Toward a Parameterized Post Friedmann Description of Cosmic Acceleration

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Cosmic Acceleration

- Cosmic acceleration, like the cosmological constant, can either be viewed as arising from
  - Missing, or dark energy, with $w \equiv \frac{\bar{p}}{\bar{\rho}} < -\frac{1}{3}$
  - Modification of gravity on large scales
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• Proof of principle models for both exist: quintessence, k-essence; DGP braneworld acceleration, $f(R)$ modified action

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- Compelling models for either explanation lacking

- Dark energy parameterized description on small scales: smooth component with a \( w(z) \) that completely defines expansion history

- Parameterized description of modified gravity acceleration?

- Previous ad-hoc attempts violate basic principles like energy-momentum conservation
Outline

• Constraints imposed by energy-momentum conservation on linear metric fluctuations around an FRW background with a given expansion history

• “Parameterized Post Friedmann” (PPF) description of remaining degrees of freedom
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• DGP braneworld acceleration example
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Outline

- Constraints imposed by energy-momentum conservation on linear metric fluctuations around an FRW background with a given expansion history
- “Parameterized Post Friedmann” (PPF) description of remaining degrees of freedom
- DGP braneworld acceleration example
- $f(R)$ modified action example
- Collaborators:
  - Dragan Huterer
  - Iggy Sawicki
  - Yong-Seon Song
  - Kendrick Smith
PPF Framework
PPF Description

- Parallel treatment of parameterized dark energy beyond the quintessence scalar field

- Demand that the model satisfies (Hu 1998)
  - Given Background Expansion
  - Gauge invariance
  - Energy-Momentum Conservation
PPF Description

- Parallel treatment of parameterized dark energy beyond the a quintessence scalar field
- Demand that the model satisfies \((\text{Hu 1998})\)
  - Given Background Expansion
  - Gauge invariance
  - Energy-Momentum Conservation

and the phenomenologically desirable property that the dark energy does not cluster with the dark matter → sound horizon

- Larger scales: energy-momentum conservation requires conservation of the comoving curvature \((\text{Bardeen 1980})\)
- Smaller scales: dark energy spatial perturbations negligible and observable phenomena depend on expansion history only
PPF Description

- Implement with a parameterized model: the sound speed in the dark energy rest frame. Quintessence sound speed $c_s = 1$

- Parameterization later shown to describe k-essence with modified scalar field kinetic term (Garriga & Mukhanov 1999)

$$\mathcal{L} = F(X, \phi) \quad X = -\frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi$$

with a sound speed

$$c_s^2 = \frac{\partial F/\partial X}{2(\partial^2 F/\partial X^2)X + (\partial F/\partial X)}$$

- Beyond single scalar fields: parameterize multiple internal degrees of freedom to allow an evolution across $w = -1$ phantom divide (Hu 2004)
PPF Description

- Modified gravity models of acceleration
- Demand that the model satisfies
  - Given Background Expansion History
  - Bianchi Identities / (FRW) Metric Theory
  - Energy-Momentum Conservation

and that modifications reach quasi-static Newtonian limit on small scales: time derivatives neglected compared with spatial gradients

- PPF description can be used to test general relativity on cosmological scales and distinguish modified gravity from smooth dark energy
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- PPF description can be used to test general relativity on cosmological scales and distinguish modified gravity from smooth dark energy

- In addition non-linear effects must bring gravity stably back to general relativity on small scales to satisfy solar system tests. Beyond the scope of this talk.
PPF Description

- On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)

- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d \ln a$ (Bardeen 1980)
• On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)

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• Gauge transformation to Newtonian gauge

$$ds^2 = -(1 + 2 \Psi)dt^2 + a^2(1 + 2 \Phi)dx^2$$

yields (Hu & Eisenstein 1999)

$$\Phi'' - \Psi' - \frac{H''}{H'} \Phi' - \left( \frac{H'}{H} - \frac{H''}{H'} \right) \Psi = 0$$

• Modified gravity theory supplies the closure relationship between $\Phi$ and $\Psi$ and expansion history $H = \dot{a}/a$ supplies rest.
A Worked Example:
DGP Braneworld Acceleration
A Worked Example: DGP Gravity

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

\[ S = \int d^5 x \sqrt{-g} \left[ \frac{(5) R}{2\kappa^2} + \delta(\chi) \left( \frac{(4) \tilde{R}}{2\mu^2} + \mathcal{L}_m \right) \right] \]

with crossover scale \( r_c = \kappa^2 / 2\mu^2 \)

- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001; see also Sawicki’s talk)

- Matter still minimally coupled and conserved

- Satisfies PFF requirements
A Worked Example: DGP Gravity

- Braneworld acceleration \( (Dvali, Gabadadze & Porrati 2000) \)
  \[
  S = \int d^5x \sqrt{-g} \left[ \frac{R^{(5)}}{2\kappa^2} + \delta(\chi) \left( \frac{R^{(4)}}{2\mu^2} + \mathcal{L}_m \right) \right]
  \]
  with crossover scale \( r_c = \kappa^2 / 2\mu^2 \)

- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk \( (Deffayet 2001; \text{see also Sawicki's talk}) \)

- Matter still minimally coupled and conserved

- Satisfies PFF requirements

- Dominance of Weyl tensor anisotropy over other components and matter sets closure relation during self acceleration \( \Psi \rightarrow \Phi \)

- Transition to this limit leads to enhancement of potential decay and large angle CMB anisotropy
DGP Expansion History

- Matching the DGP expansion history to a dark energy model with the same expansion history
- Effective equation of state $w(z)$ \([w_0 \sim -0.85, \ w_a \sim 0.35]\)

![Graph of $w(z)$ vs. redshift $z$](image)
DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$

Song, Sawicki & Hu (2006)
Standard Ruler

- Standard ruler used to measure the angular diameter distance to recombination ($z \sim 1100$; currently 2%) or any redshift for which acoustic phenomena observable.

**Formula:**

$$\frac{l(l+1)C_l}{2\pi} (\mu K^2)$$

**Graph:**

- Angular scale vs. physical scale.

**Source:** WMAP: Bennett et al (2003)
DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$
- Mismatch to CMB absolute distance $D_A$ requires curvature

Song, Sawicki & Hu (2006)
DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0 D_A$
- Mismatch to CMB absolute distance $D_A$ requires curvature
- Difference in expansion history appears as a change in local distances or the Hubble constant: $H_0$

Song, Sawicki & Hu (2006)
DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$
- Mismatch to CMB absolute distance $D_A$ requires curvature
- Compromise between SN and $H_0$ measures

Song, Sawicki & Hu (2006)
• **DGP modified gravity** is in tension with distance measures alone: CMB & SNe distances **cannot be jointly satisfied in a flat universe**

• Even fitting out curvature, **Hubble constant is too high** for Key Project measurement (and baryon oscillations)

• Joint maximization leads to a **poorer fit** even with extra curvature parameter

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**Graphical Data**

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<th>WMAP3yr+SNLS</th>
<th>WMAP3yr+SNLS+KP</th>
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<td>$\Omega_m$</td>
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<td>$H_0$</td>
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<td>82</td>
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</tbody>
</table>

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Song, Sawicki & Hu (2006) [also Fairbairn & Goobar 2005, Maartens & Majerotto 2006]
Prospects for Percent $H_0$

- Improving the distance ladder (~3-5%, Riess 2005; Macri et al 2006)
- Water maser proper motion, acceleration (~3%, VLBA Condon & Lo 2005; ~1% SKA, Greenhill 2004)

- Gravity wave sirens (~2% - 3x Adv. LIGO + GRB sat, Dalal et al 2006)
- Combination of dark energy tests: e.g. SNIa relative distances: $H_0D(z)$ and baryon acoustic oscillations $D(z)$
Flat Universe Precision

- Planck acoustic peaks, 1% $H_0$, SNAP SNe to $z = 1.7$ in a flat universe

Forecasts for CMB+$H_0$

- To complement CMB observations with $\Omega_m h^2$ to 1%, an $H_0$ of \(~1\%\) enables constant $w$ measurement to \(~2\%\) in a flat universe.

Planck: $\sigma(\ln\Omega_m h^2)=0.009$

Hu (2004)
Dark Energy Equation of State

- Marginalizing curvature degrades 68% CL area by 4.8
- CMB lensing information from SPTpol (~3% B-mode power) fully restores constraints

DGP Potential Evolution

- **Difference in expansion history** gives excess decay of grav. potential on **subhorizon scales** (Lue, Scoccimarro, Starkmann 2004; Koyama & Maartins 2005)
- **Energy-momentum conservation** and dominance of **Weyl anisotropy** leads to further decay

Sawicki, Song & Hu 2006
DGP Example

- Excess decay leads to enhanced large angle CMB anisotropy
- Requires either breaking of initial scale invariance or missing physics beyond Weyl tensor at $\sim r_c/10$ to be compatible with observations

Song, Sawicki & Hu (2006)
A Worked Example: $f(R)$ Modified Action Acceleration
A Worked Example: f(R) Gravity

- Modify the Einstein-Hilbert action (Starobinsky 1980; Carroll et al 2004)

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{2\mu^2} + \mathcal{L}_m \right] \]

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved

- Satisfies PPF requirements
A Worked Example: f(R) Gravity

- Modify the Einstein-Hilbert action (Starobinsky 1980; Carroll et al 2004)

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{2\mu^2} + L_m \right] \]

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved

- Satisfies PPF requirements

- Expansion history parameterization: Friedmann equation becomes

\[ H^2 - f_R(HH' + H^2) + \frac{1}{6} f + H^2 f_{RR} R' = \frac{\mu^2 \rho}{3} \]

where \( f_R = \frac{df}{dR}, \ f_{RR} = \frac{d^2f}{dR^2} \)

- For any desired \( H \), solve a 2nd order diffeq to find \( f(R) \)
Each expansion history, matched by dark energy model \([w(z), \Omega_{DE}, H_0]\) corresponds to a family of \(f(R)\) models due to its 4th order nature.

Parameterized by \(B \propto f_{RR} = \frac{d^2f}{dR^2}\) evaluated at \(z=0\).

(a) \(w=-1, \Omega_{DE}=0.76\)

(b) \(w=-0.9, \Omega_{DE}=0.73\)
Deviation Parameter

- Express the 4th order nature of equations as a deviation parameter
  \[ \Phi'' - \Psi' - \frac{H''}{H'} \Phi' - \left( \frac{H'}{H} - \frac{H''}{H'} \right) \Psi = \left( \frac{k}{aH} \right)^2 B \epsilon \]

- Einstein equation become a second order equation for \( \epsilon \)

- In high redshift, high curvature \( R \) limit this is
  \[ \epsilon'' + \left( \frac{7}{2} + 4 \frac{B'}{B} \right) \epsilon' + \frac{2}{B} \epsilon = \frac{1}{B} \times \text{metric sources} \]

- \( R \rightarrow \infty, B \rightarrow 0 \) and for \( B < 0 \) short time-scale tachyonic instability appears making previous models not cosmologically viable

- \( f(R) = -M^{2+2n}/R^n \)
Potential Growth

- On the stable $B>0$ branch, potential evolution reverses from decay to growth as a function of scale $B^{1/2}(k/aH)$.
- Newton constant $G$ rescaled by $1+f_R$ leading to different density and potential growth functions.
- On small scales, quasistatic equilibrium reached in linear theory with $\Psi=-2\Phi$ requiring non-linear effects restore PPN expectations.

Song, Hu & Sawicki (2006)
• Reduction of potential decay can eliminate the ISW effect at the quadrupole for $B_0 \sim 3/2$

• In conjunction with a change in the initial power spectrum can also bring the total quadrupole closer in ensemble average to the observed quadrupole

Song, Hu & Sawicki (2006)
ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: $\Delta T/T = 2\Delta \Phi$
- Effect from potential hills and wells cancel on small scales
ISW Effect

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• Contraction of spatial metric doubles the effect: $\frac{\Delta T}{T} = 2\Delta\Phi$

• Effect from potential hills and wells cancel on small scales
ISW Quadrupole

- **Reduction of large angle anisotropy** for $B_0 \sim 1$ for same expansion history and distances as $\Lambda$CDM
- **Well-tested small scale anisotropy unchanged**

\[
\frac{l(l+1)C_l}{2\pi} \propto B_0 \left( \frac{\Omega_0}{1000} \right)^{3/2}
\]

Song, Hu & Sawicki (2006)
Galaxy-ISW (Anti)Correlation

- Change in potential growth reduces galaxy-ISW correlation and for high $B_0>1$ predicts anticorrelation
- Reported positive detections place upper limit of $B_0<1$

Song, Hu & Sawicki (2006)
Linear Power Spectrum

- Linear real space power spectrum enhanced on small scales
- Degeneracy with galaxy bias and lack of non-linear predictions leave constraints from shape of power spectrum
Redshift Space Distortion

- Relationship between velocity and density field given by continuity with modified growth rate
- Redshift space power spectrum further distorted by Kaiser effect
PPF Functions
PPF Description

- On superhorizon scales, metric evolution given by conservation

\[ \Phi'' - \Psi' - \frac{H''}{H'} \Phi' - \left( \frac{H'}{H} - \frac{H''}{H'} \right) \Psi = 0 \]

requiring a closure relation between the metric fluctuations

\[ \Psi = -f_1(a) \Phi \]

- Below parameterized transition scale, modified Poisson equation

\[ k^2 \left( \frac{\Phi - \Psi}{2} \right) = \frac{1}{2} f_3(a) \mu^2 a^2 \rho \Delta \]

with a potentially different closure relation

\[ \Psi = -f_2(a) \Phi \]

and the usual quasistatic conservation laws

\[ \Delta' = \left( \frac{k}{aH} \right)^2 Hq, \quad Hq' = \Psi, \]
Summary

- Parameterized description of acceleration: background expansion history \( w(z) \) supplemented by
  - Transition scale where dark energy becomes smooth
  - Transition scale where modified gravity switches from Friedmann dynamics to quasistatic Newtonian dynamics (and a further non-linear transition to GR)
  - Consistent with energy-momentum conservation and metric theory

- Test explanations of acceleration in absence of compelling models

- Expansion history alone tests specific models: e.g. DGP by \( H_0 \)

- PPF description of DGP shows disfavored enhanced ISW effect if Weyl anisotropy dominates during self-acceleration

- PPF description of \( f(R) \) shows previous models unstable but stable models do exist and are testable with linear phenomena