Cosmological and Solar System Tests of $f(R)$ Cosmic Acceleration

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Why Study $f(R)$?

- Cosmic acceleration, like the cosmological constant, can either be viewed as arising from
  - Missing, or dark energy, with $w \equiv \bar{p}/\bar{\rho} < -1/3$
  - Modification of gravity on large scales
- Compelling models for either explanation lacking

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- Compelling models for either explanation lacking
- Dark energy parameterized description on small scales: $w(z)$ that completely defines expansion history, sound speed defines structure formation
- Parameterized description of modified gravity acceleration?
- Many ad-hoc attempts violate energy-momentum conservation, Bianchi identities, gauge invariance; others incomplete
- Study DGP braneworld acceleration and $f(R)$ modified action; learn how to generalize

AdD/C $f(R)$ Correspondence

- Necessary to take squared mass of the scalar positive so that high curvature is stable – violated in original $f(R) = -\mu^4/R$ model (stellar structure Dolgov& Kawasaki 2003, expansion history Amendola et al 2006)
- Growth of structure strongly impacted by Compton wavelength of scalar even when expansion history and distances unchanged

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- Growth of structure strongly impacted by Compton wavelength of scalar even when expansion history and distances unchanged
- Solar system test: controversy stems from two extreme spherical cow approximations: sun + cosmological background (Chiba 2003; Erikchek et al 2006), sun + infinite galaxy ($f(R)$ chameleon)
- Precision of solar system (and laboratory) tests largely irrelevant
- Viability of large deviations rests on galactic structure and evolution
- Small cosmological deviations certainly viable and are not so small in quasilinear regime
- Lessons for a Parameterized Post-Friedmann framework

Outline

- Basics and $f(R)$ as an Effective Theory
- Linear Theory Predictions and Current Constraints
- Models of $f(R)$ as Complete Theory of Gravity?
- Solar System Tests
- Parameterized Post-Friedmann Framework

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- Collaborators:
  - Hiranya Peiris (Chicago $\longrightarrow$ Cambridge)
  - Iggy Sawicki (Chicago $\longrightarrow$ NYU)
  - Yong-Seon Song (Chicago $\longrightarrow$ Portsmouth)

$f(R)$ Basics

Cast of $f(R)$ Characters

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_{\rm m} \right]$$

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- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B: Compton wavelength of f_R squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate

Modified Einstein Equation

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved

$$G_{\alpha\beta} + f_R R_{\alpha\beta} - \left(\frac{f}{2} - \Box f_R \right) g_{\alpha\beta} - \nabla_\alpha \nabla_\beta f_R = 8\pi G T_{\alpha\beta}$$

- Trace can be interpreted as a scalar field equation for f_R with a density-dependent effective potential ($p = 0$)

$$3\Box f_R + f_R R - 2f = R - 8\pi G\rho$$

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- For small deviations, $|f_R| \ll 1$ and $|f/R| \ll 1$,

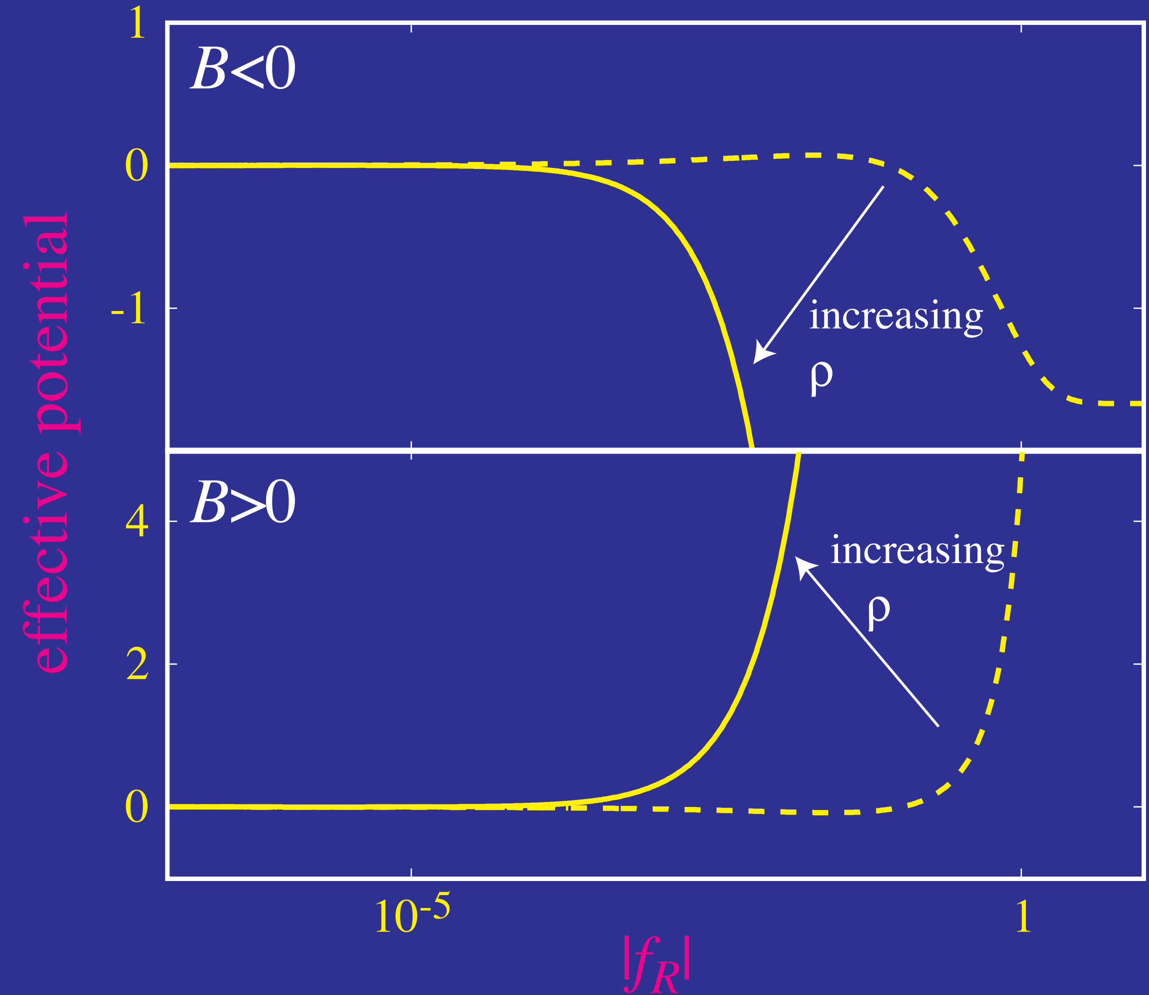
$$\Box f_R \approx \frac{1}{3}(R - 8\pi G\rho)$$

the field is sourced by the deviation from GR relation between curvature and density and has a mass

$$m_{f_R}^2 \approx \frac{1}{3}\frac{\partial R}{\partial f_R} = \frac{1}{3 f_{RR}}$$

Effective Potential

- Scalar f_R rolls in an effective potential that depends on density
- At high density, extrema is at GR $R=8\pi G\rho$
- Minimum for $B>0$, pinning field to $|f_R| \ll 1$, maximum for $B<0$



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$f(R)$ Expansion History

Modified Friedmann Equation

- Expansion history parameterization: Friedmann equation becomes

$$H^2 - f_R(HH' + H^2) + \frac{1}{6}f + H^2 f_{RR} R' = \frac{8\pi G\rho}{3}$$

- Reverse engineering $f(R)$ from the expansion history: for any desired H, solve a 2nd order diffeq to find $f(R)$
- Allows a family of $f(R)$ models, parameterized in terms of the Compton wavelength parameter B

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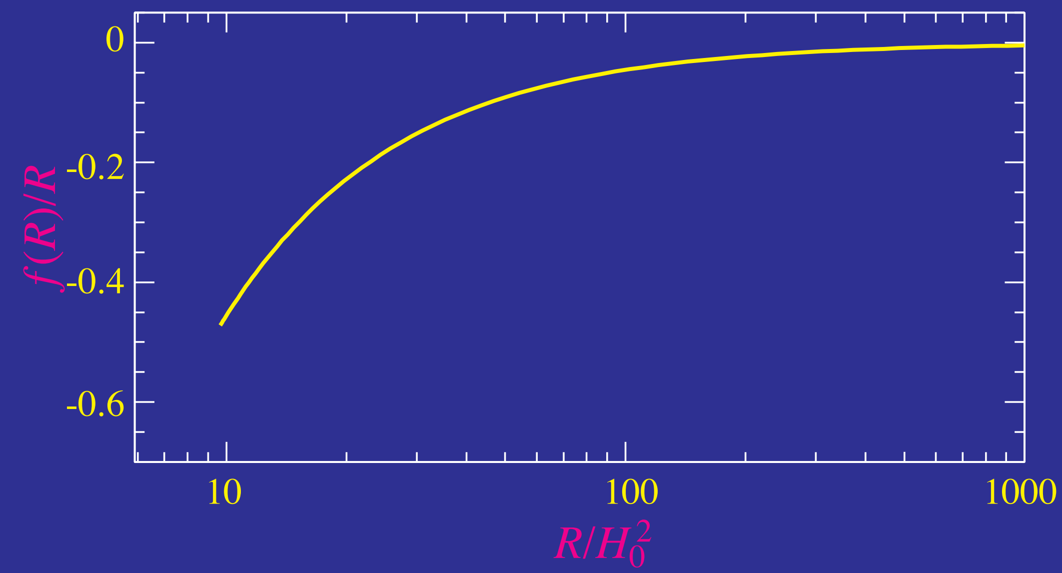
- Reverse engineering $f(R)$ from the expansion history: for any desired H, solve a 2nd order diffeq to find $f(R)$
- Allows a family of $f(R)$ models, parameterized in terms of the Compton wavelength parameter B
- Formally includes models where $B < 0$, such as $f(R) = -\mu^4/R$, leading to confusion as to whether such models provide viable expansion histories
- Answer: no these have short-time scale tachyonic instabilities at high curvature and limit as $B \to 0$ from below is not GR
- $B > 0$ family has very different implications for structure formation but with identical distance-redshift relations

Expansion History Family of $f(R)$

- Each expansion history, matched by dark energy model $[w(z),\Omega_{DE},H_0]$ corresponds to a family of $f(R)$ models due to its 4th order nature
- Parameterized by $B \propto f_{RR} = d^2f/dR^2$ evaluated at z=0



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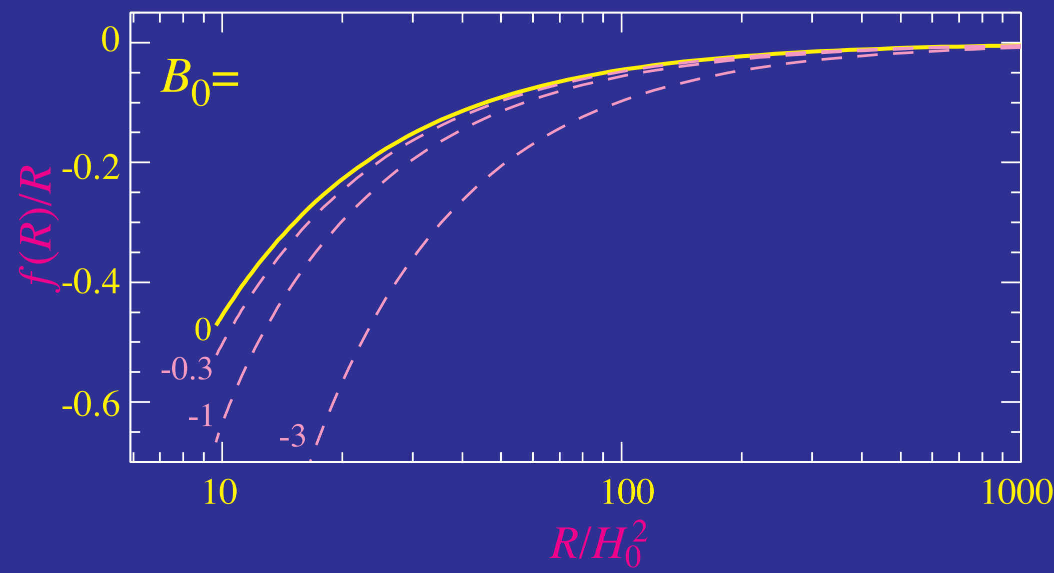

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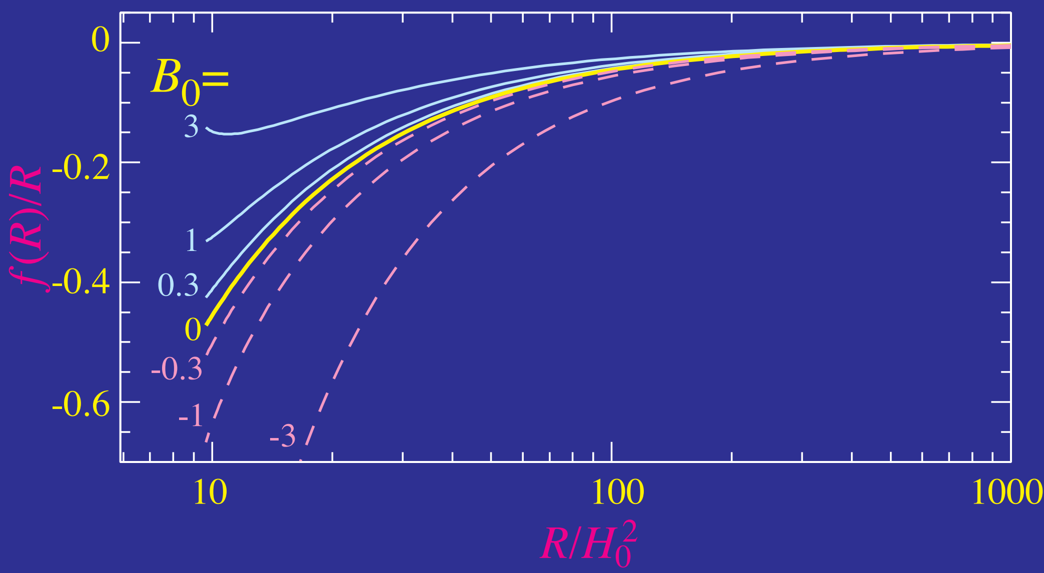

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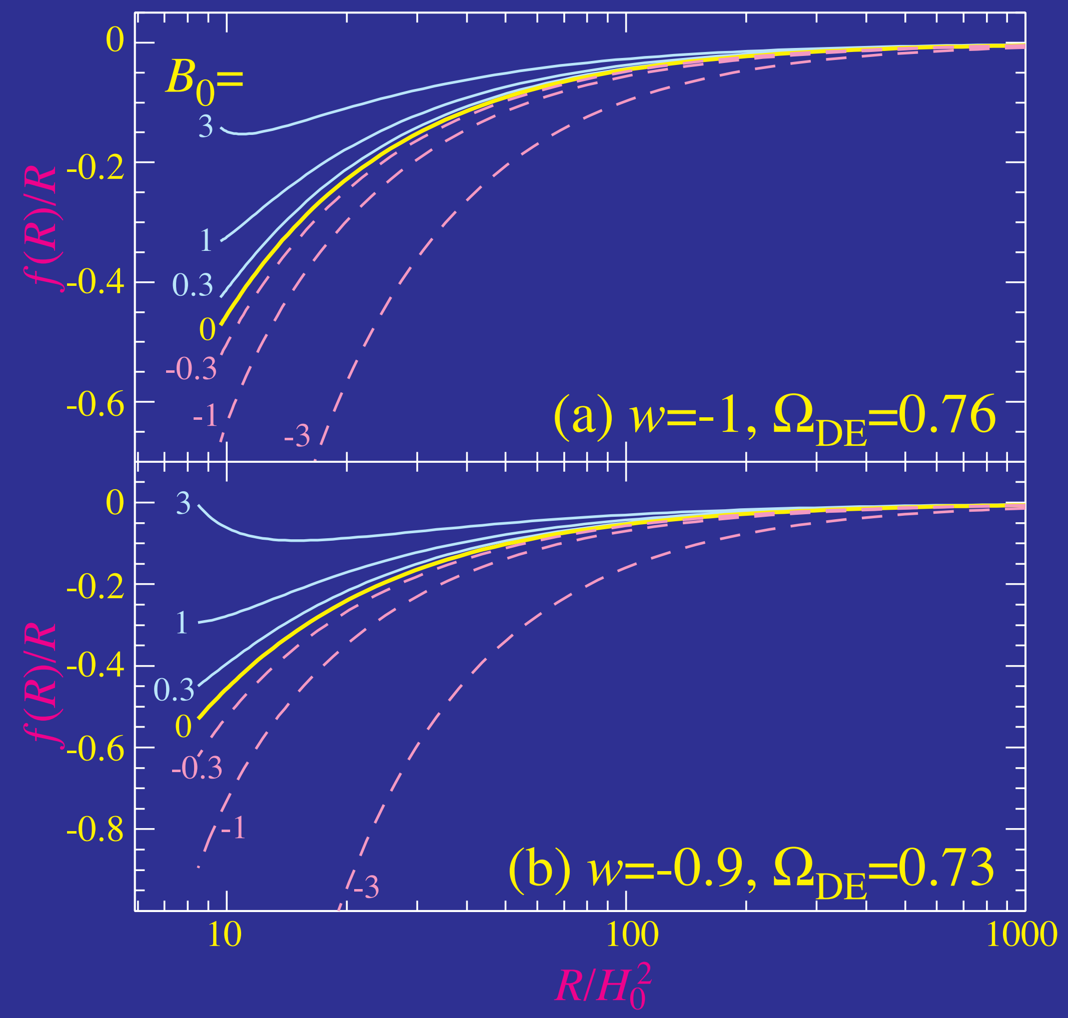

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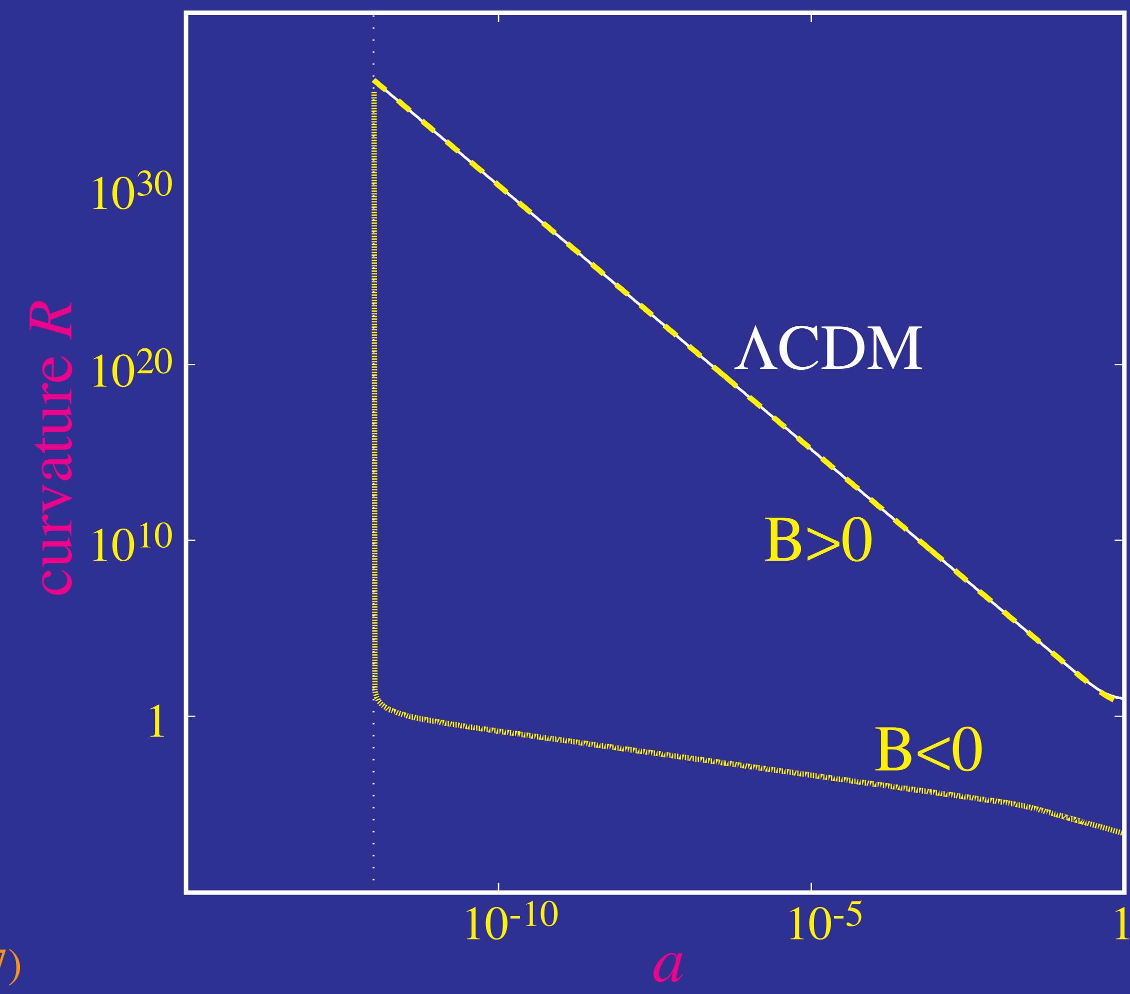
Instability at High Curvature

- Tachyonic instability for negative mass squared $B<0$ makes high curvature regime increasingly unstable: high density $\neq$ high curvature
- Linear metric perturbations immediately drop the expansion history to low curvature solution



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$f(R)$ Linear Theory

PPF Description

- On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)
- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d\ln a$ (Bardeen 1980)

Curvature Conservation

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- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d\ln a$ (Bardeen 1980)
- Gauge transformation to Newtonian gauge

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2$$

yields (Hu & Eisenstein 1999)

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = 0$$

- Modified gravity theory supplies the closure relationship $\Phi = -\gamma(\ln a)\Psi$ between and expansion history $H = \dot{a}/a$ supplies rest.

Linear Theory for $f(R)$

- In $f(R)$ model, "superhorizon" behavior persists until Compton wavelength smaller than fluctuation wavelength $B^{1/2}(k/aH) < 1$
- Once Compton wavelength becomes larger than fluctuation

$$B^{1/2}(k/aH) > 1$$

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  perturbations are in scalar-tensor regime described by $\gamma = 1/2$.
- Small scale density growth enhanced and

$$8\pi G\rho > R$$

  low curvature regime with order unity deviations from GR
- Transitions in the non-linear regime where the Compton wavelength can shrink via chameleon mechanism
- Given $k_{\rm NL}/aH \gg 1$, even very small f_R have scalar-tensor regime

Deviation Parameter

- Express the 4th order nature of equations as a deviation parameter

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = \left(\frac{k}{aH}\right)^2 B\epsilon$$

- Einstein equation become a second order equation for ϵ

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- Einstein equation become a second order equation for ϵ
- In high redshift, high curvature R limit this is

$$\epsilon'' + \left(\frac{7}{2} + 4\frac{B'}{B}\right)\epsilon' + \frac{2}{B}\epsilon = \frac{1}{B} \times \text{ metric sources}$$

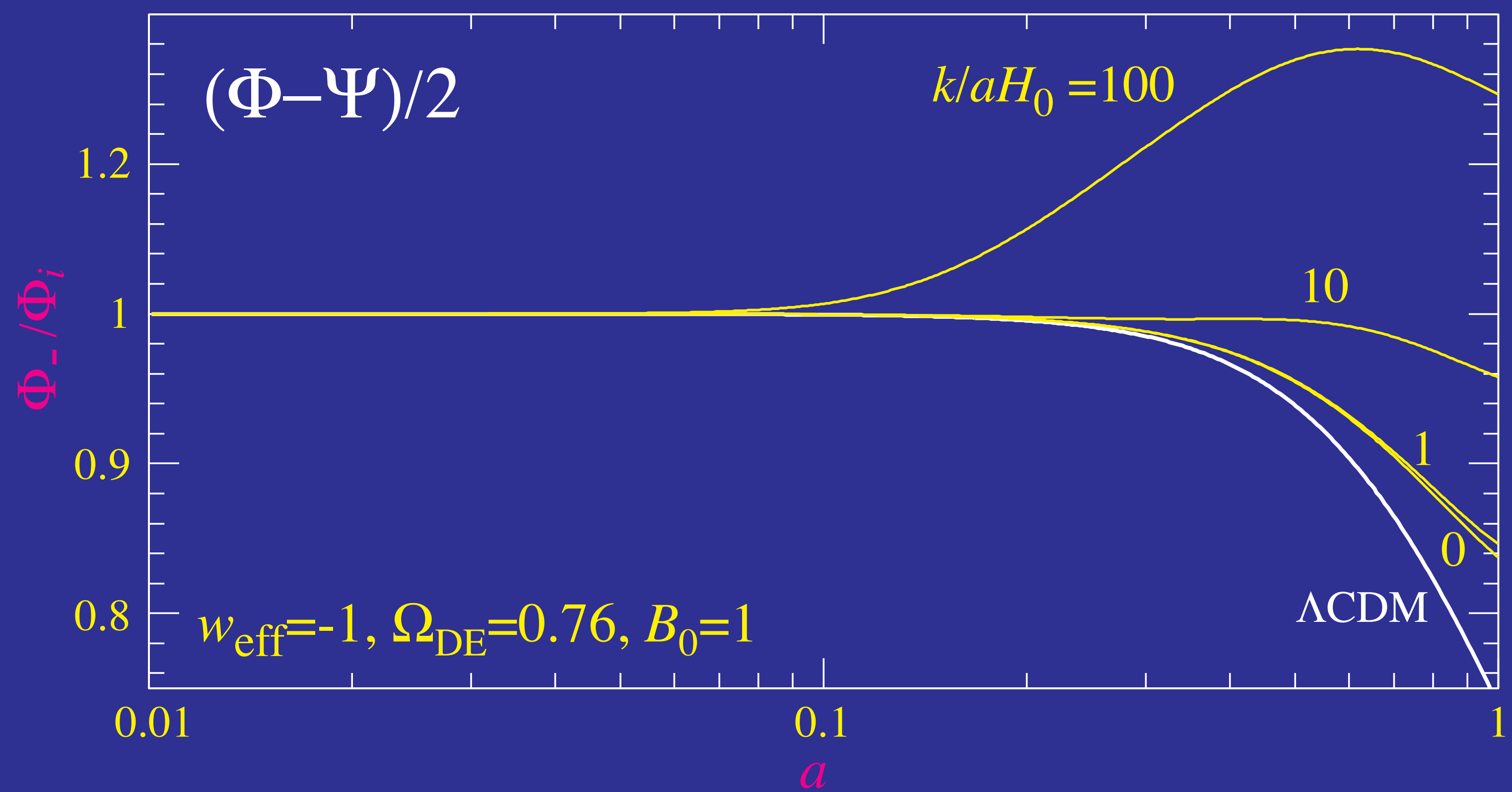
$$B = \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

- $R \to \infty$, $B \to 0$ and for $B < 0$ short time-scale tachyonic instability appears making previous models not cosmologically viable

$$f(R) = -M^{2+2n}/R^n$$

Potential Growth

- On the stable $B>0$ branch, potential evolution reverses from decay to growth as wavelength becomes smaller than Compton scale
- Quasistatic equilibrium reached in linear theory with $\gamma=-\Phi/\Psi=1/2$ until non-linear effects restore $\gamma=1$



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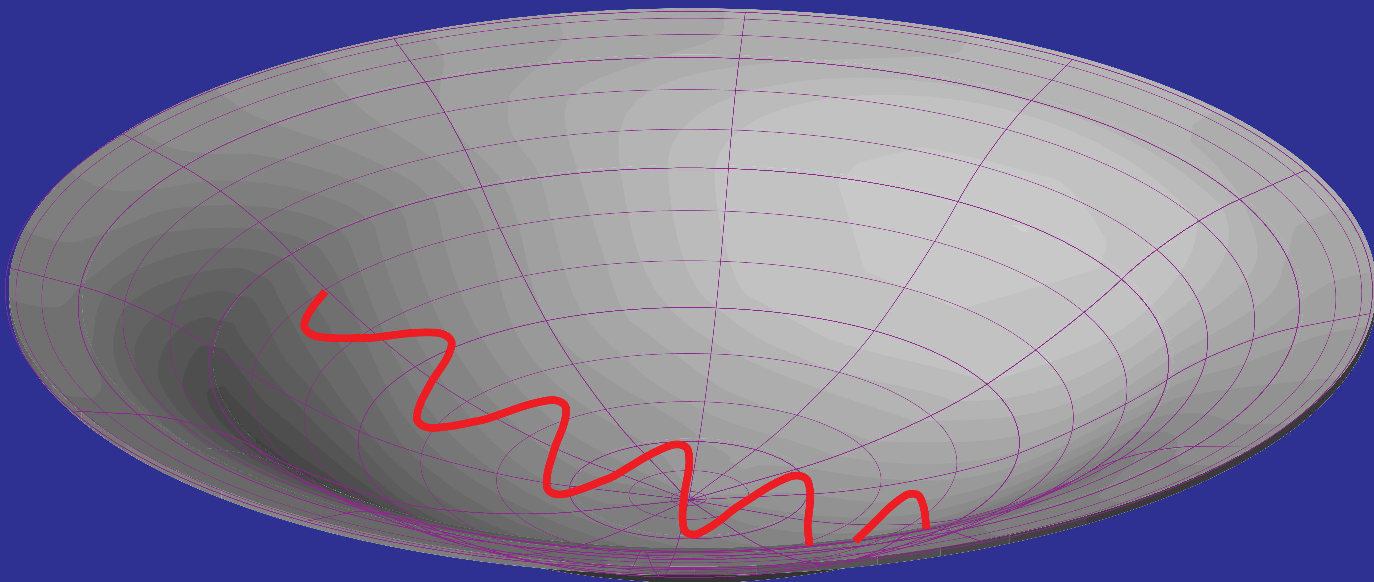
Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi - \Psi)$



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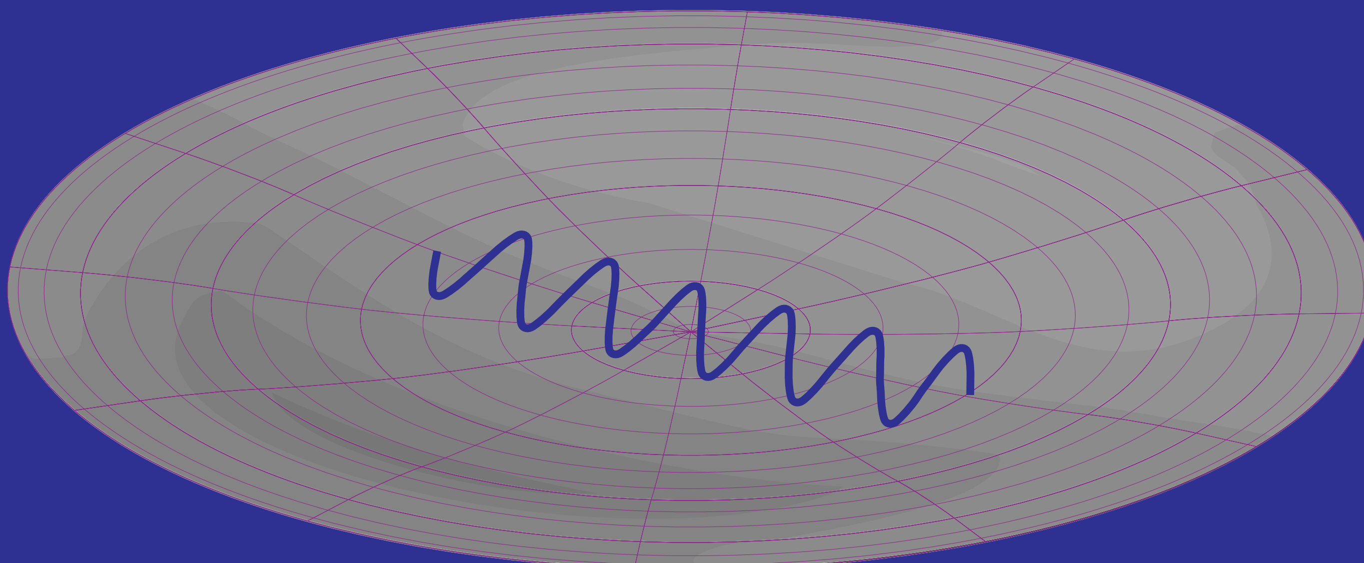
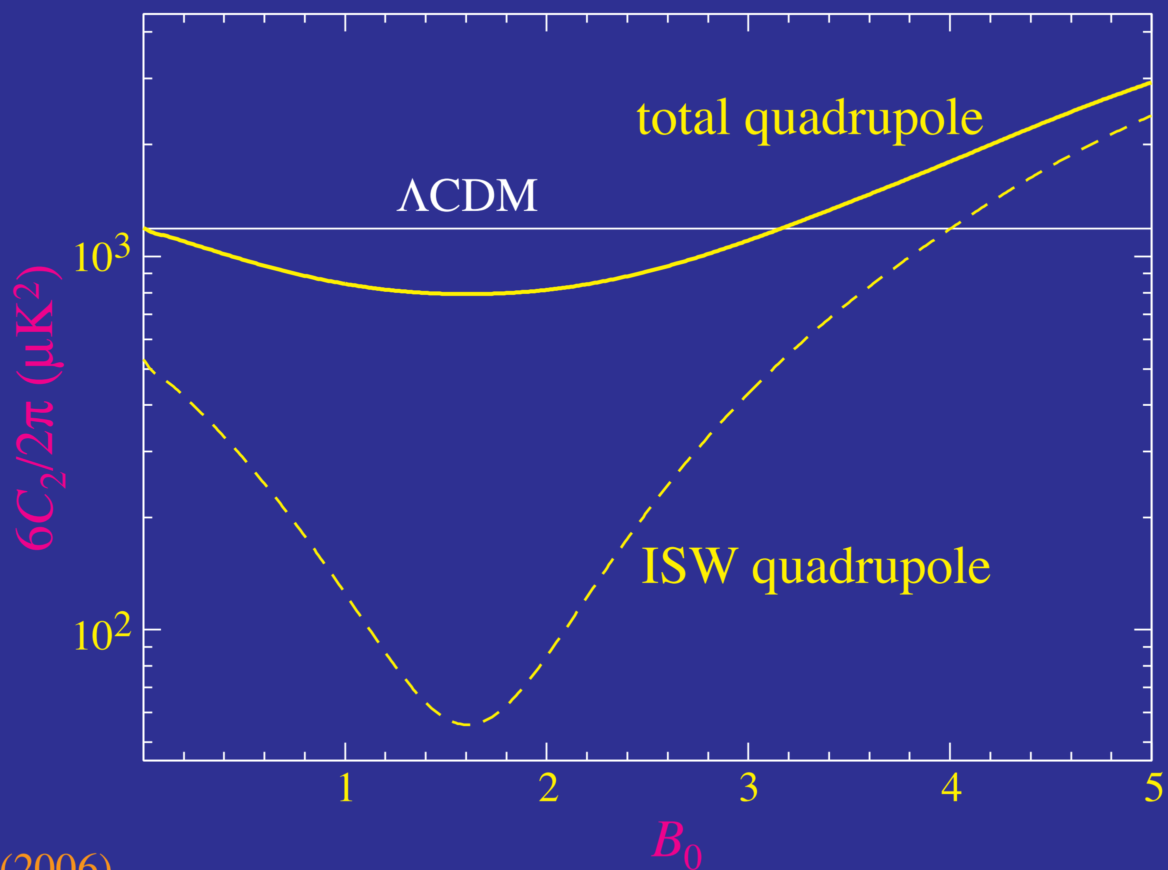


ISW Quadrupole

- Reduction of potential decay can eliminate the ISW effect at the quadrupole for B_0~3/2
- In conjunction with a change in the initial power spectrum can also bring the total quadrupole closer in ensemble average to the observed quadrupole



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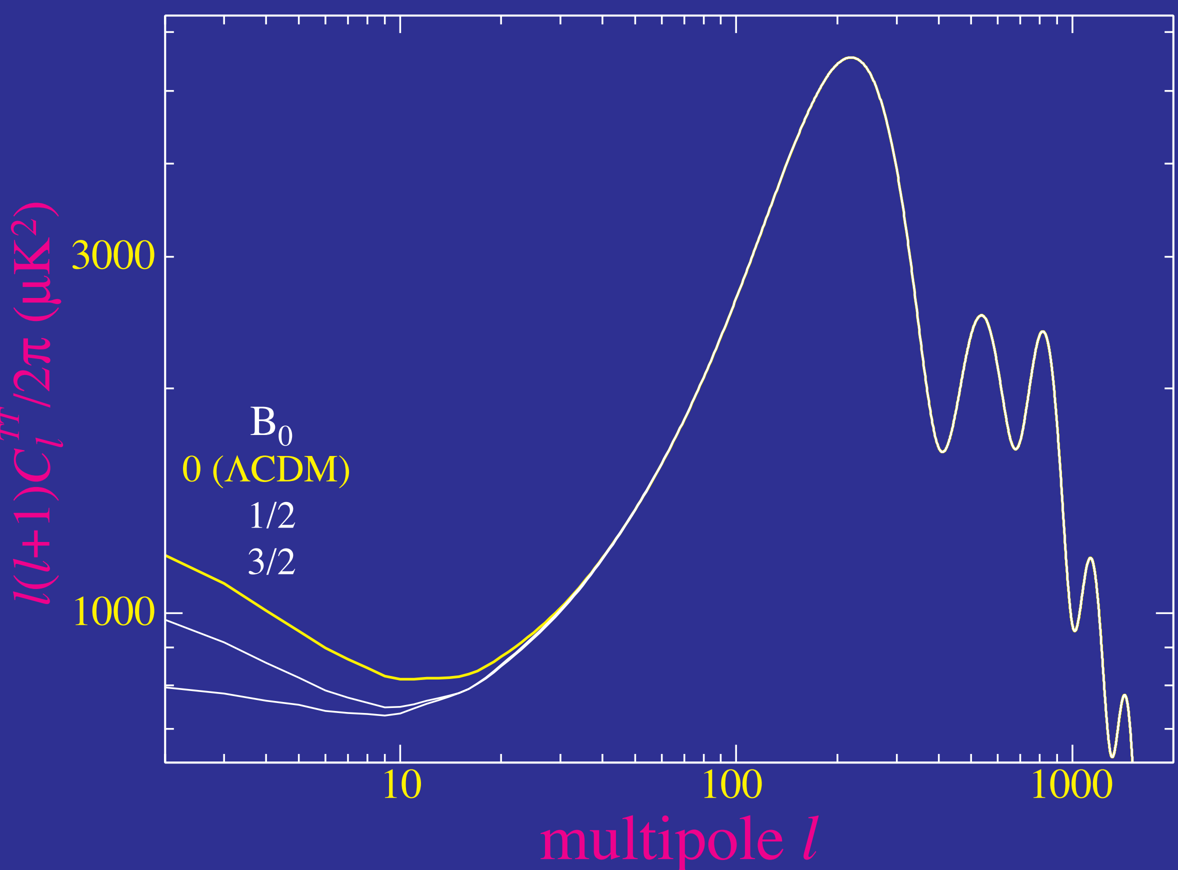
ISW Quadrupole

- Reduction of large angle anisotropy for B_0~1 for same expansion history and distances as ΛCDM
- Well-tested small scale anisotropy unchanged



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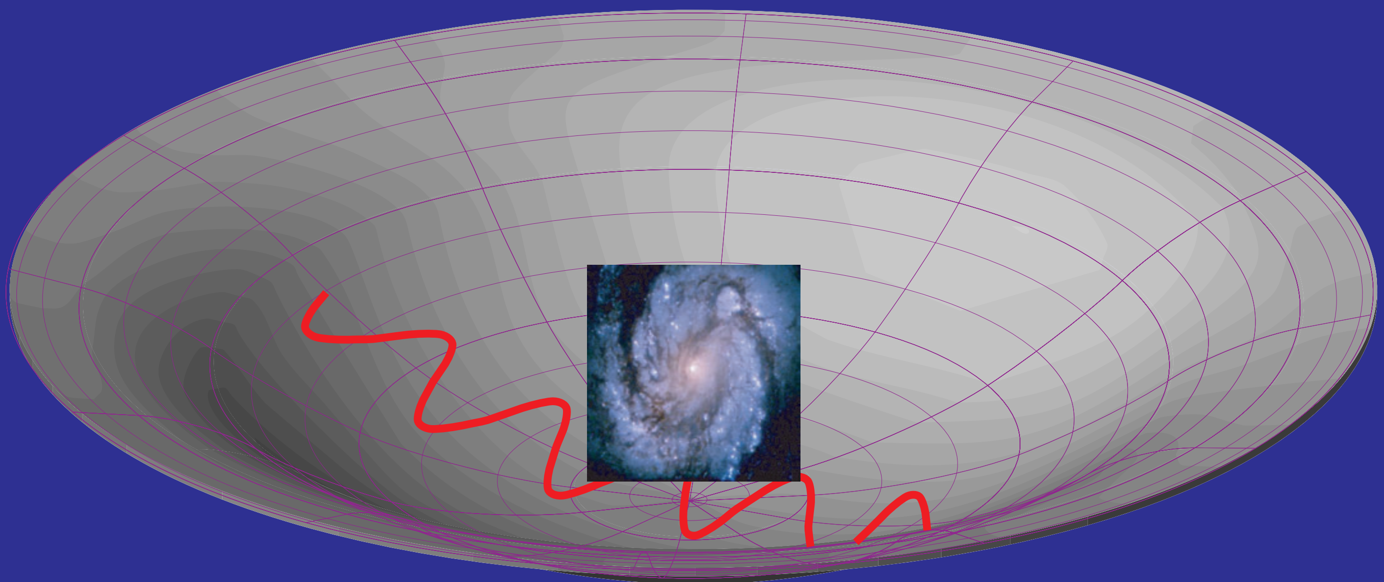
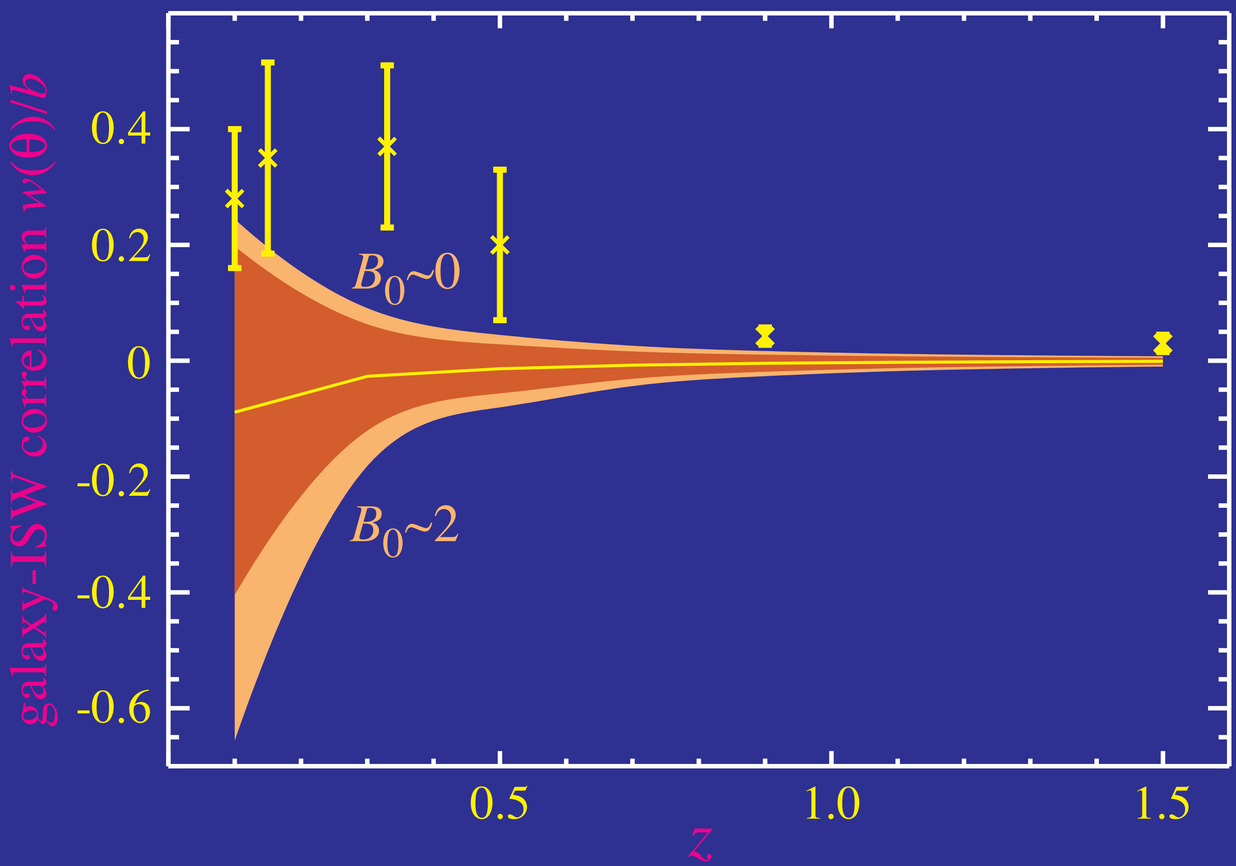
ISW-Galaxy Correlation

- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation



Galaxy-ISW Anti-Correlation

- Large Compton wavelength $B^{1/2}$ creates potential growth which can anti-correlate galaxies and the CMB
- In tension with detections of positive correlations across a range of redshifts



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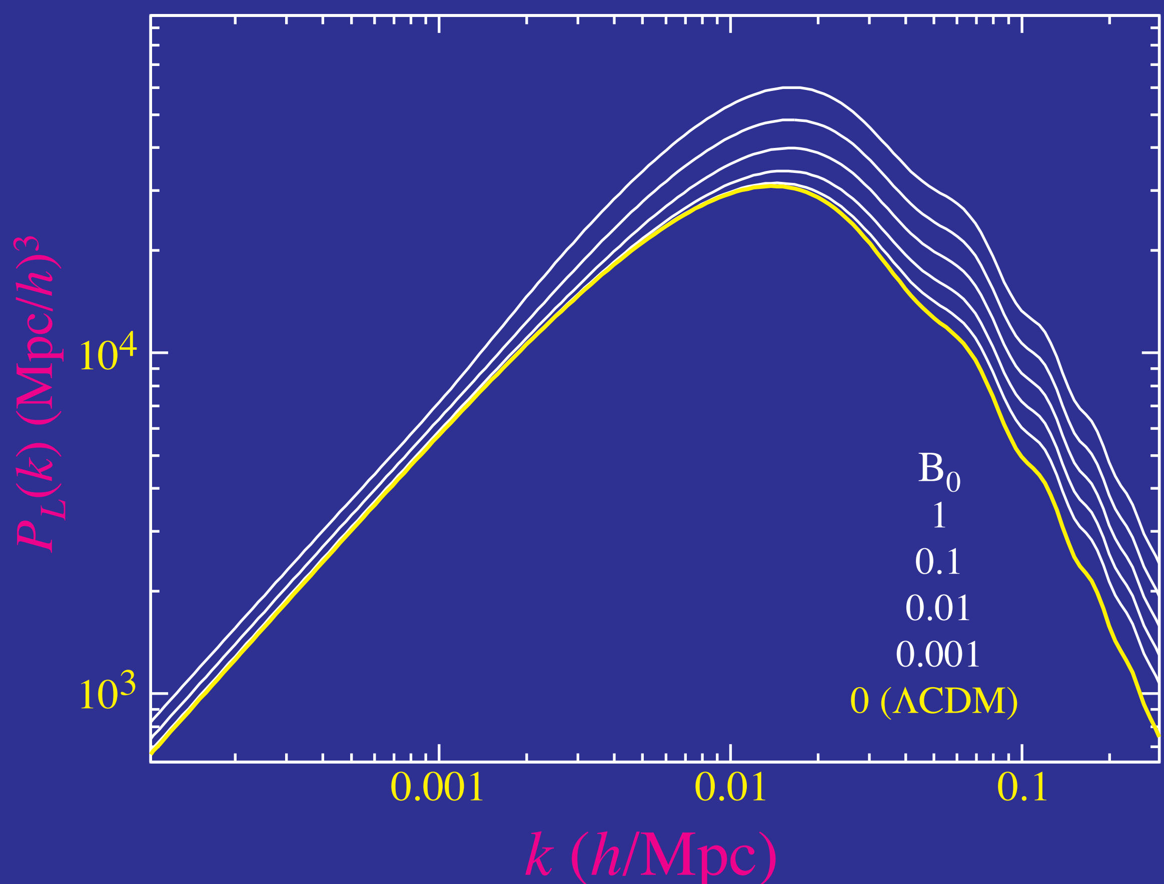
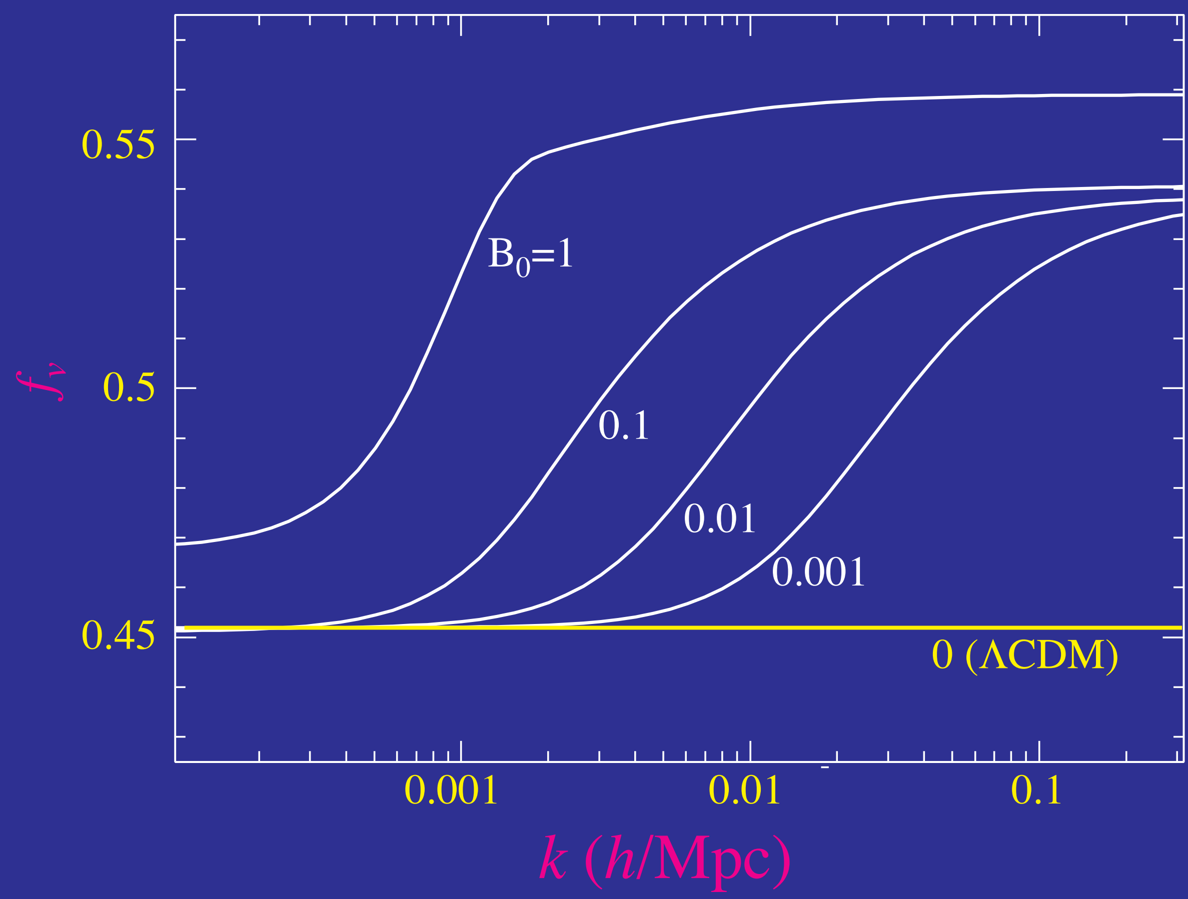
Linear Power Spectrum

- Linear real space power spectrum enhanced on small scales
- Degeneracy with galaxy bias and lack of non-linear predictions leave constraints from shape of power spectrum



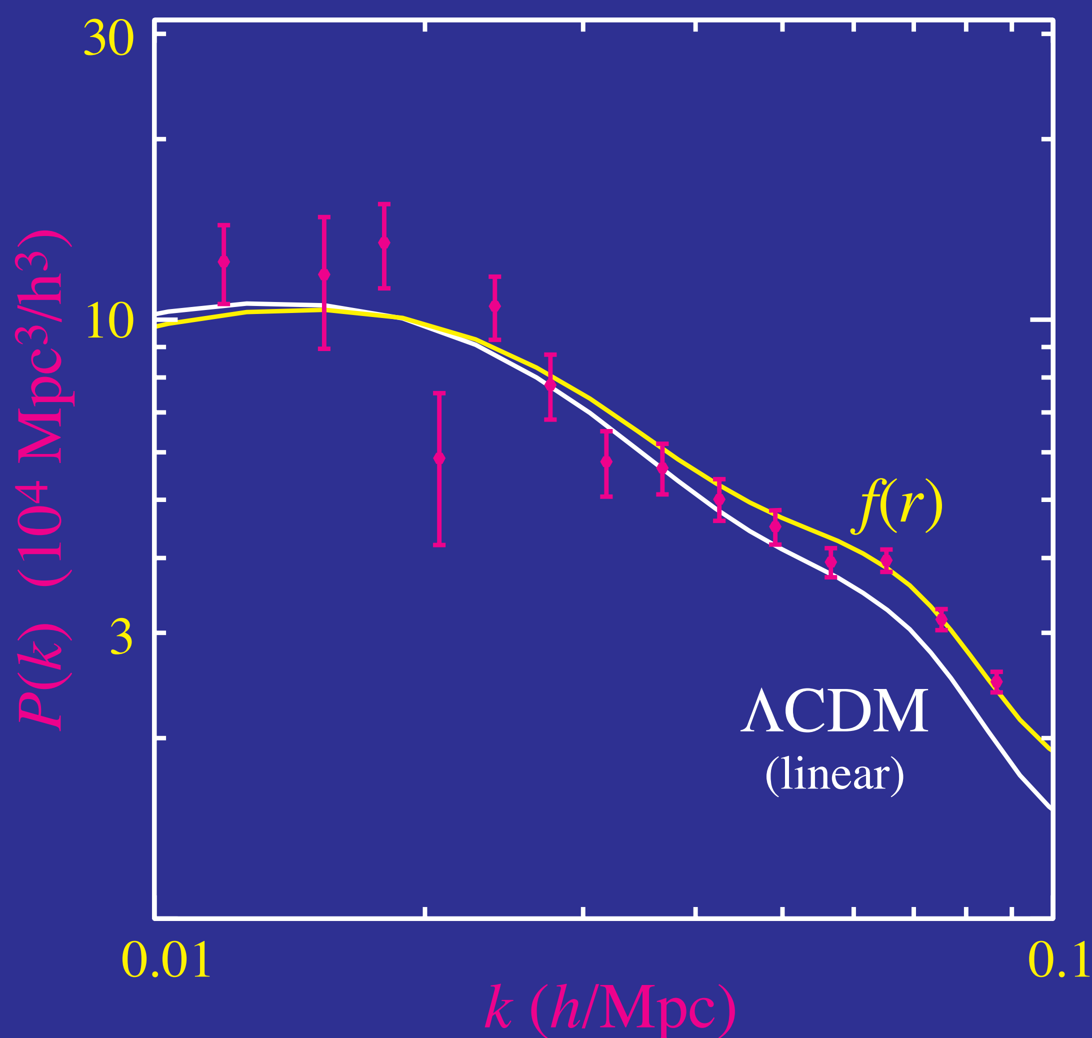
Redshift Space Distortion

- Relationship between velocity and density field given by continuity with modified growth rate
- Redshift space power spectrum further distorted by Kaiser effect



Power Spectrum Data

- Linear power spectrum enhancement fits SDSS LRG data better than ΛCDM but



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Power Spectrum Data

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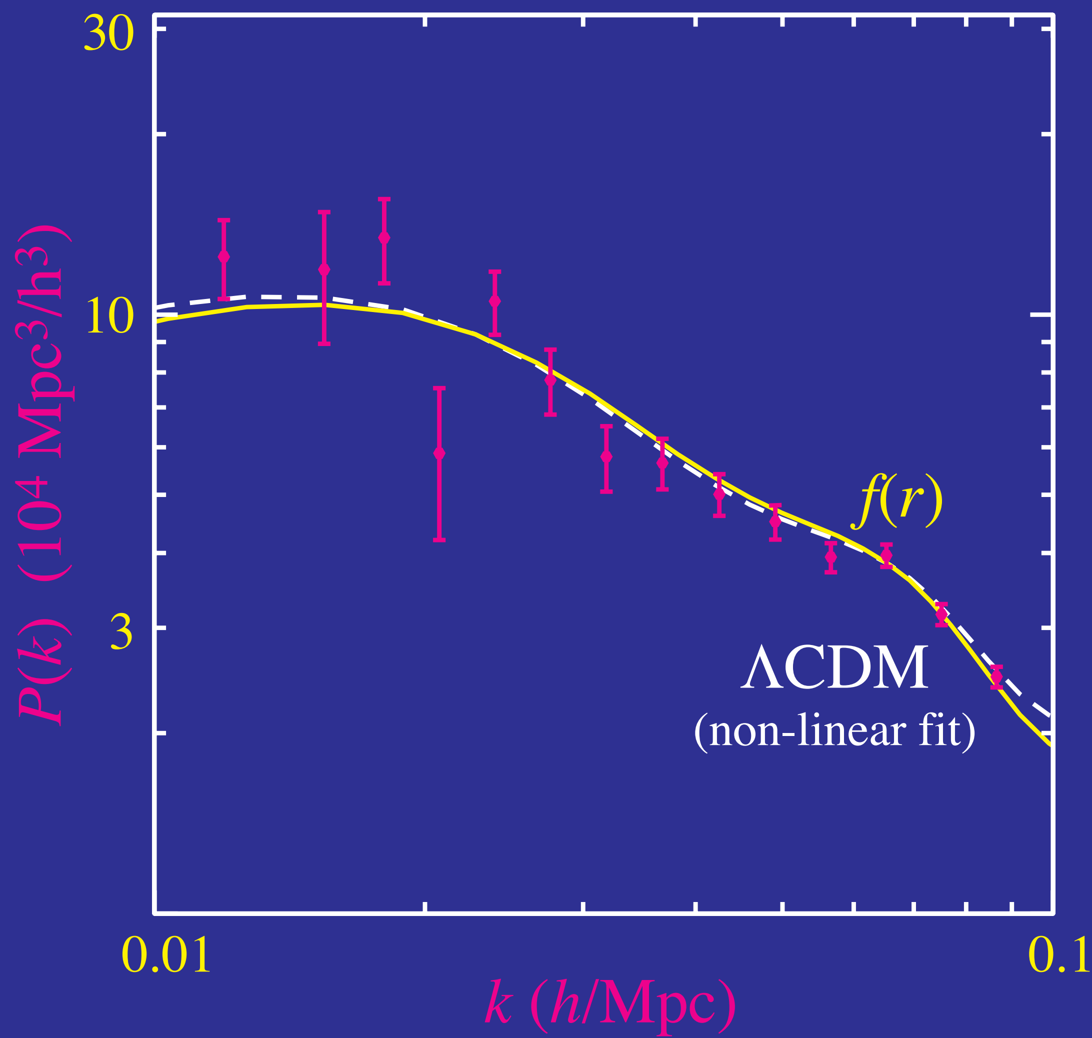
- Shape expected to be altered by non-linearities



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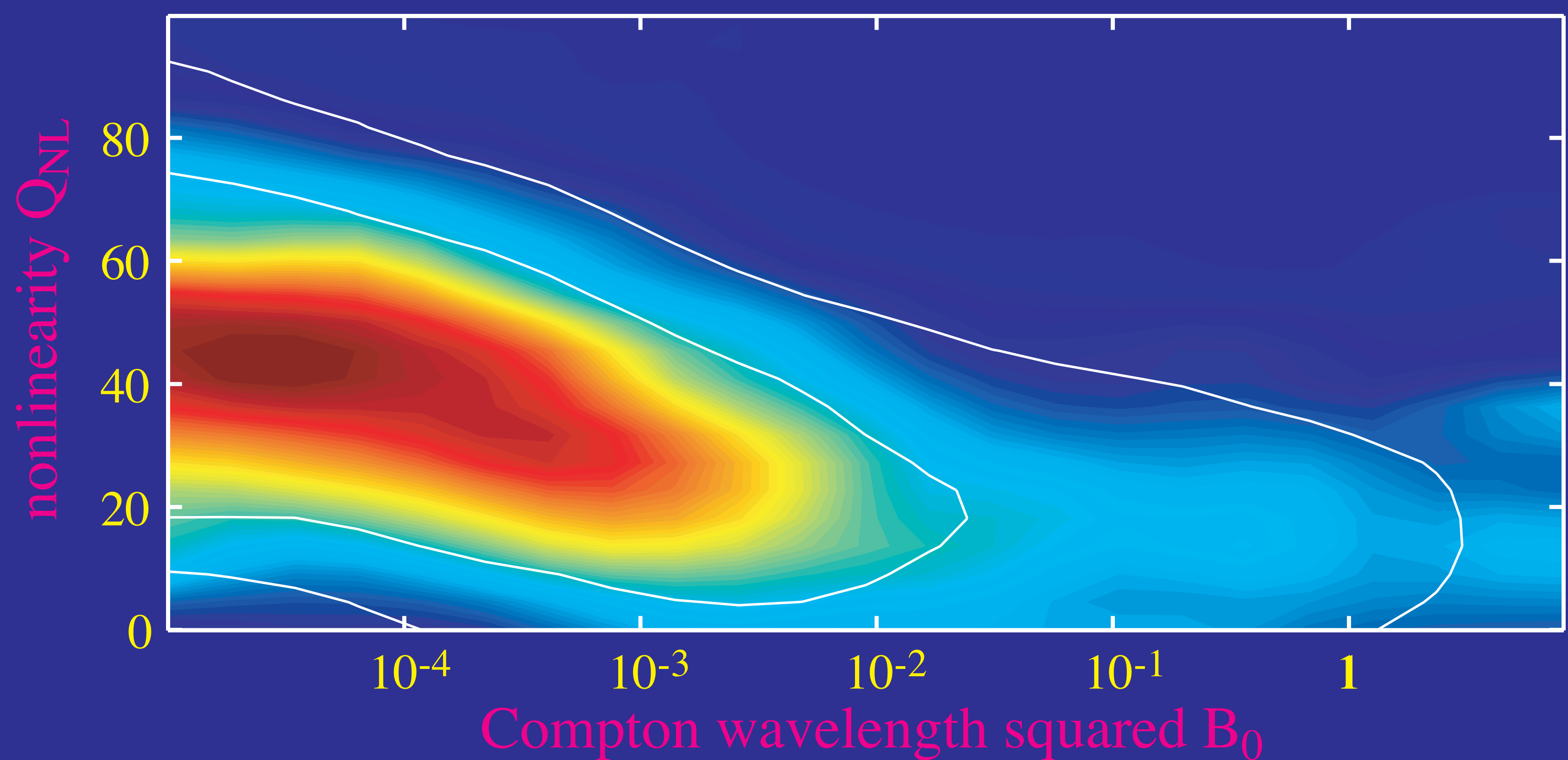
Current Constraints

- Likelihood analysis of SDSS LRG $P(k)$, WMAP C_l, SNIa d_L
- Degeneracy between non-linearity and $f(R)$ enhancement allows whole range of Compton wavelengths from infinitesimal to horizon sized
- Requires cosmological simulation of $f(R)$ to predict non-linearities



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$f(R)$ Models as
A Complete Theory of Gravity?

Engineering $f(R)$ Models

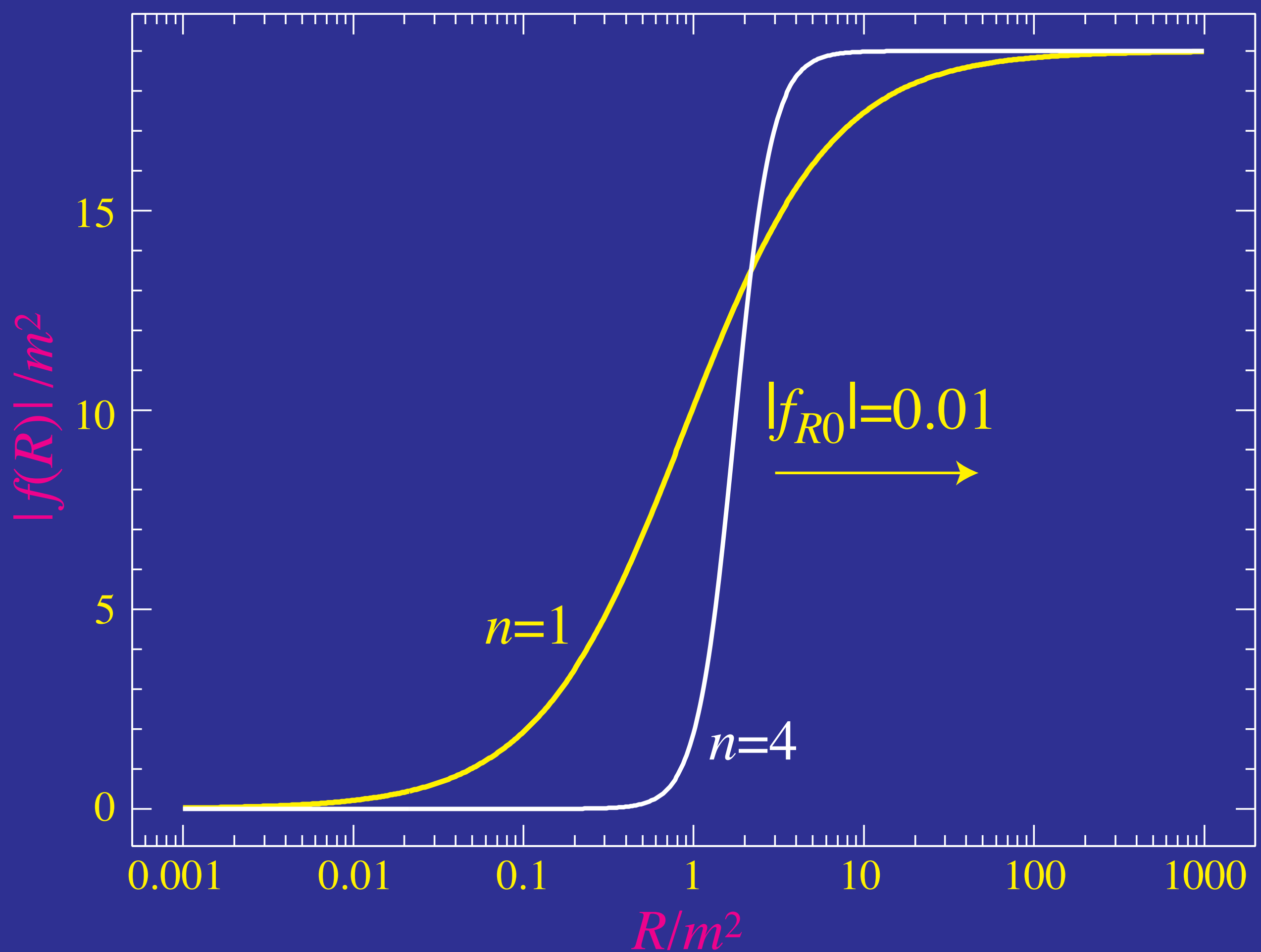
- Mimic ΛCDM at high redshift
- Accelerate the expansion at low redshift without a cosmological constant
- Sufficient freedom to vary expansion history within observationally allowed range
- Contain the phenomenology of ΛCDM in both cosmology and solar system tests as a limiting case for the purposes of constraining small deviations
- Suggests

$$f(R) \propto \frac{R^n}{R^n + \text{const.}}$$

such that modifications vanish as $R \to 0$ and go to a constant as $R \to \infty$

Form of $f(R)$ Models

- Transition from zero to constant across an adjustable curvature scale
- Slope n controls the rapidity of transition, field amplitude f_{R0} position
- Background curvature stops declining during acceleration epoch and thereafter behaves like cosmological constant



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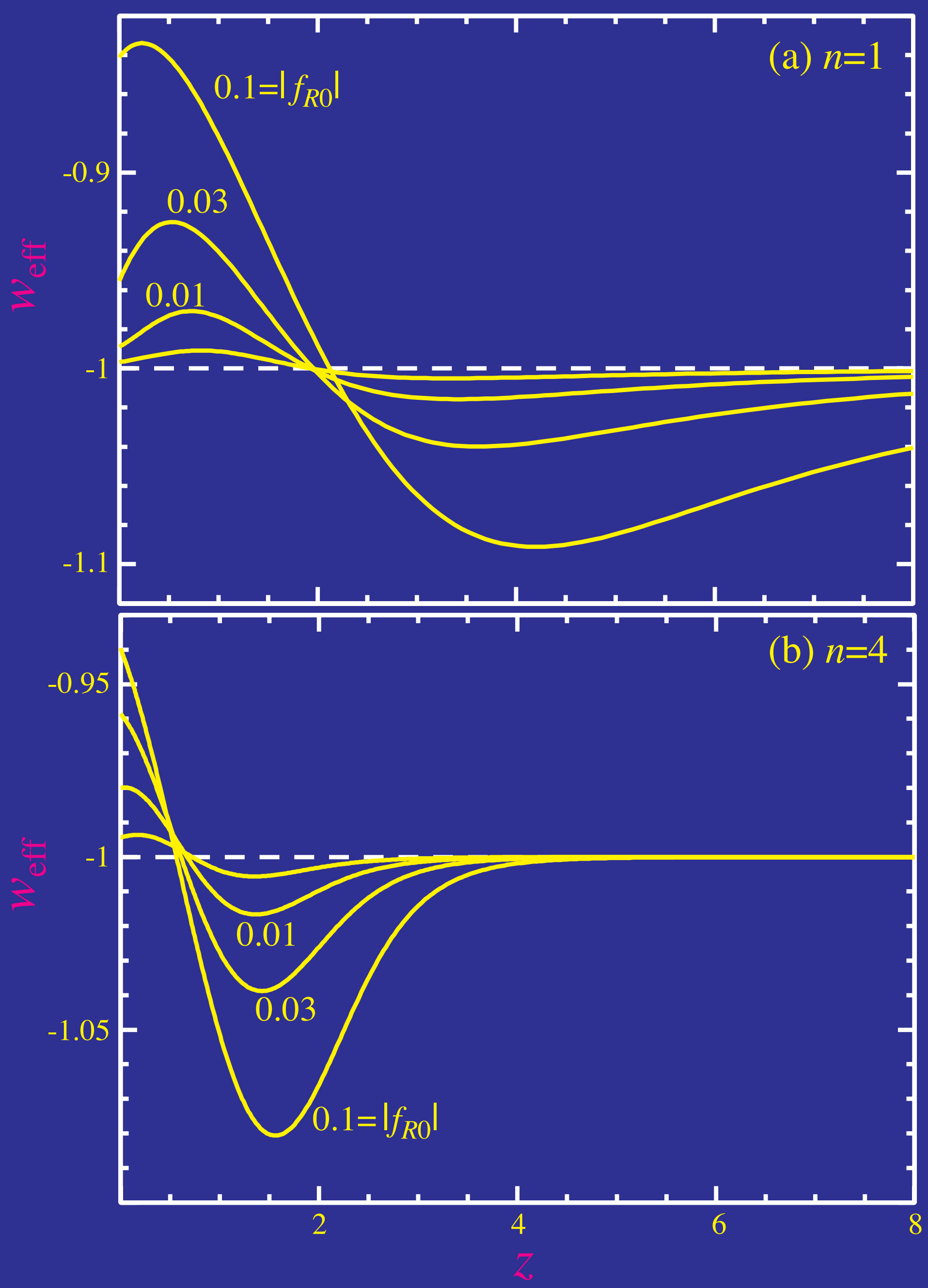
Expansion History



- Effective equation of state $w_{\rm eff}$ scales with field amplitude f_{R0}
- Crosses the phantom divide at a redshift that decreases with n
- Signature of degrees of freedom in dark energy beyond standard kinetic and potential energy of k-essence or quintessence or modified gravity

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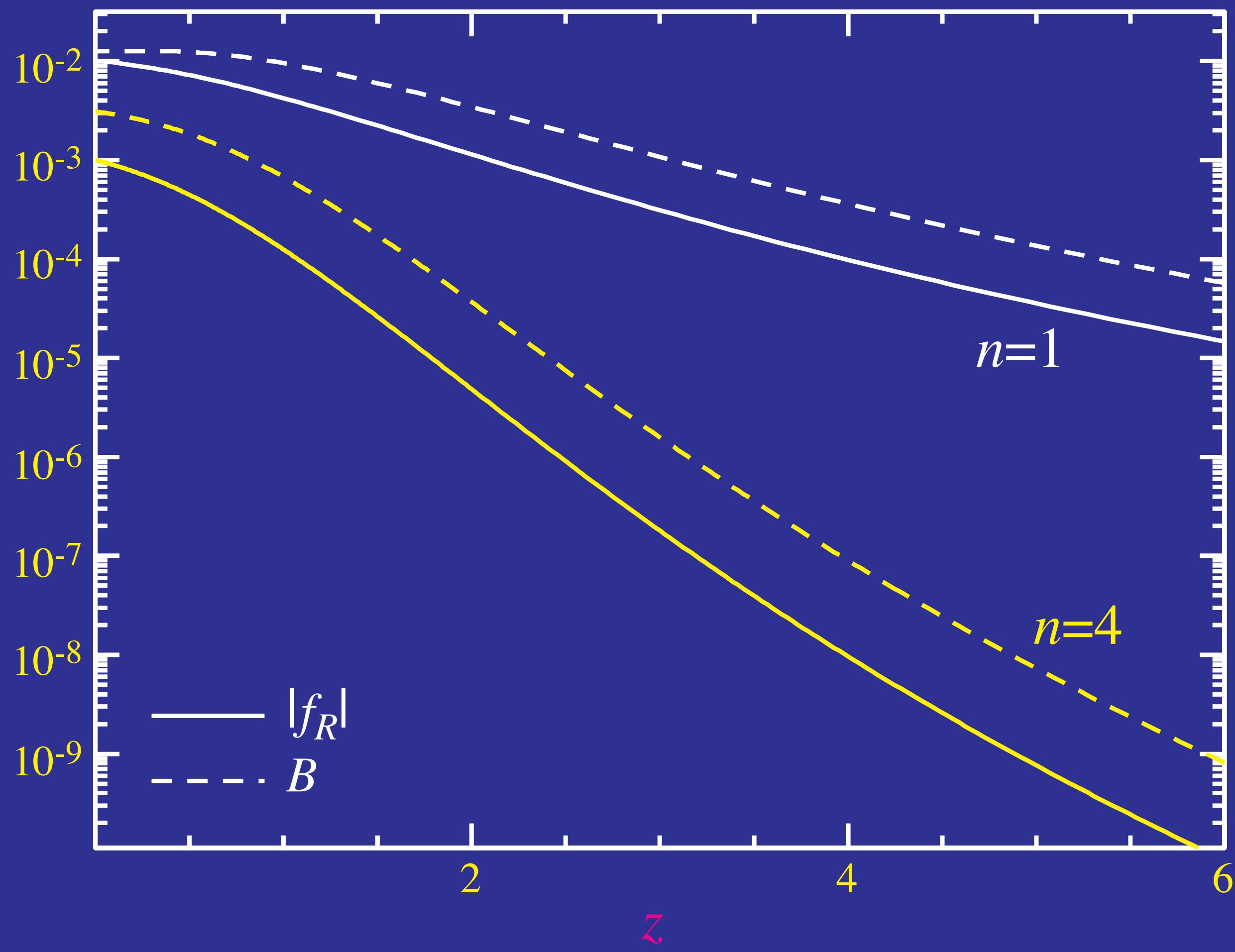
Rapid Evolution During Acceleration

- Cosmological deviations evolve rapidly and are only significant at $z<1$
- Dark matter halos like the Galaxy formed during the high curvature GR epoch



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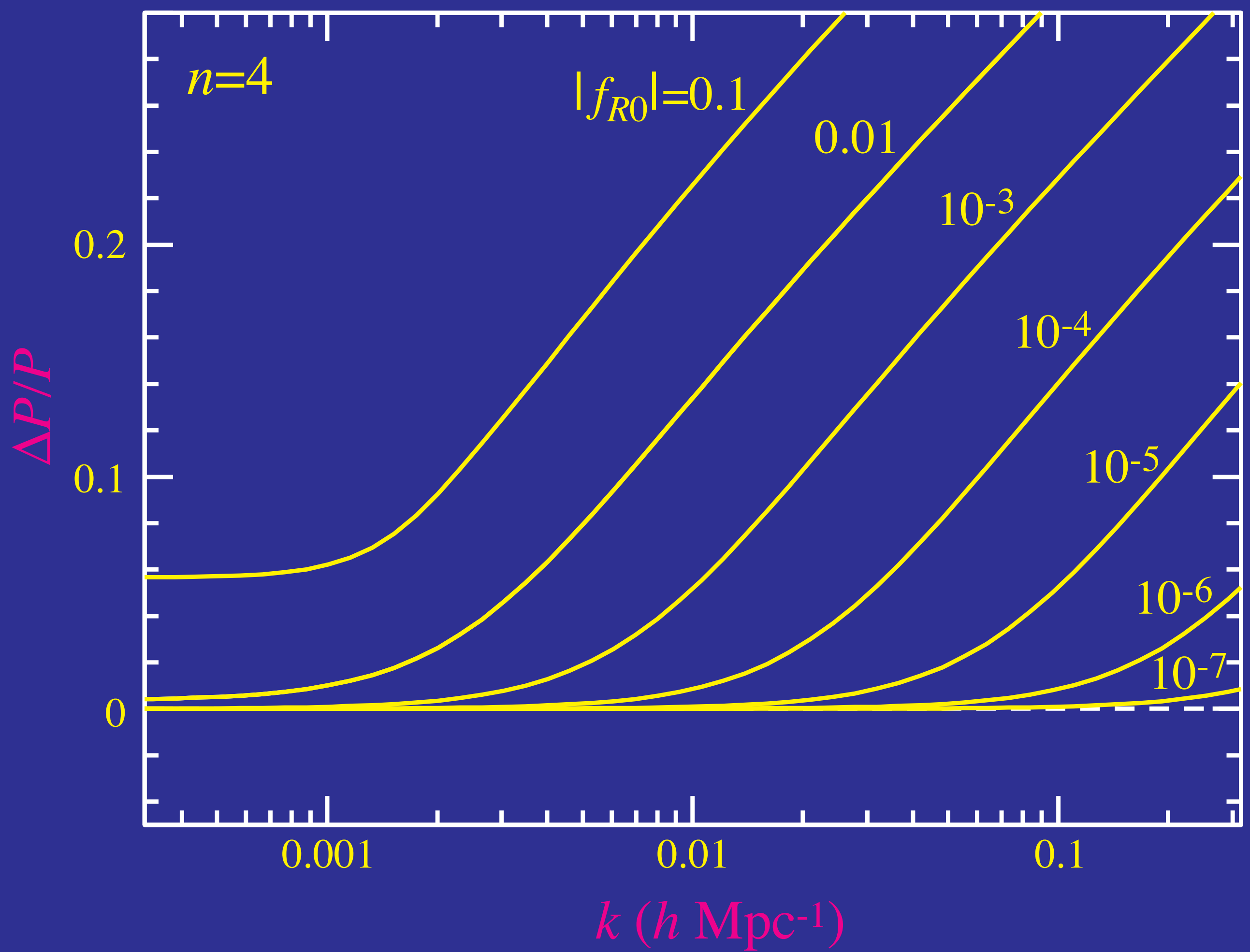
Power Spectrum Deviations

- Compton wavelength parameter B approximately field amplitude f_{R0}
- Deviations persist until $B \sim 10^{-7}$-10^{-6}



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$f(R)$ Solar System Tests

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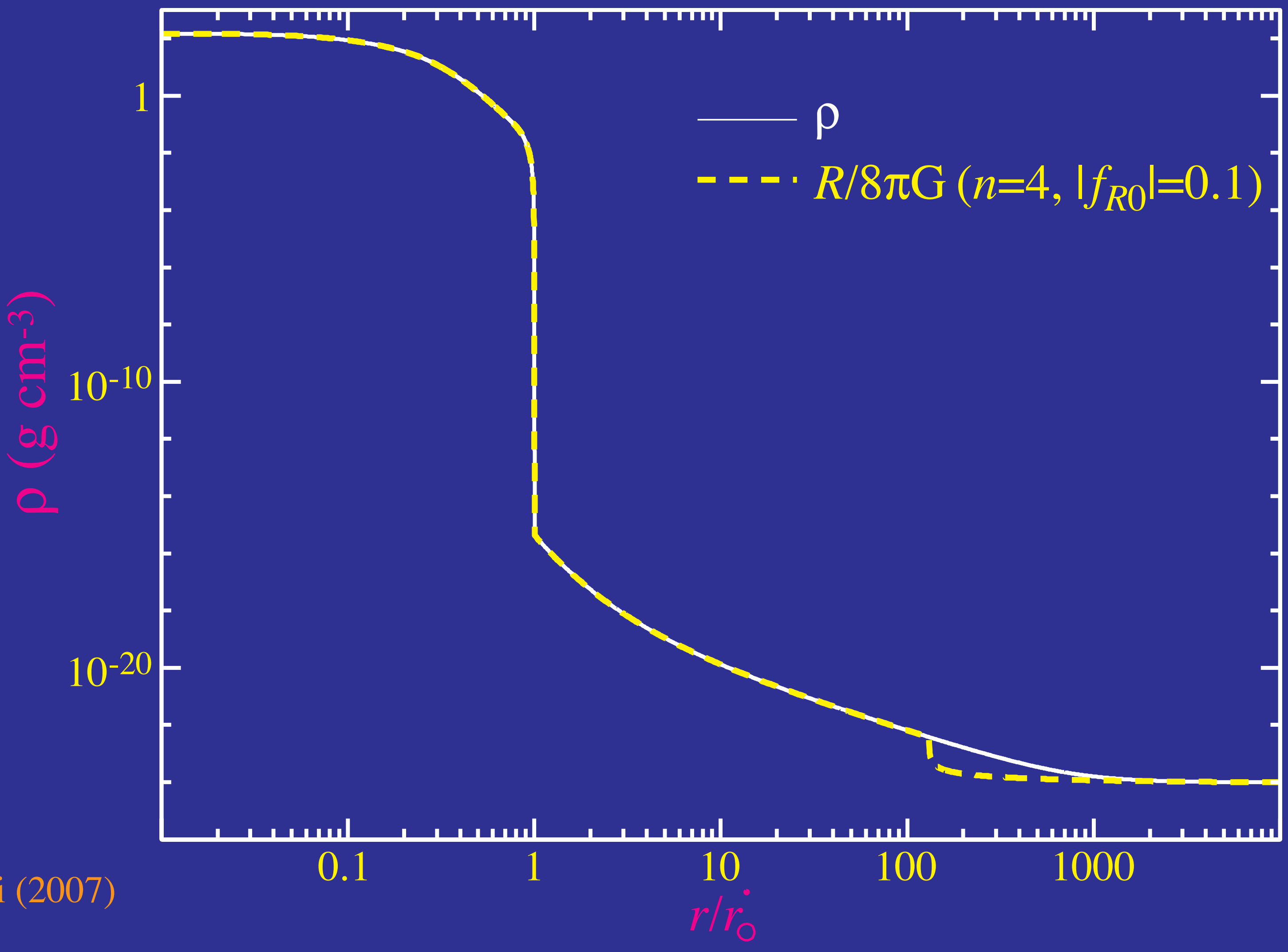
- Less wrong statement: the chameleon mechanism suppresses deviations so long as the curvature scaling is sufficiently steep – making Compton wavelength at high density sufficiently small – *gradient price paid somewhere, exterior boundary eventually must hit cosmological values*
- Overaggressive interpretation: difference between required solar system and desired cosmological field values combined with the shallow depth of solar potential rules out all $f(R)$ models – *galaxy intervenes and it determines constraint*

Solar Profile

- Density profile of Sun is not a constant density sphere - interior photosphere, chromosphere, corona
- Density drops by ~25 orders of magnitude - does curvature follow?



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$f(R)$ Chameleon

- Scalar $f(R)$ takes on a chameleon form – mass increases with density at minimum of effective potential (Khoury & Weltman 2004)

$$\nabla^2 f_R \approx \frac{1}{3}(R - 8\pi G\rho)$$

- Solutions either high curvature $R \approx 8\pi G\rho$ and small field gradient, or low curvature $R \ll 8\pi G\rho$ and large field gradient $\nabla^2 f_R \approx -8\pi G\rho/3$ depending on Compton scale vs size of object

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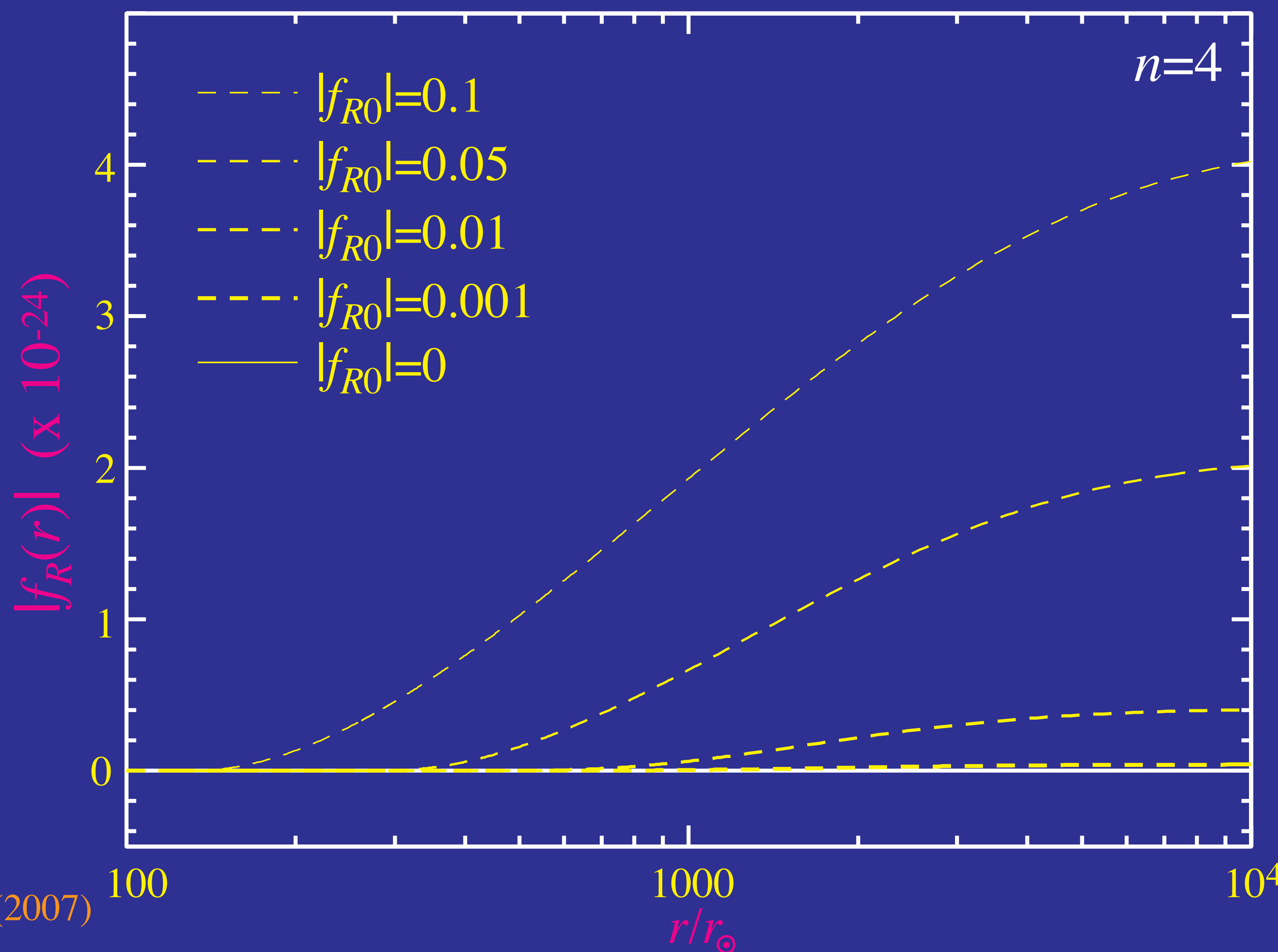
- Low curvature solution places a maximum for change in the field that is related to the gravitational potential Φ

$$\Delta f_R \le \frac{2}{3}\Phi \,,$$

- If required $|\Delta f_R| \ll \Phi$ the interior must be at high curvature to suppress the changes and hence the source $R - 8\pi G\rho \approx 0$ comes only from a thin shell of mass

Field Solution

- Field solution smoothly relaxes from exterior value to high curvature interior value $f_R \sim 0$, minimizing potential + kinetic
- Juncture is where thin-shell criterion is satisfied $|\Delta f_R| \sim \Delta\Phi$



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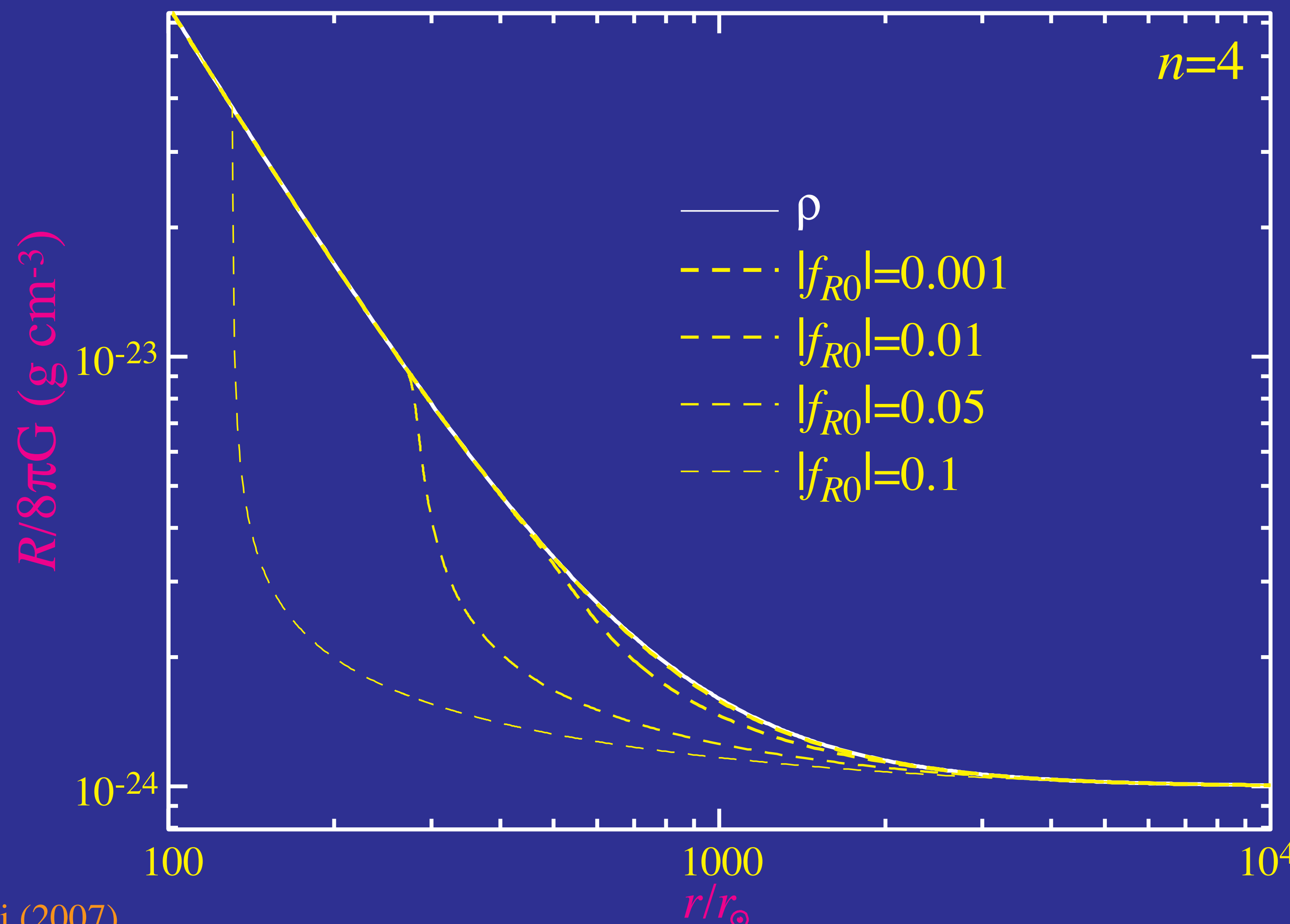
Solar Curvature

- Curvature drops suddenly as field moves slightly from zero
- Enters into low curvature regime where $R<8\pi G\rho$



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$f(R)$ Chameleon

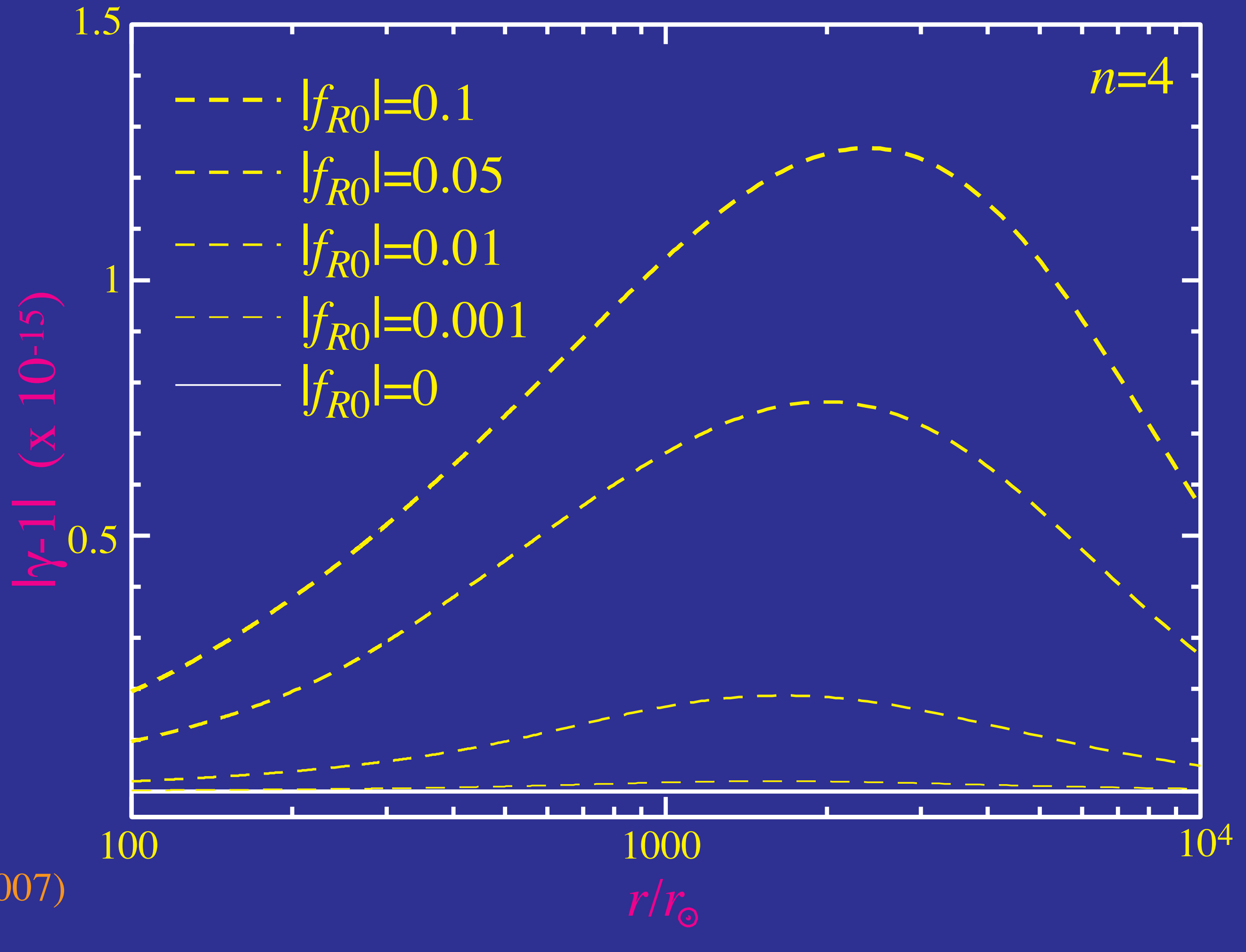
- The field f_R does not then sit at the potential minimum everywhere but instead minimizes the cost of potential and kinetic gradient energy
- A solution for f_R is a solution for R and the metric is fixed to be consistent with the curvature

$$|\gamma - 1| \approx \frac{|\Delta f_R(r)|}{\Phi(r)}$$

- Constraints on $|\gamma - 1|$ place constraints on the change in the field amplitude from the interior of the sun to the exterior of the solar system
- A second transition occurs from the field changes from in the galaxy to cosmology

Solar System Constraint

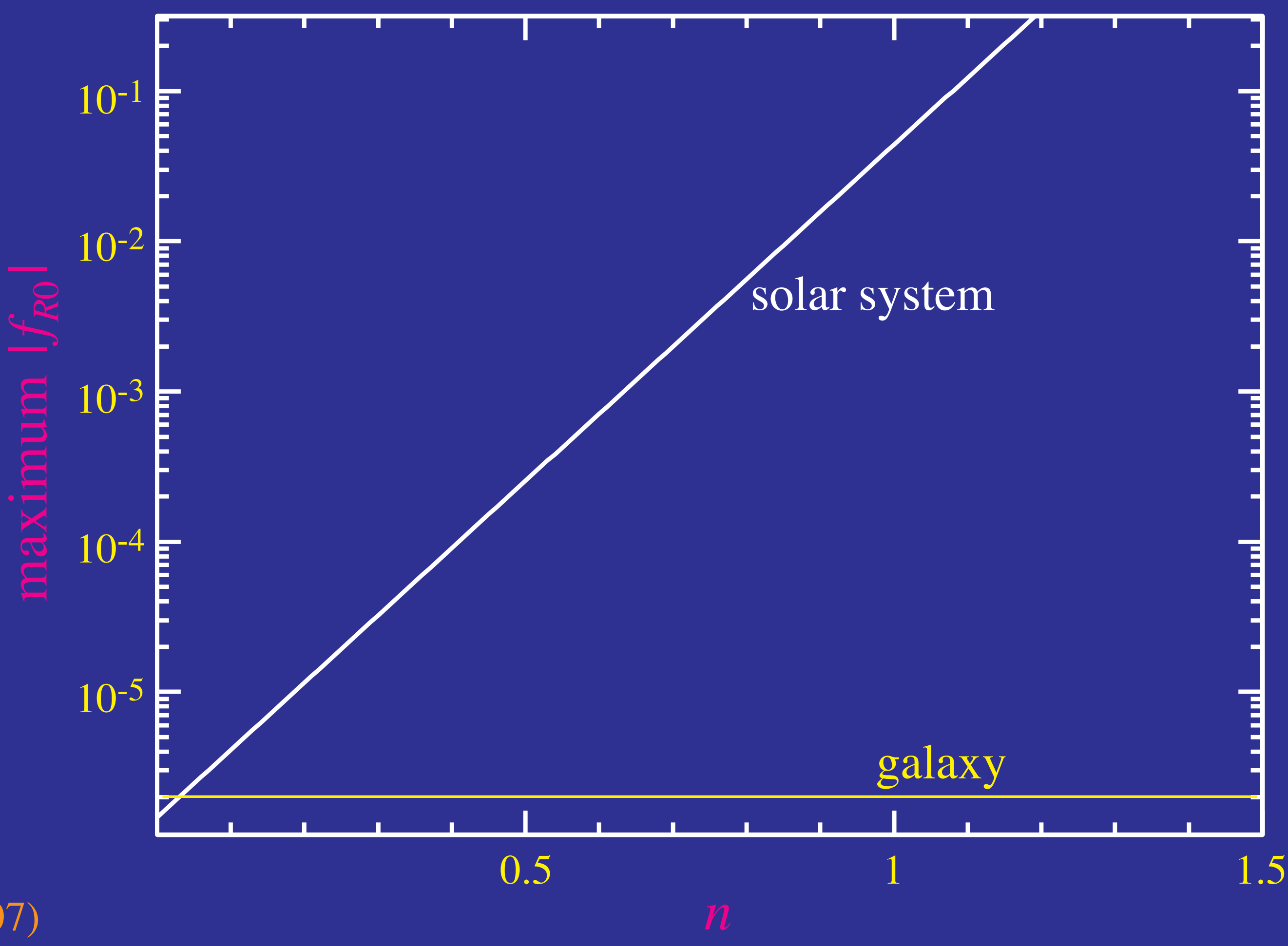
- Cassini constraint on PPN $|\gamma-1|<2.3\times10^{-5}$
- Easily satisfied if galactic field is at potential minimum $|f_{Rg}|<4.9\times10^{-11}$
- Allows even order unity cosmological fields



Hu & Sawicki (2007)

Solar System Constraint

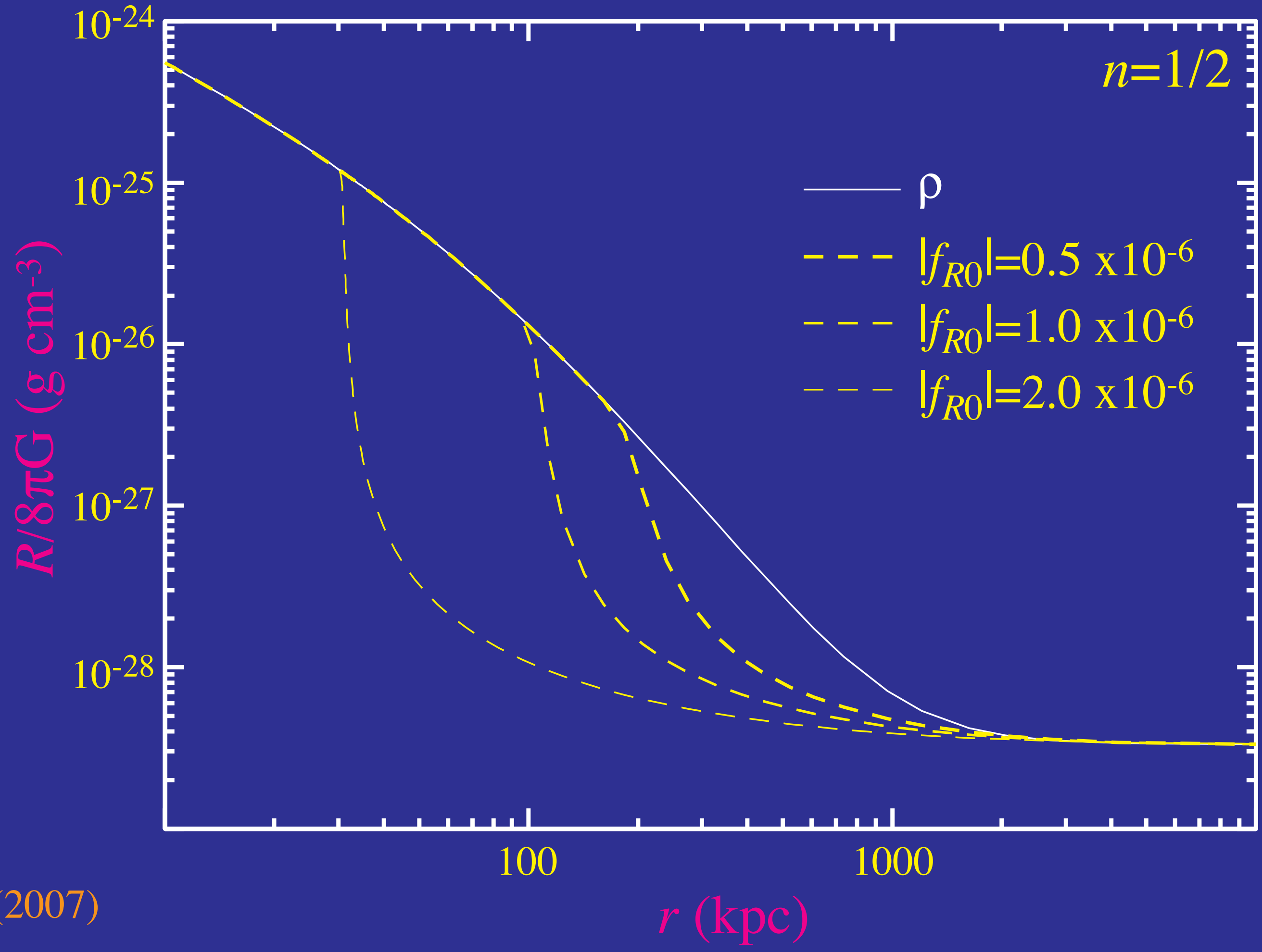
- Solar system constraint on cosmological field weakens as n increases
- Controls the strength of scaling between cosmological and galactic density



Hu & Sawicki (2007)

Galactic Thin Shell

- Galaxy must have a thin shell for interior to remain at high curvature
- Rotation curve $v/c \sim 10^{-3}$, $\Phi \sim 10^{-6} \sim |\Delta f_R|$ limits cosmological field
- Has the low cosmological curvature propagated through local group and galactic exterior?



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Three Regimes

- Three regimes defined by $\gamma = -\Phi/\Psi$
- Same division of scales as DGP braneworld acceleration
- Parameterized Post-Friedmann description of additional scalar gravitational degrees of freedom
- Challenge for theorists: sufficiently strong non-linearity to send $\gamma=1$ in the solar vicinity and interior of halos

General Relativistic Non-Linear Regime | Scalar-Tensor Regime | Conserved-Curvature Regime

Scale Radius
Galactic Compton Wavelength

Cosmological
Compton wavelength

$r \longrightarrow$

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$f(R)$: 10^{-5} simulations required 10^{-2}-1 10^{-6}-10^{-5}

General Relativistic Non-Linear Regime

Scalar-Tensor Regime

Conserved-Curvature Regime

Scale Radius Galactic Compton Wavelength

Cosmological Compton wavelength

$r \longrightarrow$

Hu & Sawicki (2007)

Summary

- Model building 101: take models where the mass squared is positive and large at high curvature with a small amplitude cosmological field
- Cosmological tests at very different range of curvature than local tests and worthwhile even in absence of viable full theory
- Solar system test alone easy to evade but not in combination with finite galaxy
- Requires cosmological simulations to study structure and evolution of dark matter halos
- Strongest deviations at intermediate scales where Compton wavelength large compared with structures, e.g. linear regime and outskirts of large halos or in small isolated halos

Summary

- Current constraints from $P(k)$ limited by theory and not observations – lack of knowledge of transition regime to 1-halo non-linear structure
- Requires cosmological simulations
- Strongest current constraint is from galaxy-ISW correlations in linear regime - lack of anti-correlation rules out order unity cosmological effects
- Lessons from $f(R)$ and DGP braneworld examples:

  Parameterized Post-Friedmann framework: 3 regimes – conservation dominated, scalar-tensor, non-linear or GR – parameterized by γ and strength of gravity