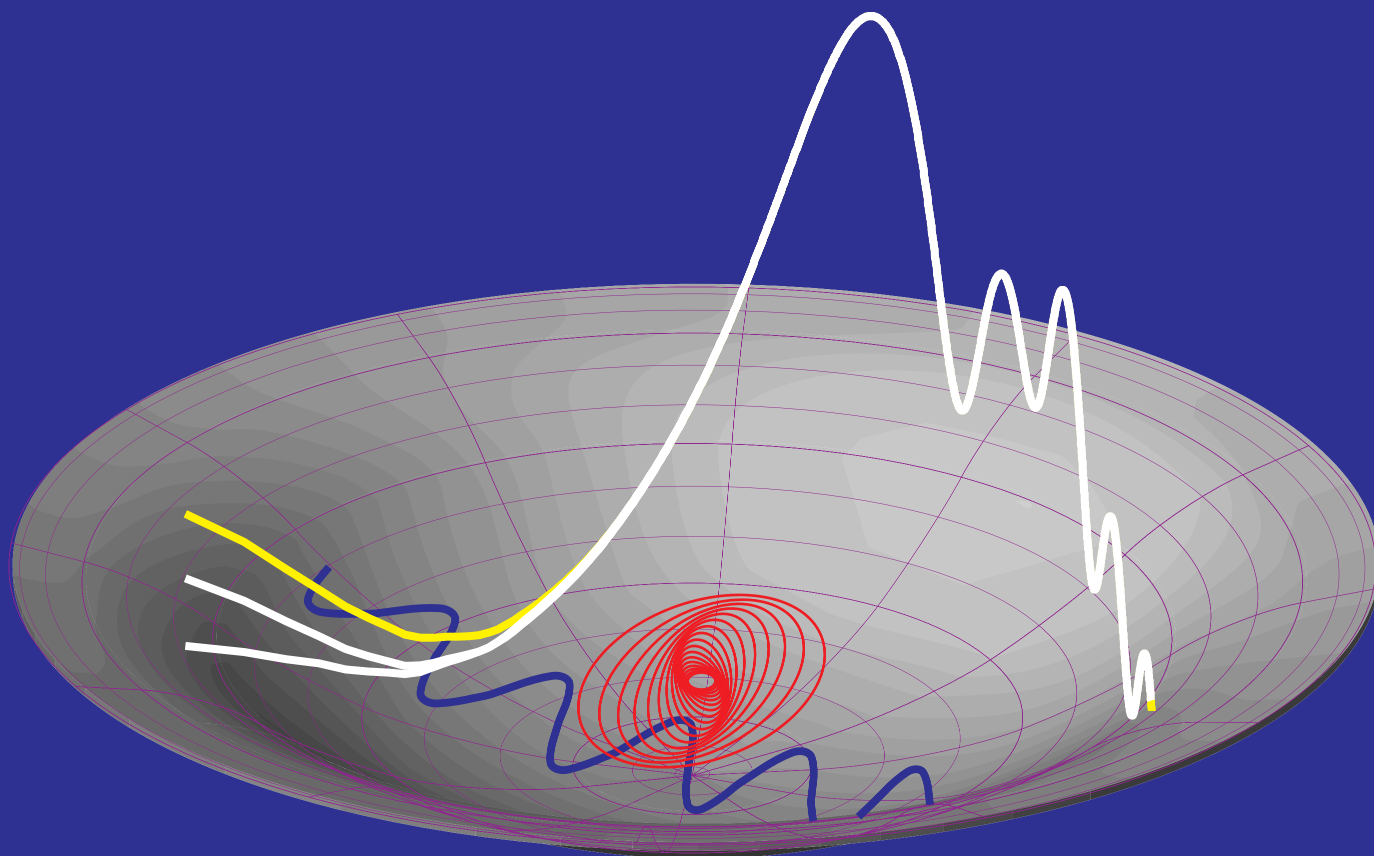
Toward a Parameterized Post Friedmann



Description of Cosmic Acceleration

Wayne Hu

NYU, November 2006

Cosmic Acceleration

- Cosmic acceleration, like the cosmological constant, can either be viewed as arising from

  Missing, or dark energy, with $w \equiv \bar{p}/\bar{\rho} < -1/3$

  Modification of gravity on large scales

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- Proof of principle models for both exist: quintessence, k-essence; DGP braneworld acceleration, $f(R)$ modified action
- Compelling models for either explanation lacking
- Dark energy parameterized description on small scales: smooth component with a $w(z)$ that completely defines expansion history
- Parameterized description of modified gravity acceleration?
- Previous ad-hoc attempts violate basic principles like energy-momentum conservation

Outline

- Constraints imposed by energy-momentum conservation
  on linear metric fluctuations
  around an FRW background
  with a given expansion history
- "Parameterized Post Friedmann" (PPF) description of remaining degrees of freedom

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- "Parameterized Post Friedmann" (PPF) description of remaining degrees of freedom
- DGP braneworld acceleration example
- $f(R)$ modified action example
- Collaborators:
  - Dragan Huterer
  - Iggy Sawicki
  - Yong-Seon Song
  - Kendrick Smith

PPF Framework

PPF Description

- Parallel treatment of parameterized dark energy beyond a quintessence scalar field
- Demand that the model satisfies (Hu 1998)

  Given Background Expansion

  Gauge Invariance

  Energy-Momentum Conservation

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  Energy-Momentum Conservation

  and the phenomenogically desirable property that the dark energy does not cluster with the dark matter $\rightarrow$ sound horizon
- Larger scales: energy-momentum conservation requires conservation of the comoving curvature (Bardeen 1980)
- Smaller scales: dark energy spatial perturbations negligible and observable phenomena depend on expansion history only

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$$\mathcal{L} = F(X, \phi) \qquad X = -\frac{1}{2}\nabla^\mu \phi \nabla_\mu \phi$$

with a sound speed

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- Beyond single scalar fields: parameterize multiple internal degrees of freedom to allow an evolution across $w = -1$ phantom divide (Hu 2004)

PPF Description

- Modified gravity models of acceleration
- Demand that the model satisfies

  Given Background Expansion History

  Bianchi Identities / (FRW) Metric Theory

  Energy-Momentum Conservation

  and that modifications reach quasi-static Newtonian limit on small scales: time derivatives neglected compared with spatial gradients
- PPF description can be used to test general relativity on cosmological scales and distinguish modified gravity from smooth dark energy

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  and that modifications reach quasi-static Newtonian limit on small scales: time derivatives neglected compared with spatial gradients
- PPF description can be used to test general relativity on cosmological scales and distinguish modified gravity from smooth dark energy
- In addition non-linear effects must bring gravity stably back to general relativity on small scales to satisfy solar system tests. Beyond the scope of this talk.

PPF Description

- On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)
- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d\ln a$ (Bardeen 1980)

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- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d\ln a$ (Bardeen 1980)
- Gauge transformation to Newtonian gauge

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2$$

yields (Hu & Eisenstein 1999)

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = 0$$

- Modified gravity theory supplies the closure relationship between Φ and Ψ and expansion history $H = \dot{a}/a$ supplies rest.

A Worked Example:
DGP Braneworld Acceleration

A Worked Example: DGP Gravity

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

$$S = \int d^5x \sqrt{-g} \left[\frac{{}^{(5)}R}{2\kappa^2} + \delta(\chi) \left(\frac{{}^{(4)}R}{2\mu^2} + \mathcal{L}_m \right) \right]$$

  with crossover scale $r_c = \kappa^2 / 2\mu^2$

- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001; see also Sawicki's talk)
- Matter still minimally coupled and conserved
- Satisfies PFF requirements

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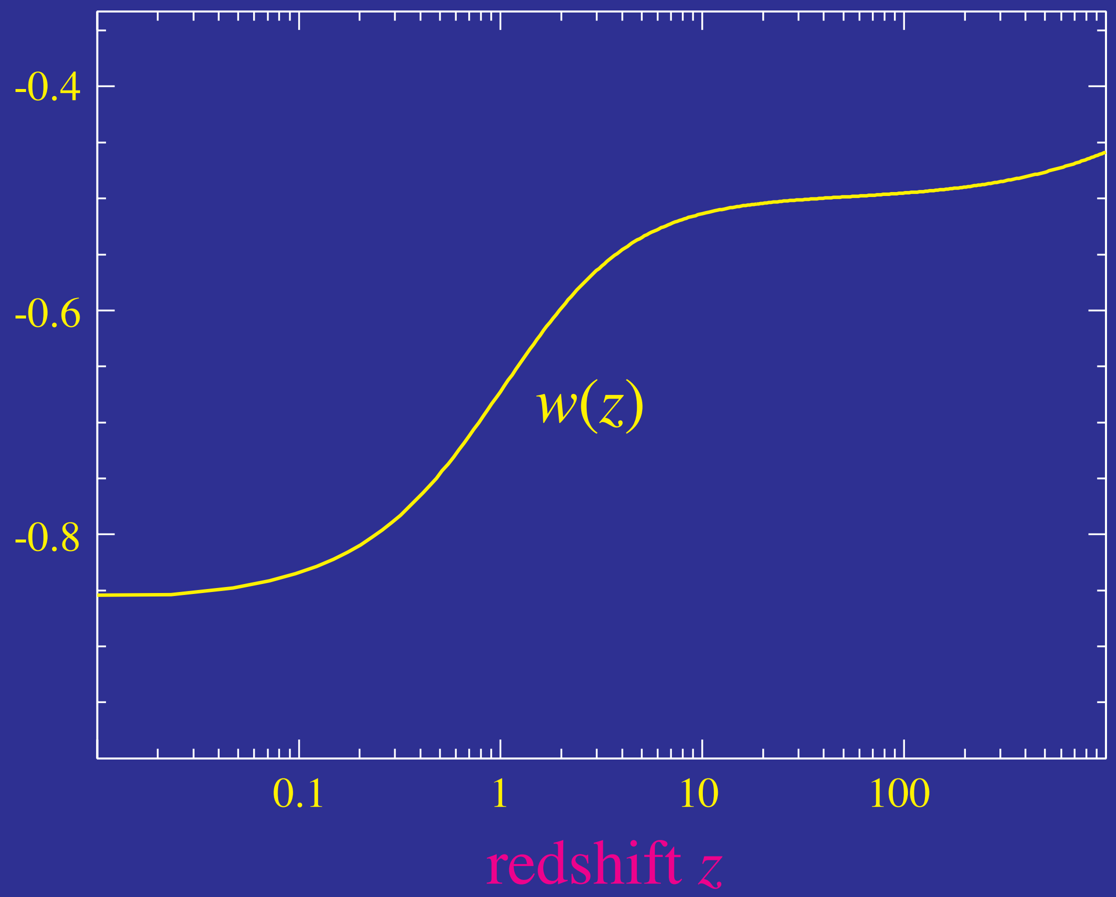
- Matter still minimally coupled and conserved
- Satisfies PFF requirements
- Dominance of Weyl tensor anisotropy over other components and matter sets closure relation during self acceleration $\Psi \to \Phi$
- Transition to this limit leads to enhancement of potential decay and large angle CMB anisotropy

DGP Expansion History

DGP Expansion History

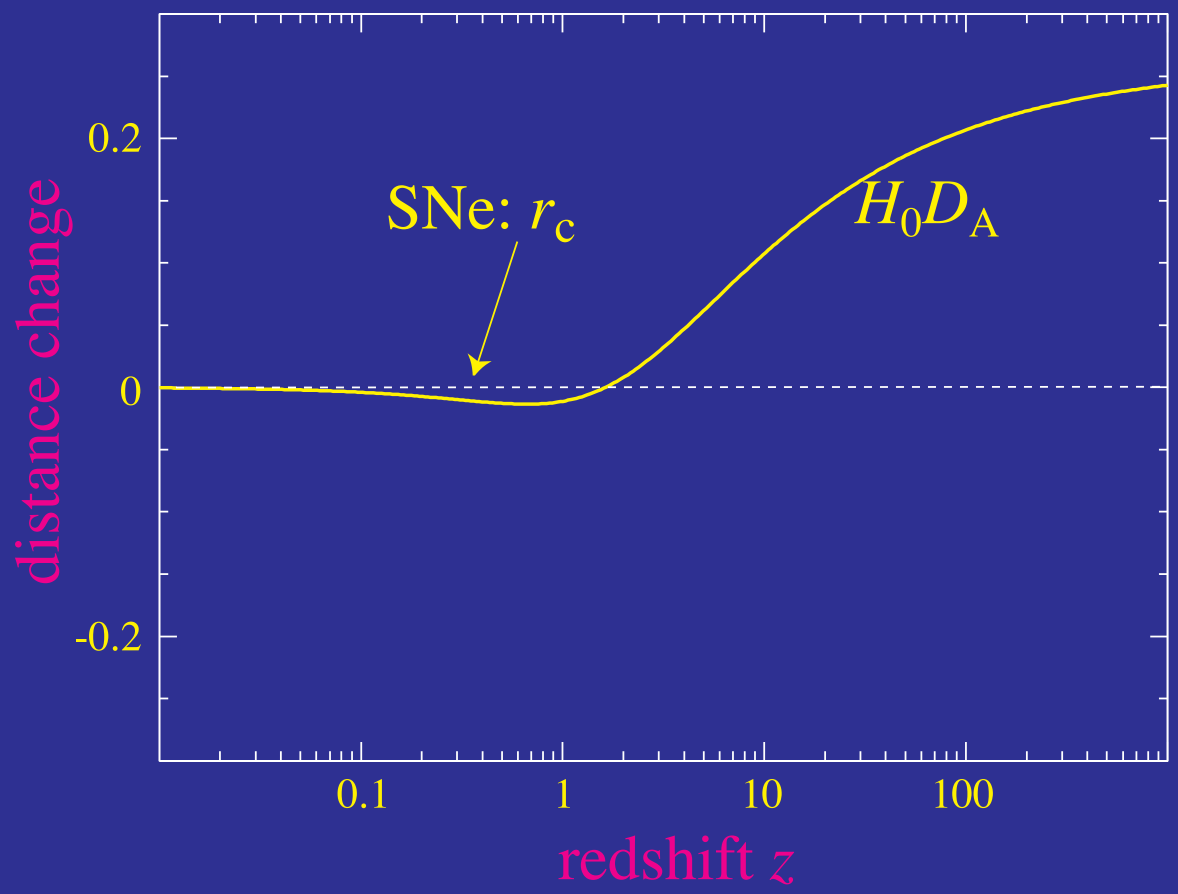
- Matching the DGP expansion history to a dark energy model with the same expansion history
- Effective equation of state $w(z)$ [w_0~-0.85, w_a~0.35]



Song, Sawicki & Hu (2006)

DGP Expansion History

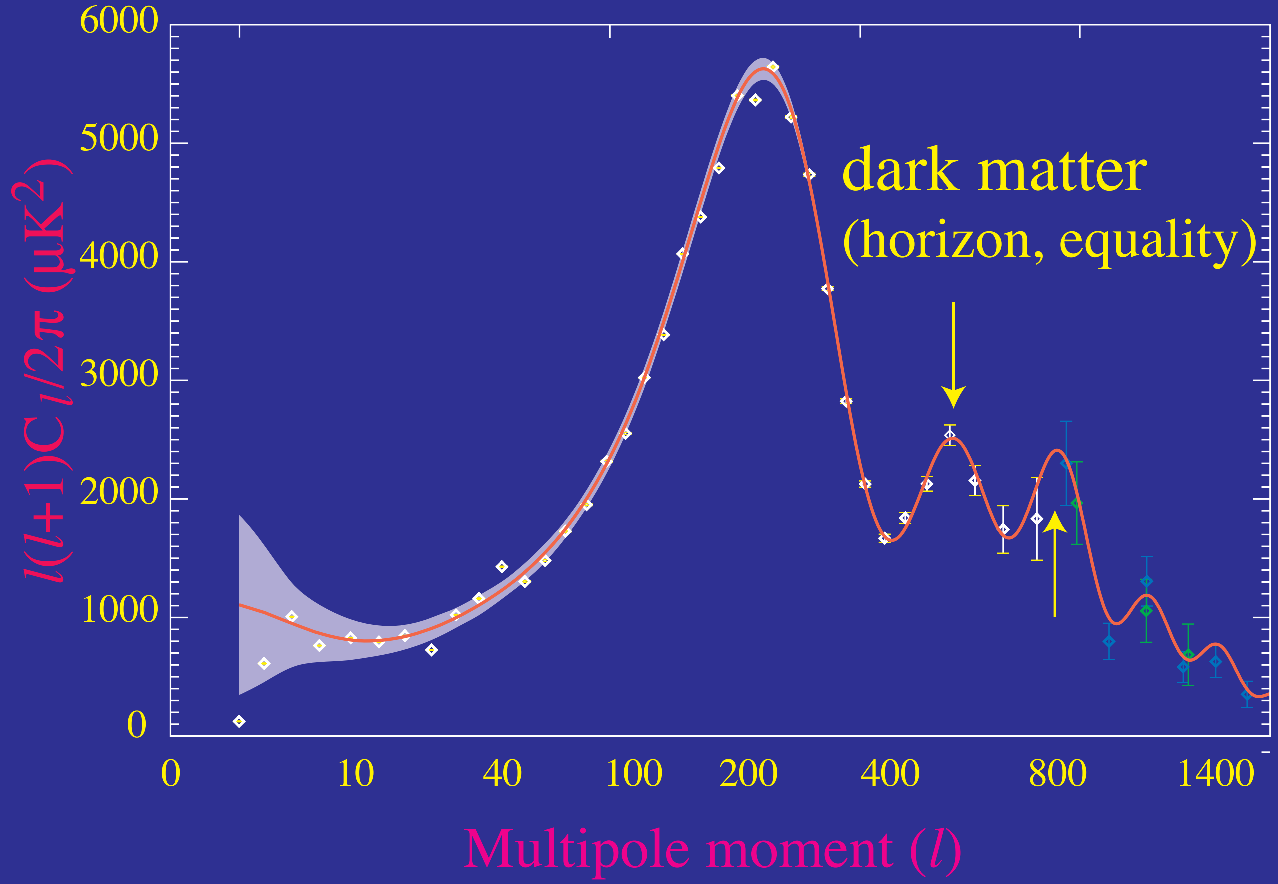
- Crossover scale r_c fit to SN relative distance from z=0: $H_0 D_A$



Song, Sawicki & Hu (2006)

Leveraging the CMB

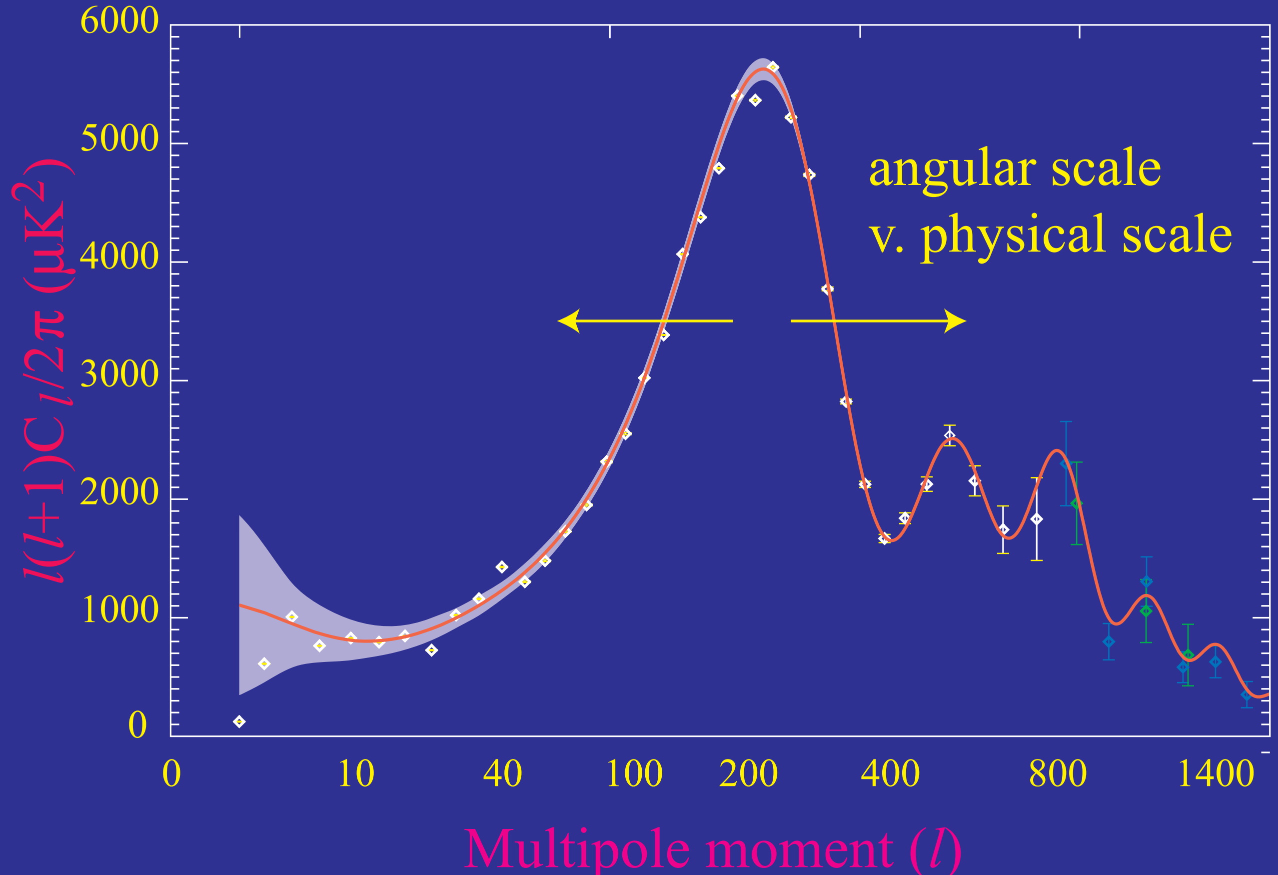
- Relative heights of the first 3 peaks calibrates sound horizon and matter radiation equality horizon: measures $\Omega_m h^2$ currently 8%



leading source of error!

Standard Ruler

- Standard ruler used to measure the angular diameter distance to recombination (z~1100; currently 2%) or any redshift for which acoustic phenomena observable



WMAP: Bennett et al (2003)

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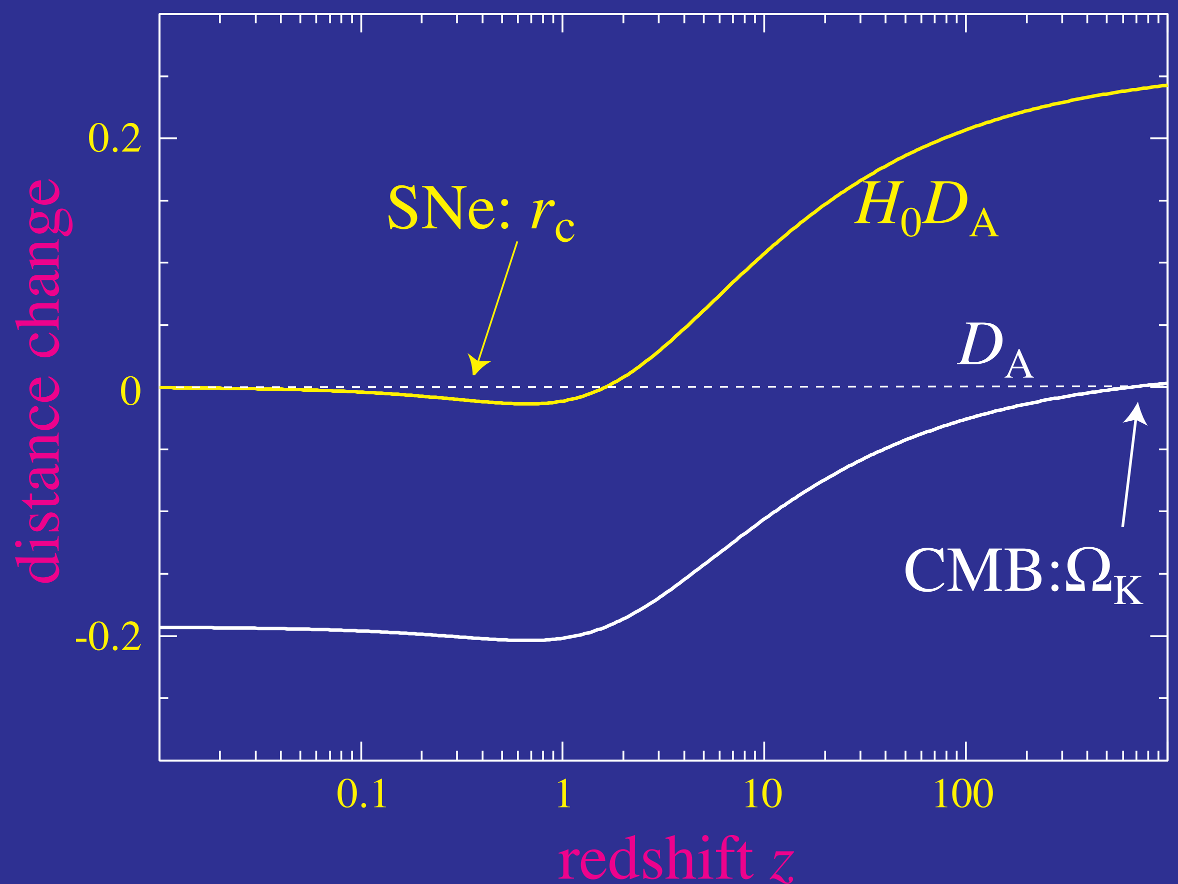
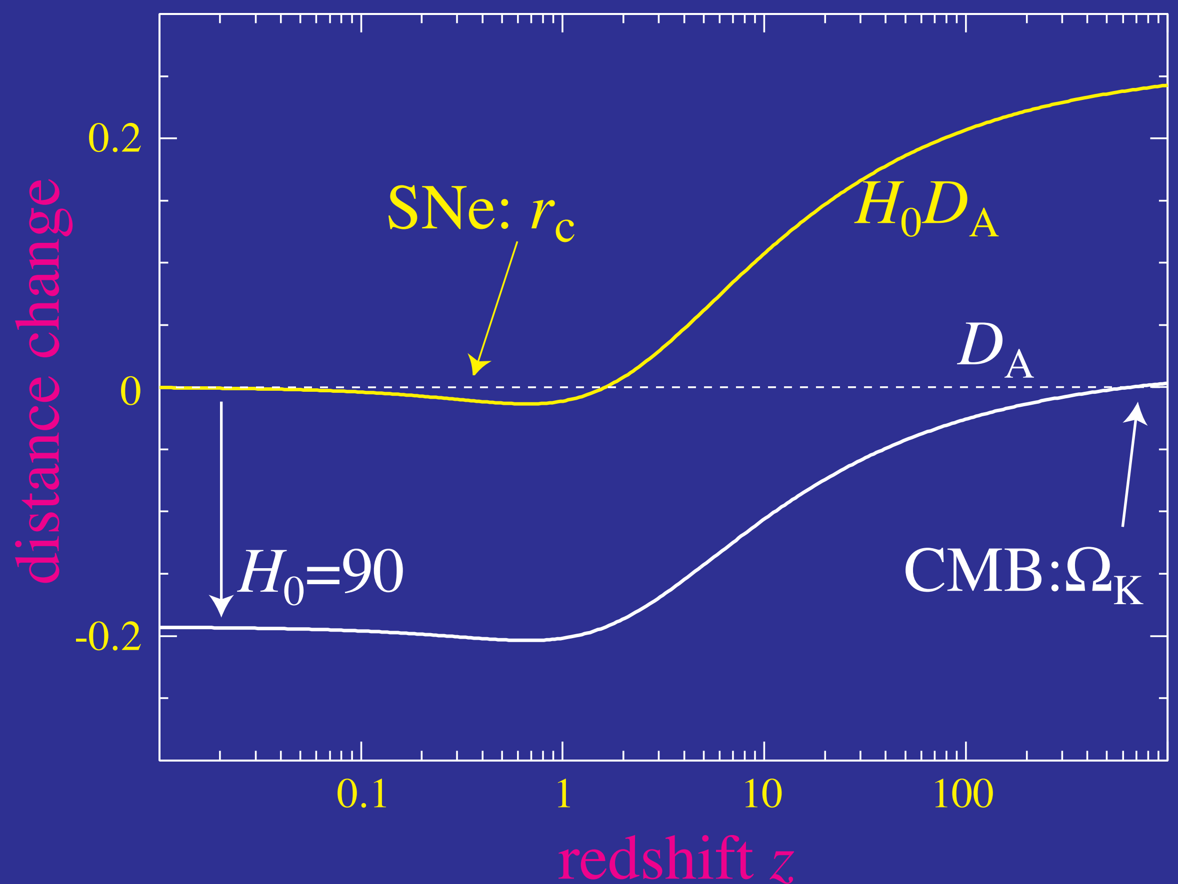


Song, Sawicki & Hu (2006)

DGP Expansion History

- Crossover scale r_c fit to SN relative distance from z=0: H_0D_A
- Mismatch to CMB absolute distance D_A requires curvature
- Difference in expansion history appears as a change in local distances or the Hubble constant: H_0



Song, Sawicki & Hu (2006)

DGP Expansion History

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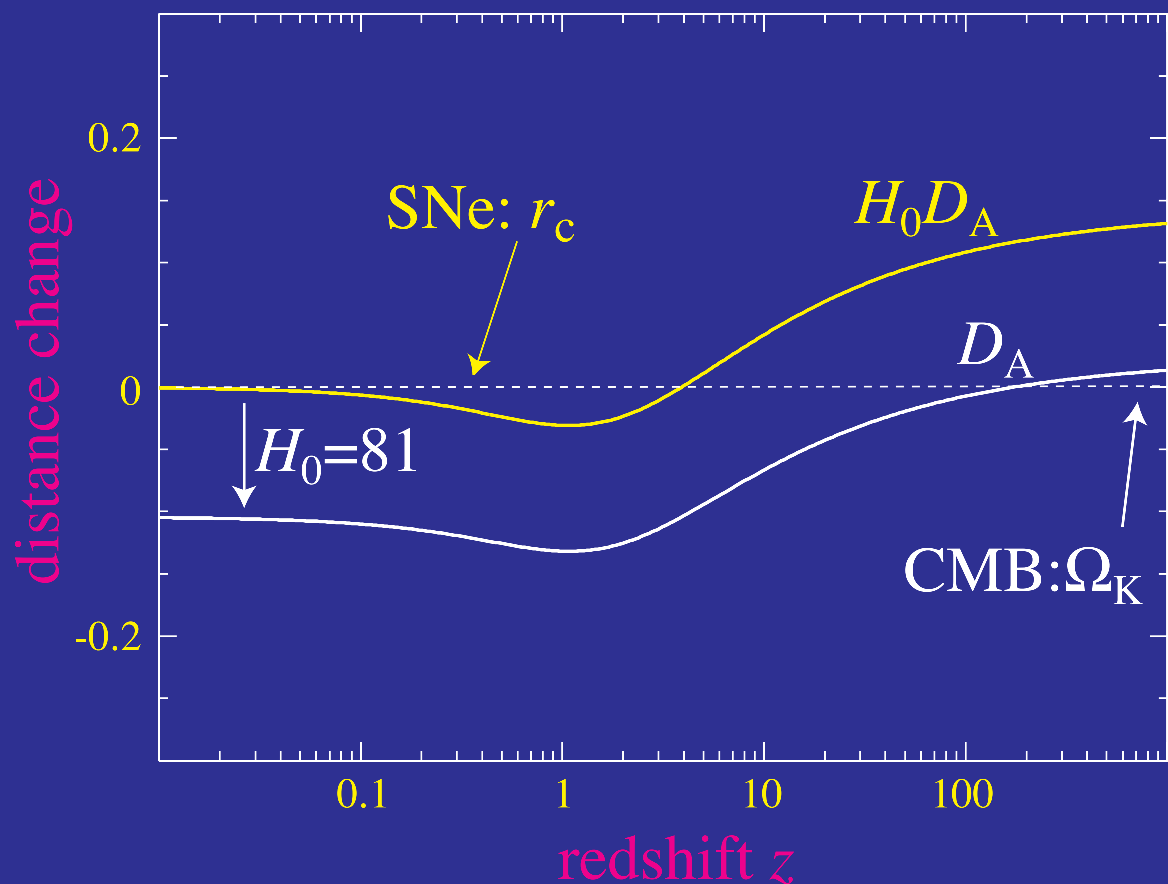
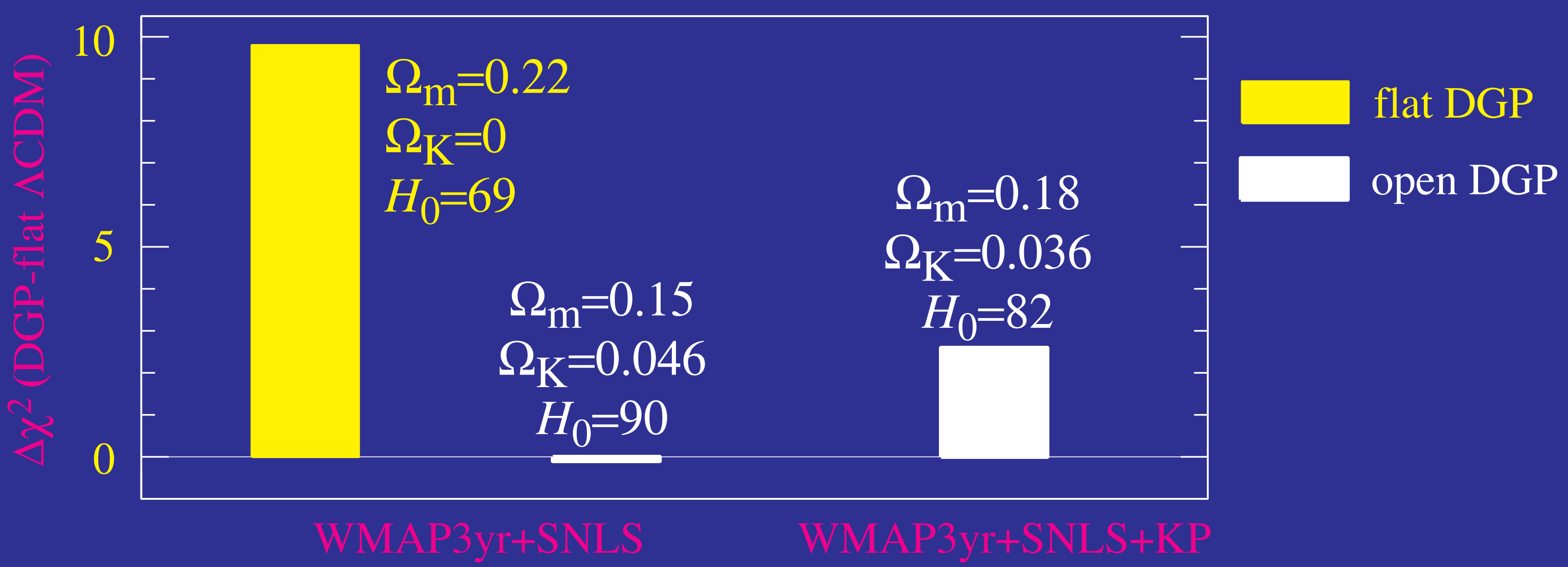
- Mismatch to CMB absolute distance D_A requires curvature
- Compromise between SN and H_0 measures



Song, Sawicki & Hu (2006)

DGP Example

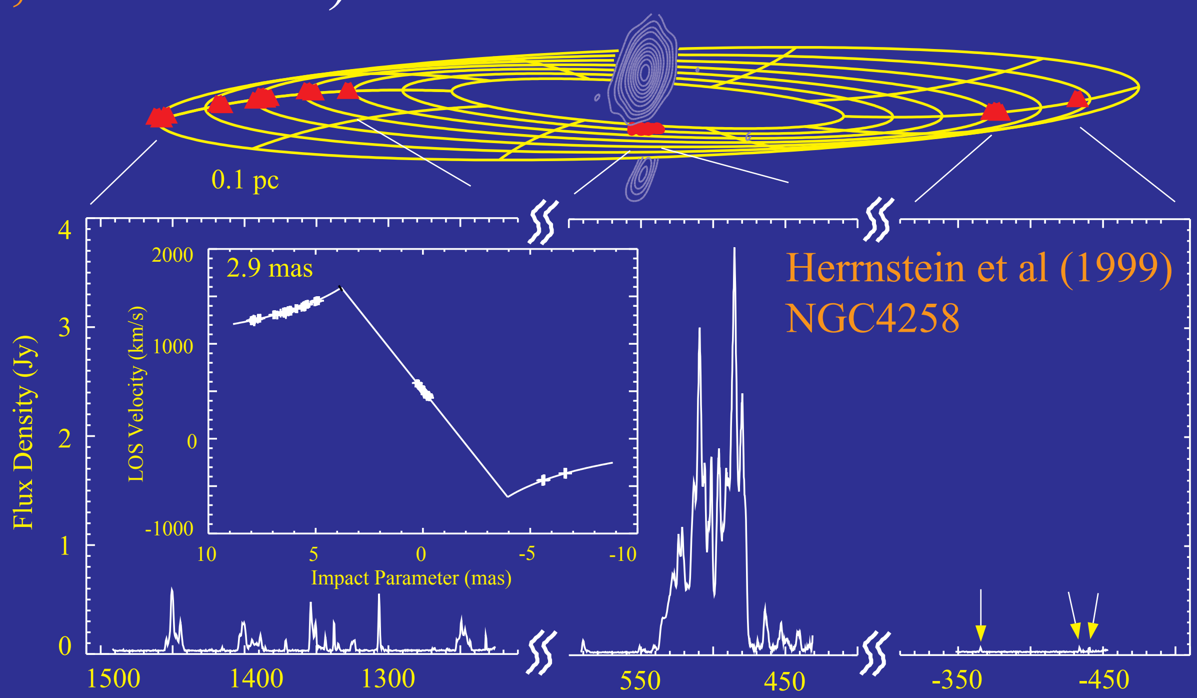
- DGP modified gravity is in tension with distance measures alone: CMB & SNe distances cannot be jointly satisfied in a flat universe
- Even fitting out curvature, Hubble constant is too high for Key Project measurement (and baryon oscillations)
- Joint maximization leads to a poorer fit even with extra curvature parameter



Song, Sawicki & Hu (2006) [also Fairbairn & Goobar 2005, Maartens & Majerotto 2006]

Prospects for Percent H_0

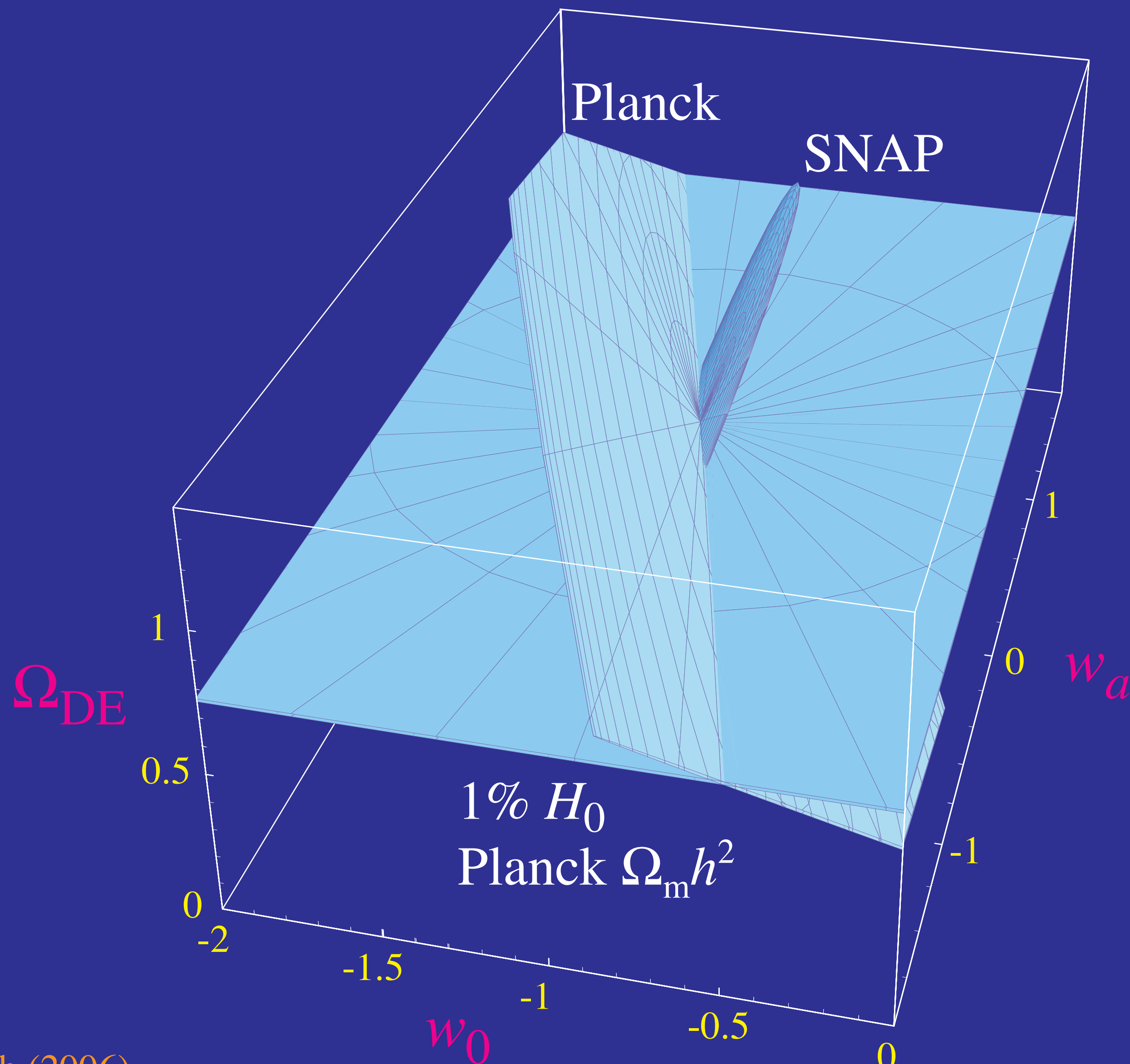
- Improving the distance ladder (~3-5%, Riess 2005; Macri et al 2006)
- Water maser proper motion, acceleration (~3%, VLBA Condon & Lo 2005; ~1% SKA, Greenhill 2004)



- Gravity wave sirens (~2% - 3x Adv. LIGO + GRB sat, Dalal et al 2006)
- Combination of dark energy tests: e.g. SNIa relative distances: $H_0 D(z)$ and baryon acoustic oscillations $D(z)$

Flat Universe Precision

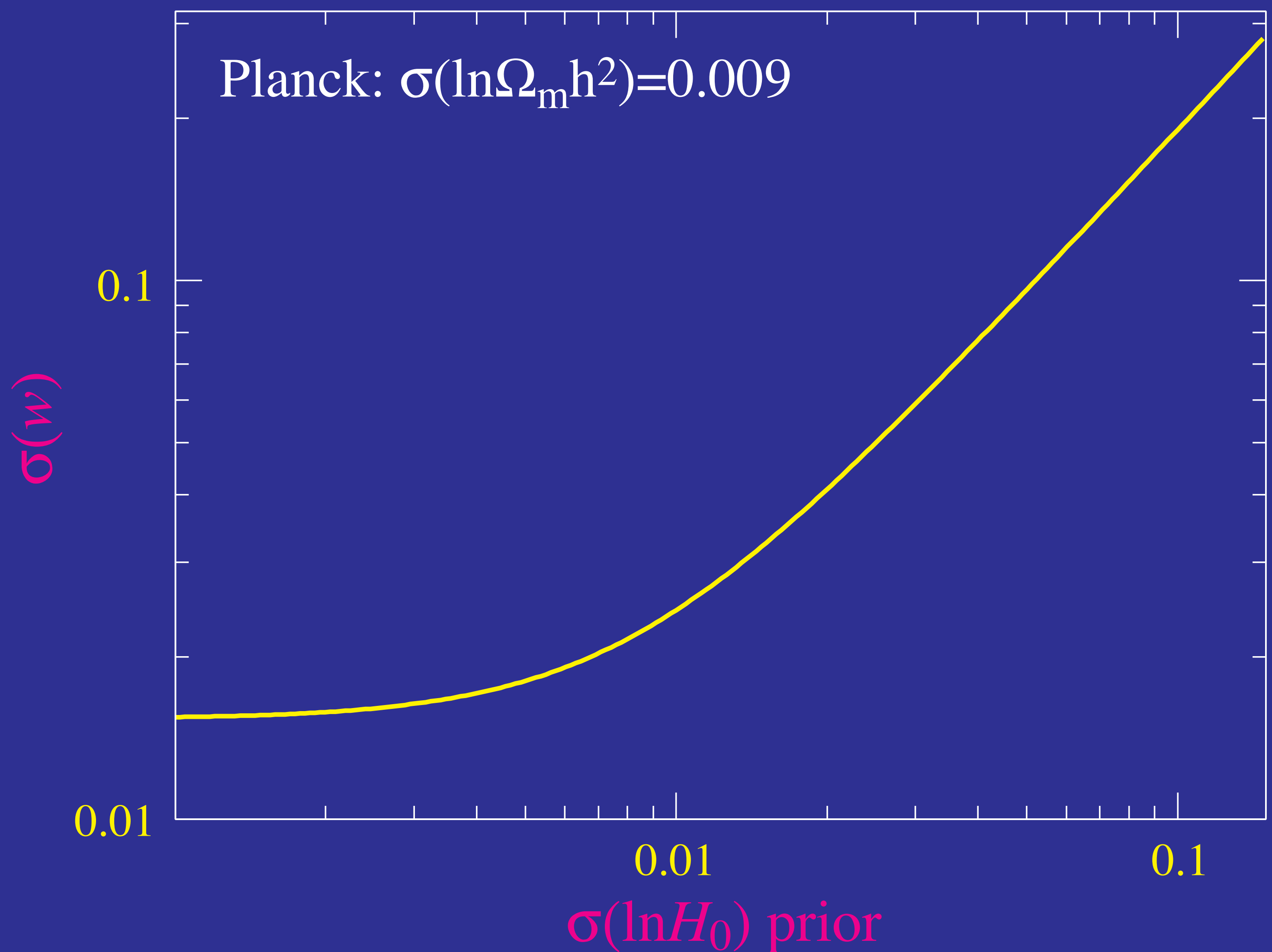
- Planck acoustic peaks, 1% H_0, SNAP SNe to z=1.7 in a flat universe



Hu, Huterer & Smith (2006)

Forecasts for CMB+H_0

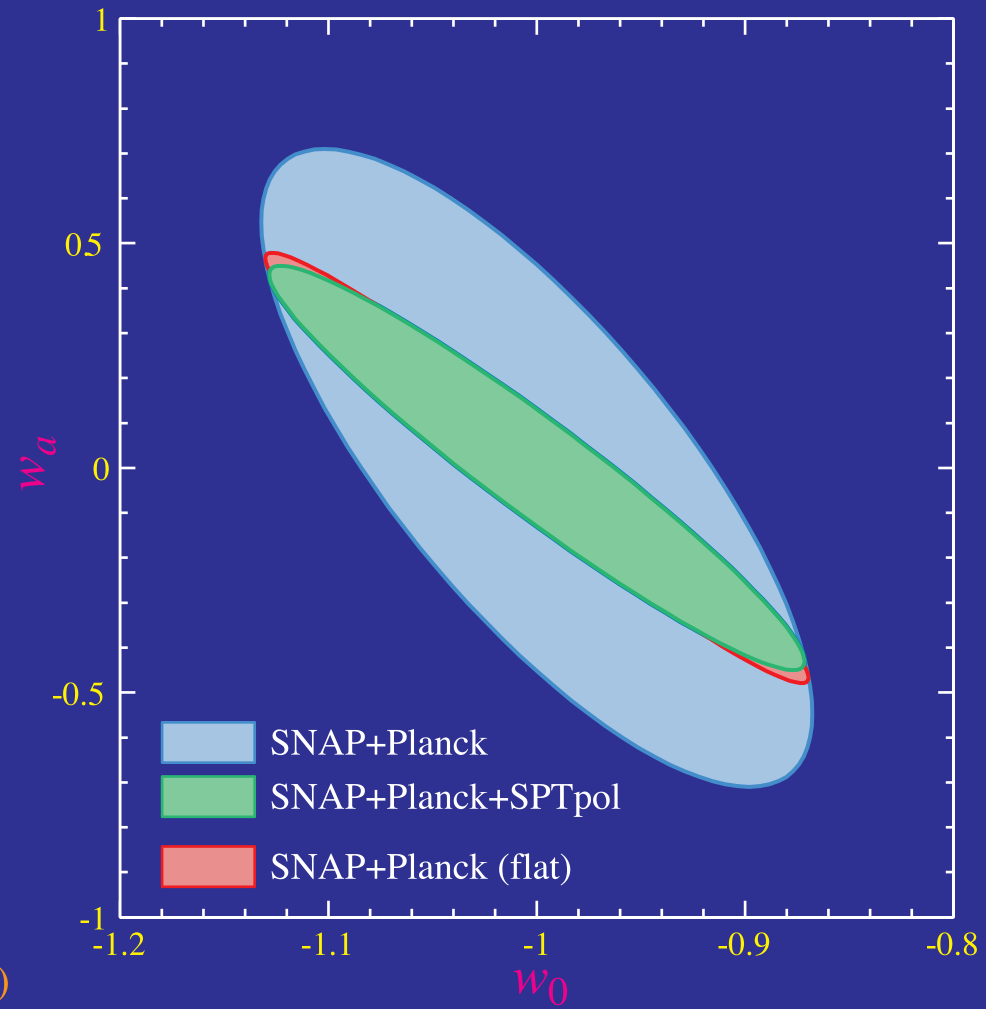
- To complement CMB observations with $\Omega_m h^2$ to 1%, an H_0 of ~1% enables constant w measurement to ~2% in a flat universe



Hu (2004)

Dark Energy Equation of State

- Marginalizing curvature degrades 68% CL area by 4.8
- CMB lensing information from SPTpol (~3% B-mode power) fully restores constraints

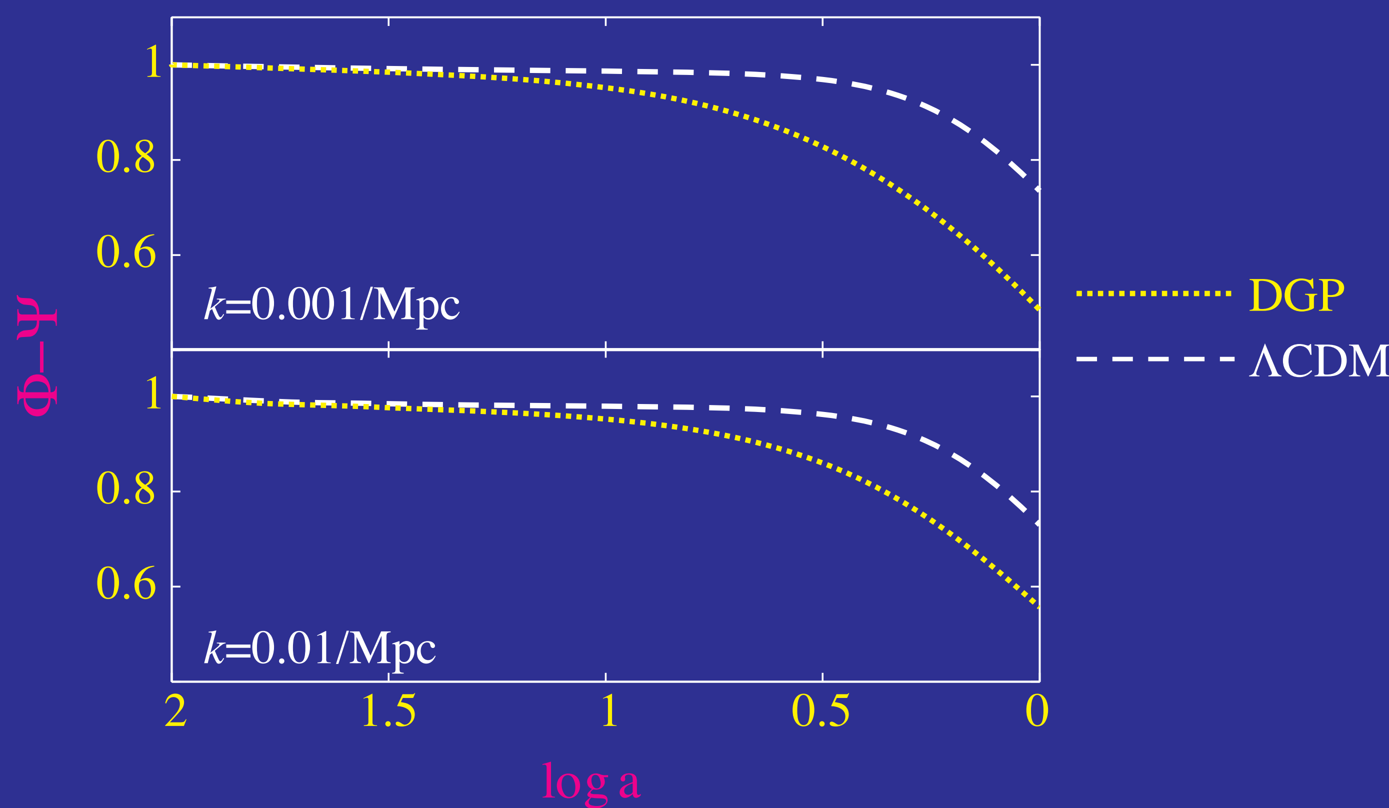


Hu, Huterer & Smith (2006)

DGP Metric Evolution

DGP Potential Evolution

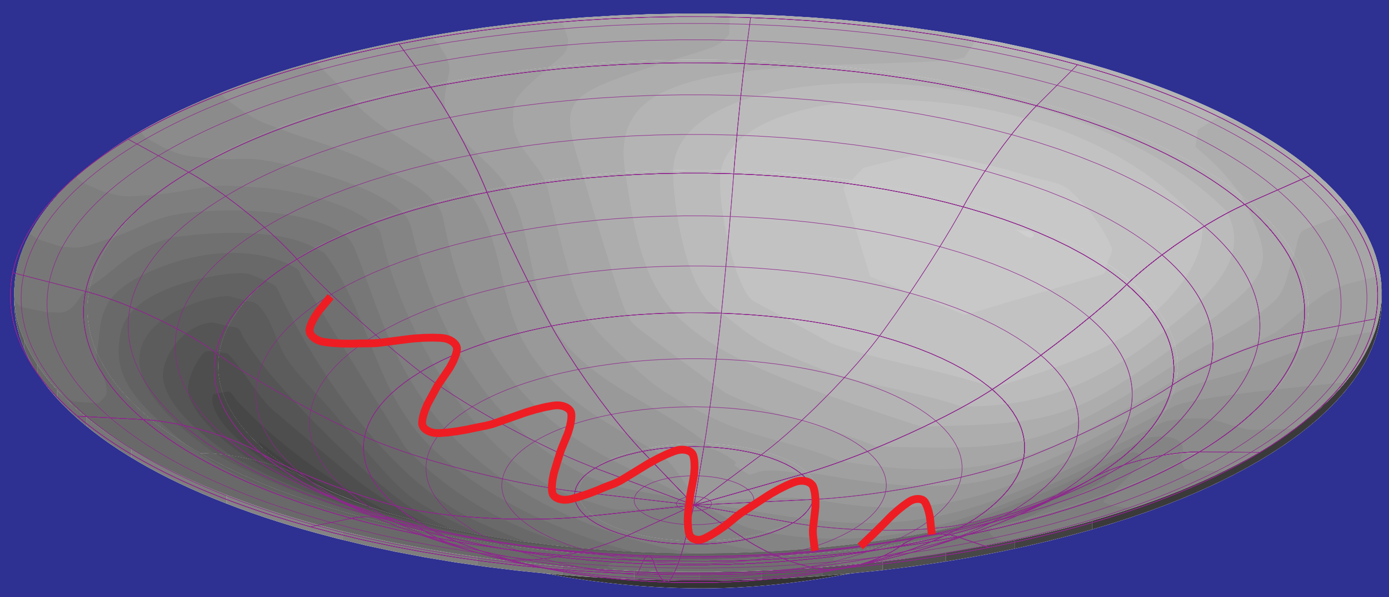
- Difference in expansion history gives excess decay of grav. potential on subhorizon scales (Lue, Scoccimarro, Starkmann 2004; Koyama & Maartins 2005)
- Energy-momentum conservation and dominance of Weyl anisotropy leads to further decay



Sawicki, Song & Hu 2006

Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi - \Psi)$



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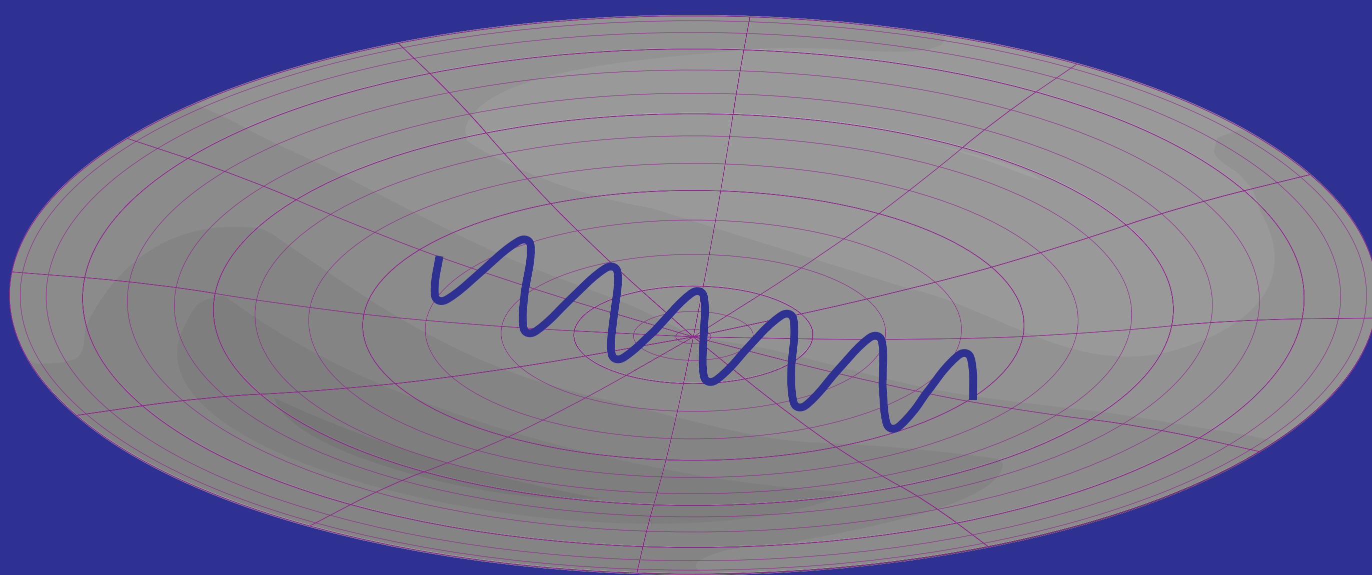
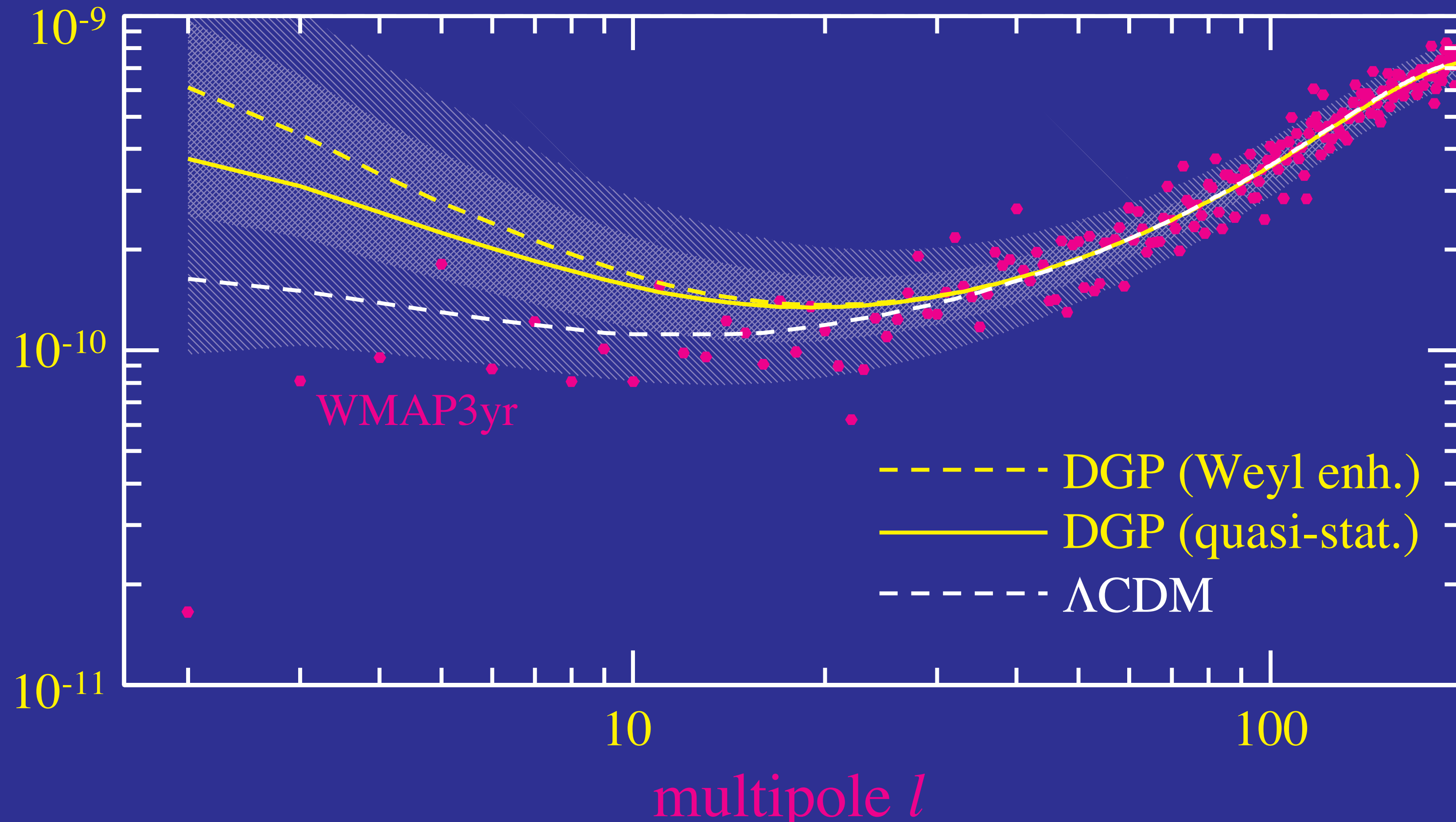


DGP Example

- Excess decay leads to enhanced large angle CMB anisotropy
- Requires either breaking of initial scale invariance or missing physics beyond Weyl tensor at $\sim r_c/10$ to be compatible with observations



Song, Sawicki & Hu (2006)

A Worked Example:
$f(R)$ Modified Action Acceleration

A Worked Example: $f(R)$ Gravity

- Modify the Einstein-Hilbert action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{2\mu^2} + \mathcal{L}_{\rm m} \right]$$

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved
- Satisifies PPF requirements

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- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved
- Satisifies PPF requirements
- Expansion history parameterization: Friedmann equation becomes

$$H^2 - f_R(HH' + H^2) + \frac{1}{6} f + H^2 f_{RR} R' = \frac{\mu^2 \rho}{3}$$

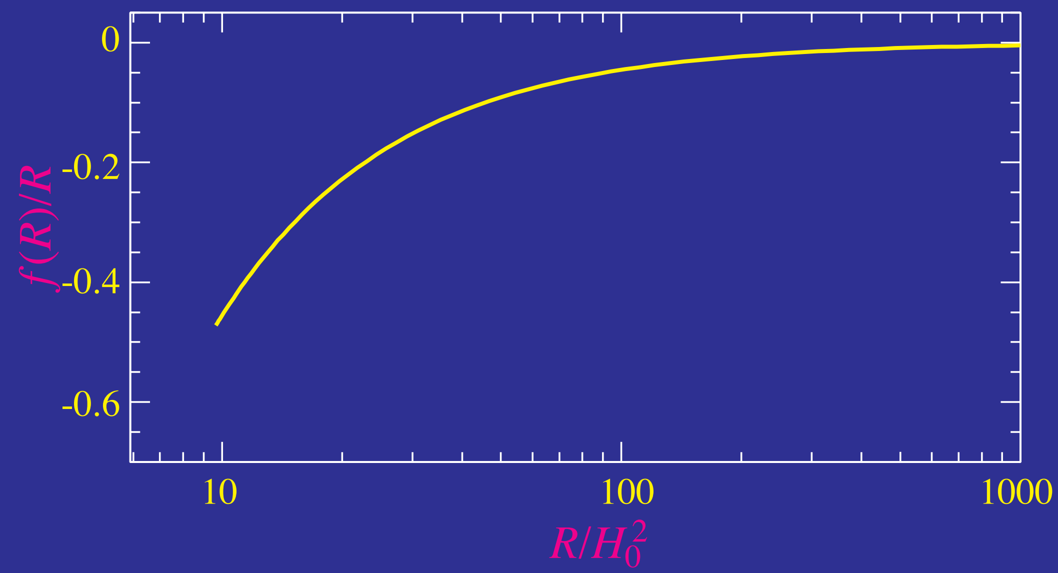
where $f_R = df/dR$, $f_{RR} = d^2 f/dR^2$

- For any desired H, solve a 2nd order diffeq to find $f(R)$

PPF Functions

Expansion History Family of $f(R)$

- Each expansion history, matched by dark energy model $[w(z),\Omega_{\rm DE},H_0]$ corresponds to a family of $f(R)$ models due to its 4th order nature
- Parameterized by $B \propto f_{RR} = {\rm d}^2 f/{\rm d}R^2$ evaluated at z=0



Song, Hu & Sawicki (2006)

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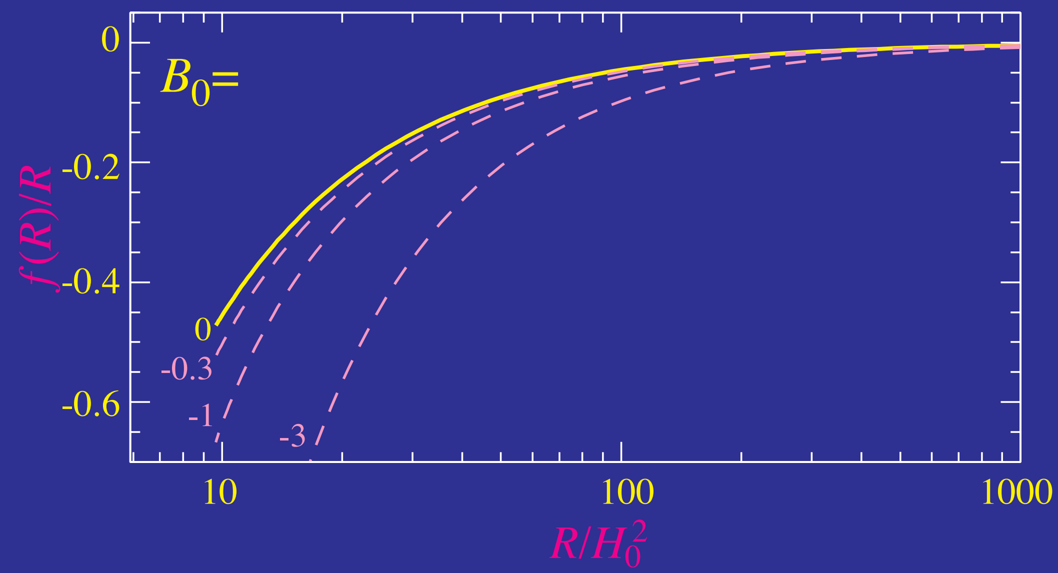

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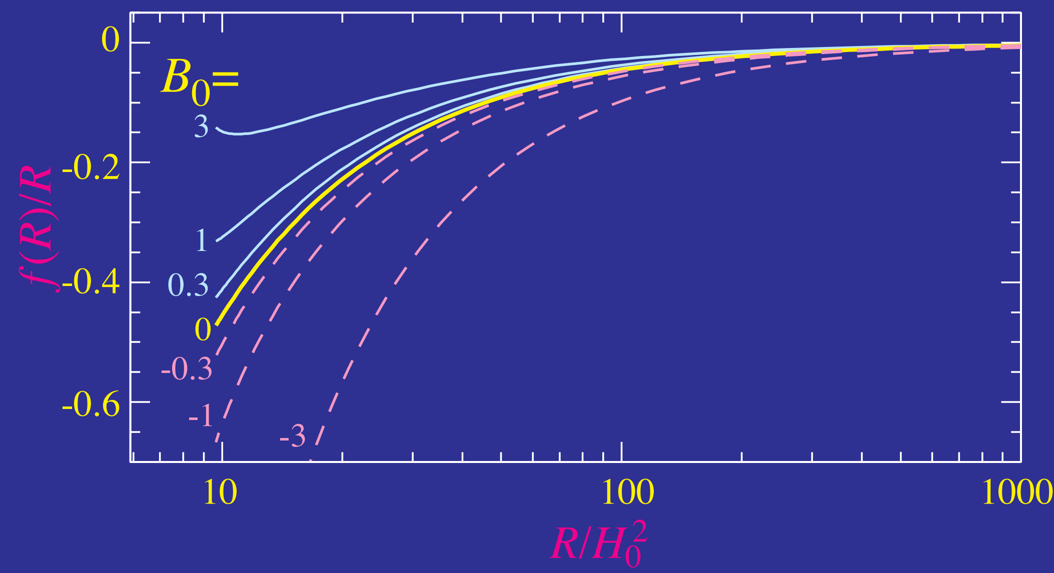

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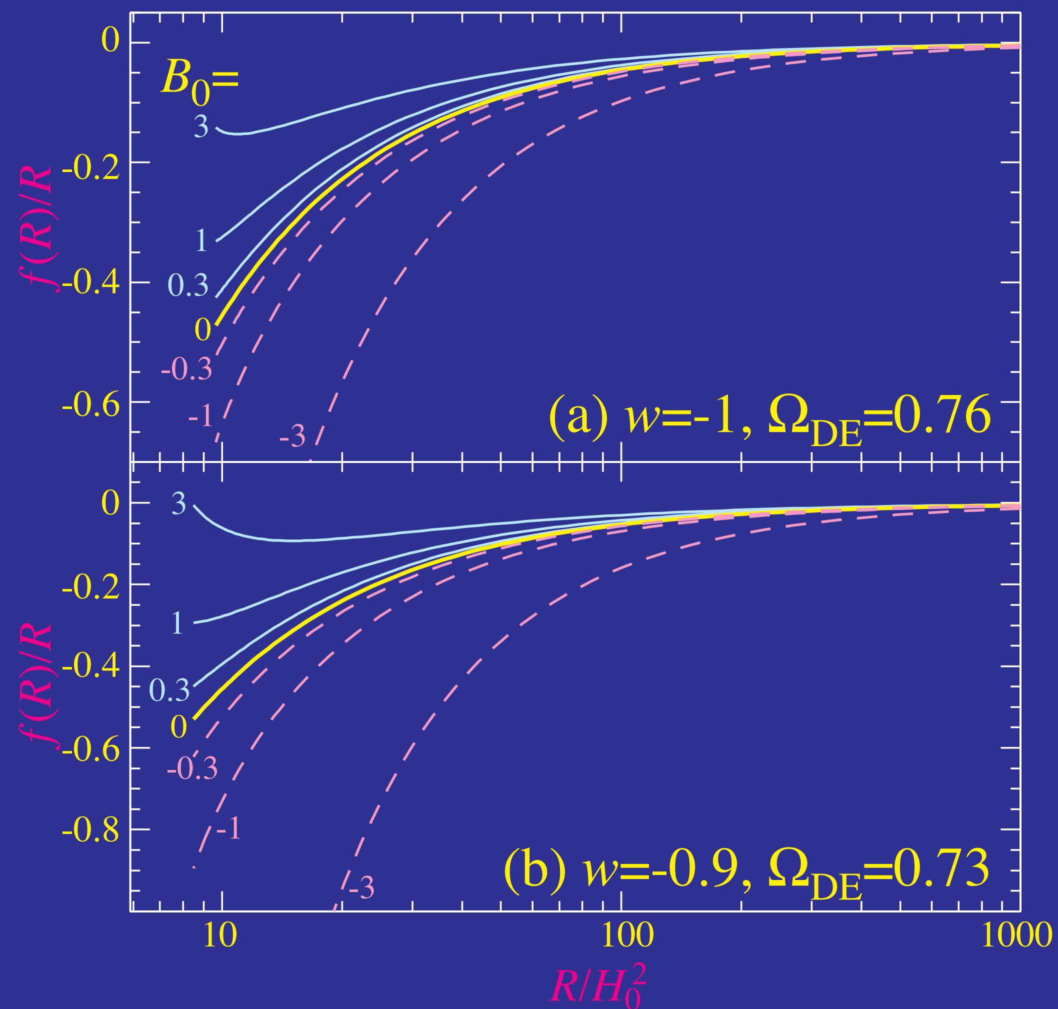

Song, Hu & Sawicki (2006)

$f(R)$ Metric Evolution

Deviation Parameter

- Express the 4th order nature of equations as a deviation parameter

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = \left(\frac{k}{aH}\right)^2 B\epsilon$$

- Einstein equation become a second order equation for ϵ

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- Einstein equation become a second order equation for ϵ
- In high redshift, high curvature R limit this is

$$\epsilon'' + \left(\frac{7}{2} + 4\frac{B'}{B}\right)\epsilon' + \frac{2}{B}\epsilon = \frac{1}{B} \times \text{ metric sources}$$

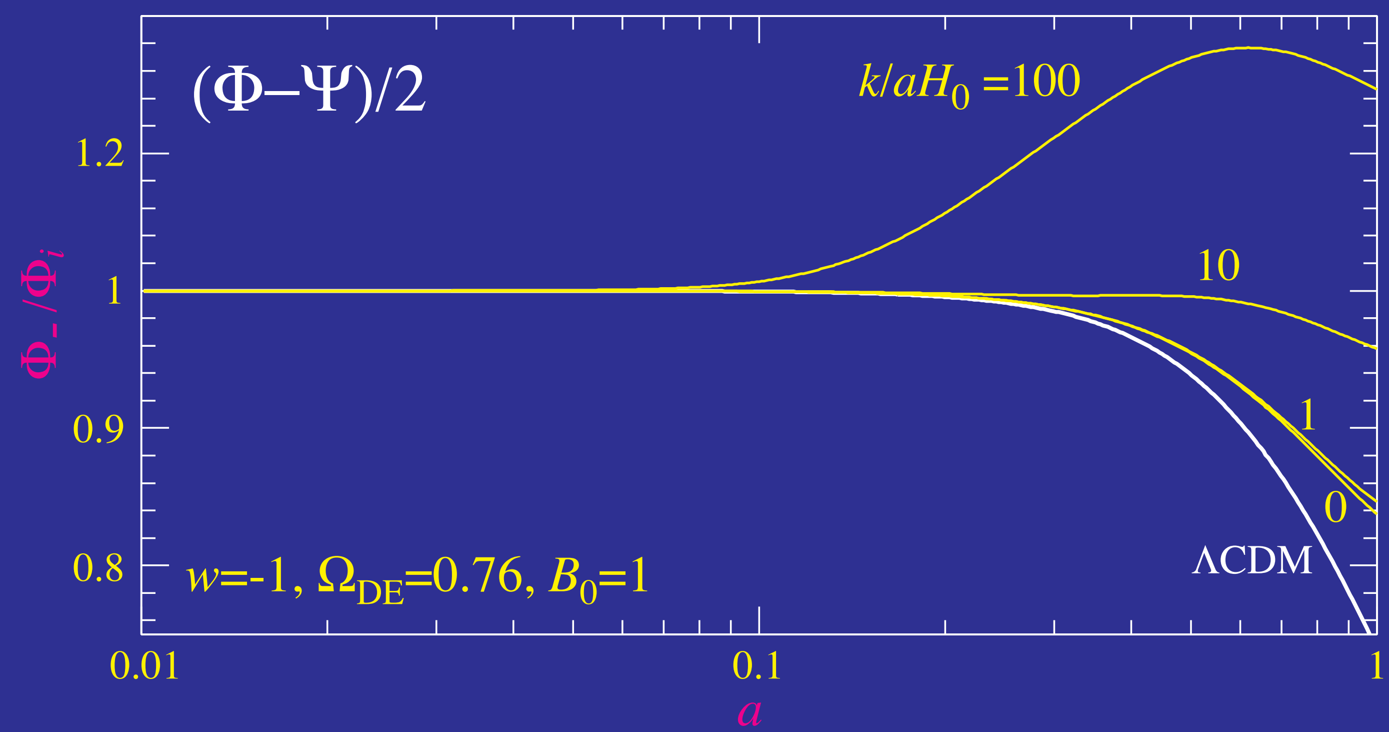
$$B = \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

- $R \to \infty$, $B \to 0$ and for $B < 0$ short time-scale tachyonic instability appears making previous models not cosmologically viable

$$f(R) = -M^{2+2n}/R^n$$

Potential Growth

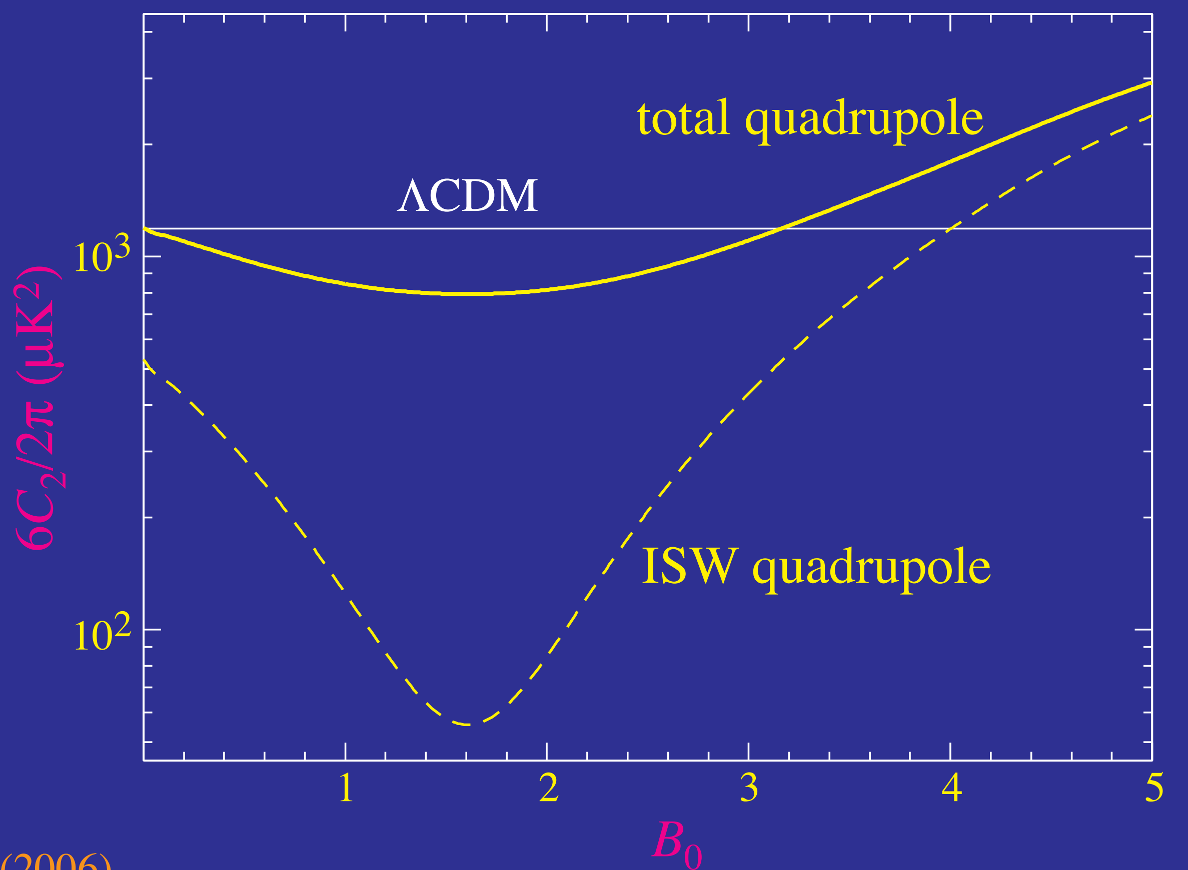
- On the stable $B>0$ branch, potential evolution reverses from decay to growth as a function of scale $B^{1/2}(k/aH)$
- Newton constant G rescaled by $1+f_R$ leading to different density and potential growth functions
- On small scales, quasistatic equilibrium reached in linear theory with $\Psi=-2\Phi$ requiring non-linear effects restore PPN expectations



Song, Hu & Sawicki (2006)

ISW Quadrupole

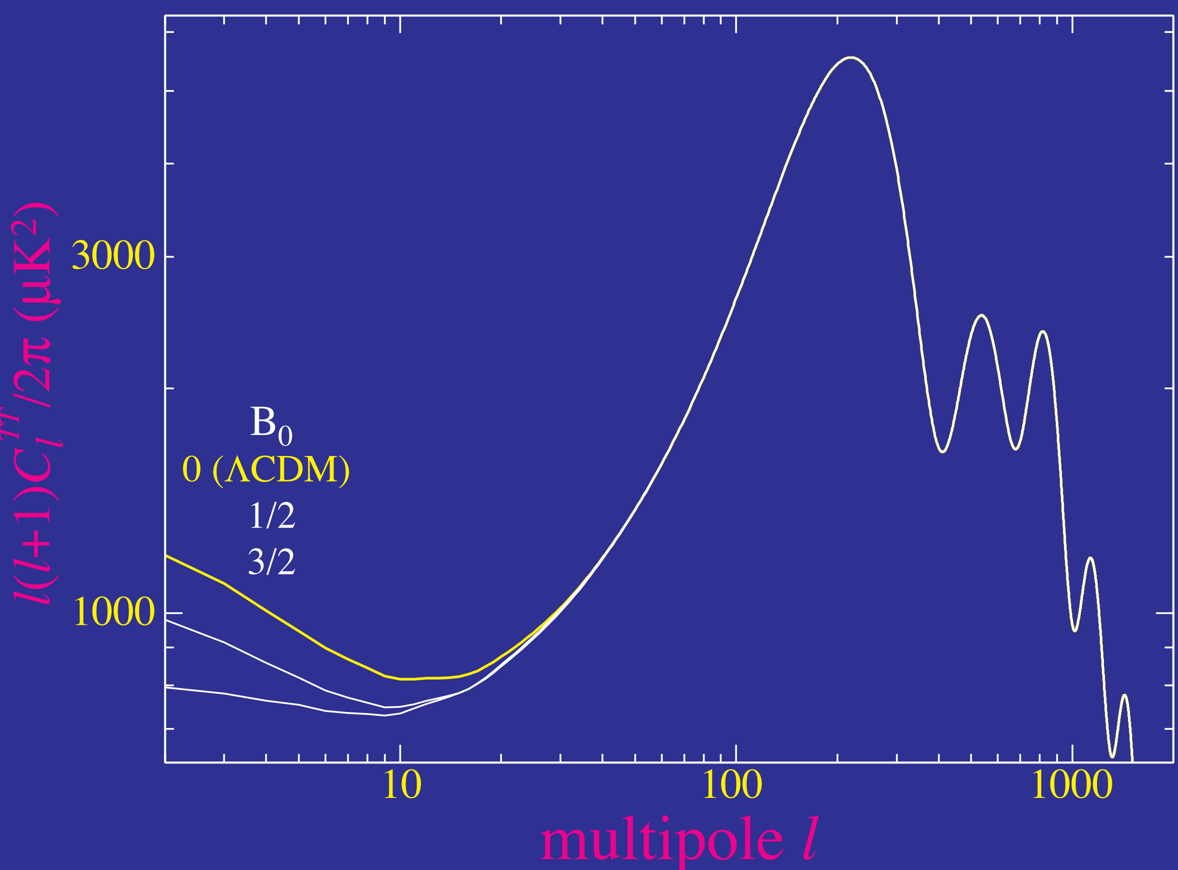
- Reduction of potential decay can eliminate the ISW effect at the quadrupole for B_0~3/2
- In conjunction with a change in the initial power spectrum can also bring the total quadrupole closer in ensemble average to the observed quadrupole



Song, Hu & Sawicki (2006)

ISW Quadrupole

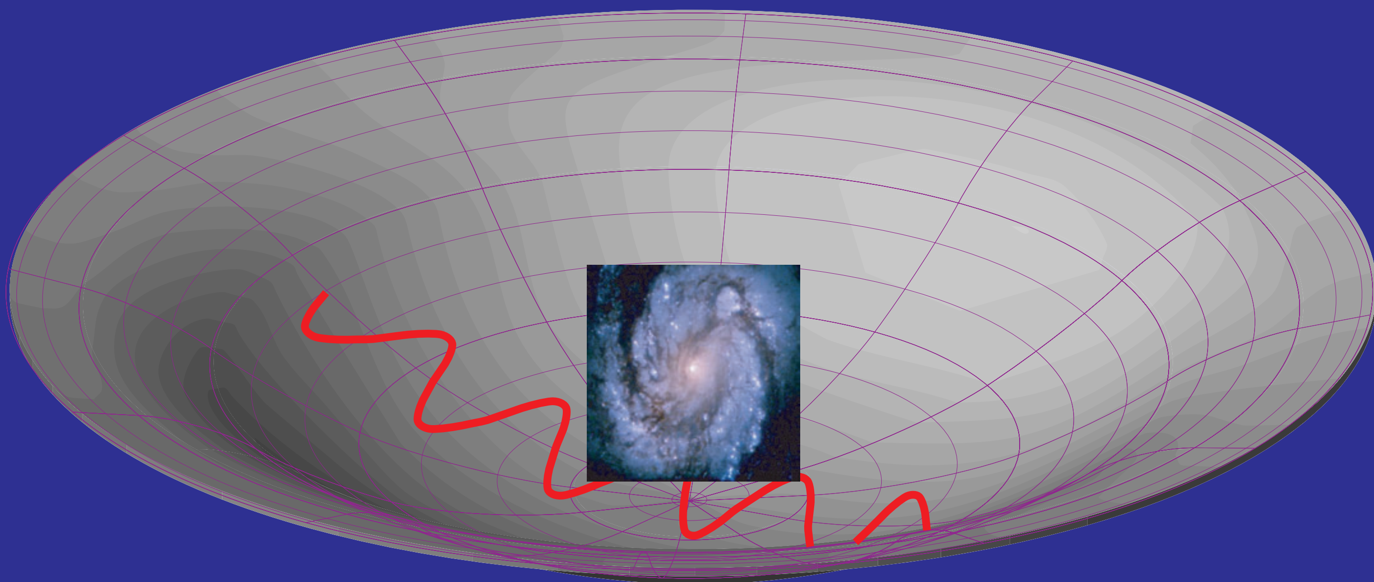
- Reduction of large angle anisotropy for B_0~1 for same expansion history and distances as ΛCDM
- Well-tested small scale anisotropy unchanged



Song, Hu & Sawicki (2006)

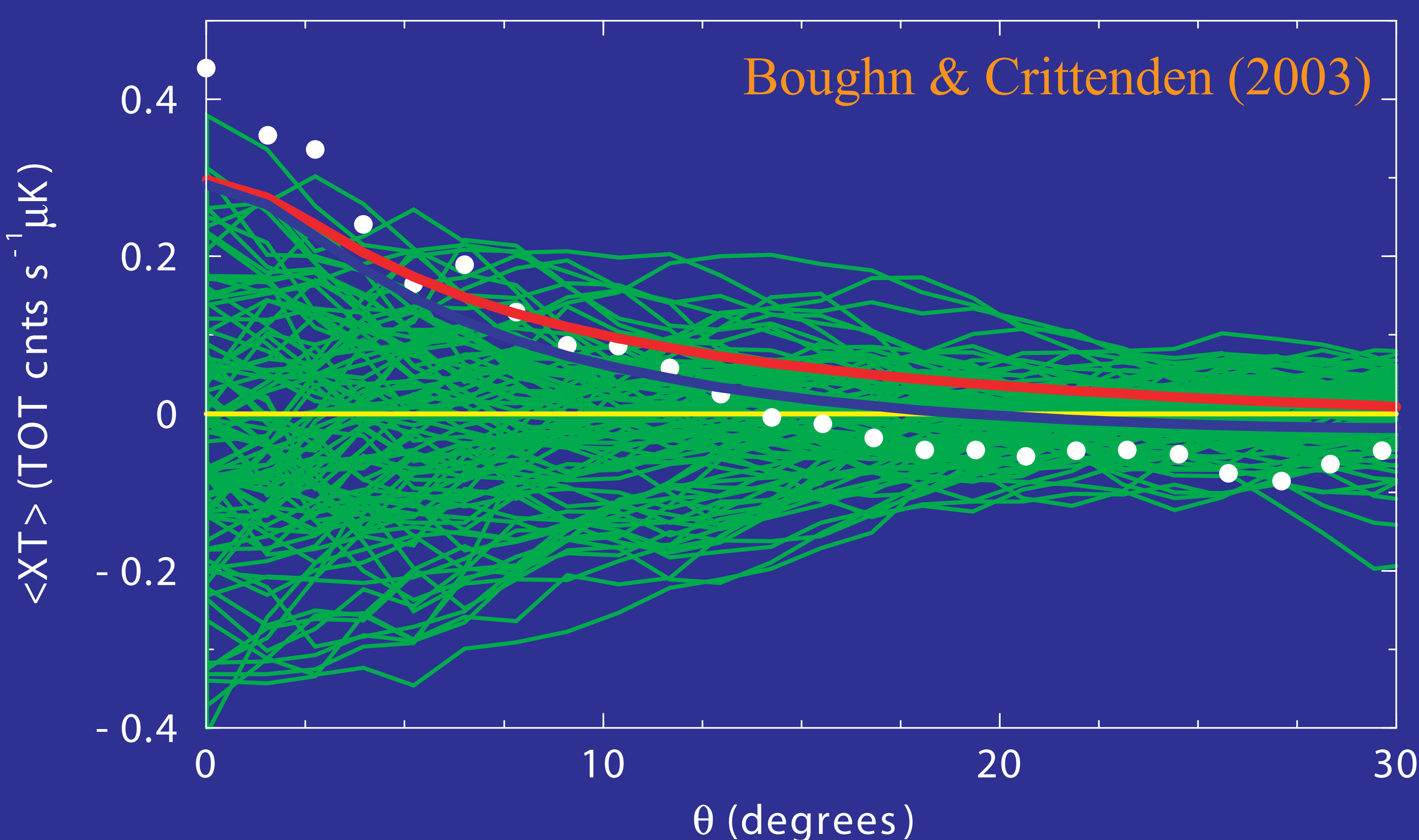
ISW-Galaxy Correlation

- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation



ISW Galaxy Correlation

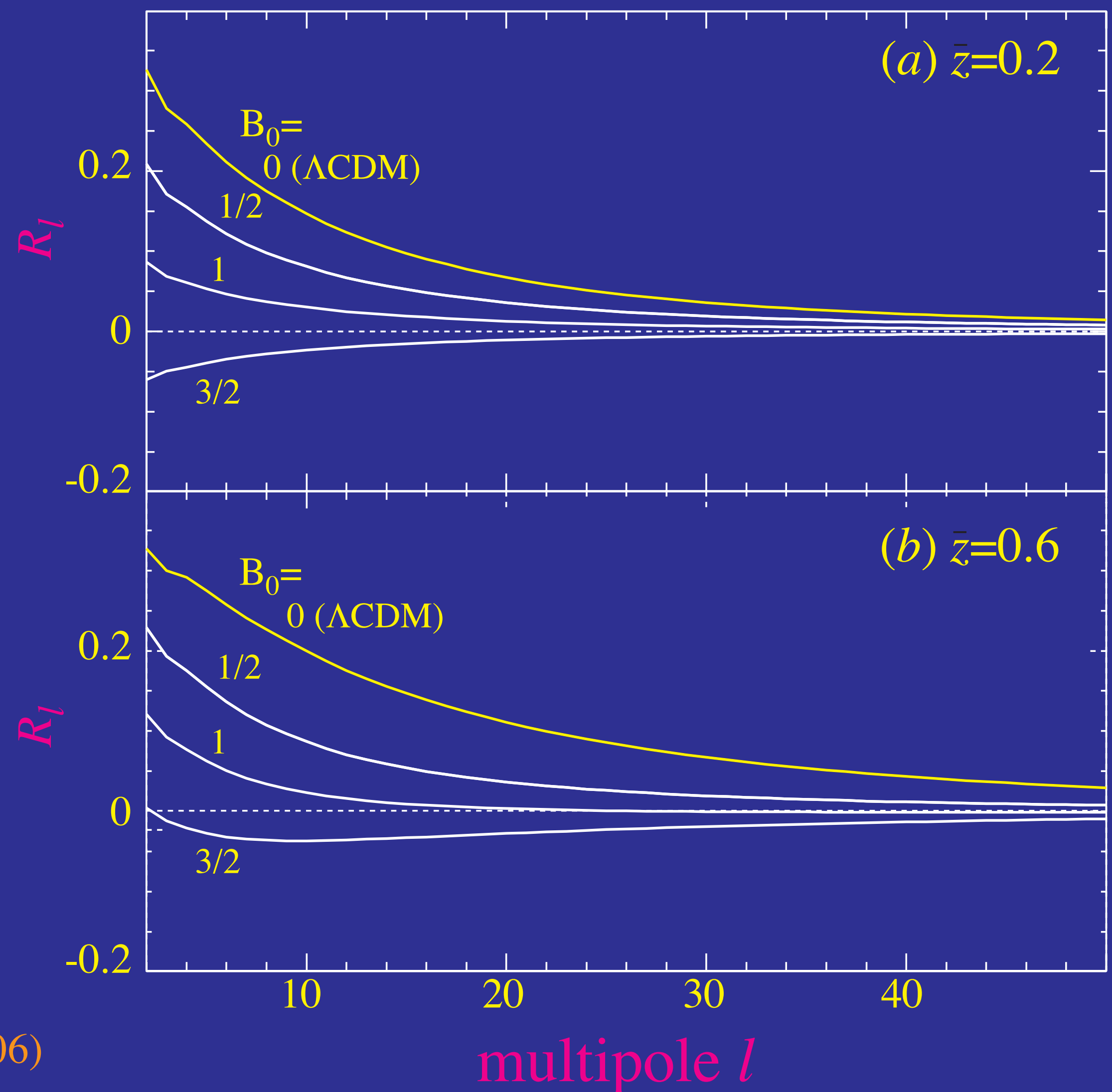
- A 2-3σ detection of the ISW effect through galaxy correlations



Boughn & Crittenden (2003); Nolte et al (2003); Fosalba & Gaztanaga (2003); Fosalba et al (2003); Afshordi et al (2003)

Galaxy-ISW (Anti)Correlation

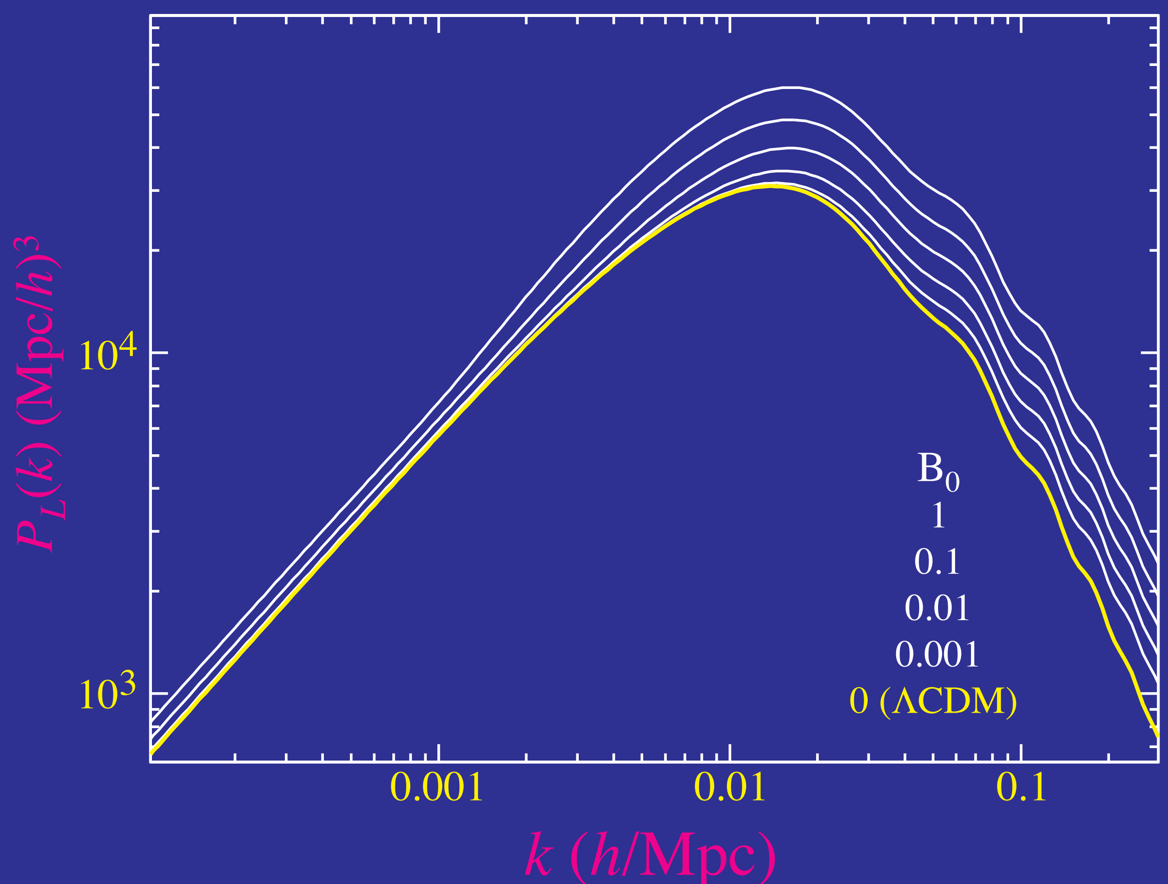
- Change in potential growth reduces galaxy-ISW correlation and for high $B_0>1$ predicts anticorrelation
- Reported positive detections place upper limit of $B_0<1$



Song, Hu & Sawicki (2006)

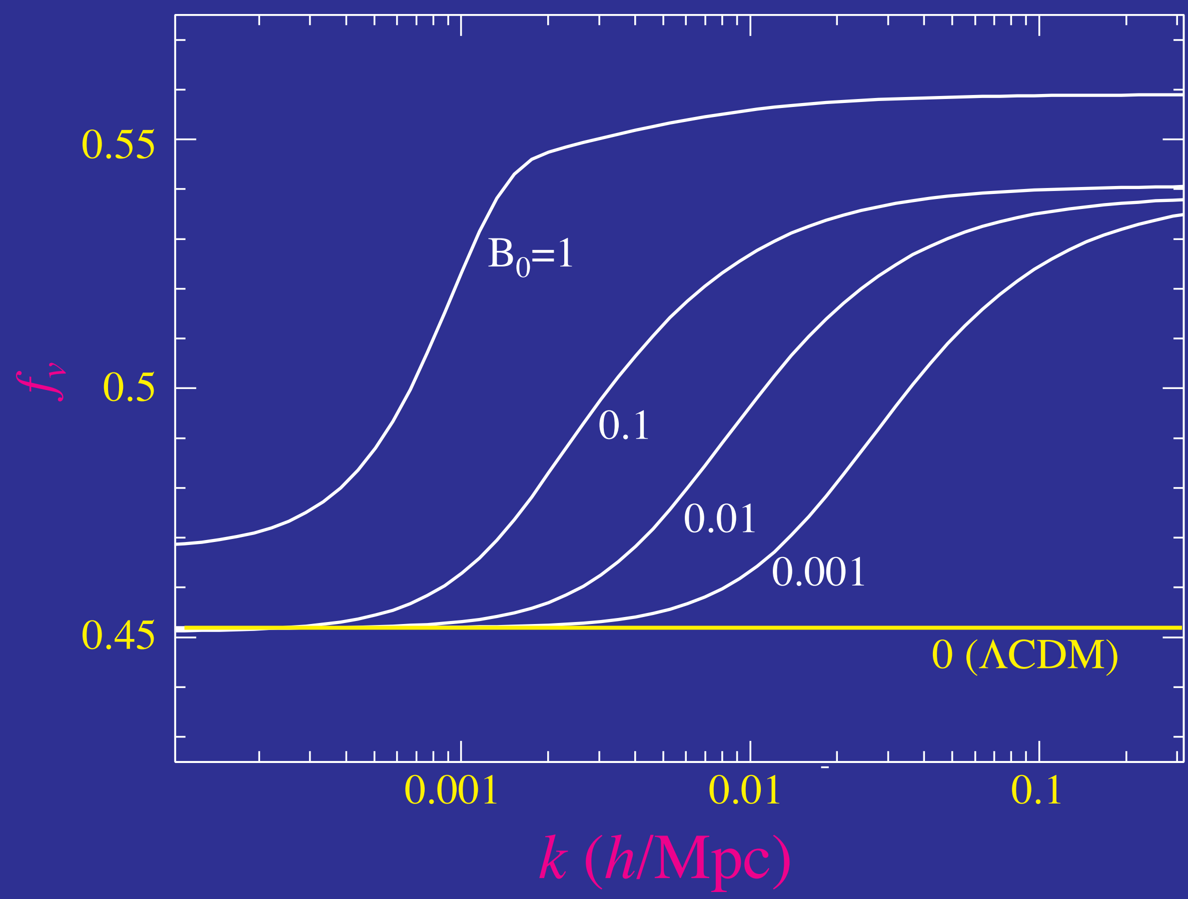
Linear Power Spectrum

- Linear real space power spectrum enhanced on small scales
- Degeneracy with galaxy bias and lack of non-linear predictions leave constraints from shape of power spectrum



Redshift Space Distortion

- Relationship between velocity and density field given by continuity with modified growth rate
- Redshift space power spectrum further distorted by Kaiser effect



PPF Description

- On superhorizon scales, metric evolution given by conservation

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = 0$$

requiring a closure relation between the metric fluctuations

$$\Psi = -f_1(a)\Phi$$

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- Below parameterized transition scale, modified Poisson equation

$$k^2\left(\frac{\Phi - \Psi}{2}\right) = \frac{1}{2} f_3(a)\mu^2 a^2 \rho\Delta$$

with a potentially different closure relation

$$\Psi = -f_2(a)\Phi$$

and the usual quasistatic conservation laws

$$\Delta' = \left(\frac{k}{aH}\right)^2 Hq\,, \quad Hq' = \Psi\,,$$

Summary

- Parameterized description of acceleration: background expansion history $w(z)$ supplemented by

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- Test explanations of acceleration in absence of compelling models
- Expansion history alone tests specific models: e.g. DGP by H_0
- PPF description of DGP shows disfavored enhanced ISW effect if Weyl anisotropy dominates during self-acceleration
- PPF description of $f(R)$ shows previous models unstable but stable models do exist and are testable with linear phenomena