Cosmic Acceleration

Dark Energy
v.
Modified Gravity

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Outline

• Dark Energy vs Modified Gravity

• Three Regimes of Modified Gravity

• Worked (Toy) Models: $f(R)$ and DGP Braneworld

• Collaborators

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Charting Out the Expansion

- **Standard candle:** apparent brightness of objects with a fixed luminosity to judge distance
- **Standard ruler:** apparent (angular) separation of objects with a fixed physical separation to judge distance

Sound waves
CMB+Galaxies
Supernovae
1998 Discovery

Supernovae
1998 Discovery

Sound waves
CMB+Galaxies
Mercury or Pluto?

- General relativity says \textit{Gravity} = \textit{Geometry}

- And \textit{Geometry} = \textit{Matter-Energy}

- Could the \textit{missing energy} required by \textit{acceleration} be an \textit{incomplete} description of how \textit{matter} determines \textit{geometry}?
Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content.
- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy.

\[
F(g_{\mu\nu}) + G_{\mu\nu} = 8\pi G T^M_{\mu\nu} - F(g_{\mu\nu}) = 8\pi G T^{DE}_{\mu\nu}
\]

\[
G_{\mu\nu} = 8\pi G [T^M_{\mu\nu} + T^{DE}_{\mu\nu}]
\]

and the Bianchi identity guarantees \( \nabla^\mu T^{DE}_{\mu\nu} = 0 \).

- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor.
- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress.
Modified Gravity ≠ “Smooth DE”

- **Scalar field** dark energy has $\delta p = \delta \rho$ (in constant field gauge) – relativistic sound speed, no anisotropic stress

- **Jeans stability** implies that its energy density is spatially smooth compared with the matter below the sound horizon

  \[
  ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2
  \]

  \[
  \nabla^2(\Phi - \Psi) \propto \text{matter density fluctuation}
  \]

- **Anisotropic stress** changes the amount of space curvature per unit dynamical mass

  \[
  \nabla^2(\Phi + \Psi) \propto \text{anisotropic stress}
  \]

  but its absence in a smooth dark energy model makes

  \[
  g = (\Phi + \Psi)/(\Phi - \Psi) = 0 \text{ for non-relativistic matter}
  \]
Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate.
- Hence geometric measurements of the expansion rate predict the growth of structure:
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations
- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm:
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)
Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way
- Deviation significantly >2% rules out $\Lambda$ with or without curvature
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in $H_0$]
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)

Cosmological Constant

Quintessence
Quintessence Falsified?

- No excess numbers of massive $z>1$ X-ray or SZ clusters with Gaussian initial conditions (Jee et al. 2009, Brodwin et al. 2010)
- No excess power in gravitational lensing at high $z$ relative to low $z$ (Bean 0909.3853)
- But would such violations favor modified gravity?
- Given astrophysical systematics, expect purported $2\sigma$ violations of smooth dark energy predictions will be common in coming years!
Dynamical vs Lensing Mass

- Newtonian potential: $\Psi = \delta g_{00} / 2g_{00}$ which non-relativistic particles feel

- Space curvature: $\Phi = \delta g_{ii} / 2g_{ii}$ which also deflects photons

- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass
Dynamical v Strong Lensing

- Comparison of strong lensing and dynamical mass assuming a density profile and velocity dispersion data
- Mean exhibits a bias from GR expectation with statistical errors only
- No mass trend detectable

T. Smith (2009)
Lensing v Dynamical Comparison

- Gravitational lensing around galaxies vs. linear velocity field (through redshift space distortions and galaxy autocorrelation)
- Consistent with GR + smooth dark energy beginning to test interesting models

**Reyes et al (2010); Lombriser et al (2010)**
Falsify in Favor of What?

- Modified gravity models change space curvature per unit dynamical mass - enhanced or reduced forces on matter
- Requires two closure relations - 1st an an effective anisotropic stress that distinguishes lensing from dynamical mass
- Viable induced modifications exhibit three separate regimes
  - Horizon Scale
  - Scalar-Tensor
  - General Relativistic
- Choice of lensing mass contribution as 2nd parameter in scalar-tensor regime favored by conformal invariance of E&M (Hu & Sawicki 2007; see also Caldwell et al 2007; Amendola et al 2007)

CAMB Package for Linearized PPF: [http://camb.info/ppf](http://camb.info/ppf)

Other uses: phantom crossing dark energy (Fang, Hu, Lewis 2009), dark energy PCs (Mortonson, Hu, Huterer 2009) cascading gravity (Afshordi, Geshnizjani, Khoury 2008)
Three Regimes

- Three regimes with different dynamics
- Examples $f(R)$ and DGP braneworld acceleration
- Parameterized Post-Friedmann description
- Non-linear regime return to General Relativity / Newtonian dynamics

General Relativistic Non-Linear Regime

Scalar-Tensor Regime

Conserved-Curvature Regime

halos, galaxy

large scale structure

CMB
Worked Examples
## Modified Action $f(R)$ Model

- **$R$**: Ricci scalar or “curvature”
- **$f(R)$**: modified action (Starobinsky 1980; Carroll et al 2004)

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]
\]

- **$f_R \equiv df/dR$**: additional propagating scalar degree of freedom (metric variation)
- **$f_{RR} \equiv d^2f/dR^2$**: Compton wavelength of $f_R$ squared, inverse mass squared
- **$B$**: Compton wavelength of $f_R$ squared in units of the Hubble length

\[
B \equiv \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}
\]

- $' \equiv d/d\ln a$: scale factor as time coordinate
Modified Einstein Equation

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved

\[
G_{\alpha\beta} + f_R R_{\alpha\beta} - \left( \frac{f}{2} - \Box f_R \right) g_{\alpha\beta} - \nabla_\alpha \nabla_\beta f_R = 8\pi G T_{\alpha\beta}
\]

- Trace can be interpreted as a scalar field equation for \( f_R \) with a density-dependent effective potential (\( p = 0 \))

\[
3\Box f_R + f_R R - 2f = R - 8\pi G \rho
\]

- For small deviations, \(|f_R| \ll 1\) and \(|f/R| \ll 1\),

\[
\Box f_R \approx \frac{1}{3} (R - 8\pi G \rho)
\]

the field is sourced by the deviation from GR relation between curvature and density and has a mass

\[
m_{f_R}^2 \approx \frac{1}{3} \frac{\partial R}{\partial f_R} = \frac{1}{3 f_{RR}}
\]
DGP Braneworld Acceleration

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

\[ S = \int d^5 x \sqrt{-g} \left[ \frac{(5) R}{2\kappa^2} + \delta(\chi) \left( \frac{(4) R}{2\mu^2} + \mathcal{L}_m \right) \right] \]

with crossover scale \( r_c = \kappa^2 / 2\mu^2 \)

- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)

- Matter still minimally coupled and conserved

- Exhibits the 3 regimes of modified gravity
  
  - Weyl tensor anisotropy dominated conserved curvature regime \( r > r_c \) (Sawicki, Song, Hu 2006; Cardoso et al 2007)
  
  - Brane bending scalar tensor regime \( r_\ast < r < r_c \) (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)

  - Strong coupling General Relativistic regime \( r < r_\ast = (r_c^2 r_g)^{1/3} \)

  where \( r_g = 2GM \) (Dvali 2006)
DGP Field Equations

- DGP field equations

\[ G_{\mu\nu} = 4r_c^2 f_{\mu\nu} - E_{\mu\nu} \]

where \( f_{\mu\nu} \) is a tensor quadratic in the 4-dimensional Einstein and energy-momentum tensors

\[ f_{\mu\nu} \equiv \frac{1}{12} AA_{\mu\nu} - \frac{1}{4} A_\alpha^\mu A_\nu^\alpha + \frac{1}{8} g_{\mu\nu} \left( A_\alpha_\beta A^{\alpha\beta} - \frac{A^2}{3} \right) \]

\[ A_{\mu\nu} \equiv G_{\mu\nu} - \mu^2 T_{\mu\nu} \]

and \( E_{\mu\nu} \) is the bulk Weyl tensor

- Background metric yields the modified Friedmann equation

\[ H^2 \pm \frac{H}{r_c} = \frac{\mu^2 \rho}{3} \]

- For perturbations, involves solving metric perturbations in the bulk through the “master equation”
• Calculation of the metric ratio $g = \Phi + \Psi / \Phi - \Psi$ requires solving for the propagation of metric fluctuations into the bulk
• Encapsulated in the off brane gradient which closes the system (e.g. normal branch $g = -1/(2Hr_C + 1)$ until deep in de Sitter)

$f(R)$ Expansion History
Engineering $f(R)$ Models

- Mimic $\Lambda$CDM at high redshift
- **Accelerate** the expansion at low redshift without a cosmological constant
- Sufficient **freedom** to vary expansion history within observationally allowed range
- **Contain** the phenomenology of $\Lambda$CDM in both cosmology and solar system tests as a **limiting case** for the purposes of constraining small deviations
- **Suggests**

$$f(R) \propto \frac{R^n}{R^n + \text{const.}}$$

such that modifications vanish as $R \to 0$ and go to a **constant** as $R \to \infty$
Form of $f(R)$ Models

- Transition from zero to constant across an adjustable curvature scale
- Slope $n$ controls the rapidity of transition, field amplitude $f_{R0}$ position
- Background curvature stops declining during acceleration epoch and thereafter behaves like cosmological constant

Hu & Sawicki (2007)
Expansion History

- Effective equation of state $w_{\text{eff}}$ scales with field amplitude $f_{R0}$
- Crosses the phantom divide at a redshift that decreases with $n$
- Signature of degrees of freedom in dark energy beyond standard kinetic and potential energy of k-essence or quintessence or modified gravity

Hu & Sawicki (2007)
DGP Expansion History
DGP Expansion History

- Matching the **DGP expansion history** to a **dark energy model with the same expansion history**
- **Effective** equation of state $w(z) \approx -0.85, \, w_a \approx 0.35$

Song, Sawicki & Hu (2006)
DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$

Song, Sawicki & Hu (2006)
DGP Normal Branch

• On the normal branch, expansion does not self-accelerate and dark energy in the form of a brane tension or scalar field necessary

\[ H^2 + \frac{H}{r_c} = \frac{\mu^2}{3} (\rho_m + \rho_{\text{DE}}) \]

• Gravity is still modified as in the self-accelerated branch (but with attractive forces)

• Ghost free in the quantum theory

• Can choose \( \rho_{\text{DE}} \) to match any desired expansion history including flat \( \Lambda \)CDM

\[ H^2 \equiv \frac{\mu^2}{3} (\rho_m + \rho_\Lambda) \rightarrow \rho_{\text{DE}} \]

• Separate out geometrical and dynamical tests of acceleration
Conserved Curvature Regime
Curvature Conservation

- On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)

- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d \ln a$ (Bardeen 1980)

- Gauge transformation to Newtonian gauge

\[
ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2
\]

yields (Hu & Eisenstein 1999)

\[
\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = 0
\]

- Modified gravity theory supplies the closure relationship $\Phi = -\gamma(\ln a)\Psi$ between and expansion history $H = \dot{a}/a$ supplies rest.
Linear Theory for $f(R)$

- In $f(R)$ model, “superhorizon” behavior persists until Compton wavelength smaller than fluctuation wavelength $B^{1/2}(k/aH) < 1$.
- Once Compton wavelength becomes larger than fluctuation, perturbations are in scalar-tensor regime described by $\gamma = 1/2$.
- Small scale density growth enhanced and $8\pi G \rho > R$, low curvature regime with order unity deviations from GR.
- Transitions in the non-linear regime where the Compton wavelength can shrink via chameleon mechanism.
- Given $k_{NL}/aH \gg 1$, even very small $f_R$ have scalar-tensor regime.
PPF $f(R)$ Description

- **Metric and matter evolution well-matched by PPF description**
- **Standard GR tools apply (CAMB), self-consistent, gauge invar.**

Hu & Sawicki (2007); Hu (2008)
Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi-\Psi)$
Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi-\Psi)$
- Reduction of large angle anisotropy for $B_0 \sim 1$ for same expansion history and distances as $\Lambda$CDM
- Well-tested small scale anisotropy unchanged
ISW-Galaxy Correlation

- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation
Galaxy-ISW Anti-Correlation

- Large Compton wavelength $B^{1/2}$ creates potential growth which can anti-correlate galaxies and the CMB
- In tension with detections of positive correlations across a range of redshifts

Song, Peiris & Hu (2007); Lombriser et al (2010) $B_0 < 0.43$
• Metric and matter evolution well-matched by PPF description
• Standard GR tools apply (CAMB), self-consistent, gauge invar.

Hu & Sawicki (2007); Hu (2008)
DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background
CMB in DGP

- Adding **cut off** as an epicycle can fix distances, ISW problem
- Suppresses **polarization** in violation of EE data - cannot save DGP!

Fang et al (2008)
Adding cut off as an epicycle can fix distances, ISW problem
Suppresses polarization in violation of EE data - cannot save DGP!

Fang et al (2008)
DGP Normal Branch

- Brane tension (cosmological constant) on normal branch allows models to pass ISW test
- Joint expansion history constraints require $Hr_c > 3$ at 95% CL

Lombriser et al (2009)
Linear Scalar Tensor Regime
Three Regimes

- **Metric**: $ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2$

- **Superhorizon regime**: $\zeta = \text{const.}, \ g(a) = (\Phi + \Psi)/(\Phi - \Psi)$

- **Linear regime - closure** ↔ “smooth” dark energy density:

  $$\nabla^2(\Phi - \Psi)/2 = -4\pi Ga^2\Delta\rho$$

  $G$ can be promoted to $G(a)$, $G(a, k)$ but for scalar degrees of freedom conformal invariance requires $G = G_N$ and

- **Non-linear regime**:

  $$\nabla^2(\Phi - \Psi)/2 = -4\pi Ga^2\Delta\rho$$

  $$\nabla^2\Psi = 4\pi Ga^2\Delta\rho + \frac{1}{2}\nabla^2\phi$$

  with non-linearity in the field equation

  $$\nabla^2\phi = g_{\text{lin}}(a)a^2(8\pi G\Delta\rho - N[\phi])$$
Linear Power Spectrum

- Linear real space power spectrum enhanced on small scales
- Degeneracy with galaxy bias and lack of non-linear predictions leave constraints from shape of power spectrum
Redshift Space Distortion

- Relationship between velocity and density field given by continuity with modified growth rate \( f_v = \frac{d\ln D}{d\ln a} \)
- Redshift space power spectrum further distorted by Kaiser effect
Lensing v Dynamical Comparison

- Gravitational lensing around galaxies vs. linear velocity field (through redshift space distortions and galaxy autocorrelation)
- Consistent with GR + smooth dark energy beginning to test interesting models


DGP Power Spectrum

- Constant suppression in the linear regime for self-acceleration

Lue, Scoccimarro, Starkman (2004); Hu & Sawicki (2007)
Non-Linear GR Regime
Three Regimes

- Fully worked $f(R)$ and DGP examples show 3 regimes
- **Superhorizon** regime: $\zeta = \text{const.}, g(a)$
- **Linear** regime - closure condition - analogue of “smooth” dark energy density:

  \[
  \nabla^2 (\Phi - \Psi) / 2 = -4\pi G a^2 \Delta \rho
  \]

  \[
  g(a, x) \leftrightarrow g(a, k)
  \]

  $G$ can be promoted to $G(a)$ but conformal invariance relates fluctuations to field fluctuation that is small

- **Non-linear** regime:

  \[
  \nabla^2 (\Phi - \Psi) / 2 = -4\pi G a^2 \Delta \rho
  \]

  \[
  \nabla^2 \Psi = 4\pi G a^2 \Delta \rho - \frac{1}{2} \nabla^2 \phi
  \]
Nonlinear Interaction

Non-linearity in the field equation

$$\nabla^2 \phi = g_{\text{lin}}(a)a^2 (8\pi G \Delta \rho - N[\phi])$$

recovers linear theory if $N[\phi] \rightarrow 0$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a non-linear function of the field

linked to gravitational potential

- For DGP, $\phi$ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$

a non-linear function of second derivatives of the field

Linked to density fluctuation
Non-Linear Chameleon

- For $f(R)$ the field equation
  \[ \nabla^2 f_R \approx \frac{1}{3} (\delta R(f_R) - 8\pi G\delta \rho) \]
  is the non-linear equation that returns general relativity

- High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$

- Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if
  \[ \Delta f_R \leq \frac{2}{3} \Phi, \]
  else required field gradients too large despite $\delta R = 8\pi G\delta \rho$ being the local minimum of effective potential
Non-Linear Dynamics

- Supplement that with the modified Poisson equation

\[ \nabla^2 \Psi = \frac{16 \pi G}{3} \delta \rho - \frac{1}{6} \delta R(f_R) \]

- Matter evolution given metric unchanged: usual motion of matter in a gravitational potential \( \Psi \)

- Prescription for \( N \)-body code

- **Particle Mesh** (PM) for the Poisson equation

- Field equation is a non-linear Poisson equation: relaxation method for \( f_R \)

- Initial conditions set to GR at high redshift
Environment Dependent Force

- Chameleon suppresses extra force (scalar field) in high density, deep potential regions

\[ f_{R0} = 10^{-6} \]

\[ \text{density: } \max[\ln(1+\delta)] \]
\[ \text{potential: } \min[\Psi'] \]
\[ \text{field: } \min[f_R/f_{R0}] \]

Environment Dependent Force

- For large background field, gradients in the scalar prevent the chameleon from appearing.

512\(^3\) PM-relaxation code resolves the chameleon transition to GR: greatly reduced non-linear effect

\[ \frac{P(k)}{P_{GR}(k)} - 1 \]

Artificially turning off the chameleon mechanism restores much of enhancement.
N-body Power Spectrum

- Models where the chameleon absent today (large field models) show residual effects from a high redshift chameleon

![Graph showing the power spectrum](image)

\[ P(k)/P_{GR}(k) = \begin{cases} 
1 & \text{No chameleon}, \\
\text{Full } f_R \text{ simulation} & \text{simulation} \\
\text{Linear} & 
\end{cases} \]

\( |f_{R0}| = 10^{-4} \)
\( |f_{R0}| = 10^{-6} \)

Enhanced abundance of rare dark matter halos (clusters) with extra force

Mass Function

- Local cluster abundance (Chandra sample) current best cosmological constraint (~4 orders of magnitude better than ISW)

Schmidt, Vikhlinin, Hu (2009)
• Halos at a fixed mass less rare and less highly biased

Halo Mass Correlation

- Enhanced forces vs lower bias

Halo Model

- Power spectrum trends also consistent with halos and modified collapse

\[ |f_{R0}| = 10^{-4} \]

Nonlinear Interaction

Non-linearity in the field equation

$$\nabla^2 \phi = g_{\text{lin}}(a)a^2 \left(8\pi G \Delta \rho - N[\phi]\right)$$

recovers linear theory if $N[\phi] \to 0$

• For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a non-linear function of the field

Linked to gravitational potential

• For DGP, $\phi$ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$

a non-linear function of second derivatives of the field

Linked to density fluctuation
DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential  Brane Bending Mode

**Apparent Equivalence Principle Violation**

- Self-field of a “test mass” can saturate an external field (for $f(R)$ in the gradient, for DGP in the second derivatives)

Hui, Nicolis, Stubbs (2009); Hu (2009)
Summary

- Lessons from the $f(R)$ and DGP worked examples – 3 regimes:
  - large scales: conservation determined
  - intermediate scales: scalar-tensor
  - small scales: GR in high density regions, modified in low

- Large scales: expansion history and metric ratio
  \[ g = (\Phi + \Psi)/(\Phi - \Psi) \] through curvature conservation

- Intermediate scales: scalar tensor modified Newtonian regime, $g$ and Poisson equation

- Small scales: nonlinear interaction of modification field makes $g$ depend on local environment (not scale) - density or potential - suppressing deviations

- $N$-body (PM-relaxation) simulations show halo model framework can describe observables in the nonlinear regime
Solar System Tests
Solar Profile

- Density profile of Sun is not a constant density sphere - interior photosphere, chromosphere, corona
- Density drops by ~25 orders of magnitude - does curvature follow?

\[ \rho \sim \frac{R}{8\pi G} \quad (n=4, |f_R|_{0.1}) \]

Hu & Sawicki (2007)
Field Solution

- Field solution smoothly relaxes from exterior value to high curvature interior value $f_R \sim 0$, minimizing potential + kinetic
- Juncture is where thin-shell criterion is satisfied $|\Delta f_R| \sim \Delta \Phi$

![Graph showing field solution with various $|f_{R0}|$ values](image)
Solar Curvature

- Curvature drops suddenly as field moves slightly from zero
- Enters into low curvature regime where $R<8\pi G\rho$

Hu & Sawicki (2007)
Solar System Constraint

- **Cassini constraint on PPN** $|\gamma-1| < 2.3 \times 10^{-5}$
- Easily satisfied if **galactic field is at potential minimum** $|f_R| < 4.9 \times 10^{-11}$
- Allows even **order unity** cosmological fields

Hu & Sawicki (2007)