# Generalized Slow Roll
## in the
# Effective Field Theory of Inflation


Wayne Hu
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# Outline

- Beyond Canonical Slow-Roll Inflation
- Single Clock and ADM
- EFT of Single Field Inflation
- Power Spectra
- Generalized Slow Roll

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- Polarization and Bispectrum
- Reconstructing the EFT

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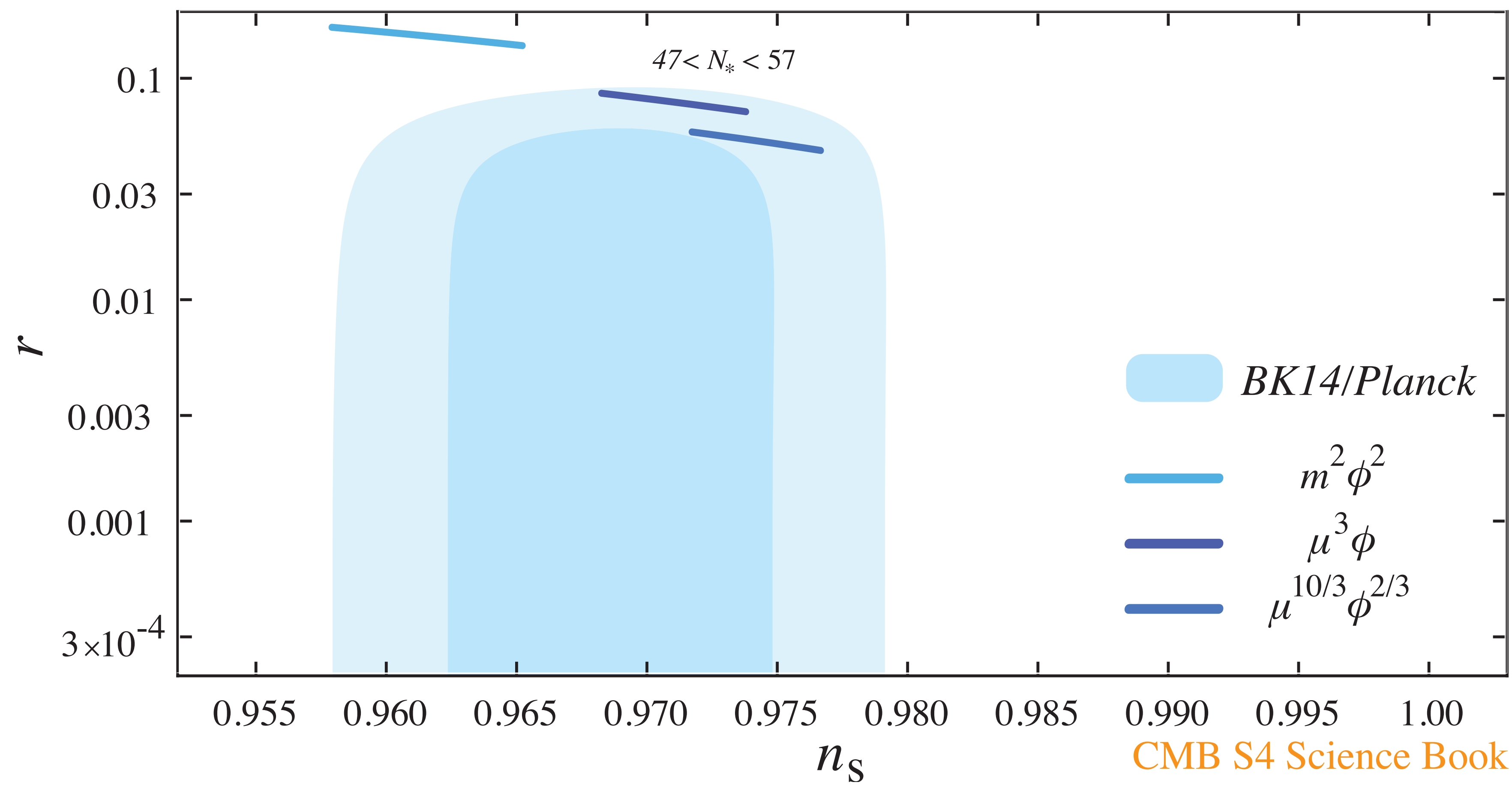
- Thanks to many collaborators: Hayato Motohashi, Peter Adshead, Cora Dvorkin, Chen Heinrich, Vivian Miranda, Georges Obied, Sam Passaglia, Hector Ramirez...

# Beyond Canonical Slow Roll

# Constraints on Inflation Models

- Constraints on the scalar tilt $n_s$ and tensor-scalar ratio $r$
- Simple featureless potentials like $m^2\phi^2$ disfavored




CMB S4 Science Book

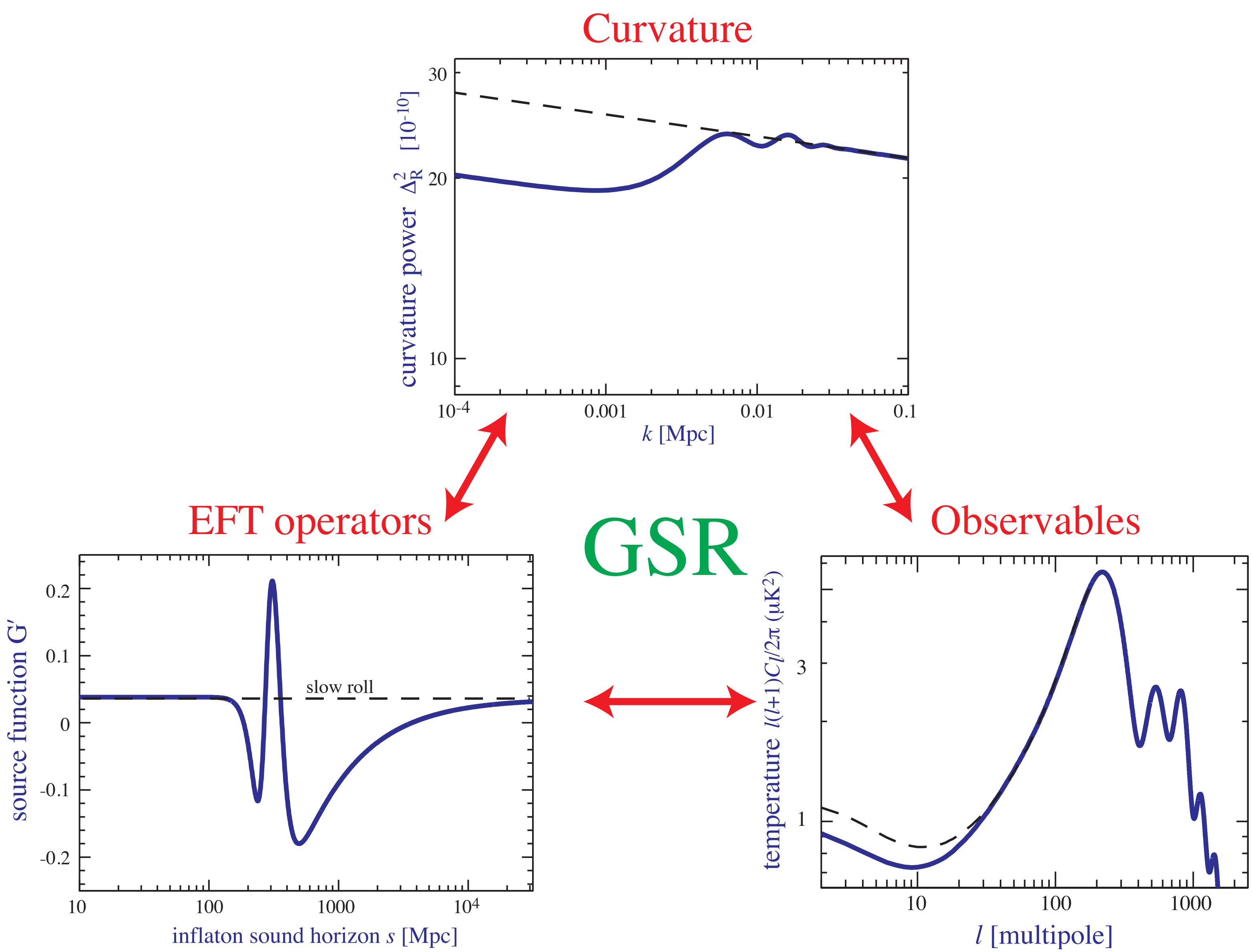
# Beyond Canonical Slow Roll

- Simplest scale-free monomial potentials in canonical inflation coming under increasing observational pressure
- More complicated single-field models involve non-standard kinetic terms and (temporal) features with hints in large scale observables
- Requires a more general framework for the inflationary paradigm that does not assume:
  - canonical Lagrangian for scalar field
  - scale-free behavior for the full 60 efolds of inflation

  and allows model building from observations to theory
- Effective Field Theory (EFT)
  + Generalized Slow Roll (GSR)...

# Operators to Observables

- From operators to observables and back



# Single Clock and ADM

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- Geometric objects and their dynamics in this $3+1$ split is best characterized in the ADM (Arnowitt-Deser-Misner) formalism
- Define most general line element: lapse $N$, shift $N^i$, 3-metric $h_{ij}$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

or equivalently the metric

$$g_{00} = -N^2 + N^i N_i, \quad g_{0i} = h_{ij} N^j \equiv N_i, \quad g_{ij} = h_{ij}$$

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or equivalently the metric (inverse: $g^{00}$ depends only on lapse)

$$g_{00} = -N^2 + N^i N_i, \quad g_{0i} = h_{ij} N^j \equiv N_i, \quad g_{ij} = h_{ij}$$

$$g^{00} = -1/N^2, \quad g^{0i} = N^i/N^2, \quad g^{ij} = h^{ij} - N^i N^j / N^2$$

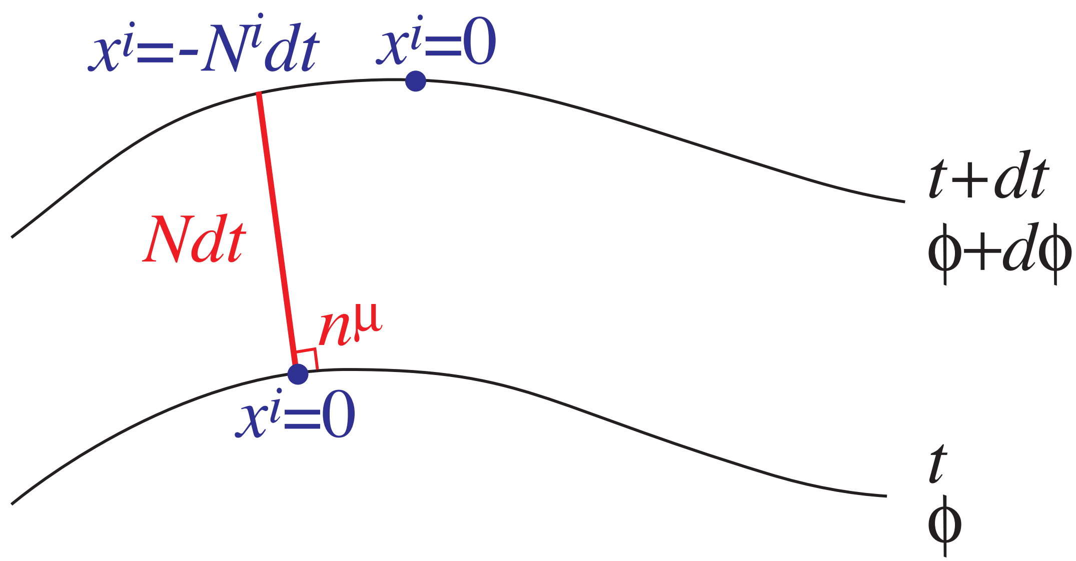
# ADM 3+1 Split

- Useful to define the unit normal timelike vector $n_\mu n^\mu = -1$, orthogonal to constant time surfaces $n_\mu \propto \partial_\mu \phi$

$$n_\mu = (-N, 0, 0, 0), \quad n^\mu = (1/N, -N^i/N)$$

  where we have used $n^\mu = g^{\mu\nu} n_\nu$

- Interpretation: lapse of proper time along normal, shift of spatial coordinates with respect to normal



# ADM 3+1 Split

- Projecting 4D tensors onto the normal direction utilizes $n^\mu n_\nu$, e.g.

$$-n^\mu n_\nu V^\nu$$

- Projecting 4D tensors onto the 3D tensors involves the complement through the induced metric

$$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu,$$
$$h^\mu{}_\nu V^\nu = (\delta^\mu{}_\nu + n^\mu n_\nu)V^\nu = V^\mu + n^\mu n_\nu V^\nu$$

e.g. in the preferred slicing

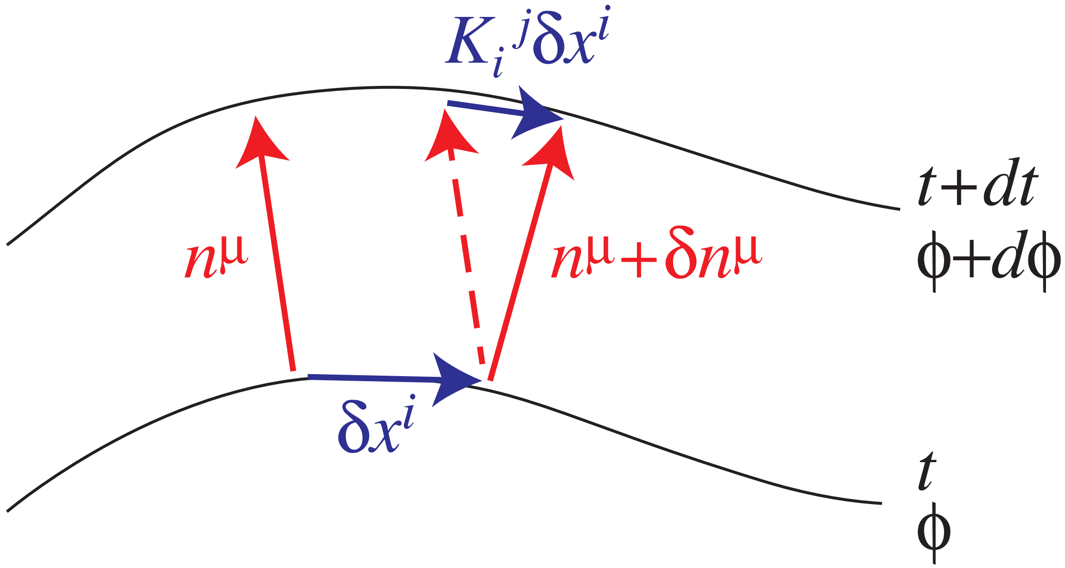
$$\tilde{V}^\mu = h^\mu{}_\nu V^\nu = (\delta^\mu{}_\nu + n^\mu n_\nu)V^\nu = (0, V^i + N^i V^0)$$

whose spatial indices are raised an lowered by $h_{ij}$:
$\tilde{V}_i = g_{i\nu}\tilde{V}^\nu = h_{ij}\tilde{V}^j$

# Extrinsic Curvature

- 3-surface embedded in 4D, so there is both an intrinsic curvature associated with $h_{ij}$ and an extrinsic curvature
- Extrinsic curvature $K_{\mu\nu}$ is the spatial projection of the gradient of $n^\mu$

$$K_{\mu\nu} = h_\mu{}^\alpha h_\nu{}^\beta n_{\alpha;\beta}$$



# Spacetime Curvature

- Likewise split the spacetime curvature ${}^{(4)}R$ into intrinsic ${}^{(3)}R = R$ and extrinsic pieces via Gauss-Codazzi relation

$$ {}^{(4)}R = K_{\mu\nu}K^{\mu\nu} - (K_{\mu}{}^{\mu})^2 + R + 2(K_{\nu}{}^{\nu}n^{\mu} - n^{\alpha}n^{\mu}{}_{;\alpha})_{;\mu} $$

- Last piece is total derivative so Einstein Hilbert $L_{\rm EH} = {}^{(4)}R/2$ action is equivalent to keeping first three pieces

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- Last piece is total derivative so Einstein Hilbert $L_{\rm EH} = {}^{(4)}R/2$ action is equivalent to keeping first three pieces
- Gravitational action of GR composed of extrinsic $K_\mu{}^\nu$, and intrinsic $R_i^{\,j}$ curvatures
- No dependence on slicing and threading $N$, $N^i$ $\leftrightarrow$ full diffs
- In GR any preferred slicing is picked out by the matter distribution $L = L_{\rm EH} + L_{\rm matter}$ not gravity
- Alternately view "matter" with preferred homogeneous slicing + "gravitational" Lagrangian $\rightarrow$ total depend on $N, t$

# $K_{ij}$ as Metric "Velocity"

- In terms of the ADM variables

$$K_{ij} = \frac{1}{2N}(\partial_t h_{ij} - N_{j|i} - N_{i|j})$$

  where $|$ denotes the covariant derivative with respect to $h_{ij}$

- Extrinsic curvature acts like a "velocity" term for $h_{ij}$ moving the metric from one slice to another with the coordinate freedom of the lapse and shift
- In GR: define $h_{ij}$ and $\dot{h}_{ij}$ on the spacelike surface and integrate forwards, with lapse and shift defining the temporal and spatial coordinates

# Beyond GR

- In a general scalar-tensor theory in addition to functions of $N, t$ Lagrangian involves curvatures

$$L(K_i{}^j, R_i{}^j, N; t)$$

  in non EH combinations leading to different kinetic structure for spatial metric: Horndeski/GLPV

- Extra spatial derivatives without temporal derivatives typically imply extra degrees of freedom hidden by the preferred slicing

- Derivatives of $N$ usually make it dynamical: Ostrogradsky instability

- Special degenerate theories ("DHOST") propagate one combination of the lapse and spatial metric…

# Beyond Beyond GR

- DHOST adds degenerate combinations built out of the acceleration: directional derivative of the normal along the normal

$$a_\mu = (n_{\mu;\beta})n^\beta$$

which contains spatial derivatives of the lapse and/or

$$\beta = n^\mu(\ln N)_{,\mu}$$

which contains temporal derivatives as well

- Define EFT as the most general spatially covariant combination of these ADM quantities, expanded around an FRW backgroundd
- EFT of inflation as this EFT of "dark sector" under the assumption that other components of matter are negligible and background is near de Sitter …

# EFT of Single Field Inflation

# Single Field Inflation

- General Lagrangian for metric: preferred slicing but unbroken spatial diffs Cheung et al 2008,...,Hu & Motohashi 2017

$$S = \int d^4x N\sqrt{h}\, L(N, K^i{}_j, R^i{}_j; t)$$

  where the function $L$ can be any spatially covariant contractions of the ADM geometric objects

- Can extended to include covariant spatial derivatives $\nabla_i$ Gleyzes et al 2015, dynamical lapse $(a_\mu, \beta)$ Langois et al 2017, Hu & Motohashi, in prep and shift, but these generally introduce extra dofs
- In preferred "unitary" slicing, inflaton degree of freedom is in the metric

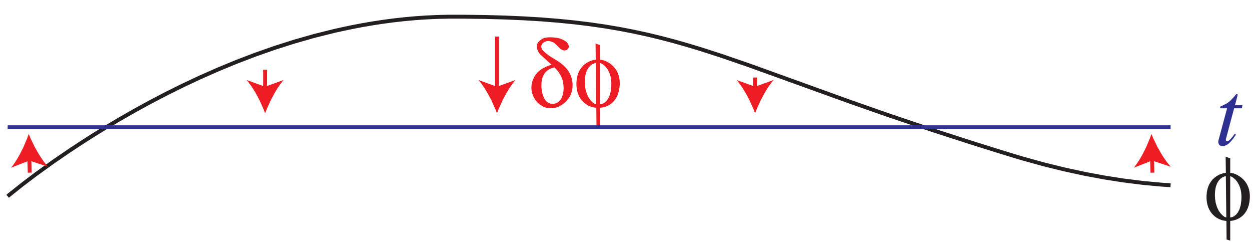
# Inflaton as Stuckelberg Field

- Restore temporal diffs by introducing $\phi(\mathbf{x}, t)$ as a Stuckelberg field, or equivalently a gauge transformation out of unitary gauge
- On a new time slicing of constant

$$t \to t - \pi(\mathbf{x}, t) \quad \text{or} \quad \delta\phi(\mathbf{x}, t) = \dot{\phi}\pi$$

- Work with $\pi$ (or $\delta\phi$) in an arbitrary gauge

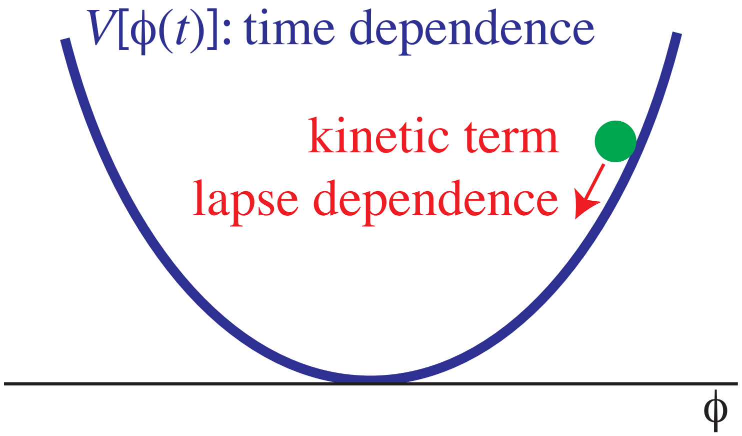
  Spatially flat gauge: scalar $\delta h_{ij} = 0$, non-dynamical lapse and shift
- Or alternately stick with unitary gauge and work with $\delta h_{ij}$ as the dynamical variables as we will continue to do



# Examples

- Canonical scalar field

  $X = \nabla_\mu \phi \nabla^\mu \phi$, potential $V(\phi)$




$$L_\phi = -\frac{X}{2} - V$$

- In unitary gauge $\phi(t)$ so accounting for the lapse $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

$$X = -\frac{\dot{\phi}^2}{N^2} \qquad \text{and} \qquad V[\phi(t)] = V(t)$$

and

$$\begin{aligned} L_{\rm EH} + L_\phi &= L_{\rm EH}(K^i_{\ j}, R) + \frac{\dot{\phi}^2(t)}{2N^2} - V(t) \\ &= L(K^i_{\ j}, R, N, t) \end{aligned}$$

# Examples

- K-essence $L_\phi = P(X,\phi)$ also gives

$$L(K^i{}_j, R, N, t)$$

but with a more general functional form for the $N$ dependence

- Horndeski and GLPV theories: specific contractions of $K^i{}_j$, $R^i{}_j$

$$\begin{aligned} L &= A_2(N,t) + A_3(N,t)K + A_4(N,t)(K^2 - K^i{}_jK^j{}_i) \\ &\quad + B_4(N,t)R + A_5(N,t)(K^3 - 3KK^i{}_jK^j{}_i + 2K^i{}_jK^j{}_kK^k{}_i) \\ &\quad + B_5(N,t)(K^i{}_jR^j{}_i - \tfrac{1}{2}KR) \\ &= L(K^i{}_j, R^i{}_j, N, t) \end{aligned}$$

- DHOST adds dependence on $a_i$ and $\beta$ for lapse that is dynamical but subsumed into a single scalar degree of freedom

# EFT Coefficients

- These general models can all be described by EFT coefficients representing the Taylor expansion of $L$ around a spatially flat FRW background

$$\bar{N} = 1, \quad \bar{N}^i = 0, \quad \bar{h}_{ij} = a^2 \delta_{ij}$$

where given $H = d \ln a / dt$

$$\bar{K}^i{}_j = H \delta^i{}_j, \quad \bar{R}^i{}_j = 0$$

with time dependent expansion coefficients for $X, Y, Z \in K, R, N$

$$L\Big|_{\rm b} = \mathcal{C}, \quad \frac{\partial L}{\partial Y^i{}_j}\Big|_{\rm b} = \mathcal{C}_Y \delta^j{}_i,$$

$$\frac{\partial^2 L}{\partial Y^i{}_j \partial Z^k{}_\ell}\Big|_{\rm b} = \mathcal{C}_{YZ} \delta^j{}_i \delta^\ell{}_k + \frac{\tilde{\mathcal{C}}_{YZ}}{2} (\delta^\ell{}_i \delta^j{}_k + \delta_{ik} \delta^{j\ell}), \quad \ldots$$

# Power Spectra

# Quadratic Action

- Expand Lagrangian in the metric fluctuations: scalar and tensor

$$N = 1 + \delta N, \quad N_i = \partial_i \psi, \quad h_{ij} = a^2 e^{2\mathcal{R}} (\delta_{ij} + \gamma_{ij})$$

- In linear theory, scalars and tensors decouple due to the symmetry of background
- Consider their quadratic Lagrangians separately

$$\mathcal{L} = \sqrt{-g} L = N \sqrt{h} L$$

- For tensors

$$\mathcal{L}_2 = a^3 \left[ \frac{\tilde{\mathcal{C}}_{KK}}{8} \dot{\gamma}_{ij}^2 - \frac{\mathcal{C}_R}{4a^2} (\partial_k \gamma_{ij})^2 \right].$$

# Gravitational Waves

- For plane wave fluctuations in the two polarization states $\gamma_{+,\times}$

$$\gamma_{ij}(t,z) = \gamma_+(t)e^{ikz}(\delta_{ix}\delta_{jx} - \delta_{iy}\delta_{jy}) + \gamma_\times(t)e^{ikz}(\delta_{ix}\delta_{jy} + \delta_{jx}\delta_{iy})$$

- Quadratic Lagrangian

$$\mathcal{L}_2 = \sum_{\lambda=+,\times} \frac{a^3 b_t}{4c_t^2}\left(\dot{\gamma}_\lambda^2 - \frac{c_t^2 k^2}{a^2}\gamma_\lambda^2\right)$$

- Modified propagation speed

$$c_t^2 = \frac{2\mathcal{C}_R}{\tilde{\mathcal{C}}_{KK}}$$

- Non-canonical normalization

$$b_t = 2\mathcal{C}_R$$

- In $P(X,\phi)$ and canonical, $c_t = b_t = 1$

# Scalar Quadratic Action

- For scalars, varying with respect to the lapse and shift yield constraints by which they can be eliminated
- Quadratic Lagrangian for $k$-modes of $\mathcal{R}$ (for second order in space and time derivs)

$$\mathcal{L}_2 = \frac{a^3 b_s \epsilon_H}{c_s^2}\left(\dot{\mathcal{R}}^2 - \frac{c_s^2 k^2}{a^2}\mathcal{R}^2\right)$$

  where the slow roll parameter

$$\epsilon_H = -\frac{d\ln H}{d\ln a}$$

- Sound speed and normalization can be written in terms of the EFT coefficients $\mathcal{C}$ (see Motohashi & Hu 2017 for specific form)
- For $P(X,\phi)$, $b_s = 1$, $c_s =$ arbitrary and for canonical scalar $b_s = c_s = 1$

# Equations of Motion

- Tensor and scalar quadratic Lagrangians follow the same form but with different normalization and sound speeds
- Equations of motion follow the general form with different sound horizons

$$s_{s,t} = \int d\ln a \frac{c_{s,t}}{aH}$$

and different normalizations

$$f_{s,t} = 2\pi z_{s,t}\sqrt{c_{s,t}}s_{s,t}$$

determined by the source to the Mukhanov-Sasaki variable $z_{s,t}$

$$z_s = \frac{a}{c_s}\sqrt{2b_s\epsilon_H}, \qquad z_t = \frac{a}{c_t}\sqrt{b_t/2}$$

# Modefunctions & Power Spectra

- Mukhanov-Sasaki variable or modefunctions of the scalar curvature and tensor polarization states

$$\sqrt{\frac{k^3}{2\pi^2}}\mathcal{R} = \frac{x_s y_s}{f_s}, \qquad \sqrt{\frac{k^3}{2\pi^2}}\gamma_{+,\times} = \frac{x_t y_t}{f_t}$$

obey the general form with $x = ks$

$$\frac{d^2 y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f}\frac{y}{x^2}, \qquad ' = \frac{d}{d\ln s}$$

with Bunch-Davies initial conditions where $\lim_{x\to\infty} y = e^{ix}$

- Power spectra are evaluated at $x \ll 1$ where the perturbations are well outside their sound horizons

$$\Delta^2_{\mathcal{R}} = \lim_{x_s \to 0} \left| \frac{x_s y_s}{f_s} \right|^2, \qquad \Delta^2_{\gamma} = \lim_{x_t \to 0} \left| \frac{x_t y_t}{f_t} \right|^2,$$

# Generalized Slow Roll

# Mukhanov-Sasaki Equation

- Solutions to the modefunction evolution in general require numerical solutions to the Mukhanov-Sasaki equation
- Approximations involve assuming that the net fractional change in $f$ or $\Delta \ln f$ across the efolds of evolution is small in amplitude
- Usual lowest order slow roll approximation takes $f$=const., evaluated around sound horizon crossing so

$$y \to y_0 = \left(1 + \frac{i}{x}\right) e^{ix}$$

and

$$\Delta^2_{\mathcal{R}} \approx \frac{1}{f_s^2}\Big|_{x_s=1}, \qquad \Delta^2_{\gamma} \approx \frac{1}{f_t^2}\Big|_{x_t=1}$$

- Focus on scalar sector, so hereafter drop subscript "$s$" for compactness

# Beyond Slow Roll

- Sufficient inflation requires only that $\epsilon_H \ll 1$
- Evolution of $f$ on timescales shorter than the 60 efolds of inflation lead to non-power law (constant tilt $n_s - 1$) spectra
  - – running of tilt that is of order the tilt
  - – monodromy oscillations in the power spectrum
  - – step features that cause power spectrum glitches...
- As long as the modefunction remains sufficiently close to the leading order solution $y_0$, "generalized slow-roll" applies
- Iteratively correct solution $y$ to the Mukhanov-Sasaki equation

$$y = y_0 + y_1 + \ldots$$

# Generalized Slow Roll

- Construct Green function out of the source free solution $y_0$ and consider the $f$-terms as external source

$$\frac{d^2 y_1}{dx^2} + \left(1 - \frac{2}{x^2}\right) y_1 = \frac{f'' - 3f'}{f} \frac{y_0}{x^2}$$

to obtain the first order correction Stewart 2002

$$y_1(x) = -\int_x^\infty \frac{du}{u^2} \left(\frac{f'' - 3f'}{f}\right) y_0 \mathrm{Im}[y_0^*(u) y_0(x)]$$

and iteratively improve

$$y_n(x) = -\int_x^\infty \frac{du}{u^2} \left(\frac{f'' - 3f'}{f}\right) y_{n-1} \mathrm{Im}[y_0^*(u) y_0(x)]$$

- Series converges if the amplitude of the deviation of $y$ from $y_0$ is small – does not necessarily require evolution of $f$ is slow

# Generalized Slow Roll

- This basic technique holds for general set of cases
- Includes ultra slow roll where $f'/f = 3$ and the curvature does not freeze out
- Flaws for cases when the curvature does freeze out based on further assumptions:

  If we additionally assume $|f'/f| \ll 1$ the first order power spectrum becomes Stewart 2002

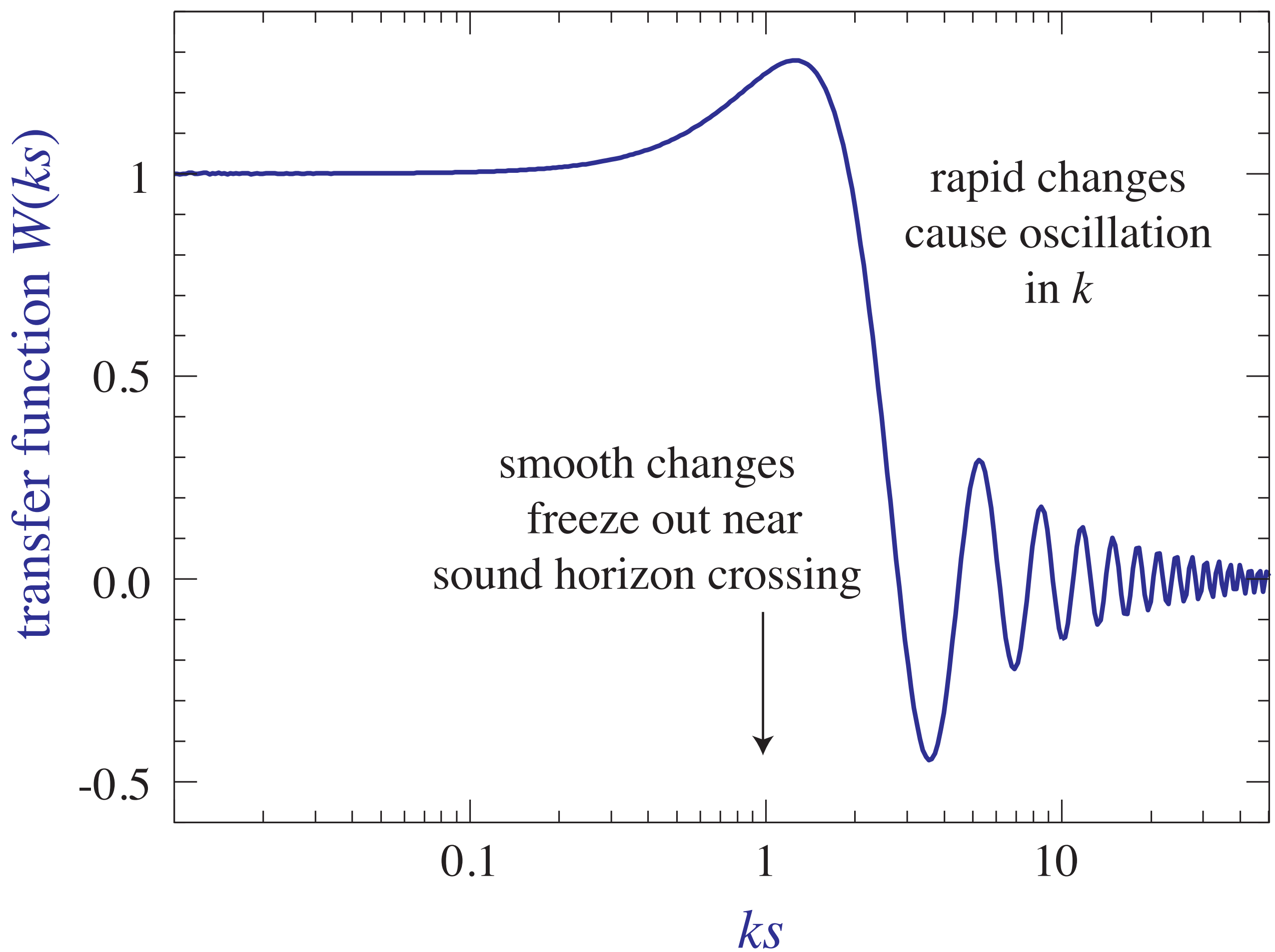
$$\Delta^2_{\mathcal{R}} = \frac{1}{f^2}\left[1 + \frac{2}{3}\frac{f'}{f} + \frac{2}{3}\int_x^\infty \frac{du}{u} W(u) \left(\frac{f'' - 3f'}{f}\right)\right]$$

  $W$ is a window function that determines when excitations from the source freeze out: not instantanously at horizon crossing...

# Freezeout Window

- Window determines how excitations freeze out (see second lecture)

$$W(x) \equiv \frac{3\sin 2x}{2x^3} - \frac{3\cos 2x}{x^2} - \frac{3\sin 2x}{2x}$$



# Flaws

- This form of GSR suffers from notable flaws

$$\Delta^2_{\mathcal{R}} = \frac{1}{f^2}\left[1 + \frac{2}{3}\frac{f'}{f} + \frac{2}{3}\int_x^\infty \frac{du}{u} W(u) \left(\frac{f'' - 3f'}{f}\right)\right]$$

  – power spectrum is not necessarily positive definite

  – depends on arbitrary superhorizon $x \ll 1$ epoch for evaluation

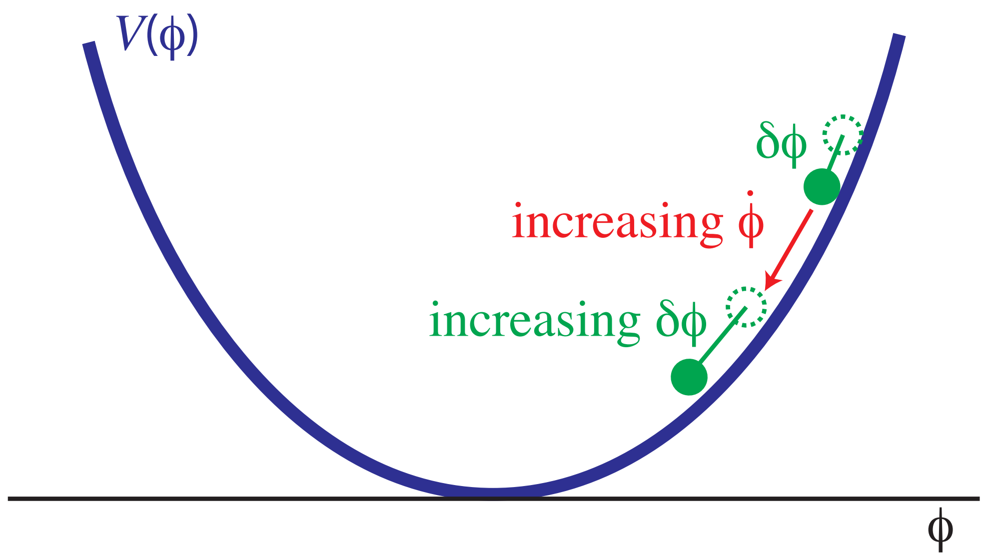
  – does not enforce constant superhorizon curvature

- Can make GSR less accurate than ordinary slow roll if $x$ too small
- Worse yet, these flaws very apparent when $f'/f$ becomes large

- Rectify these flaws to construct a practically useful approach
- First recall how and why superhorizon curvature is constant...

# Curvature Freezeout

- Transformation $\mathcal{R} = xy/f$ lies at the heart of poor convergence of GSR, especially if evaluated well after horizon crossing $x \ll 1$
- Canonical field highlights problem: normalized field $y$ is related to scalar field fluctuation $\delta\phi$ in spatially flat gauge $y = -\sqrt{2k}a\delta\phi$
- Field fluctuation does not freezeout but follows the rolling of the background field: $\delta\phi \propto \dot{\phi}$ [exception: ultra slow roll where $f''/f = 3f'/f$ on a flat potential]



# Curvature Freezeout

- For a canonical field where

$$\mathcal{R} = -\frac{\delta\phi}{d\phi/dN}$$

  generally freezes out on superhorizon scales

- More generally consequence of the separate universe approximation

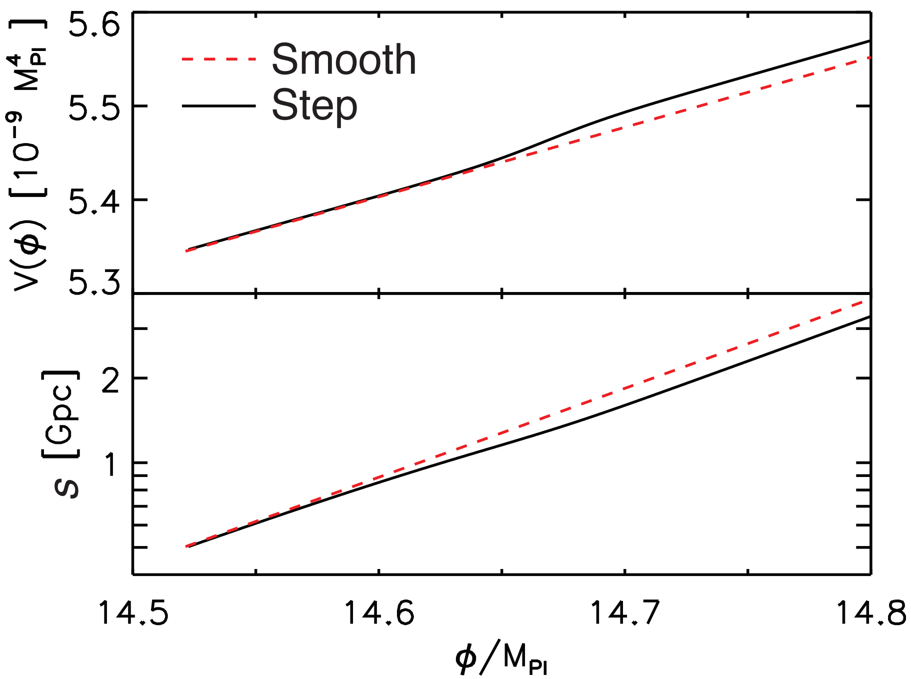
- If superhorizon fluctuations behave as the background of a local FRW expansion, the local curvature measured by freely falling observers $K_{\rm local} = \text{const}$ Hu & Joyce 2016

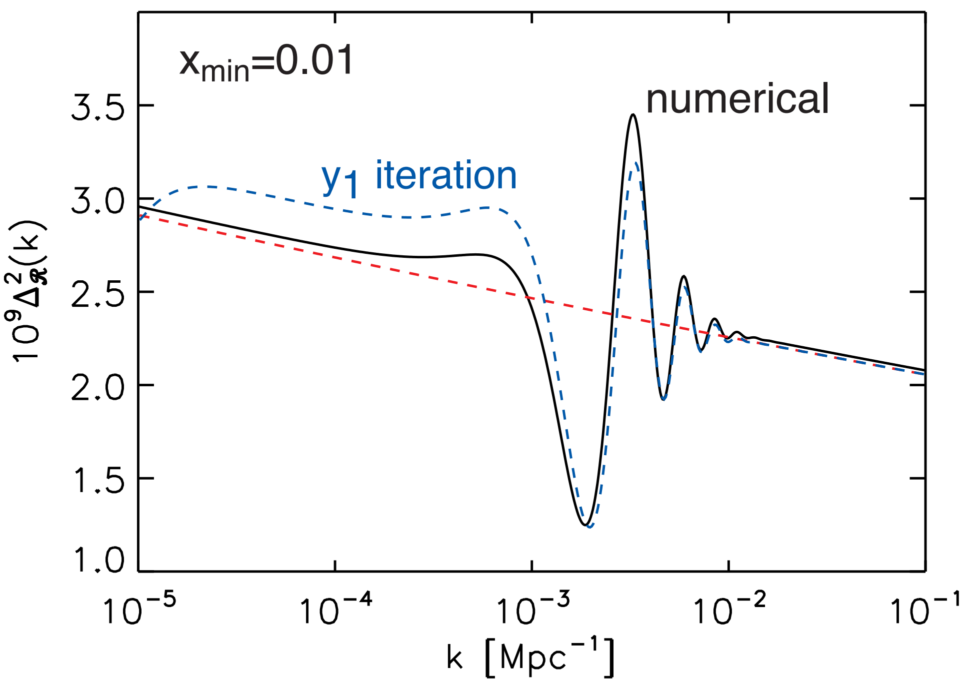
- It is sufficient that unitary gauge observers see $\mathcal{R}$=const. for separate universe approximation to hold

- Iterative approach in $y$ mixes order in $\mathcal{R}$ when $f$ evolves

# Step Example with $y_1$

- Example: step in potential - evolution of $f$ causes noticeable discrepancy for superhorizon modes at the step Dvorkin & Hu 2010
- Error can be arbitrarily large as evaluation point $x_{\rm min} \to 0$



# GSR for Large Power Spectrum Features

- Solution: reorganize iterations in terms of $\mathcal{R}_n$ instead of $y_n$ by including $f'/f$ corrections that appear at next order Dvorkin & Hu 2010

$$\ln \Delta^2_{\mathcal{R}} \approx G(\ln x) + \int_x^\infty \frac{du}{u} W(u) G'(\ln u)$$
$$\approx - \int_x^\infty \frac{du}{u} W'(x) G(\ln u),$$

where integration by parts $\rightarrow x$ independence, $W'(x \rightarrow 0) = 0$

$$G \equiv -2 \ln f + \frac{2}{3} (\ln f)'$$

and the replacement $g \rightarrow G'$ involves $(f'/f)^2$ corrections from the next order

$$G' = \frac{2}{3} \left[ g - \left( \frac{f'}{f} \right)^2 \right]$$

# GSR for Large Power Spectrum Features

- Superhorizon evolution of $G$ no longer changes $\mathcal{R}$ and the evaluation point can now be taken to zero $x \to 0$
- Exponential form guarantees positive definite power spectrum, controlled approximation even for large features
- Derivation can be formalized by directly iterating in $\mathcal{R}$ Miranda, Hu, (Heinrich née) He, Motohashi 2015

$$\mathcal{R} = \mathcal{R}_0 + \mathcal{R}_1 + \mathcal{R}_2$$

with the tradeoff being that the Bunch-Davies initial condition $\mathcal{R}_0(x) = xy_0/f_0$ depends on $f_0 = f(x_0)$

$$\mathcal{R}_n(x) = 2\int_x^{x_0} \frac{du}{u}\frac{f'}{f}\frac{x}{u}\frac{d\mathcal{R}_{n-1}}{du}\mathrm{Im}[y_0^*(u)y_0(x)]$$

# GSR for Large Power Spectrum Features

- Result is the same structure introduced in Dvorkin & Hu 2010 but with a more systematic order counting
- Exponentiation appears because a modefunction excitation generates further excitations Miranda, Hu, He, Motohashi 2016
- Functional form even in the nonlinear regime is highly constrained by relation between Bogoliubov coefficients ($|\alpha|^2 = |\beta|^2 + 1$) for subhorizon sources

$$\Delta^2_{\mathcal{R}} = A(\cosh B - \sinh B \cos\varphi)$$

  where $B$ can be related to the first order excitation for an impulse source and explains the origin of exponentiation
- Second order expression is sufficiently accurate in current observables up to order unity deviations in power spectra

# Second Order in GSR

- Iterating to second order Choe, Gong, Stewart 2004; Dvorkin & Hu 2010

$$\Delta^2_{\mathcal{R}} \approx e^{I_0}\left[\left(1+\frac{1}{4}I_1^2+\frac{1}{2}I_2\right)^2+\frac{1}{2}I_1^2\right]$$

where $I_0$ gives the first order piece and the second order corrections are

$$I_1 = \frac{1}{\sqrt{2}}\int_0^\infty \frac{dx}{x} G'(\ln x) X(x),$$

$$I_2 = -4\int_0^\infty \frac{dx}{x}\left(X+\frac{1}{3}X'\right)\frac{f'}{f}\int_x^\infty \frac{du}{u^2}\frac{f'}{f}$$

with

$$X(x) = \frac{3}{x^3}(\sin x - x\cos x)^2$$

# Single Source Function $G'$

- $I_1$ represents the square of first order excitations (imaginary vs real component)
- $I_2$ represents an excitation from an excitation and is suppressed before horizon crossing Miranda, Hu, He, Motohashi 2015
- For large, rapid, power spectrum features keeping only $I_1$ often suffices; magnitude is a control parameter for iterative expansion

$$I_1 < 1/\sqrt{2}$$

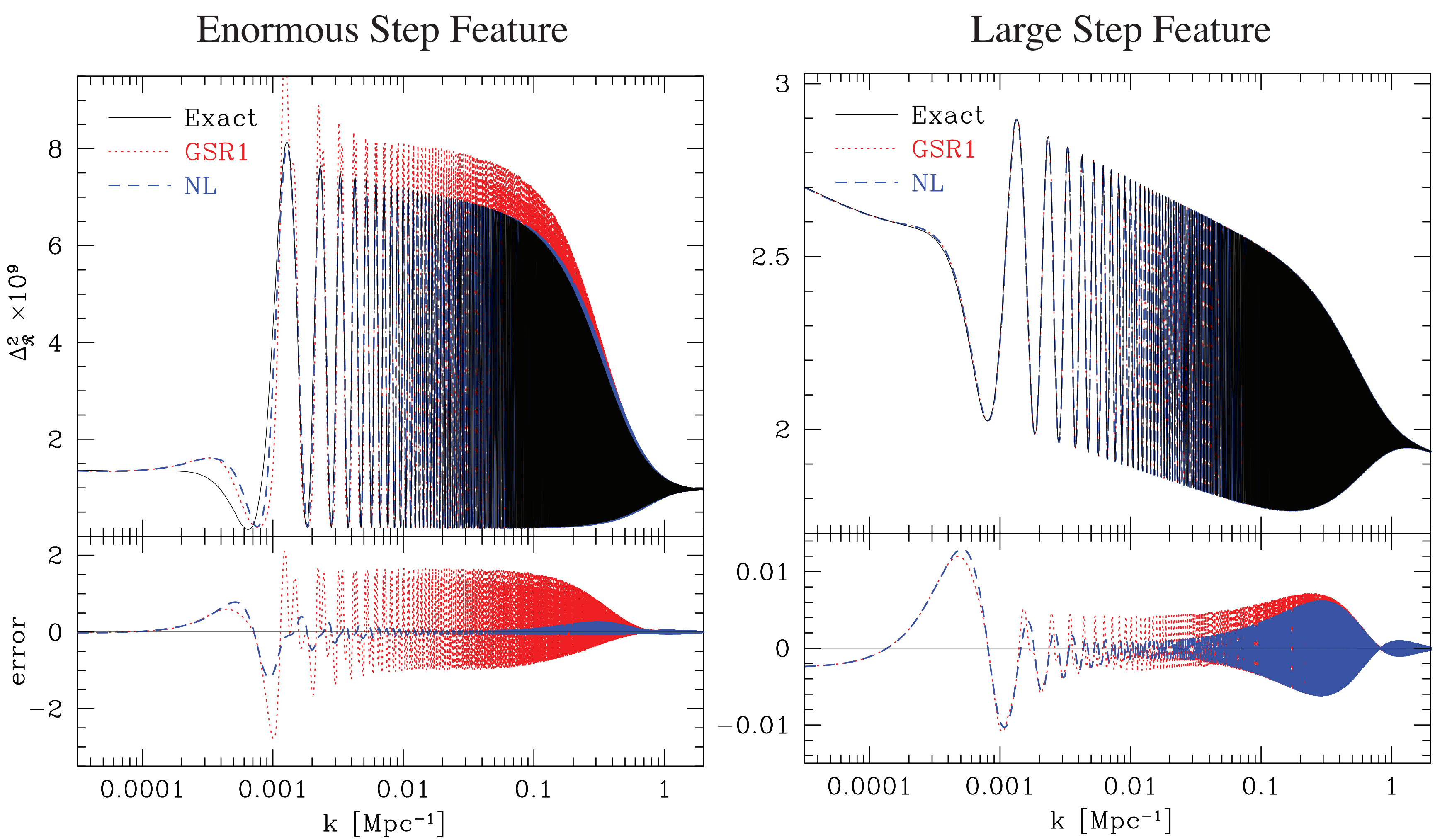
- For slowly varying power spectrum features $I_1$ and $I_2$ partially cancel but their net effect then too small to observe
- Power spectrum becomes a functional of $G'$ alone

$$\Delta^2_{\mathcal{R}}[G'] \to G'[\Delta^2_{\mathcal{R}}]$$

allowing the data to reconstruct $G'$

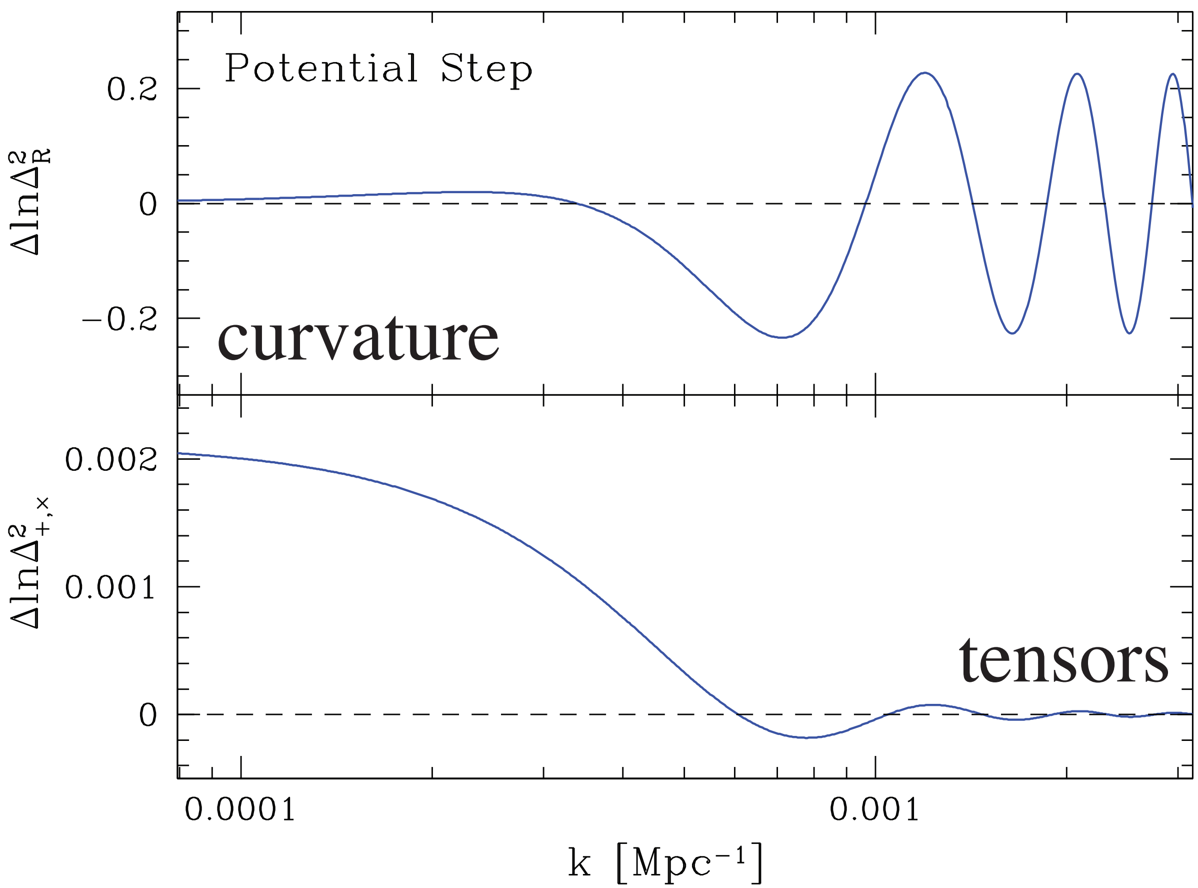
# Single Source Function $G'$

- Single source approximation vs. subhorizon resumation vs numerical for large potential step Miranda, Hu, He, Motohashi 2015



# Tensor Spectrum

- Tensor fluctuations follow the same rules but with the single function being $G'_t$
- Tensor features usually small compared with scalars for canonical scalar since $H$ typically is smoothly varying Gong 2004; Hu 2014



# Summary of Lecture I

- Single field inflation is defined by having a single clock or preferred ADM time slicing
- EFT constructs all models consistent with unbroken spatial diffs on the slice
- Lagrangian from spatially covariant functions of intrinsic and extrinsic curvature, lapse (and their covariant derivatives, acceleration)
- Second order theories in both space and time derivatives lead to Horndeski/GLPV Lagrangian
- All such cases give quadratic action for scalars and tensors in their normal form with sound speed and normalization as parameters
- Parameters can have arbitrary time dependence in EFT

# Summary of Lecture I

- Generalized slow roll provides an iterative approach to solving Mukhanov-Sasaki equation for
  - Modefunctions
  - Power spectra
  - Bispectra (next lecture), $\ldots$
- Characterized by
  - Source of excitations from de Sitter modefunctions
  - Window function for freezeout
- For up to order unity excitations, GSR characterized by single source function
  - $G$ for scalars and tensors separately
- Tensor usually suppressed compared with scalars

# Generalized Slow Roll
in the
# Effective Field Theory of Inflation

Wayne Hu
July 2018, YITP Kyoto

# Outline

- Beyond Canonical Slow-Roll Inflation
- Single Clock and ADM
- EFT of Single Field Inflation
- Power Spectra
- Generalized Slow Roll
- Optimized Slow Roll
- Features and their Templates
- Polarization and Bispectrum
- Reconstructing the EFT

# Optimized Slow Roll

# Deviations from Scale Invariance

- If $f \approx$ const.

$$\Delta^2_{\mathcal{R}} \approx \left.\frac{1}{f^2}\right|_{\frac{c_s k}{aH} \approx 1}$$

Nearly scale independent power spectrum in ordinary slow roll approximation

- Net deviations from scale invariance in amplitude observationally small across $k$-efolds $\delta \ln k$ in CMB

$$\frac{\delta \ln \Delta^2_{\mathcal{R}}}{\delta \ln k} = n_s - 1$$

and for models, typically for inflation to end in $N \sim 60$ efolds

$$1 - n_s \approx \frac{1}{N}$$

- Deviations need not vary equally slowly in $\Delta \ln k$

# Generalized Slow-Roll Deviations

- In EFT, coefficients can have arbitrary time dependence as long as they don't cause inflation to end
- GSR allows us to separate these two senses of deviations from scale-invariance: amplitude and temporal frequency
- Two pieces of the slow roll approximation:
  - Average amplitude of modefunction deviations

$$\bar{G}' = 1 - n_s = \mathcal{O}(1/N)$$

  - Slowness of the temporal variation

$$G''/G' = \mathcal{O}(1/\Delta N) \to G^{(p)} = \mathcal{O}(1/N\Delta N^{p-1})$$

- Usual slow roll approximation conflates $N \sim 60$ and $\Delta N$ by implicitly assuming that inflation has only one feature: its end

# Generalized Slow Roll

- For $1 \lesssim \Delta N \ll N$ need to keep $\mathcal{O}(1/N\Delta N^{p-1})$ as leading order
- In GSR, this means we can use the leading order in modefunction amplitude deviation Motohashi & Hu 2015

$$\ln \Delta^2_{\mathcal{R}} \approx -\int_0^\infty \frac{dx}{x} W'(x) G(\ln x) + \ldots$$

and Taylor expand $G$ around the epoch of horizon crossing $x_f$

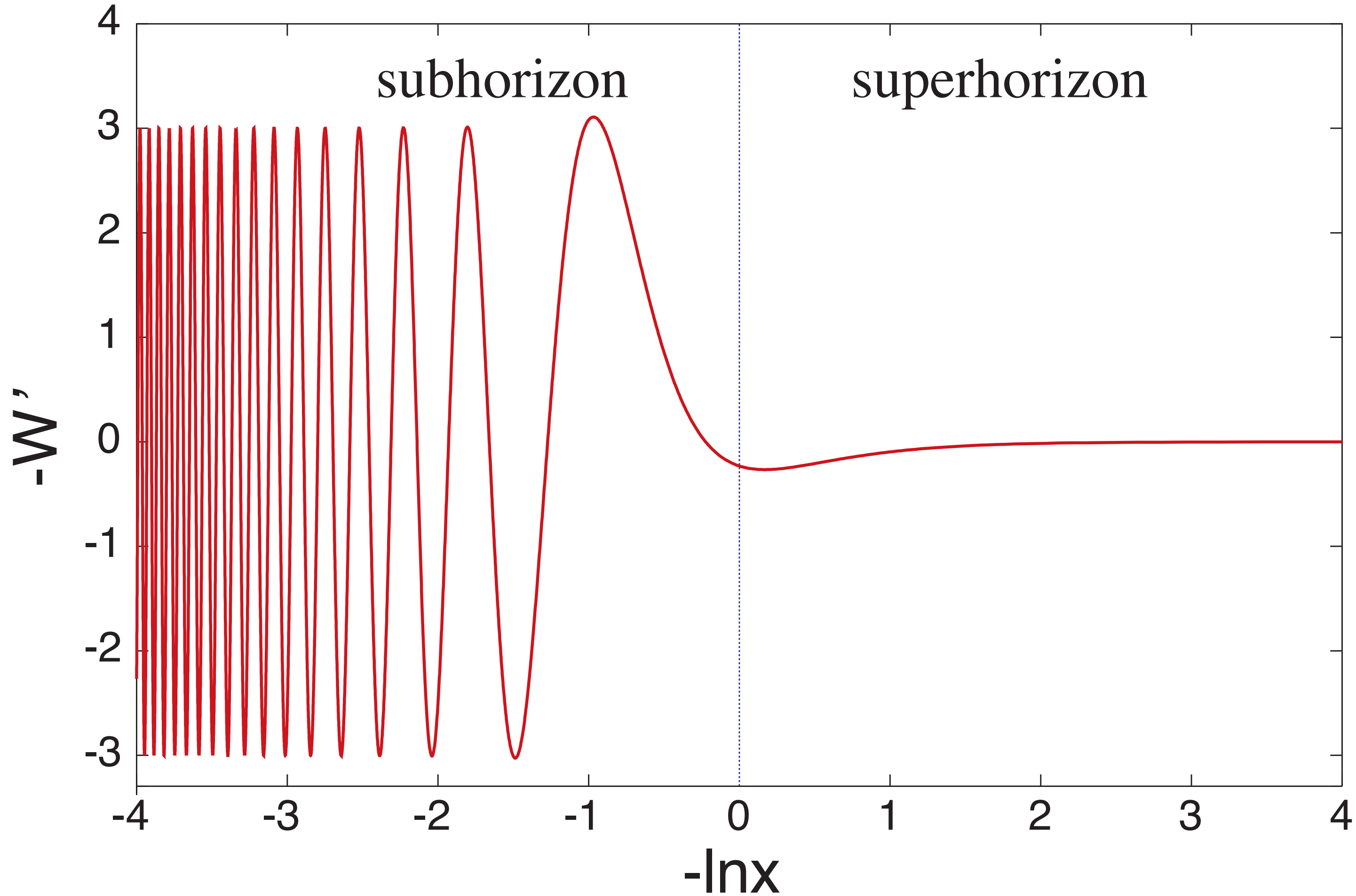
$$G(\ln x) = \sum_0^\infty \frac{G^{(p)}(\ln x_f)}{p!}(\ln x - \ln x_f)^p$$

- Integrals can be precomputed

$$q_p(\ln x_f) = -\frac{1}{p!}\int_0^\infty \frac{dx}{x} W'(x)(\ln x - \ln x_f)^p$$

# Freezeout Window

- $W'$ decreases rapidly for $x \ll 1$ or $-\ln x > 0$, freezing out $G$ according to the pretabulated $q_p$ coefficients



# Generalized Slow Roll

- Leads to a series expansion of the power spectrum that converges if $\Delta N > 1$

$$\ln \Delta^2_{\mathcal{R}} \approx G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

- Taylor expansion in $G$ then defines tilt and running of tilt $d/d\ln k = -d/d\ln s$

$$\frac{dG^{(p)}(\ln x_f)}{d\ln k} = -G^{(p+1)}(\ln x_f)$$

- Tilt associated with $G'$ to leading order in $1/\Delta N$

$$n_s - 1 \equiv \frac{d\ln \Delta^2_{\mathcal{R}}}{d\ln k} \approx -G'(\ln x_f) - \sum_{p=1}^{\infty} q_p G^{(p+1)}(\ln x_f)$$

# Running of tilt

- Running of tilt $G''$ to leading order in $1/\Delta N$

$$\alpha \equiv \frac{dn_s}{d\ln k} \approx G''(\ln x_f) + \sum_{p=1}^{\infty} q_p G^{(p+2)}(\ln x_f)$$

- Since $G''/G' = \mathcal{O}(1/\Delta N)$, running of the tilt is only suppressed vs tilt by $1/\Delta N$ not $1/N$ as usually assumed
- With $\Delta N \sim$ few, running can be observably large if $\Delta N$ relatively small
- If $\Delta N$ small, then higher order terms in evaluating tilt and running become relatively more important too

- Taylor series still leaves unspecified the epoch around horizon crossing of the expansion which can be optimized...

# Optimized Slow Roll

- Weights $q_p$ on Taylor coefficients depend only $x_f$
- Enhance the accuracy of the Taylor expansion by choosing the freezeout epoch $x_f$ to zero out next $q_p$ in the series

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- Leading Order:

  – Keep only leading order term, set $q_1(\ln x_f) = 0$ by choosing $\ln x_f = 1.06$, i.e. around 1-efold before horizon crossing

  – As accurate as retaining next order term but leaving $\ln x_f = 0$

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  – Keep only leading order term, set $q_1(\ln x_f) = 0$ by choosing $\ln x_f = 1.06$, i.e. around 1-efold before horizon crossing

  – As accurate as retaining next order term but leaving $\ln x_f = 0$
- Next Order:

  – Retain $q_1$ and set $q_2(\ln x_f) = 0$ by choosing $\ln x_f = 0.22$

  – As accurate as retaining next-to-next order term for generic $\ln x_f$.
- Self consistent order counting between observables (remaining error mainly in $\ln k \leftrightarrow N$ not between observables)

# Slow Roll Parameters

- GSR parameters $G^{(n)} \leftrightarrow$ more familiar slow-roll parameters

$$G \equiv -2\ln f + \frac{2}{3}(\ln f)', \quad f \propto \sqrt{\frac{b_s \epsilon_H c_s}{H^2}}\frac{aHs}{c_s}$$

$$' \equiv \frac{d}{d\ln s} \leftrightarrow -\frac{d}{dN}$$

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$$' \equiv \frac{d}{d\ln s} \leftrightarrow -\frac{d}{dN}$$

- Evolution of $H$ (and $\epsilon_H$)

$$\epsilon_H \equiv -\frac{d\ln H}{dN}, \quad \delta_1 \equiv \frac{1}{2}\frac{d\ln \epsilon_H}{dN} - \epsilon_H,$$

$$\delta_{p+1} \equiv \frac{d\delta_p}{dN} + \delta_p(\delta_1 - p\epsilon_H)$$

- Evolution of $c_s$

$$\sigma_1 \equiv \frac{d\ln c_s}{dN}, \quad \sigma_{p+1} \equiv \frac{d\sigma_p}{dN},$$

# Slow Roll Parameters

- Evolution of normalization $b_s$

$$\xi_1 \equiv \frac{d\ln b_s}{dN}, \quad \xi_{p+1} \equiv \frac{d\xi_p}{dN},$$

- Similar hierarchies for $c_t$ and $b_t$ tensor functions
- For explicit relations: Motohashi & Hu 2017

# Slow Roll Parameters

- Evolution of normalization $b_s$

$$\xi_1 \equiv \frac{d \ln b_s}{dN}, \qquad \xi_{p+1} \equiv \frac{d\xi_p}{dN},$$

- Similar hierarchies for $c_t$ and $b_t$ tensor functions
- For explicit relations: Motohashi & Hu 2017

- For canonical scalar only $\epsilon_H, \delta_p$
- For $P(X, \phi)$ add $\sigma_p$
- For Horndeski/GLPV add $\xi_p$

- GSR expansion involves keeping higher order in $p$ but still dropping products of slow roll parameters

# Slow Roll Parameters

- In canonical inflation, can also relate $G^{(p)}$ to derivatives of the potential $V^{(p)}$

$$\mathcal{V}_p = \left(\frac{V^{(1)}}{V}\right)^{p-1} \frac{V^{(p+1)}}{V}$$

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- Ordinary slow roll approximation assumes

$$\{\epsilon_H, \delta_1, \sigma_{i,1}, \xi_1\} = \mathcal{O}\left(\frac{1}{N}\right), \quad \{\delta_p, \sigma_p, \xi_p, \mathcal{V}_p\} = \mathcal{O}\left(\frac{1}{N^p}\right)$$

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so that to leading order we need only keep the first set

- More generally, evolution of the first set of slow roll parameters can take place on a different, shorter time scale $\Delta N < N$

$$\{\epsilon_H, \delta_1, \sigma_1, \xi_1\} = \mathcal{O}\left(\frac{1}{N}\right), \quad \{\delta_p, \sigma_p, \xi_p, \mathcal{V}_p\} = \mathcal{O}\left(\frac{1}{N\Delta N^{p-1}}\right)$$

# Slow Roll Parameters

- If one uses the ordinary slow roll approximation to decide which parameters to keep can lead to very wrong relationships between tilt $n_s$ and running $\alpha$.
- For example Hubble flow parameters

$$\frac{d\epsilon_p}{dN} = \epsilon_p \epsilon_{p+1}, \quad \epsilon_1 = \epsilon_H$$

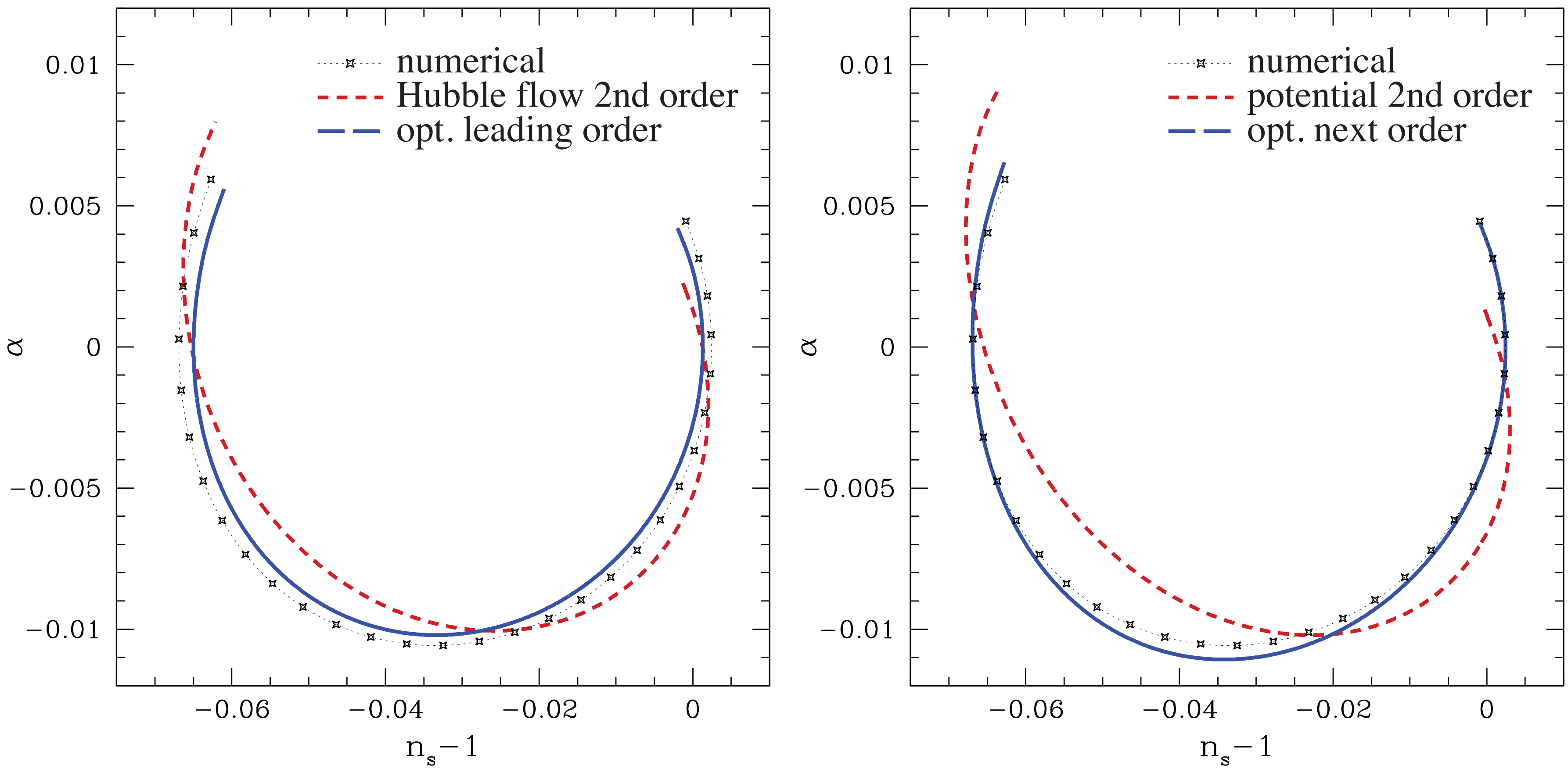
- Constructs the time derivative of each order as the product of two slow parameters and builds in a counting procedure where $\Delta N \approx N$ and $\epsilon_n = \mathcal{O}(1/N)$.
- Inconsistent to truncate based on keeping a fixed order in Hubble flow parameters
- Can falsely rule out true model because of inconsistent evaluation between observables

# Monodromy

- Consider a monodromy potential Silverstein et al 2008

$$V(\phi) = \lambda\phi + \Lambda^4 \cos\left(\frac{\phi}{f} + \theta\right)$$

- Inflaton rolls over oscillations in $60 \gg \Delta N > 1$ Motohashi & Hu 2015

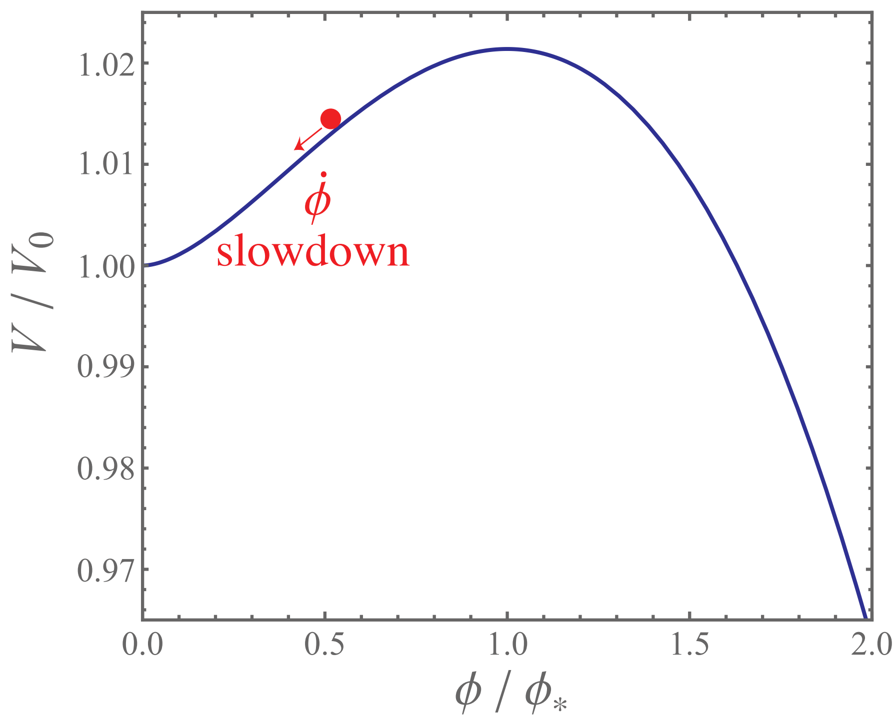


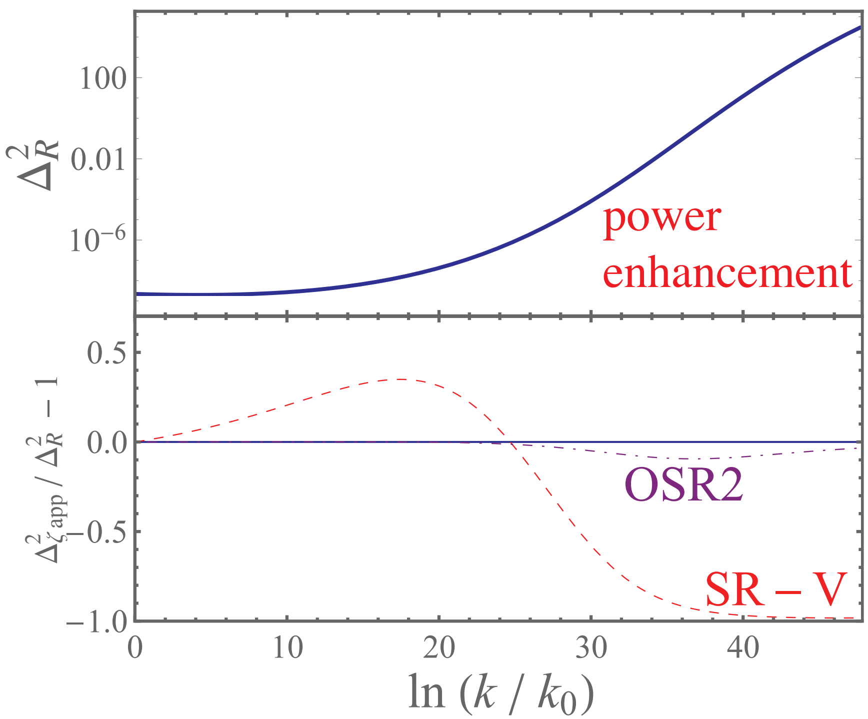
# Primordial Black Holes

- Amplify power on small scales via running of the mass

$$V(\phi) = V_0 + \frac{1}{2}m^2(\ln\phi)\phi^2$$

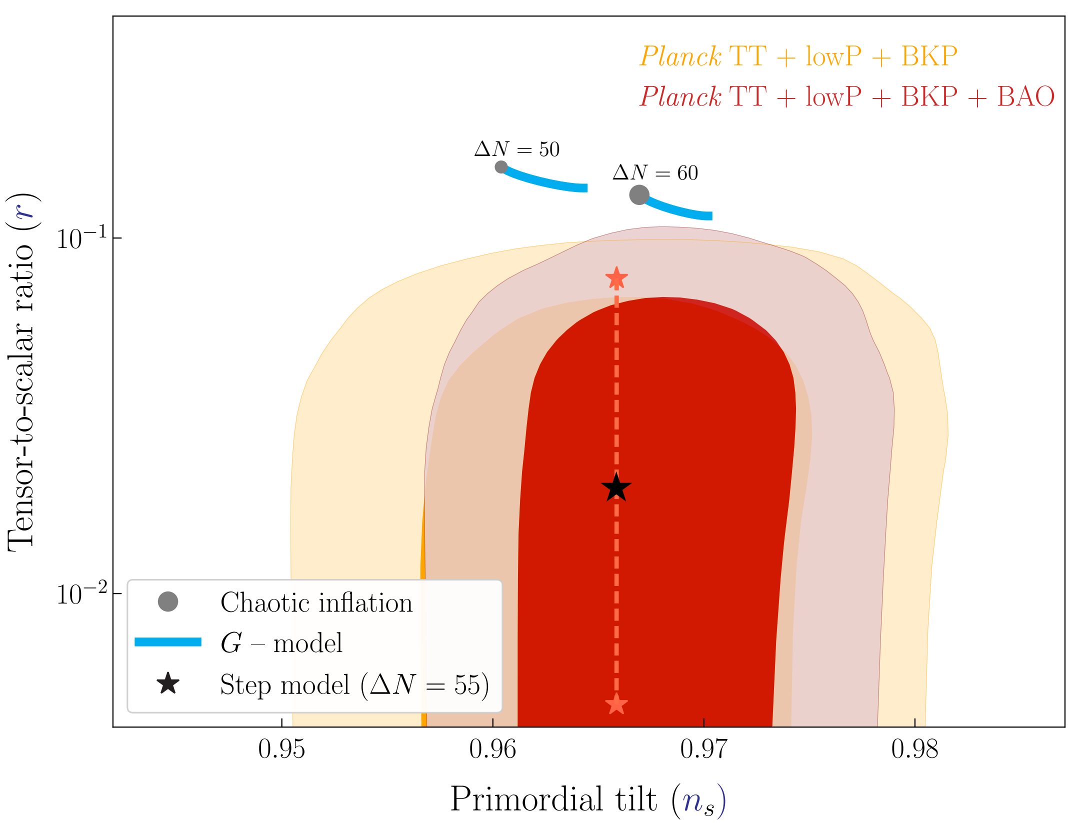
- For primordial black holes to be all the dark matter, a large feature during inflation is required violating the ordinary slow roll approximation Motohashi & Hu 2017

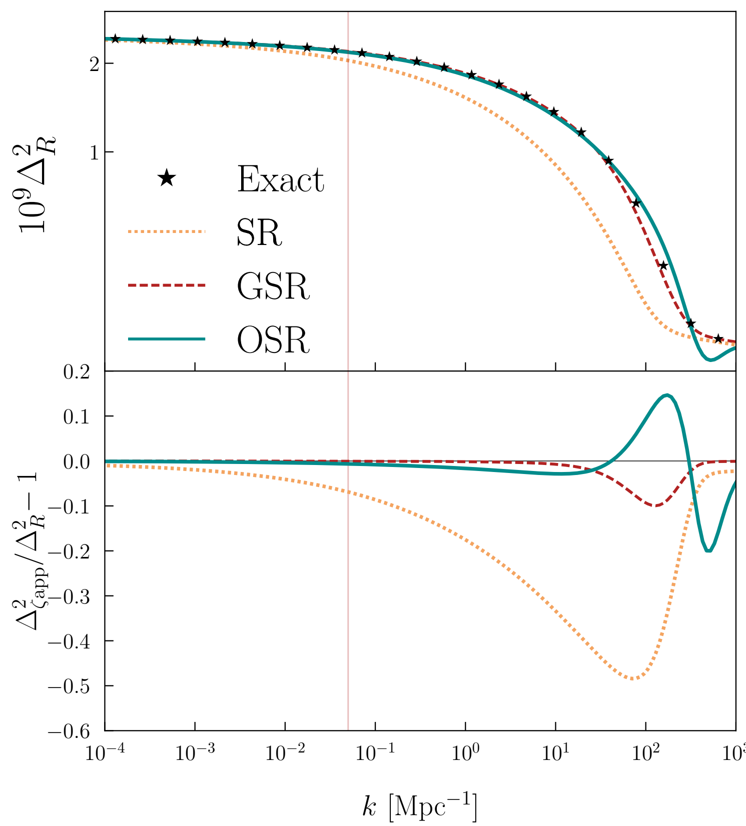


# G-inflation

- Transition from cubic galileon potential-driven inflation to canonical inflation $\rightarrow n_s$ induced by transition not end of inflation
- Large but allowed running, reduced tensors given $n_s$ Ramirez, Passaglia, Motohashi, Hu, Mena 2018



# Features and their Templates

# Features

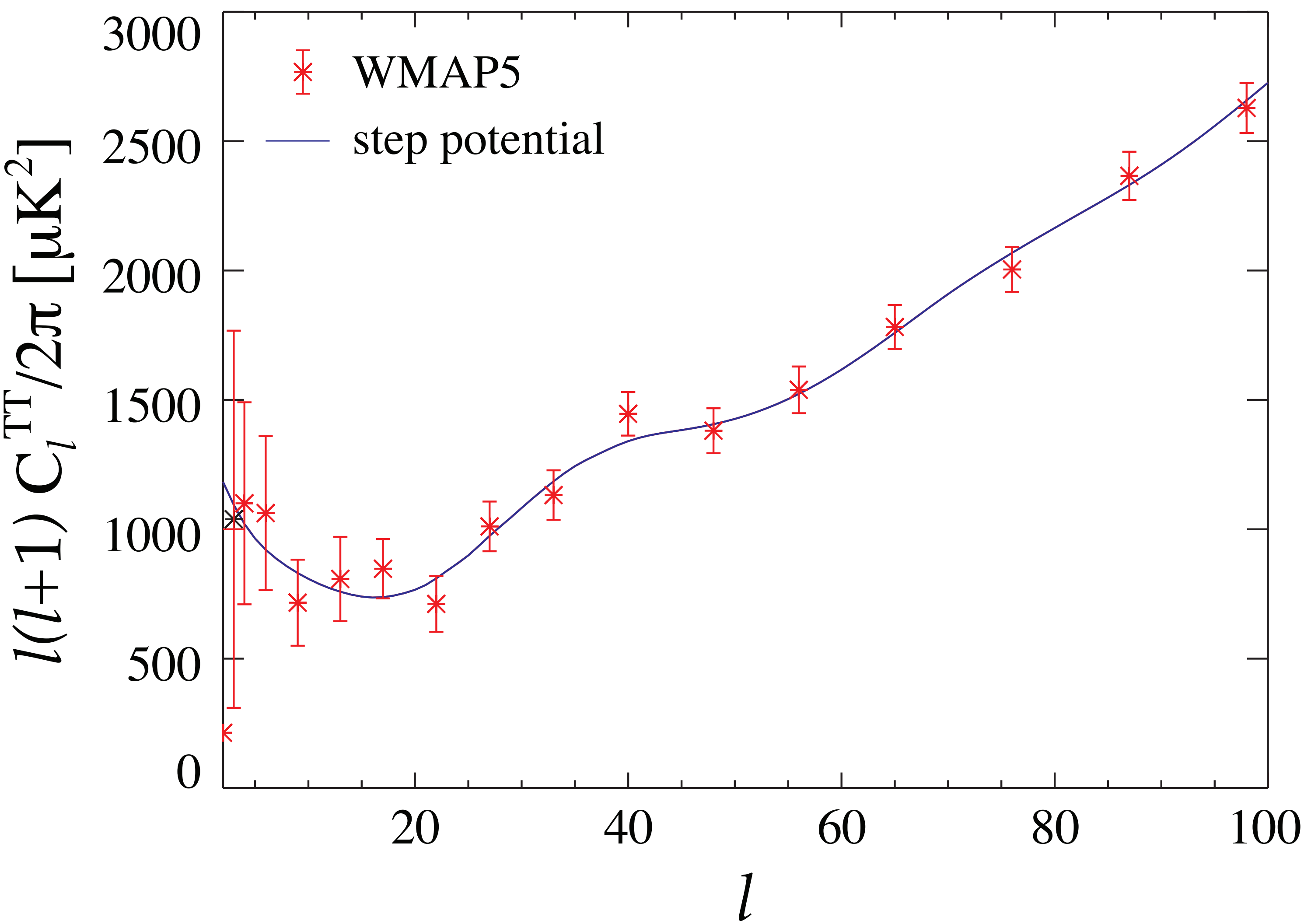
- If the timescale for variations in the EFT coefficients is $\Delta N \ll 1$ all generic constructions based on slow-roll parameters (including optimized slow roll) fail
- GSR itself can handle high frequency cases so long as the amplitude of the features is still less than $\mathcal{O}(1)$
- Useful in constructing parameterized templates for observational features in the power spectrum: examples

  Power spectrum steps

  Monodromy
- Templates enable extensive likelihood analyses (MCMC) where numerical computation of each inflationary model impractical
- Templates enable matching predictions in multiple observables for confirmation/refutation

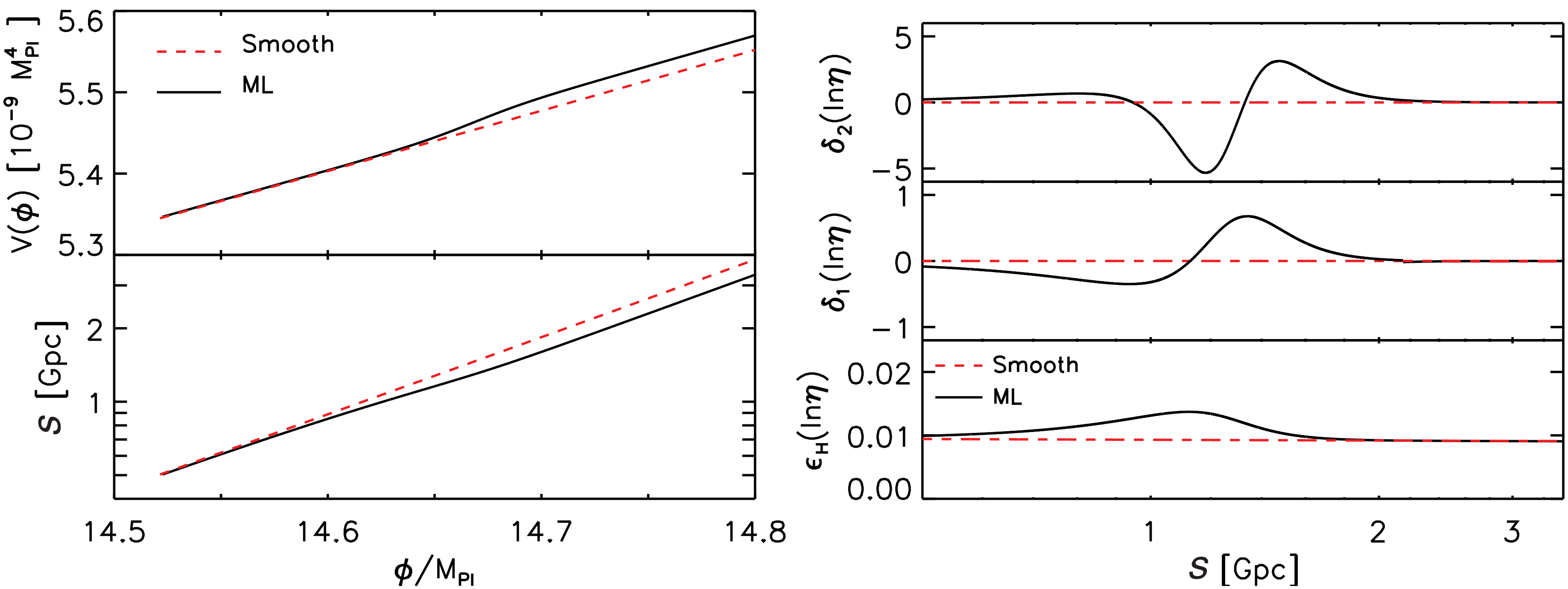
# Low Multipole Glitch

- Feature in the low-$\ell$ CMB power spectrum first seen in WMAP, confirmed in Planck, responsible for cosmological parameter shifts



# Step Potential

- Fits a steplike potential with transition $\Delta N < 1$, causing increasingly large slow roll parameters
- Causes oscillations in the power spectrum, not $\Delta^2_{\mathcal{R}} \propto H^2/\epsilon_H$
- SR qualitatively wrong; requires GSR approach to capture Dvorkin & Hu 2010

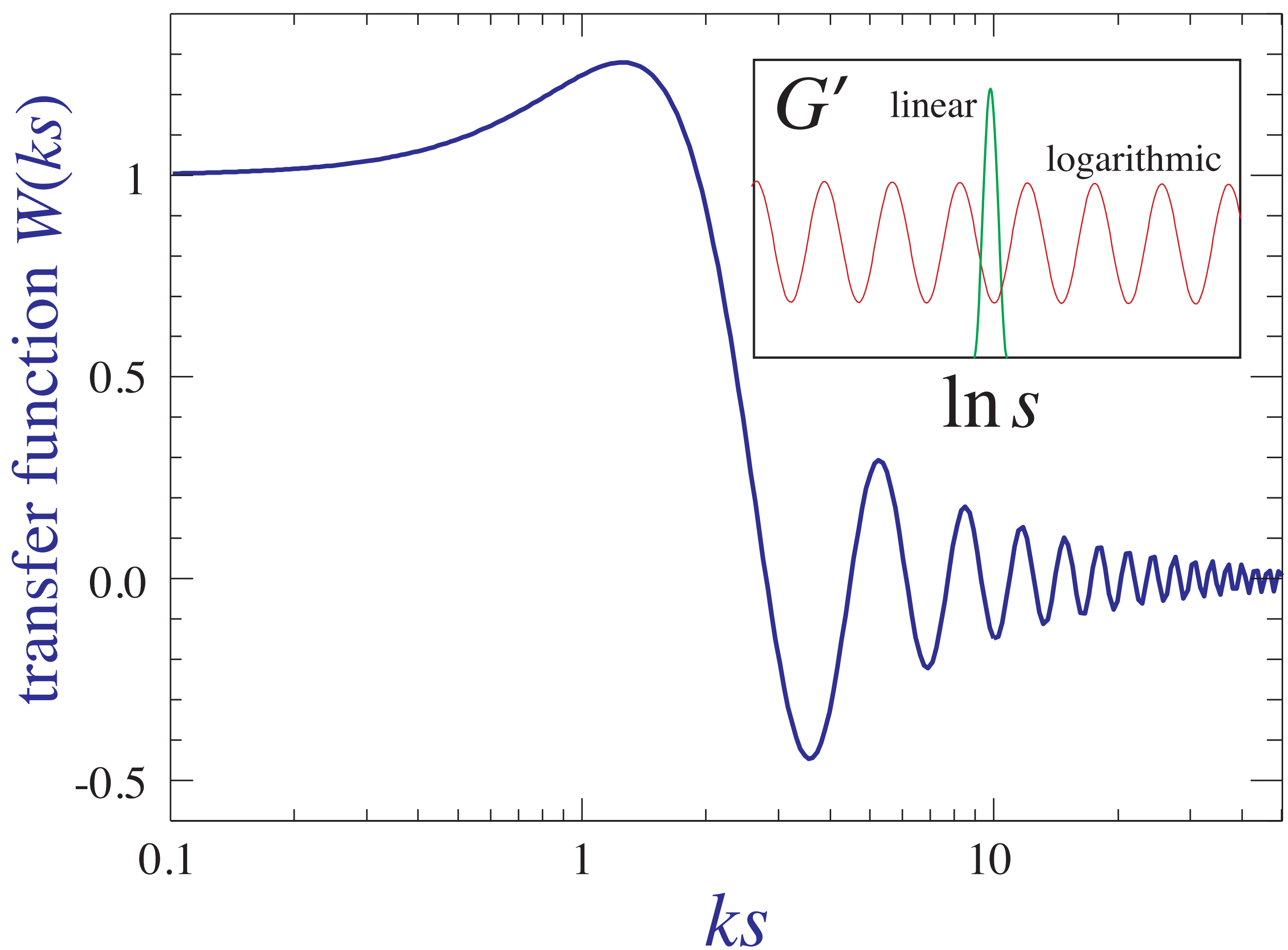


# Oscillations

- Rapid changes represent sharp temporal features
- Imprint sharp features in spatial correlation
- A single sharp temporal feature leads to linear ringing in Fourier space, damped by the width
  - Example: steps
- Periodic features generate resonances with the window, leads to logarithmic ringing in Fourier space
  - Example: monodromy

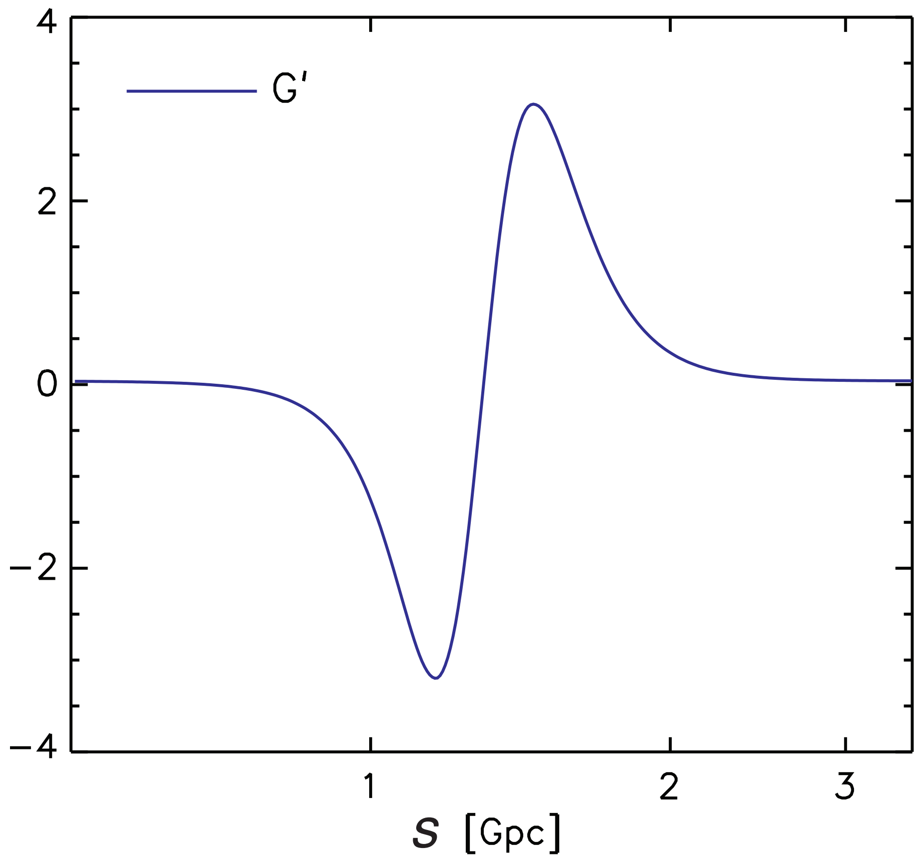
# Freezeout Window

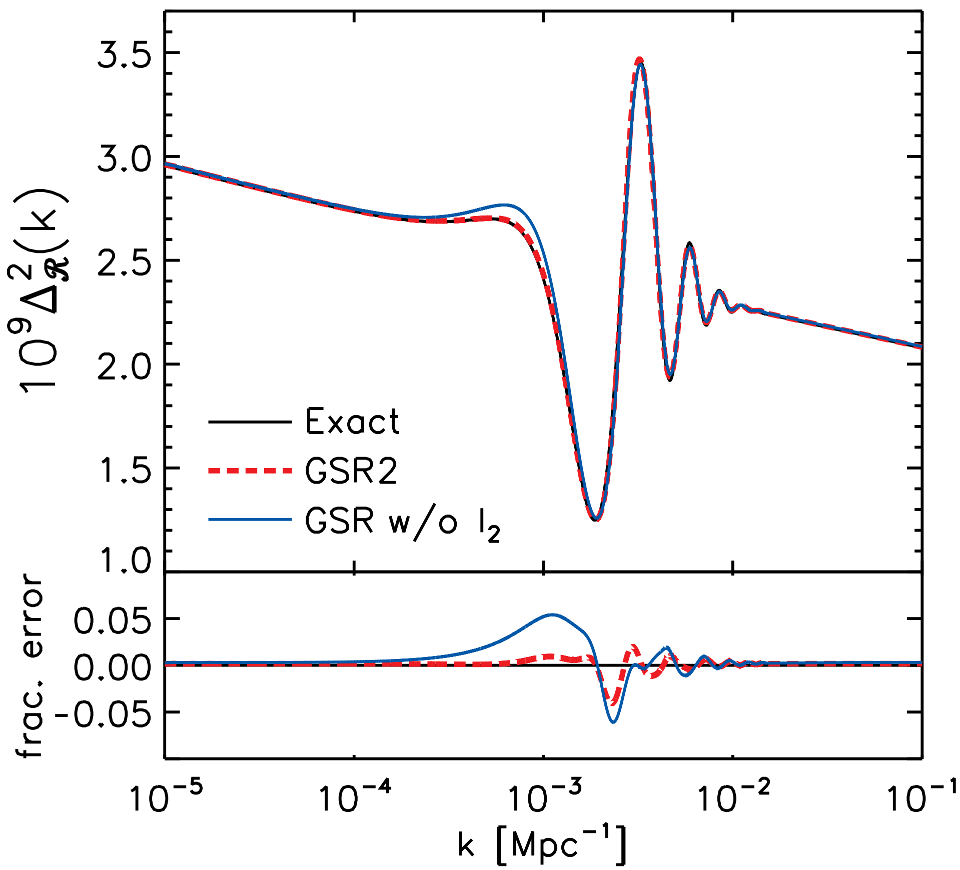
- Convolve $G'(\ln s)$ source with $W(ks)$ window
- Single sharp temporal feature leads to damped linear oscillations
- Periodic source leads to resonant logarithmic oscillations



# Step Potential

- Second derivative of $f$ makes step potential look like derivative of delta function in source $G'$
- Dip and bump and damped oscillations in curvature power

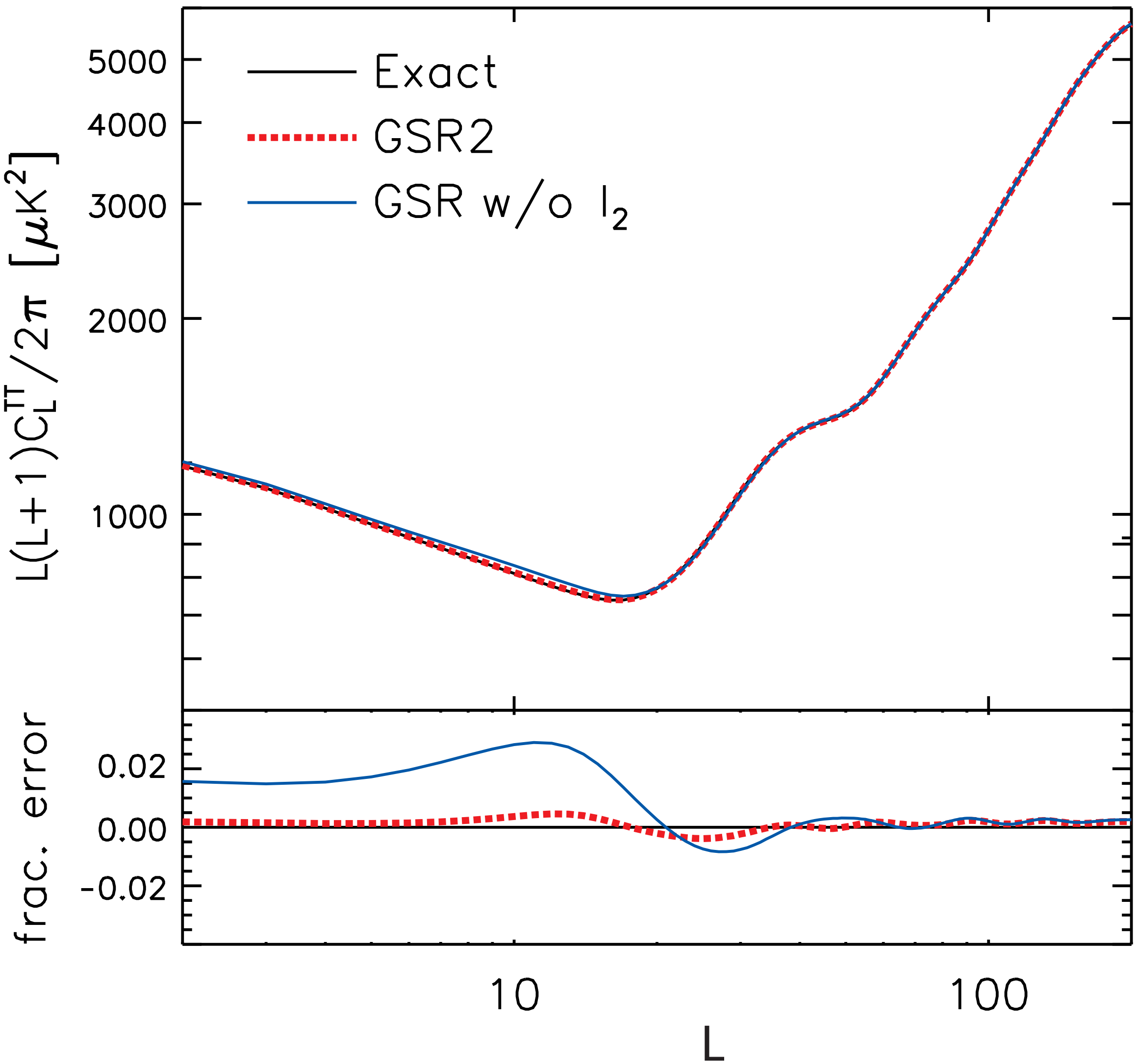


# CMB Glitch

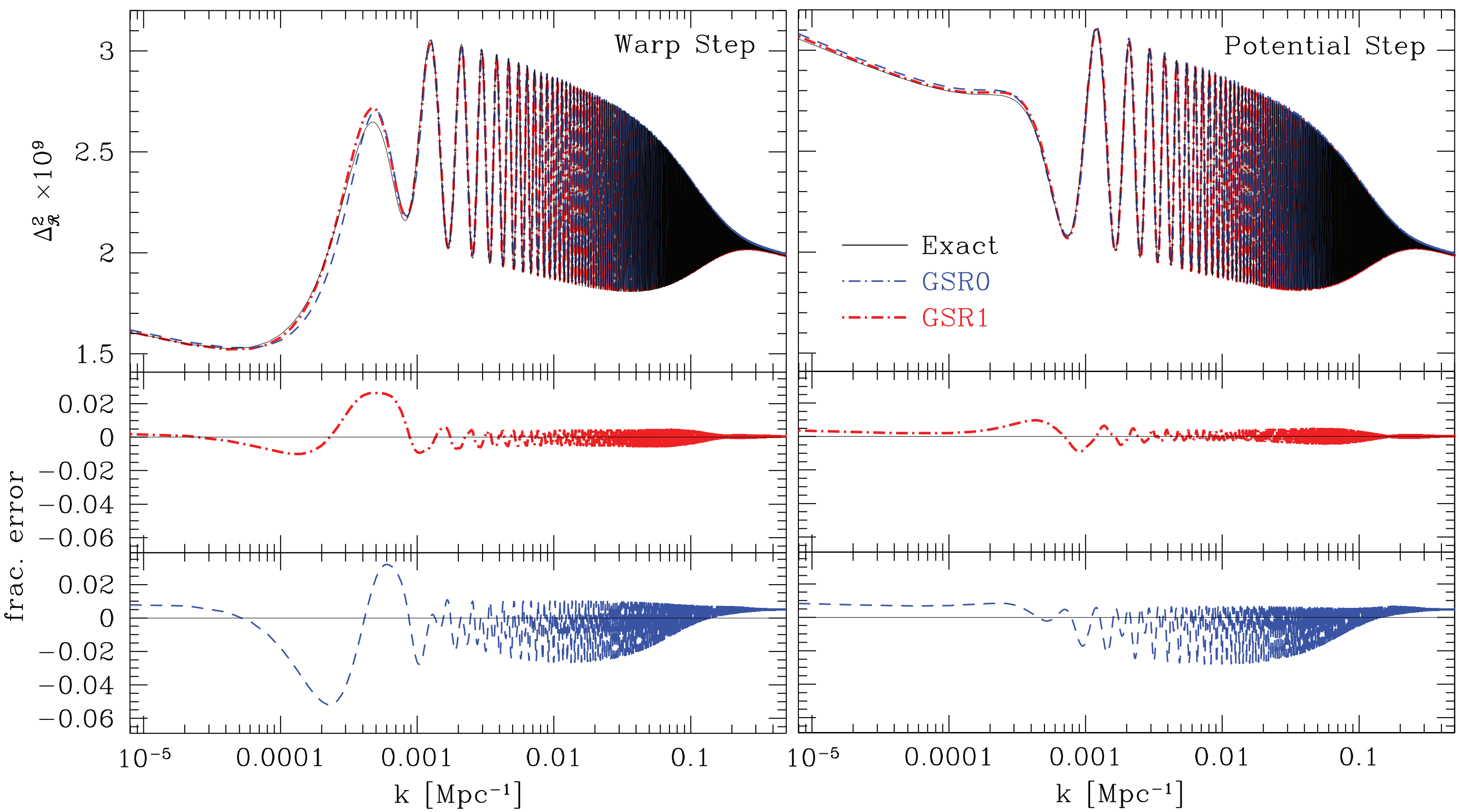
- GSR sufficiently accurate for testing step model against CMB data

Dvorkin & Hu 2010



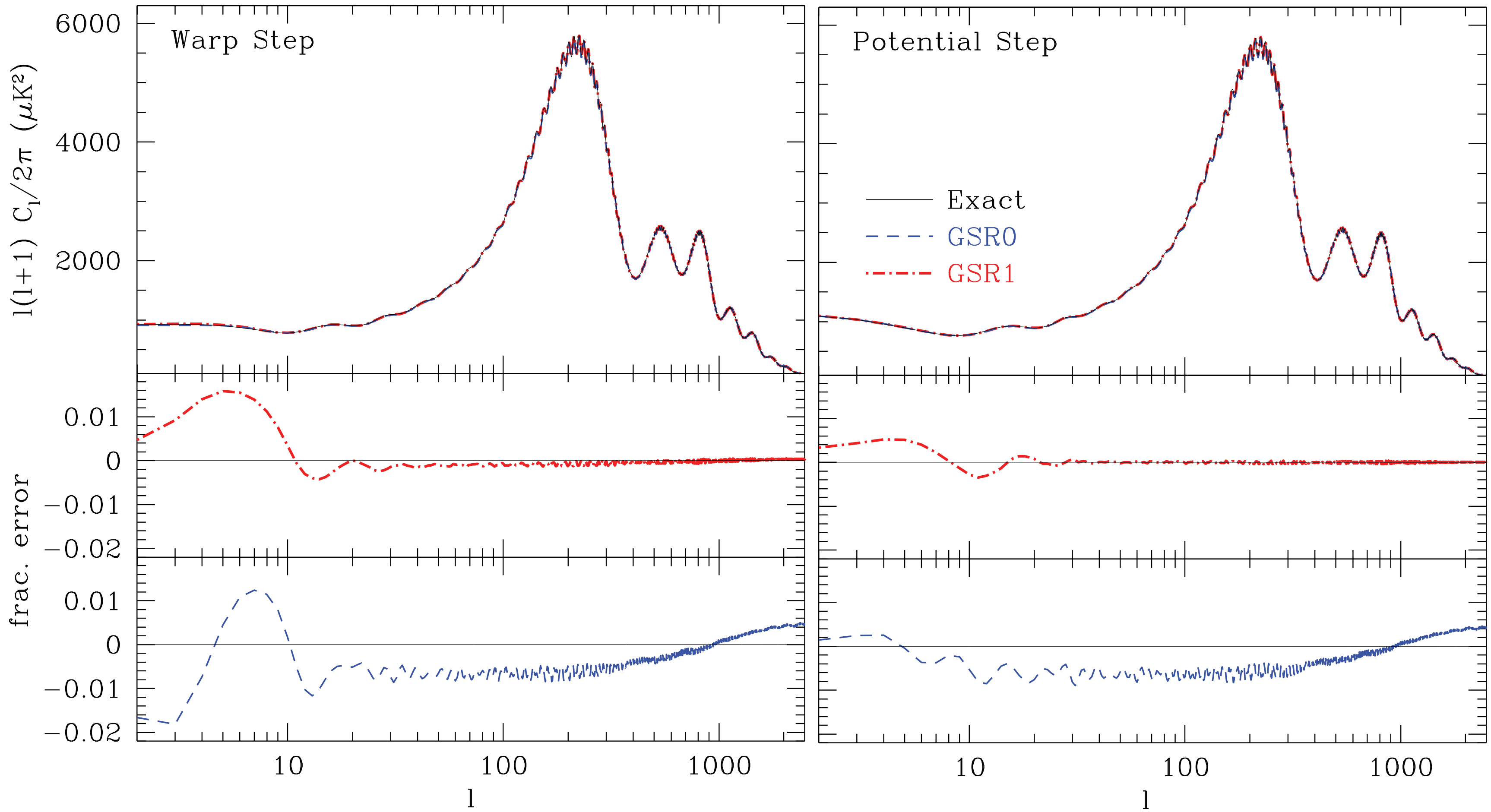
# Sharp step

- With sharp step $G'$ is mainly derivative of $\delta$ functions $\rightarrow$ integrate by parts analytically
- Analytic template for fast searches for features Adshead, Hu, Dvorkin, Peiris 2011, Miranda & Hu 2013



# Search for Features

- Early indications of oscillatory features in WMAP and Planck were followed up with these detailed templates
- Preference for features weakened with higher multipole data



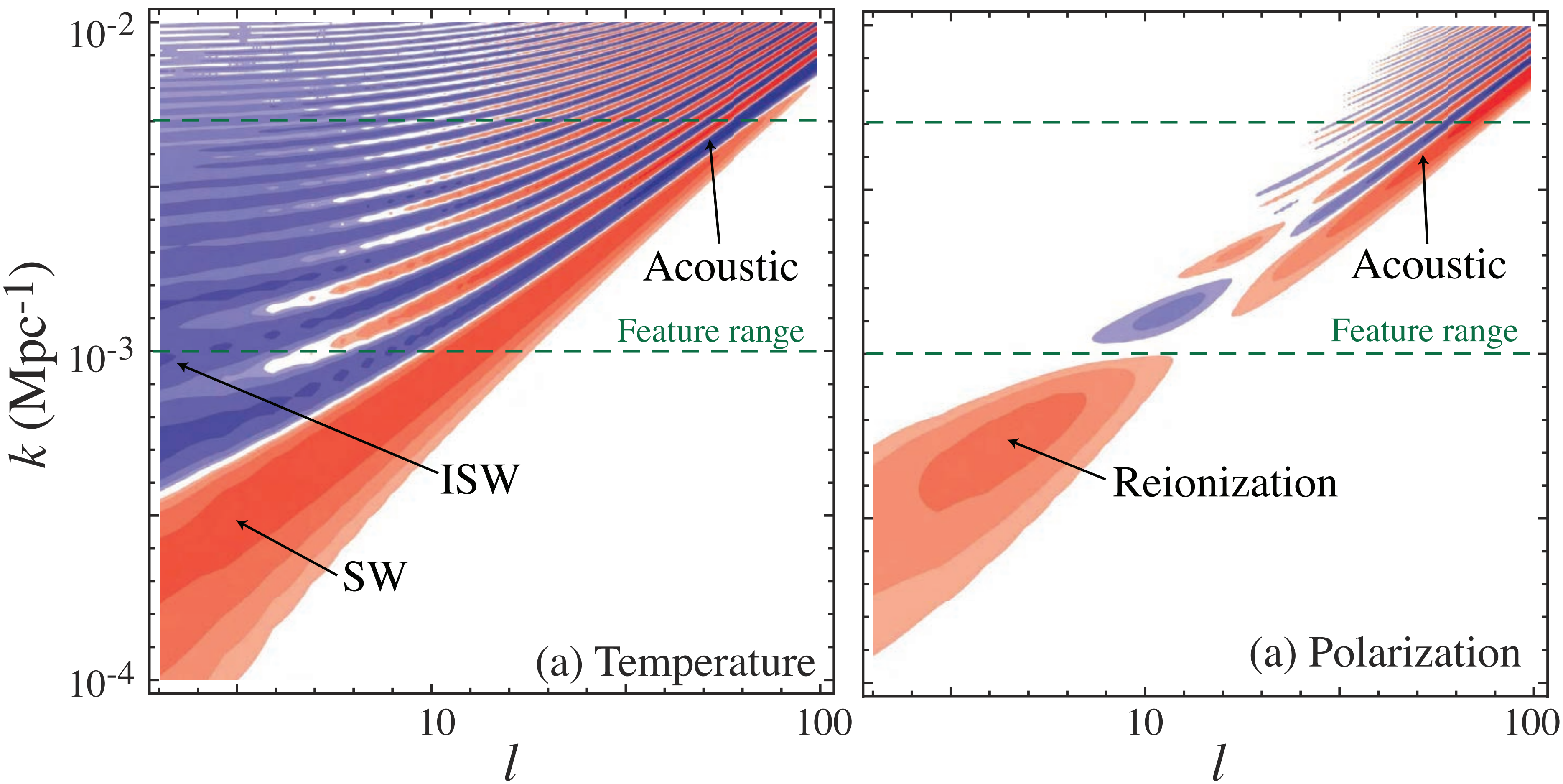
# Polarization and Bispectrum

# Lesson from Feature Searches

- Inflationary features can fit random noise fluctuations in any one data set
- Important to check matching features in different observables
  - Different multipoles of temperature power spectrum
  - Matter power spectrum
  - Polarization power spectrum
  - Bispectrum

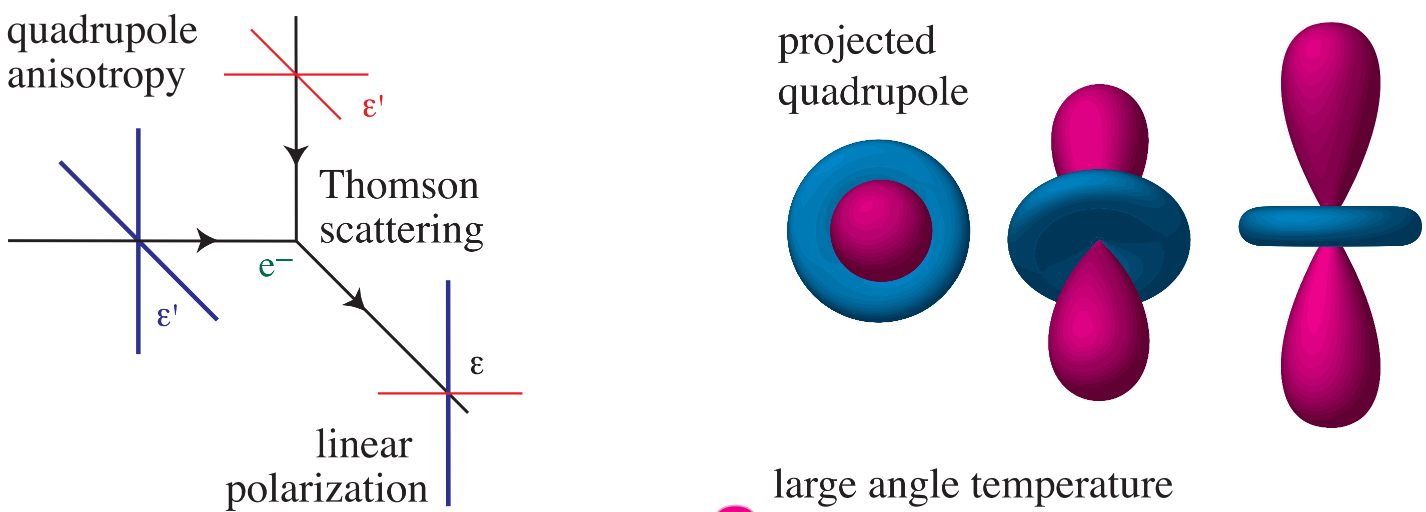
# Polarization Transfer

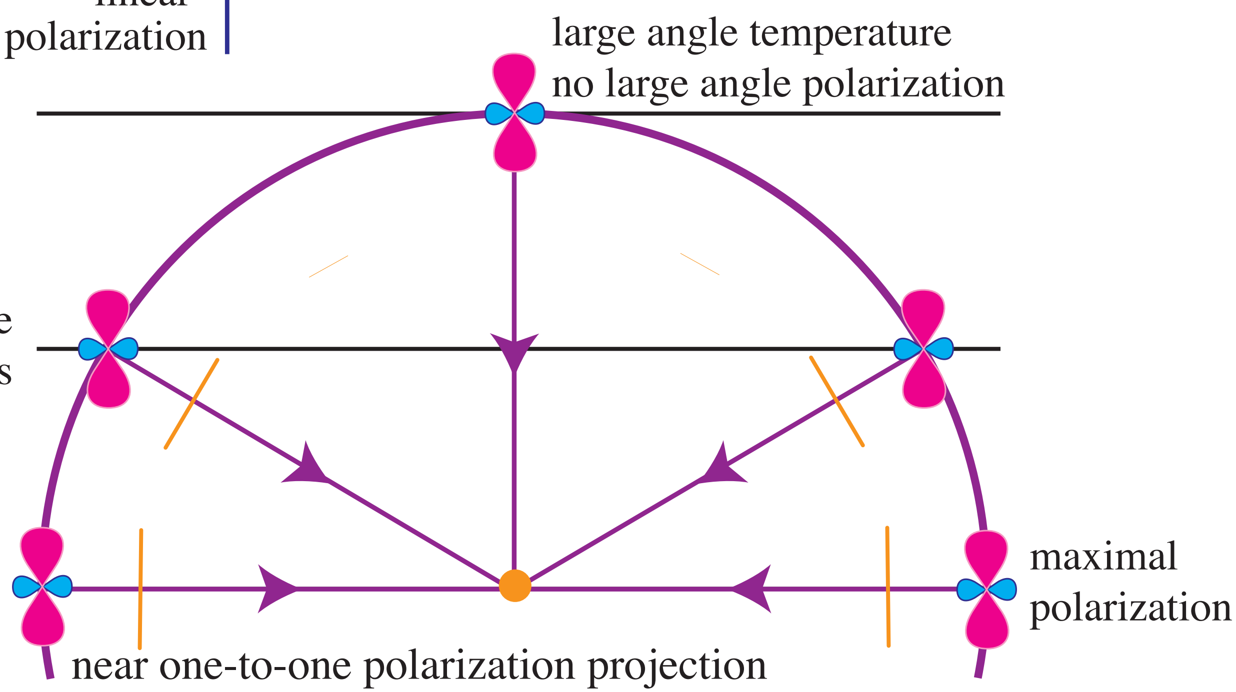
- Due to projection, polarization features in the acoustic regime are sharper and weighted to slightly higher $\ell$



# Quadrupole Projection

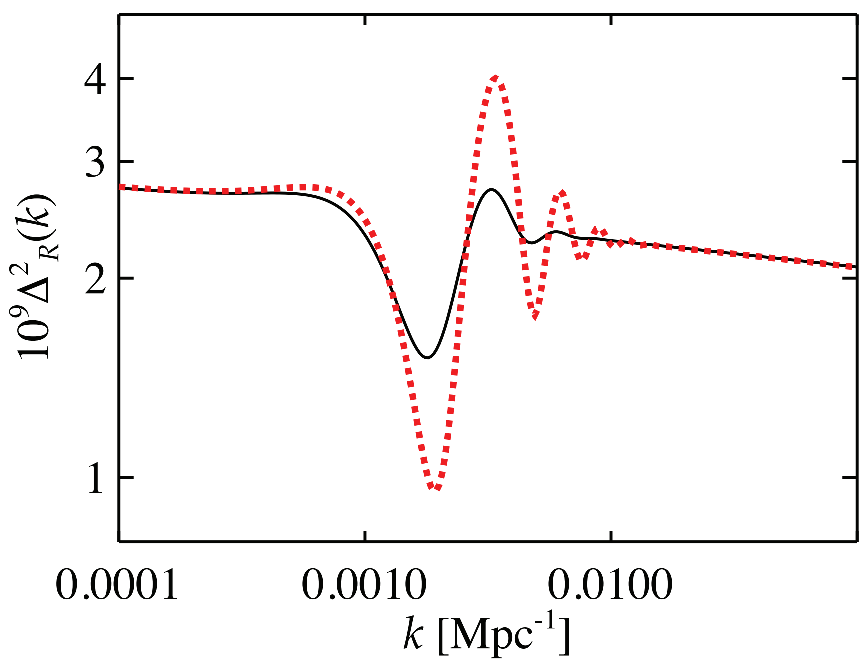
- Polarization features follow the projection of quadrupole moments

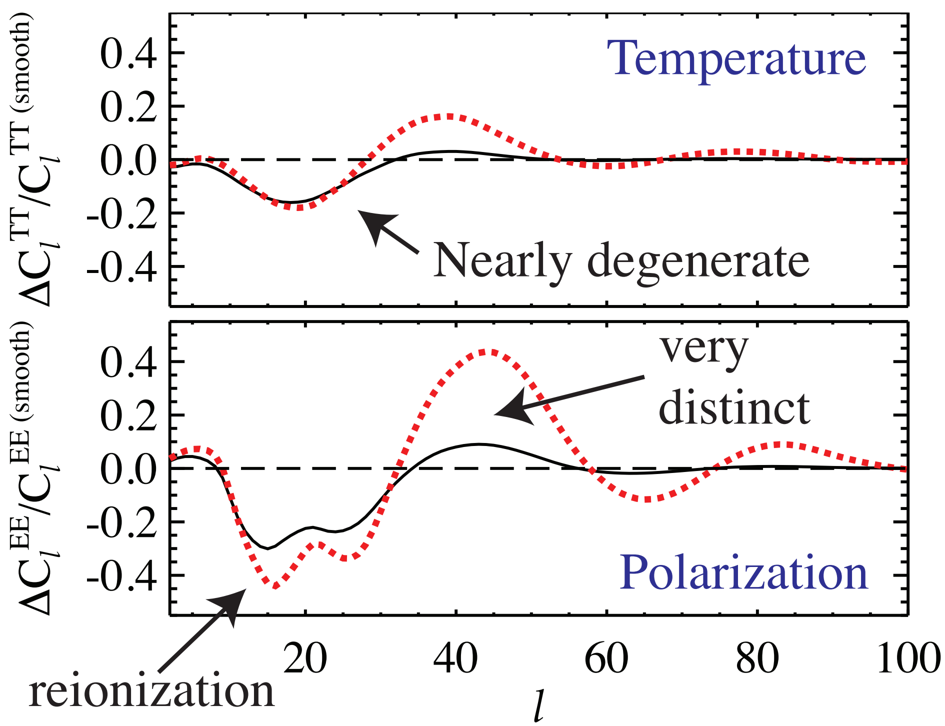


# Polarization Features

- For step models that fit the glitch, sharper matching polarization features that can verify/falsify them Mortonson, Dvorkin, Peiris, Hu 2009
- Requires high signal-to-noise EE measurements at $\ell = 20 - 60$
- Can be separated from high-$z$ reionization Obied et al 2018



# GSR EFT Bispectrum

- Expand EFT Lagrangian to cubic order: $\mathcal{L}_3$
- Cubic operators represent interactions with interaction Hamiltonian $H_I = -\int d^3x \mathcal{L}_3$
- 10 distinct cubic EFT operators Passaglia & Hu 2018
- Calculate bispectrum in the interaction picture using in-in formalism

$$\langle \hat{\mathcal{R}}_{\mathbf{k_1}}(t_*)\hat{\mathcal{R}}_{\mathbf{k_2}}(t_*)\hat{\mathcal{R}}_{\mathbf{k_3}}(t_*)\rangle = 2\Re\left[-i\int_{-\infty}^{t_*} dt \langle \hat{\mathcal{R}}_{\mathbf{k_1}}(t_*)\hat{\mathcal{R}}_{\mathbf{k_2}}(t_*)\hat{\mathcal{R}}_{\mathbf{k_3}}(t_*) H_I(t)\rangle\right]$$

- Iteratively expand mode functions to define GSR integral expressions for the bispectrum Adshead, Hu, Miranda 2012

# GSR EFT Bispectrum

- Leading order GSR efficiently computes all triangle configurations of the bispectrum in the form

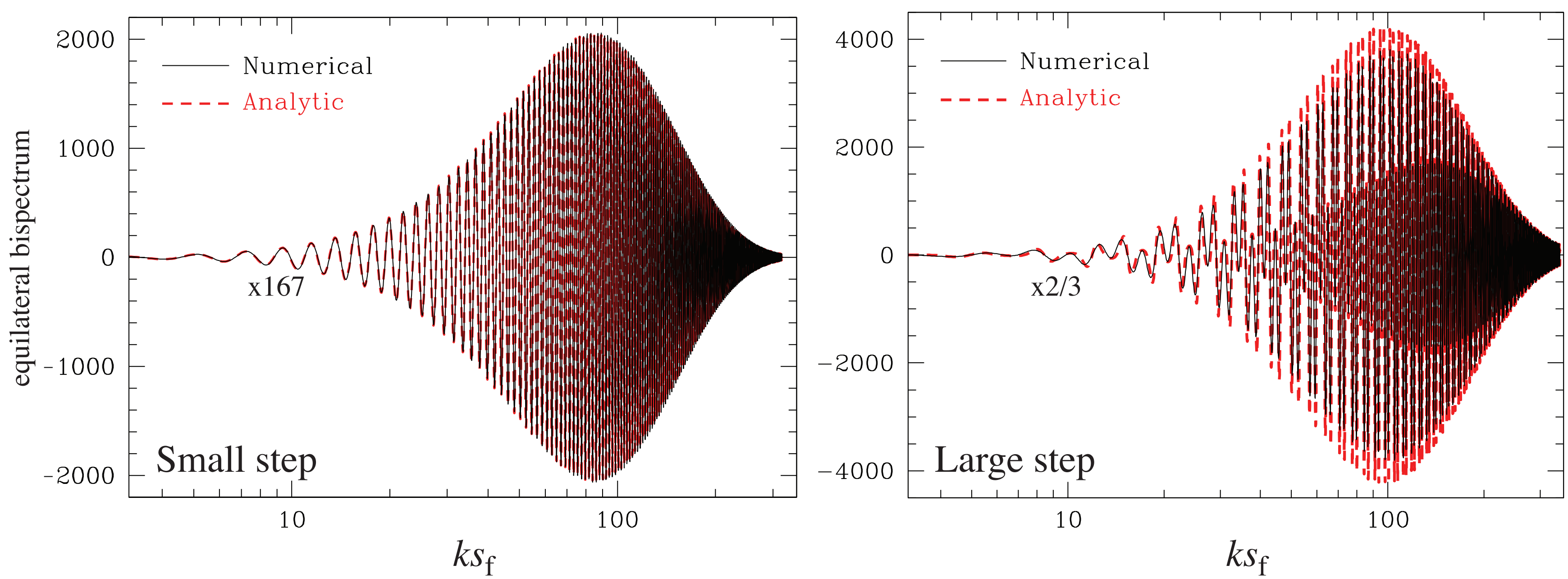
$$B_{\mathcal{R}}(k_1, k_2, k_3) = \Delta_{\mathcal{R}}(k_1)\Delta_{\mathcal{R}}(k_2)\Delta_{\mathcal{R}}(k_3) \sum_i T_i(k_1, k_2, k_3) \times \int \frac{ds}{s} S_i(\ln s) W_i'(Ks)$$

  $K = k_1 + k_2 + k_3$ is the perimeter of the triangle

- Elements
  - $\Delta_{\mathcal{R}}$: $\mathcal{R}$-modefunction or square-root of power spectrum
  - $T_i$: source independent configuration shape
  - $S_i$: source from the EFT cubic interactions
  - $W_i$: freezeout window for the source

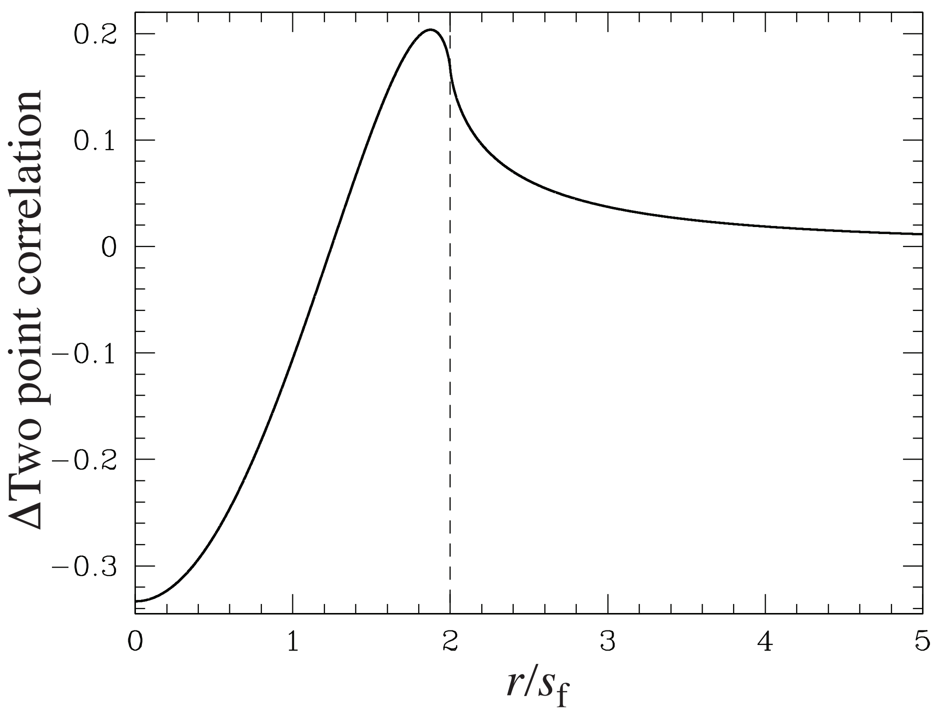
# Sharp Step Bispectrum

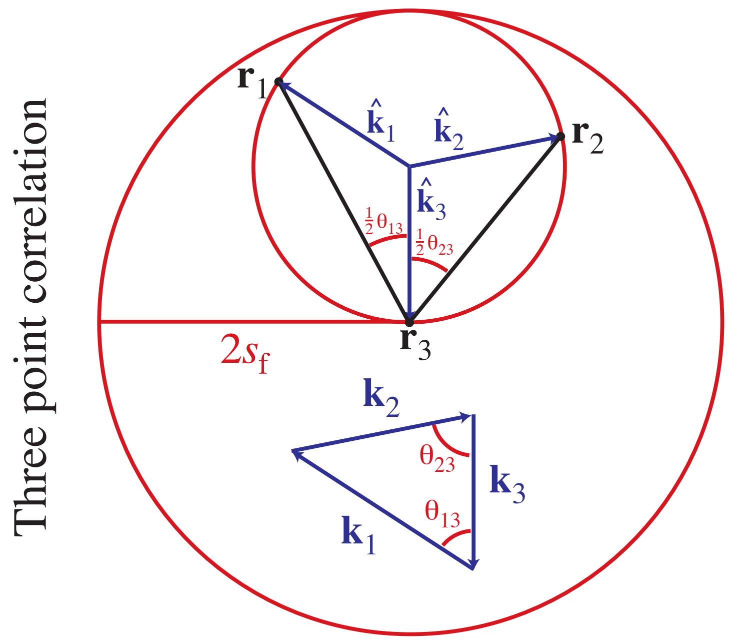
- Sharp step leads to rising equilateral bispectrum until finite-width damping scale Adshead, Dvorkin, Hu, Lim 2011
- Eventually so large that excitations are strongly coupled, beyond EFT Adshead & Hu 2014
- Oscillatory shape requires new template forms – GSR provides accurate analytic expressions



# Sharp Correlation Function Features

- Sharp step: ringing at high $k$ represents sharp feature in physical space at much larger scales Adshead, Dvorkin, Hu, Lim 2011
- Sharpness blurred out by temporal width of feature
- CMB anomalies at large angular scales may have subtle signatures at high multipole



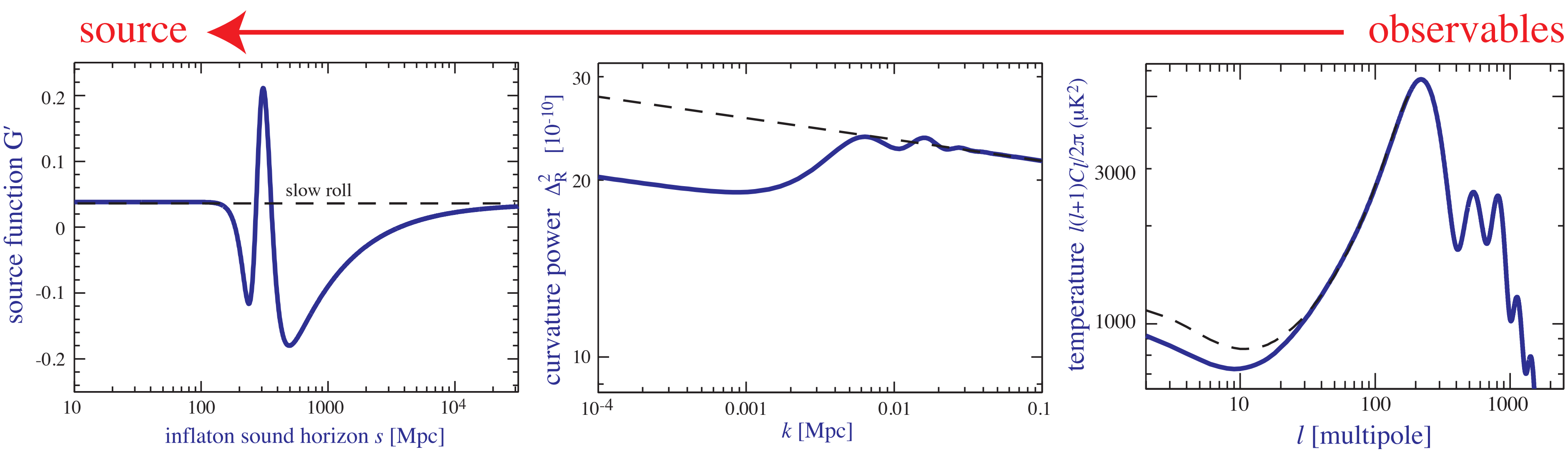
# Reconstructing EFT

# Power Spectrum Features

- Step and monodromy examples highlight that features in power spectrum are highly constrained by inflationary origin
- Sharp features in $k$-space must be accompanied by ringing
- Ringing must appear stronger in CMB polarization than temperature from recombination
- Matching, but lower signal to noise, features in the temperature and polarization bispectra
- Features can easily fit random noise in any one of these measurements but not all
- Inverse problem of reconstructing the EFT source function(s) $[G', \ldots]$

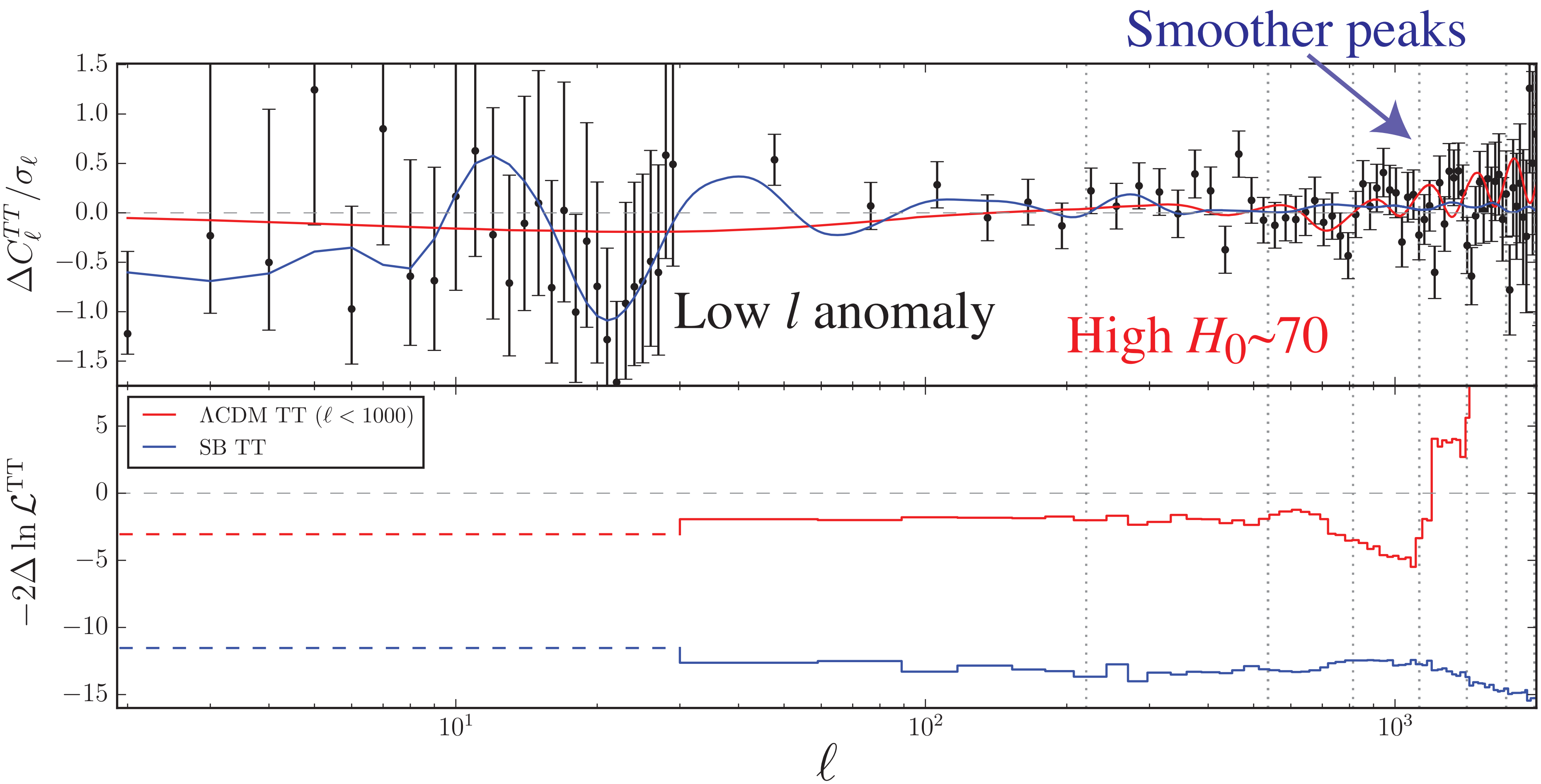
# Reconstruction

- By going directly from observables to the inflationary source we guarantee that inferred features are consistent with single field inflation
- If we reconstruct $\Delta^2_{\mathcal{R}}$ first instead, a sharp $k$-space feature with no ringing pattern would be inconsistent with single field EFT



# Low Multipole Anomaly

- Using 10 parameters per decade for $G'(\ln s)$ from $200 \leq s/\mathrm{Mpc} \leq 20000$ to fit out low multipole residuals from a pure tilt Dvorkin & Hu 2011, Obied et al 2017,2018

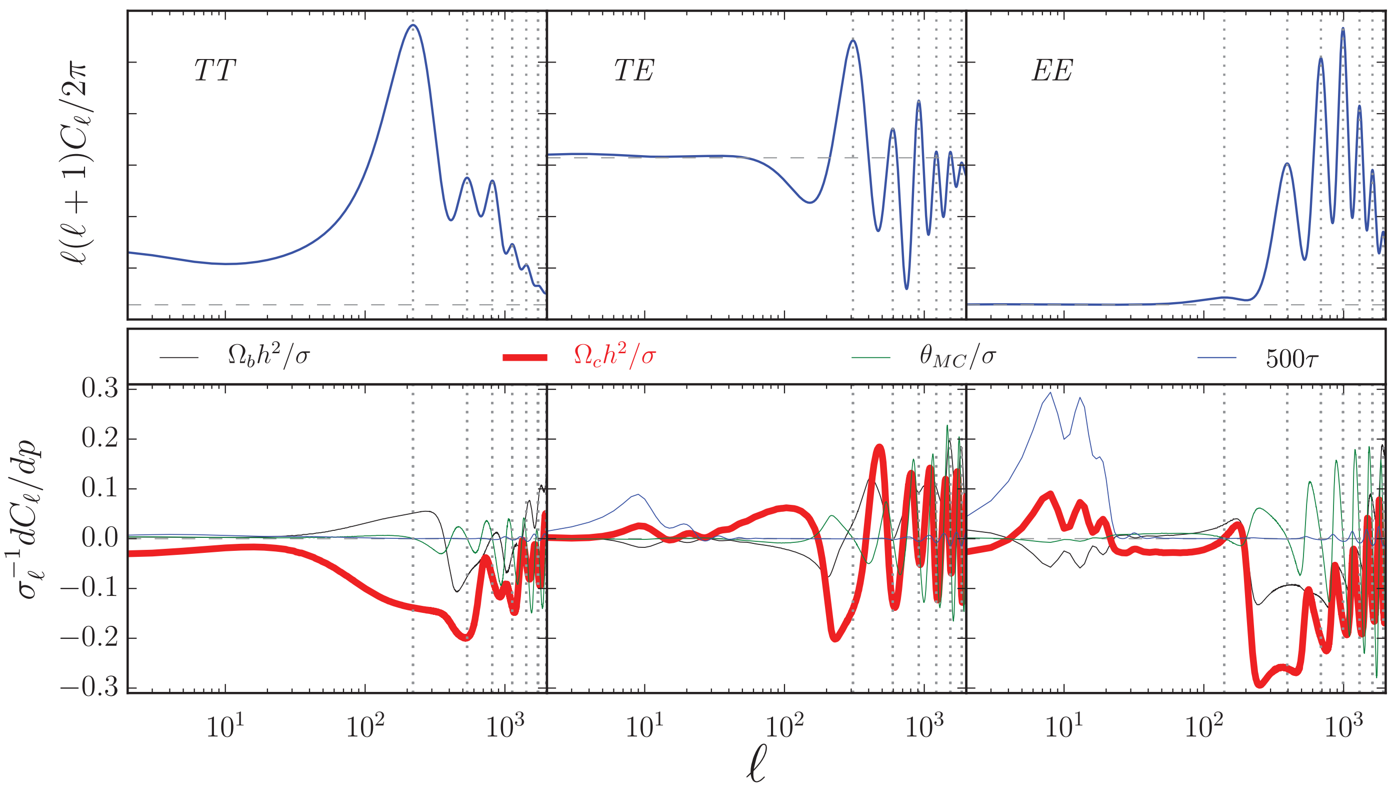


# Low Multipole Anomaly

- Reconstruction parameters also allow one to marginalize impact of non-power law power spectra on cosmological parameters
- Low multipole anomaly influences $H_0$ - with only $\ell < 1000$ data, $H_0$ shifted higher to fit anomaly Addison et al 2015; Aghanim et al 2016
- After marginalizing $G'$ source parameters, even $\ell < 1000$ WMAP data compatible with low $H_0$
- Planck data at $\ell > 1000$ have smoother temperature peaks than allowed by high $H_0$

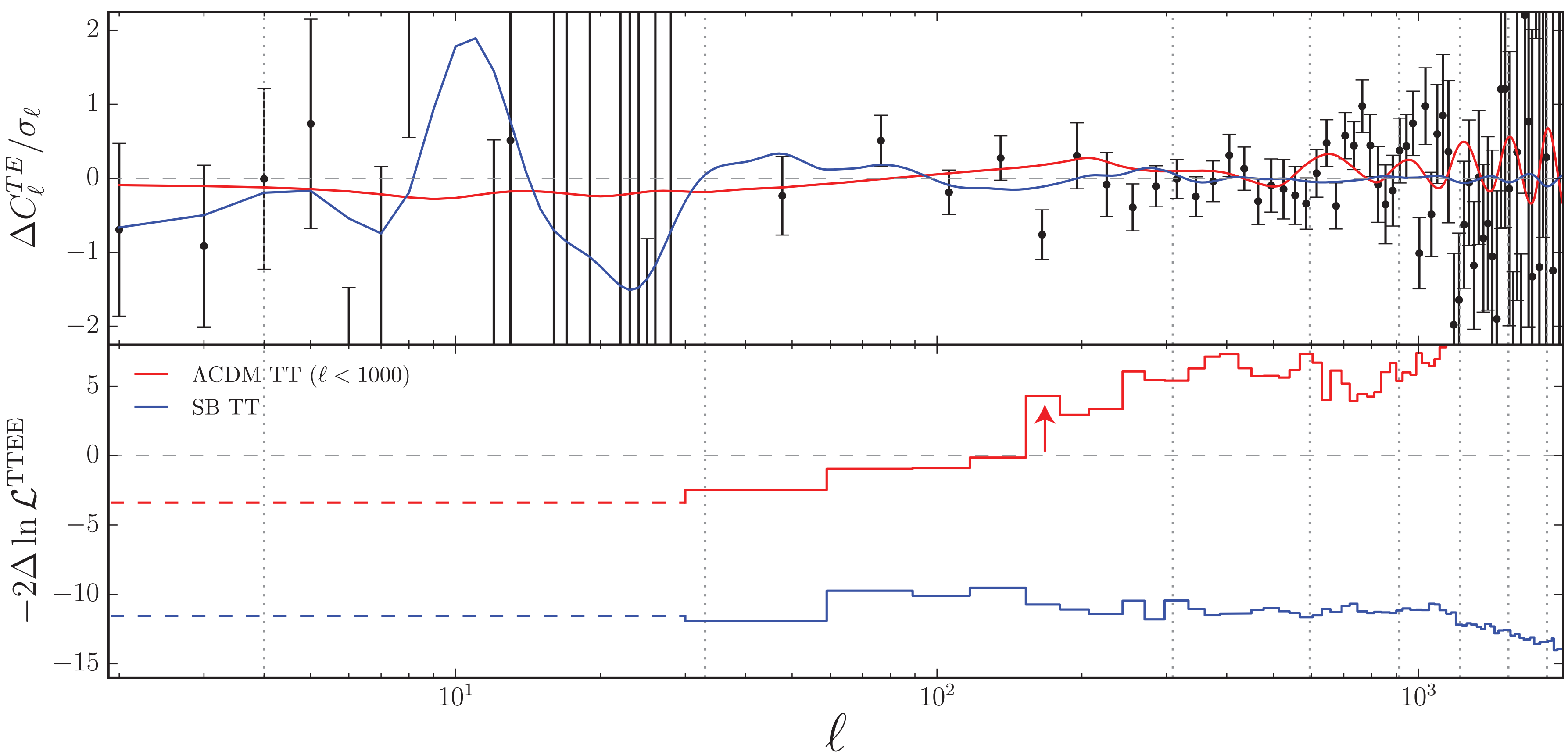
# $H_0$ Tension

- Shift to lower $H_0$ indicates more CDM relative to radiation from driving effect of potential decay
- Increased angular scale of sound horizon compensated by larger distance to recombination through lower $H_0$
- $> 3\sigma$ tension with direct $H_0$ distance ladder measurements



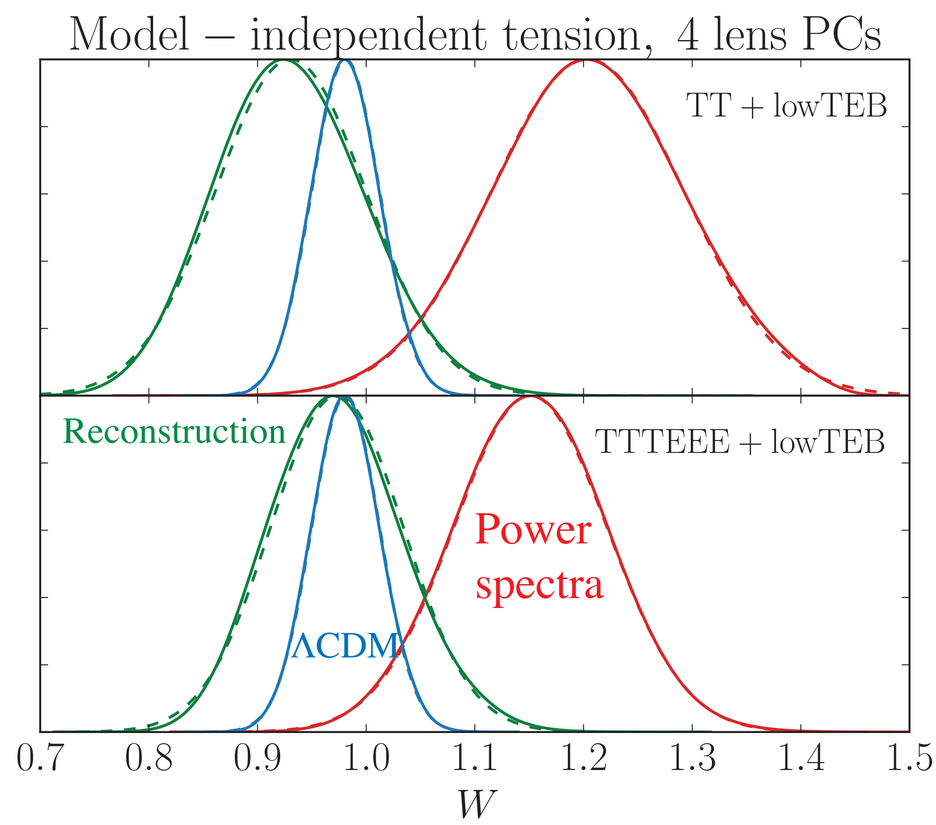
# Polarization

- Polarization $EE$ and $TE$ should provide matching inflationary features and also distinct signatures of low $H_0$ solution
- Planck 2015 $TE$ spectrum anomalously sensitive to $H_0$ due to a single deviant multipole band



# Lensing Anomaly

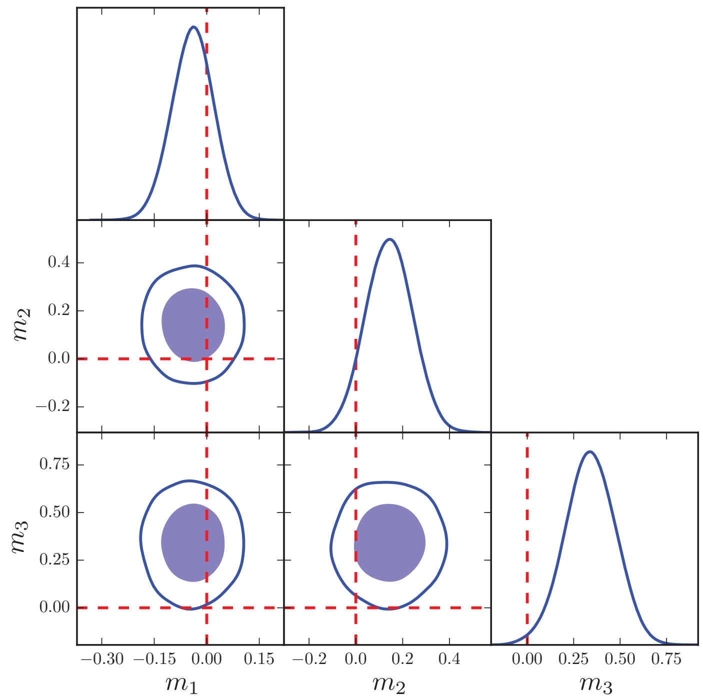
- Planck TT power specta want even smoother peaks than low $H_0$ achieves leaving remaining oscillatory residuals
- If lensing amplitude is allowed to vary, then residuals can be better fit since lensing smooths peaks

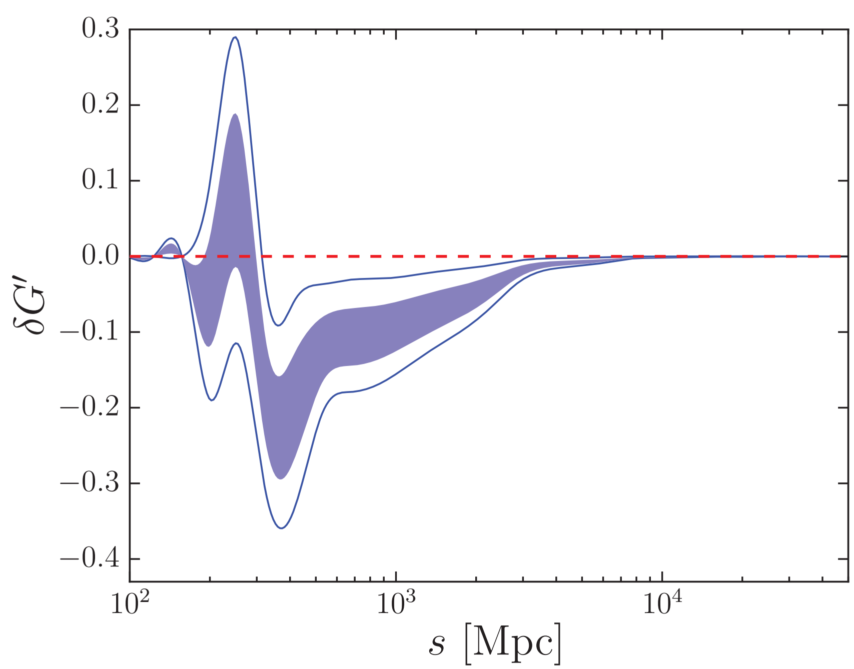



- Lens reconstruction from quadratic estimators do not show higher lensing
- Using principal components, this tension is independent of cosmological model at low-$z$ (dark energy, etc) Motloch & Hu 2018

# EFT Source Reconstruction

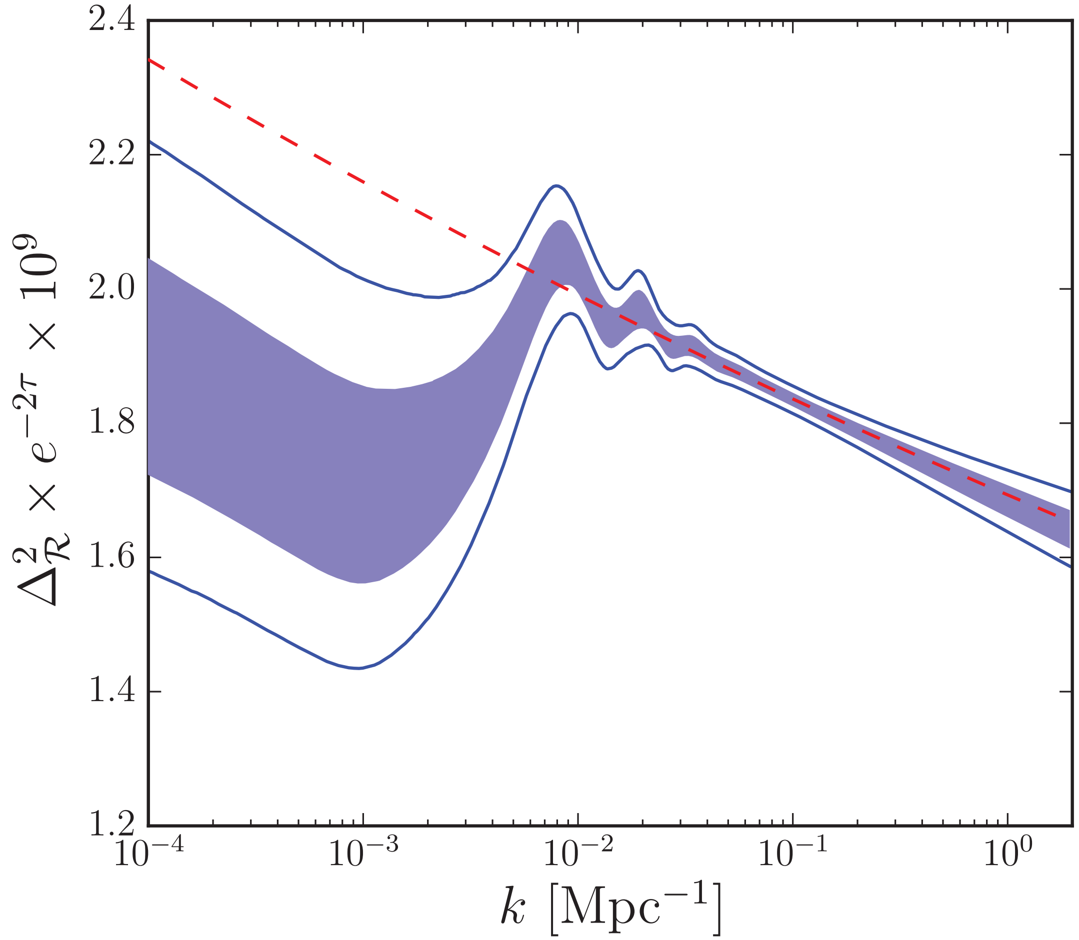
- In source function space, 20 parameters currently mainly fit noise
  - – good for marginalizing impact on cosmological parameters
  - – bad for trying to interpret implications for inflation
- Filter out noise by constructing principal components, rank ordered to best constrained modes, of $G'$ parameter covariance matrix
- 3 PCs constrained with 95% local CL deviations in $m_1 - m_3$



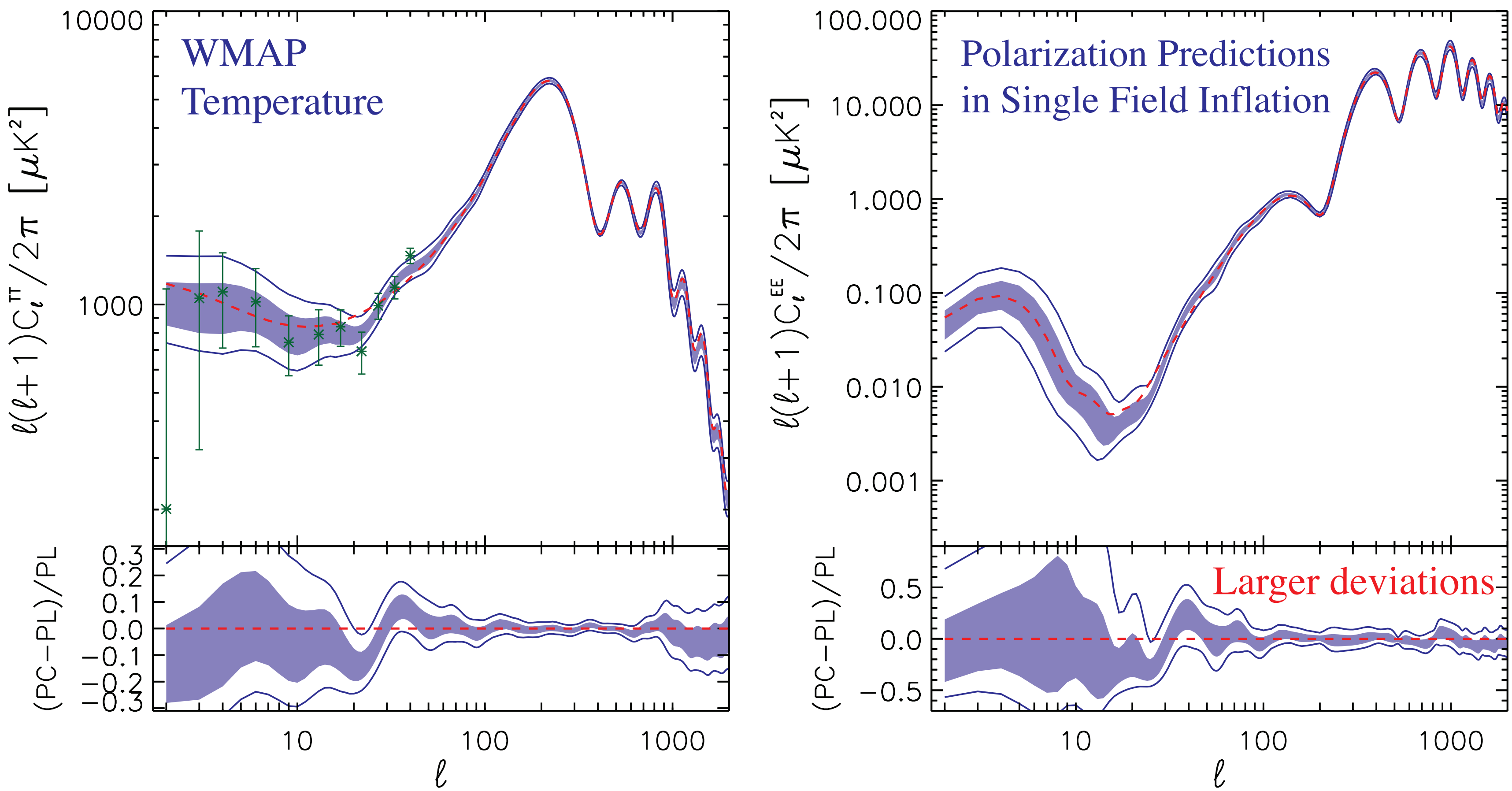
# Suppression of Large Scale Power

- Inflationary source corresponds to sharp suppression of large scale power
- Predicts $EE$ polarization feature sharper and at slightly higher multipole
- More generally, reconstruction from $TT$ makes predictions for polarization
- Testing polarization predictions immune to look elsewhere effect...



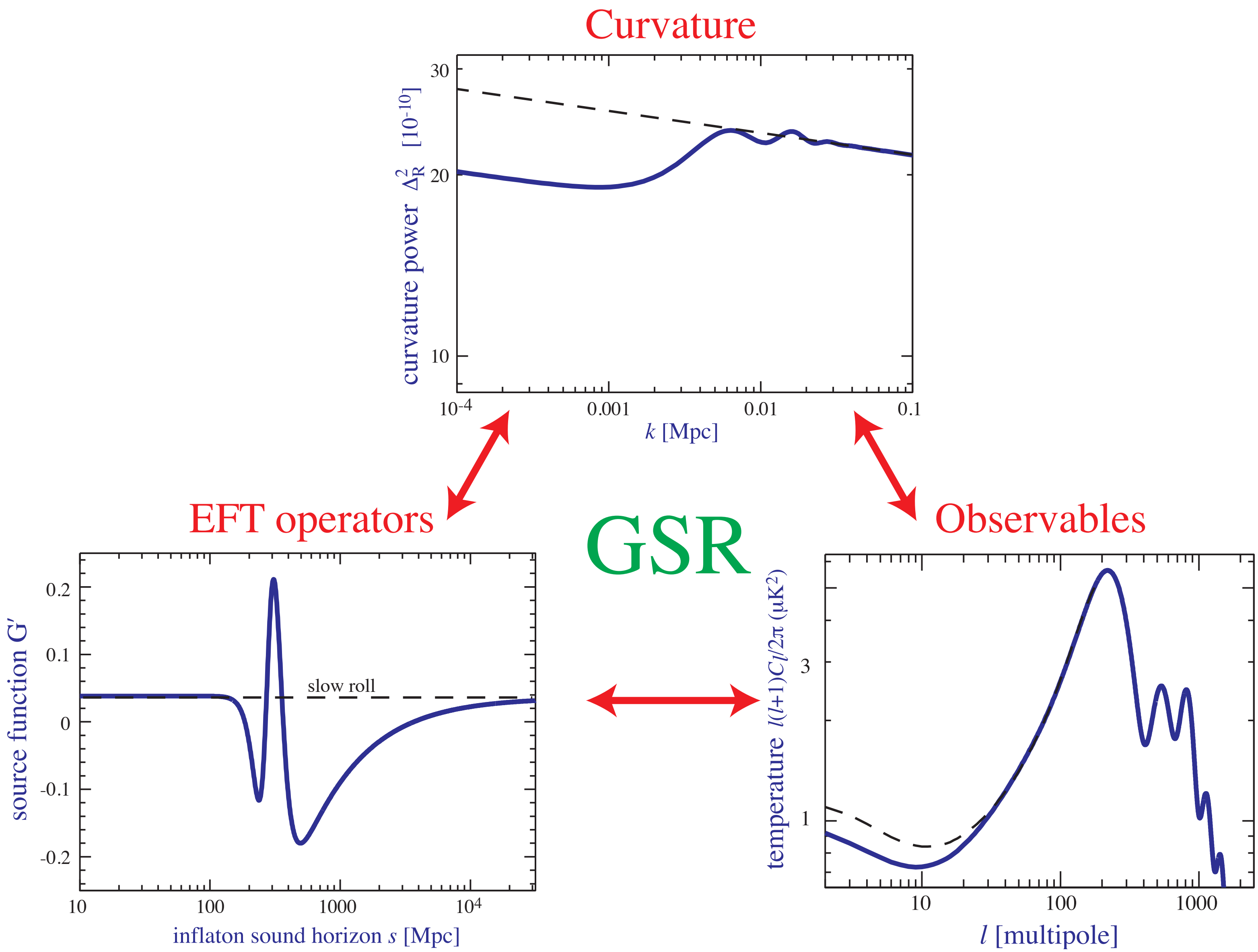
# Polarization Consistency

- Polarization predictions under single field inflation from WMAP temperature power spectrum
  - Test origin of temperature features
  - Violations could even falsify single field EFT inflation itself



# Operators to Observables

- From operators to observables and back



# Summary of Lecture II

- GSR allows for temporal features during (rather than purely associated with the end) of inflation
- New scale $\Delta N$ in efolds breaks the ordinary slow roll hierarchy assumption that higher parameters are suppressed by increasing powers of $N \sim 60$
- If $\Delta N > 1$, GSR is solved by a generalization of slow roll hierarchy
  - Taylor expansion of $G$ source of exitations
  - Optimized evaluation point to zero out next term in series
  - Consistent predictions between observables $n_s - 1, \alpha, \ldots$
  - Necessary when $\Delta N \ll N$, $\alpha$ comparable to $n_s - 1$

# Summary of Lecture II

- If $\Delta N < 1$, GSR predicts ringing of power spectra in a form that must be specialized to individual cases
  - Monodromy
  - Steps

  which usually leads to analytic templates for $\Delta N \ll 1$, enabling fast MCMC searches
- Allowing $\Delta N < 1$ leads to noise in CMB TT being fit by inflationary features but also predicts consistency relations with
  - Polarization (and tensors)
  - Bispectrum

# Summary of Lecture II

- Reconstructing the EFT of inflation source $G$ (or $G'$) directly from observations
  - – Enforces inflationary prediction: sharp features $\rightarrow$ oscillations (cf. power spectrum reconstruction)
  - – Marginalize inflationary assumptions for cosmo params ($H_0$)
  - – Highlights low-$\ell$ power anomaly as locally significant
  - – Testable predictions for polarization
  - – Ultimately test validity of whole single field paradigm