Testing Gravity with Galaxy Clusters

Wayne Hu
KITP
March 2011
Outline

• Falsifying $\Lambda$CDM and Smooth Dark Energy
• In favor of Modified Gravity?

• Collaborators:
  • Simone Ferraro
  • Dragan Huterer
  • Yin Li
  • Marcos Lima
  • Hiro Oyaizu
  • Michael Mortonson
  • Fabian Schmidt
Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate.

- Hence geometric measurements of the expansion rate predict the growth of structure:
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations

- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm:
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)
Falsifying $\Lambda$CDM

- Geometric measures of distance redshift from SN, CMB, BAO

Supernova Cosmology Project

Standard (izable)
Candle
Supernovae
Luminosity v Flux

Standard Ruler
Sound Horizon
v CMB, BAO angular and redshift separation
Falsifying ΛCDM

- Λ slows growth of structure in highly predictive way
Falsifying Quintessence

- **Dark energy slows growth of structure in highly predictive way**

- Deviation significantly $>2\%$ rules out $\Lambda$ with or without curvature

- Excess $>2\%$ rules out quintessence with or without curvature and early dark energy [as does $>2\%$ excess in $H_0$]
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)
Elephantine Predictions

- Geometric constraints on the cosmological parameters of $\Lambda$CDM
- Convert to distributions for the predicted average number of clusters above a given mass and redshift
\( \Lambda \text{CDM} \) Falsified?

- 95% of \( \Lambda \text{CDM} \) parameter space predicts less than 1 cluster in 95% of samples of the survey area above the \( M(z) \) curve
- No currently known high mass, high redshift cluster violates this bound

Mortonson, Hu, Huterer (2010)

\[ M = 10^{15} \text{ M}_\odot \]

SPT–CL J0546–5345

XMMU J2235.3–2557

Eddington Bias Correction

Lima & Hu (2005)

300 deg²
\( \Lambda \)CDM Falsified?

- 95% of \( \Lambda \)CDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the \( M(z) \) curve
- Convenient fitting formulae for future elephants: http://background.uchicago.edu/abundance

![Graph showing M vs. z for SPT-CL J0546-5345 and XMMU J2235.3-2557 with Eddington Bias Correction](Mortonson, Hu, Huterer (2010))
Number Bias

- For $>M_{\text{obs}}$, scatter and steep mass function gives excess over $>M$
- Equate the number $>M_{\text{obs}}$ to $>M_{\text{eff}}$
- Not the same as best estimate of true mass given model!

Lima & Hu (2005)
Number Bias

- For $>M_{\text{obs}}$, scatter and steep mass function gives excess over $>M$
- Equate the number $>M_{\text{obs}}$ to $>M_{\text{eff}}$
- Not the same as best estimate of true mass given model!

Lima & Hu (2005)
Pink Elephant Parade

- SPT catalogue on 2500 sq degrees

Williamson et al (2011)
Falsify in Favor of What?
Mercury or Pluto?

- General relativity says $\text{Gravity} = \text{Geometry}$

- And $\text{Geometry} = \text{Matter-Energy}$

- Could the missing energy required by acceleration be an incomplete description of how matter determines geometry?
Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content

- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy

\[
F(g_{\mu\nu}) + G_{\mu\nu} = 8\pi G T^M_{\mu\nu} \quad - F(g_{\mu\nu}) = 8\pi G T^{DE}_{\mu\nu}
\]

\[
G_{\mu\nu} = 8\pi G [T^M_{\mu\nu} + T^{DE}_{\mu\nu}]
\]

and the Bianchi identity guarantees \( \nabla^\mu T^{DE}_{\mu\nu} = 0 \)

- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor

- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress
Modified Gravity $\neq$ “Smooth DE”

- **Scalar field** dark energy has $\delta p = \delta \rho$ (in constant field gauge) – relativistic sound speed, no **anisotropic** stress.
- **Jeans stability** implies that its energy density is spatially smooth compared with the matter below the sound horizon.

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2$$

$$\nabla^2 (\Phi - \Psi) \propto \text{matter density fluctuation}$$

- **Anisotropic stress** changes the amount of **space curvature** per unit dynamical mass.

$$\nabla^2 (\Phi + \Psi) \propto \text{anisotropic stress}$$

but its absence in a **smooth dark energy** model makes

$$g = (\Phi + \Psi)/(\Phi - \Psi) = 0$$ for non-relativistic matter.
Dynamical vs Lensing Mass

- Newtonian potential: $\Psi = \delta g_{00}/2g_{00}$ which non-relativistic particles feel

- Space curvature: $\Phi = \delta g_{ii}/2g_{ii}$ which also deflects photons

- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass
Growth of Structure

- Alteration in how density sources Newtonian potential $\Psi$

- Changes the growth of structure and hence the masses of dark matter halos or the abundance at fixed mass

- Requires solution of the dynamical structure formation problem in the context of a model
Modified Action $f(R)$ Model

- $R$: Ricci scalar or “curvature”
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f / dR^2$: Compton wavelength of $f_R$ squared, inverse mass squared
- $B$: Compton wavelength of $f_R$ squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R R'} \frac{H}{H'}$$

- $' \equiv d/d \ln a$: scale factor as time coordinate
DGP Braneworld Acceleration

• Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

\[ S = \int d^5 x \sqrt{-g} \left[ \frac{(5) R}{2\kappa^2} + \delta(\chi) \left( \frac{(4) R}{2\mu^2} + \mathcal{L}_m \right) \right] \]

with crossover scale \( r_c = \kappa^2 / 2\mu^2 \)

• Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)

• Matter still minimally coupled and conserved

• Exhibits the 3 regimes of modified gravity

• Weyl tensor anisotropy dominated conserved curvature regime \( r > r_c \) (Sawicki, Song, Hu 2006; Cardoso et al 2007)

• Brane bending scalar tensor regime \( r_* < r < r_c \) (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)

• Strong coupling General Relativistic regime \( r < r_* = (r_c^2 r_g)^{1/3} \)

where \( r_g = 2GM \) (Dvali 2006)
Three Regimes

- Fully worked $f(R)$ and DGP examples show 3 regimes
- **Superhorizon** regime: $\zeta = \text{const.}, g(a)$
- **Linear** regime - closure condition - analogue of “smooth” dark energy density:

  \[
  \nabla^2 (\Phi - \Psi)/2 = -4\pi G a^2 \Delta \rho \\
g(a, x) \leftrightarrow g(a, k)
  \]

$G$ can be promoted to $G(a)$ but conformal invariance relates fluctuations to field fluctuation that is small

- **Non-linear** regime:

  \[
  \nabla^2 (\Phi - \Psi)/2 = -4\pi G a^2 \Delta \rho \\
  \nabla^2 \Psi = 4\pi G a^2 \Delta \rho - \frac{1}{2} \nabla^2 \phi
  \]
Nonlinear Interaction

Non-linearity in the field equation

$$\nabla^2 \phi = g_{\text{lin}}(a)a^2 (8\pi G \Delta \rho - N[\phi])$$

recovers linear theory if $N[\phi] \to 0$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a non-linear function of the field

Linked to gravitational potential

- For DGP, $\phi$ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$

a non-linear function of second derivatives of the field

Linked to density fluctuation
Environment Dependent Force

- **Chameleon suppresses extra force** (scalar field) in high density, deep potential regions

Environment Dependent Force

- For large background field, gradients in the scalar prevent the chameleon from appearing
Cluster Abundance

- Enhanced **abundance** of rare dark matter halos (clusters) with extra force

\[ \frac{d}{d\log M} M_{300} \left( h^{-1} M_\odot \right) \]

\[ |f_{R0}| = 10^{-4} \]

- **Full simulation**
- **No chameleon**
- **Spherical collapse**

Lima, Schmidt, Oyaizu, Hu (2008)
Cluster $f(R)$ Constraints

- Clusters provide best current cosmological constraints on $f(R)$ models.
- Spherical collapse rescaling to place constraints on full range of inverse power law models of index $n$.

Cluster $f(R)$ Constraints

- Approaching competitiveness with solar system + Galaxy constraints of few $10^{-6}$ at low $n$
- Vastly different scale

Chameleon Mass Function

- **Chameleon effect suppresses the enhancement at high masses**
- **Pile up of abundance at intermediate group scale**

![Graph showing the Chameleon Mass Function](chart)

$Lima, Schmidt, Oyaizu, Hu (2008)$
Chameleon Mass Function

- Simple single parameter extension covers variety of models
- Basis of a halo model based post Friedmann parameterization of chameleon

\[ \frac{\Delta n_M}{n_M} \]

\[ M_{300} \ [h^{-1} M_\odot] \]

\[ n=1, \ |f_{R0}|=10^{-6} \]

Full Simulations

Fit

Li & Hu (2011)
Halo Bias

- Halos at a fixed mass less rare and less highly biased

Halo Mass Correlation

- Enhanced forces vs lower bias

Nonlinear Interaction

Non-linearity in the field equation

$$\nabla^2 \phi = g_{\text{lin}}(a)a^2 (8\pi G \Delta \rho - N[\phi])$$

recovers linear theory if $N[\phi] \to 0$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a non-linear function of the field

Linked to gravitational potential

- For DGP, $\phi$ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$

a non-linear function of second derivatives of the field

Linked to density fluctuation
DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential

Brane Bending Mode

Summary

- Given current geometric data, $\Lambda$CDM and quintessence ($w > -1$) are highly predictive and falsifiable.
- Linear growth at all $z$ cannot exceed fiducial > few percent.
- With Gaussian fluctuations, exponential sensitivity of cluster abundance exploits this test: e.g. high $M$, high $z$ pink elephants.
- No currently known single cluster falsifies $\Lambda$CDM.
- Places currently the strongest cosmological constraints on modified gravity models with enhanced forces but $\Lambda$CDM expansion history, e.g. $f(R)$.
- Future tests which complement the solar system constraints will need to move down the mass function.
- Parameterized approaches should take into account that the force modifications depend on local environment, potential or density.