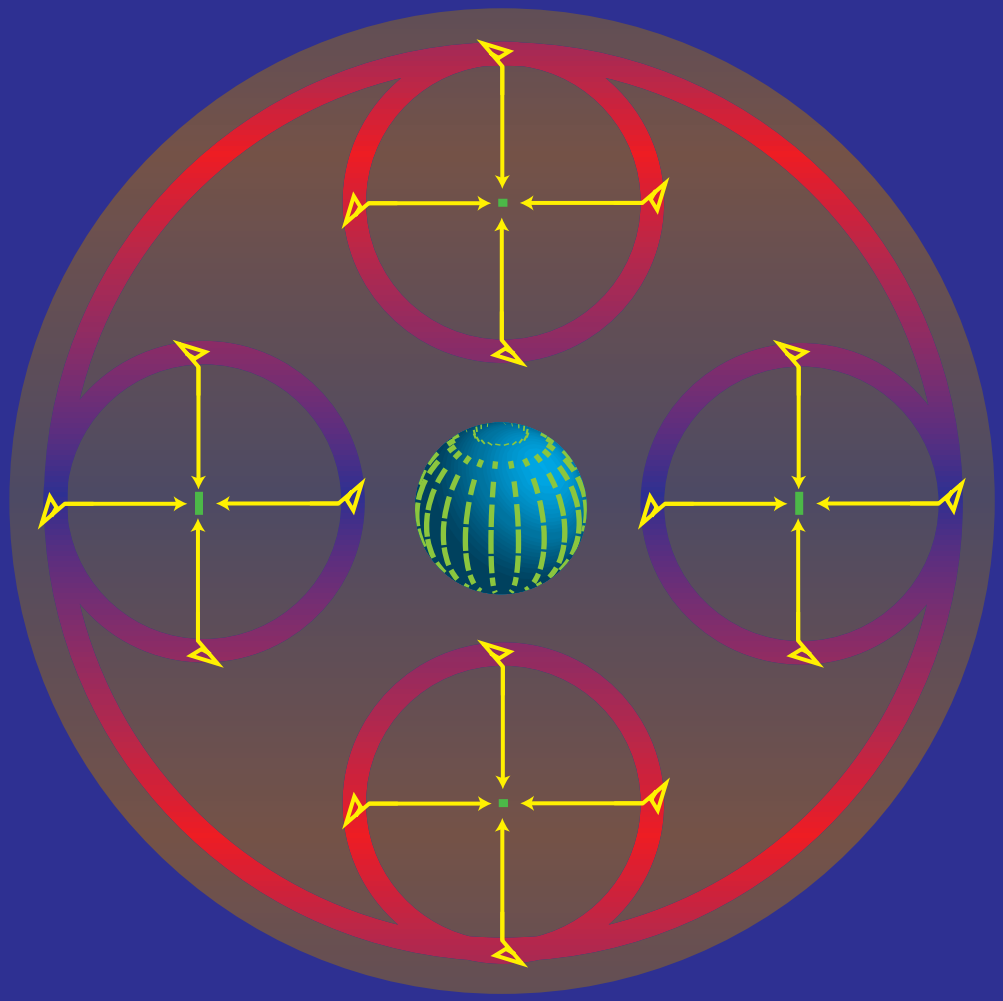
The Physics of CMB Polarization



Wayne Hu

Chicago, March 2004

Chicago's Polarization Orientation

- Thomson Radiative Transfer (Chandrashekhar 1960 ;-)
- Reionization (WMAP 2003; Planck?)
- Gravitational Waves (Future? Beyond Einstein Satellite)
- Acoustic Waves (DASI 2002; CAPMAP 2004?)
- Gravitational Lensing (Next Generation? QUad, QUiet, SPT-Pol...)

- Recent Collaborators:

  Christopher Gordon

  Matt Hedman

  Gil Holder

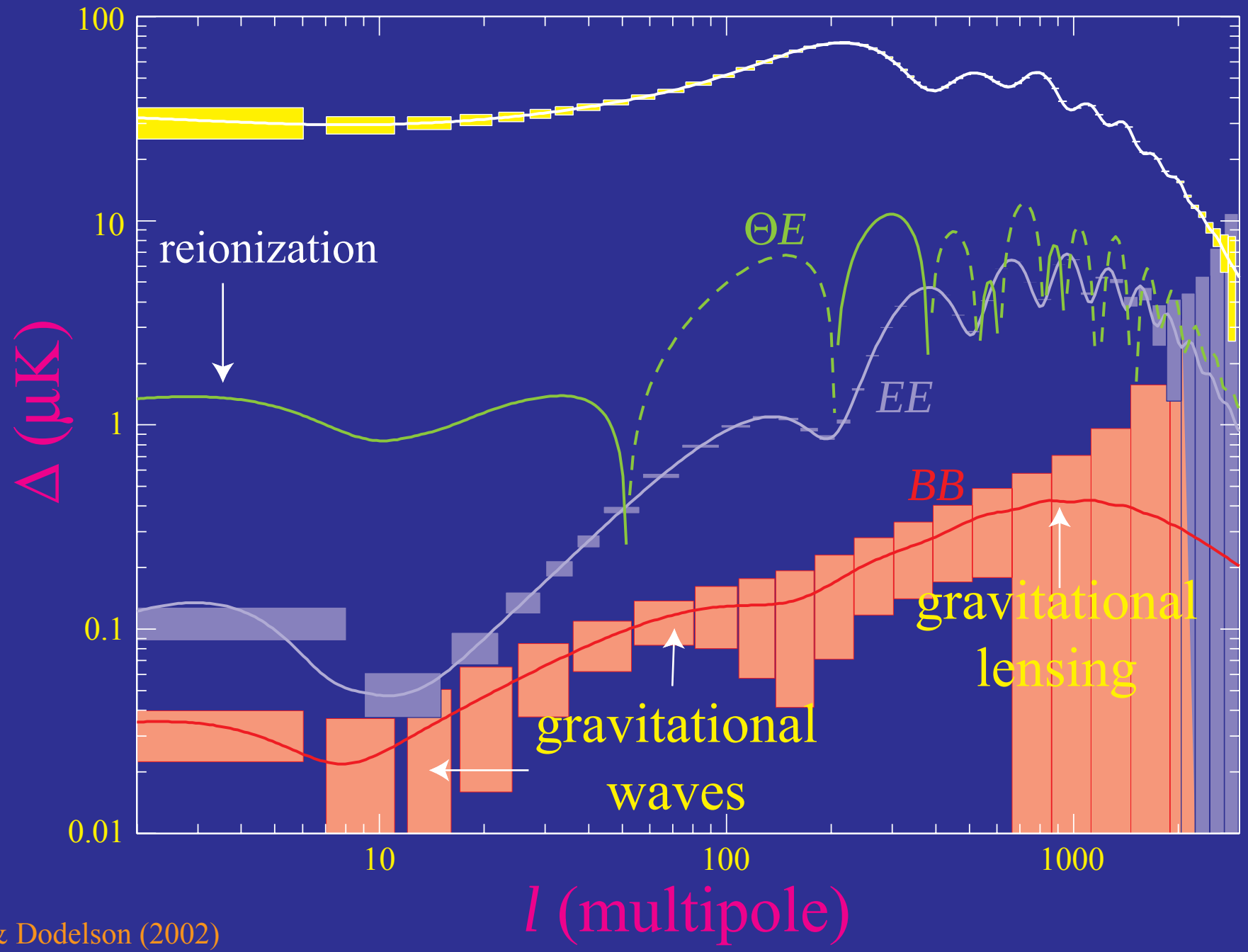
  Manoj Kaplinghat

  Takemi Okamoto

  Kendrick Smith

http://background.uchicago.edu
under Presentations

The Colloquial Landscape

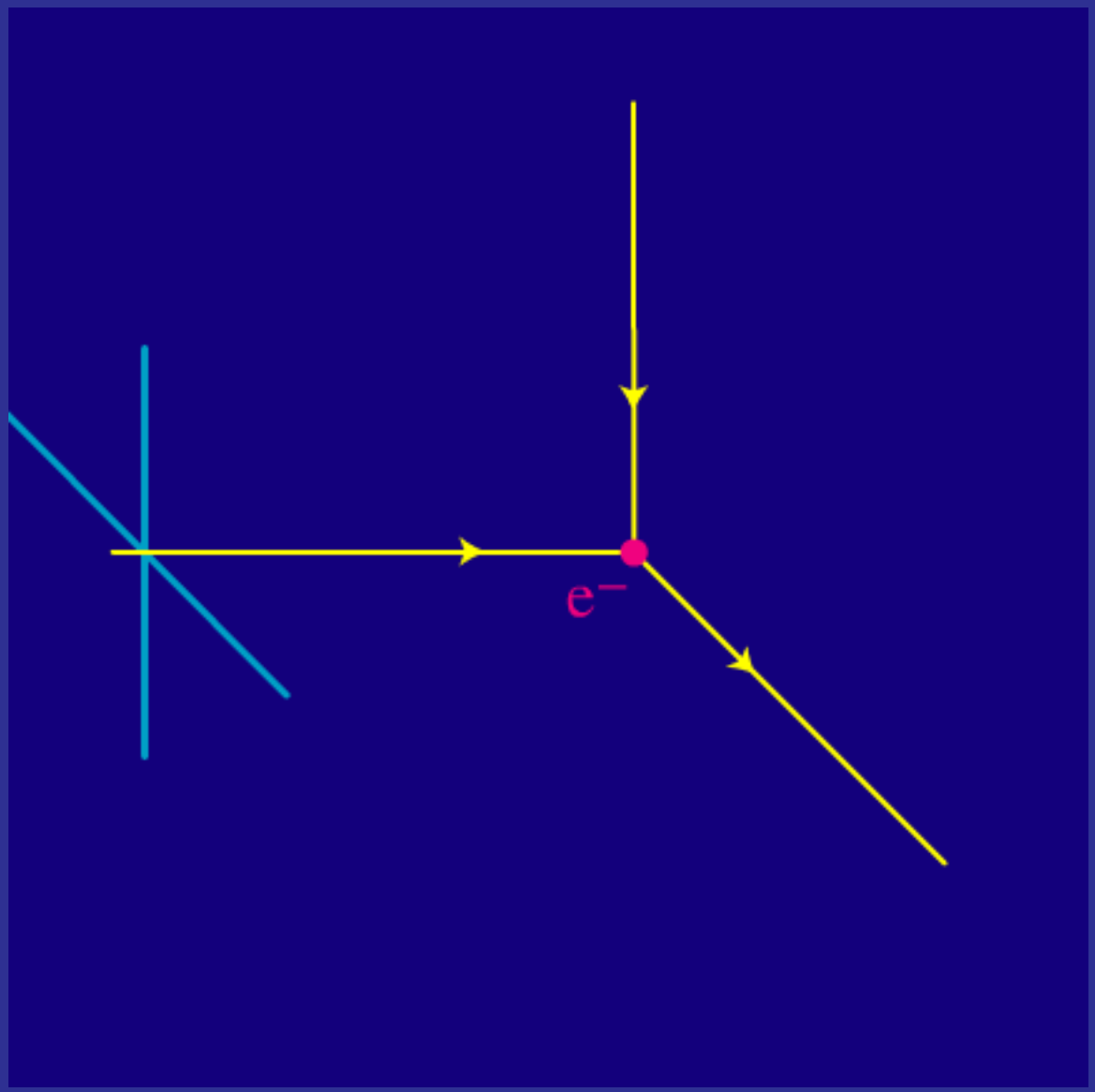


Hu & Dodelson (2002)

Thomson Scattering

Polarization from Thomson Scattering

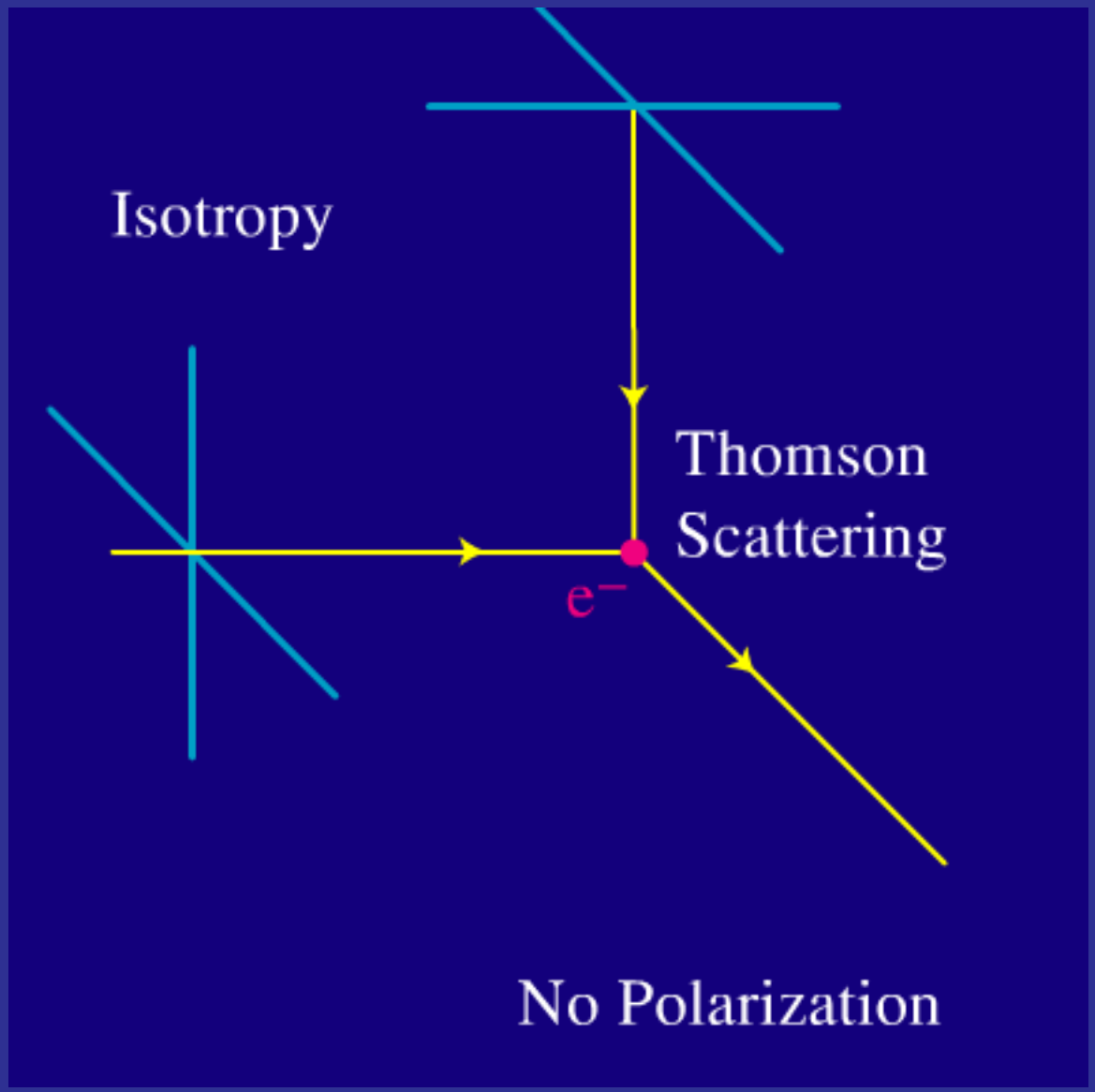
- Differential cross section depends on polarization and angle



$$\frac{d\sigma}{d\Omega} = \frac{3}{8\pi}|\hat{\varepsilon}' \cdot \hat{\varepsilon}|^2 \sigma_T$$

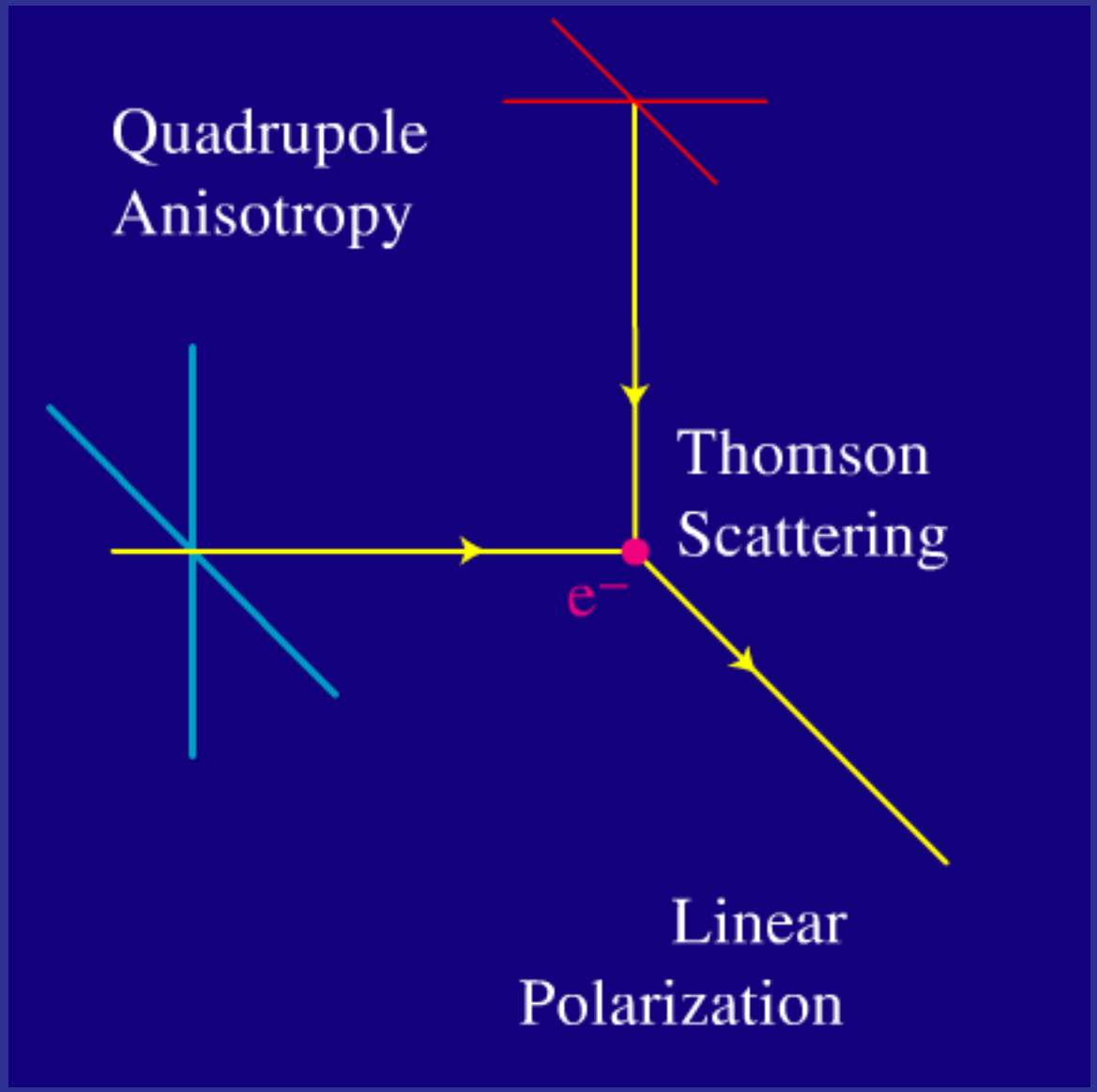
Polarization from Thomson Scattering

- Isotropic radiation scatters into unpolarized radiation



Polarization from Thomson Scattering

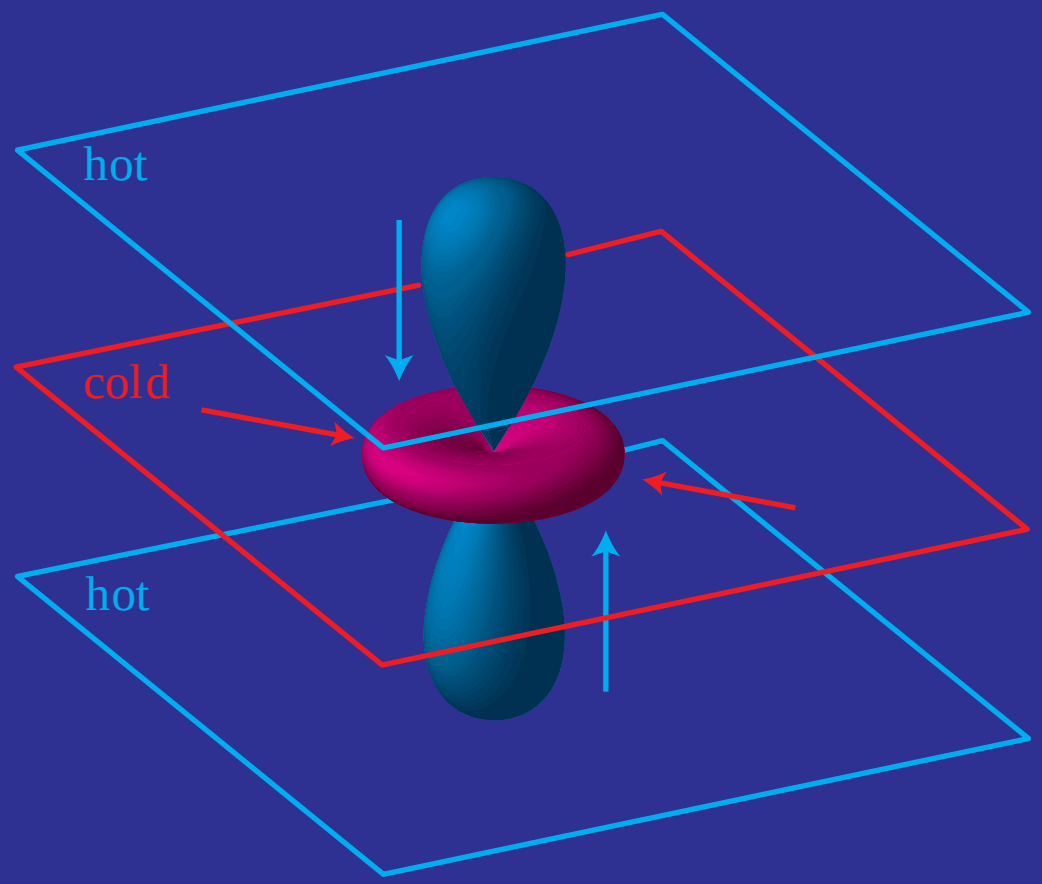
- Quadrupole anisotropies scatter into linear polarization



aligned with
cold lobe

Whence Quadrupoles?

- Temperature inhomogeneities in a medium
- Photons arrive from different regions producing an anisotropy

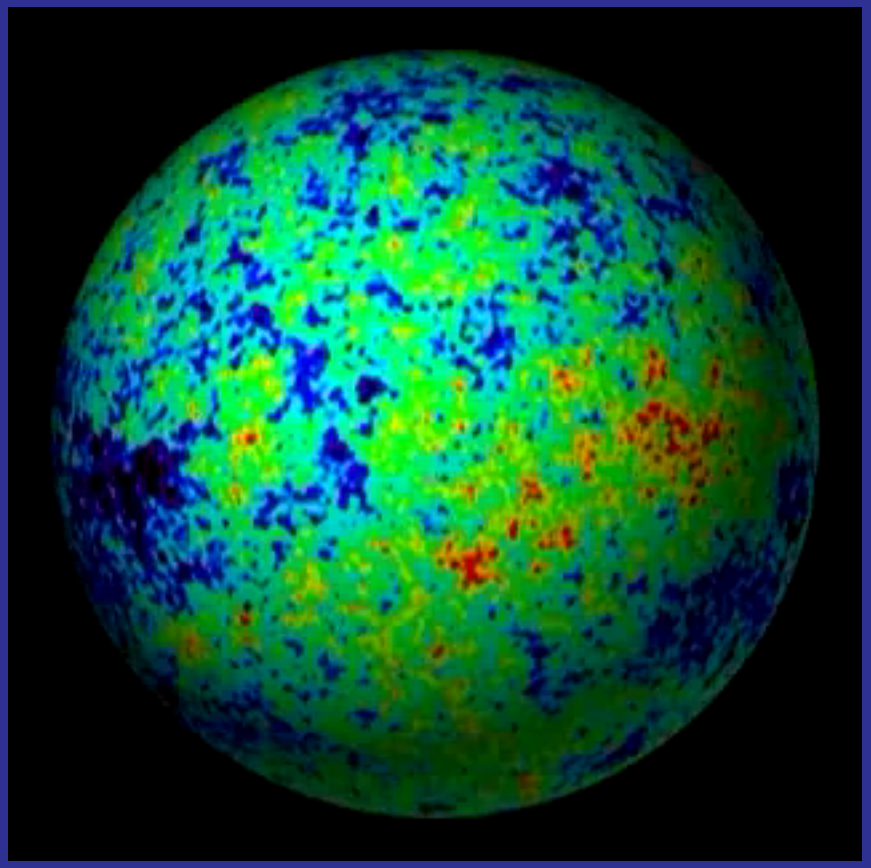


(Scalar) Temperature Inhomogeneity

Hu & White (1997)

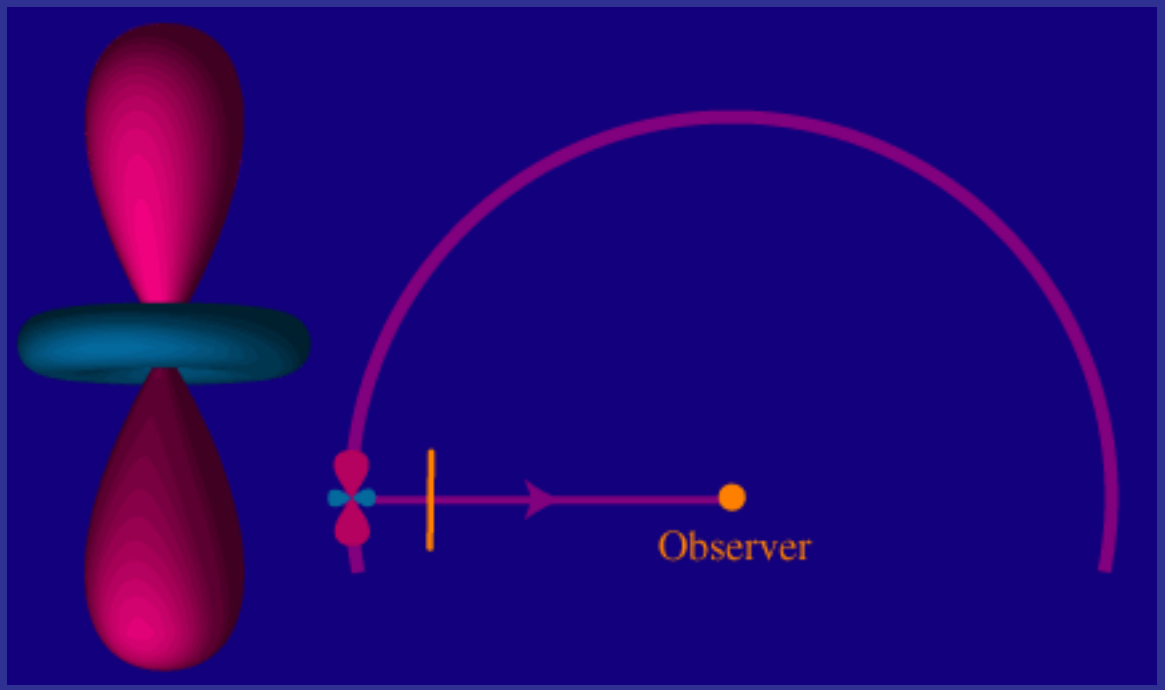
CMB Anisotropy

- WMAP map of the CMB temperature anisotropy



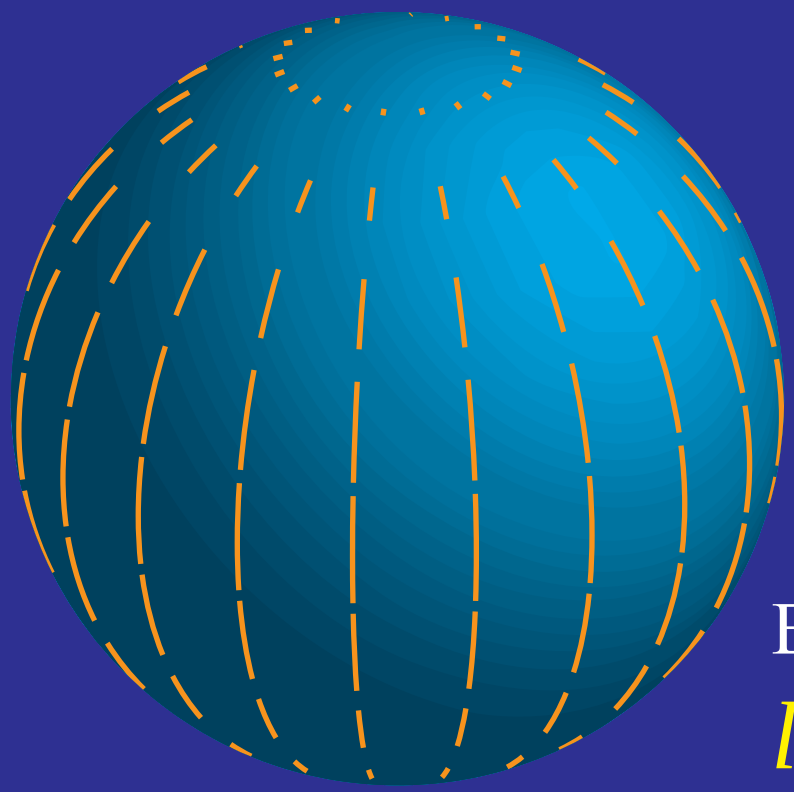
Whence Polarization Anisotropy?

- Observed photons scatter into the line of sight
- Polarization arises from the projection of the quadrupole on the transverse plane



Polarization Multipoles

- Mathematically pattern is described by the tensor (spin-2) spherical harmonics [eigenfunctions of Laplacian on trace-free 2 tensor]
- Correspondence with scalar spherical harmonics established via Clebsch-Gordan coefficients (spin x orbital)
- Amplitude of the coefficients in the spherical harmonic expansion are the multipole moments; averaged square is the power

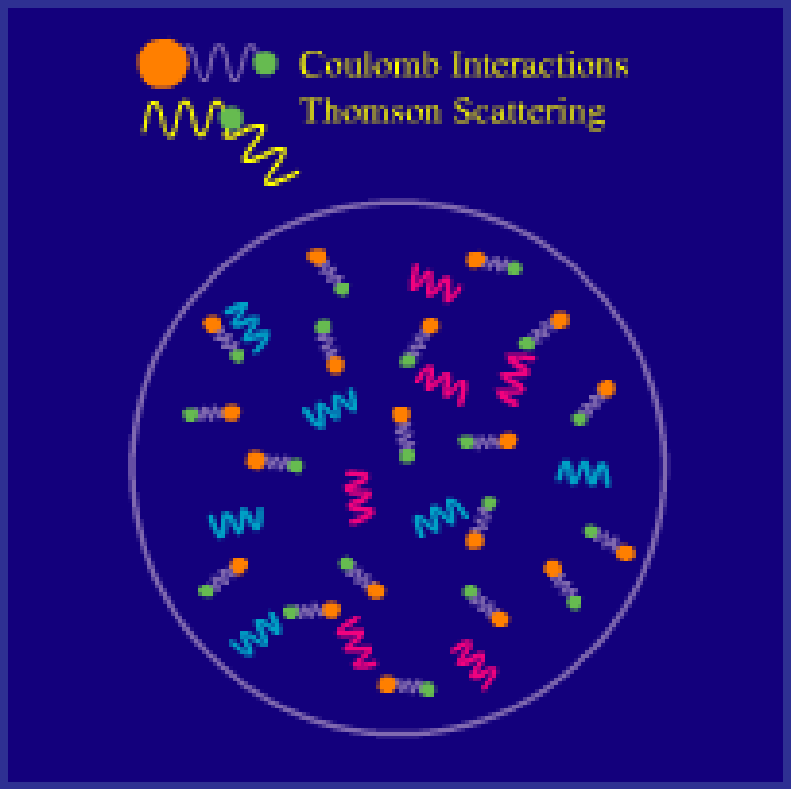


E-tensor harmonic

$l=2,\ m=0$

A Catch-22

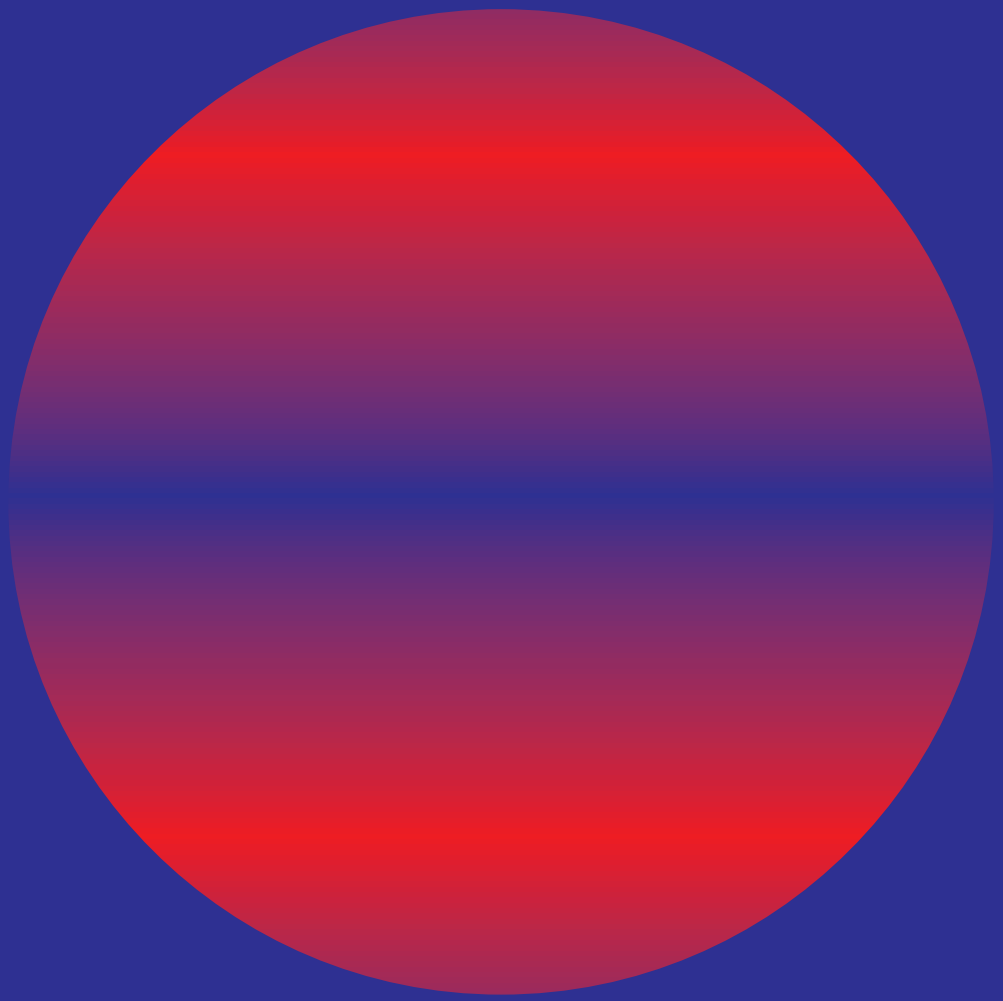
- Polarization is generated by scattering of anisotropic radiation
- Scattering isotropizes radiation
- Polarization only arises in optically thin conditions: reionization and end of recombination
- Polarization fraction is at best a small fraction of the 10^{-5} anisotropy: $\sim 10^{-6}$ or μK in amplitude



Reionization

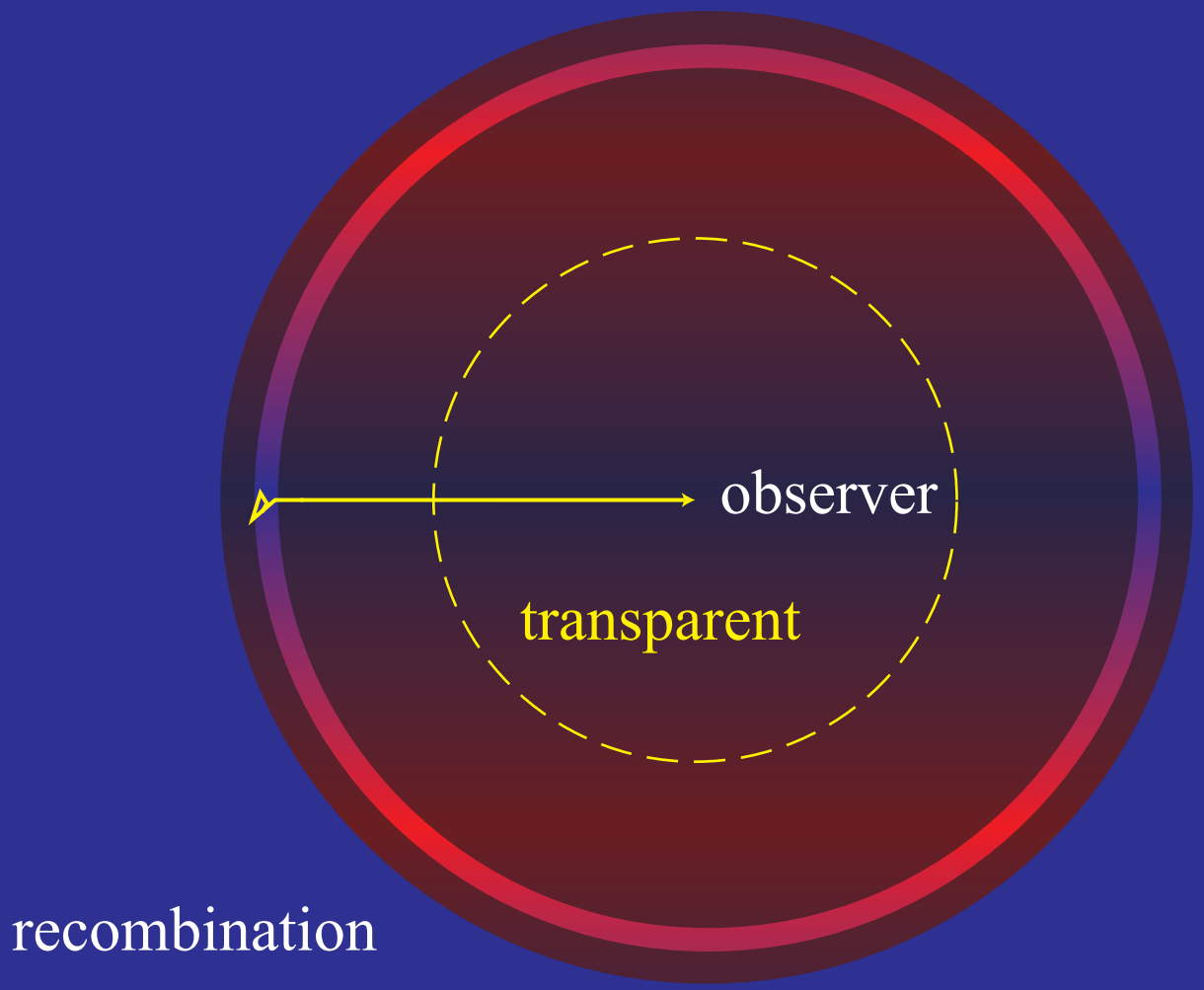
Temperature Inhomogeneity

- Temperature inhomogeneity reflects initial density perturbation on large scales
- Consider a single Fourier moment:



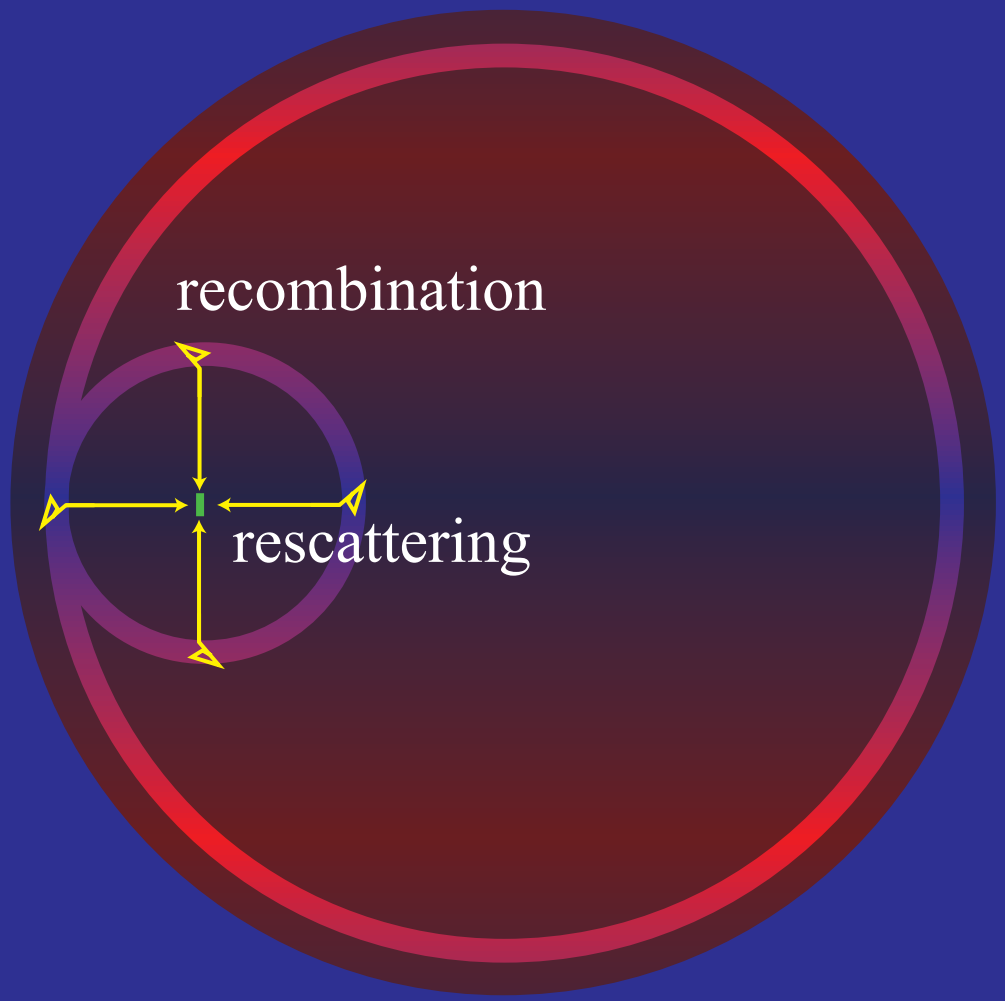
Locally Transparent

- Presently, the matter density is so low that a typical CMB photon will not scatter in a Hubble time (~age of universe)



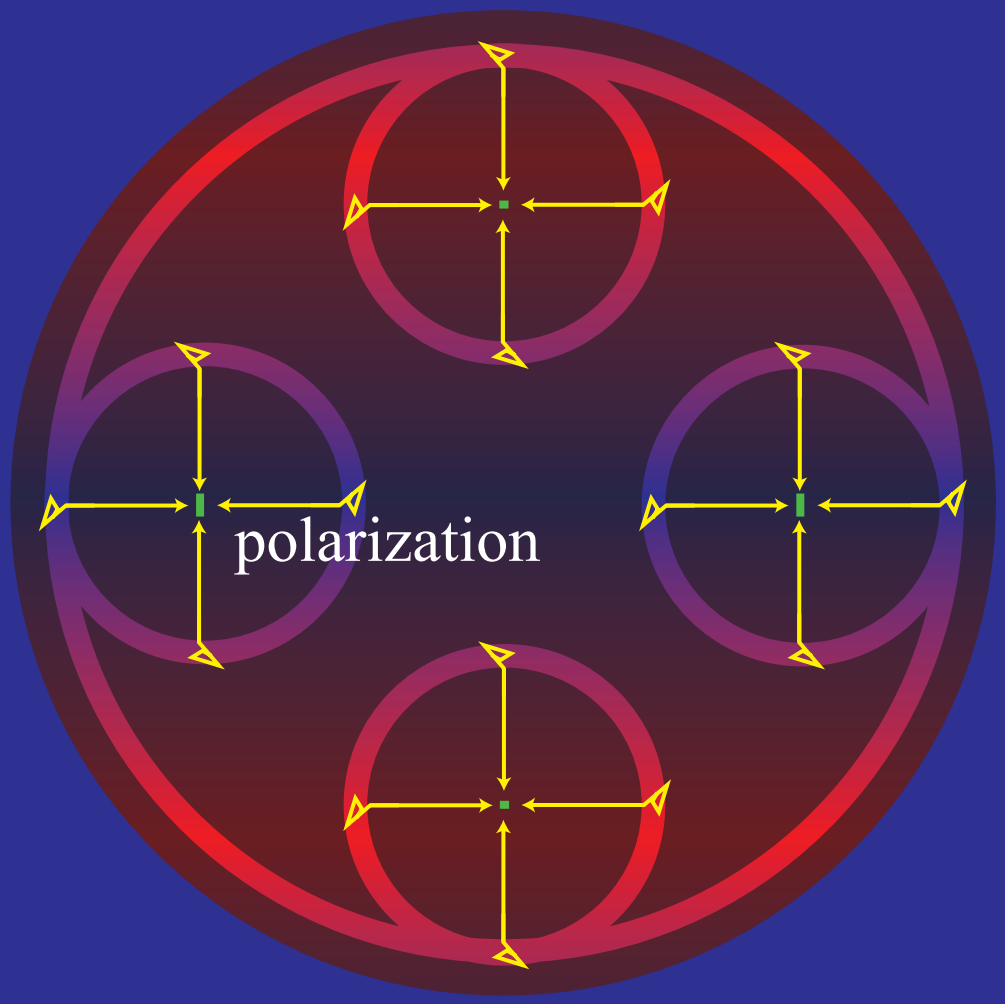
Reversed Expansion

- Free electron density in an ionized medium increases as scale factor a^{-3}; when the universe was a tenth of its current size CMB photons have a finite (~10%) chance to scatter



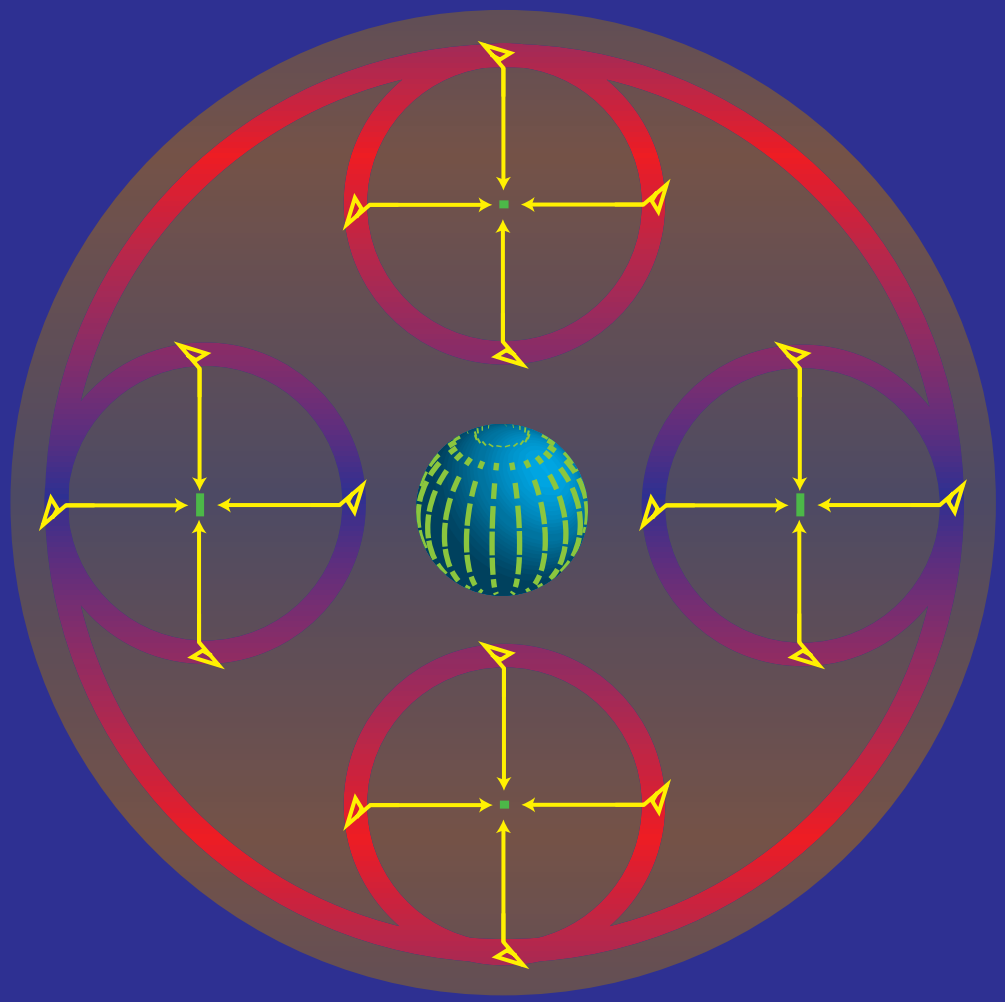
Polarization Anisotropy

- Electron sees the temperature anisotropy on its recombination surface and scatters it into a polarization



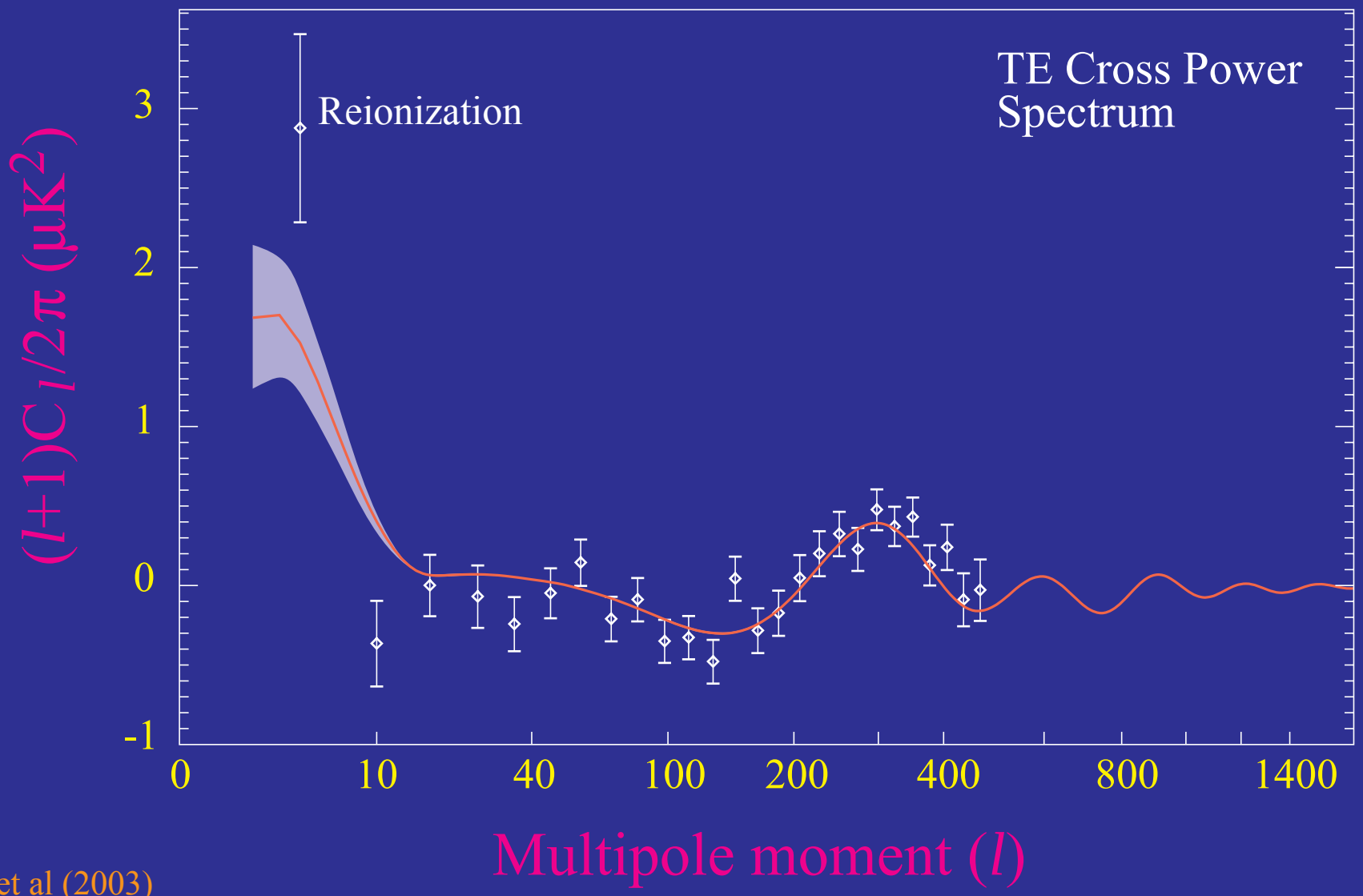
Temperature Correlation

- Pattern correlated with the temperature anisotropy that generates it; here an m=0 quadrupole



WMAP Correlation

- Measured correlation indicates the universe remained at least partially ionized to a surprisingly large redshift or early time (z>10)



Hinshaw et al (2003)

Why Care?

- Early ionization is puzzling if due to ionizing radiation from normal stars; may indicate more exotic physics is involved

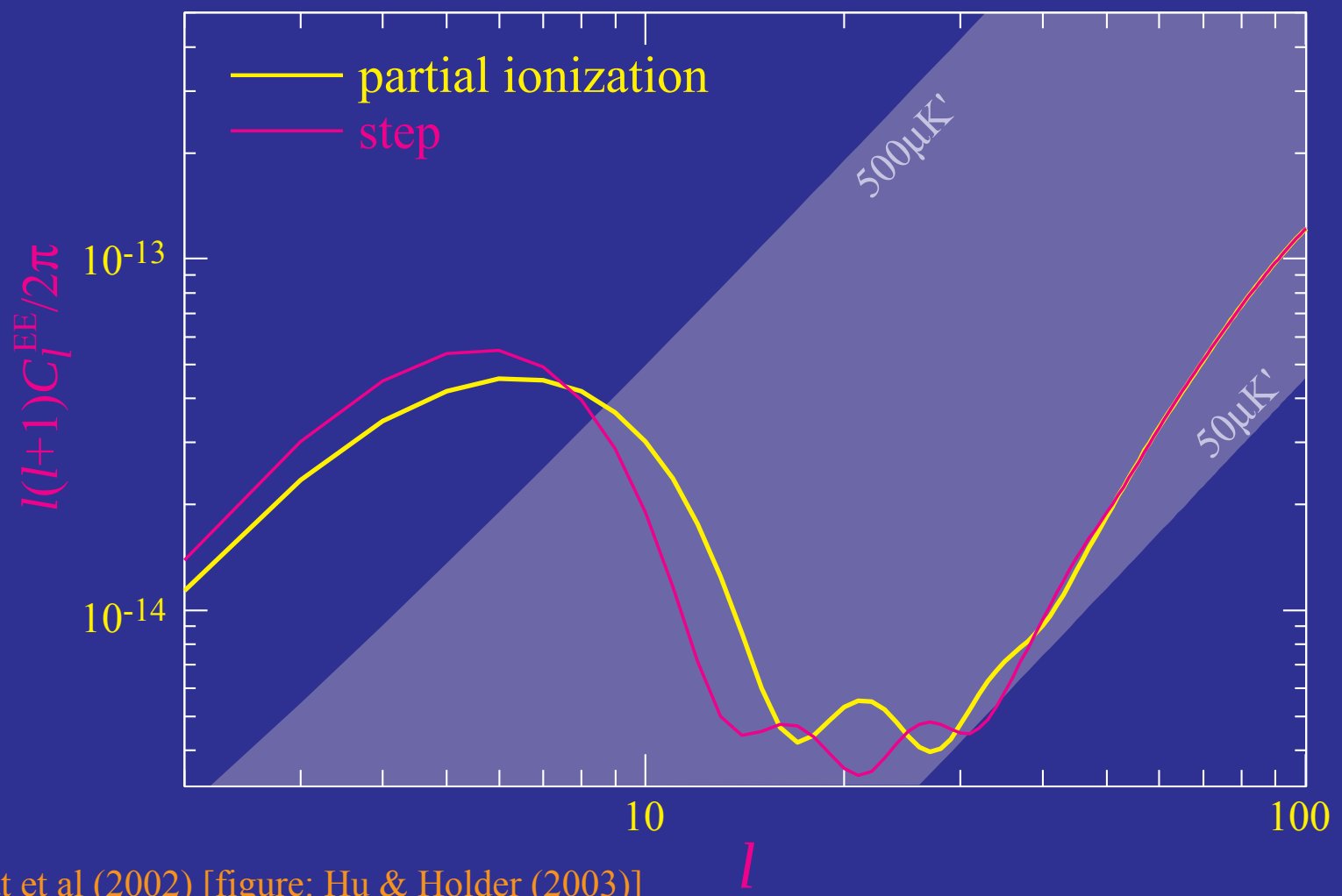
- Reionization screens temperature anisotropy on small scales making the true amplitude of initial fluctuations larger by e^{τ}
- Measuring the growth of fluctuations is one of the best ways of determining the neutrino masses and the dark energy

- Offers an opportunity to study the origin of the low multipole statistical anomalies
- Presents a second, and statistically cleaner, window on gravitational waves from the early universe

Ionization History

Polarization Power Spectrum

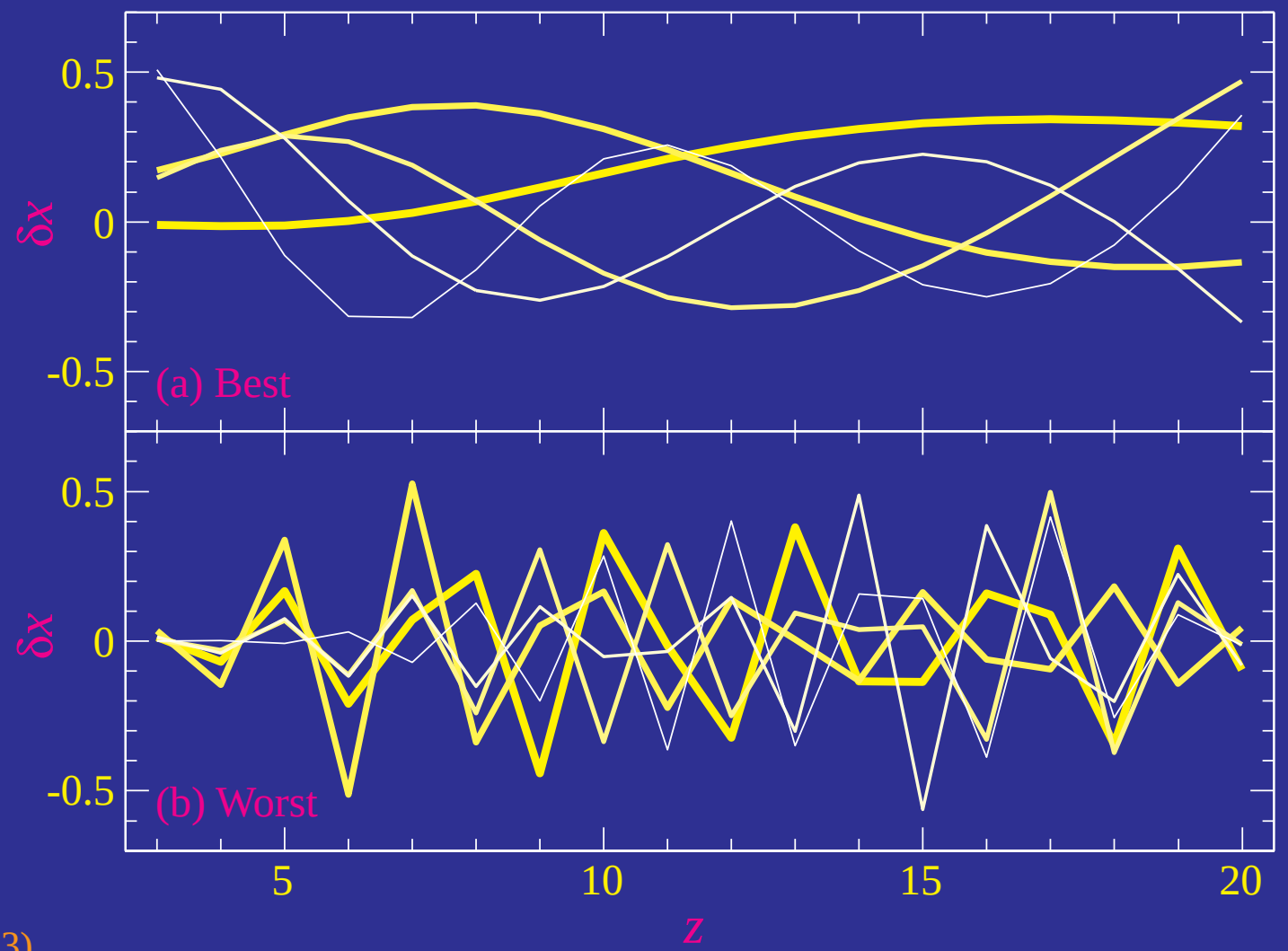
- Most of the information on ionization history is in the polarization (auto) power spectrum - two models with same optical depth but different ionization fraction



Kaplinghat et al (2002) [figure: Hu & Holder (2003)]

Principal Components

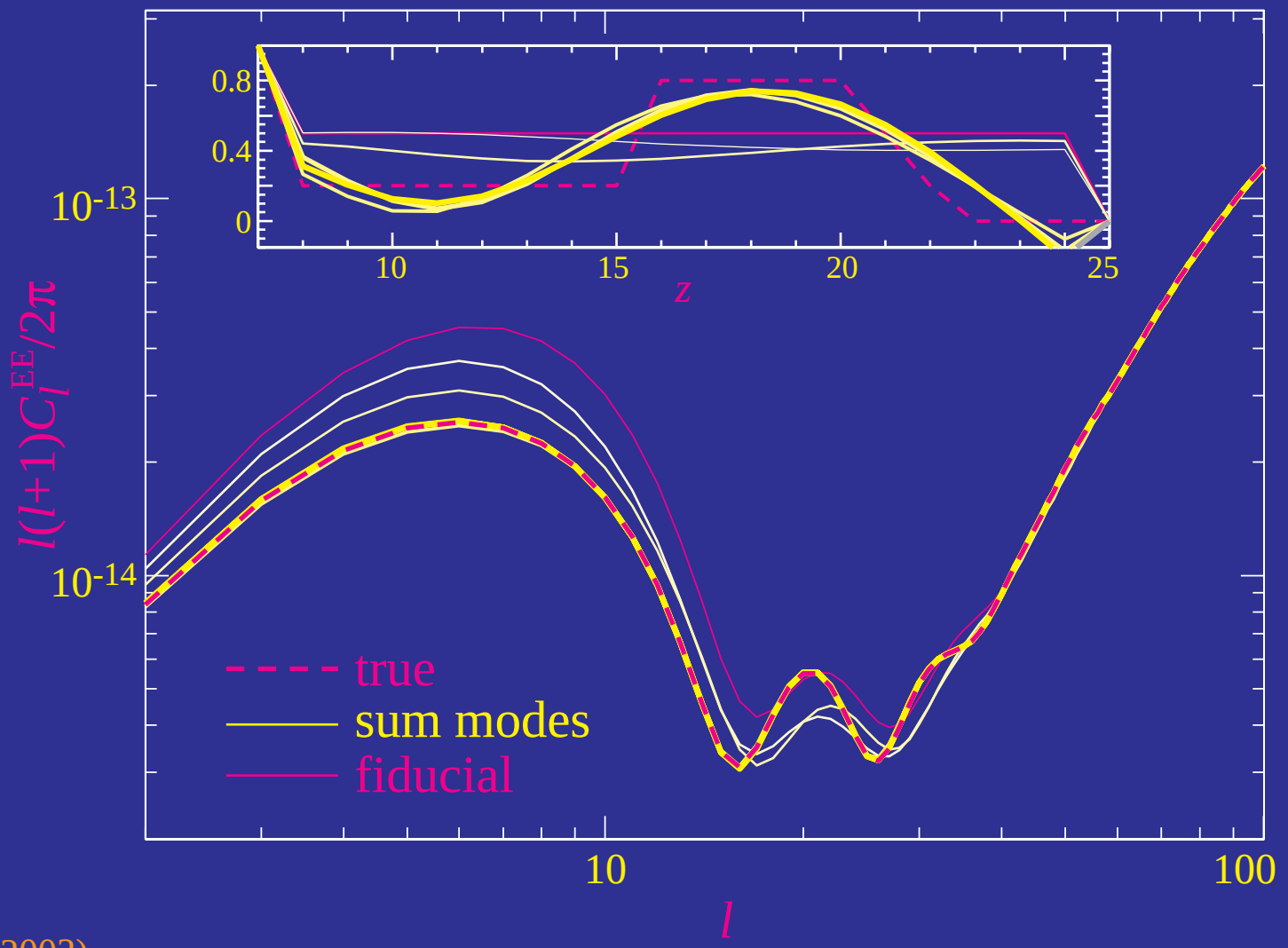
- Information on the ionization history is contained in ~5 numbers - essentially coefficients of first few Fourier modes



Hu & Holder (2003)

Representation in Modes

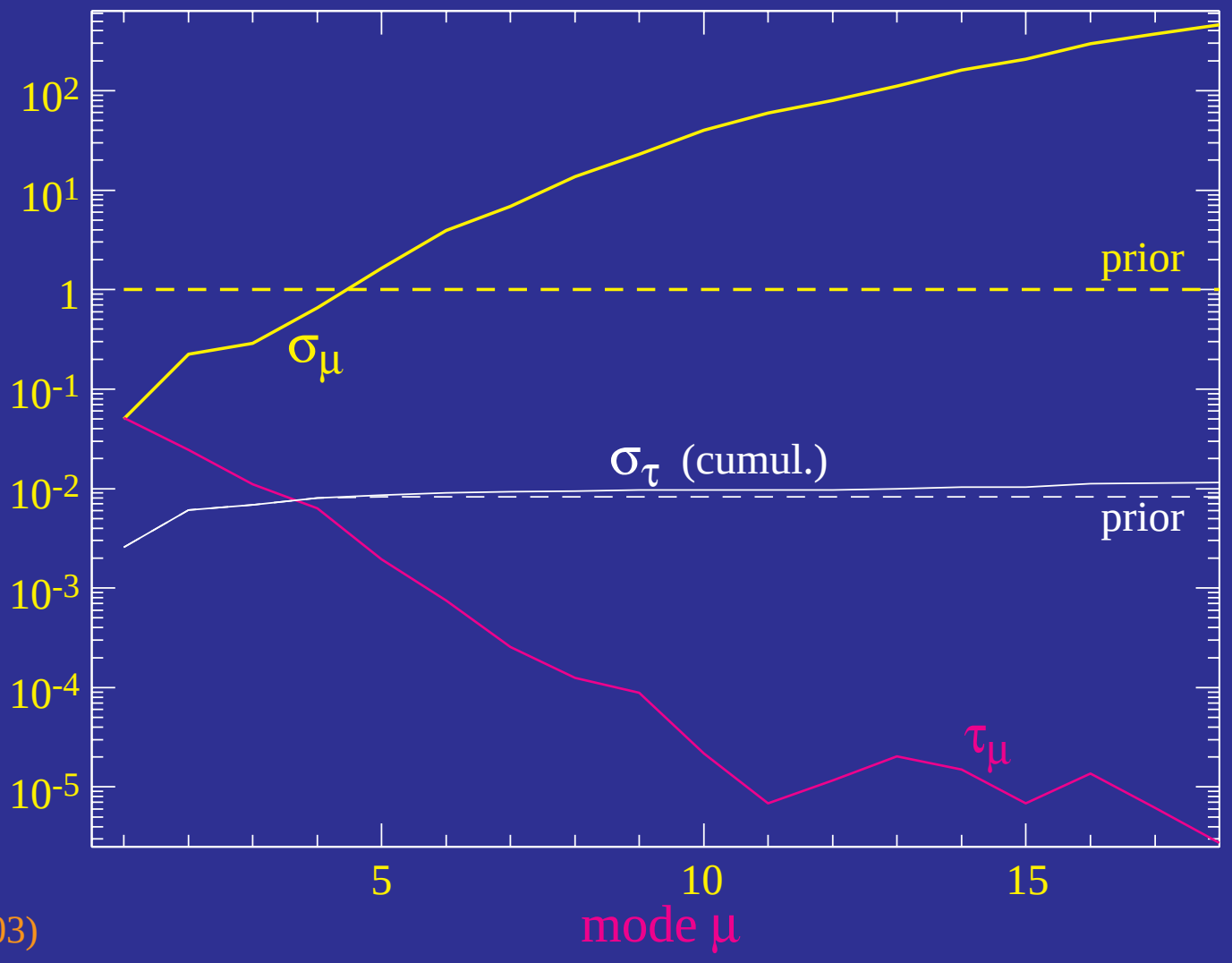
- Reproduces the power spectrum and net optical depth (actual τ=0.1375 vs 0.1377); indicates whether multiple physical mechanisms suggested



Hu & Holder (2003)

Total Optical Depth

- Optical depth measurement unbiased
- Ultimate errors set by cosmic variance here 0.01
- Equivalently 1% determination of initial amplitude for dark energy

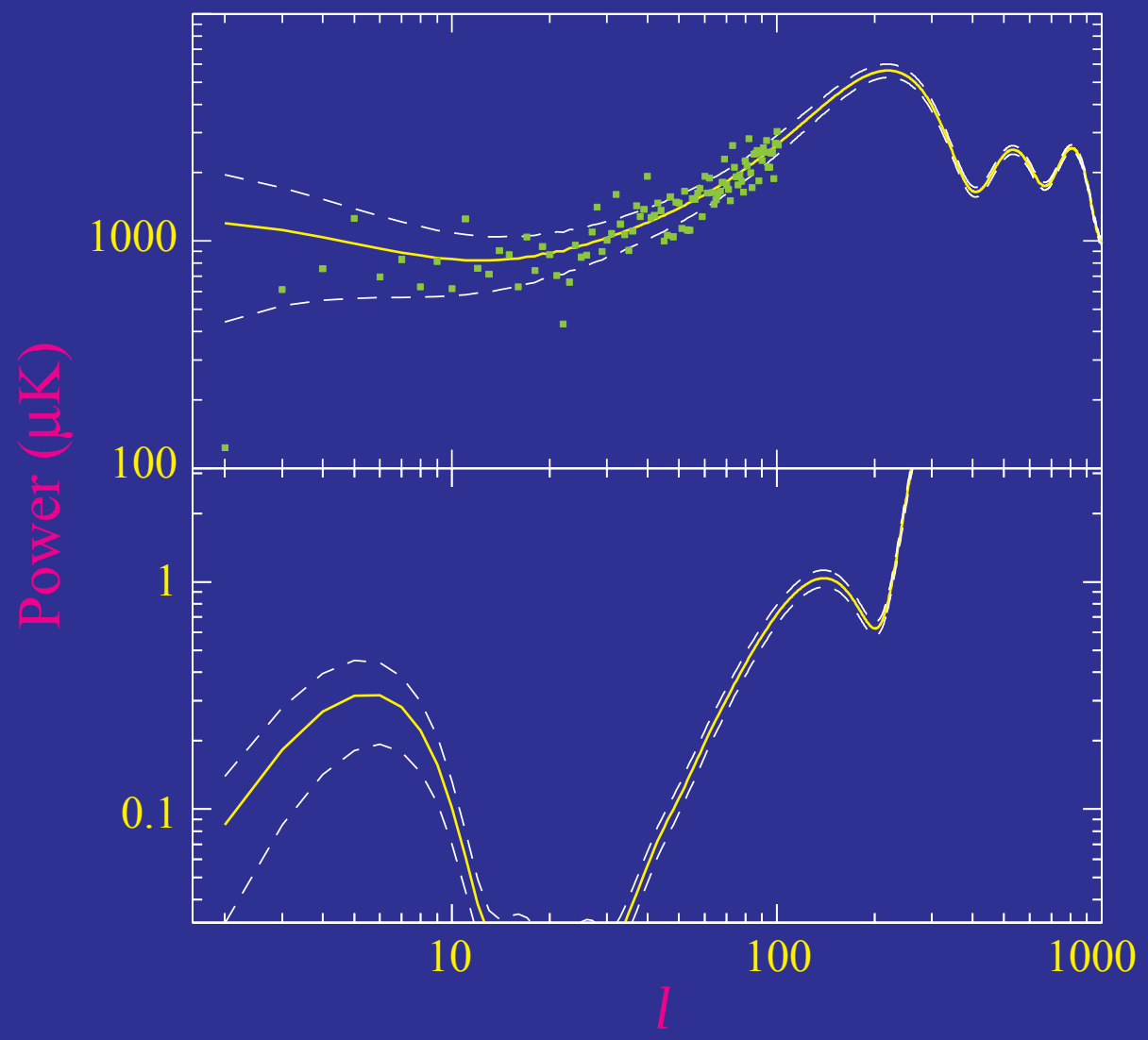


Hu & Holder (2003)

Quadrupole Aside

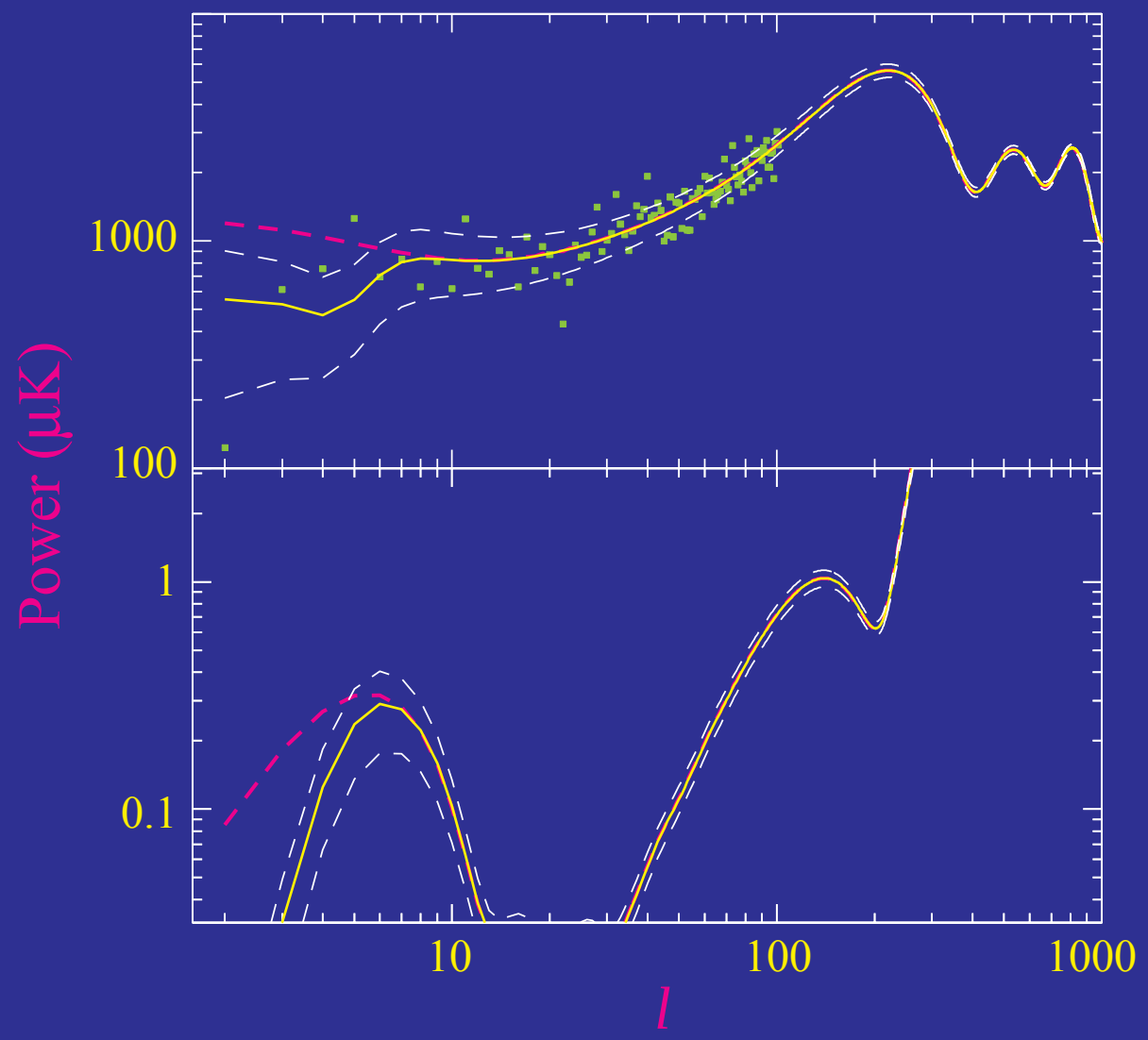
The Quadrupole

- Our quadrupole is up to an order of magnitude smaller than the expected ensemble average (known since COBE)



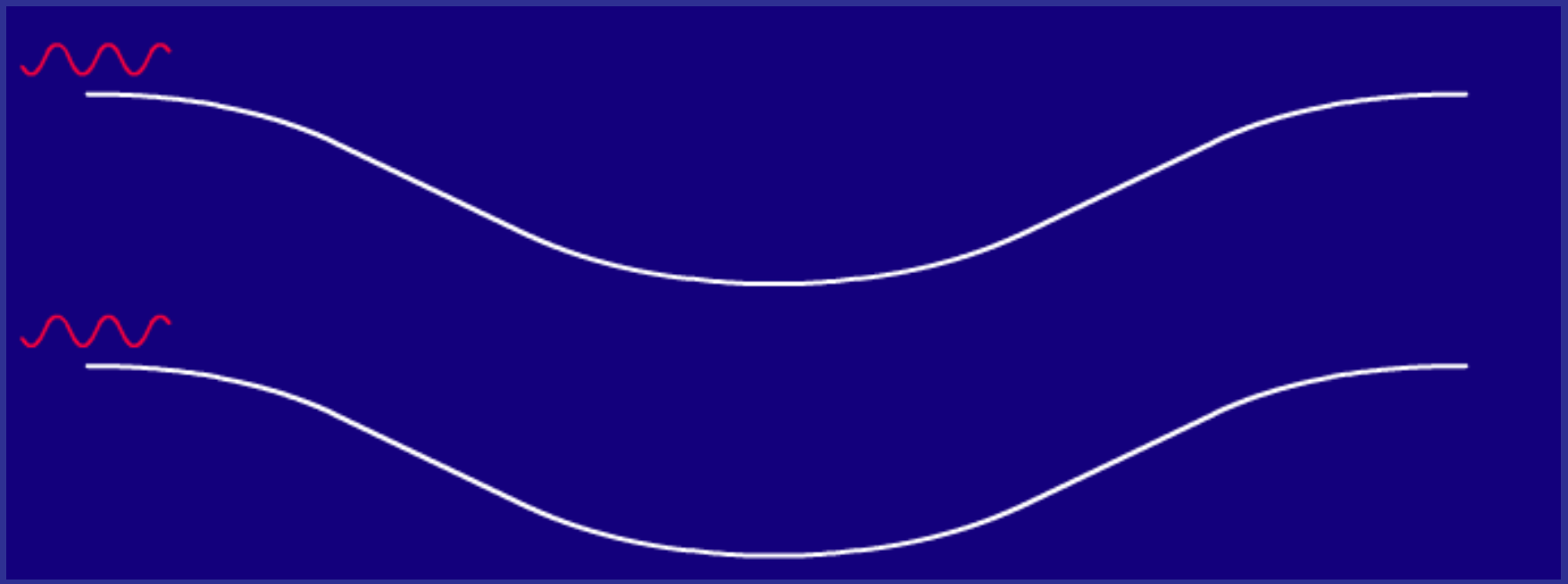
Naive Interpretation?

- No long-wavelength power? cut off near horizon scale $k=0.005\ \mathrm{Mpc}^{-1}$ - problematic due to ISW & polarization



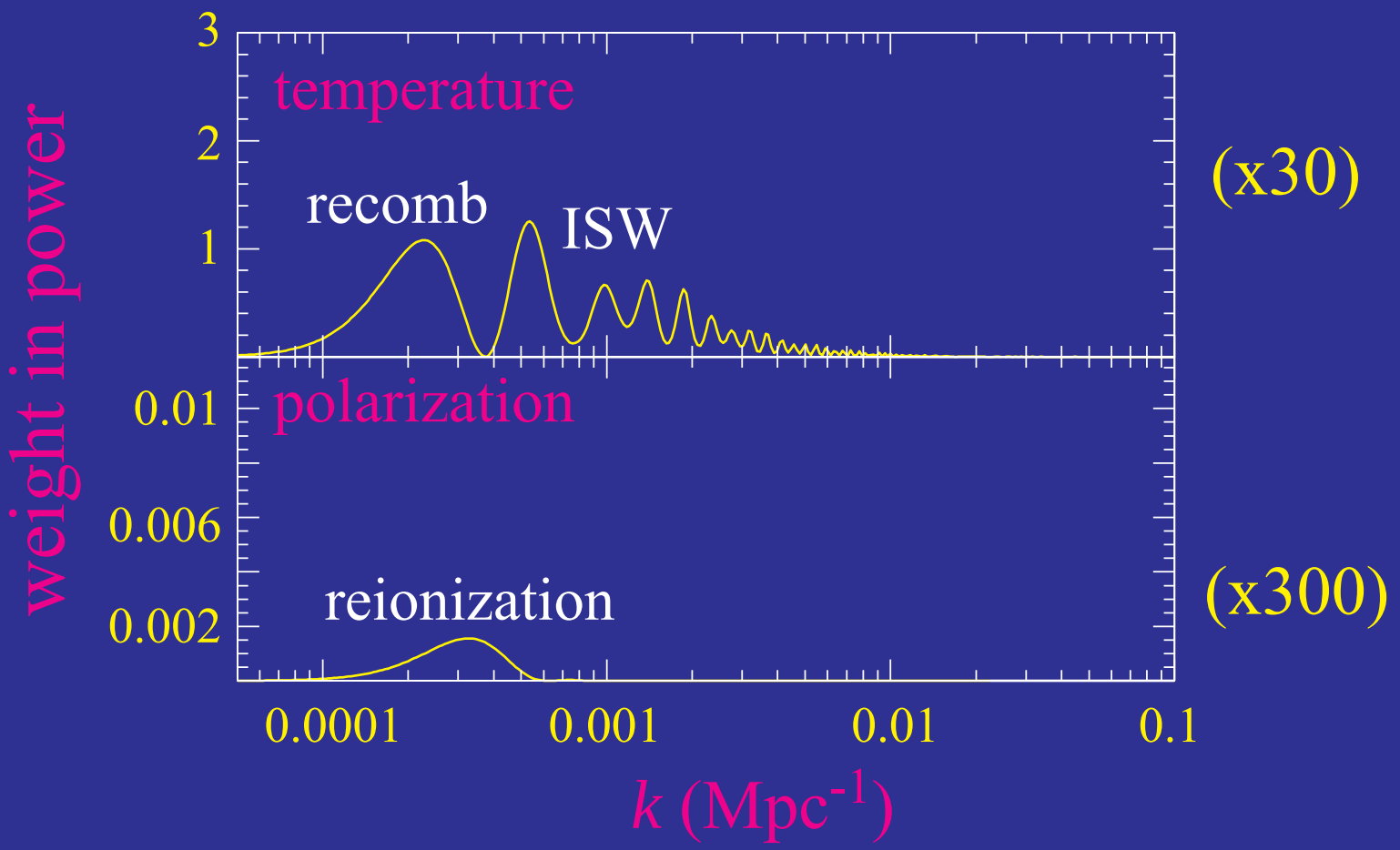
ISW Effect

- Cosmological constant is a spatially constant energy density and does not cluster with dark matter
- Gravitational potentials decay with the expansion in the dark energy dominated era
- Differential gravitational redshift integrated along the line of sight yields the Integrated Sachs-Wolfe effect



Temperature v. Polarization

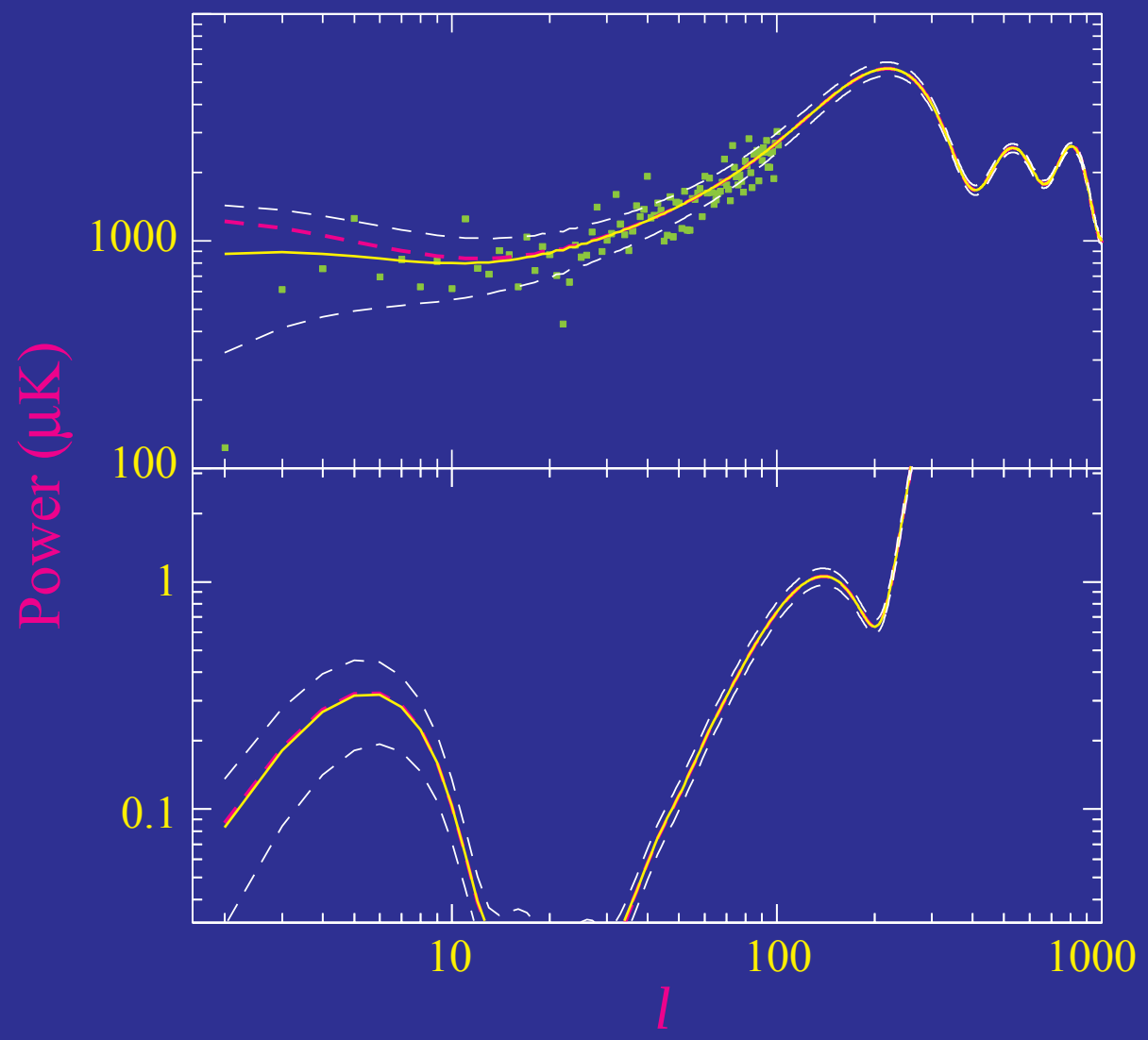
- Quadrupole in polarization originates from a tight range of scales around the current horizon
- Quadrupole in temperature gets contributions from 2 decades in scale



Hu & Okamoto (2003)

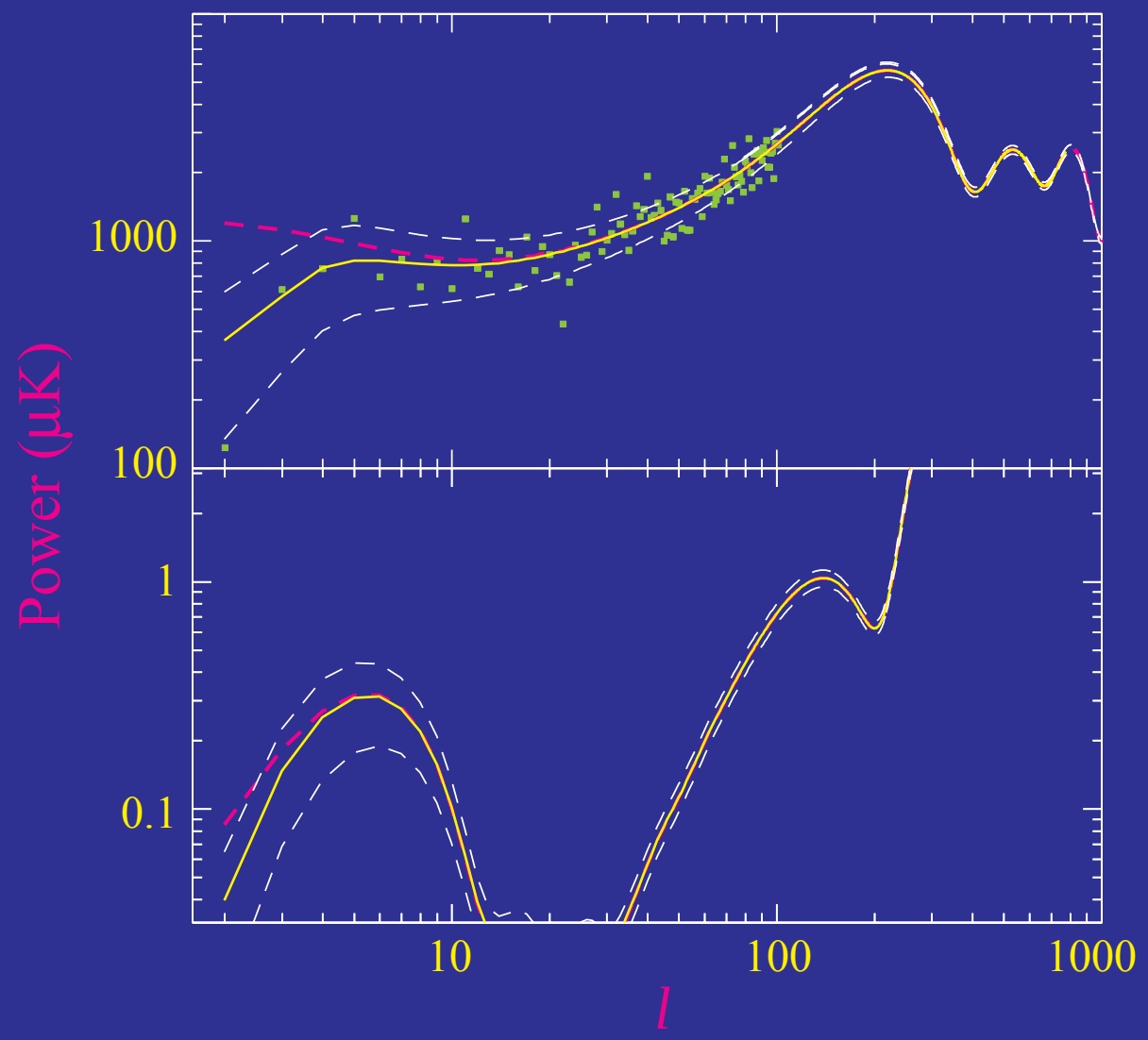
Exotic Dark Energy?

- Modify the clustering of the dark energy to eliminate low-*l* ISW effect - helps moderately, leaves polarization unaffected



Both?

- Can some change in gravity introduce a cut off and modify the large-scale dark energy? toy FRW model:



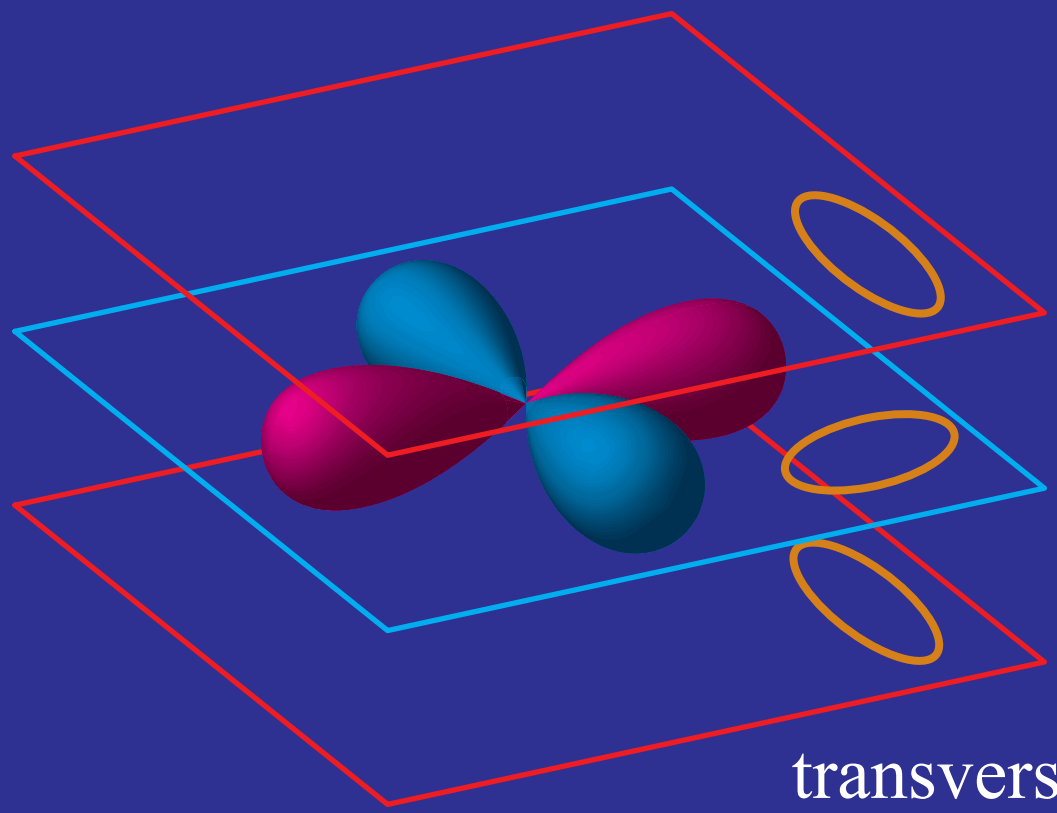
Neither

- A statistical fluke, e.g. short wavelength ISW cancels long wavelength SW - again polarization unaffected

Gravitational Waves

Gravitational Waves

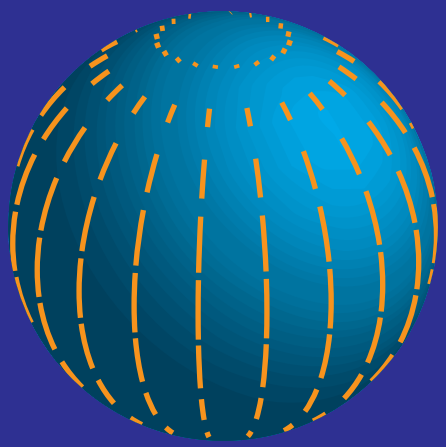
- Inflation predicts near scale invariant spectrum of gravitational waves
- Amplitude proportional to the square of the $E_I = V^{1/4}$ energy scale
- If inflation is associated with the grand unification $E_I \sim 10^{16}$ GeV and potentially observable



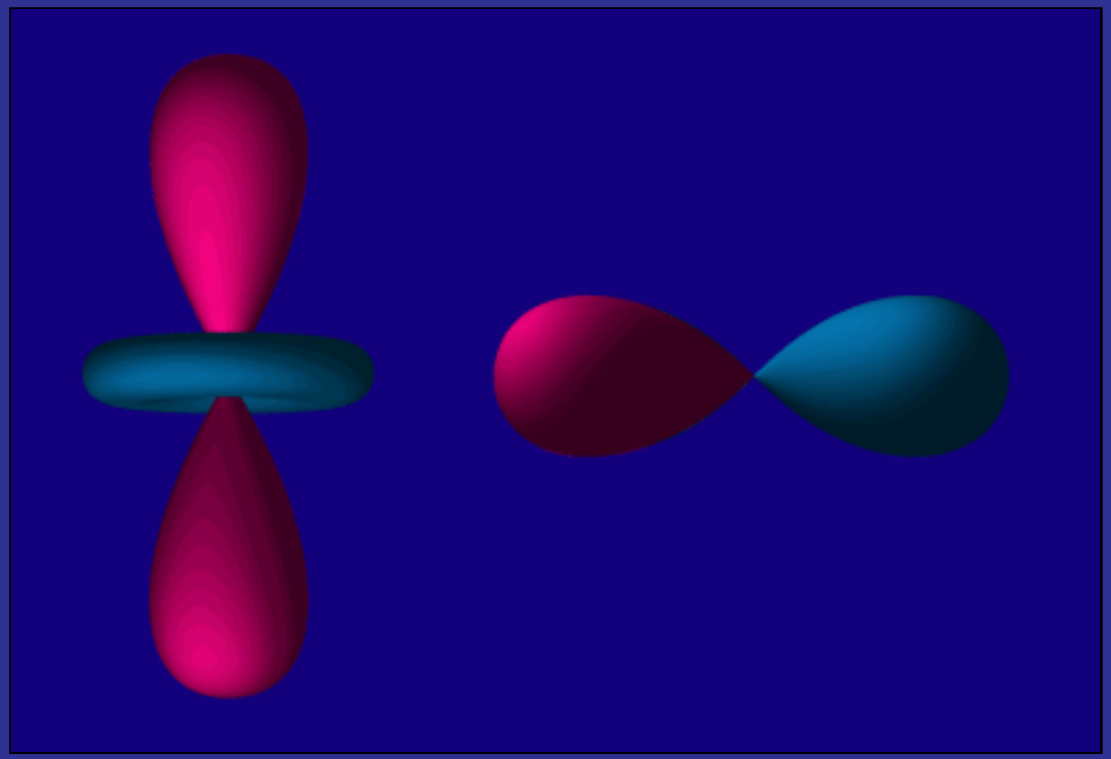
transverse-traceless distortion

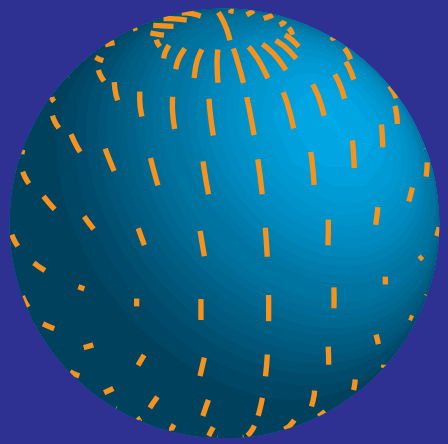
Gravitational Wave Pattern

- Projection of the quadrupole anisotropy gives polarization pattern
- Transverse polarization of gravitational waves breaks azimuthal symmetry



density perturbation



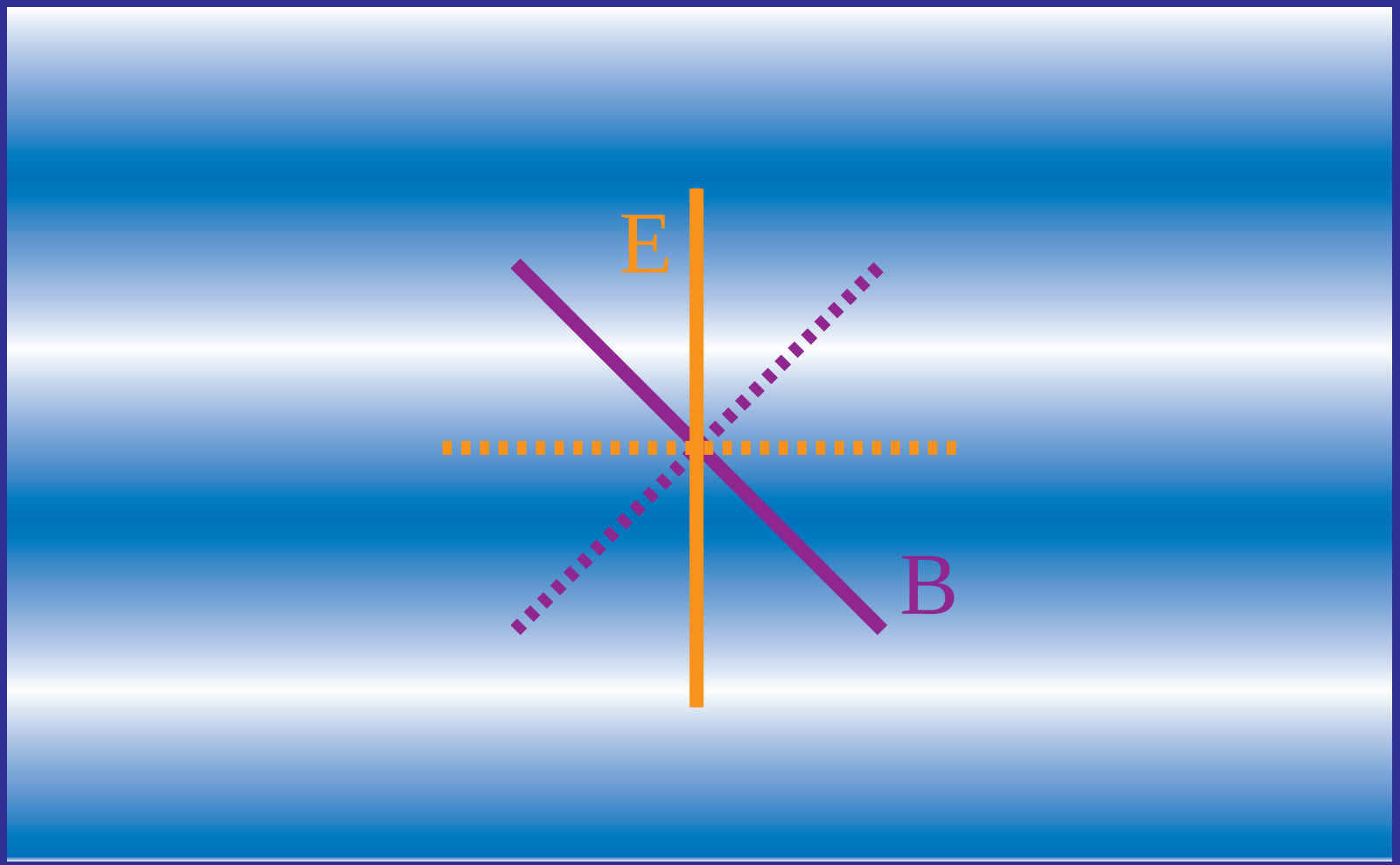


gravitational wave

Electric & Magnetic Polarization

(a.k.a. gradient & curl)

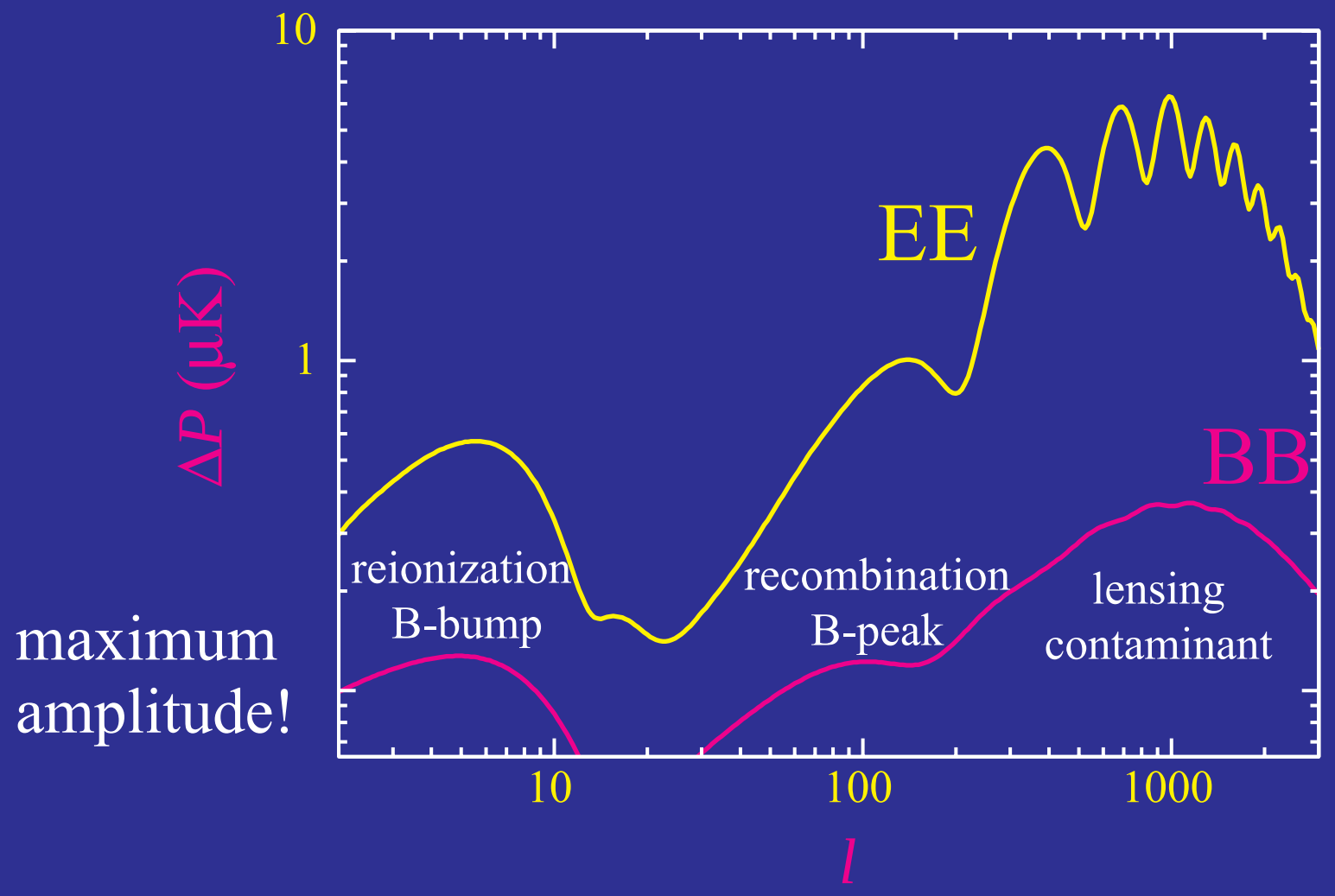
- Alignment of principal vs polarization axes (curvature matrix vs polarization direction)



Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)

The B-Bump

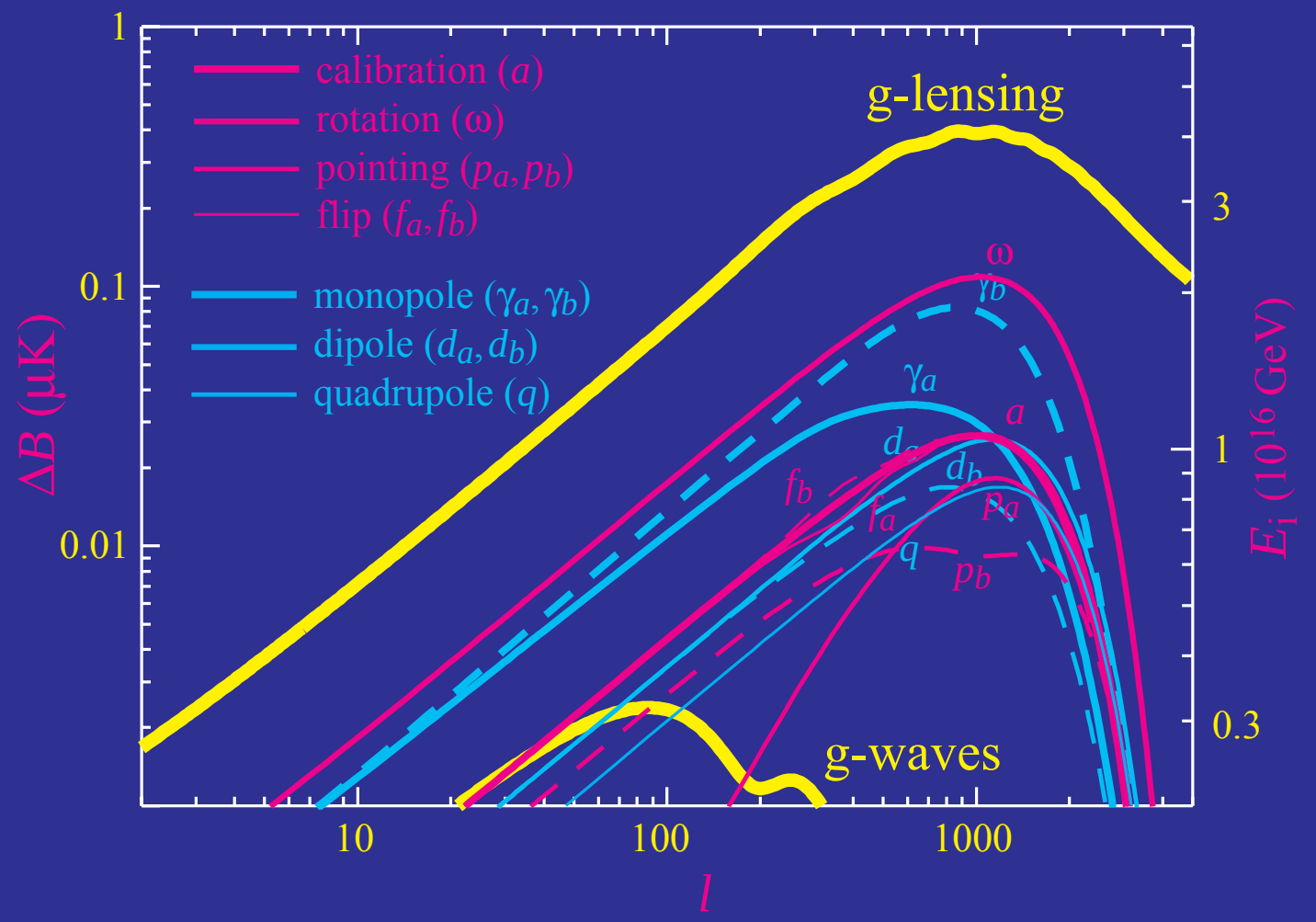
- Rescattering of gravitational wave anisotropy generates the B-bump
- Potentially the most sensitive probe of inflationary energy scale



maximum amplitude!

Contamination from E-mode Distortions

- Even small distortion (cosmological and instrumental) of the much larger polarization from density will will contaminate test below 10^{16} GeV - better for B-bump



Stability!
Space?

Foregrounds?

10^{-3} rms

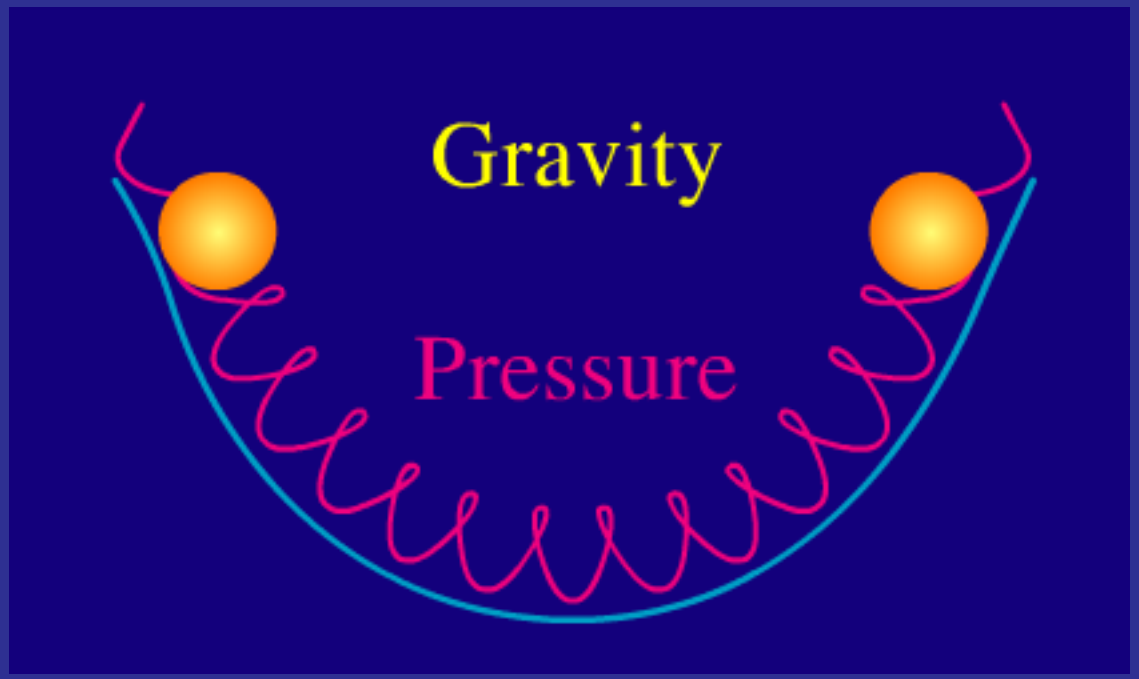
10^{-2} rms

Hu, Hedman & Zaldarriaga (2002)

Acoustic Waves

Acoustic Oscillations

- When T>3000K, medium ionized
- Photons tightly coupled to free electrons via Thomson scattering; electrons to protons via Coulomb interactions
- Medium behaves as a perfect fluid
- Radiation pressure competes with gravitational attraction causing perturbations to oscillate



Acoustic Basics

- Continuity Equation: (number conservation)

$$\dot{\Theta} = -\frac{1}{3} k v_\gamma$$

where $\Theta = \delta n_\gamma / 3 n_\gamma$ is the temperature fluctuation with $n_\gamma \propto T^3$

Acoustic Basics

- Continuity Equation: (number conservation)

$$\dot{\Theta} = -\frac{1}{3} k v_\gamma$$

where $\Theta = \delta n_\gamma / 3 n_\gamma$ is the temperature fluctuation with $n_\gamma \propto T^3$

- Euler Equation: (momentum conservation)

$$\dot{v}_\gamma = k(\Theta + \Psi)$$

with force provided by pressure gradients $k\delta p_\gamma/(\rho_\gamma + p_\gamma) = k\delta\rho_\gamma/4\rho_\gamma = k\Theta$ and potential gradients $k\Psi$.

Acoustic Basics

- Continuity Equation: (number conservation)

$$\dot{\Theta} = -\frac{1}{3} k v_\gamma$$

where $\Theta = \delta n_\gamma / 3 n_\gamma$ is the temperature fluctuation with $n_\gamma \propto T^3$

- Euler Equation: (momentum conservation)

$$\dot{v}_\gamma = k(\Theta + \Psi)$$

with force provided by pressure gradients $k\delta p_\gamma/(\rho_\gamma + p_\gamma) = k\delta\rho_\gamma/4\rho_\gamma = k\Theta$ and potential gradients $k\Psi$.

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi$$

where $c_s^2 \equiv \dot{p}/\dot{\rho}$ is the sound speed squared

Harmonic Peaks

- Adiabatic (Curvature) Mode Solution

$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0)\ \cos(ks)$$

where the sound horizon $s \equiv \int c_s d\eta$ and $\Theta + \Psi$ is also the observed temperature fluctuation after gravitational redshift

- All modes are frozen in at recombination

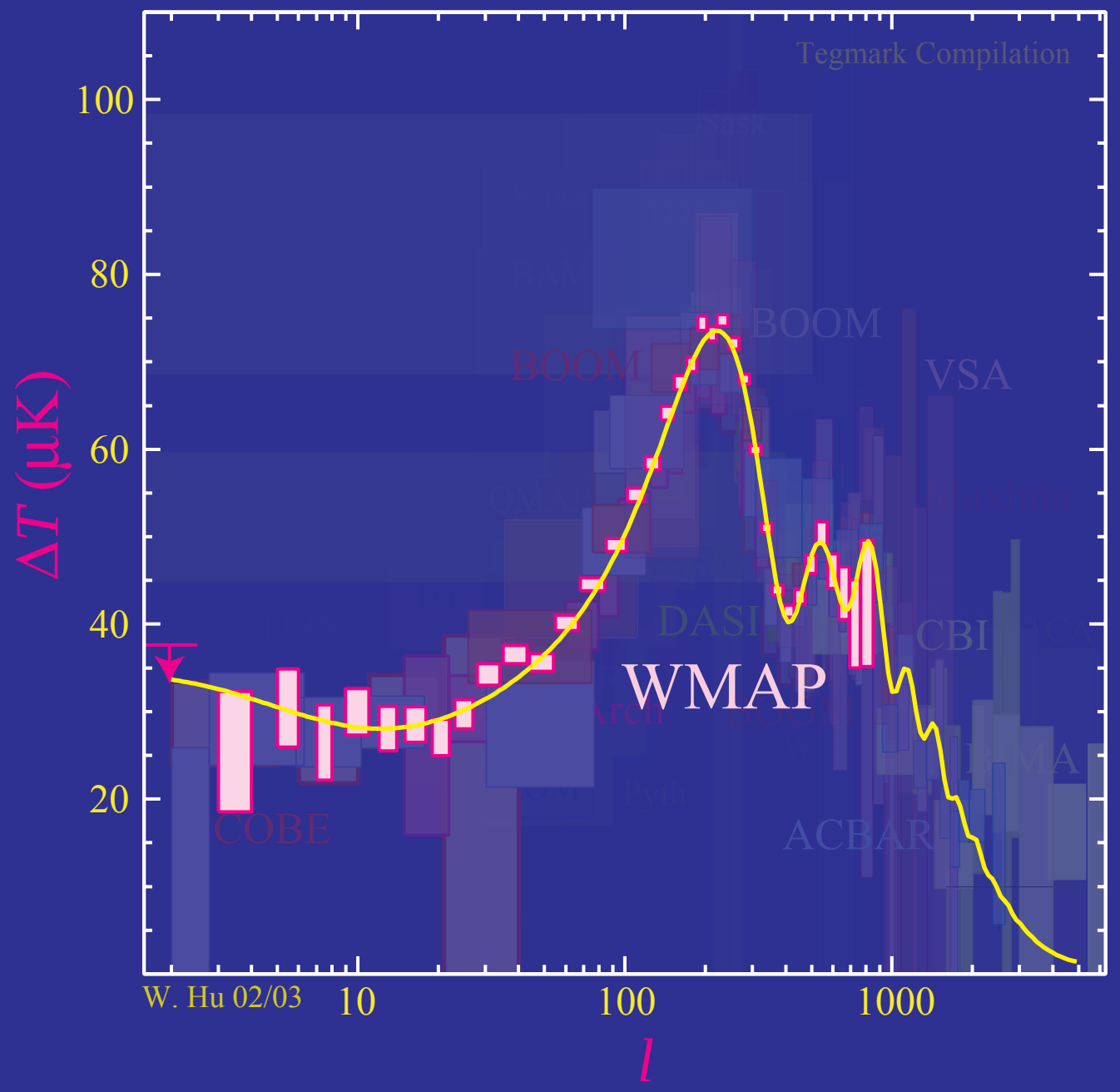
$$[\Theta + \Psi](\eta_*) = [\Theta + \Psi](0)\ \cos(ks_*)$$

- Modes caught in the extrema of their oscillation will have enhanced fluctuations

$$k_n s_* = n\pi$$

yielding a fundamental scale or frequency, related to the inverse sound horizon and series dependent on adiabatic assumption

Temperature Spectrum



Viscosity & Polarization

Fluid Imperfections

- Perfect fluid: no anisotropic stresses due to scattering isotropization; baryons and photons move as single fluid
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

  is the conformal opacity to Thomson scattering
- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

  the geometric mean between the horizon and mean free path
- $\lambda_D/\eta_* \sim$ few %, so expect the peaks >3 to be affected by dissipation

Equations of Motion

- Continuity

$$\dot{\Theta} = -\frac{k}{3} v_\gamma \,, \quad \dot{\delta}_b = -k v_b$$

where gravitational effects ignored and $\Theta \equiv \Delta T/T$.

- Euler

$$\dot{v}_\gamma = k\Theta - \frac{k}{6}\pi_\gamma - \dot{\tau}(v_\gamma - v_b)$$

$$\dot{v}_b = -\frac{\dot{a}}{a} v_b + \dot{\tau}(v_\gamma - v_b)/R$$

where $k\Theta$ is the pressure gradient term, $k\pi_\gamma$ is the viscous stress term, and $v_\gamma - v_b$ is the momentum exchange term with $R \equiv 3\rho_b/4\rho_\gamma$ the baryon-photon momentum ratio.

Damping

- Perfect fluid: no anisotropic stresses due to scattering isotropization; baryons and photons move as single fluid
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

is the conformal opacity to Thomson scattering

Damping

- Perfect fluid: no anisotropic stresses due to scattering isotropization; baryons and photons move as single fluid
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

is the conformal opacity to Thomson scattering

- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

the geometric mean between the horizon and mean free path

Damping

- Perfect fluid: no anisotropic stresses due to scattering isotropization; baryons and photons move as single fluid
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

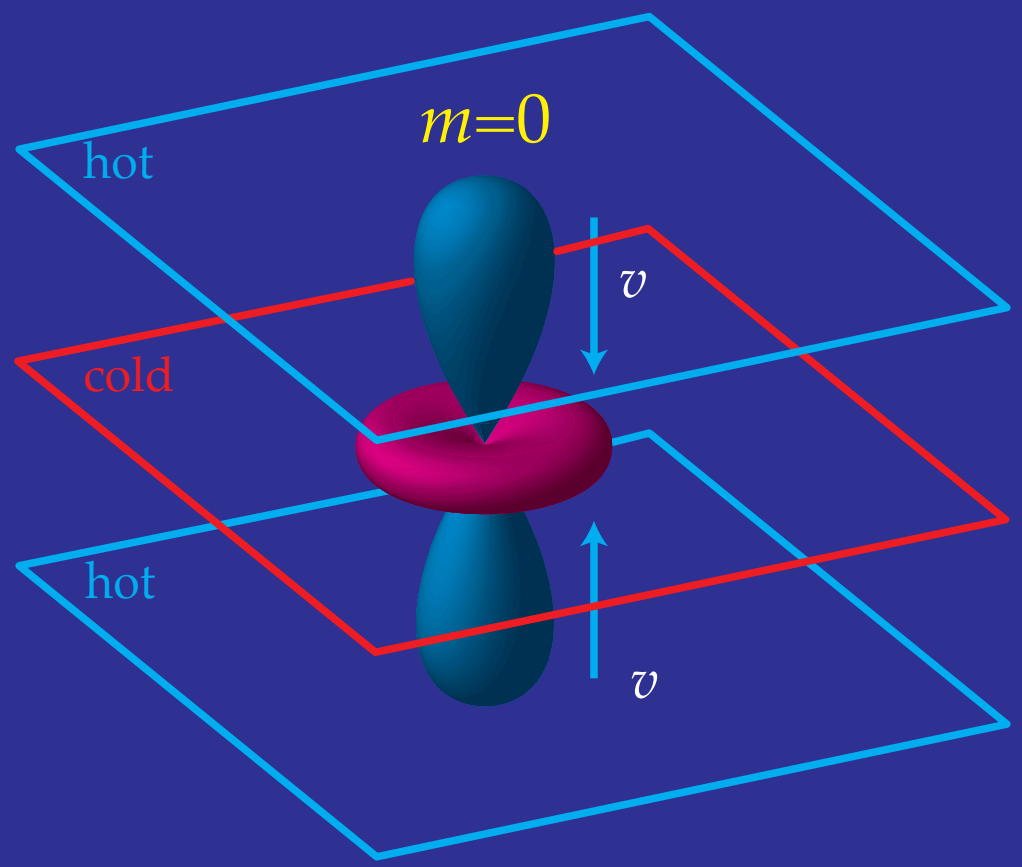
  is the conformal opacity to Thomson scattering
- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

  the geometric mean between the horizon and mean free path
- $\lambda_D/\eta_* \sim$ few %, so expect the peaks $>$ 3rd to be affected by dissipation

Viscosity & Heat Conduction

- Both fluid imperfections are related to the gradient of the velocity kv_γ by opacity $\dot{\tau}$: slippage of fluids $v_\gamma - v_b$.
- Viscosity is an anisotropic stress or quadrupole moment formed by radiation streaming from hot to cold regions



Viscosity

- Viscosity is generated from radiation streaming from hot to cold regions
- Expect

$$\pi_\gamma \sim v_\gamma \frac{k}{\dot{\tau}}$$

generated by streaming, suppressed by scattering in a wavelength of the fluctuation. Radiative transfer says

$$\pi_\gamma \approx 2 A_v v_\gamma \frac{k}{\dot{\tau}}$$

where $A_v = 16/15$

$$\dot{v}_\gamma = k(\Theta + \Psi) - \frac{k}{3} A_v \frac{k}{\dot{\tau}} v_\gamma$$

Damping Term

- Oscillator equation contains a $\dot{\Theta}$ damping term

$$\ddot{\Theta} + \frac{k^2}{\dot{\tau}} A_{\rm d} \dot{\Theta} + k^2 c_s^2 \Theta = 0$$

- Solve in the adiabatic approximation

$$\Theta \propto \exp(i \int \omega d\eta)$$

$$\exp(i \int \omega d\eta) = e^{\pm ik \int c_s d\eta} \exp[-(k/k_D)^2]$$

- Diffusion wavenumber, geometric mean between horizon and mfp:

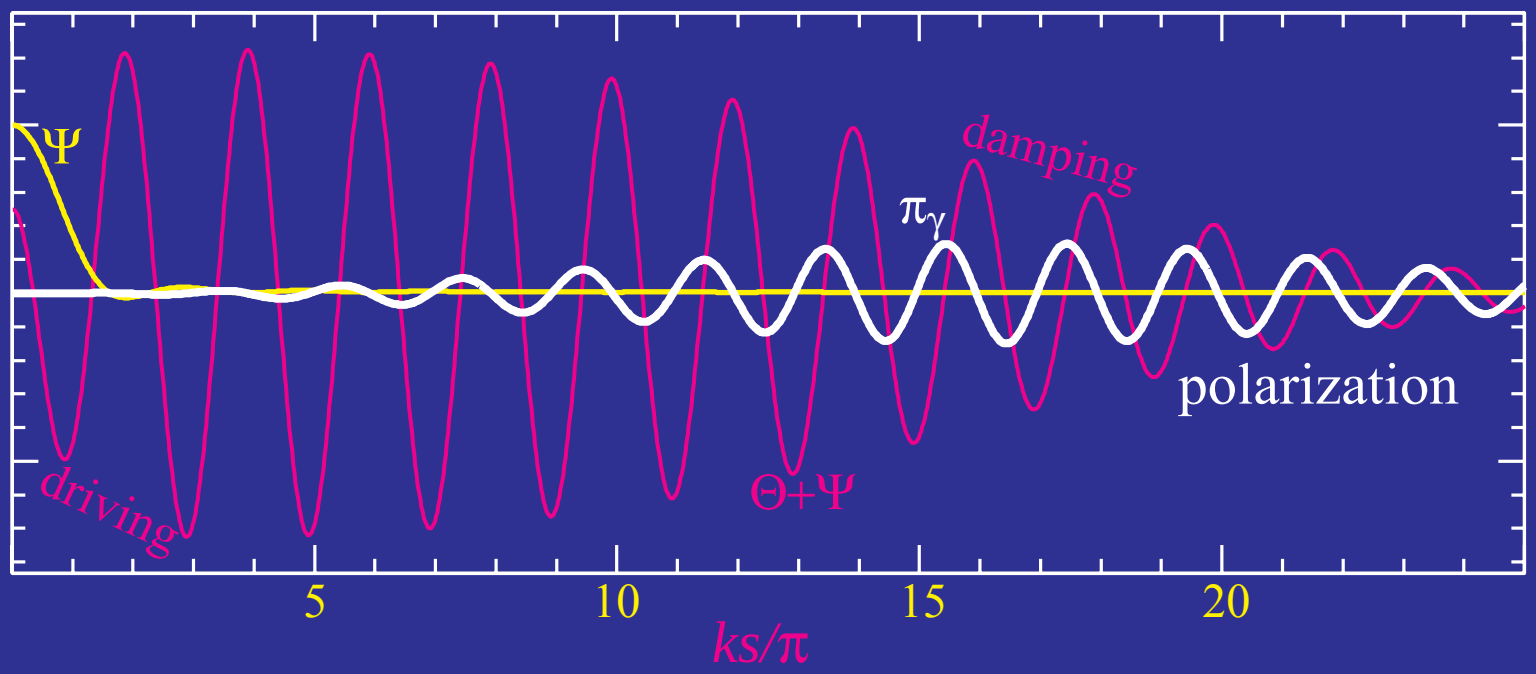
$$k_D^{-2} = \frac{1}{2} \int \frac{d\eta}{\dot{\tau}} A_{\rm d} \sim \frac{\eta}{\dot{\tau}}$$

Damping & Polarization

- Quadrupole moments:

  damp acoustic oscillations from fluid viscosity
  generates polarization from scattering

- Rise in polarization power coincides with fall in temperature power – $l \sim 1000$



Dimensional Analysis

- Viscosity= quadrupole anisotropy that follows the fluid velocity

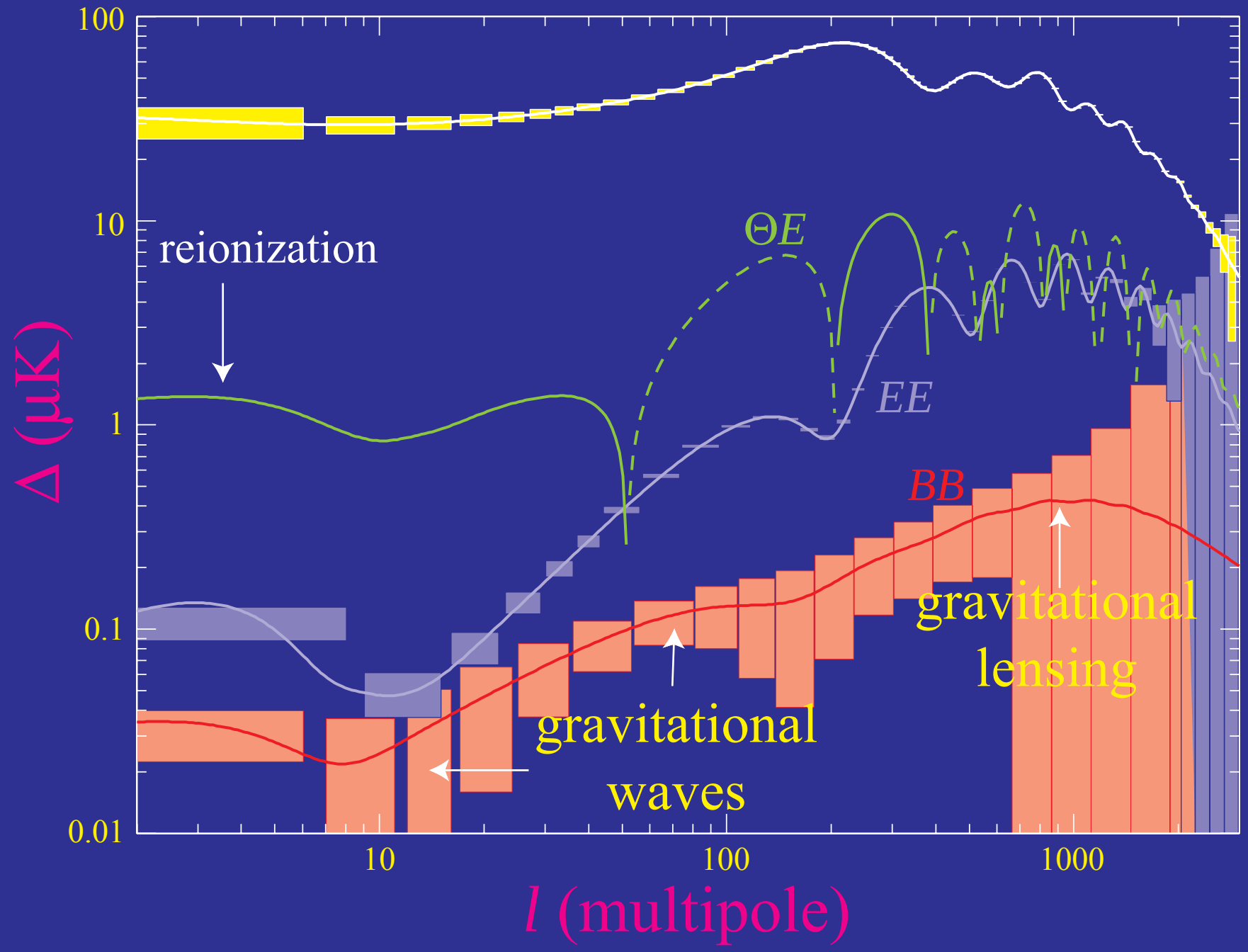
$$\pi_\gamma \approx \frac{k}{\dot{\tau}} v_\gamma$$

- Mean free path related to the damping scale via the random walk $k_D = (\dot{\tau}/\eta_*)^{1/2} \rightarrow \dot{\tau} = k_D^2 \eta_*$
- Damping scale at $\ell \sim 1000$ vs horizon scale at $\ell \sim 100$ so $k_D \eta_* \approx 10$
- Polarization amplitude rises to the damping scale to be $\sim 10\%$ of anisotropy

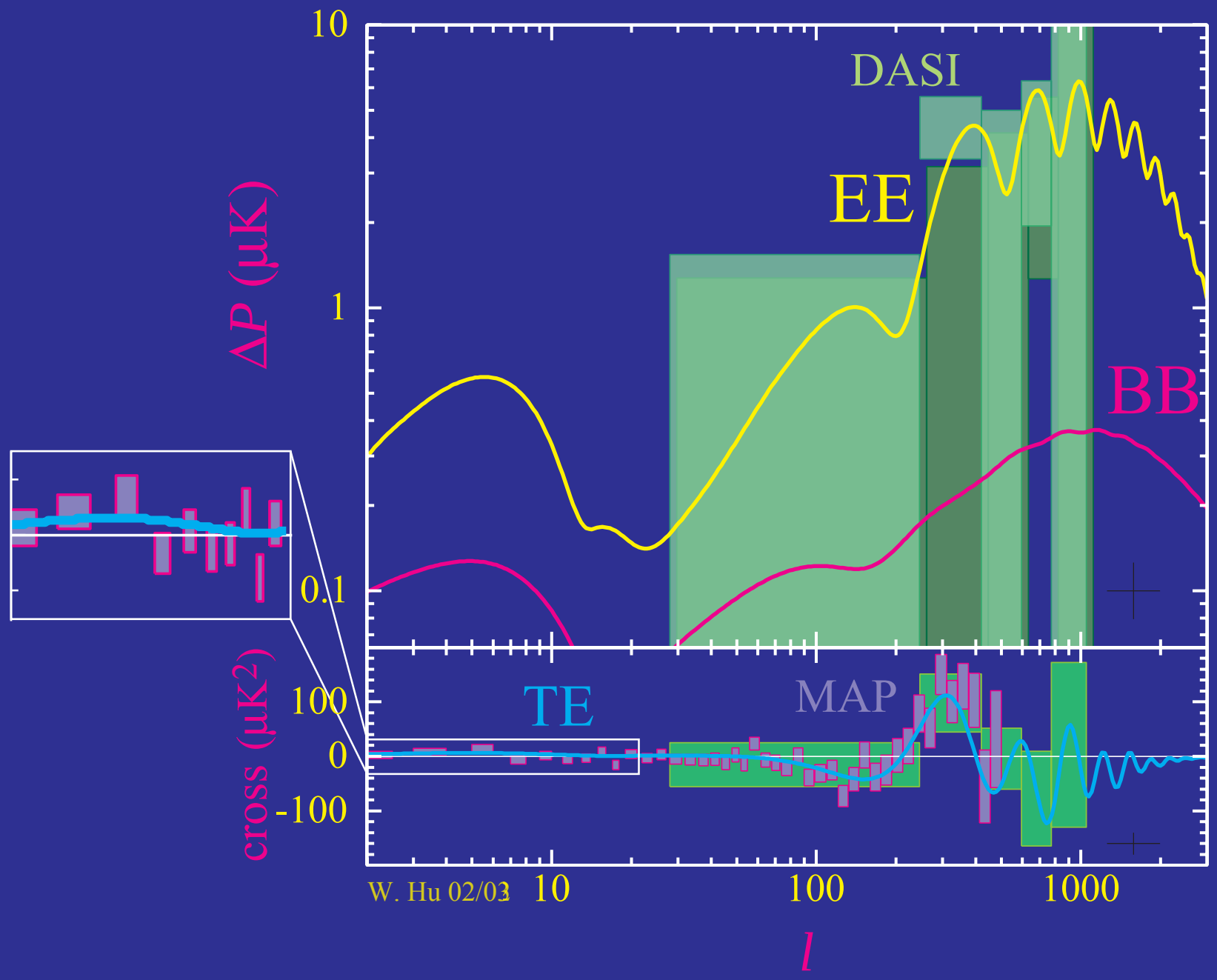
$$\pi_\gamma \approx \frac{k}{k_D} \frac{1}{10} v_\gamma \qquad \Delta_P \approx \frac{\ell}{\ell_D} \frac{1}{10} \Delta_T$$

- Polarization phase follows fluid velocity

Temperature and Polarization Spectra



Current Status



Why Care?

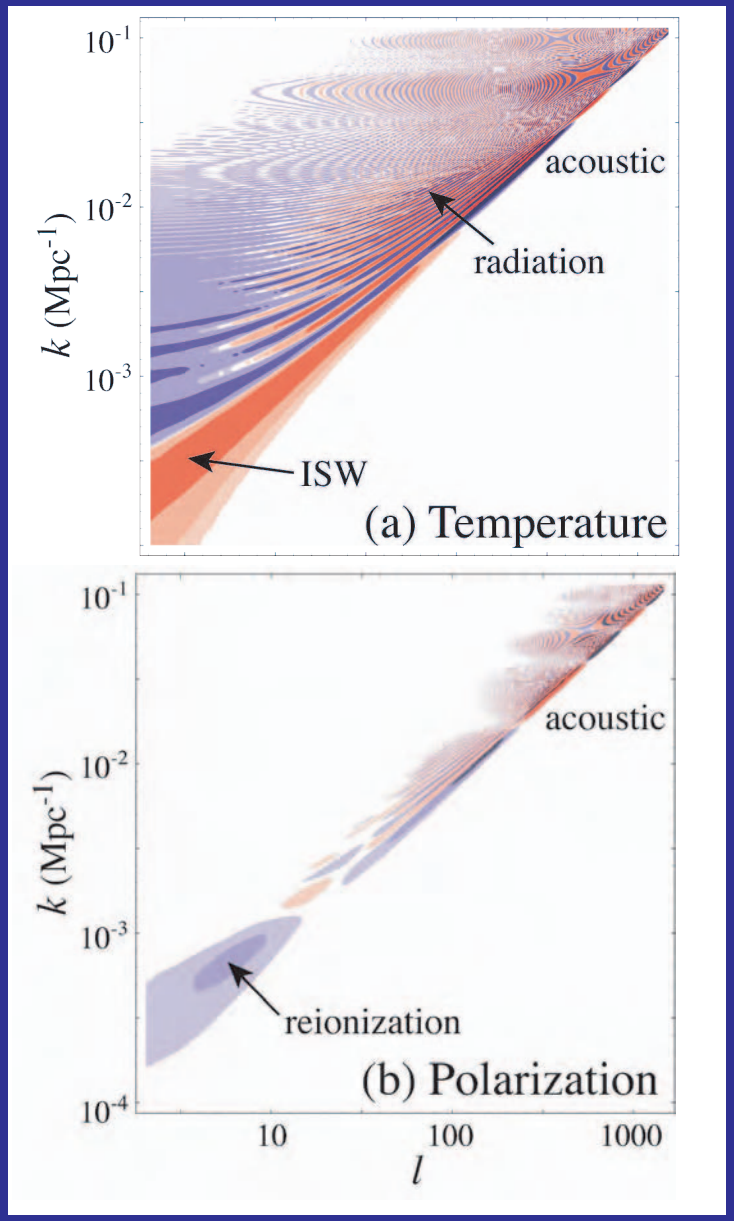
- In the standard model, acoustic polarization spectra uniquely predicted by same parameters that control temperature spectra
- Validation of standard model
- Improved statistics on cosmological parameters controlling peaks

- Polarization is a complementary and intrinsically more incisive probe of the initial power spectrum and hence inflationary (or alternate) models

- Acoustic polarization is lensed by the large scale structure into B-modes
- Lensing B-modes sensitive to the growth of structure and hence neutrino mass and dark energy
- Contaminate the gravitational wave B-mode signature

Initial Spectrum

Transfer of Initial Power

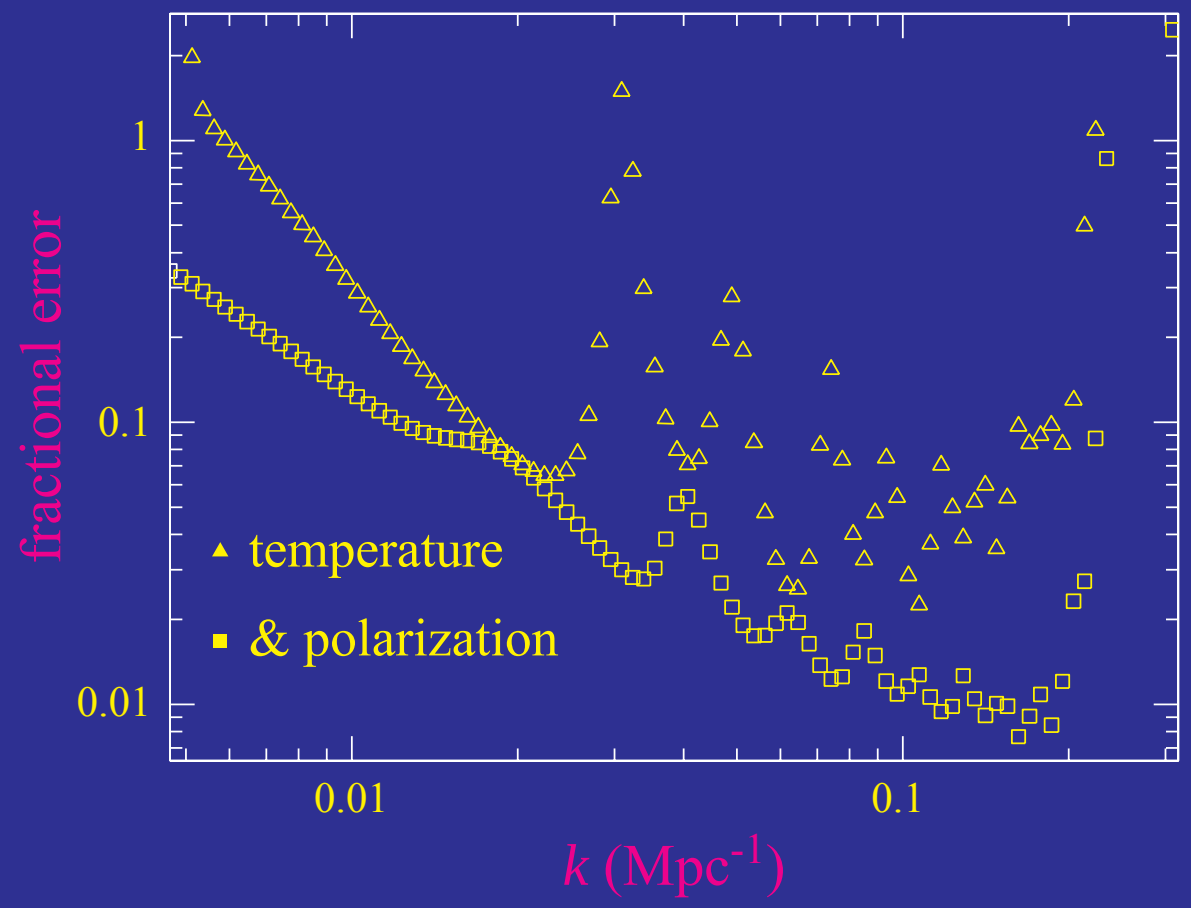


Hu & Okamoto (2003)

Prospects for Initial Conditions

- Polarization crucial for detailed study of initial conditions, decade in scale of the acoustic peaks can provide exquisite tests of scale free initial conditions

Hu & Okamoto (2003)



cosmological parameters fixed

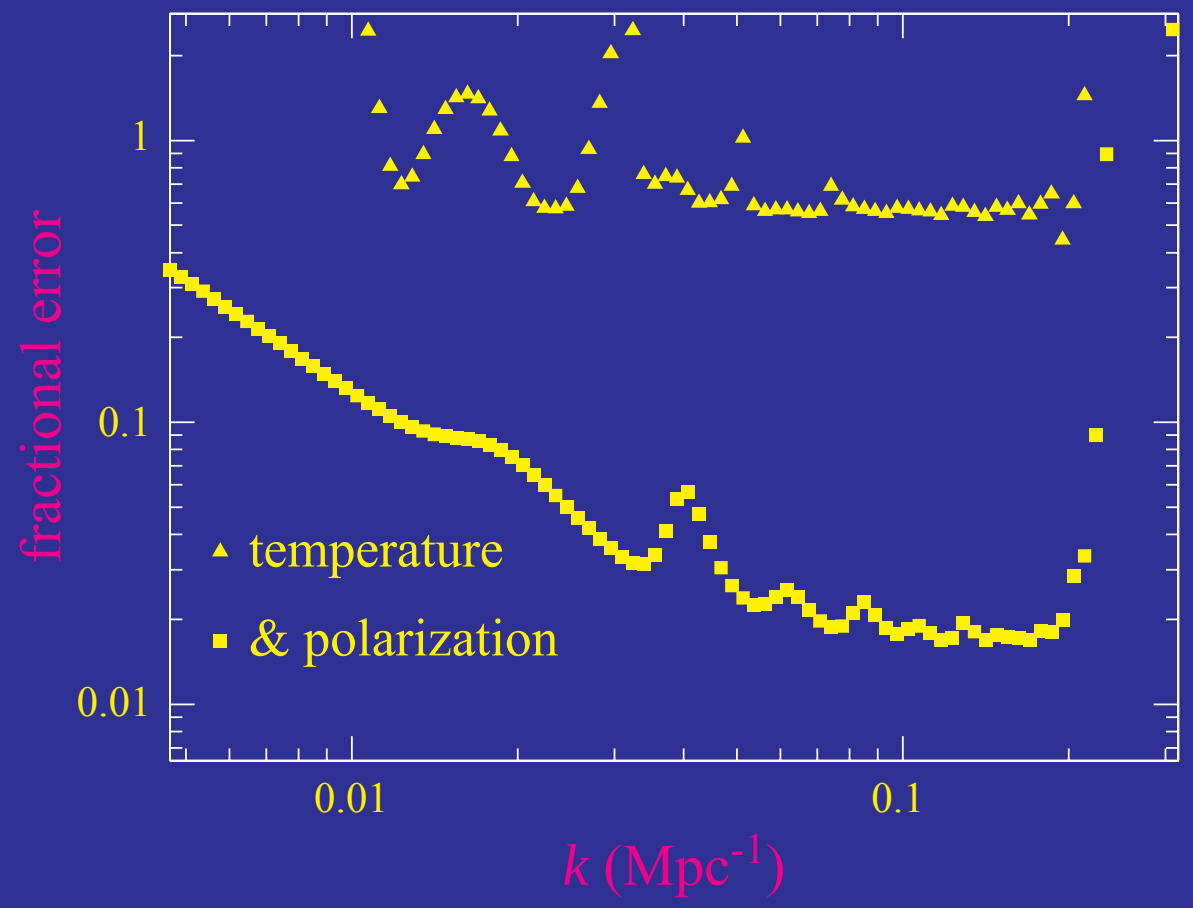
even with large lever arm other tests must match accuracy

Wang et al (1999); Kinney (2001); Miller et al (2002); Tegmark & Zaldarriaga (2002); Bridle et al (2003)

Prospects for Initial Conditions

- Polarization crucial for detailed study of initial conditions, decade in scale of the acoustic peaks can provide exquisite tests of scale free initial conditions

Hu & Okamoto (2003)

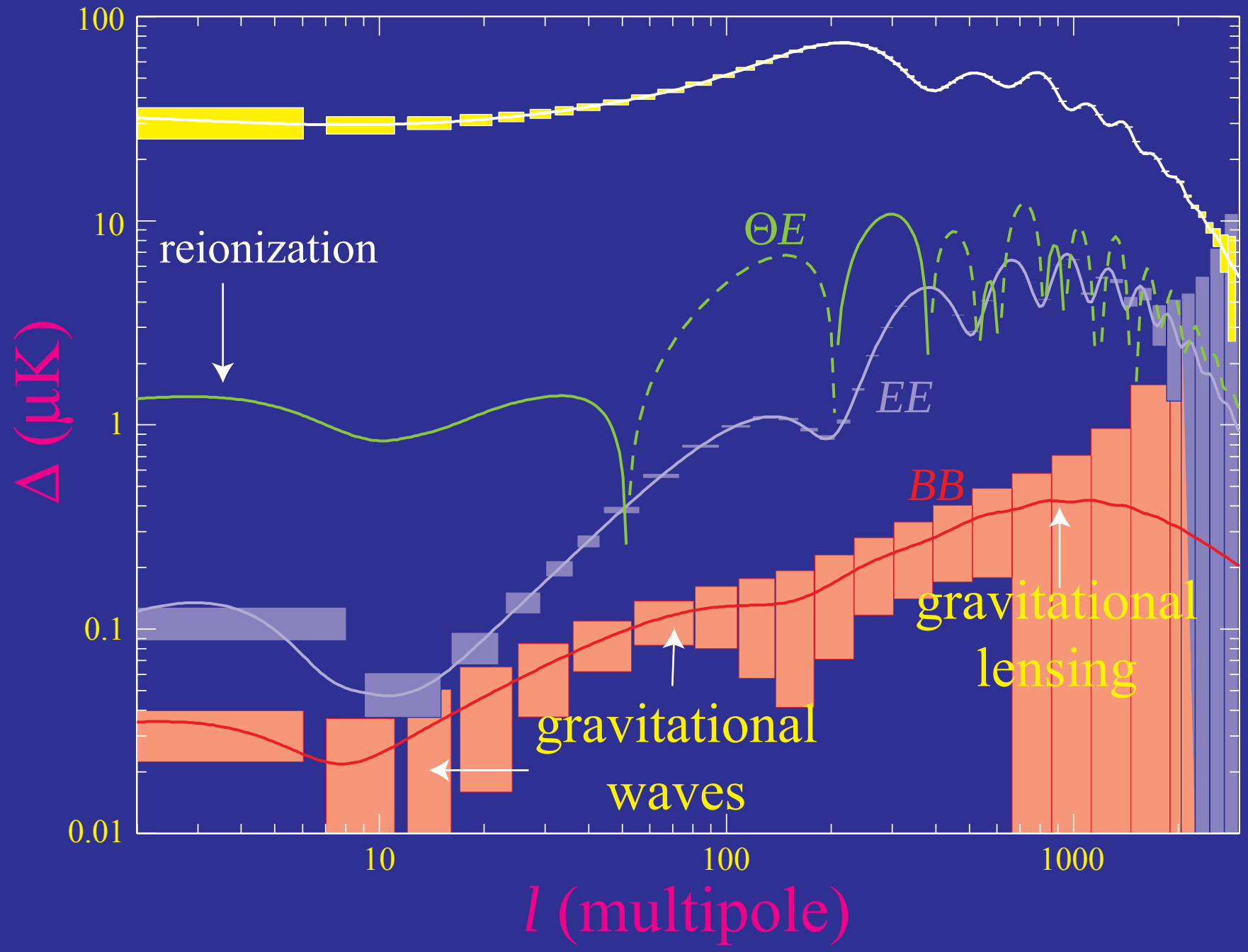


cosmological parameters marginalized

even with large lever arm other tests must match accuracy

Wang et al (1999); Kinney (2001); Miller et al (2002); Tegmark & Zaldarriaga (2002); Bridle et al (2003)

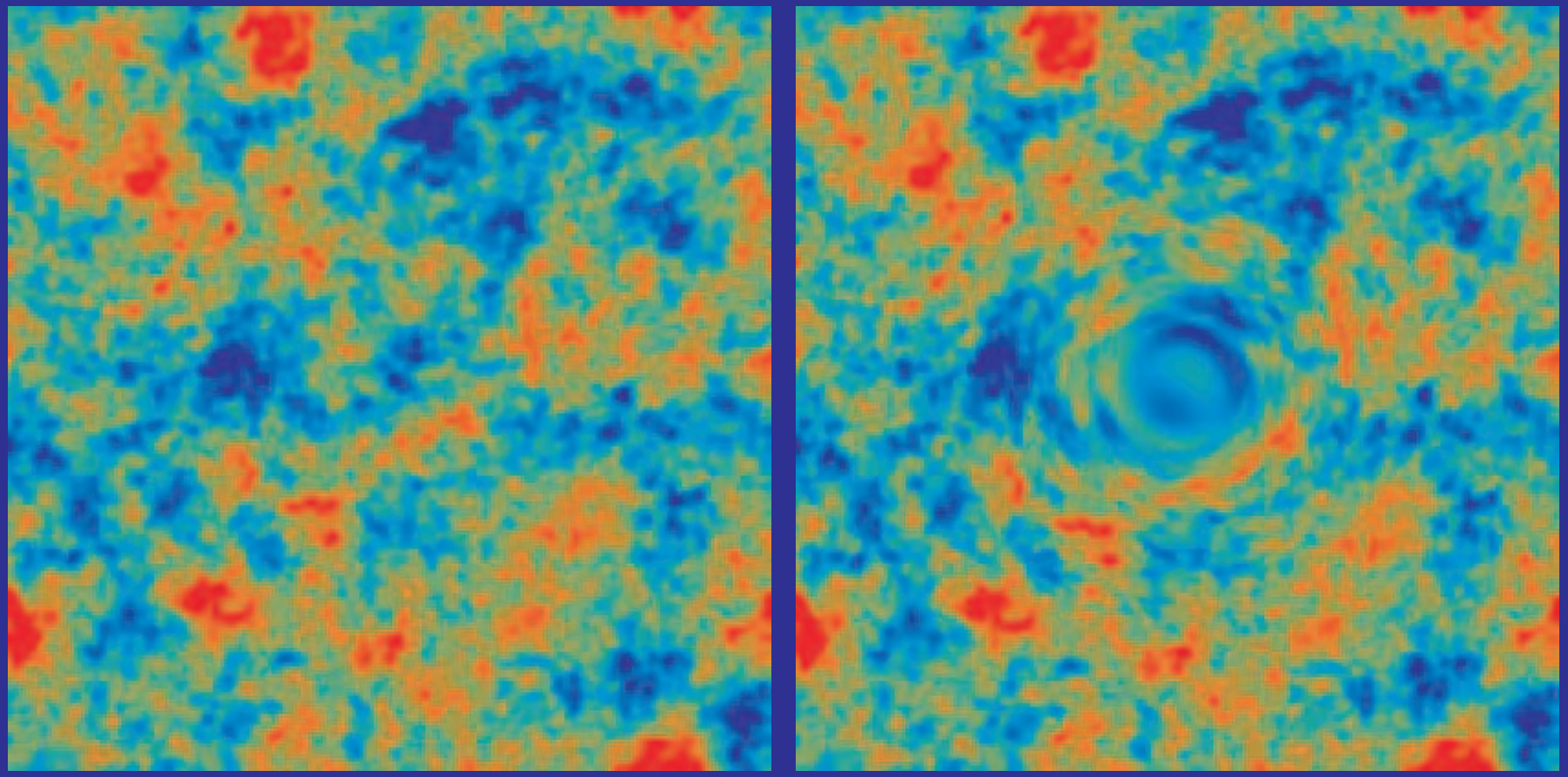
Temperature and Polarization Spectra



Gravitational Lensing

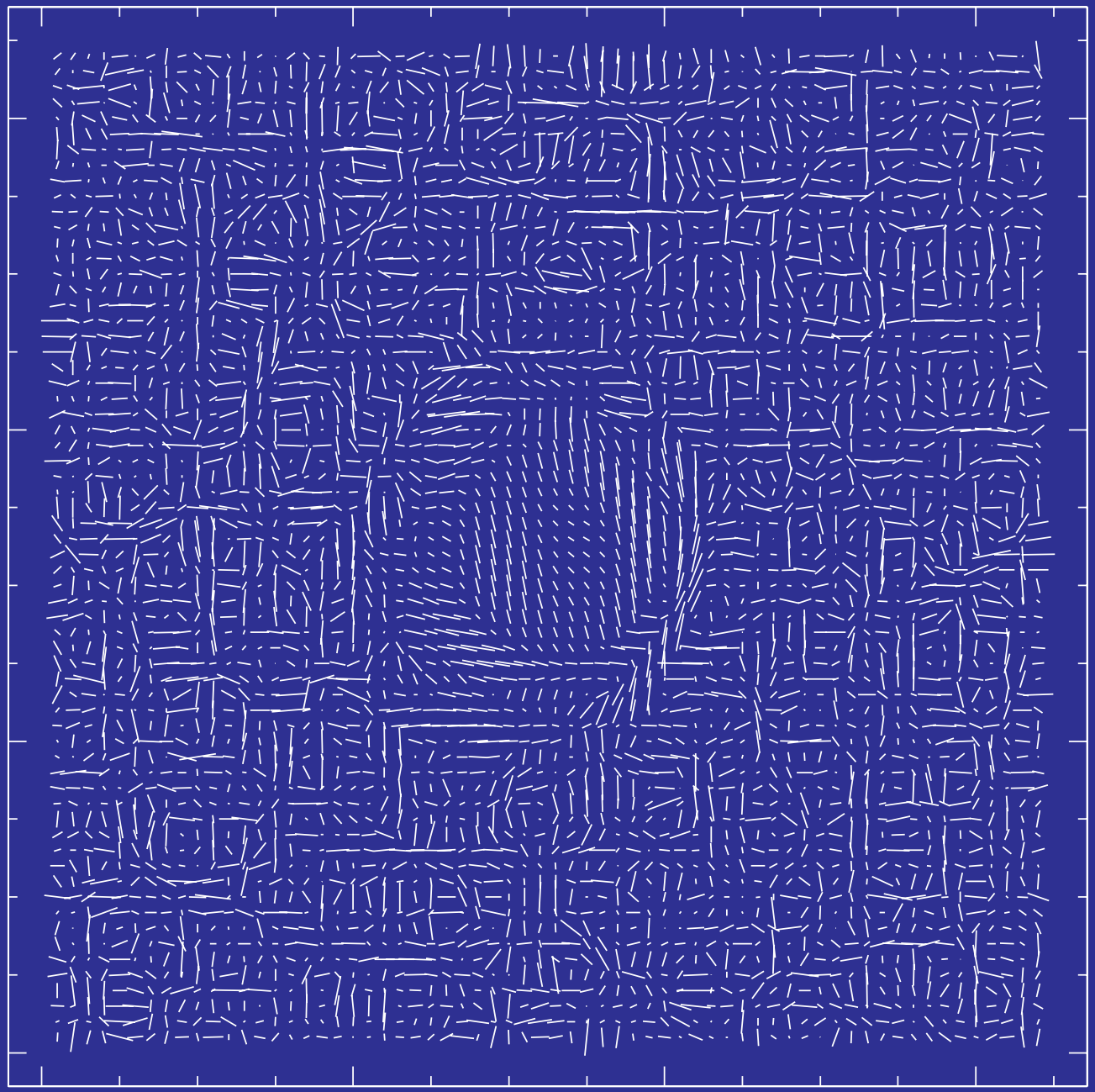
Gravitational Lensing

- Gravitational lensing by large scale structure distorts the observed temperature and polarization fields
- Exaggerated example for the temperature



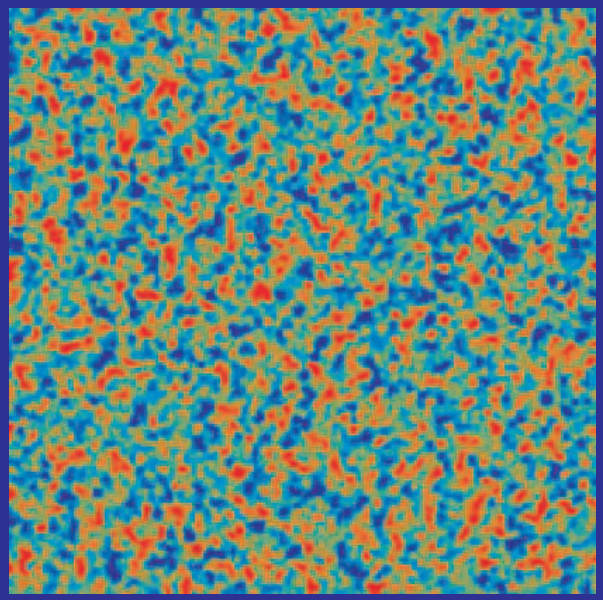
Original Lensed

Polarization Lensing

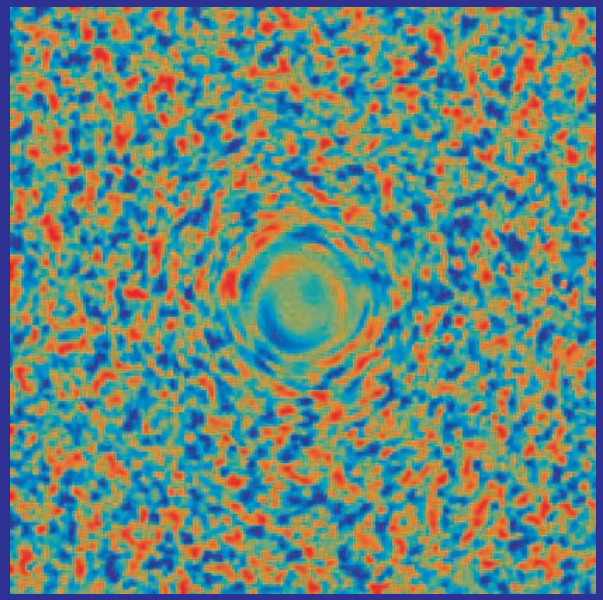


Polarization Lensing

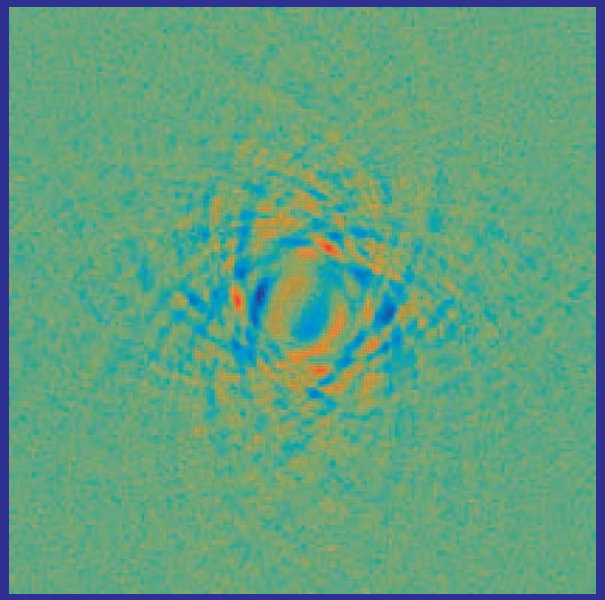
- Since E and B denote the relationship between the polarization amplitude and direction, warping due to lensing creates B-modes



Original



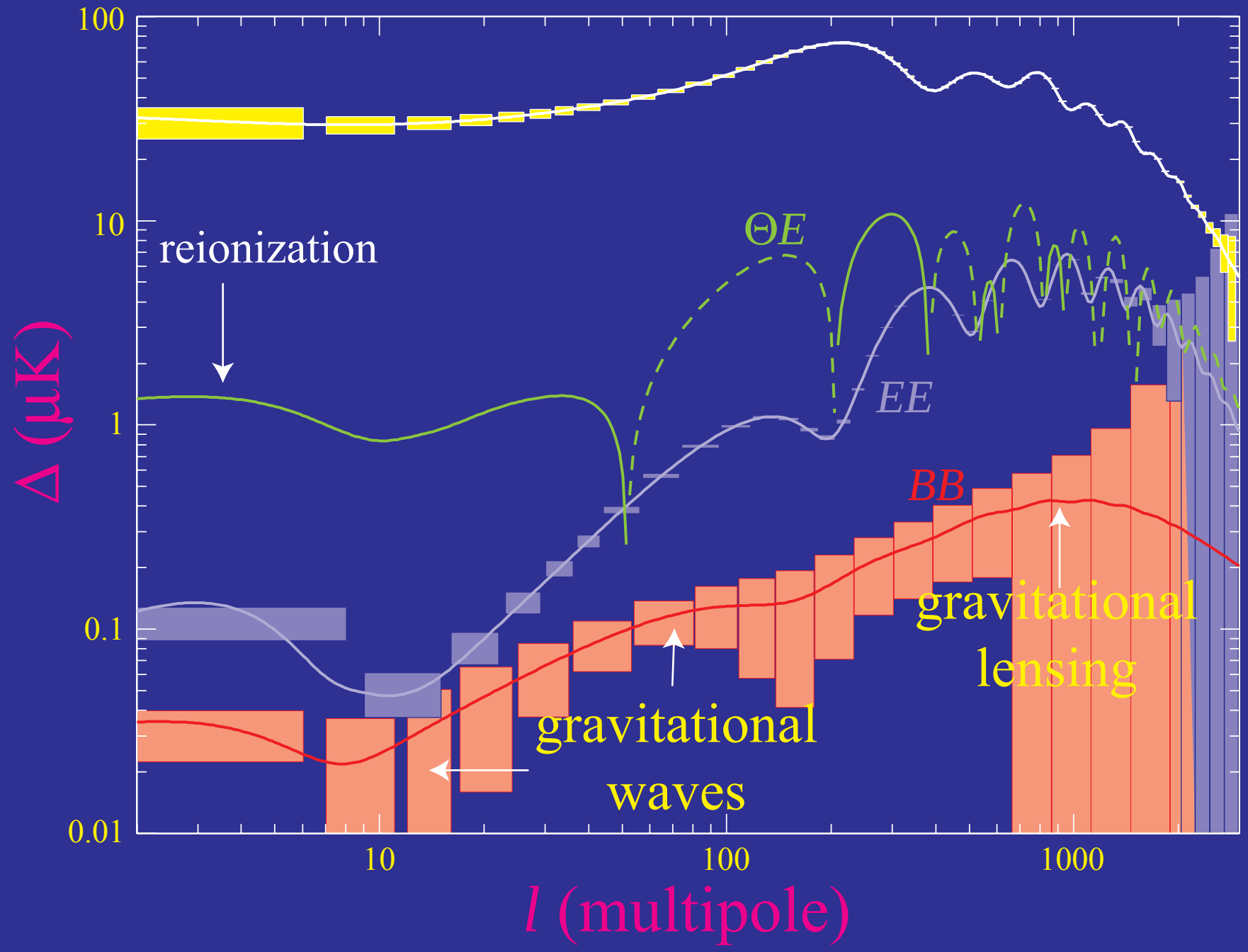
Lensed E



Lensed B

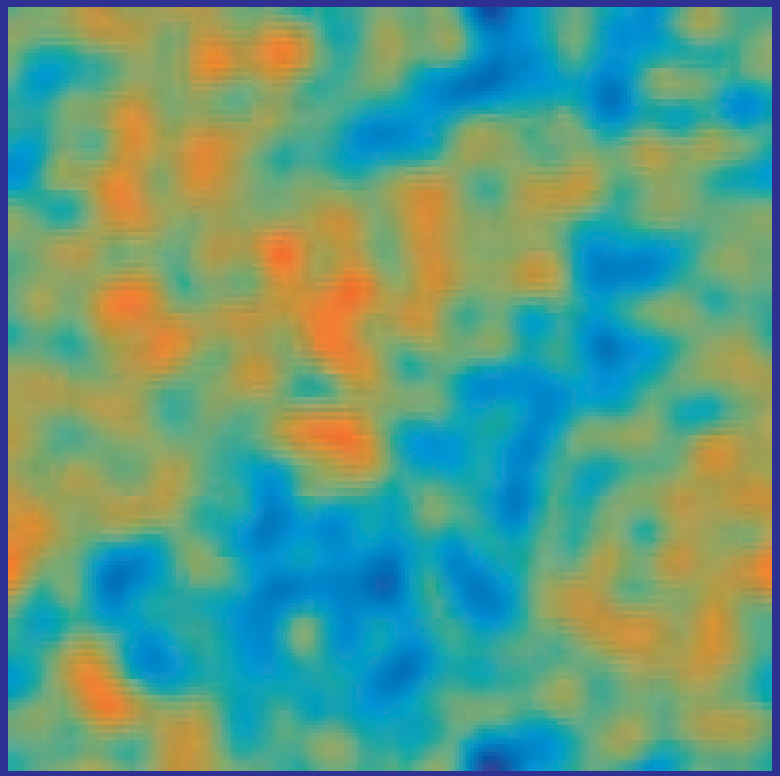
Zaldarriaga & Seljak (1998) [figure: Hu & Okamoto (2001)]

Temperature and Polarization Spectra



Lensing by a Gaussian Random Field

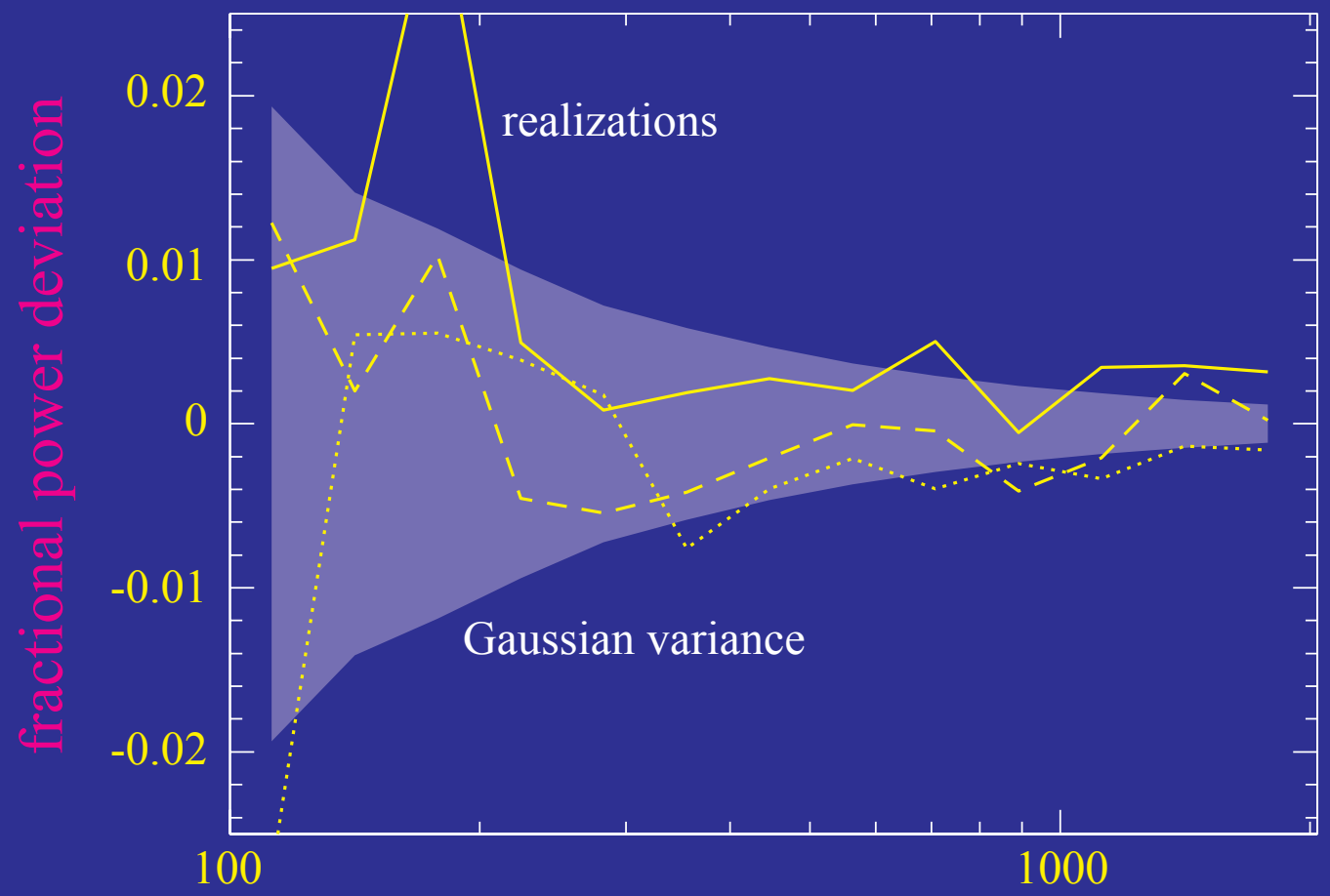
- Mass distribution at large angles and high redshift in in the linear regime
- Projected mass distribution (low pass filtered reflecting deflection angles): 1000 sq. deg



rms deflection
2.6'
deflection coherence
10°

Power Spectrum Measurements

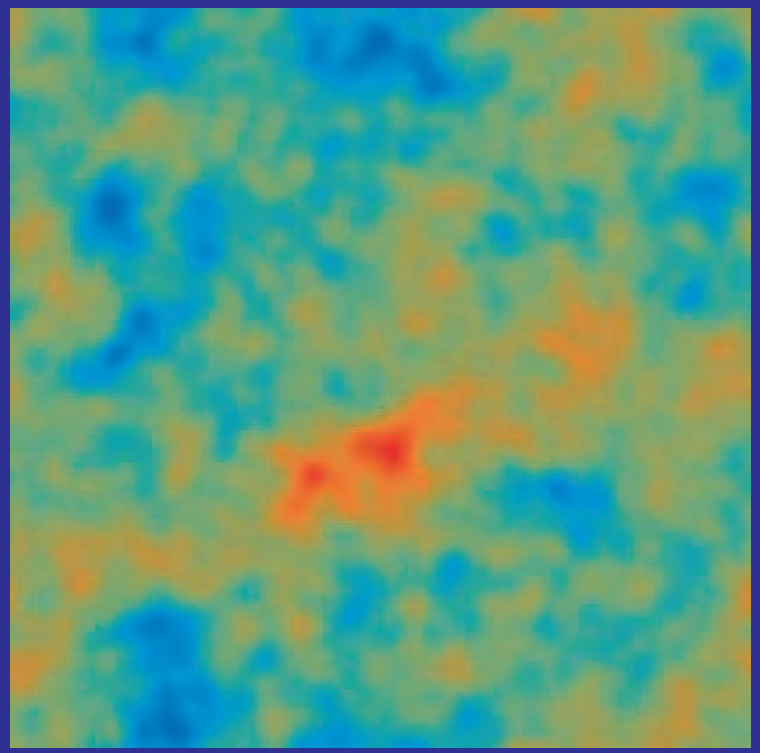
- Lensed field is non-Gaussian in that a single degree scale lens controls the polarization at arcminutes
- Increased variance and covariance implies that 10x as much sky needed compared with Gaussian fields



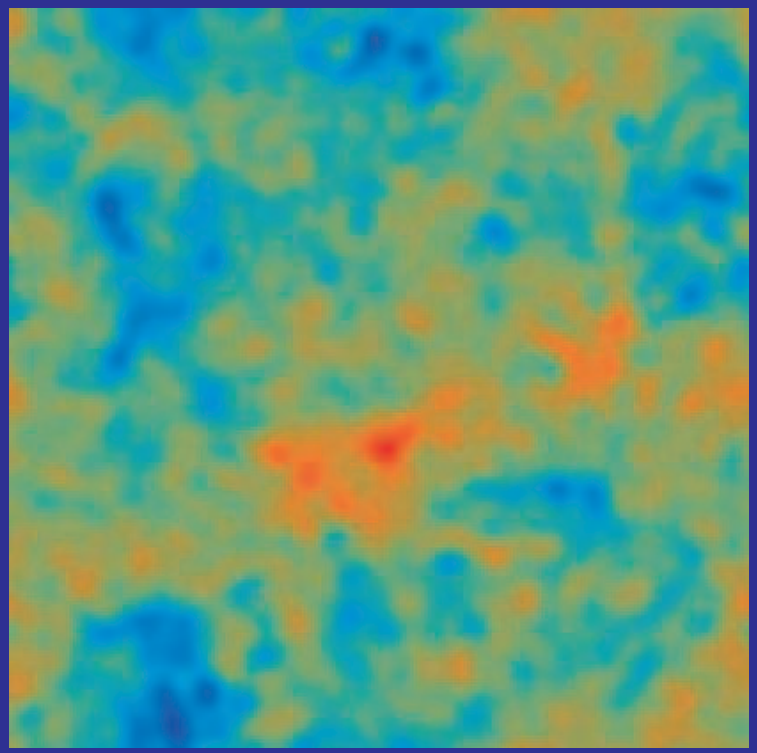
Smith, Hu & Kaplinghat (2004)

Reconstruction from Polarization

- Lensing B-modes correlated to the orignal E-modes in a specific way
- Correlation of E and B allows for a reconstruction of the lens
- Reference experiment of 4' beam, 1μK' noise and 100 deg^2



Original Mass Map

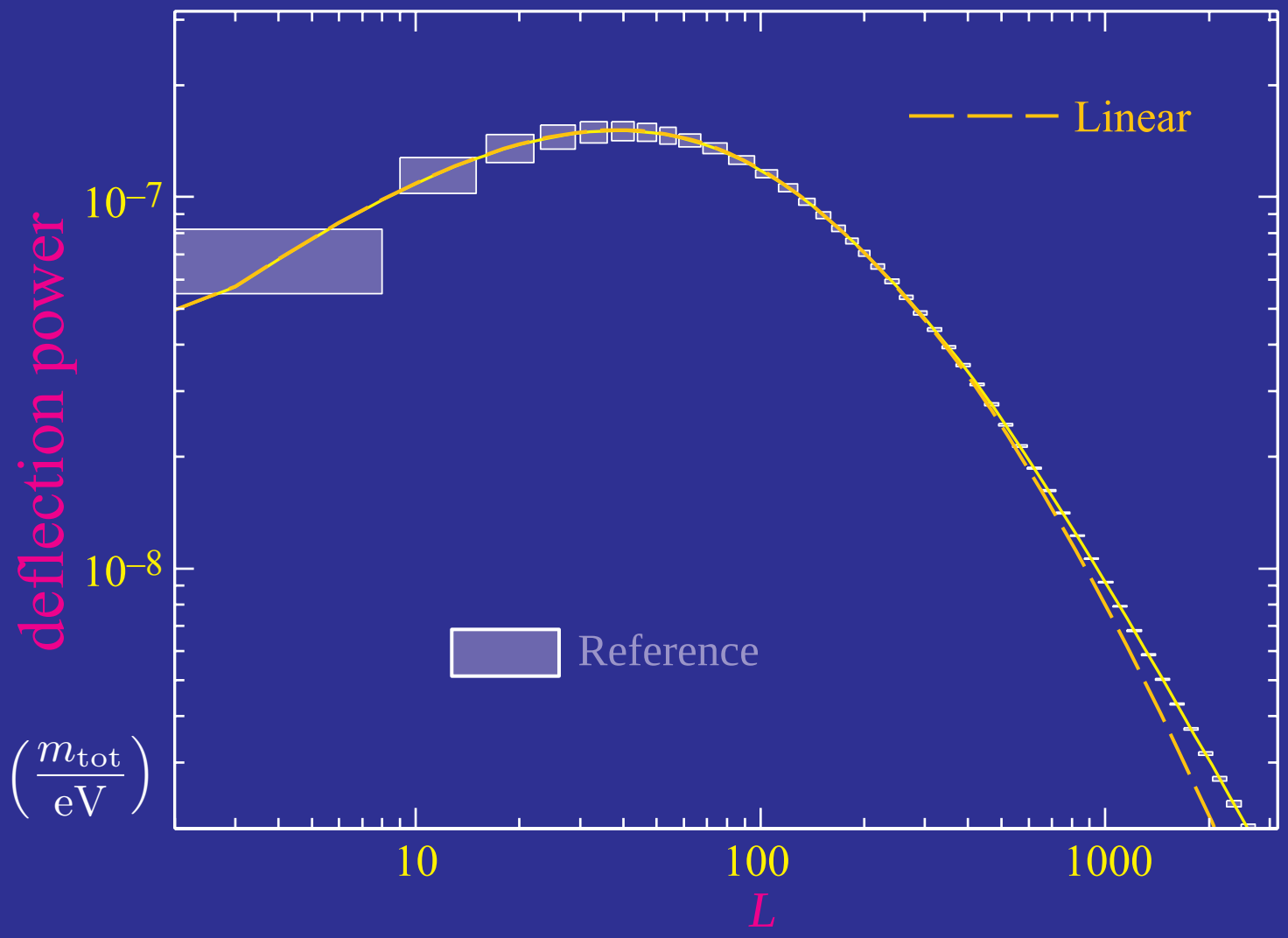


Reconstructed Mass Map

Hu & Okamoto (2001) [iterative improvement Hirata & Seljak (2003)]

Matter Power Spectrum

- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime 0.002< k < 0.2 *h*/Mpc



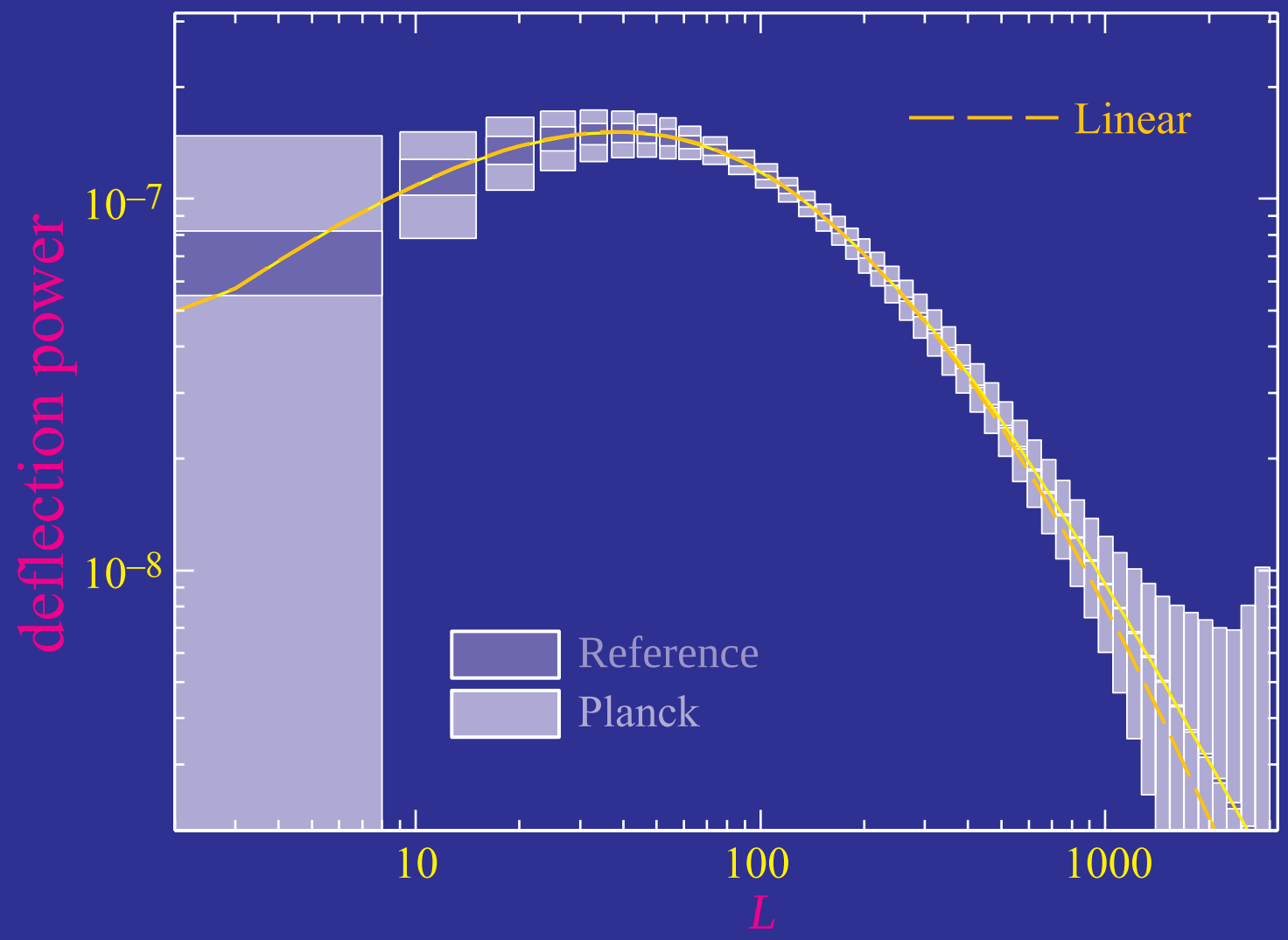
$$\frac{\Delta P}{P} \approx -0.6 \left(\frac{m_{\rm tot}}{\rm eV}\right)$$

Hu & Okamoto (2001) [parameter forecasts: Kaplighat et al 2003]

$\sigma(w)$~0.06

Matter Power Spectrum

- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime 0.002< k < 0.2 *h*/Mpc



Hu & Okamoto (2001)

$\sigma(w)$~0.06; 0.14

Summary

- CMB polarization generated by scattering alone and hence provides probes that are well localized in time and space
- Polarization carries a direction and hence can separate linear density and gravitational wave perturbations [E vs. B modes]
- Early reionization detected by WMAP provides a new window not only on the first generation of structure but also on gravitational waves and statistical anomalies on large scales
- Acoustic polarization detected by DASI eventually can provide exceedingly precise measurements of the initial power spectrum and any features that might exist in the decade of the peaks
- Lensing of the acoustic polarization provides a means of reconstructing the mass distribution and hence constrain the neutrino mass and the dark energy