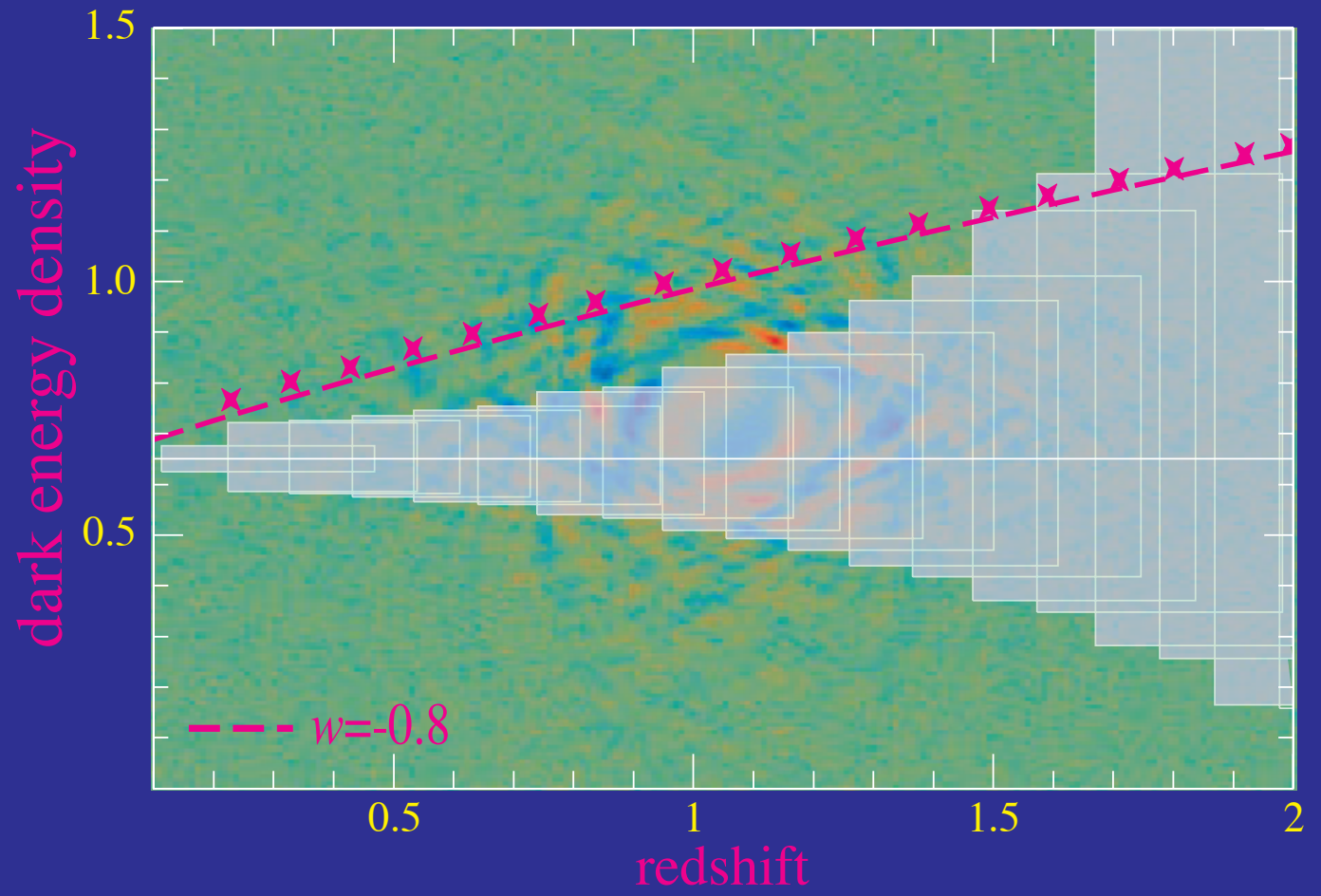
Gravitational Lensing and the Dark Energy



Future Prospects

Wayne Hu
U. Penn, April 2003

Outline

- Evidence for Dark Energy in the Background
- Dark Energy Evolution from Gravitational Lensing
- Dark Energy Clustering from Gravitational Lensing

- Collaborators:

Chuck Keeton Zhaoming Ma

Takemi Okamoto

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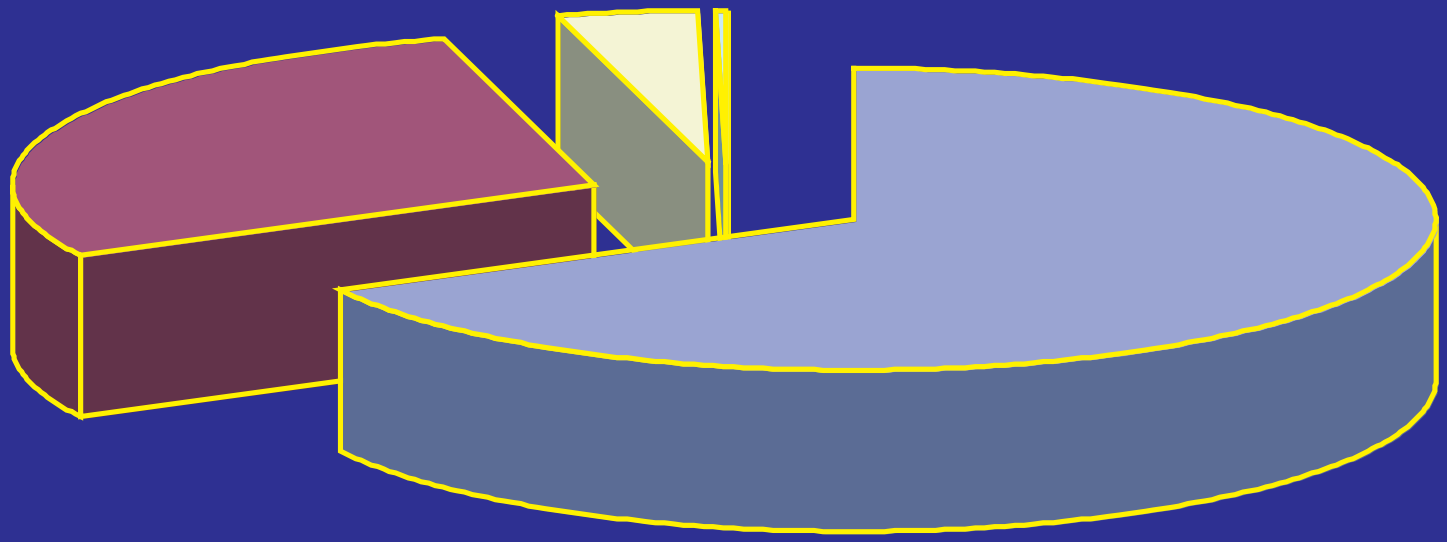
http://background.uchicago.edu

If its not dark, it doesn't matter!

- Cosmic matter-energy budget:



Dark Energy
Dark Matter
Dark Baryons
Visible Matter
Dark Neutrinos

Making Light of the Dark Side

- Line element:

$$ds^2 = a^2[(1-2\Phi)d\eta^2 - (1+2\Phi)(dD^2 + D_A^2 d\Omega)]$$

where η is the conformal time, D comoving distance, D_A the comoving angular diameter distance and Φ the gravitational potential

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- Light propagates on null geodesics: in the background $\Delta D = \Delta\eta$; around structures according to lensing by Φ
- Dark components visible in their influence on the metric elements a and Φ
- Relation given by Einstein equations

Friedmann and Poisson Equations

- Friedmann equation:

$$\left(\frac{d\ln a}{d\eta}\right)^2 = \frac{8\pi G}{3}a^2\sum\rho \equiv [aH(a)]^2$$

implying that the densities of dark components are visible in the distance-redshift relation ($a = (1+z)^{-1}$)

$$D = \int d\eta = \int \frac{d\ln a}{aH(a)} = \int \frac{dz}{H}$$

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- Poisson equation:

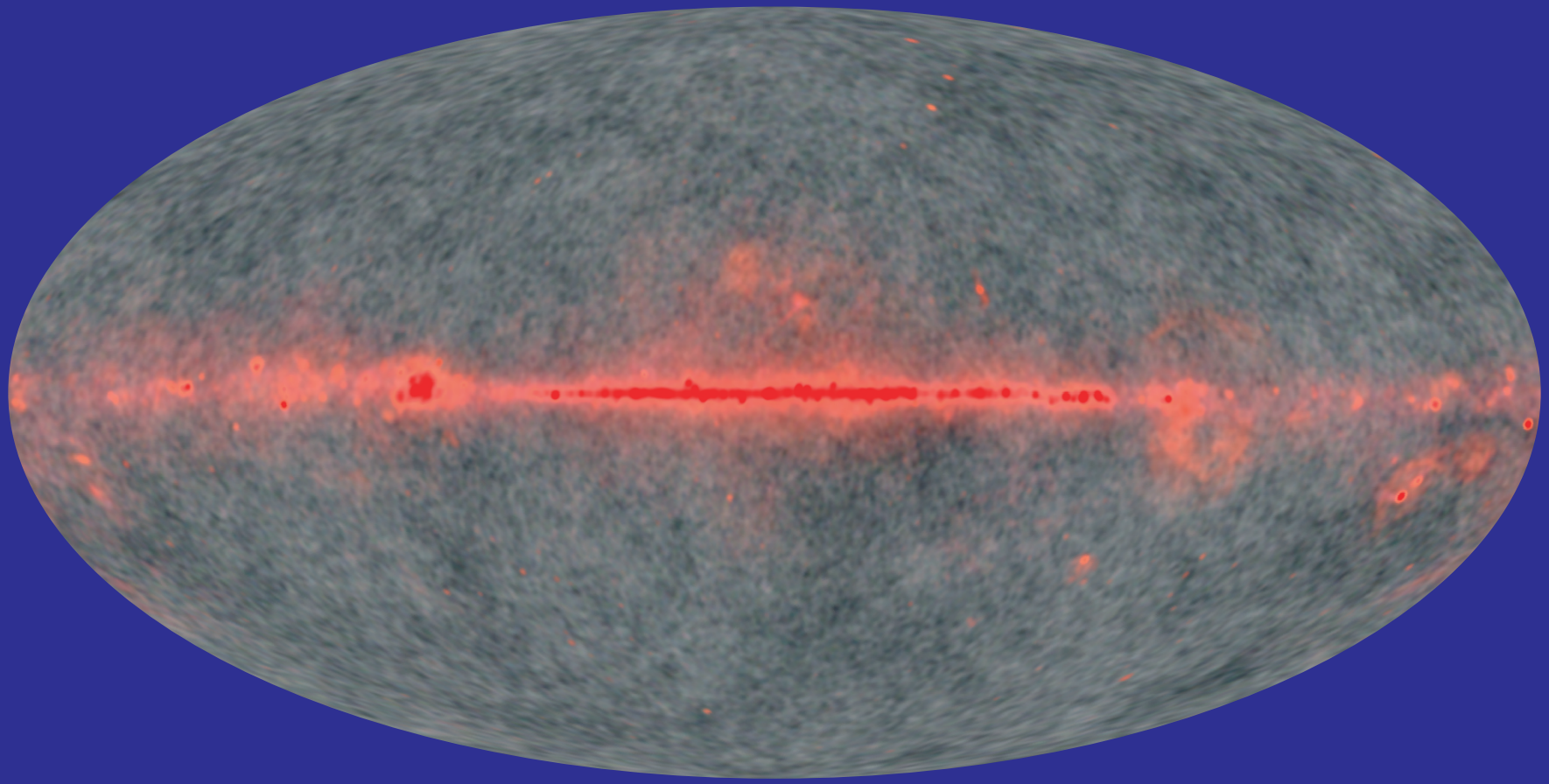
$$\nabla^2\Phi = -4\pi G a^2 \Delta\rho$$

implying that the spatial clustering of the dark components is visible in the potential: dark energy has relativistic pressure, stabilizing perturbations.

Dark Energy in the Background

Dark Energy in the CMB

- Gravitational lensing test in the background: acoustic spot sizes



WMAP Collaboration

Time Ordered CMB Data

A Freshman Physics Problem

- Simple harmonic oscillator
- Gravitational force from initial potential fluctuations
- Photon pressure restoring force

Peebles & Yu (1970)

Acoustic Basics

- Continuity Equation: (number conservation)

$$\dot{\Theta} = -\frac{1}{3} k v_\gamma$$

where $\Theta = \delta n_\gamma / 3 n_\gamma$ is the temperature fluctuation with $n_\gamma \propto T^3$

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- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi$$

where $c_s^2 \equiv \dot{p}/\dot{\rho}$ is the sound speed squared

Harmonic Peaks

- Adiabatic (Curvature) Mode Solution

$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0)\ \cos(ks)$$

where the sound horizon $s \equiv \int c_s d\eta$ and $\Theta + \Psi$ is also the observed temperature fluctuation after gravitational redshift

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- Modes caught in the extrema of their oscillation will have enhanced fluctuations

$$k_n s_* = n\pi$$

yielding a fundamental scale or frequency, related to the inverse sound horizon and series dependent on adiabatic assumption

Peak Location

- Fundmental physical scale, the distance sound travels, becomes an angular scale by simple projection according to the angular diameter distance D_A

$$\begin{aligned} \theta_A &= \lambda_A / D_A \\ \ell_A &= k_A D_A \end{aligned}$$

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- In a flat universe, the distance is simply $D_A = D \equiv \eta_0 - \eta_* \approx \eta_0$, the horizon distance, and $k_A = \pi / s_* = \sqrt{3}\pi/\eta_*$ so

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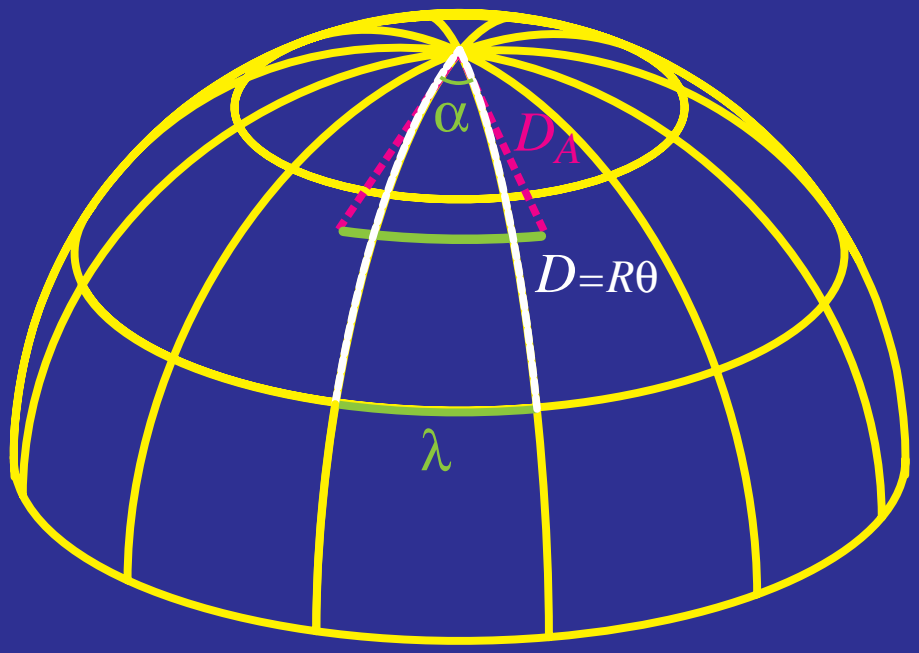
- In a matter-dominated universe $\eta \propto a^{1/2}$ so $\theta_A \approx 1/30 \approx 2^\circ$ or

$$\ell_A \approx 200$$

Curvature

- In a curved universe, the apparent or angular diameter distance is no longer the conformal distance $D_{\rm A} = R\sin(D/R) \neq D$



- Comoving objects in a closed universe are further than they appear! gravitational lensing by the background...
- Future: gravitational lensing of CMB by large-scale structure

CMB Dark Energy Observable

- Universe is neither fully matter dominated at recombination nor at the present due to radiation and dark energy
- Given a recombination epoch a_* (depends mainly on temperature and atomic physics) calculate the sound horizon

$$s_* \equiv \int c_s d\eta = \int_0^{a_*} da \frac{c_s}{a^2 H}$$

  note that this depends mainly on the expansion rate H at a_*, i.e. the matter-radiation ratio.

- Given a dark energy model, calculate the comoving distance to a_*

$$D = \int_{a_*}^1 da \frac{1}{a^2 H}$$

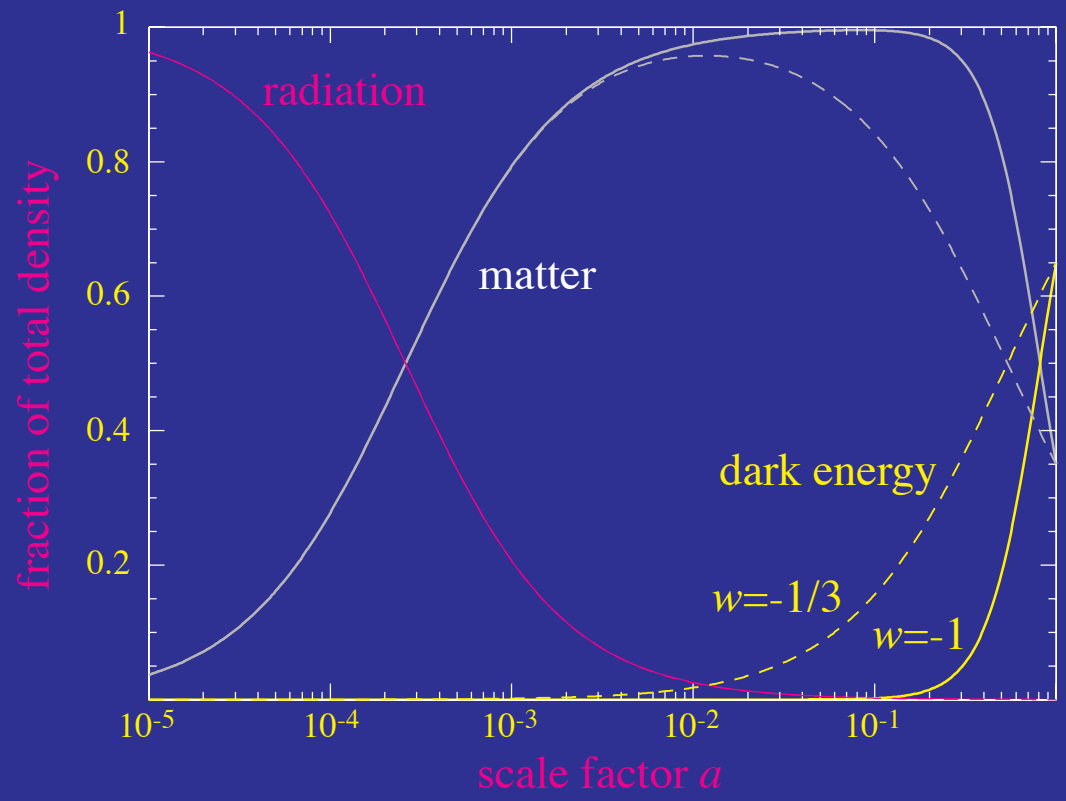
- Given a curvature calculate the angular diameter distance $D_A = R \sin(D/R)$

Dark Energy Parameterization

- Energy conservation (continuity) equation

$$\dot{\rho}_{\rm DE} = -3(\rho_{\rm DE} + p_{\rm DE})\frac{\dot{a}}{a}$$

- Energy density given by a present value $\Omega_{\rm DE} = 8\pi G\rho_{\rm DE}/3H_0^2$ and an equation of state $w(z) = p_{\rm DE}/\rho_{\rm DE}$, often assumed constant.



A Parameterized Example

- Put it together:

$$\ell_A \equiv \frac{\pi D_A}{s_*}$$

- Note that H_0 always cancels in the ratio, but that with both radiation and dark energy
- Simple model: around $\Omega_m h^2 = 0.14$, $\Omega_b h^2 = 0.024$, $\Omega_{DE} = 0.73$, $w_{DE} = -1$, $\alpha = \alpha_0$

$$\frac{\Delta\ell_A}{\ell_A} = 1.2\frac{\Delta\Omega_{\rm tot}}{\Omega_{\rm tot}} - 0.11\frac{\Delta\Omega_{DE}}{\Omega_m} + 0.1\Delta w_{DE}$$
$$- 0.25\frac{\Delta\Omega_m h^2}{\Omega_m h^2} + 0.083\frac{\Delta\Omega_b h^2}{\Omega_b h^2} + 2.3\frac{\Delta\alpha}{\alpha}$$

- Leading source of error to $\Omega_{\rm tot} = \Omega_m + \Omega_{DE}$, Ω_{DE} is from $\Omega_m h^2$

Before and AfterMap

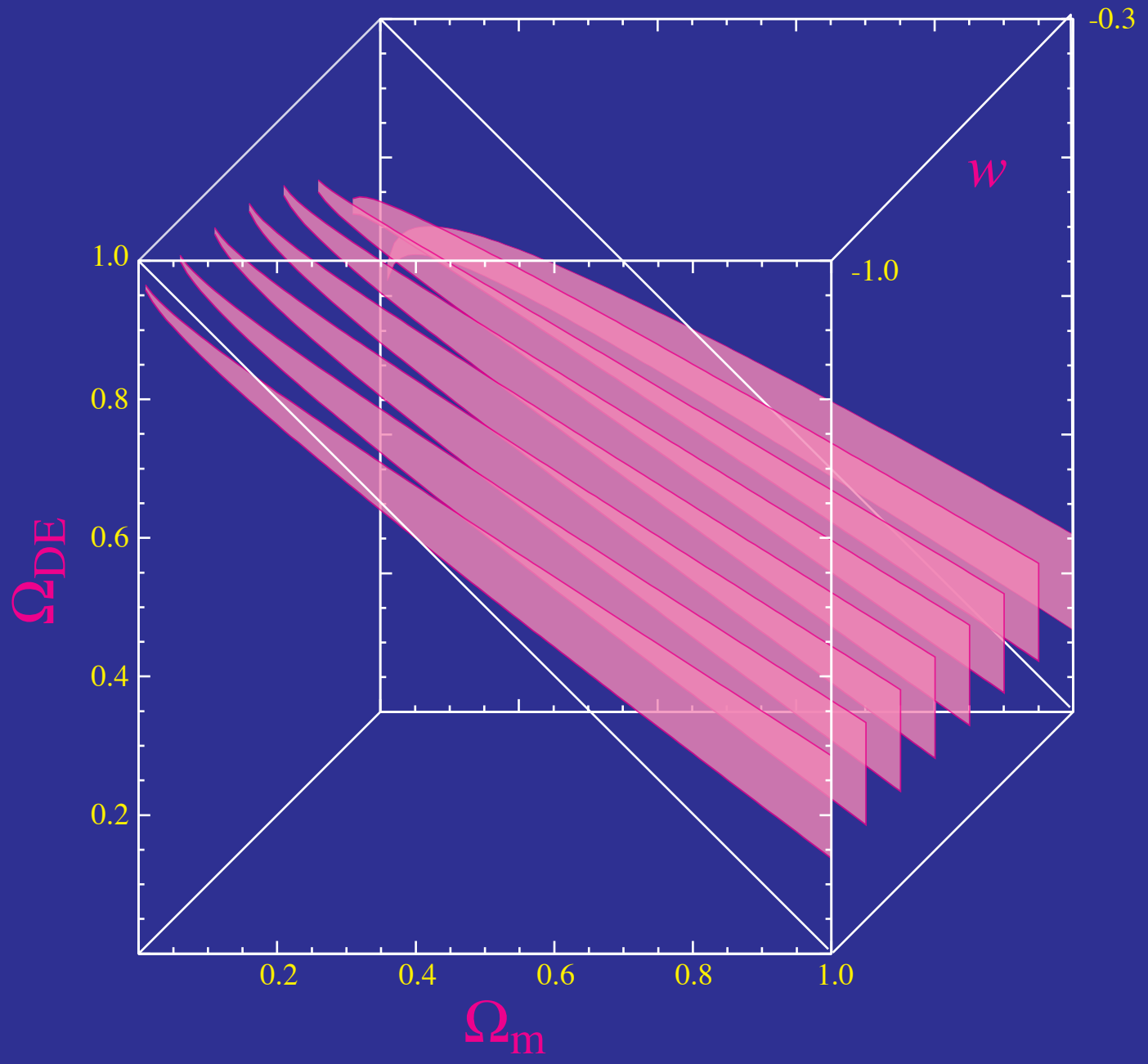
- Parameter Estimates (1σ range)

Peak Parameters	Before (Wang et al; Knox et al)	MAP (Spergel et al)
ℓ_A	$300 - 308$	$297 - 301$
$\Omega_m h^2$	$0.118 - 0.135$	$0.12 - 0.16$
$\Omega_b h^2$	$0.020 - 0.026$	$0.023 - 0.025$
Initial Spectrum		
n_s	$0.93 - 1.05$	$0.95 - 1.03$
Reionization		
τ	$0 - 0.1$	$0.1 - 0.24$

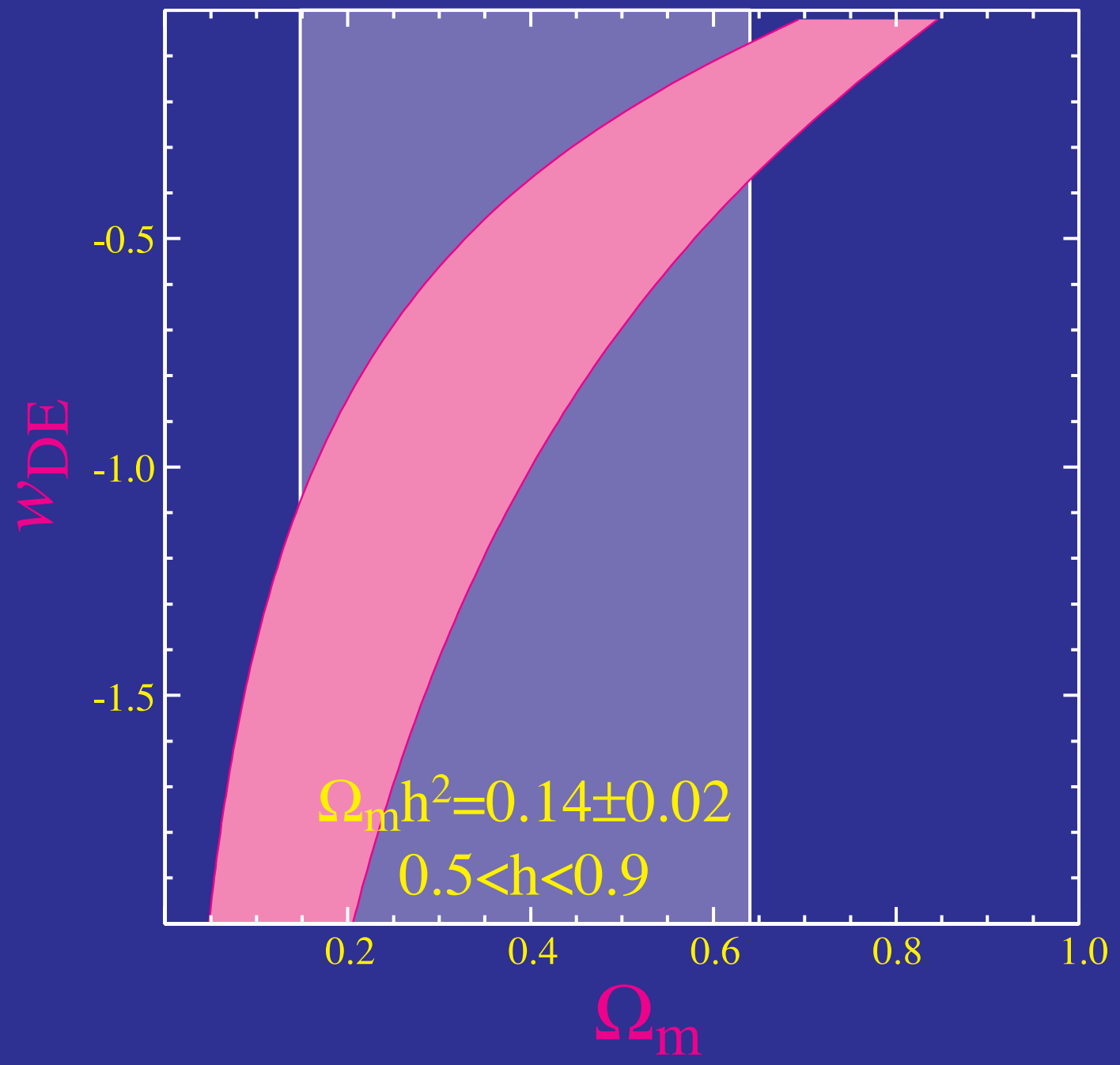
- Previous experiments both precise and accurate on peak parameters!
- Importantly, the extended range in previous experiments ℓ improves $\Omega_m h^2$, consequences for dark energy

Dark Energy

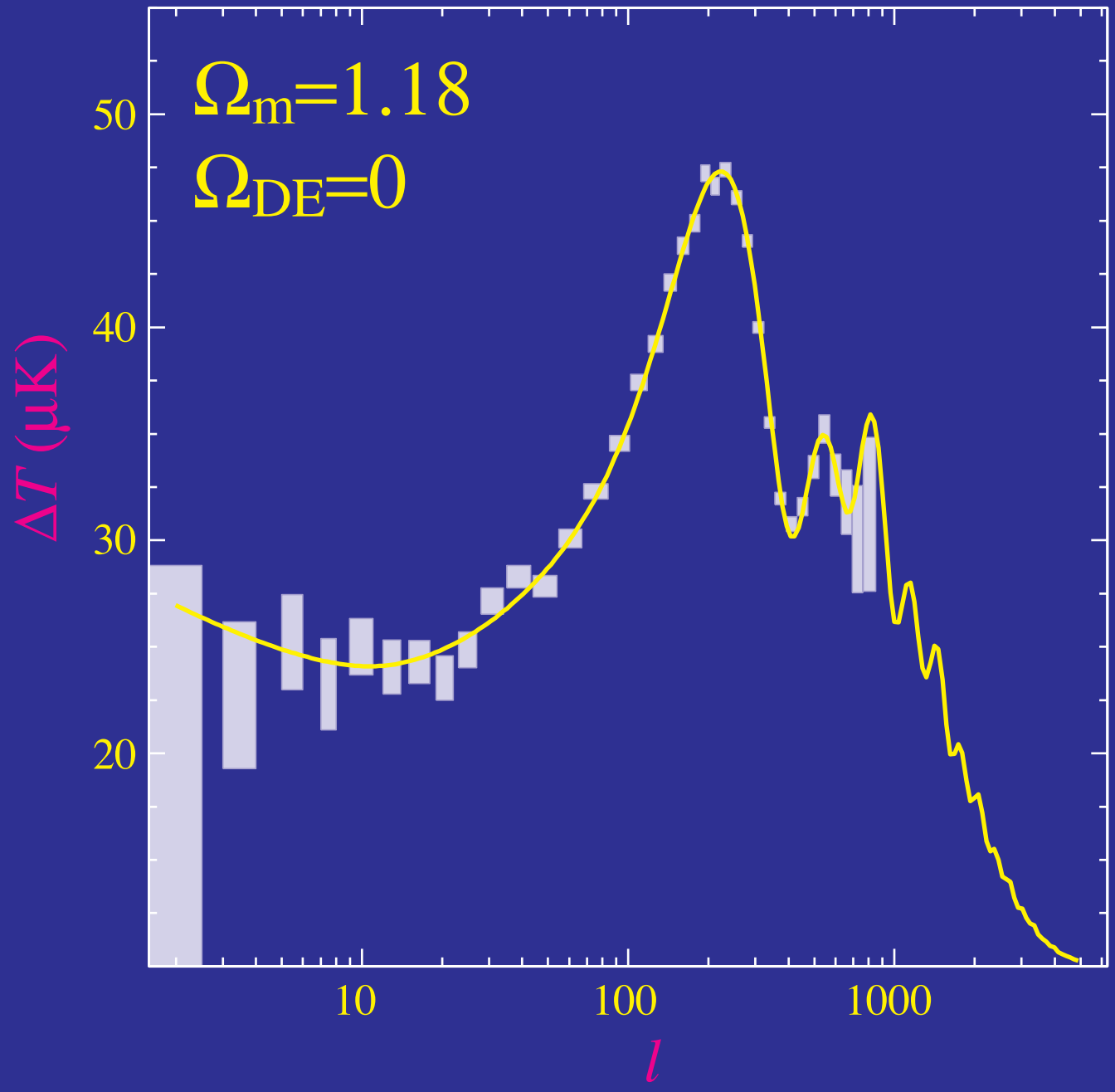
Dark Cube



Equation of State

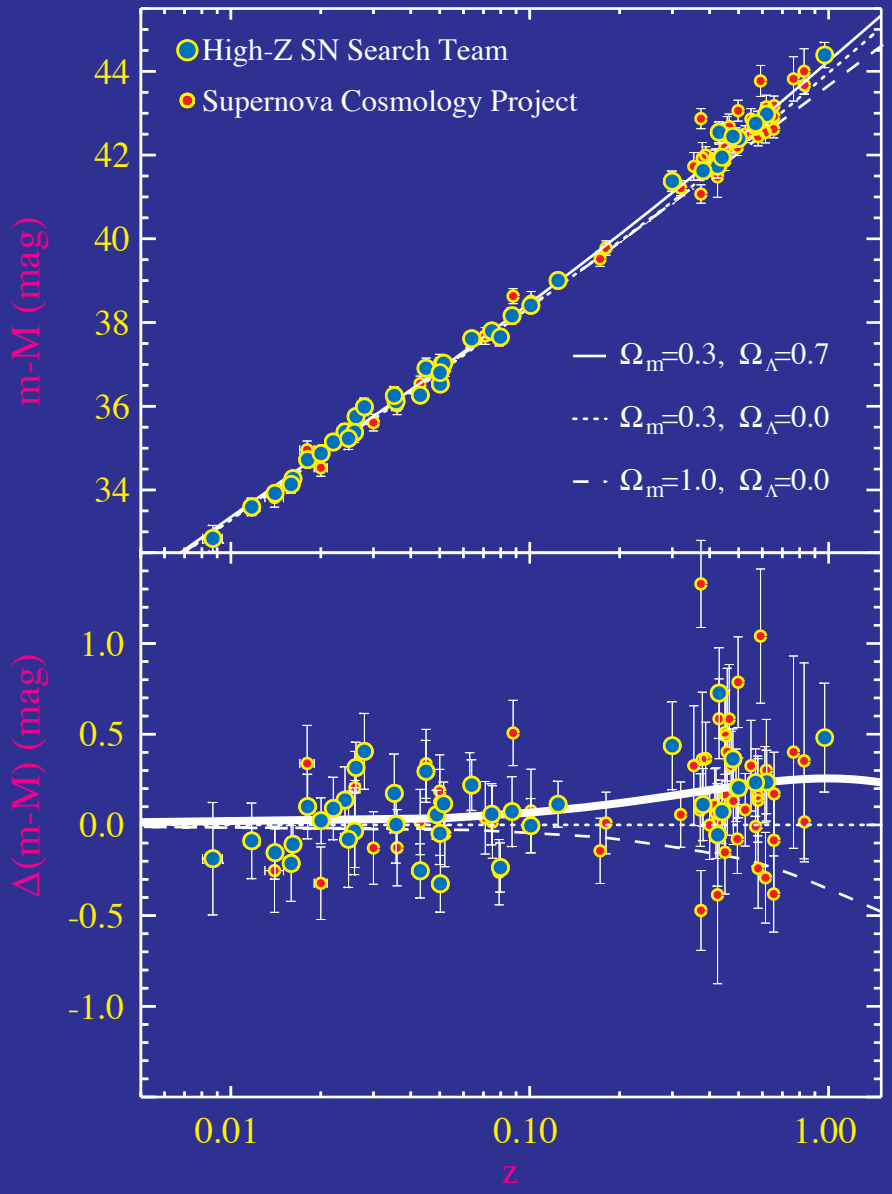


Can Dark Energy be Absent?



Negative Pressure from SNe

- Distance-redshift to supernovae Ia indicates negative pressure (acceleration)



compilation from High-z team

Dark Energy
and
Gravitational Lensing

Growth of Structure

- Simulation (by A. Kravstov)

Clustering Probes of Dark Energy

- Dark energy affects gravitational potentials through the Poisson equation, providing a new handle on the dark energy.
- As in the equation of state $p = w\rho$, define a sound speed, for the fluctuations $\delta p = c_s^2 \delta\rho$ (in rest frame (Hu 1998))
- Different particle candidates for dark energy predict different values: quintessence (canonical scalar field) $c_s^2 = 1$; k-essence c_s^2 variable...
- Below sound horizon $s_{\rm DE} = \int d\eta c_s$ dark energy density stabilizes and becomes smooth relative to dark matter, $\Phi \propto g$ decays

$$\frac{d^2 g}{d\ln a^2} + \left[\frac{5}{2} - \frac{3}{2} w(z)\Omega_{DE}(z)\right] \frac{dg}{d\ln a} + \frac{3}{2}[1 - w(z)]\Omega_{DE}(z) g = 0\,,$$

Gravitational Potentials Lens Images

- Lensing of any background image, e.g. CMB primary anisotropy
- Shearing of the image according to deflection remapping

Lensing Observables

- Image distortion described by Jacobian matrix of the remapping

$$\mathbf{A} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix},$$

where κ is the convergence, γ_1, γ_2 are the shear components

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- related to the gravitational potential Φ by spatial derivatives

$$\psi_{ij}(z_s) = 2\int_0^{z_s} dz \frac{dD}{dz}\frac{D(D_s-D)}{D_s}\Phi_{,ij}\,,$$

$\psi_{ij} = \delta_{ij} - A_{ij}$, i.e. via Poisson equation

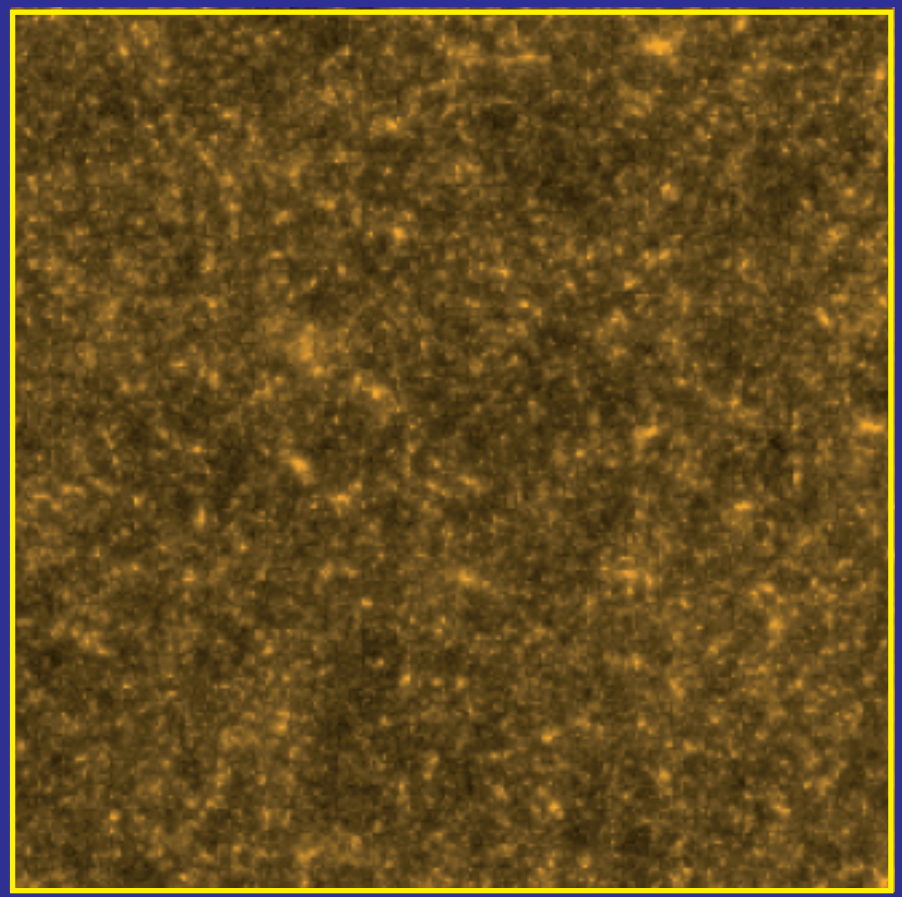
$$\kappa(z_s) = \frac{3}{2}H_0^2\Omega_m \int_0^{z_s} dz\, \frac{dD}{dz}\,\frac{D(D_s-D)}{D_s}\,\delta/a\,,$$

Blandford et al. (1991); Miralda-Escude (1991); Kaiser (1992)

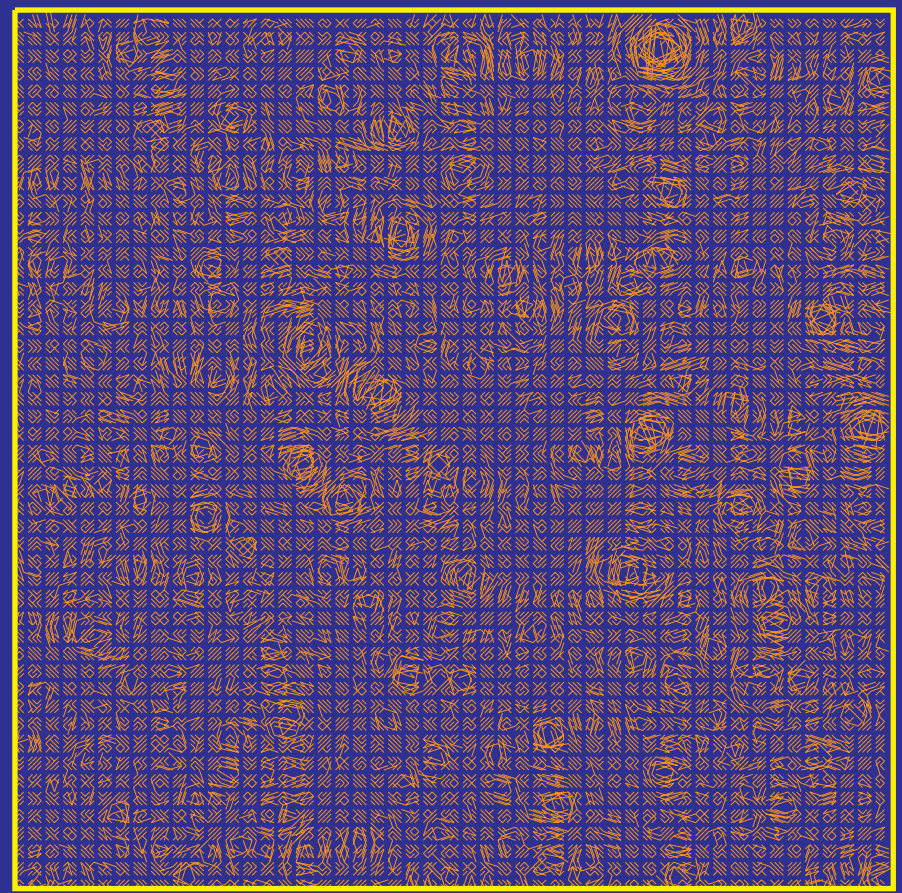
Cosmic Shear

- Simulating mass distribution is a routine exercise

Convergence



Shear

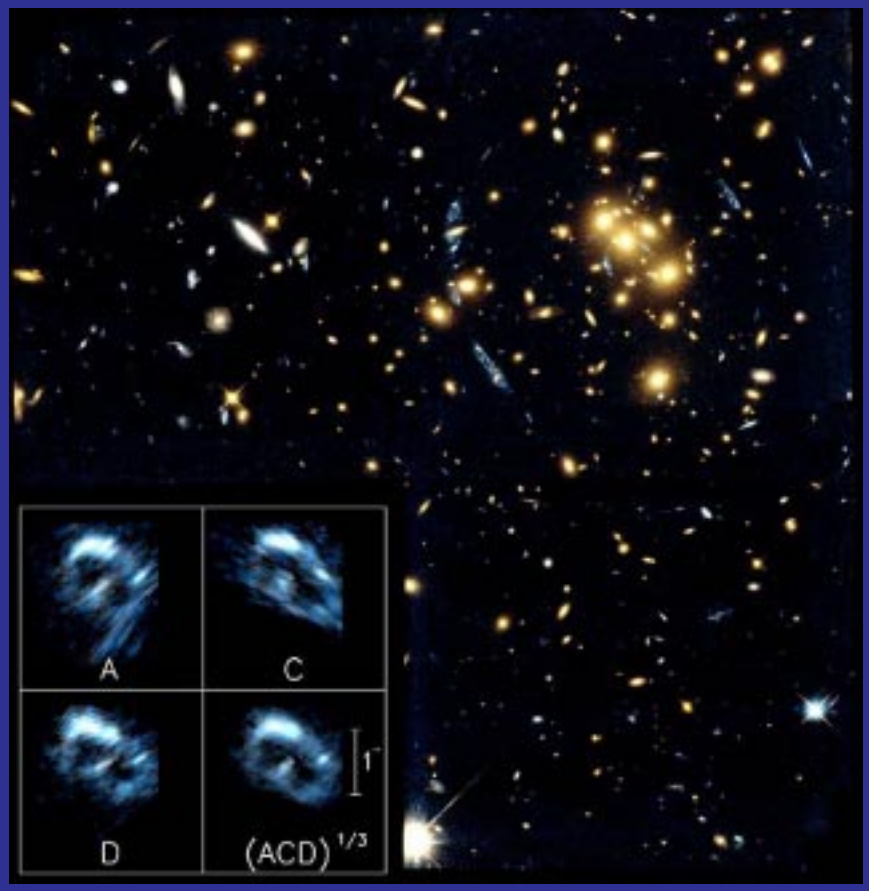


$36\mathrm{deg}^2$ FOV; 2' Res.; 245–75 h^{-1}Mpc box; 480–145 h^{-1}kpc mesh; 2–70 10^9 $M_\odot$

Jain, Seljak & White (1998); White & Hu (1999)

Weak Gravitational Lensing

- Shearing of galaxy images, similar to strong lensing but few % level
- Main systematic effects are instrumental rather than astrophysical

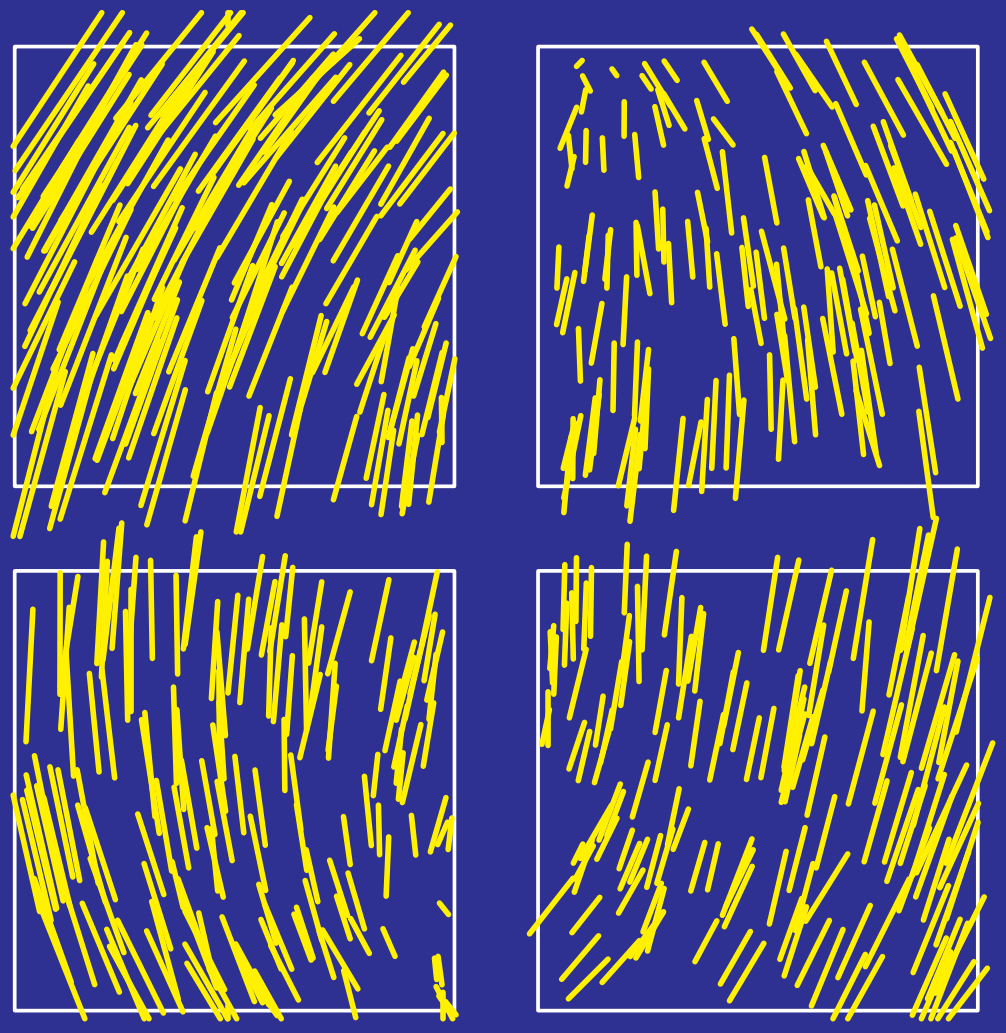


Cluster (Strong) Lensing: 0024+1654

Colley, Turner, & Tyson (1996)

Instrumental Systematics

- Raw data has instrumental systematics (PSF anisotropy) larger than signal, removed by demanding stars be round



1% ellipticity

Jarvis et al. (2002)

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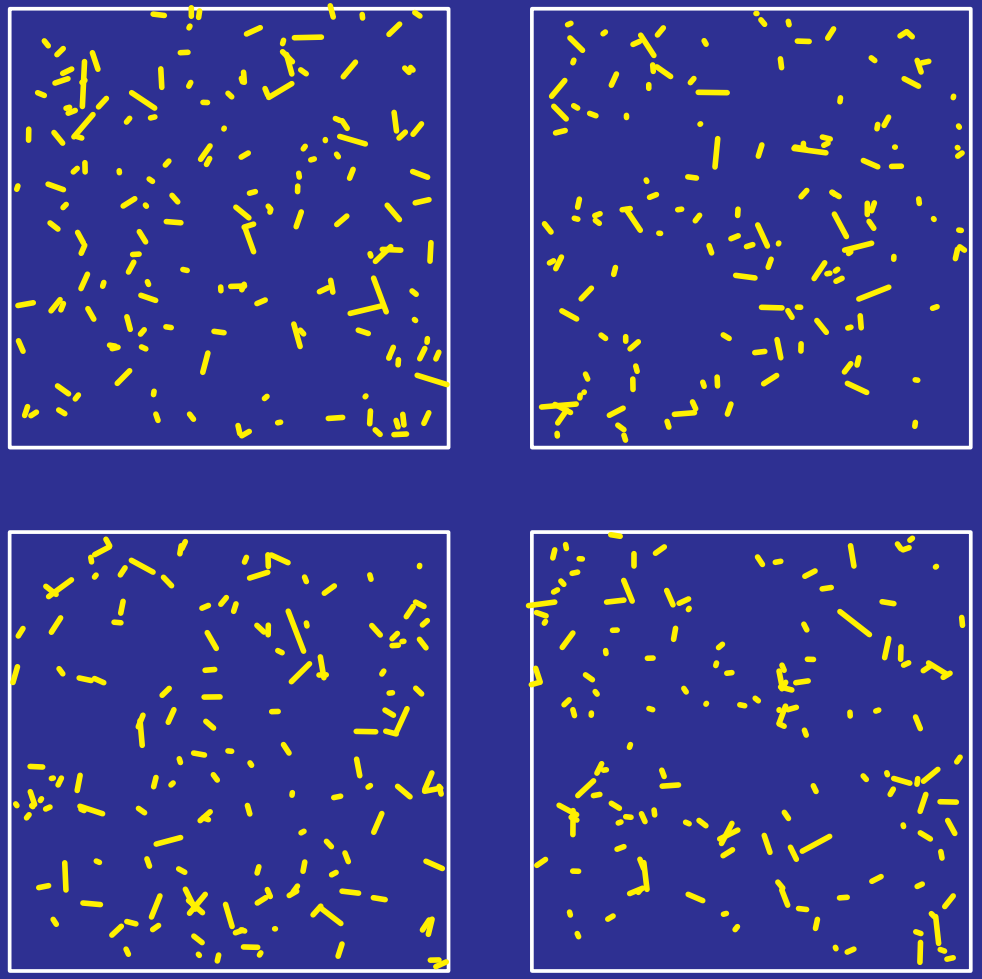

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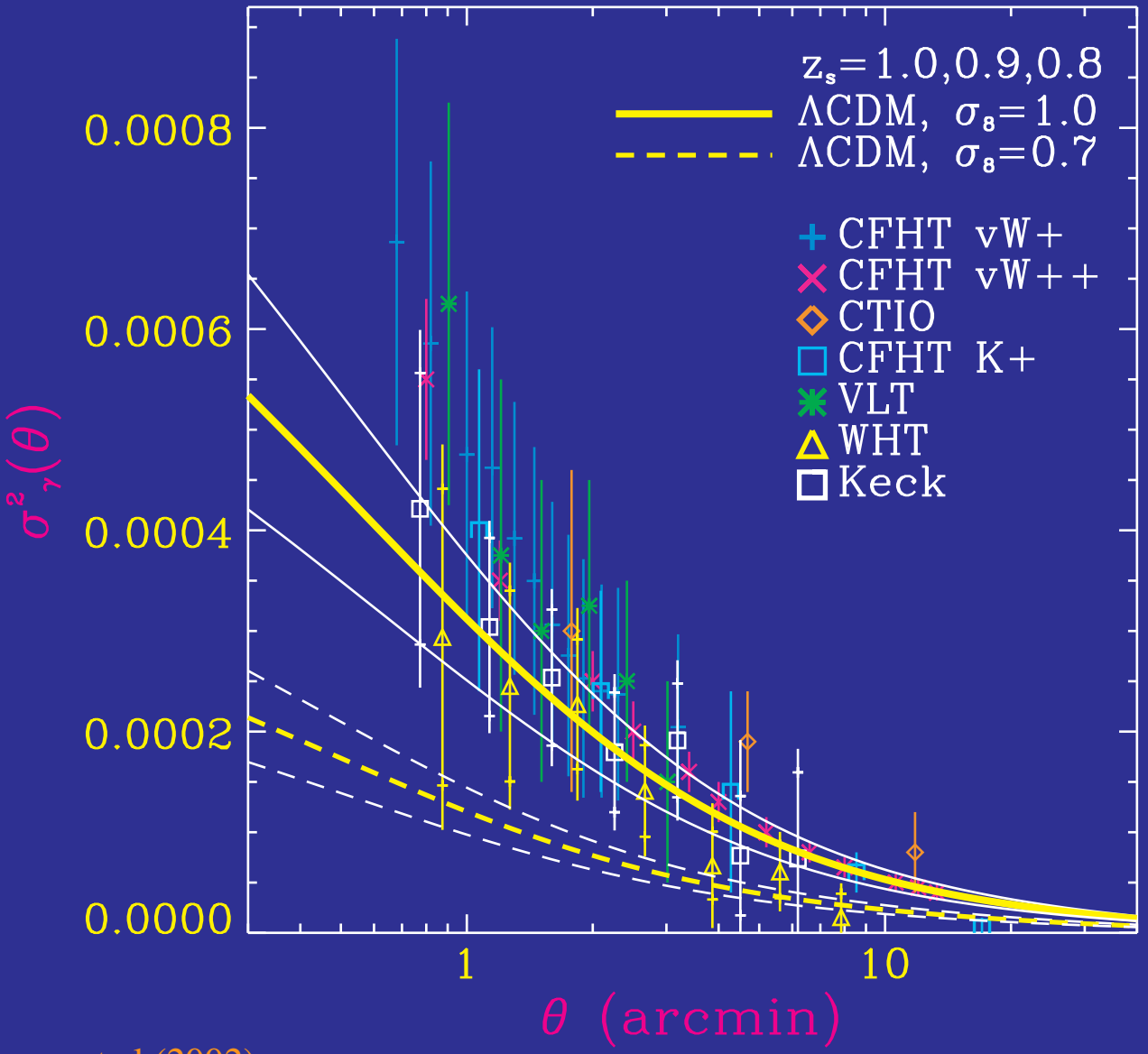
Cosmic Shear Data

- Shear variance as a function of smoothing scale



compilation from Bacon et al (2002)

Hidden Dark Energy Information

- Most of the information on the dark energy is hidden in the temporal or radial dimension
- Evolution of growth rate (dark energy pressure slows growth)
- Evolution of distance-redshift relation

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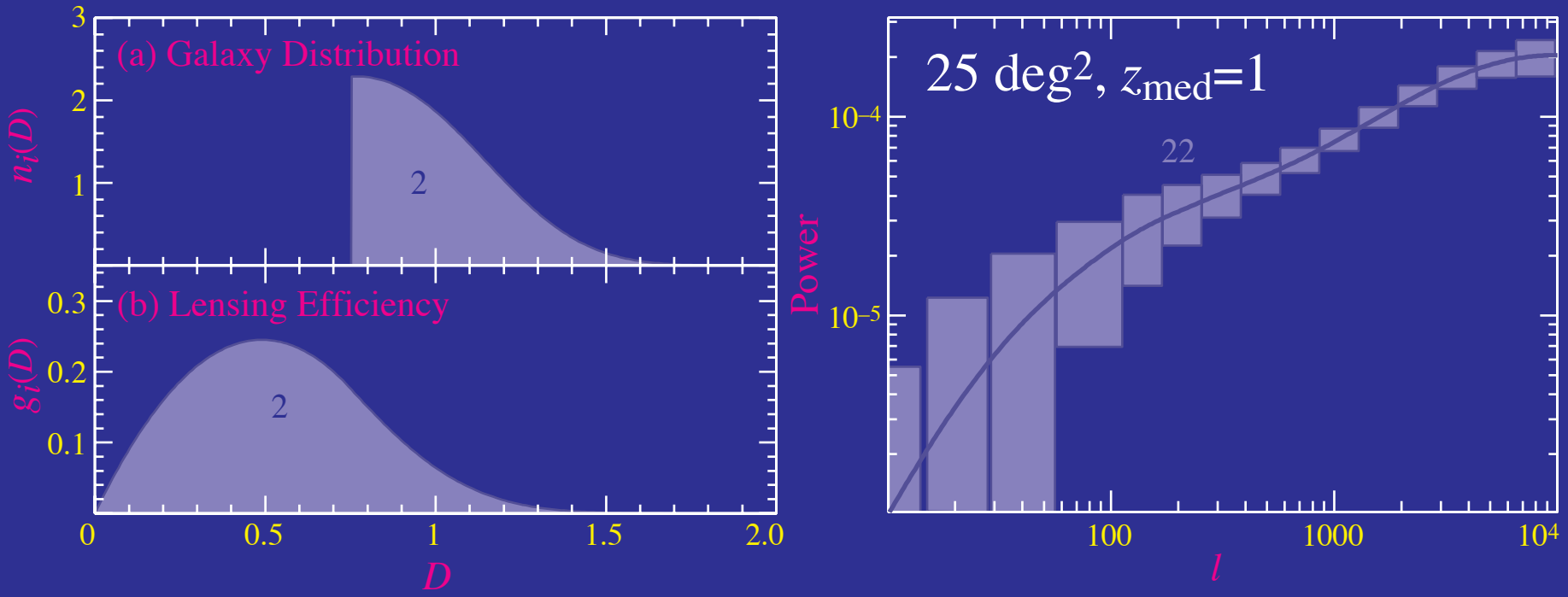
- Lensing is inherently two dimensional: all mass along the line of sight lenses
- Tomography implicitly or explicitly reconstructs radial dimension with source redshifts
- Photometric redshift errors currently $\Delta z < 0.1$ out to $z \sim 1$ and allow for tomography

Crude Tomography

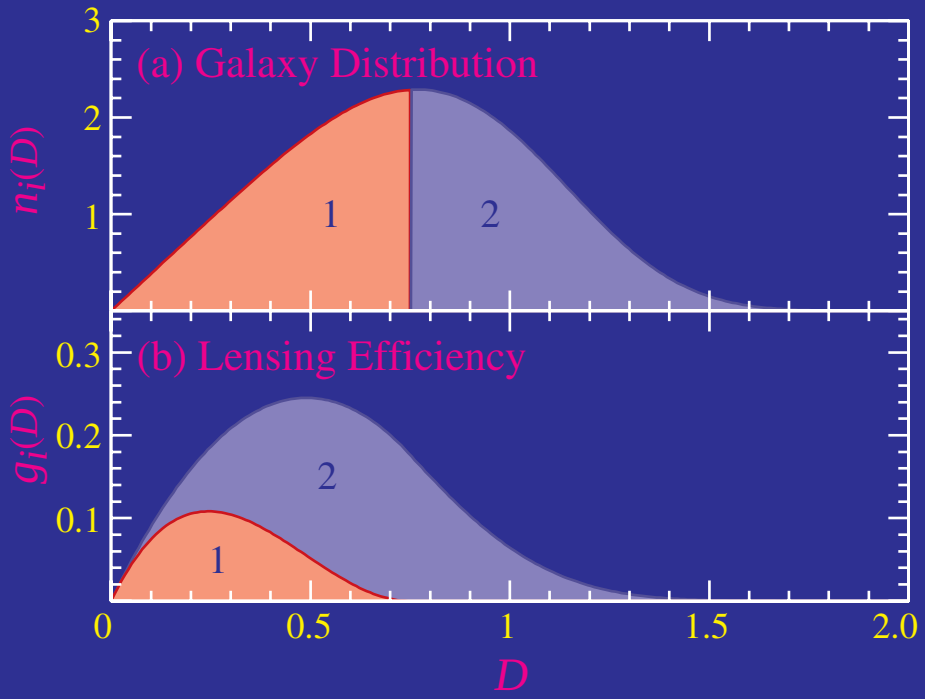
- Divide sample by photometric redshifts

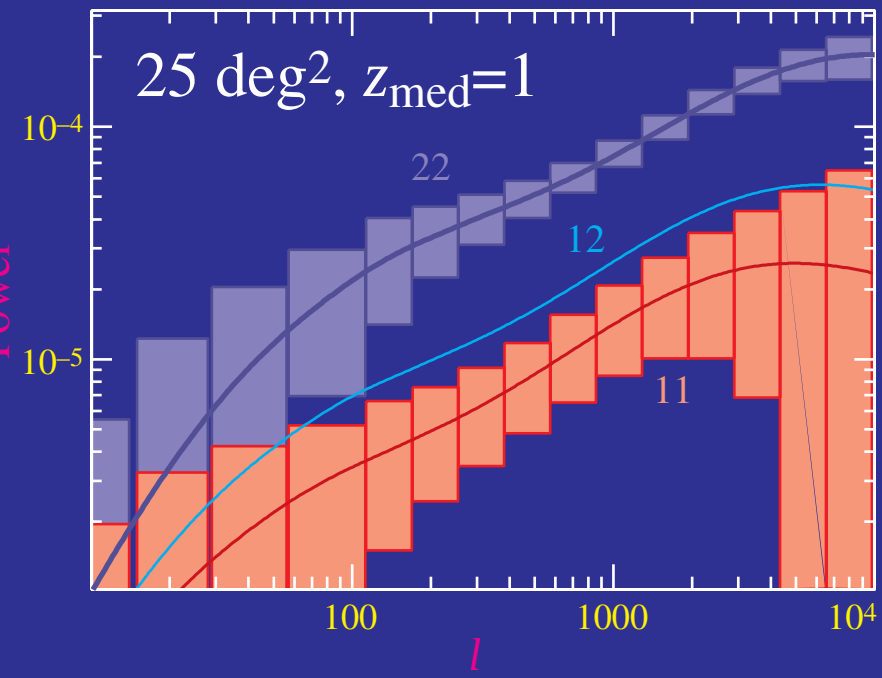


Hu (1999)

Crude Tomography

- Divide sample by photometric redshifts
- Cross correlate samples



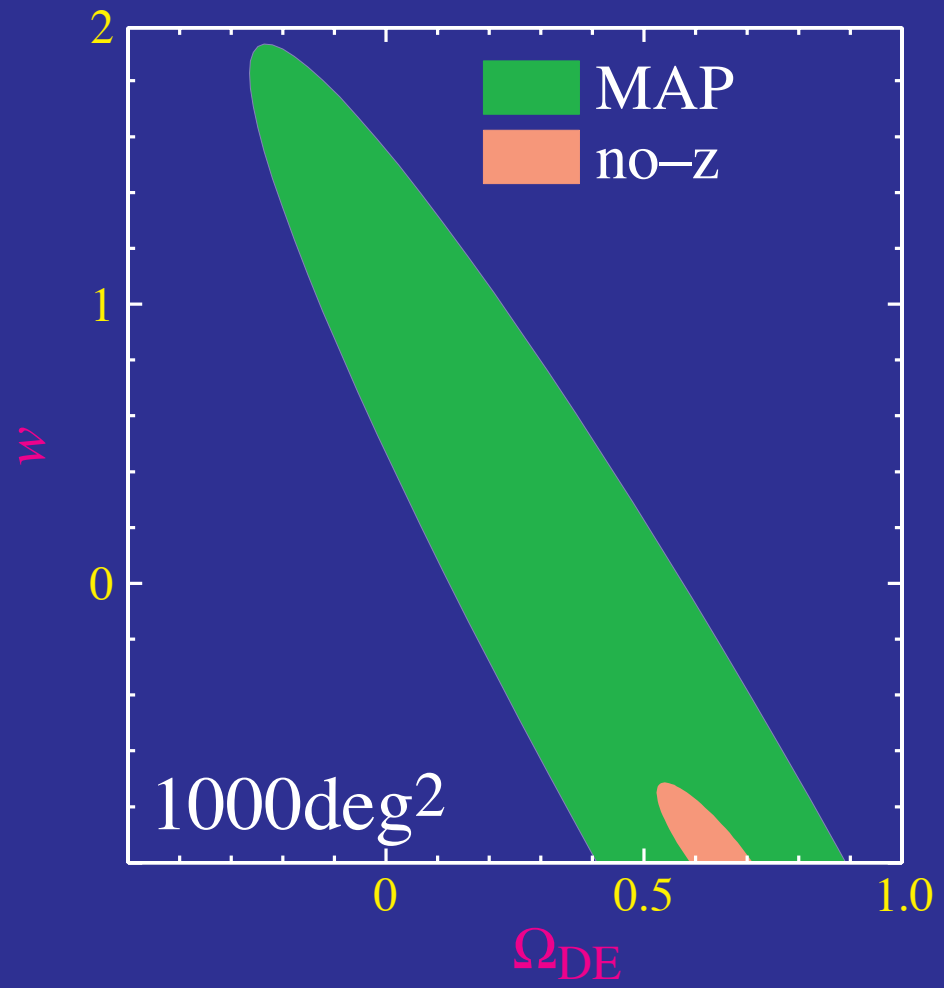


- Order of magnitude increase in precision even after CMB breaks degeneracies

Hu (1999)

Dark Energy & Tomography

- Both CMB and tomography help lensing provide interesting constraints on dark energy



l<3000; 56 gal/deg^2

Hu (2001)

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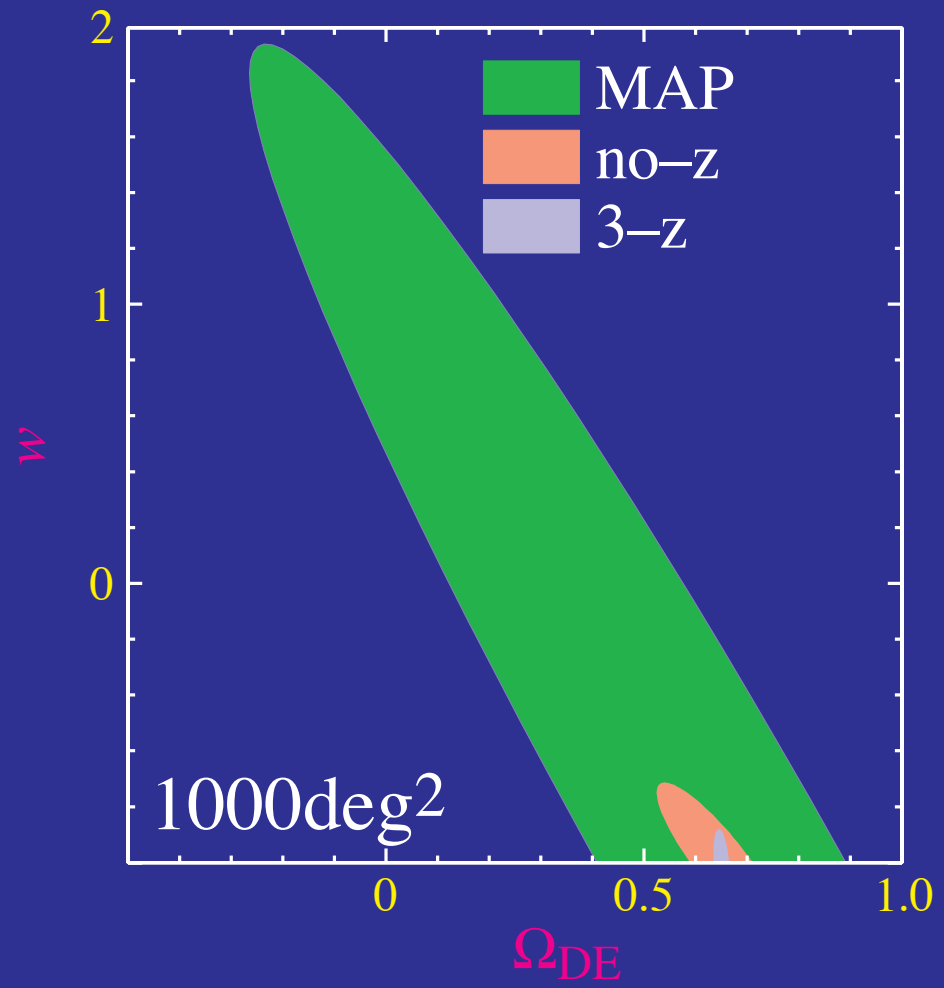

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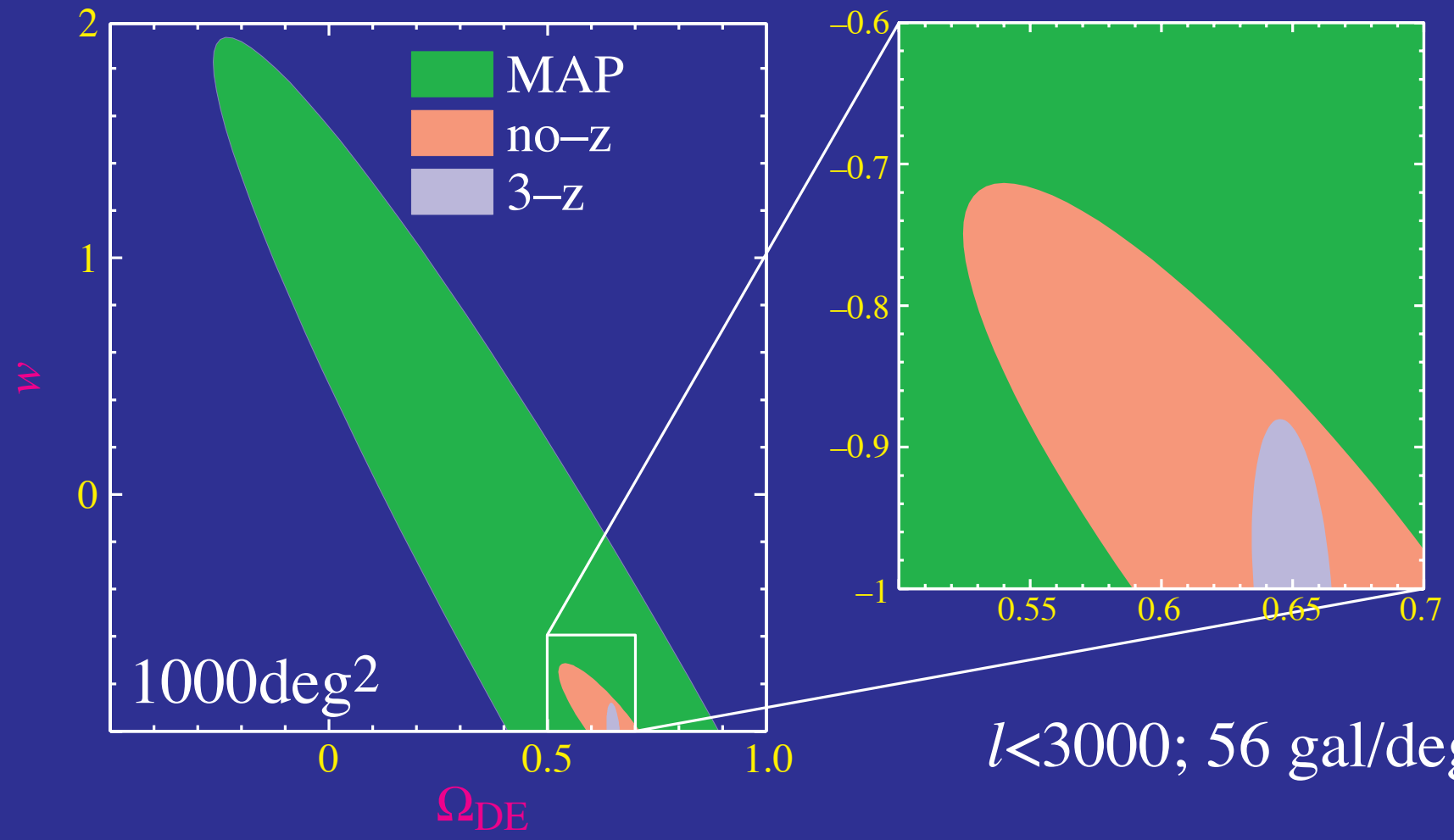

Hu (2001)

Fine Tomography

- Convergence – projection of $\Delta = \delta/a$ for each z_s

$$\kappa(z_s) = \frac{3}{2} H_0^2 \Omega_m \int_0^{z_s} dz \, \frac{dD}{dz} \, \frac{D(D_s - D)}{D_s} \, \Delta \, ,$$

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- Data is linear combination of signal + noise

$$\mathbf{d}_\kappa = \mathbf{P}_{\kappa\Delta} \mathbf{s}_\Delta + \mathbf{n}_\kappa \, ,$$

$$[\mathbf{P}_{\kappa\Delta}]_{ij} = \begin{cases} \frac{3}{2} H_0^2 \Omega_m \delta D_j \frac{(D_{i+1} - D_j) D_j}{D_{i+1}} & D_{i+1} > D_j \, , \\ 0 & D_{i+1} \leq D_j \, , \end{cases}$$

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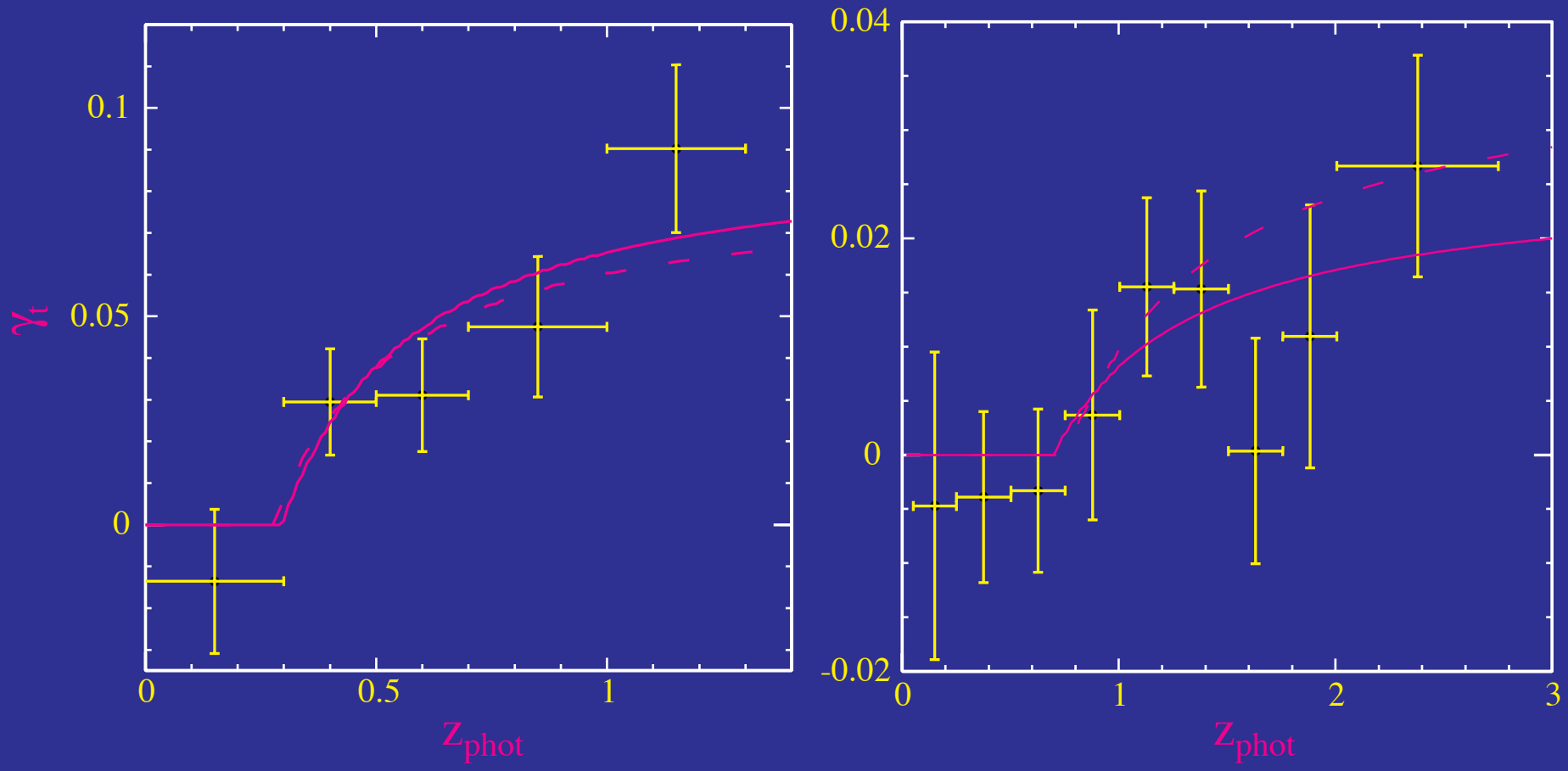
- Well-posed (Taylor 2002) but noisy inversion (Hu & Keeton 2002)
- Noise properties differ from signal properties $\rightarrow$ optimal filters

Tomography in Practice

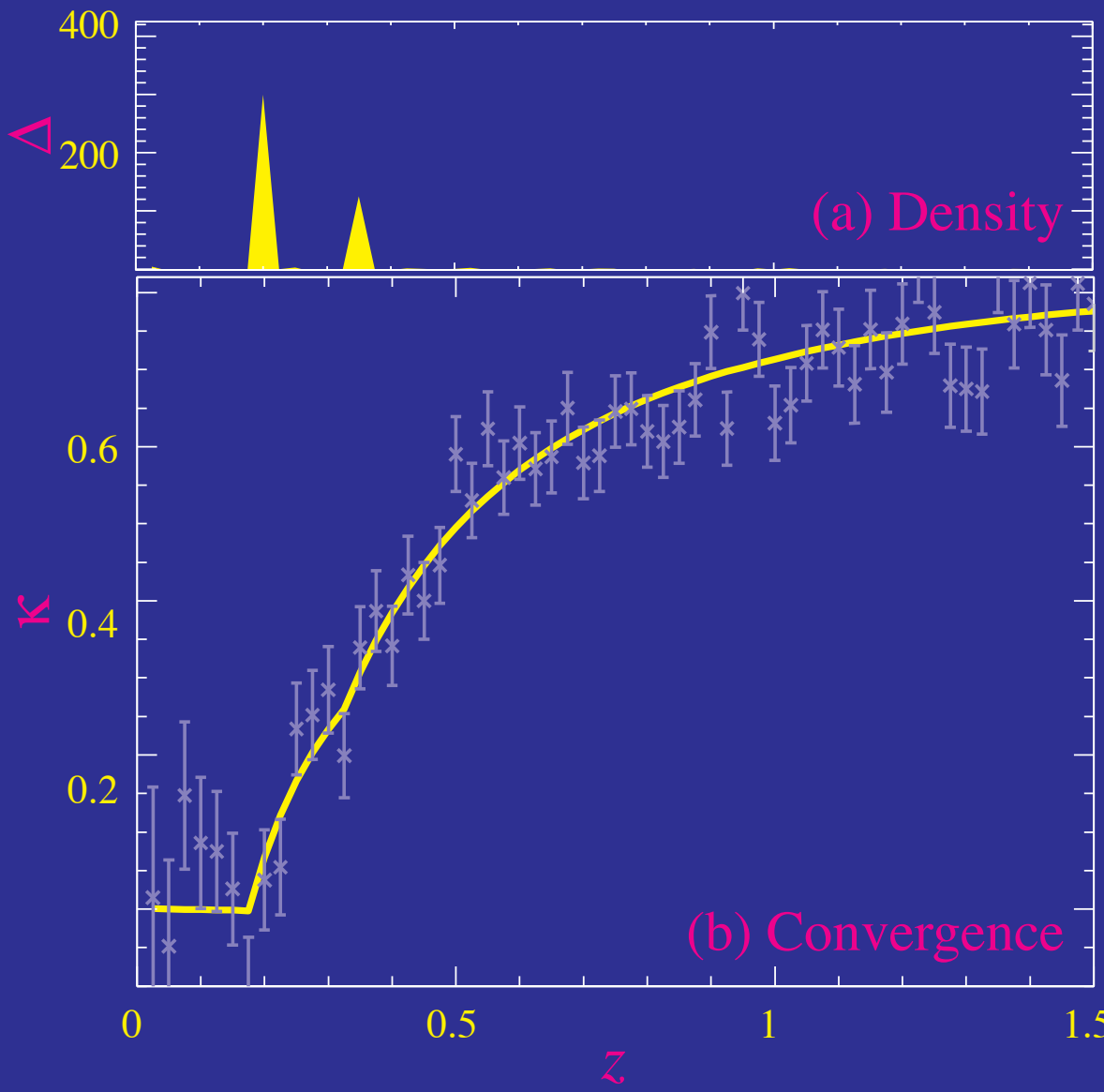
- Localization and selection of clusters



Wittman et al. (2001; 2002)

Tomography in Principle

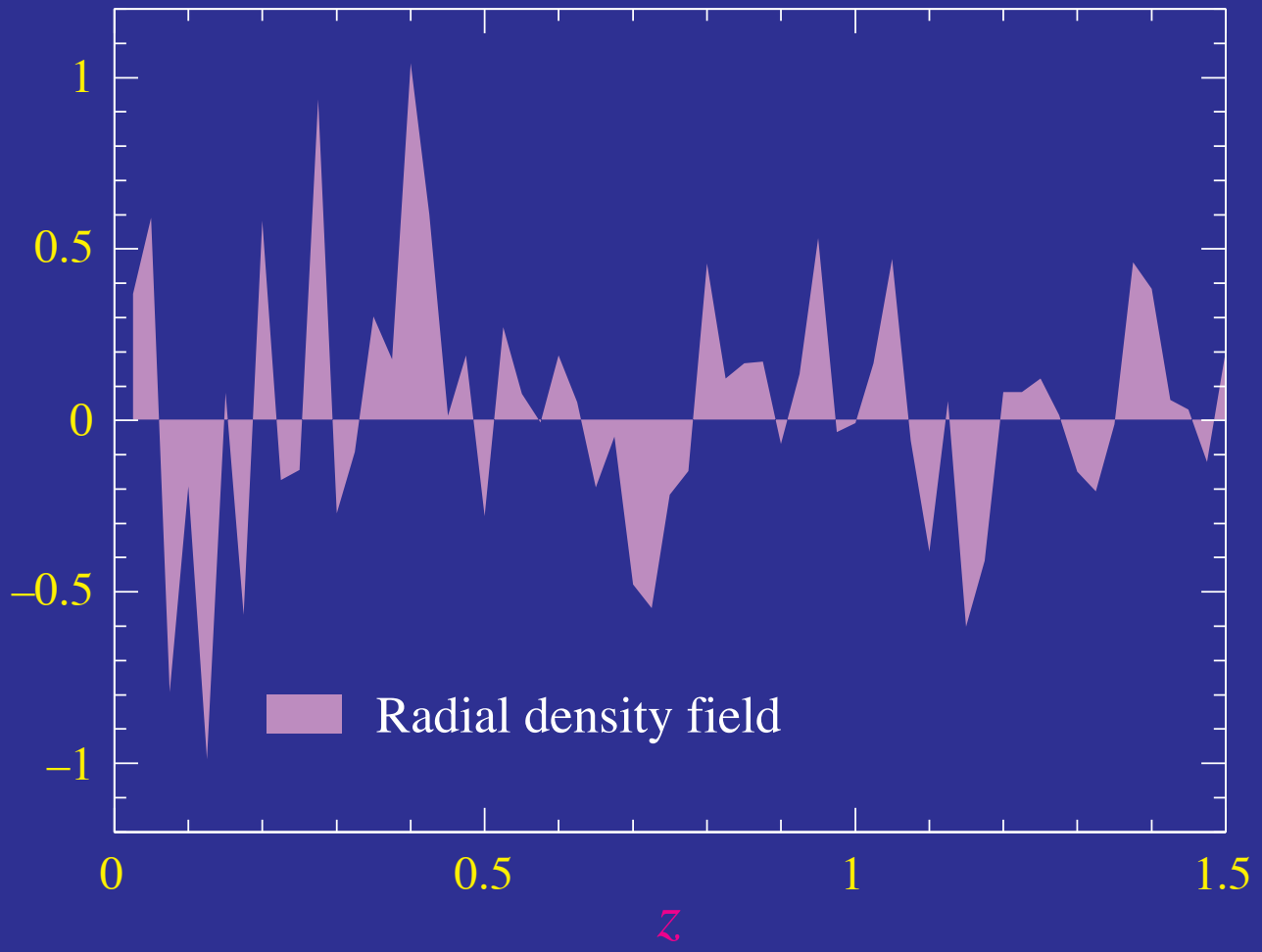
- Derivatives of noisy convergence isolate radial structures



Hu & Keeton (2001)

Fine Tomography

- Tomography can produce direct 3D dark matter maps, but realistically only broad features (Hu & Keeton 2002)



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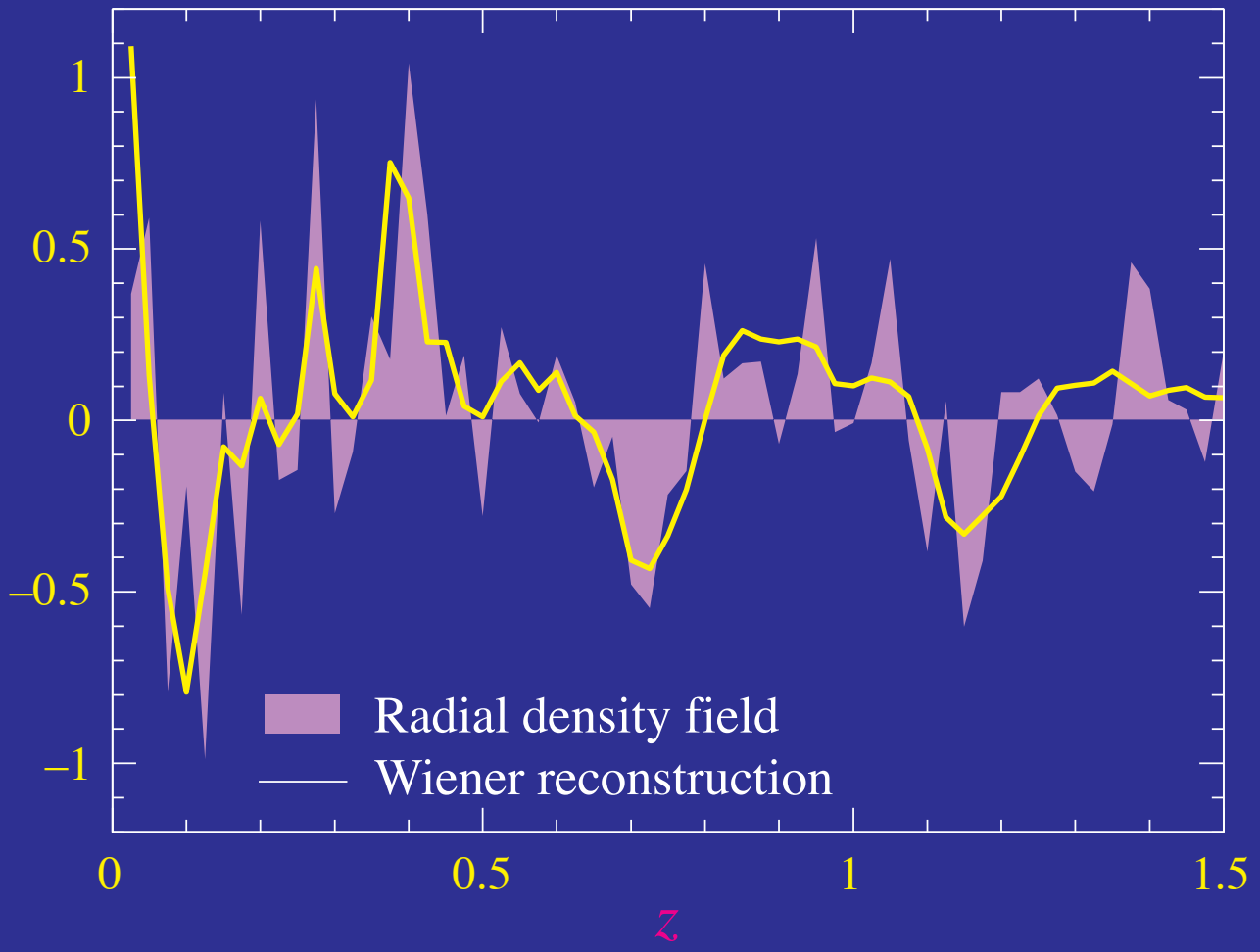

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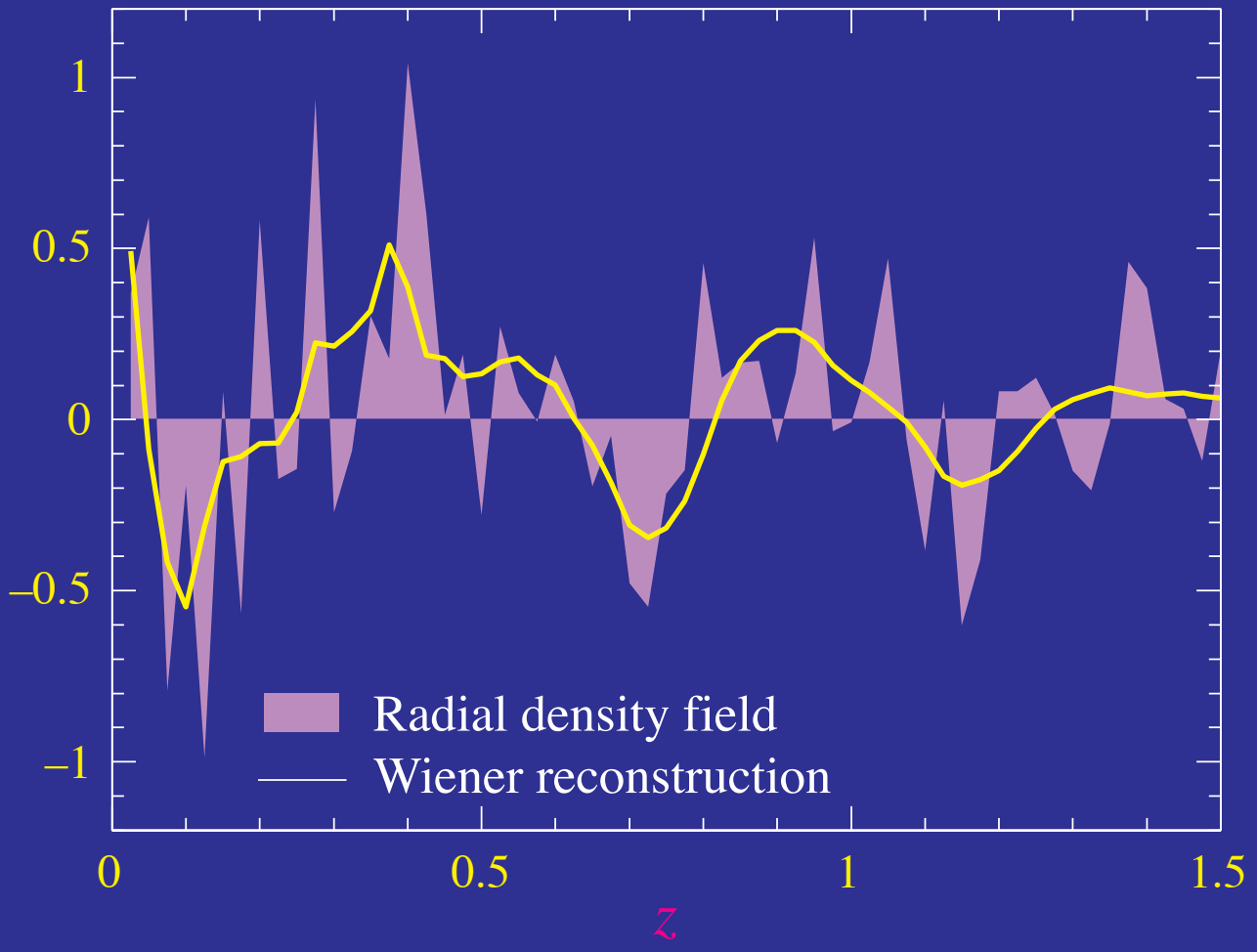

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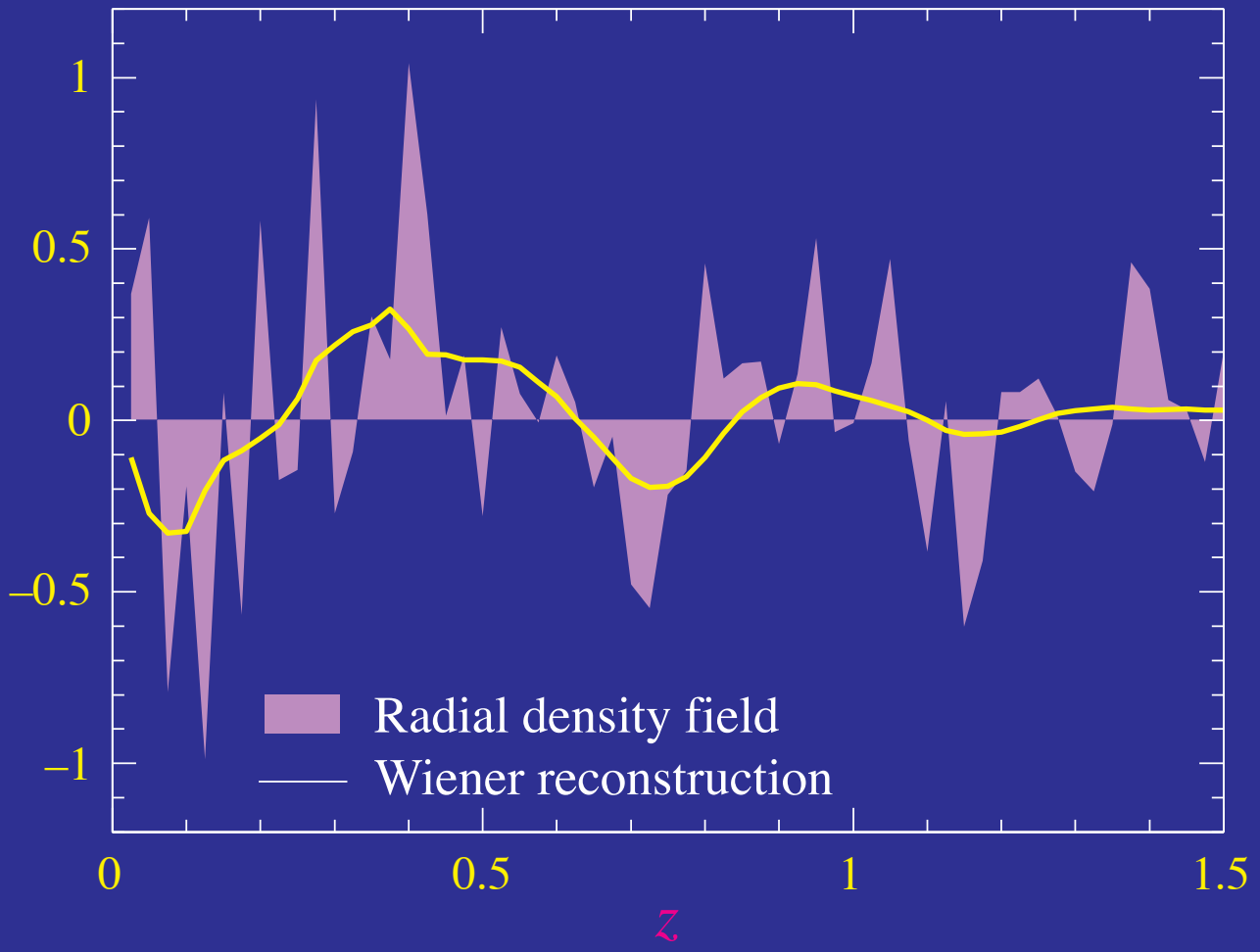

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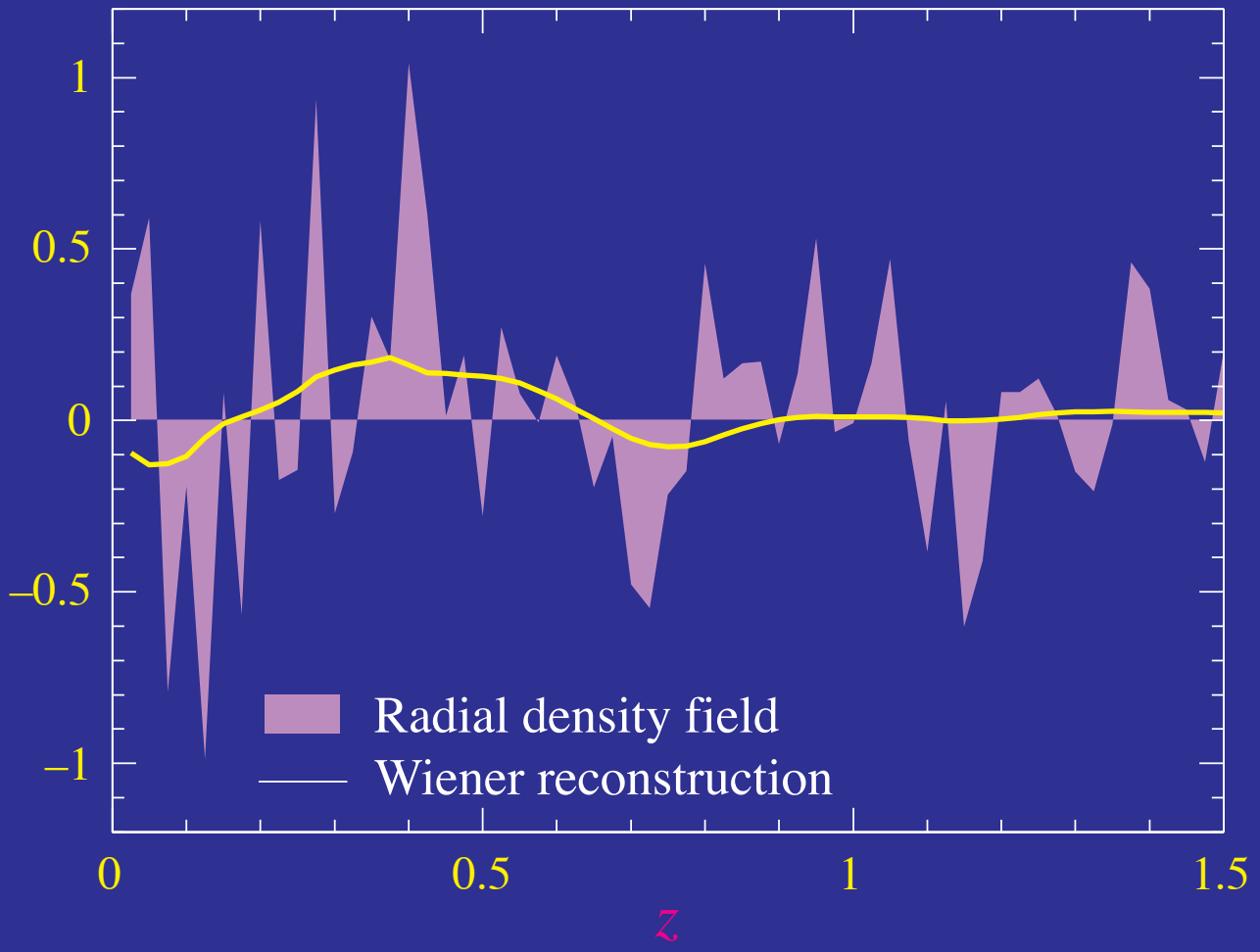
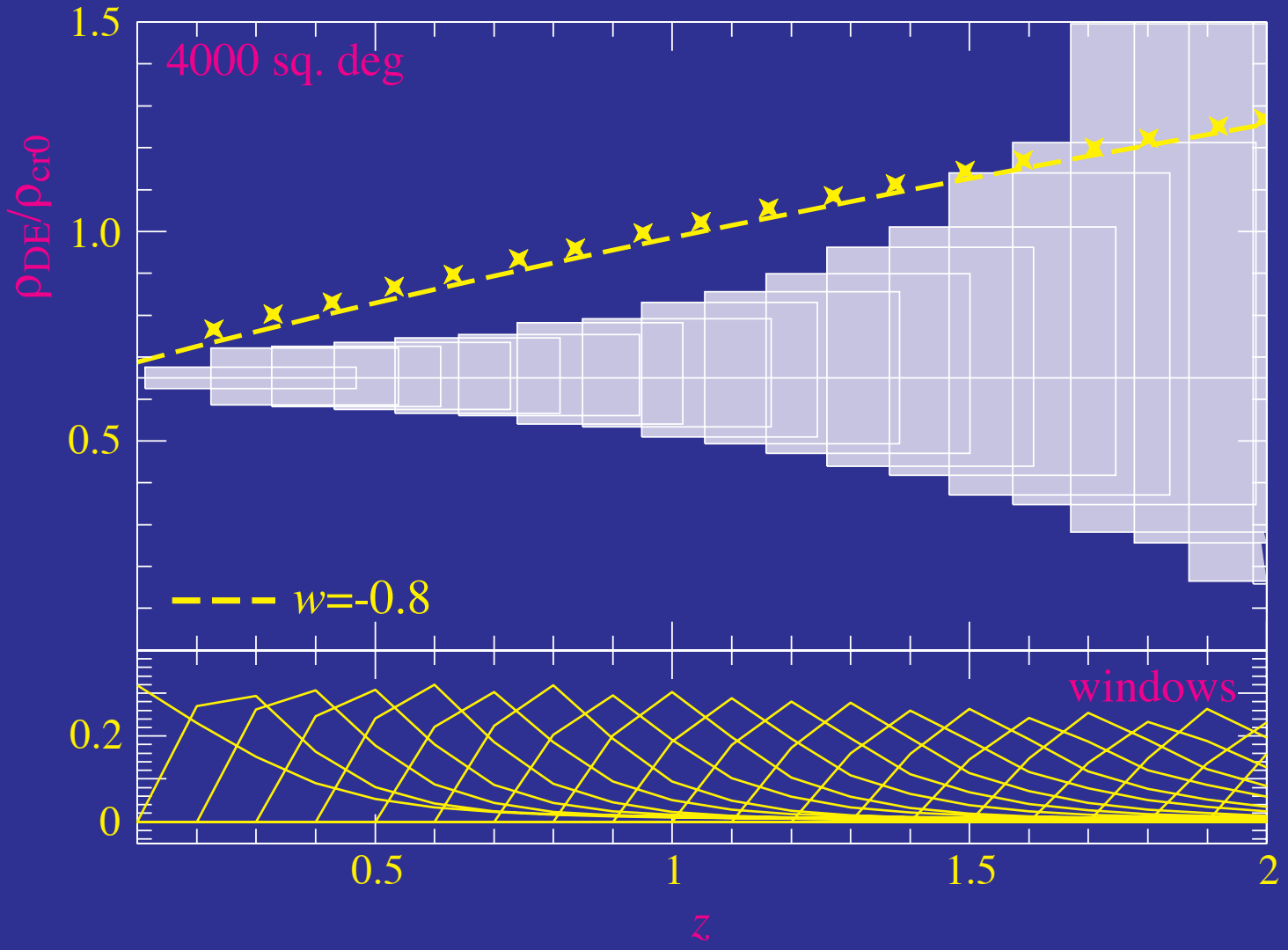


Dark Energy Evolution

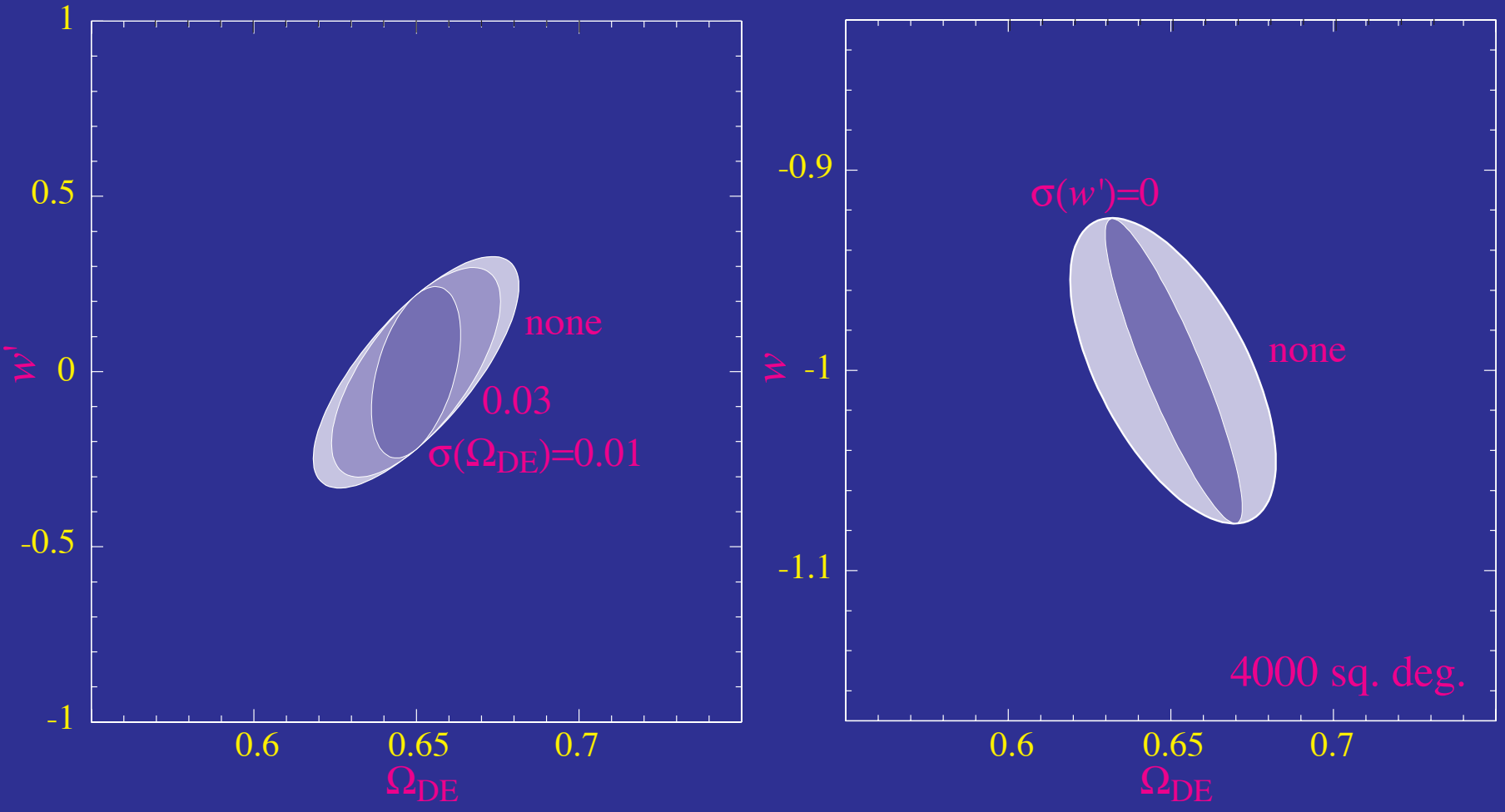
- Crude binning still allows dark energy probe



Hu (2002)

Dark Energy Parameters

- Three parameter dark energy model (Ω_{DE}, w, $dw/dz=w'$)



Hu (2002)

Weak Lensing of CMB Anisotropy

CMB Lensing

- A unique opportunity to probe the large scale mass distribution and so the particle properties of dark energy
- The most distant source plane for tomographic studies or cross-correlations with other tracers
- Relatively large signal by lensing standards $3'$ deflections coherent across 10°

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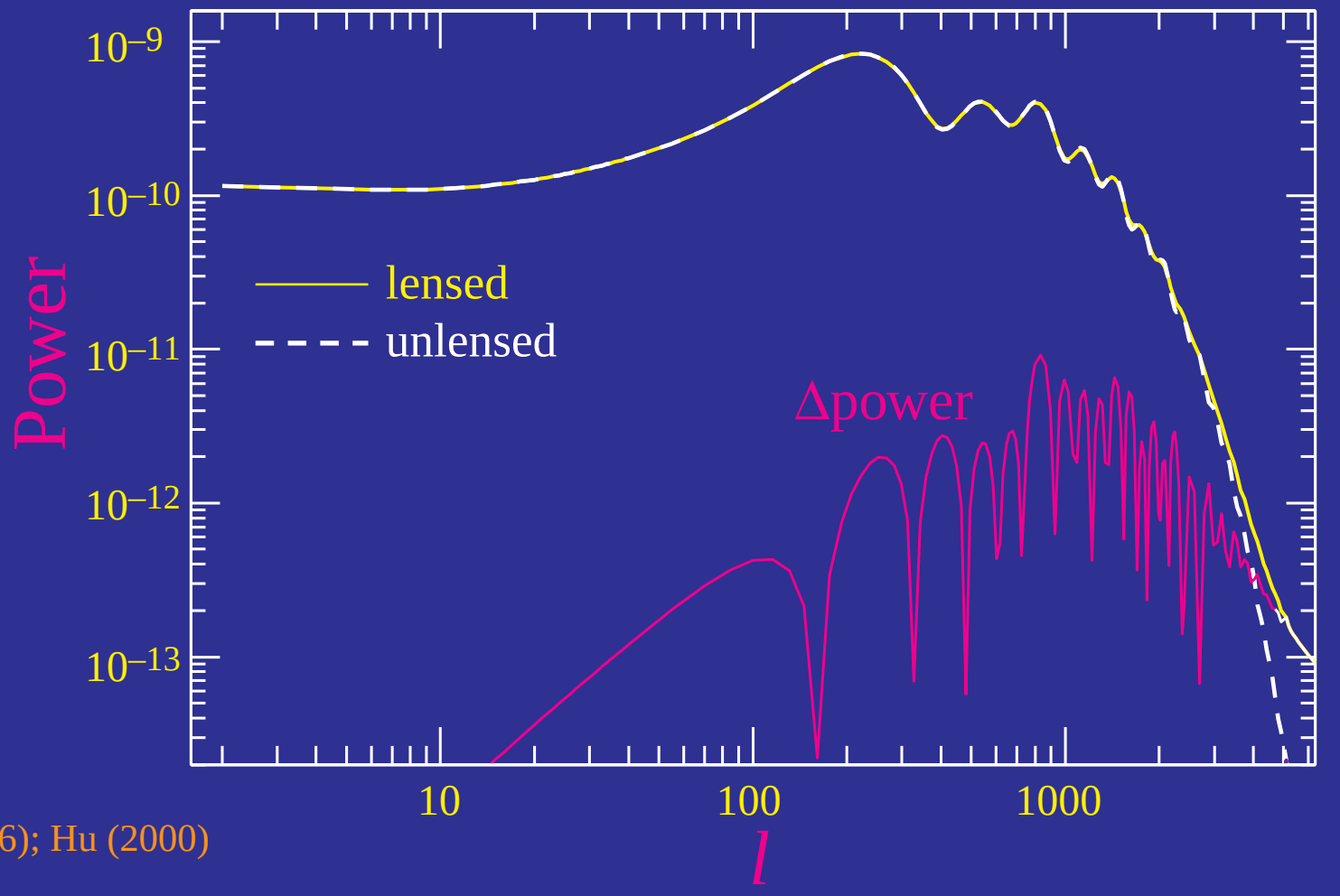
- Conceptual problem? both image and lens are Gaussian random fields
- Practical problem: requires both high angular resolution and wide fields

Lensing in the Power Spectrum

- Lensing smooths the power spectrum with a width $\Delta l \sim 60$
- Convolution with specific kernel: higher order correlations between multipole moments – not apparent in power



Seljak (1996); Hu (2000)

Reconstruction from the CMB

- Correlation between Fourier moments reflect lensing potential $\kappa = \nabla^2\phi$

$$\langle x(\mathbf{l})x'(\mathbf{l}')\rangle_{\rm CMB} = f_\alpha(\mathbf{l},\mathbf{l}')\phi(\mathbf{l}+\mathbf{l}')\,,$$

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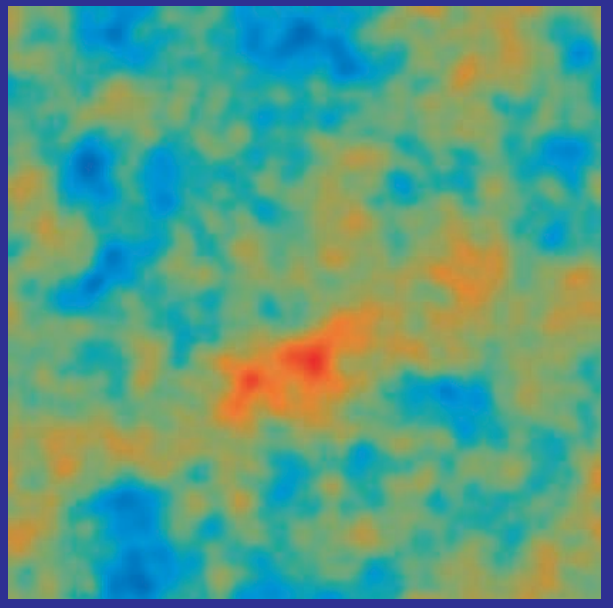
  where $x \in$ temperature, polarization fields and f_α is a fixed weight that reflects geometry

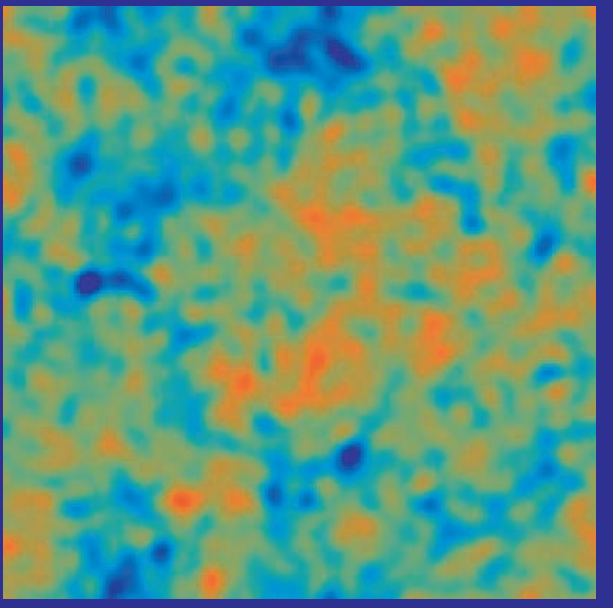
- Each pair forms a noisy estimate of the potential or projected mass - just like a pair of galaxy shears
- Minimum variance weight all pairs to form an estimator of the lensing mass

Ultimate (Cosmic Variance) Limit

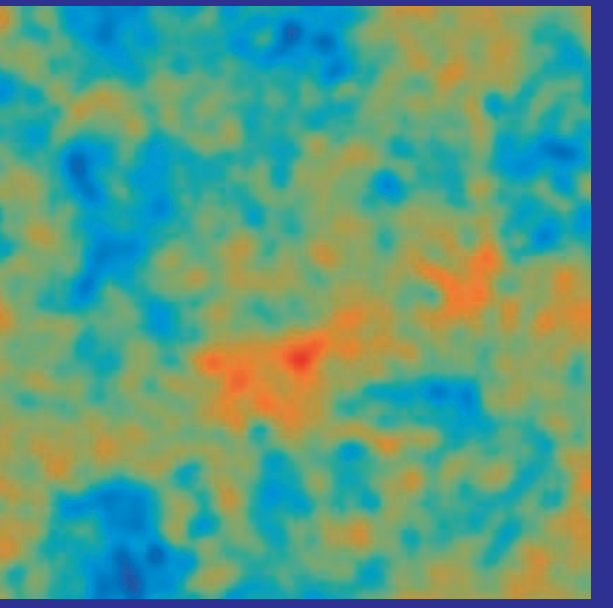
- Cosmic variance of CMB fields sets ultimate limit
- Polarization allows mapping to finer scales (~10')



mass



temp. reconstruction



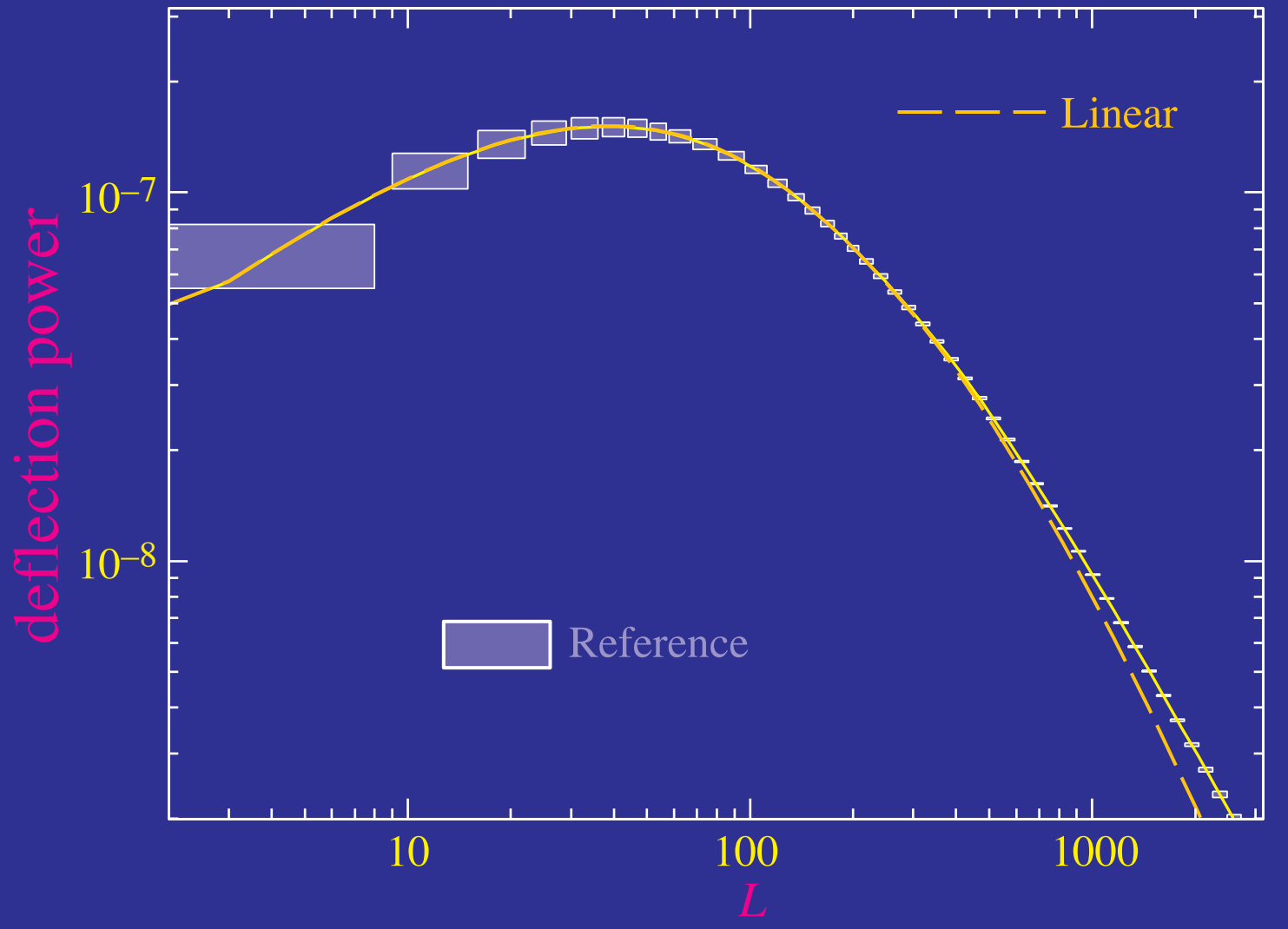
EB pol. reconstruction

100 sq. deg; 4' beam; 1μK-arcmin

Hu & Okamoto (2001)

Matter Power Spectrum

- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime $0.002 < k < 0.2\ h/\mathrm{Mpc}$



Hu & Okamoto (2001)

$\sigma(w)\sim 0.06$

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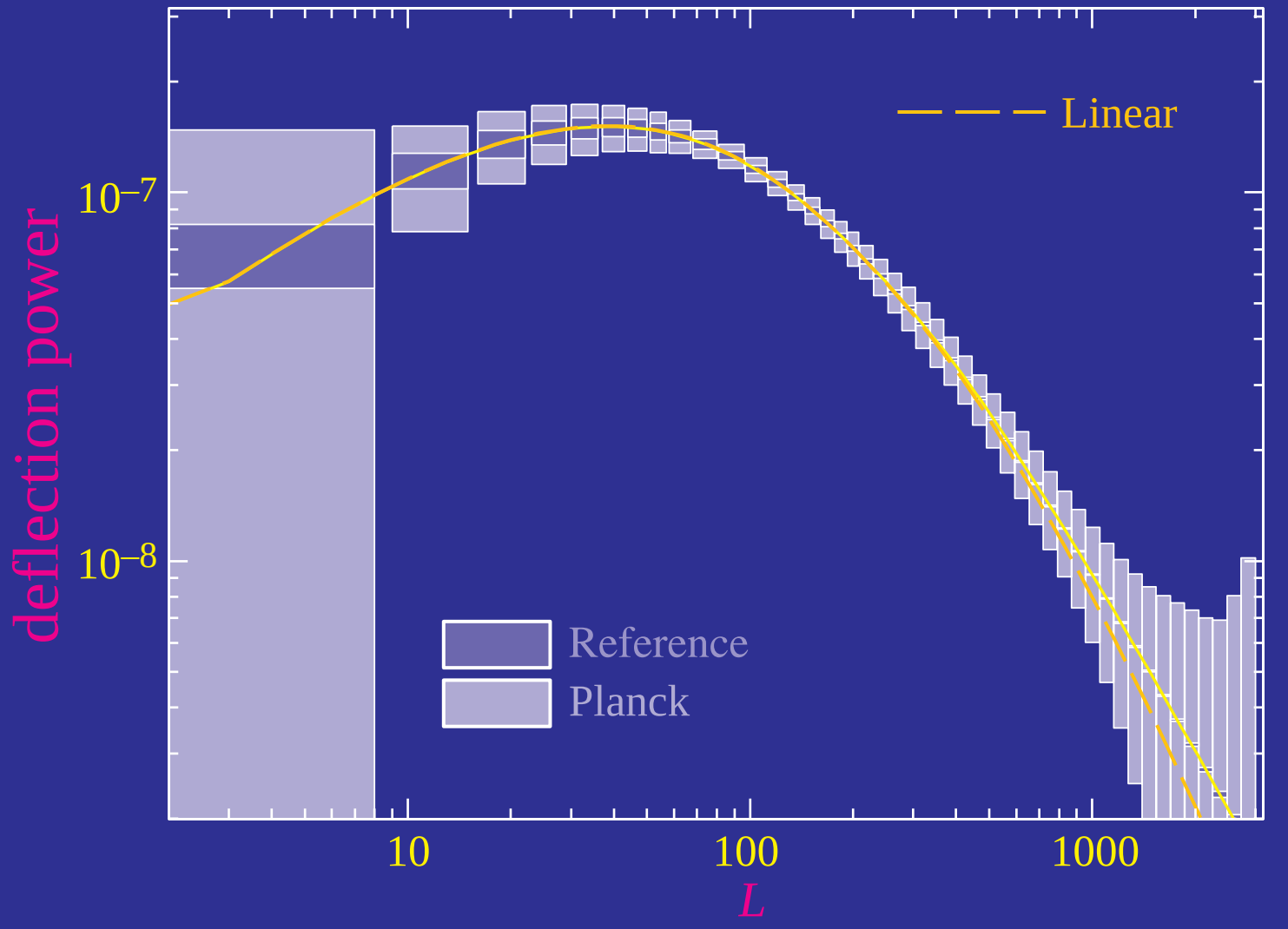
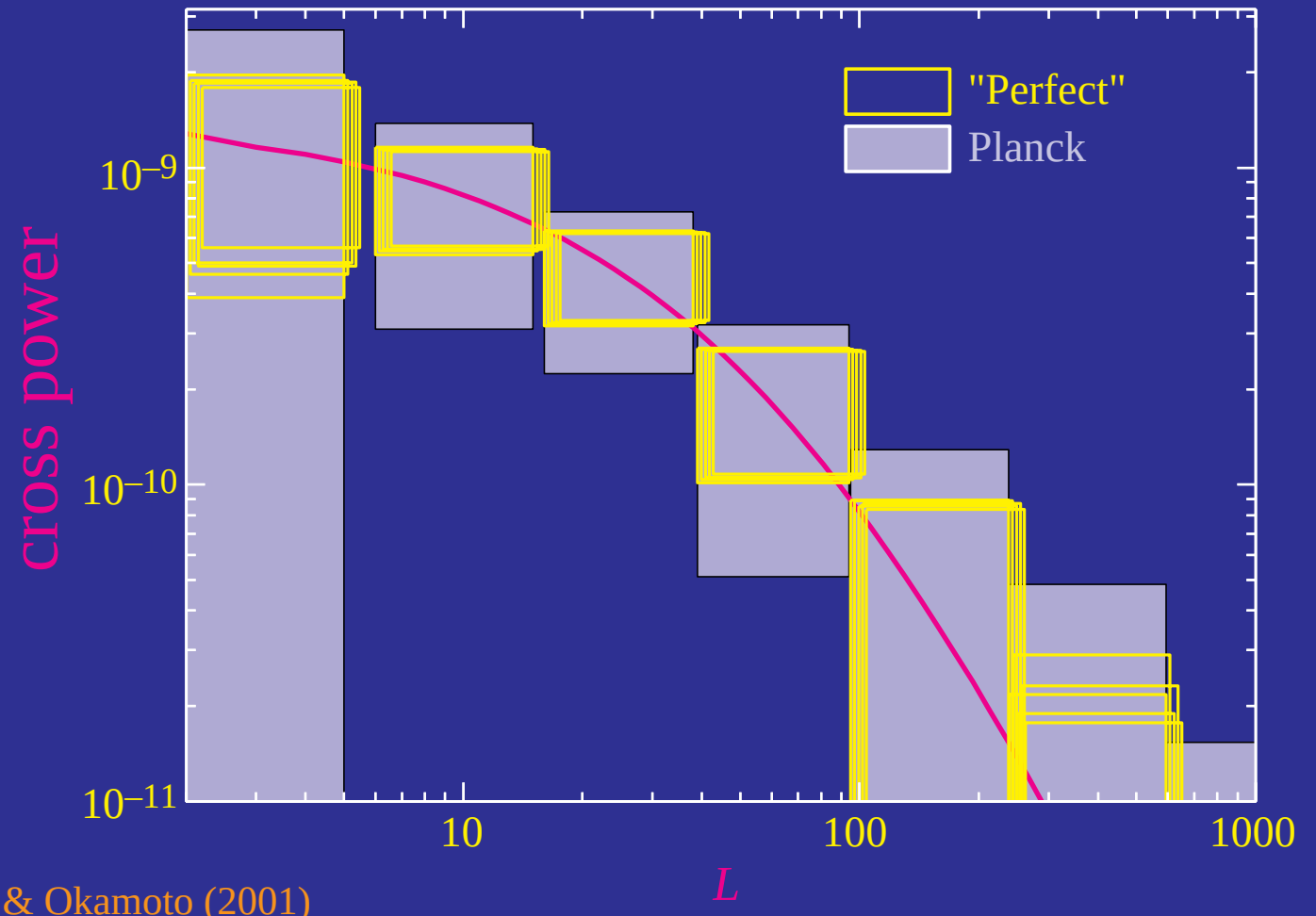


Hu & Okamoto (2001)

$\sigma(w)$~0.06; 0.14

Particle Properties of the Dark Energy

- Any correlation is a direct detection of a smooth energy density component through the ISW effect
- 5 nearly independent measures in temperature & polarization



Hu (2001); Hu & Okamoto (2001)

Particle Properties of the Dark Energy

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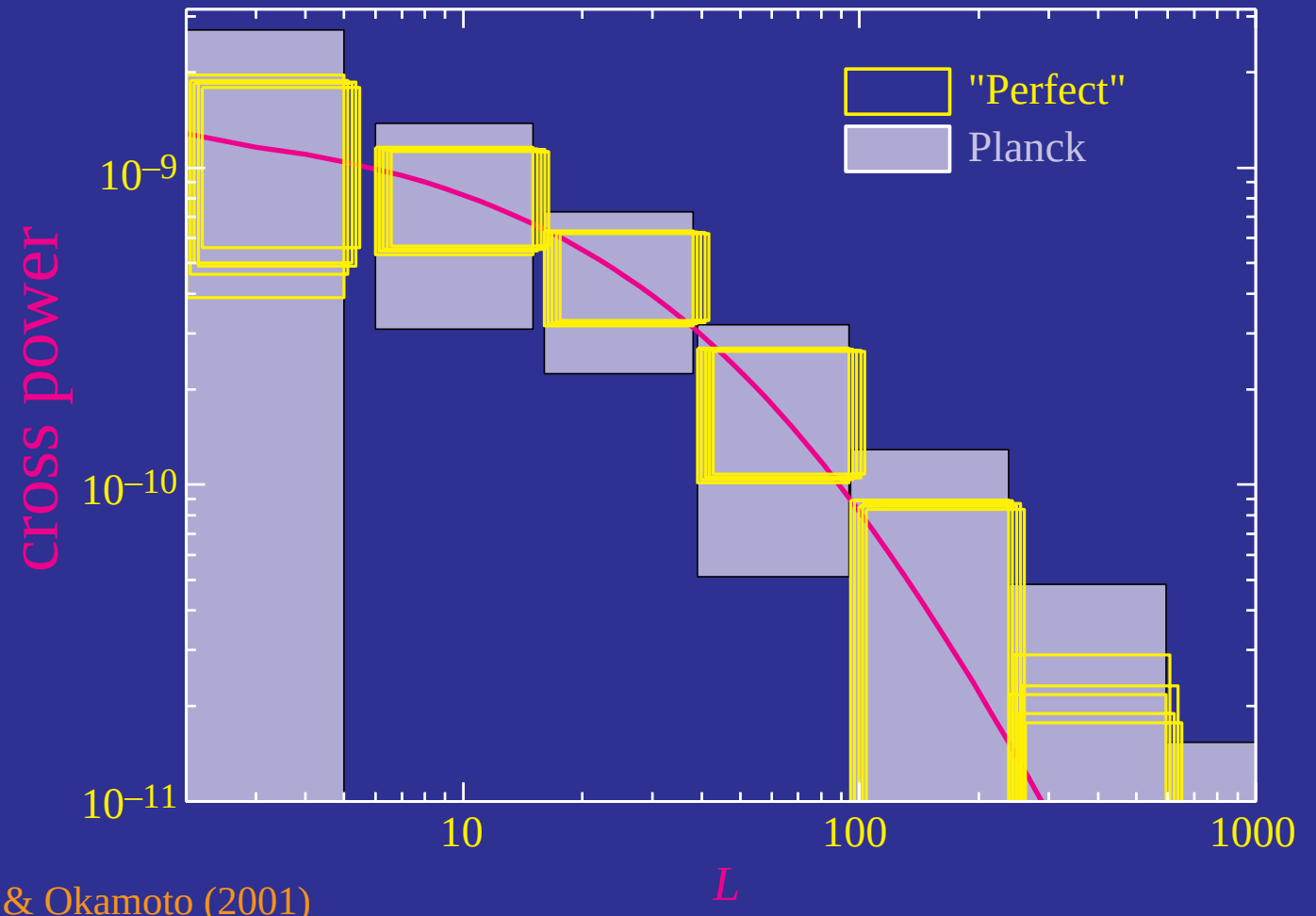
- Show dark energy smooth >5-6 Gpc scale, test quintesence



Hu (2001); Hu & Okamoto (2001)

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- Gravitational lensing avoids ambiguity, utilizes luminous objects only as background image

- Evolution of dark energy can be extracted tomographically
- Clustering of dark energy (test of scalar field paradigm) extractable from wide-field CMB lensing