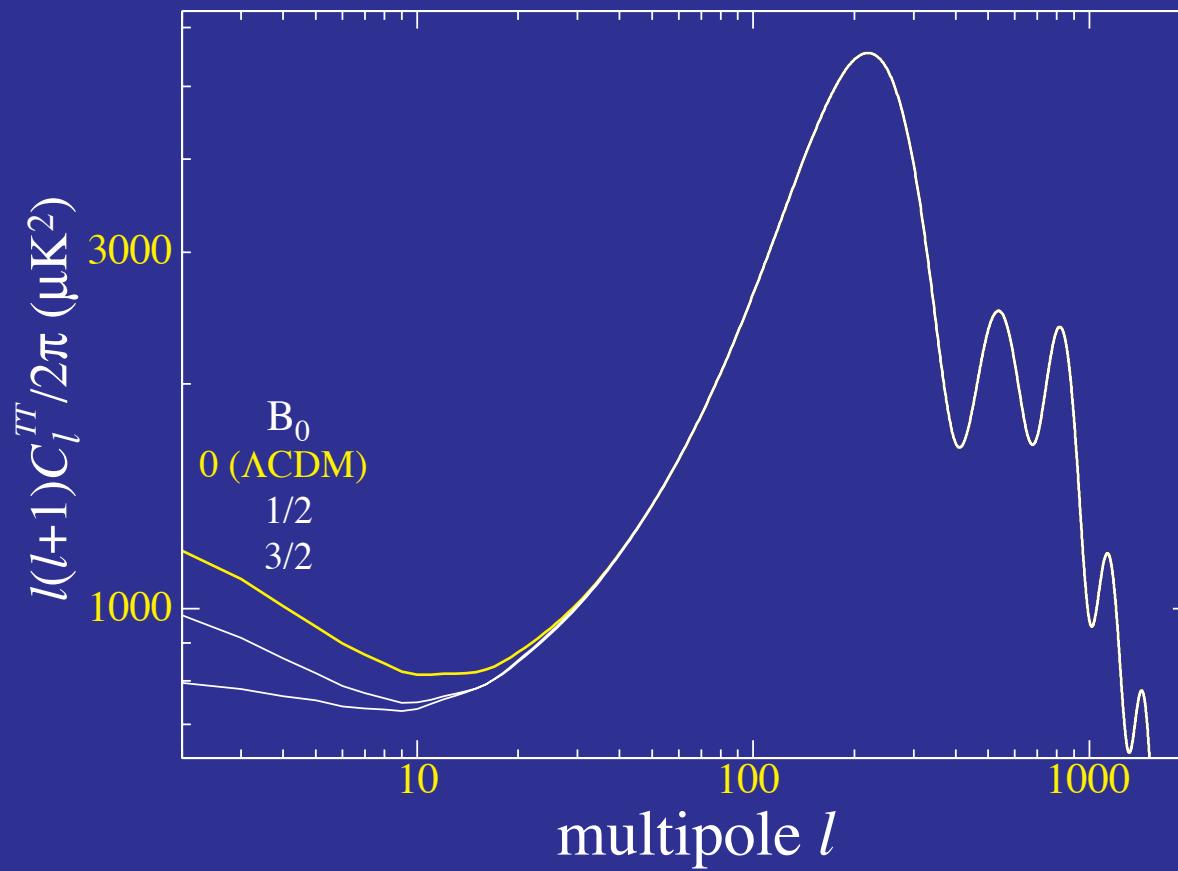
Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi - \Psi)$



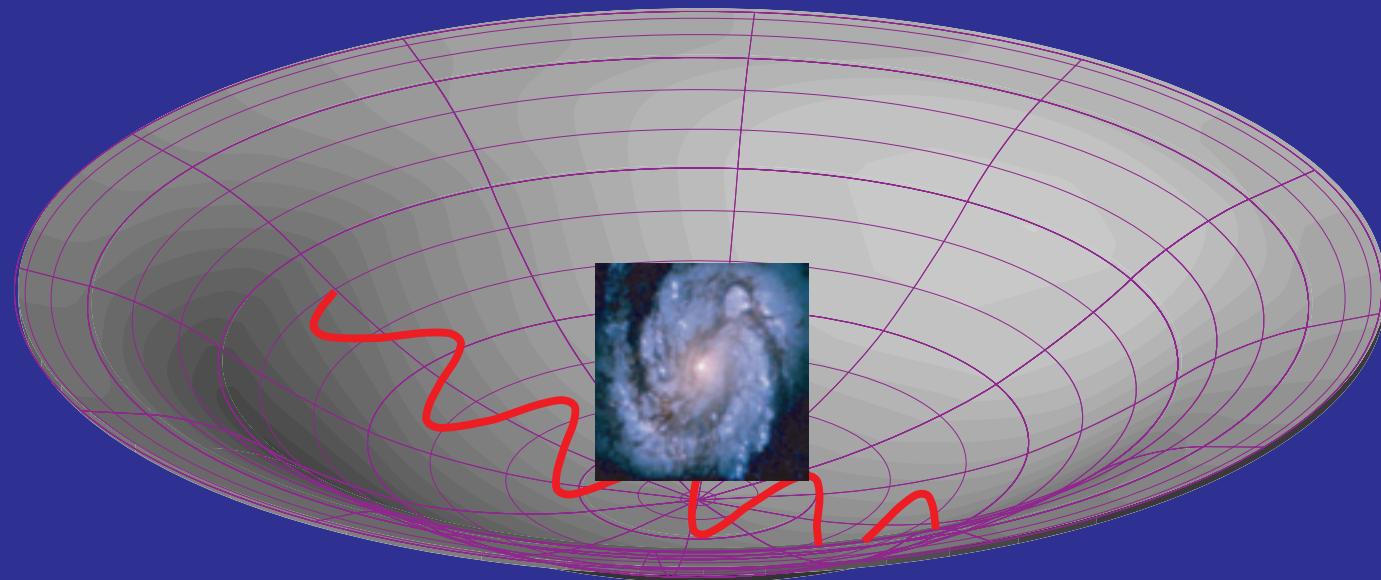
ISW Quadrupole

- Reduction of large angle anisotropy for $B_0 \sim 1$ for same expansion history and distances as Λ CDM
- Well-tested small scale anisotropy unchanged



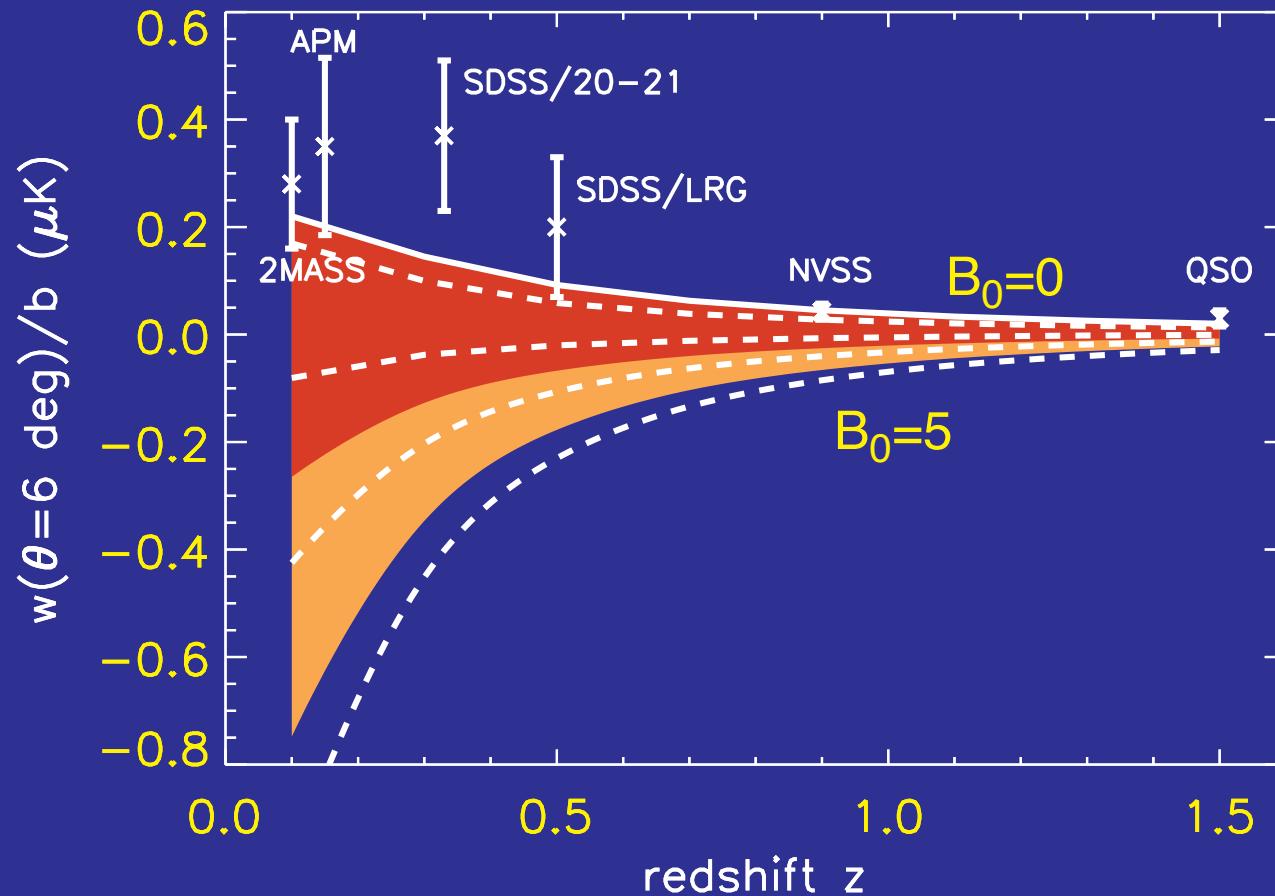
ISW-Galaxy Correlation

- Decaying potential: galaxy positions **correlated** with CMB
- Growing potential: galaxy positions **anticorrelated** with CMB
- Observations indicate **correlation**



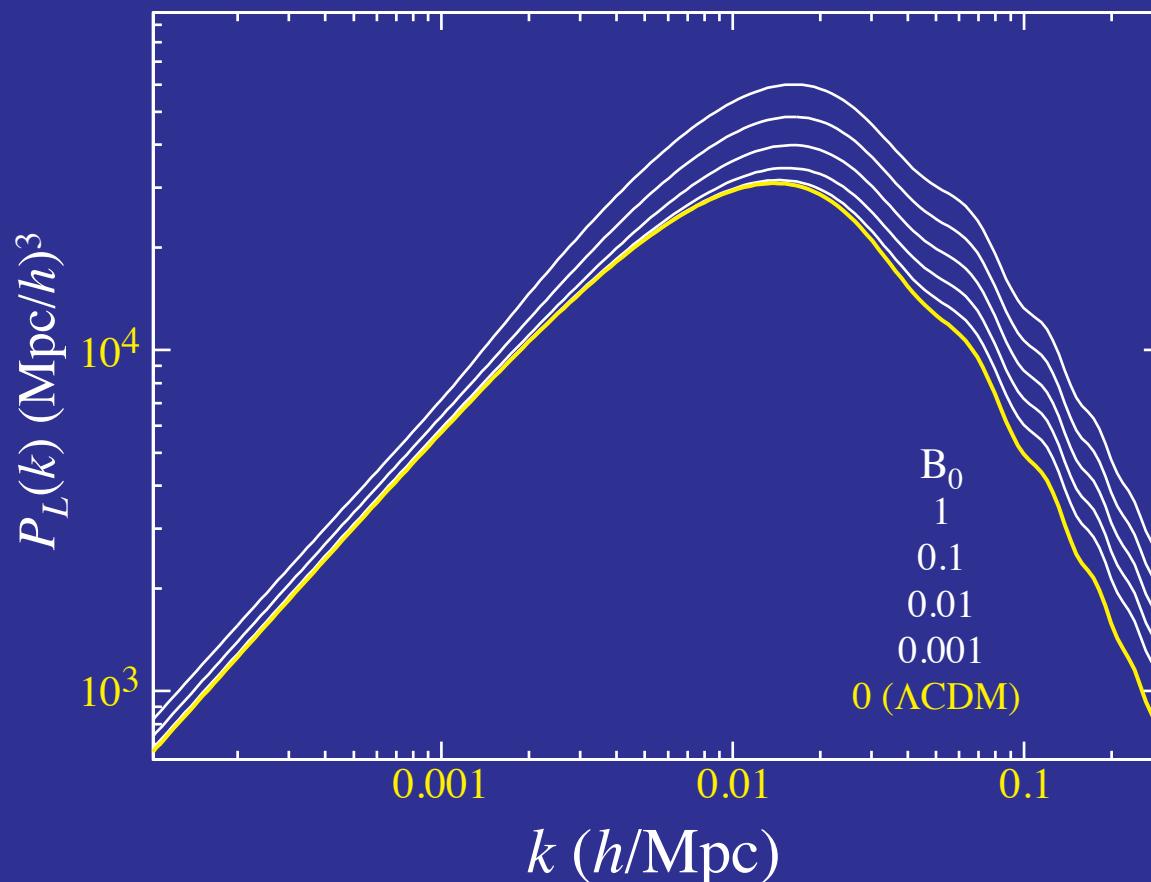
Galaxy-ISW Anti-Correlation

- Large Compton wavelength $B^{1/2}$ creates potential growth which can anti-correlate galaxies and the CMB
- In tension with detections of positive correlations across a range of redshifts



Linear Power Spectrum

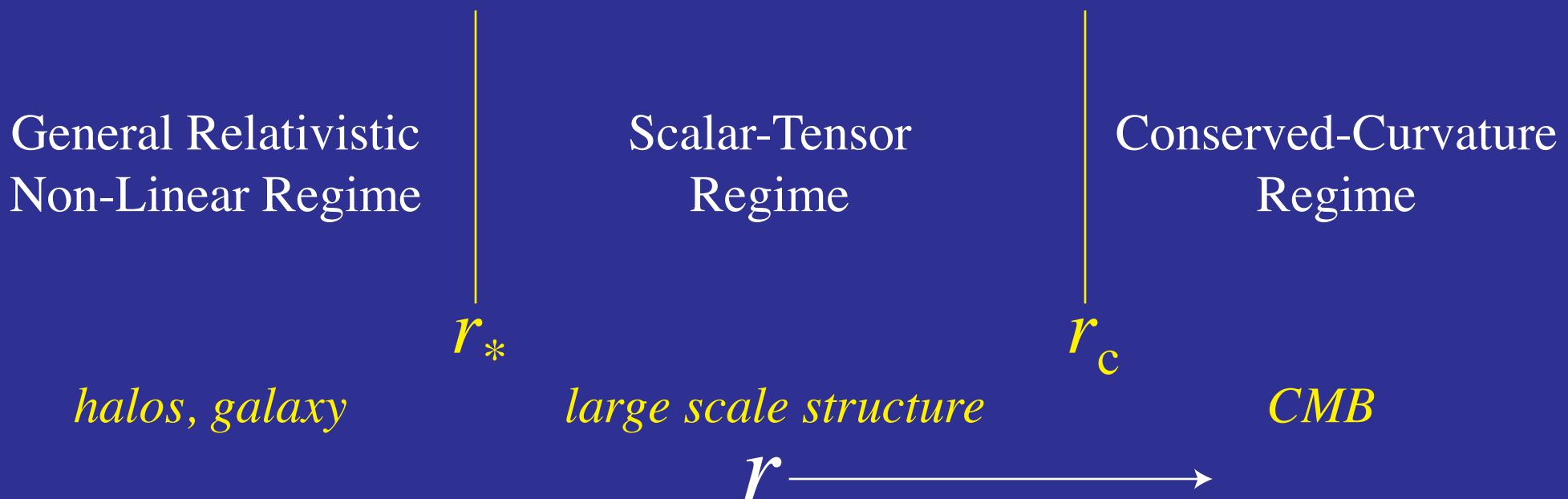
- Linear real space power spectrum enhanced on scales below Compton scale in the background
- Scale-dependent growth rate and potentially large deviations on small scales



$f(R)$ Non-Linear Evolution

Three Regimes

- Three regimes defined by $\gamma = -\Phi/\Psi$ BUT with different dynamics
- Examples $f(R)$ and DGP braneworld acceleration
- Parameterized Post-Friedmann description
- Non-linear regime follows a halo paradigm but a full parameterization still lacking and theoretical, examples few: $f(R)$ now fully worked



Non-Linear Chameleon

- For $f(R)$ the field equation

$$\nabla^2 f_R \approx \frac{1}{3}(\delta R(f_R) - 8\pi G \delta\rho)$$

is the **non-linear** equation that returns **general relativity**

- High curvature implies short Compton wavelength and suppressed deviations but requires a **change** in the **field** from the background value $\delta R(f_R)$
- Change in field is generated by **density perturbations** just like **gravitational potential** so that the chameleon appears only if

$$\Delta f_R \leq \frac{2}{3}\Phi ,$$

else required **field** gradients **too large** despite $\delta R = 8\pi G \delta\rho$ being the **local minimum** of effective potential

Non-Linear Dynamics

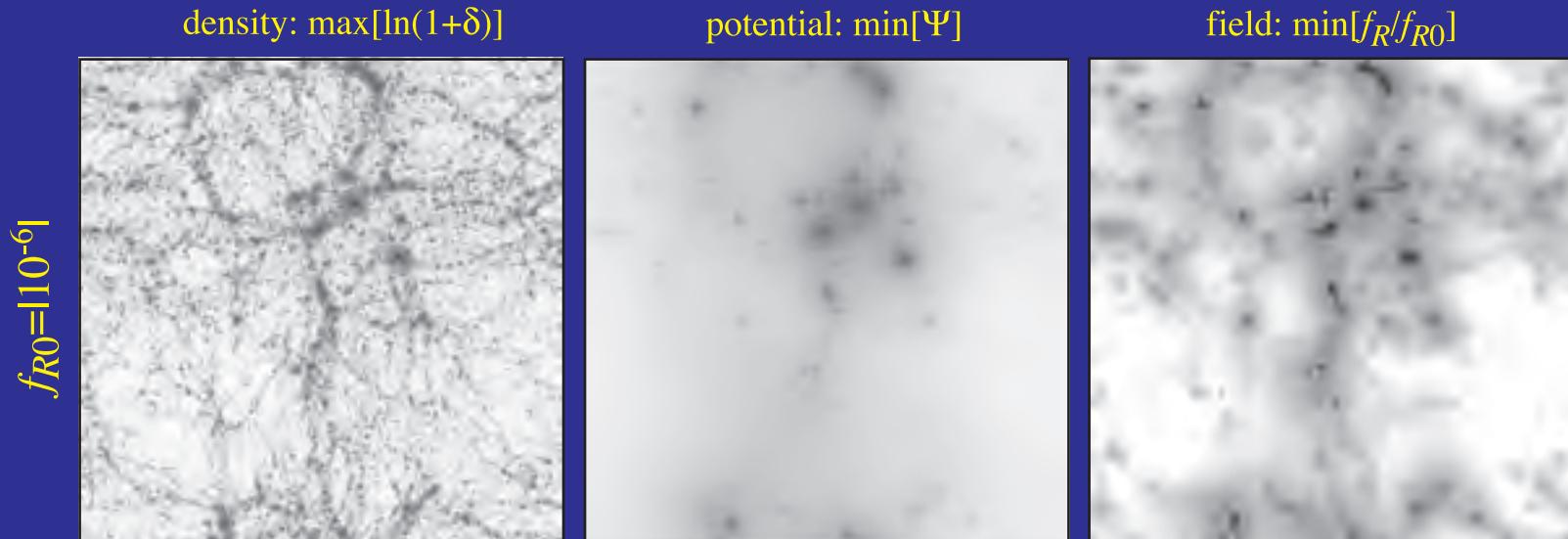
- Supplement that with the modified Poisson equation

$$\nabla^2 \Psi = \frac{16\pi G}{3} \delta\rho - \frac{1}{6} \delta R(f_R)$$

- Matter evolution given metric unchanged: usual motion of matter in a gravitational potential Ψ
- Prescription for N -body code
- Particle Mesh (PM) for the Poisson equation
- Field equation is a non-linear Poisson equation: relaxation method for f_R
- Initial conditions set to GR at high redshift

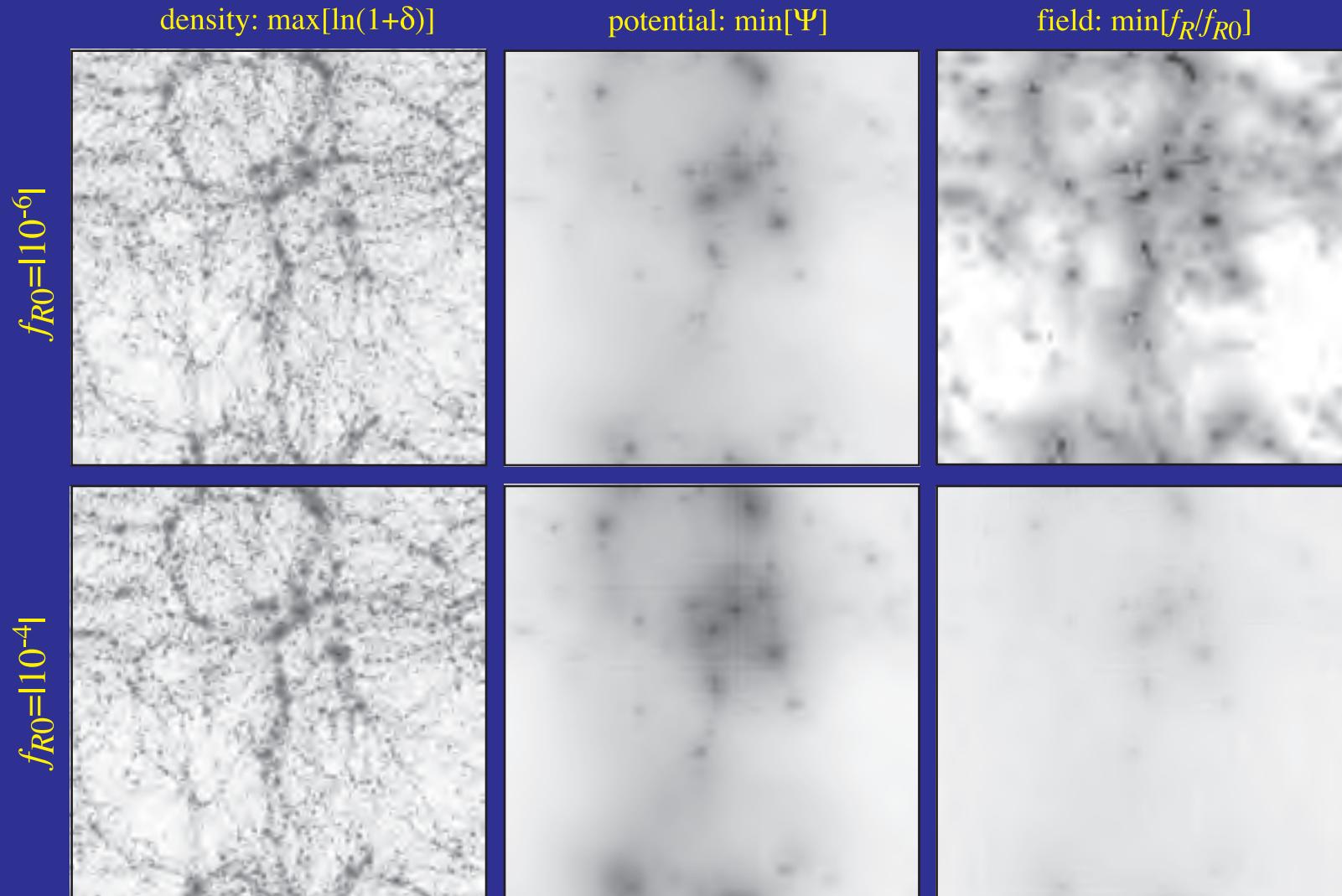
Environment Dependent Force

- Chameleon suppresses extra force (scalar field) in high density, deep potential regions



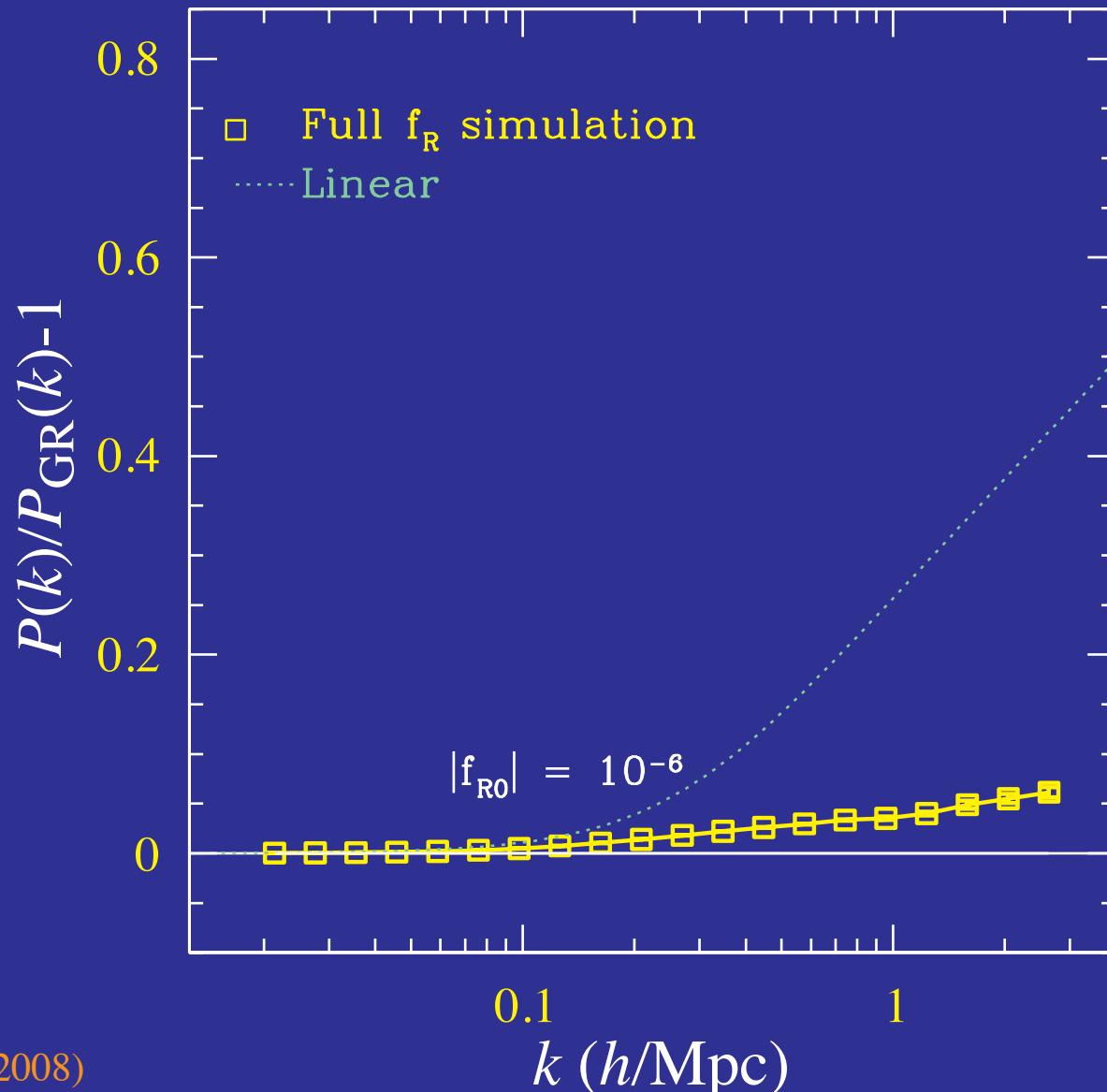
Environment Dependent Force

- For large background field, gradients in the scalar prevent the chameleon from appearing



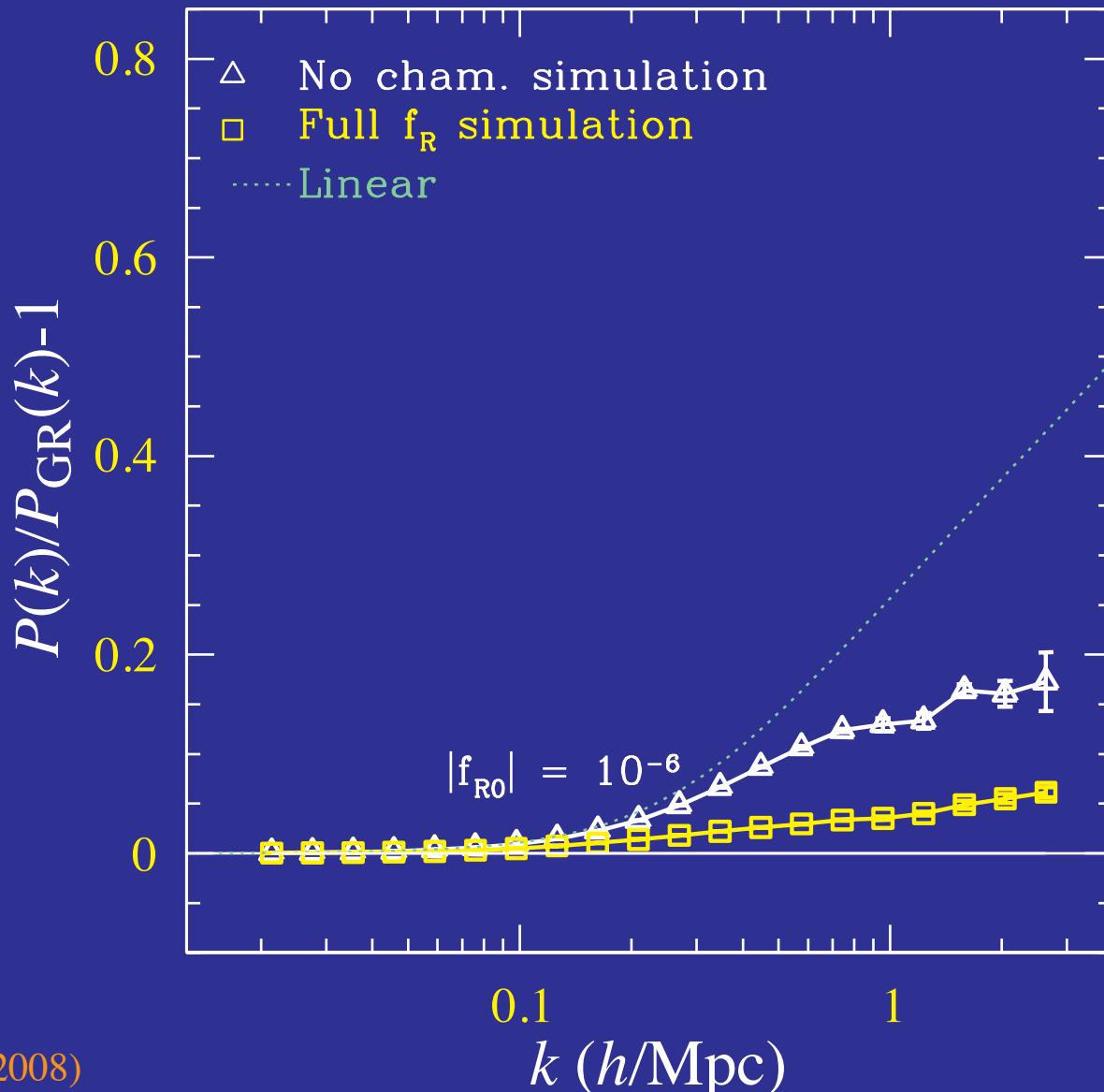
N-body Power Spectrum

- 512^3 PM-relaxation code resolves the chameleon transition to GR: greatly reduced non-linear effect



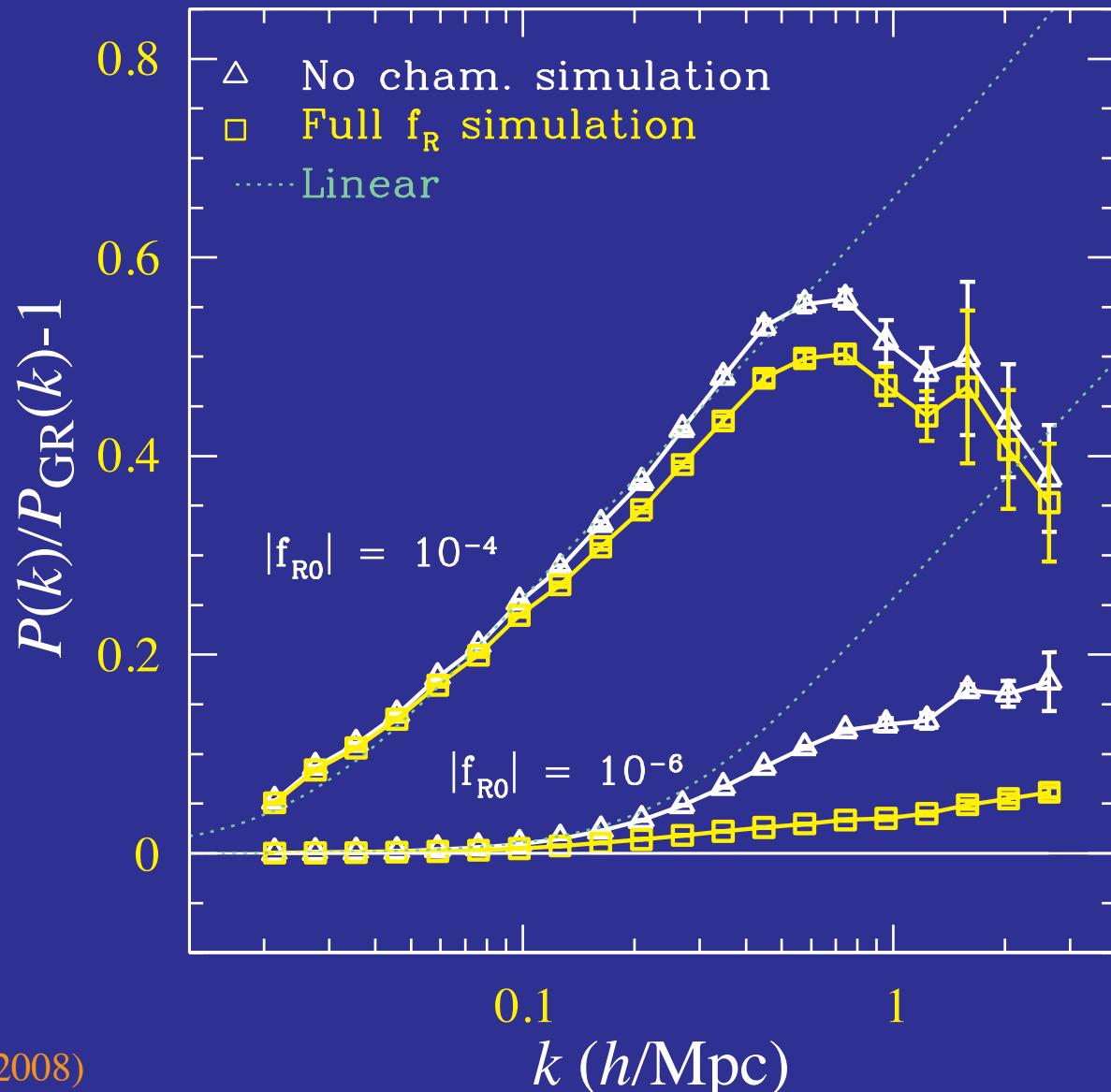
N-body Power Spectrum

- Artificially turning off the chameleon mechanism restores much of enhancement



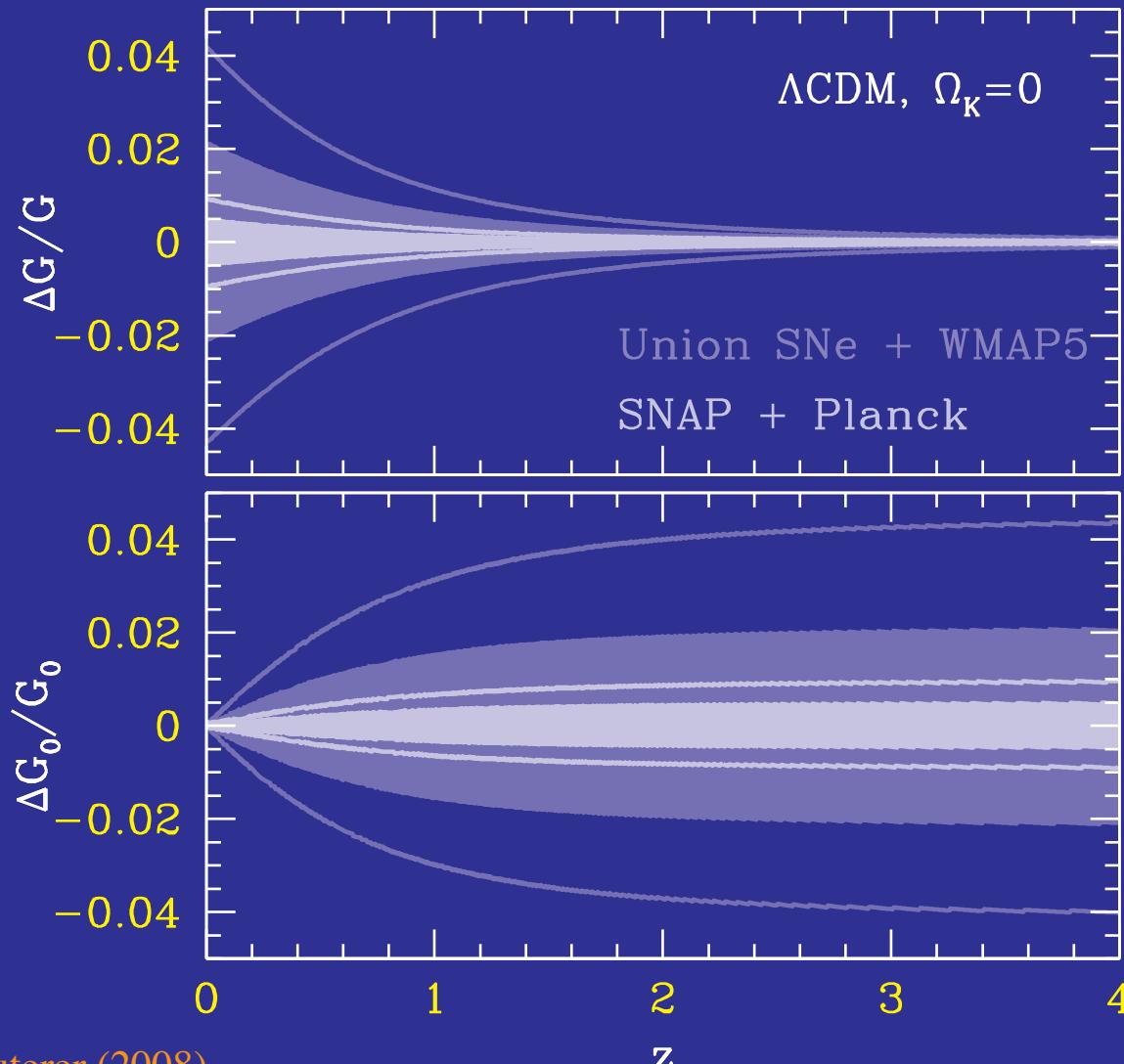
N-body Power Spectrum

- Models where the chameleon absent today (large field models) show residual effects from a high redshift chameleon



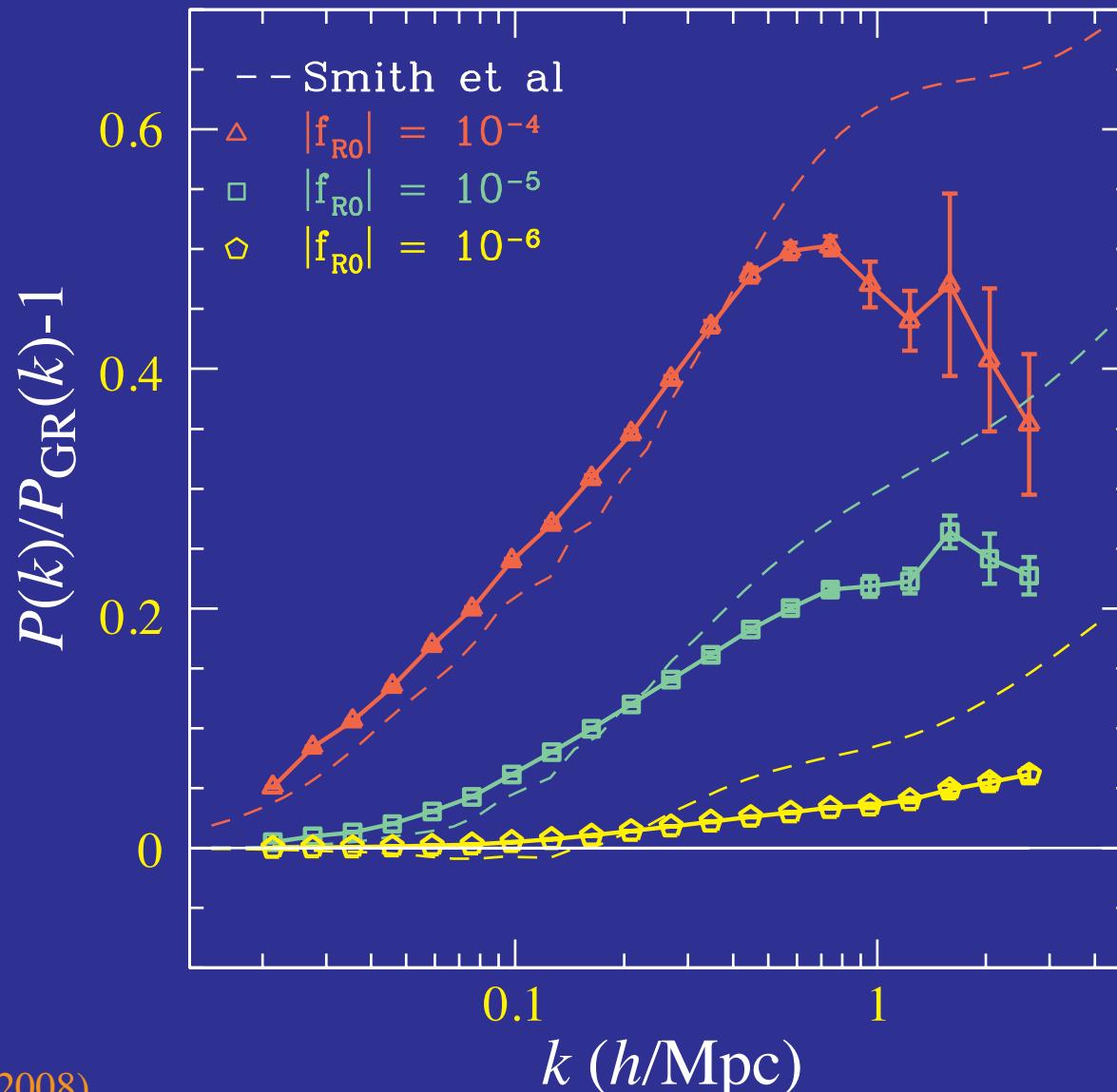
Distance Predicts Growth

- With smooth dark energy, distance predicts scale-invariant growth to a few percent - a falsifiable prediction



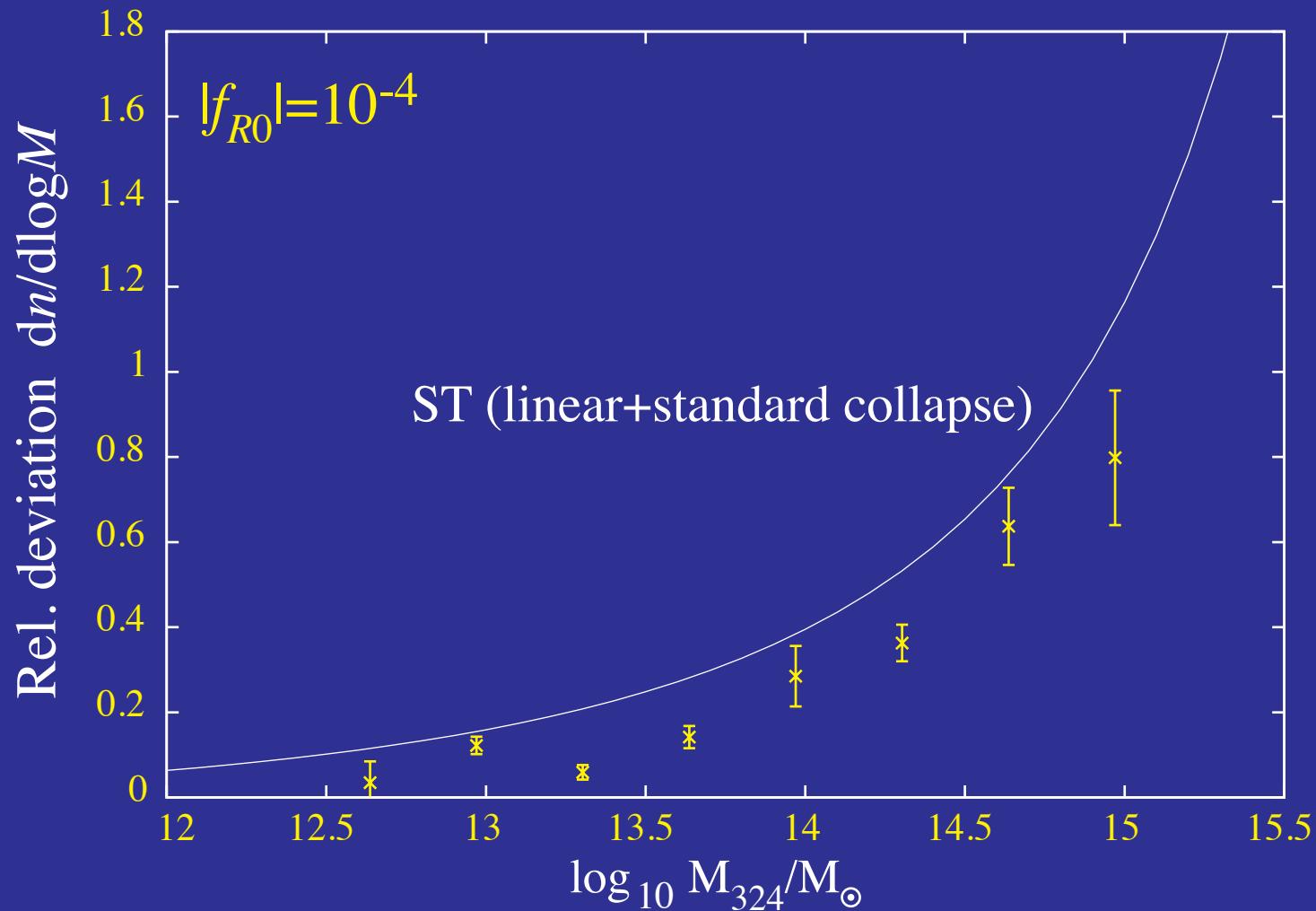
Scaling Relations

- Fitting functions based on normal gravity fail to capture chameleon and effect of extra forces on dark matter halos



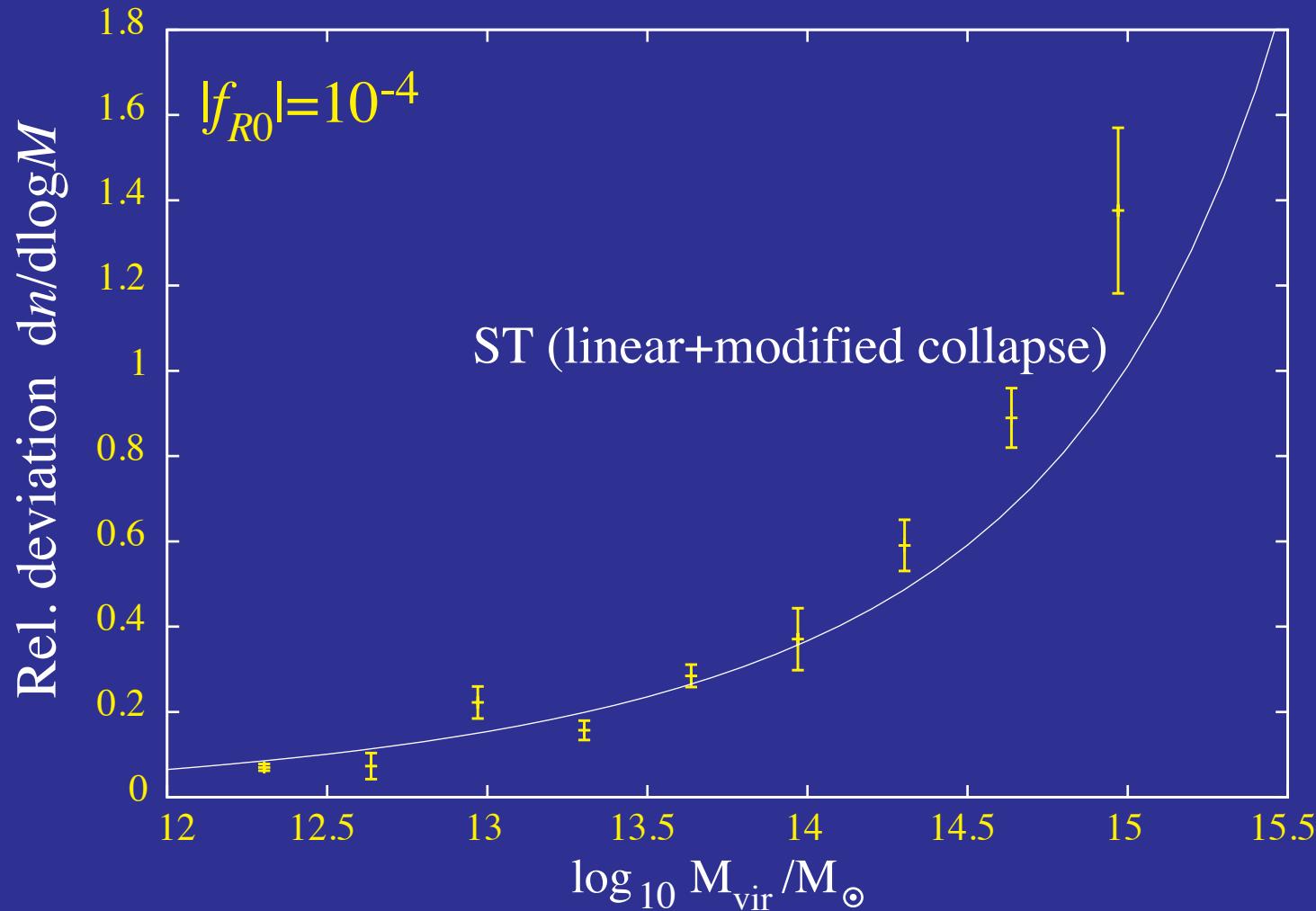
Mass Function

- Enhanced abundance of rare dark matter halos (clusters) with extra force



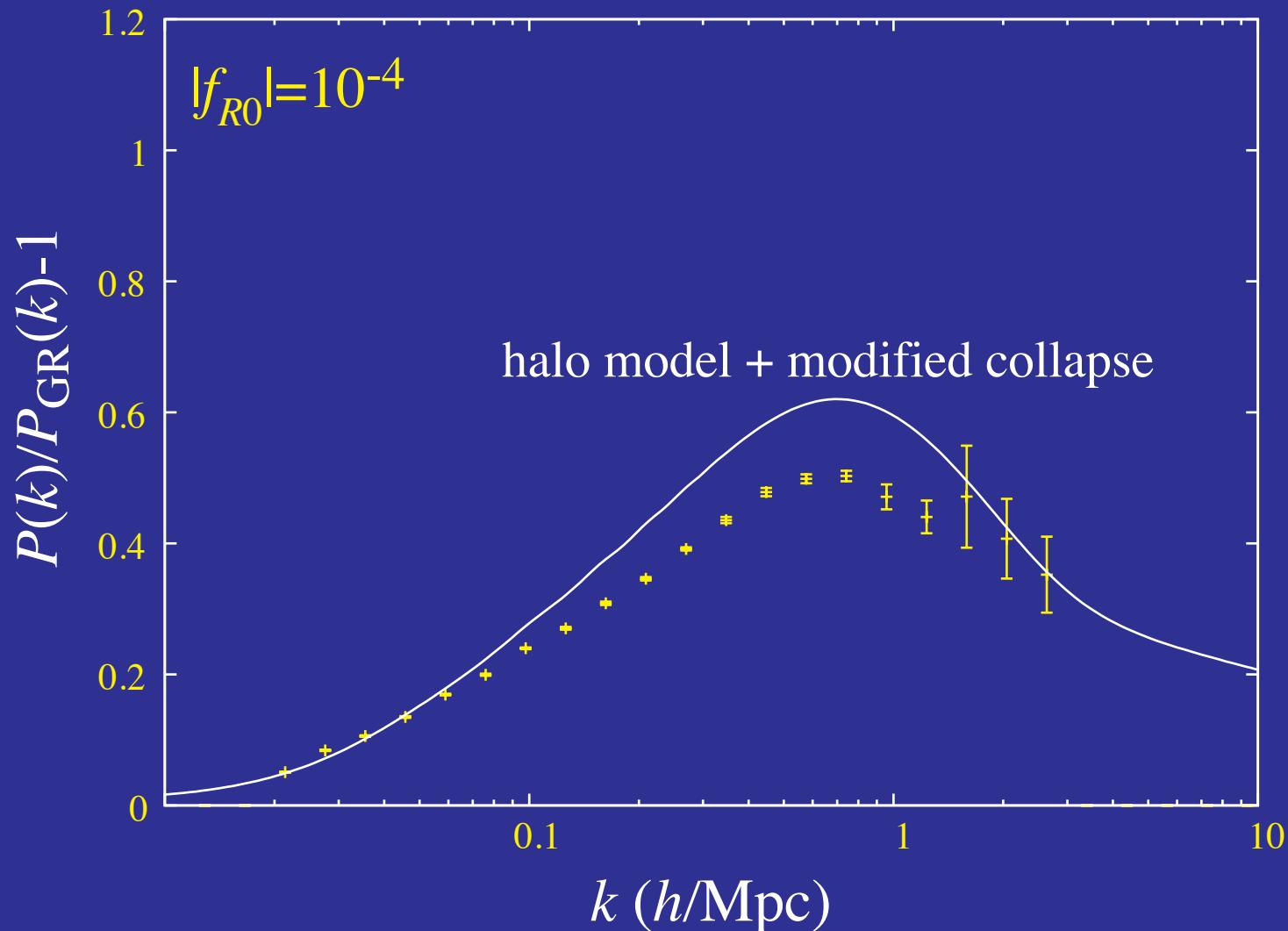
Mass Function

- Small trends consistent with modified collapse from extra force



Halo Model

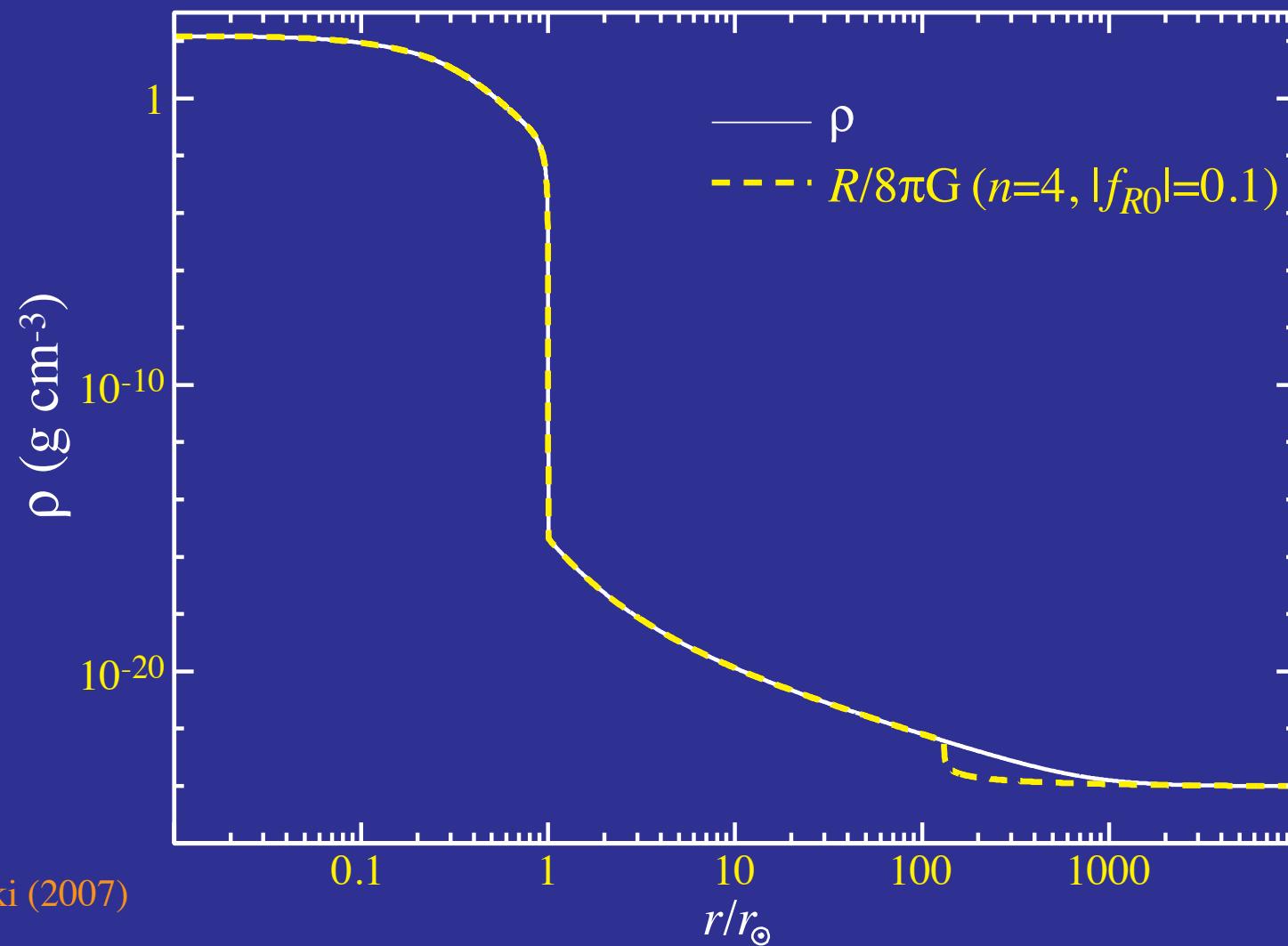
- Power spectrum trends also consistent with halos and modified collapse



$f(R)$ Solar System Tests

Solar Profile

- Density profile of Sun is not a constant density sphere - interior photosphere, chromosphere, corona
- Density drops by ~ 25 orders of magnitude - does curvature follow?



$f(R)$ Chameleon

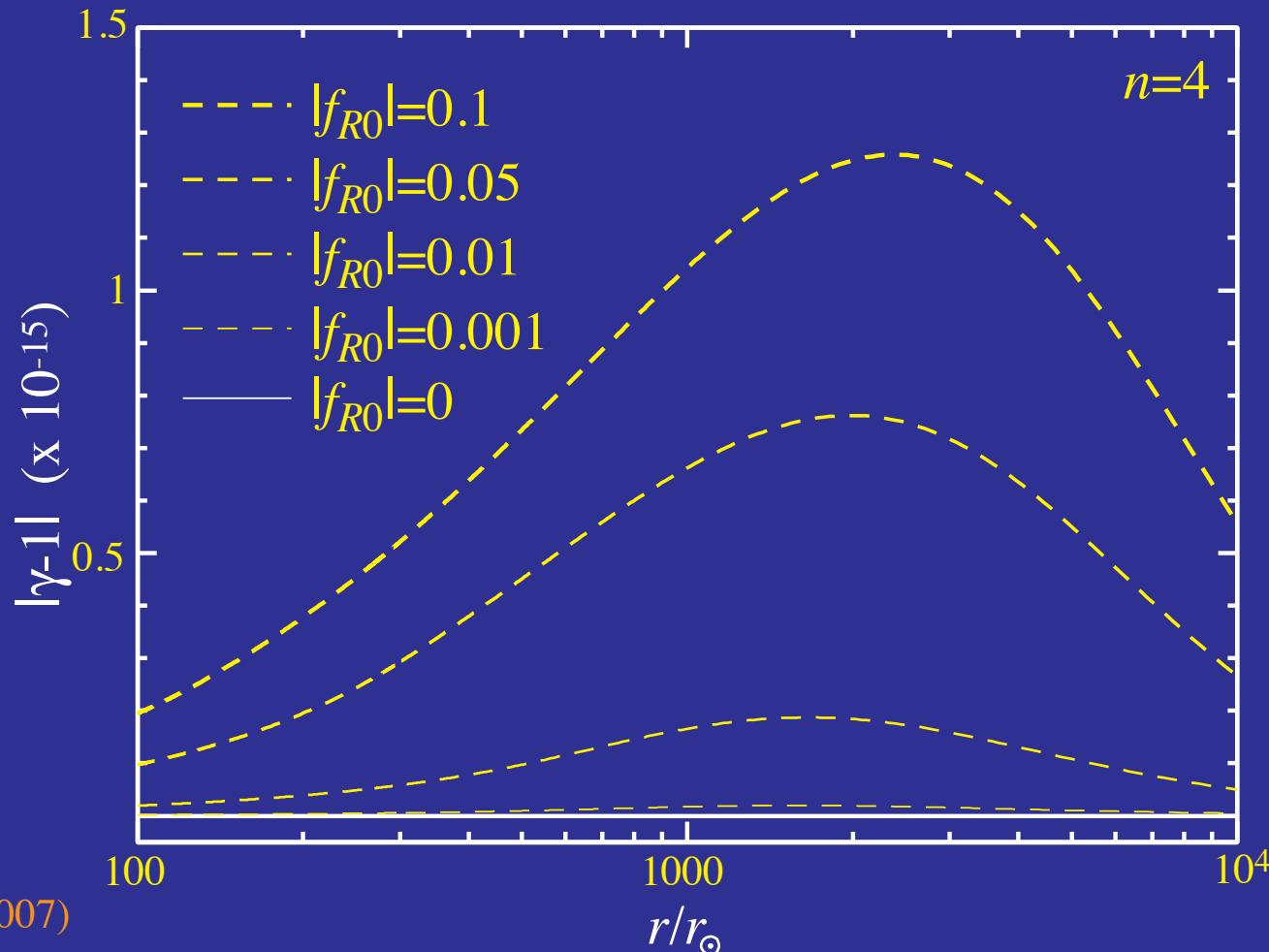
- The field f_R does not then sit at the potential minimum everywhere but instead minimizes the cost of potential and kinetic gradient energy
- A solution for f_R is a solution for R and the metric is fixed to be consistent with the curvature

$$|\gamma - 1| \approx \frac{|\Delta f_R(r)|}{\Phi(r)}$$

- Constraints on $|\gamma - 1|$ place constraints on the change in the field amplitude from the interior of the sun to the exterior of the solar system
- A second transition occurs from the field changes from in the galaxy to cosmology

Solar System Constraint

- Cassini constraint on PPN $|\gamma-1| < 2.3 \times 10^{-5}$
- Easily satisfied if galactic field is at potential minimum
 $|f_{Rg}| < 4.9 \times 10^{-11}$
- Allows even order unity cosmological fields



Summary

- General lessons from $f(R)$ example – 3 regimes:
 - large scales: conservation determined
 - intermediate scales: scalar-tensor
 - small scales: GR in high density regions, modified in low
- Given fixed expansion history $f(R)$ has additional continuous parameter: Compton wavelength
- Enhanced gravitational forces below environment-dependent Compton scale affect growth of structure
- Enhancement hidden by non-linear chameleon mechanism at high curvature \neq high density)
- N -body (PM-relaxation) simulations show potentially observable differences in the power spectrum and mass function