Shear Power of Weak Lensing

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Why Power Spectrum?

Pros:

• Direct observable: shear-based
• Statistical properties simple on large-scales (cf. variance, higher order)
• Complete statistical information on large-scales
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Cons:

- Relatively featureless
- Non-linear below degree scale
- Degeneracies: initial spectrum, transfer function shape, density growth, and angular diameter distance + source redshift distribution
- Computationally expensive: extraction by likelihood analysis $N_{\text{pix}}^3$
Statistics and Modelling
Shear Power Modes

- Alignment of shear and wavevector defines modes
- Lensing weighted **Limber projection of density power spectrum**
- \( \varepsilon \)-shear power = \( \kappa \)-power

![Graph showing power spectrum](image)

- Linear
- Non-linear
- Limber approximation

Kaiser (1992)
Jain & Seljak (1997)
Hu (2000)
Statistics & Simulations

- Measurement of power in each multipole is independent if the field is Gaussian
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- Need many simulations to test statistical properties of the field in particular: sample variance

White & Hu (1999)
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- Non-linearities make the power spectrum non-Gaussian.
- Need many simulations to test statistical properties of the field, in particular: sample variance.
- PM simulations ideal if sufficient angular resolution can be achieved.
PM Simulations

- Hundreds of independent simulations

Convergence

Shear

$6^\circ \times 6^\circ$ FOV; 2' Res.; 245–75 $h^{-1}\text{Mpc}$ box; 480–145 $h^{-1}\text{kpc}$ mesh; 2–70 $10^9 \text{M}_\odot$

White & Hu (1999)
Mean & Sampling Errors

- Mean agrees well with PD96 + Limber (Jain, Seljak & White 1999)
- Sampling errors per 6° x 6° field:

![Graph showing power ratio and shot noise](image-url)
• Sampling errors ~Gaussian at $l<1000$
• Non-Gaussianity increases sampling errors on binned power spectrum
• At current survey depths, shot noise dominates in non-Gaussian regime

White & Hu (1999)
Mean & Sampling Errors

- Correlation in Band Powers:

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Halo Model

- Model density field as (linearly) clustered NFW halos of PS abundance:

Simulation

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White & Hu (1999)
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Power Spectrum Statistics in Halo Model

- Power spectrum as a function of largest halo mass included
- Non-linear regime dominated by halo profile / individual halos increased power spectrum variance and covariance

Cooray, Hu, Miralda-Escude (2000)
Halo Model vs. Simulations

- Halo model for the trispectrum: power spectrum correlation

Cooray & Hu (2001)
Halo Model vs. Simulations

- Halo model for the bispectrum: $S_3$ dominated by massive halos

Cooray & Hu (2001)

Cooray & Hu (2001)
Halo Model vs. Simulations

- Explains large sampling errors:

White & Hu (1999)

\[ \sigma (\text{arcmin}) \]

\[ S^3 \]

\[ 6^\circ \times 6^\circ \]
Power Spectrum Estimation
Likelihood Analysis

Pros:

• **Optimal** power spectrum estimator for Gaussian field
• Automatically accounts for **irregular (sparse sampled)** geometries & varying sampling densities
• **Arbitrary noise correlations**
• Marginalize over **systematic error templates**
• (ε, β, cross) checks for non-gravitational effects
• **Rigorous error analysis** in linear regime
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Test iterated **quadratic estimator** of maximum likelihood **solution** and **error matrix**
Testing the Likelihood Method

- Input: pixelized shear data $\gamma_1(n_i), \gamma_2(n_i)$; pixel-pixel noise correlation
- Iterate in band power parameter space to maximum likelihood...
- Output: maximum likelihood band powers and local curvature for error estimate including covariance

Hu & White (2001)
Testing the Likelihood Method

- Sparse sampling test
- Mean and errors correctly recovered

\[ \frac{l(l+1)C_l^\Sigma}{2\pi} \]

Hu & White (2001)
Testing the Likelihood Method

- Systematic error monitoring: example underestimated shot noise
- $\beta$-channel appearance in power
- Systematic error template: jointly estimate or marginalize

Hu & White (2001)
Cosmological Parameter Forecasts
Degeneracies

- All parameters of ICs, transfer function, growth, angular diam. distance
- Power spectrum lacks strong features: degeneracies

\[ \frac{l(l+1)C_{\ell}^{\text{ee}}}{2\pi} \]

- Linear
- Non-linear
- Limber approximation

\[ z_s = 1 \]

\[ k = 0.02 \, h/\text{Mpc} \]
Degeneracies

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**CMB**: provides high redshift side
IC's, transfer fn., ang. diameter to $z=1000$

**Lensing**: dark energy, dark matter
Hu & Tegmark (1999)

**Redshifts**: source (and lens)
Breaks degeneracies by tomography
Hu (1999)
Tomography

- Divide sample by photometric redshifts
- Cross correlate samples

- Order of magnitude increase in precision even after CMB breaks degeneracies

Hu (1999)
Error Improvement: 25deg²

Cosmological Parameters

MAP +
3-z
no-z

Hu (1999; 2001)
Error Improvement: 1000deg$^2$

Cosmological Parameters

MAP +

Hu (1999; 2001)
Both CMB and tomography help lensing provide interesting constraints on dark energy.

\begin{itemize}
  \item MAP no–z
  \item 3–z
\end{itemize}

\textit{Hu (2001)}
Direct Detection of Dark Energy?

- In the presence of dark energy, shear is correlated with CMB temperature via ISW effect.

\[ \frac{(l+1)C_l}{2\pi} \leq 1 \quad \text{for} \quad z < 1 \]

\[ 1 < z < 1.5 \]

\[ z > 1.5 \]

Hu (2001)
CMB Lensing: Tomography Anchor

- CMB acoustic waves are the ultimate high redshift source!

Hu (2001)
Mass Reconstruction

- CMB acoustic waves are the ultimate high redshift source!

original
mass (deflection) map

reconstructed
1.5' beam; 27μK-arcmin noise
Power Spectra

- Power spectrum of deflection and cross correlation with CMB ISW

- Extend tomography to larger scales and higher redshift
- Constrain clustering properties of dark energy through ISW correlation

Hu (2001)
**Lens Redshifts**

- Shear selected halos: *clustered* according to well-understood halo bias

Shear

Halo Identification

Mo & White (1996)  
e.g. aperture mass: Schneider (1996)
Lens Redshifts

- Shifting of feature(s) in the angular power spectrum: $d_A(z)$

$C_l / 2\pi$

$z = 0.3 - 0.4$

$z = 1.0 - 1.2$

$4000 \text{deg}^2, M > 10^{14}$

Cooray, Hu, Huterer, Joffre (2001)
• If baryon features not detected, 10% distances to $z \approx 0.7$ possible

Cooray, Hu, Huterer, Joffre (2001)
Lens Redshifts

• If baryon features are detected, ultimate standard ruler!

\[ l(l+1)C_l^{hh}/2\pi \]

\begin{align*}
z = 0.3 &– 0.4 \\
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\end{align*}
Summary

• N-body sims and halo model for understanding power spectrum stats. and higher order correlations

• Likelihood analysis is feasible and near optimal for current generation of surveys
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- *Likelihood analysis* is feasible and near optimal for current generation of surveys
  
  properly accounts for *sparse* or *uneven sampling* and *correlated noise*

  built in monitors for *systematic errors*

  techniques for *marginalizing* known systematics w. templates
Summary

• **N-body sims and halo model** for understanding **power spectrum stats.**
  and higher order correlations

• **Likelihood analysis** is feasible and near optimal for current generation
  of surveys
  
  properly accounts for **sparse or uneven sampling** and
  correlated noise

  built in monitors for **systematic errors**
  
  techniques for **marginalizing** known systematics w. templates

• Power spectrum **complements CMB** information –
  order of magnitude increase in precision

• **Tomography** (including CMB lensing) assists in identifying
  dark energy (including possible clustering)

• **Lens redshifts** yield halo number counts and halo power spectra