Toward a Parameterized Post Friedmann Description of Cosmic Acceleration

Description of Cosmic Acceleration

Wayne Hu
FRS, U Chicago, March 2007
Cosmic Acceleration

- Cosmic acceleration, like the cosmological constant, can either be viewed as arising from
  
  Missing, or dark energy, with $w \equiv \frac{\bar{p}}{\bar{\rho}} < -1/3$
  
  Modification of gravity on large scales
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- Proof of principle models for both exist: quintessence, k-essence; DGP braneworld acceleration, $f(R)$ modified action

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- Dark energy parameterized description on small scales: smooth component with a \( w(z) \) that completely defines expansion history

- Parameterized description of modified gravity acceleration?

- Previous ad-hoc attempts violate basic principles like energy-momentum conservation
Outline

- Constraints imposed by energy-momentum conservation on linear metric fluctuations around an FRW background with a given expansion history
- “Parameterized Post Friedmann” (PPF) description of remaining degrees of freedom
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- DGP braneworld acceleration example
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• “Parameterized Post Friedmann” (PPF) description of remaining degrees of freedom

• DGP braneworld acceleration example

• $f(R)$ modified action example

• Collaborators:
  • Dragan Huterer
  • Iggy Sawicki
  • Yong-Seon Song
  • Kendrick Smith
PPF Framework
PPF Description

- Parallel treatment of parameterized dark energy beyond a quintessence scalar field
- Demand that the model satisfies (Hu 1998)
  - Given Background Expansion
  - Gauge Invariance
  - Energy-Momentum Conservation
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- Demand that the model satisfies (Hu 1998)
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and the phenomenologically desirable property that the dark energy does not cluster with the dark matter → sound horizon

- Larger scales: energy-momentum conservation requires conservation of the comoving curvature (Bardeen 1980)

- Smaller scales: dark energy spatial perturbations negligible and observable phenomena depend on expansion history only
PPF Description

- Implement with a parameterized model: the sound speed in the dark energy rest frame. Quintessence sound speed $c_s = 1$
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• Parameterization later shown to describe \textit{k-essence} with modified scalar field kinetic term (Garriga & Mukhanov 1999)

\[
\mathcal{L} = F(X, \phi) \quad X = -\frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi
\]

with a sound speed

\[
c_s^2 = \frac{\partial F/\partial X}{2(\partial^2 F/\partial X^2)X + (\partial F/\partial X)}
\]
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$$c_s^2 = \frac{\partial F/\partial X}{2(\partial^2 F/\partial X^2)X + (\partial F/\partial X)}$$

- Beyond single scalar fields: parameterize multiple internal degrees of freedom to allow an evolution across $w = -1$ phantom divide (Hu 2004)
PPF Description

- Modified gravity models of acceleration
- Demand that the model satisfies
  
  Given Background Expansion History
  
  Bianchi Identities / (FRW) Metric Theory
  
  Energy-Momentum Conservation

  and that modifications reach quasi-static Newtonian limit on small scales: time derivatives neglected compared with spatial gradients

- PPF description can be used to test general relativity on cosmological scales and distinguish modified gravity from smooth dark energy
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- PPF description can be used to test general relativity on cosmological scales and distinguish modified gravity from smooth dark energy

- In addition non-linear effects must bring gravity stably back to general relativity on small scales to satisfy solar system tests. Beyond the scope of this talk.
PPF Description

- On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)

- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d \ln a$ (Bardeen 1980)
On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006).

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Gauge transformation to Newtonian gauge

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2$$

yields (Hu & Eisenstein 1999)

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = 0$$

Modified gravity theory supplies the closure relationship between $\Phi$ and $\Psi$ and expansion history $H = \dot{a}/a$ supplies rest.
A Worked Example:
DGP Braneworld Acceleration
A Worked Example: DGP Gravity

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)
  
  \[ S = \int d^5 x \sqrt{-g} \left[ \frac{(5) R}{2\kappa^2} + \delta(\chi) \left( \frac{(4) R}{2\mu^2} + \mathcal{L}_m \right) \right] \]

  with crossover scale \( r_c = \frac{\kappa^2}{2\mu^2} \)

- Influence of bulk through \textbf{Weyl tensor anisotropy} - solve \textbf{master equation in bulk} (Deffayet 2001; see also Sawicki’s talk)

- Matter still \textbf{minimally coupled} and conserved

- \textbf{Satisfies PFF requirements}
A Worked Example: DGP Gravity

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- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001; see also Sawicki’s talk)

- Matter still minimally coupled and conserved

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- Dominance of Weyl tensor anisotropy over other components and matter sets closure relation during self acceleration \( \Psi \rightarrow \Phi \)

- Transition to this limit leads to enhancement of potential decay and large angle CMB anisotropy
DGP Expansion History
DGP Expansion History

- Matching the DGP expansion history to a dark energy model with the same expansion history
- Effective equation of state \( w(z) \) \([w_0 \sim -0.85, \, w_a \sim 0.35]\)
DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$

Song, Sawicki & Hu (2006)
Leveraging the CMB

- Relative heights of the first 3 peaks calibrates sound horizon and matter radiation equality horizon: measures $\Omega_m h^2$ currently 8%

![Graph showing the multipole moment ($l$) versus the $l(l+1)C_l/2\pi$ (uK²) for dark matter (horizon, equality) as the leading source of error!](image)
Standard Ruler

- Standard ruler used to measure the angular diameter distance to recombination (z~1100; currently 2%) or any redshift for which acoustic phenomena observable

![Graph showing angular scale vs. physical scale](image)

$\ell\left(\ell+1\right)C_\ell / 2\pi$ (µK²)

Multipole moment ($\ell$)

DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$
- Mismatch to CMB absolute distance $D_A$ requires curvature

Song, Sawicki & Hu (2006)
DGP Expansion History

- Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$
- Mismatch to CMB absolute distance $D_A$ requires curvature
- Difference in expansion history appears as a change in local distances or the Hubble constant: $H_0$

![Graph showing DGP expansion history with distance change against redshift $z$.]
• Crossover scale $r_c$ fit to SN relative distance from $z=0$: $H_0D_A$
• Mismatch to CMB absolute distance $D_A$ requires curvature
• Compromise between SN and $H_0$ measures

Song, Sawicki & Hu (2006)
DGP Example

- **DGP modified gravity** is in tension with distance measures alone: CMB & SNe distances cannot be jointly satisfied in a flat universe.
- Even fitting out curvature, **Hubble constant is too high** for Key Project measurement (and baryon oscillations).
- Joint maximization leads to a **poorer fit** even with extra curvature parameter.

### Figure

<table>
<thead>
<tr>
<th>WMAP3yr+SNLS</th>
<th>WMAP3yr+SNLS+KP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_m=0.22$</td>
<td>$\Omega_m=0.18$</td>
</tr>
<tr>
<td>$\Omega_K=0$</td>
<td>$\Omega_K=0.036$</td>
</tr>
<tr>
<td>$H_0=69$</td>
<td>$H_0=82$</td>
</tr>
</tbody>
</table>

Song, Sawicki & Hu (2006) [also Fairbairn & Goobar 2005, Maartens & Majerotto 2006]
Prospects for Percent $H_0$

- **Improving the distance ladder** (~3-5%, Riess 2005; Macri et al 2006)
- **Water maser proper motion, acceleration** (~3%, VLBA Condon & Lo 2005; ~1% SKA, Greenhill 2004)

- **Gravity wave sirens** (~2% - 3x Adv. LIGO + GRB sat, Dalal et al 2006)
- **Combination of dark energy tests:** e.g. SNIa relative distances: $H_0D(z)$ and baryon acoustic oscillations $D(z)$
Flat Universe Precision

- Planck acoustic peaks, 1\% $H_0$, SNAP SNe to $z=1.7$ in a flat universe

Forecasts for CMB+$H_0$

- To complement CMB observations with $\Omega_m h^2$ to 1%, an $H_0$ of ~1% enables constant $w$ measurement to ~2% in a flat universe

Planck: $\sigma(\ln \Omega_m h^2) = 0.009$
Dark Energy Equation of State

- Marginalizing curvature degrades 68% CL area by 4.8
- CMB lensing information from SPTpol (~3% B-mode power) fully restores constraints

DGP Metric Evolution
DGP Potential Evolution

- Difference in expansion history gives excess decay of grav. potential on subhorizon scales (Lue, Scoccimarro, Starkmann 2004; Koyama & Maartins 2005)
- Energy-momentum conservation and dominance of Weyl anisotropy leads to further decay
Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure.
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit.
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi - \Psi)$. 

\[ D \frac{T}{T} = -D(F - Y) \]
**Integrated Sachs-Wolfe Effect**

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DGP Example

• Excess decay leads to enhanced large angle CMB anisotropy
• Requires either breaking of initial scale invariance or missing physics beyond Weyl tensor at $\sim r_c/10$ to be compatible with observations

Song, Sawicki & Hu (2006)
A Worked Example: \( f(R) \) Modified Action Acceleration
A Worked Example: $f(R)$ Gravity

- Modify the **Einstein-Hilbert action** (Starobinsky 1980; Carroll et al 2004)

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{2\mu^2} + \mathcal{L}_m \right] \]

- In the **Jordan frame**, gravity becomes 4th order but matter remains minimally coupled and separately conserved

- Satisfies **PPF requirements**
A Worked Example: $f(R)$ Gravity

- Modify the **Einstein-Hilbert action** (Starobinsky 1980; Carroll et al 2004)

$$ S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{2\mu^2} + L_m \right] $$

- In the **Jordan frame**, gravity becomes 4th order but matter remains minimally coupled and separately conserved

- Satisfies **PPF requirements**

- Expansion history parameterization: **Friedmann equation** becomes

$$ H^2 - f_R(\dot{H}H' + H^2) + \frac{1}{6}f + H^2 f_{RR}R' = \frac{\mu^2 \rho}{3} $$

where $f_R = df/dR$, $f_{RR} = d^2f/dR^2$

- For any desired $H$, solve a **2nd order diffeq** to find $f(R)$
Expansion History Family of $f(R)$

- Each expansion history, matched by dark energy model [$w(z), \Omega_{DE}, H_0$] corresponds to a family of $f(R)$ models due to its 4th order nature.
- Parameterized by $B \propto f_{RR} = d^2f/dR^2$ evaluated at $z=0$.
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Song, Hu & Sawicki (2006)
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![Graph showing the expansion history family of $f(R)$ with different values of $B_0$.](attachment:image.png)

Song, Hu & Sawicki (2006)
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Parameterized by \(B \propto f_{RR} = d^2f/dR^2\) evaluated at \(z=0\)

(a) \(w=-1, \Omega_{DE}=0.76\)

(b) \(w=-0.9, \Omega_{DE}=0.73\)
$f(R)$ Metric Evolution
Deviation Parameter

- Express the 4th order nature of equations as a deviation parameter

\[ \Phi'' - \Psi' - \frac{H''}{H'} \Phi' - \left( \frac{H'}{H} - \frac{H''}{H'} \right) \Psi = \left( \frac{k}{aH} \right)^2 B \epsilon \]

- Einstein equation become a second order equation for \( \epsilon \)
Deviation Parameter

- Express the 4th order nature of equations as a deviation parameter

\[ \Phi'' - \Psi' - \frac{H''}{H'} \Phi' - \left( \frac{H'}{H} - \frac{H''}{H'} \right) \Psi = \left( \frac{k}{aH} \right)^2 B \epsilon \]

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- In high redshift, **high curvature** \( R \) limit this is

\[ \epsilon'' + \left( \frac{7}{2} + 4 \frac{B'}{B} \right) \epsilon' + \frac{2}{B} \epsilon = \frac{1}{B} \times \text{metric sources} \]

\[ B = \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'} \]

- \( R \to \infty, B \to 0 \) and for \( B < 0 \) short time-scale **tachyonic instability** appears making previous models **not cosmologically viable**

\[ f(R) = -M^{2+2n} / R^n \]
Potential Growth

- On the stable $B>0$ branch, potential evolution reverses from decay to growth as a function of scale $B^{1/2}(k/aH)$
- Newton constant $G$ rescaled by $1+f_R$ leading to different density and potential growth functions
- On small scales, quasistatic equilibrium reached in linear theory with $\Psi=-2\Phi$ requiring non-linear effects restore PPN expectations

\[ (\Phi-\Psi)/2 \]
ISW Quadrupole

- Reduction of potential decay can eliminate the ISW effect at the quadrupole for $B_0 \sim 3/2$
- In conjunction with a change in the initial power spectrum can also bring the total quadrupole closer in ensemble average to the observed quadrupole

![Graph showing ISW quadrupole and total quadrupole](image-url)
ISW Quadrupole

- Reduction of large angle anisotropy for $B_0 \approx 1$ for same expansion history and distances as $\Lambda$CDM
- Well-tested small scale anisotropy unchanged

Song, Hu & Sawicki (2006)
ISW-Galaxy Correlation

- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation
ISW Galaxy Correlation

- A $2-3\sigma$ detection of the ISW effect through galaxy correlations

Galaxy-ISW (Anti)Correlation

- Change in potential growth reduces galaxy-ISW correlation and for high $B_0 > 1$ predicts anticorrelation
- Reported positive detections place upper limit of $B_0 < 1$

Song, Hu & Sawicki (2006)
Linear Power Spectrum

- Linear real space **power spectrum** enhanced on small scales
- Degeneracy with **galaxy bias** and lack of non-linear predictions leave constraints from **shape** of power spectrum
### Redshift Space Distortion

- Relationship between velocity and density field given by continuity with modified growth rate
- Redshift space power spectrum further distorted by Kaiser effect

![Graph showing the relationship between $f_{\nu}$ and $k (h/\text{Mpc})$.](image)

- Parameters:
  - $B_0 = 1$,
  - $0.1$,
  - $0.01$,
  - $0.001$,
  - $0 (\Lambda\text{CDM})$
PPF Functions
On superhorizon scales, metric evolution given by conservation:

\[ \Phi'' - \Psi' - \frac{H''}{H'} \Phi' - \left( \frac{H'}{H} - \frac{H''}{H'} \right) \Psi = 0 \]

requiring a closure relation between the metric fluctuations:

\[ \Psi = -f_1(a) \Phi \]
PPF Description

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requiring a closure relation between the metric fluctuations

\[ \Psi = -f_1(a)\Phi \]

- Below parameterized transition scale, modified Poisson equation

\[ k^2 \left( \frac{\Phi - \Psi}{2} \right) = \frac{1}{2} f_3(a) \mu^2 a^2 \rho \Delta \]

with a potentially different closure relation

\[ \Psi = -f_2(a)\Phi \]

and the usual quasistatic conservation laws

\[ \Delta' = \left( \frac{k}{aH} \right)^2 H q, \quad H q' = \Psi, \]
Summary

• Parameterized description of acceleration: background expansion history $w(z)$ supplemented by
  Transition scale where dark energy becomes smooth
  Transition scale where modified gravity switches from Friedmann dynamics to quasistatic Newtonian dynamics (and a further non-linear transition to GR)
  consistent with energy-momentum conservation and metric theory

• Test explanations of acceleration in absence of compelling models
Summary

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• Test explanations of acceleration in absence of compelling models

• Expansion history alone tests specific models: e.g. DGP by $H_0$

• PPF description of DGP shows disfavored enhanced ISW effect if Weyl anisotropy dominates during self-acceleration

• PPF description of $f(R)$ shows previous models unstable but stable models do exist and are testable with linear phenomena