Outline

• Dark Energy vs Modified Gravity

• Three Regimes of Modified Gravity

• Worked (Toy) Models: $f(R)$ and DGP Braneworld

• Collaborators

  Wenjuan Fang  Dragan Huterer
  Marcos Lima  Lucas Lombriser
  Michael Mortonson  Hiro Oyaizu
  Fabian Schmidt  Hiranya Peiris
  Iggy Sawicki  Sanjeev Seehra
  Uros Seljak  Yong-Seon Song
  Anze Slosar  Amol Upadhye
  Sheng Wang  Alexey Vikhlinin
Charting Out the Expansion

- **Standard candle:** apparent brightness of objects with a fixed luminosity to judge distance
- **Standard ruler:** apparent (angular) separation of objects with a fixed physical separation to judge distance

Supernovae

1998 Discovery

Sound waves
CMB+Galaxies
Mercury or Pluto?

- General relativity says $\text{Gravity} = \text{Geometry}$

- And $\text{Geometry} = \text{Matter-Energy}$

- Could the **missing energy** required by **acceleration** be an **incomplete** description of how **matter determines geometry**?
Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content.

- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy.

\[
F(g_{\mu\nu}) + G_{\mu\nu} = 8\pi G T^M_{\mu\nu} - F(g_{\mu\nu}) = 8\pi G T^{DE}_{\mu\nu}
\]

\[
G_{\mu\nu} = 8\pi G [T^M_{\mu\nu} + T^{DE}_{\mu\nu}]
\]

and the Bianchi identity guarantees \( \nabla^\mu T^{DE}_{\mu\nu} = 0 \)

- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor.

- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress.
Modified Gravity ≠ “Smooth DE”

- **Scalar field** dark energy has $\delta p = \delta \rho$ (in constant field gauge) – relativistic sound speed, no anisotropic stress

- **Jeans stability** implies that its energy density is spatially smooth compared with the matter below the sound horizon

$$ds^2 = -(1 + 2\Psi) dt^2 + a^2 (1 + 2\Phi) dx^2$$

$$\nabla^2 (\Phi - \Psi) \propto \text{matter density fluctuation}$$

- **Anisotropic stress** changes the amount of space curvature per unit dynamical mass

$$\nabla^2 (\Phi + \Psi) \propto \text{anisotropic stress}$$

but its absence in a smooth dark energy model makes

$$g = (\Phi + \Psi)/(\Phi - \Psi) = 0$$

for non-relativistic matter.
Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate

- Hence geometric measurements of the expansion rate predict the growth of structure
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations

- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)
Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way

Cosmological Constant
- Deviation significantly >2% rules out $\Lambda$ with or without curvature

Quintessence
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in $H_0$]
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in a highly predictive way

Mortonson, Hu, Huterer (2009)
Quintessence Falsified?

• No excess numbers of massive $z \geq 1$ X-ray or SZ clusters with Gaussian initial conditions (Jee et al 2009, Brodwin et al 2010)

• No excess power in gravitational lensing at high $z$ relative to low $z$ (Bean 0909.3853)

• But would such violations favor modified gravity?

• Given astrophysical systematics, expect purported $2\sigma$ violations of smooth dark energy predictions will be common in coming years!
Dynamical vs Lensing Mass

- Newtonian potential: $\Psi = \delta g_{00}/2g_{00}$ which non-relativistic particles feel

- Space curvature: $\Phi = \delta g_{ii}/2g_{ii}$ which also deflects photons

- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass
Dynamical v Strong Lensing

- Comparison of strong lensing and dynamical mass assuming a density profile and velocity dispersion data
- Mean exhibits a bias from GR expectation with statistical errors only
- No mass trend detectable
Lensing v Dynamical Comparison

- Gravitational lensing around galaxies vs. linear velocity field (through redshift space distortions and galaxy autocorrelation)
- Consistent with GR + smooth dark energy beginning to test interesting models


Falsify in Favor of What?

- Modified gravity models change space curvature per unit dynamical mass - enhanced or reduced forces on matter
- Requires two closure relations - 1st an effective anisotropic stress that distinguishes lensing from dynamical mass
- Viable induced modifications exhibit three separate regimes
  - Horizon Scale
  - Scalar-Tensor
  - General Relativistic
- Choice of lensing mass contribution as 2nd parameter in scalar-tensor regime favored by conformal invariance of E&M (Hu & Sawicki 2007; see also Caldwell et al 2007; Amendola et al 2007)

CAMB Package for Linearized PPF: http://camb.info/ppf

Other uses: phantom crossing dark energy (Fang, Hu, Lewis 2009), dark energy PCs (Mortonson, Hu, Huterer 2009) cascading gravity (Afshordi, Geshnizjani, Khoury 2008)
Three Regimes

- Three regimes with different dynamics
- Examples $f(R)$ and DGP braneworld acceleration
- Parameterized Post-Friedmann description
- Non-linear regime return to General Relativity / Newtonian dynamics

General Relativistic Non-Linear Regime
Scalar-Tensor Regime
Conserved-Curvature Regime

$r_*$

$halos, galaxy$

$large scale structure$

$CMB$
Worked Examples
Modified Action $f(R)$ Model

- $R$: Ricci scalar or “curvature”
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2f/dR^2$: Compton wavelength of $f_R$ squared, inverse mass squared
- $B$: Compton wavelength of $f_R$ squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R R'} \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate
Modified Einstein Equation

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved

\[ G_{\alpha\beta} + f_R R_{\alpha\beta} - \left( \frac{f}{2} - \Box f_R \right) g_{\alpha\beta} - \nabla_\alpha \nabla_\beta f_R = 8\pi G T_{\alpha\beta} \]

- Trace can be interpreted as a scalar field equation for \( f_R \) with a density-dependent effective potential (\( \rho = 0 \))

\[ 3\Box f_R + f_R R - 2f = R - 8\pi G \rho \]

- For small deviations, \( |f_R| \ll 1 \) and \( |f/R| \ll 1 \),

\[ \Box f_R \approx \frac{1}{3} (R - 8\pi G \rho) \]

the field is sourced by the deviation from GR relation between curvature and density and has a mass

\[ m_{fR}^2 \approx \frac{1}{3} \frac{\partial R}{\partial f_R} = \frac{1}{3 f_{RR}} \]
DGP Braneworld Acceleration

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

\[ S = \int d^5 x \sqrt{-g} \left[ \frac{(5) R}{2\kappa^2} + \delta(\chi) \left( \frac{(4) R}{2\mu^2} + \mathcal{L}_m \right) \right] \]

with crossover scale \( r_c = \kappa^2 / 2\mu^2 \)

- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)

- Matter still minimally coupled and conserved

- Exhibits the 3 regimes of modified gravity

- Weyl tensor anisotropy dominated conserved curvature regime \( r > r_c \) (Sawicki, Song, Hu 2006; Cardoso et al 2007)

- Brane bending scalar tensor regime \( r_\ast < r < r_c \) (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)

- Strong coupling General Relativistic regime \( r < r_\ast = (r_c^2 r_g)^{1/3} \)

where \( r_g = 2GM \) (Dvali 2006)
DGP Field Equations

- **DGP field equations**

\[ G_{\mu\nu} = 4r_c^2 f_{\mu\nu} - E_{\mu\nu} \]

where \( f_{\mu\nu} \) is a tensor quadratic in the 4-dimensional Einstein and energy-momentum tensors

\[ f_{\mu\nu} \equiv \frac{1}{12} A A_{\mu\nu} - \frac{1}{4} A^\alpha_\mu A^\alpha_{\nu} + \frac{1}{8} g_{\mu\nu} \left( A_{\alpha\beta} A^{\alpha\beta} - \frac{A^2}{3} \right) \]

\[ A_{\mu\nu} \equiv G_{\mu\nu} - \mu^2 T_{\mu\nu} \]

and \( E_{\mu\nu} \) is the bulk Weyl tensor

- **Background metric yields the modified Friedmann equation**

\[ H^2 + \frac{H}{r_c} = \frac{\mu^2 \rho}{3} \]

- For perturbations, involves solving metric perturbations in the bulk through the “master equation”
• Calculation of the metric ratio \( g = \frac{\Phi + \Psi}{\Phi - \Psi} \) requires solving for the propagation of metric fluctuations into the bulk

• Encapsulated in the off brane gradient which closes the system (e.g. normal branch \( g = -\frac{1}{(2Hr_C + 1)} \) until deep in de Sitter)

$f(R)$ Expansion History
Engineering $f(R)$ Models

- Mimic $\Lambda$CDM at high redshift
- Accelerate the expansion at low redshift without a cosmological constant
- Sufficient freedom to vary expansion history within observationally allowed range
- Contain the phenomenology of $\Lambda$CDM in both cosmology and solar system tests as a limiting case for the purposes of constraining small deviations
- Suggests

$$f(R) \propto \frac{R^n}{R^n + \text{const.}}$$

such that modifications vanish as $R \to 0$ and go to a constant as $R \to \infty$
Form of $f(R)$ Models

- Transition from **zero** to **constant** across an adjustable curvature scale
- Slope $n$ controls the **rapidity** of transition, field amplitude $f_{R0}$ **position**
- Background **curvature** stops declining during acceleration epoch and thereafter behaves like **cosmological constant**

Hu & Sawicki (2007)
Expansion History

- Effective equation of state $w_{\text{eff}}$ scales with field amplitude $f_{R0}$

- Crosses the phantom divide at a redshift that decreases with $n$

- Signature of degrees of freedom in dark energy beyond standard kinetic and potential energy of k-essence or quintessence or modified gravity

Hu & Sawicki (2007)
DGP Expansion History
DGP Expansion History

- Matching the DGP expansion history to a dark energy model with the same expansion history
- Effective equation of state \( w(z) \) \([w_0 \sim -0.85, \, w_a \sim 0.35]\)

Song, Sawicki & Hu (2006)
DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$
DGP Normal Branch

- On the normal branch, expansion does not self-accelerate and dark energy in the form of a brane tension or scalar field necessary

\[ H^2 + \frac{H}{r_c} = \frac{\mu^2}{3} (\rho_m + \rho_{DE}) \]

- Gravity is still modified as in the self-accelerated branch (but with attractive forces)

- Ghost free in the quantum theory

- Can choose \( \rho_{DE} \) to match any desired expansion history including flat \( \Lambda \) CDM

\[ H^2 \equiv \frac{\mu^2}{3} (\rho_m + \rho_\Lambda) \to \rho_{DE} \]

- Separate out geometrical and dynamical tests of acceleration
Conserved Curvature Regime
Curvature Conservation

- On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)

- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d \ln a$ (Bardeen 1980)

- Gauge transformation to Newtonian gauge

$$ds^2 = -(1 + 2\Psi) dt^2 + a^2 (1 + 2\Phi) dx^2$$

yields (Hu & Eisenstein 1999)

$$\Phi'' - \Psi' - \frac{H''}{H'} \Phi' - \left( \frac{H'}{H} - \frac{H''}{H'} \right) \Psi = 0$$

- Modified gravity theory supplies the closure relationship $\Phi = -\gamma (\ln a) \Psi$ between and expansion history $H = \dot{a}/a$ supplies rest.
Linear Theory for $f(R)$

- In $f(R)$ model, “superhorizon” behavior persists until Compton wavelength smaller than fluctuation wavelength $B^{1/2}(k/aH) < 1$
- Once Compton wavelength becomes larger than fluctuation

$$B^{1/2}(k/aH) > 1$$

perturbations are in scalar-tensor regime described by $\gamma = 1/2$.
- Small scale density growth enhanced and

$$8\pi G \rho > R$$

low curvature regime with order unity deviations from GR
- Transitions in the non-linear regime where the Compton wavelength can shrink via chameleon mechanism
- Given $k_{NL}/aH \gg 1$, even very small $f_R$ have scalar-tensor regime
PPF $f(R)$ Description

- **Metric and matter evolution well-matched by PPF description**
- **Standard GR tools apply (CAMB), self-consistent, gauge invar.**

Hu & Sawicki (2007); Hu (2008)
Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi-\Psi)$
Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure.
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit.
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi-\Psi)$.
ISW Quadrupole

- **Reduction of large angle anisotropy** for $B_0 \sim 1$ for same expansion history and distances as $\Lambda$CDM
- **Well-tested small scale anisotropy unchanged**

Song, Hu & Sawicki (2006)
ISW-Galaxy Correlation

- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation
Galaxy-ISW Anti-Correlation

- Large Compton wavelength $B^{1/2}$ creates potential growth which can anti-correlate galaxies and the CMB
- In tension with detections of positive correlations across a range of redshifts

Song, Peiris & Hu (2007); Lombriser et al (2010) $B_0 < 0.43$
DGP Horizon Scales

- Metric and matter evolution well-matched by PPF description
- Standard GR tools apply (CAMB), self-consistent, gauge invar.

Hu & Sawicki (2007); Hu (2008)
DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background
CMB in DGP

- Adding **cut off** as an epicycle can fix distances, ISW problem
- Suppresses **polarization** in violation of EE data - cannot save DGP!

Fang et al (2008)
CMB in DGP

- Adding **cut off** as an epicycle can fix distances, ISW problem
- Suppresses **polarization** in violation of EE data - cannot save DGP!

Fang et al (2008)
DGP Normal Branch

- Brane tension (cosmological constant) on normal branch allows models to pass ISW test
- Joint expansion history constraints require $Hr_C > 3$ at 95% CL

Lombriser et al. (2009)
Linear Scalar Tensor Regime
Three Regimes

- **Metric**: \( ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2 \)

- **Superhorizon regime**: \( \zeta = \text{const.}, g(a) = (\Phi + \Psi)/(\Phi - \Psi) \)

- **Linear regime - closure** ↔ “smooth” dark energy density:

\[
\nabla^2(\Phi - \Psi)/2 = -4\pi Ga^2 \Delta \rho
\]

\( G \) can be promoted to \( G(a) \), \( G(a, k) \) but for scalar degrees of freedom conformal invariance requires \( G = G_N \) and

- **Non-linear regime**:

\[
\nabla^2(\Phi - \Psi)/2 = -4\pi Ga^2 \Delta \rho
\]

\[
\nabla^2 \Psi = 4\pi Ga^2 \Delta \rho + \frac{1}{2} \nabla^2 \phi
\]

with non-linearity in the field equation

\[
\nabla^2 \phi = g_{\text{lin}}(a)a^2 (8\pi G \Delta \rho - N[\phi])
\]
Linear Power Spectrum

- Linear real space **power spectrum enhanced on small scales**
- Degeneracy with **galaxy bias and lack** of non-linear predictions leave constraints from **shape** of power spectrum

![Graph of Linear Power Spectrum](image)
Redshift Space Distortion

- Relationship between velocity and density field given by continuity with modified growth rate \( f_v = \frac{d \ln D}{d \ln a} \)
- Redshift space power spectrum further distorted by Kaiser effect
Lensing v Dynamical Comparison

- **Gravitational lensing** around galaxies vs. **linear velocity field** (through redshift space distortions and galaxy autocorrelation)
- **Consistent with GR** + smooth dark energy beginning to test interesting models


DGP Power Spectrum

- Constant suppression in the linear regime for self-acceleration

Lue, Scoccimarro, Starkman (2004); Hu & Sawicki (2007)
Non-Linear GR Regime
Three Regimes

- Fully worked $f(R)$ and DGP examples show 3 regimes
- **Superhorizon** regime: $\zeta = \text{const.}, g(a)$
- **Linear** regime - closure condition - analogue of “smooth” dark energy density:

\[
\nabla^2 (\Phi - \Psi)/2 = -4\pi Ga^2 \Delta \rho
\]

\[
g(a, x) \leftrightarrow g(a, k)
\]

$G$ can be promoted to $G(a)$ but conformal invariance relates fluctuations to field fluctuation that is small

- **Non-linear** regime:

\[
\nabla^2 (\Phi - \Psi)/2 = -4\pi Ga^2 \Delta \rho
\]

\[
\nabla^2 \Psi = 4\pi Ga^2 \Delta \rho - \frac{1}{2} \nabla^2 \phi
\]
Nonlinear Interaction

Non-linearity in the field equation

\[ \nabla^2 \phi = g_{\text{lin}}(a)a^2 \left( 8\pi G \Delta \rho - N[\phi] \right) \]

recovers linear theory if \( N[\phi] \to 0 \)

- For \( f(R) \), \( \phi = f_R \) and

\[ N[\phi] = \delta R(\phi) \]

a non-linear function of the field

Linked to gravitational potential

- For DGP, \( \phi \) is the brane-bending mode and

\[ N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right] \]

a non-linear function of second derivatives of the field

Linked to density fluctuation
Non-Linear Chameleon

• For $f(R)$ the field equation

$$\nabla^2 f_R \approx \frac{1}{3} (\delta R(f_R) - 8\pi G \delta \rho)$$

is the non-linear equation that returns general relativity

• High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$

• Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if

$$\Delta f_R \leq \frac{2}{3} \Phi,$$

else required field gradients too large despite $\delta R = 8\pi G \delta \rho$ being the local minimum of effective potential
Non-Linear Dynamics

- Supplement that with the **modified Poisson equation**
  \[ \nabla^2 \Psi = \frac{16\pi G}{3} \delta \rho - \frac{1}{6} \delta R(f_R) \]
- Matter evolution given metric unchanged: usual **motion of matter** in a gravitational potential \( \Psi \)
- Prescription for **N-body** code
- **Particle Mesh (PM)** for the Poisson equation
- Field equation is a non-linear Poisson equation: **relaxation** method for \( f_R \)
- **Initial conditions** set to GR at high redshift
Chameleon suppresses extra force (scalar field) in high density, deep potential regions

Environment Dependent Force

- For large background field, gradients in the scalar prevent the chameleon from appearing.
\textbf{N-body Power Spectrum}

- $512^3$ PM-relaxation code resolves the chameleon transition to GR: greatly reduced non-linear effect

![Graph showing the power spectrum comparison between Full $f_R$ simulation and linear predictions](image)

$|f_{R0}| = 10^{-6}$

Artificially **turning off the chameleon mechanism** restores much of enhancement.
N-body Power Spectrum

- Models where the chameleon absent today (large field models) show residual effects from a high redshift chameleon

\[
P(k)/P_{GR}(k) = 1 - \left| f_R^0 \right|
\]

\( k \) (\( h/\text{Mpc} \))

0.1 1

0 0.2 0.4 0.6 0.8

Enhanced abundance of rare dark matter halos (clusters) with extra force
Mass Function

- Local cluster abundance (Chandra sample) current best cosmological constraint (~4 orders of magnitude better than ISW)

Schmidt, Vikhlinin, Hu (2009)
Halo Bias

- Halos at a fixed mass less rare and less highly biased

Halo Mass Correlation

- Enhanced forces vs lower bias

Halo Model

- Power spectrum trends also consistent with halos and modified collapse

$$|f_{R0}| = 10^{-4}$$

Nonlinear Interaction

Non-linearity in the field equation

\[ \nabla^2 \phi = g_{\text{lin}}(a)a^2 \left( 8\pi G \Delta \rho - N[\phi] \right) \]

recovers linear theory if \( N[\phi] \rightarrow 0 \)

- For \( f(R) \), \( \phi = f_R \) and

\[ N[\phi] = \delta R(\phi) \]

a non-linear function of the field

Linked to gravitational potential

- For DGP, \( \phi \) is the brane-bending mode and

\[ N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right] \]

a non-linear function of second derivatives of the field

Linked to density fluctuation
DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential

Brane Bending Mode

Apparent Equivalence Principle Violation

- Self-field of a “test mass” can saturate an external field (for $f(R)$ in the gradient, for DGP in the second derivatives)

Hui, Nicolis, Stubbs (2009); Hu (2009)
Summary

- Lessons from the $f(R)$ and DGP worked examples – 3 regimes:
  - large scales: conservation determined
  - intermediate scales: scalar-tensor
  - small scales: GR in high density regions, modified in low

- Large scales: expansion history and metric ratio
  \[ g = (\Phi + \Psi)/(\Phi - \Psi) \] through curvature conservation

- Intermediate scales: scalar tensor modified Newtonian regime, $g$ and Poisson equation

- Small scales: nonlinear interaction of modification field makes $g$ depend on local environment (not scale) - density or potential - suppressing deviations

- $N$-body (PM-relaxation) simulations show halo model framework can describe observables in the nonlinear regime
Solar System Tests
Solar Profile

- **Density profile of Sun is not a constant density sphere** - interior, photosphere, chromosphere, corona
- **Density drops by ~25 orders of magnitude** - does curvature follow?

![Graph showing the density profile of the Sun](image)

Hu & Sawicki (2007)
Field Solution

- Field solution smoothly relaxes from exterior value to high curvature interior value $f_R \sim 0$, minimizing potential + kinetic
- Juncture is where thin-shell criterion is satisfied $|\Delta f_R| \sim \Delta \Phi$

Hu & Sawicki (2007)
Solar Curvature

- Curvature drops suddenly as field moves slightly from zero
- Enters into low curvature regime where $R < 8\pi G\rho$

![Graph showing the relationship between $R/8\pi G\rho$ and $r/r_\odot$ with different values of $|f_{R0}|$.](image)

Hu & Sawicki (2007)
**Solar System Constraint**

- **Cassini constraint on PPN** $|\gamma - 1| < 2.3 \times 10^{-5}$
- Easily satisfied if galactic field is at potential minimum $|f_{Rg}| < 4.9 \times 10^{-11}$
- Allows even **order unity** cosmological fields

Hu & Sawicki (2007)