Physics of CMB Anisotropies

Moriond 2000

Wayne Hu
Current CMB Quilt

\[ \Delta T (\text{\(\mu\)K}) \]

\[ \theta \text{ (degrees)} \]

\[ l \text{ (multipole)} \]

Projected Boomerang Errors

$\Delta T$ ($\mu$K)

$\theta$ (degrees)

$l$ (multipole)

W. Hu – May 1999
What We Have Already Learned

• Adiabatic CDM models have survived the onslaught of data

• Dark energy is not all in curvature $\Omega_K \leq 0.3$
  robust to model unless: $h > 1$, recombination substantially delayed, or closed + isocurvature

For the Skeptic: confirm with 2nd peak

Lineweaver (1998); Bond & Jaffe (1998); Dodelson & Knox (1999); Tegmark & Zaldarriaga (1999)
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- Assuming adiabatic CDM:
  Baryonic dark matter necessary ($\Omega_b h^2 \geq 0.015$)
  For the Skeptic: confirm with 2nd peak; measure with 3rd peak
  Hints of low $\Omega_m \approx 0.3$ hence a cosmological constant $\Omega_{\Lambda} \approx 0.7$
  For the Skeptic: confirm with relative peak heights
  Optically thin during reionization $\tau \leq 0.5$

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- Inflationary origin implied
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- Consistent with LSS, cluster abundance, SNIa, BBN, $h$

Lineweaver (1998); Bond & Jaffe (1998); Dodelson & Knox (1999); Tegmark & Zaldarriaga (1999)
Acoustic Peak
Preliminaries
Thermal History

- $z > 1000; T_\gamma > 3000K$
- Hydrogen ionized
- Free electrons glue photons to baryons

Photon–baryon fluid
Potential wells that later form structure

![Diagram showing tightly coupled fluid with mean free path much less than wavelength, indicating a hot/cold gradient.](image-url)
Thermal History

- $z > 1000; \ T_\gamma > 3000K$
  - Hydrogen ionized
  - Free electrons glue photons to baryons

  $\gamma e^- p$

  Compton scattering
  Coulomb interactions

- $z \sim 1000; \ T_\gamma \sim 3000K$
  - Recombination
  - Fluid breakdown

- $z < 1000; \ T_\gamma < 3000K$
  - Gravitational redshifts & lensing
  - Reionization; rescattering

Photon–baryon fluid
Potential wells that later form structure

\[ \lambda \sim k^{-1} \]
\[ \theta \sim l^{-1} \]

Observer

Last scattering surface

$z=1000$ recombination
Acoustic Oscillations

- Photon pressure resists compression in potential wells
- Acoustic oscillations

Peebles & Yu (1970)  
Hu & Suguama (1995)
Acoustic Oscillations

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- Gravity displaces zero point
  \[ \Theta \equiv \frac{\delta T}{T} = -\Psi \]

Oscillation amplitude = initial displacement from zero point
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- Gravitational redshift: observed
  \[ (\frac{\delta T}{T})_{\text{obs}} = \Theta + \Psi \]
  oscillates around zero

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First Extrema

Peebles & Yu (1970)

Hu & Sugyama (1995)
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Harmonic Peaks

- Oscillations frozen at last scattering
- Wavenumbers at extrema = peaks
- Sound speed $c_s$

Doroshkevich, Zel'dovich & Sunyaev (1978); Bond & Efstathiou (1984); Hu & Sugiyama (1995)
Harmonic Peaks

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- Frequency $\omega = kc_s$; conformal time $\eta$
- Phase $\propto k$; $\phi = \int_0^{\text{last scattering}} d\eta \omega = k$ sound horizon
- Harmonic series in sound horizon
  $\phi_n = n\pi \rightarrow k_n = n\pi/\text{sound horizon}$

Doroshkevich, Zel'dovich & Sunyaev (1978); Bond & Efstathiou (1984); Hu & Sugiyama (1995)
Curvature and the Cosmological Constant
Angular Diameter Distance

- **A Classical Test**
  Standard(ized) comoving ruler
  Measure angular extent
  Absolute scale drops out
  Infer curvature

- **Upper limit 1st Peak Scale (Horizon)**
  Upper limit on Curvature

- **Calibrate 2 Physical Scales**
  Sound horizon (peak spacing)
  Diffusion scale (damping tail)

Houston, Spergel & Sugiyama (1994)
Hu & White (1996)
Curvature and the Cosmological Constant

Gravitational Redshift

Shifted Acoustic Signature

Power

$\Omega_K$

$\Omega_\Lambda$

$\Omega_K$

$\Omega_\Lambda$

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Curvature & $\Lambda$: Constraints

$\Omega_m$ vs $\Omega_\Lambda$

Boomerang & COBE

Melchiorri et al. (1999)
Curvature & $\Lambda$: Constraints

- Boomerang & COBE
- Melchiorri et al. (1999)
- Riess et al. (1998)
- Perlmutter et al. (1998)
Dark Energy: Future Prospects

Dark Baryons
Baryon Drag

• Baryons provide **inertia**

• Relative momentum density
  
  \[ R = (\rho_b + p_b) V_b / (\rho_\gamma + p_\gamma) V_\gamma \propto \Omega_b h^2 \]

• Effective **mass** \( m_{\text{eff}} = (1 + R) \)

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Hu & Sugiyama (1995)
Baryon Drag

- Baryons provide inertia
- Relative momentum density
  \[ R = \frac{(\rho_b + p_b)V_b}{(\rho_\gamma + p_\gamma)V_\gamma} \propto \Omega_b h^2 \]
- Effective mass \( m_{\text{eff}} = (1 + R) \)
- Baryons drag photons into potential wells → zero point ↑
- Amplitude ↑
- Frequency ↓ (\( \omega \propto m_{\text{eff}}^{-1/2} \))

- Constant \( R, \Psi \):
  \[
  (1+R)\ddot{\Theta} + \left(\frac{k^2}{3}\right)\Theta = -(1+R)\left(\frac{k^2}{3}\right)\Psi
  \]
  \[ \Theta + \Psi = [\Theta(0) + (1+R)\Psi(0)] \cos \left[ \frac{k\eta}{\sqrt{3}} (1+R) \right] - R\Psi \]

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Dissipation / Diffusion Damping

- Imperfections in the coupled fluid $\rightarrow$ mean free path $\lambda_C$ in the baryons
- Random walk over diffusion scale: $\lambda_D \sim \lambda_C \sqrt{N} \sim \sqrt{\lambda_C \eta} \gg \lambda_C$
  
  Viscous damping for $R<1$; heat conduction damping for $R>1$

$N = \eta / \lambda_C$

Silk (1968)
Dissipation / Diffusion Damping

- Imperfections in the coupled fluid $\rightarrow$ mean free path $\lambda_C$ in the baryons
- Random walk over diffusion scale: $\lambda_D \sim \lambda_C \sqrt{N} \sim \sqrt{\lambda_C \eta} \gg \lambda_C$
- Rapid increase at recombination as mfp $\uparrow$

- Peak/Damping angular scale: calibrate $\Omega_b h^2$ or test recombination
- Robust physical scale for angular diameter distance test ($\Omega_K$, $\Omega_\Lambda$)

Recombination

Silk (1968); Hu & White (1996)
Baryons in the CMB

- High odd peaks

- Additional Effects
  - Time–varying potential
  - Dissipation/Fluid imperfections

\[ \Omega_b h^2 \]

\[ \text{Power} \]

\[ l \]
Matter–Radiation Ratio
Driving Effects and Matter/Radiation

- Potential perturbation: \( k^2 \Psi = -4\pi G a^2 \delta \rho \) generated by radiation
- **Radiation \rightarrow Potential:** inside sound horizon \( \delta \rho / \rho \) pressure supported \( \delta \rho \) hence \( \Psi \) decays with expansion

Hu & Sugiyama (1995)
Driving Effects and Matter/Radiation

- Potential perturbation: $k^2 \Psi = -4\pi G a^2 \delta \rho$ generated by radiation

- Radiation → Potential: inside sound horizon $\delta \rho / \rho$ pressure supported $\delta \rho$ hence $\Psi$ decays with expansion

- Potential → Radiation: $\Psi$–decay timed to drive oscillation $-2\Psi + (1/3)\Psi = -(5/3)\Psi \rightarrow 5x$ boost

- Feedback stops at matter domination

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Hu & Sugiyama (1995)
Matter Density in the CMB

- Amplitude ramp across matter–radiation equality
- Radiation density fixed by CMB temperature & thermal history

- Measure $\Omega_m h^2$ from peak heights
Inflation & The Origin of Perturbations
Inflation as Source of Perturbations

- Superluminal expansion (inflation) required to generate superhorizon potential (density) perturbations

- Potential perturbations drive oscillations

- (Nearly) unique prediction for phase

- Ratio of peak locations
  - Inflation: 1:2:3...
  - Passive causal models: 1:3:5...
  - Active causal models: no peaks

Hu & White (1996)
Inflation as Source of Perturbations

- Superluminal expansion (inflation) required to generate superhorizon potential (density) perturbations

- Potential perturbations drive oscillations

- (Nearly) unique prediction for phase

- Ratio of peak locations 1:2:3 strongly suggests inflation but not necessarily the adiabatic or isocurvature nature of initial conditions

(Hu 1998; Hu & Peebles 1999)
Summary of Acoustic Phenomenology

- Fluid + Gravity
  $\rightarrow$ harmonic series:
  inflationary origin
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  - harmonic series:
    - inflationary origin
  - alternating peaks:
    - photon/baryon $\Omega_b h^2$
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  - $\text{harmonic series:}$ inflationary origin
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- Ruler Calibration
  - sound horizon
  - damping scale

- Geometry
  - angular diameter distance $f(\Omega_\Lambda, \Omega_K)$
  - flatness or no $\Omega_\Lambda$,
  - $\Omega_\Lambda$ or $\Omega_K$

\[
l_A = d_A \times \pi k_A \quad \text{ISW}
\]

\[
l_D = d_A \times k_D
\]
Secondary Anisotropies
Physics of Secondary Anisotropies

Primary Anisotropies

recombination
$z \sim 1000$

reionization
$z \sim 10$

$\Lambda$–domination
$z \sim 1$
Secondary Anisotropies: Power Spectra

- Gravitational Effects
  - ISW Effect
    (redshift from decaying potentials)
  - Weak Lensing
    (smoothes peaks and generates power <1')

- Scattering Effects
  - Doppler Effect
  - Vishniac Effect
    (LSS kinetic SZ effect)
  - Patchy Reionization
    (LSS thermal)
  - SZ effect
    (LSS thermal)
Recent Work on Isolating Secondary Anisotropies

- **Subarcminute Power Spectrum**
  - Vishniac Effect; Kinetic SZ Effect;
  - Patchy Reionization Hu (1999)
  - Bruscoli et al. (1999)
- **SZ in Clusters**
  - Komatsu & Kitayama (1999)
- **SZ in Radio Galaxies**
- **Polarization**
  - Weak Lensing
    - Zaldarriaga & Seljak (1999)
    - Guzik, Seljak & Zaldarriaga (1999)
  - Secondary Scattering
    - Hu (1999); Weller (1999)
- **Frequency spectrum**
  - SZ Effect
    - Bouchet & Gispert (1999); Tegmark et al. (1999)
    - Cooray, Hu & Tegmark (2000)
- **Temperature non-Gaussianity**
  - Weak Lensing & Secondaries
    - 3pt function (bispectrum)
    - Weak Lensing: 4pt function (trispectrum)
      - Zaldarriaga (1999)
    - spot ellipticity & correlation
      - Van Waerbeke, Bernardeau & Benabed (1999)
  - SZ Effect: hydro-simulations
    - da Silva et al. (1999); Refrigier et al. (1999), Seljak, Burwell, Pen (2000);
    - Press-Schechter Aghanim & Forni (1999);
- **Polarization non-Gaussianity**
  - Hu (2000)
Polarization
Polarization Diagnostics

- CMB polarization generated by scattering of quadrupole anisotropies
Polarization Diagnostics

- CMB polarization generated by scattering of quadrupole anisotropies
- Isolates the last scattering surface
  → tests causal generation (inflation vs. defects)

Current Constraints
< 20–40 μK
Saskatoon
TOCO

Hu & White (1997)
Zaldarriaga & Spergel (1997)
Polarization Diagnostics

- **CMB polarization generated by scattering of quadrupole anisotropies**
- **Isolates the last scattering surface**
  → measures the reionization epoch / optical depth (first structures)

Hogan, Kaiser, & Rees (1982)
Efstathiou & Bond (1987)
Perturbations & Their Quadrupoles

- Orientation of quadrupole relative to wave ($\mathbf{k}$) determines pattern
- Scalars (density) $m=0$
- Vectors (vorticity) $m=\pm 1$
- Tensors (gravity waves) $m=\pm 2$

Hu & White (1997)
Polarization Patterns

Scalars

$E, B$

$l=2, m=0$

$l=2, m=1$

$l=2, m=2$

Vectors

$\phi = \pi/2$

Tensors
Foregrounds and Tensors

- 257–561 Foreground Parameters Simultaneously Estimated
- Foreground power spectra, frequency dependence, frequency coherence
  - free-free, synchrotron, vibrating dust, rotating dust, thermal SZ, radio point sources, IR point sources
- 10 Cosmological Parameters
- Potentially significant degradation: better prior knowledge; more frequencies

Bouchet & Gispert (1999); Knox (1999)
Foregound and Baryons

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- Foreground power spectra, frequency dependence, frequency coherence
  - free-free, synchrotron, vibrating dust, rotating dust, thermal SZ, radio point sources, IR point sources
- 10 Cosmological Parameters
- Degradation of less than 2 in errors

Tegmark, Eisenstein, Hu, de Oliviera-Costa (1999)
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• Simple adiabatic CDM models have survived the onslaught of data to date
• The dark energy is not curvature
• Baryonic dark matter and low density cold dark matter indicated
• First peak location inconsistent with most non-inflationary models (unless universe is closed or recombination delayed)
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• Nature of the dark energy can be revealed by precision data
• First objects and reionization revealed by polarization and sub-arcminute scale anisotropy
• Large-scale structure, hot gas via non-Gaussianity, cross correlation
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- First objects and **reionization** revealed by **polarization** and **sub-arcminute scale** anisotropy
- Large-scale structure, hot gas via **non-Gaussianity**, **cross correlation**

- Foregrounds **not** expected to be a problem for **power spectrum** estimation in the **acoustic regime** but will be a serious issue for polarization, **sub-arcminute anisotropy** and **non-Gaussianity**.