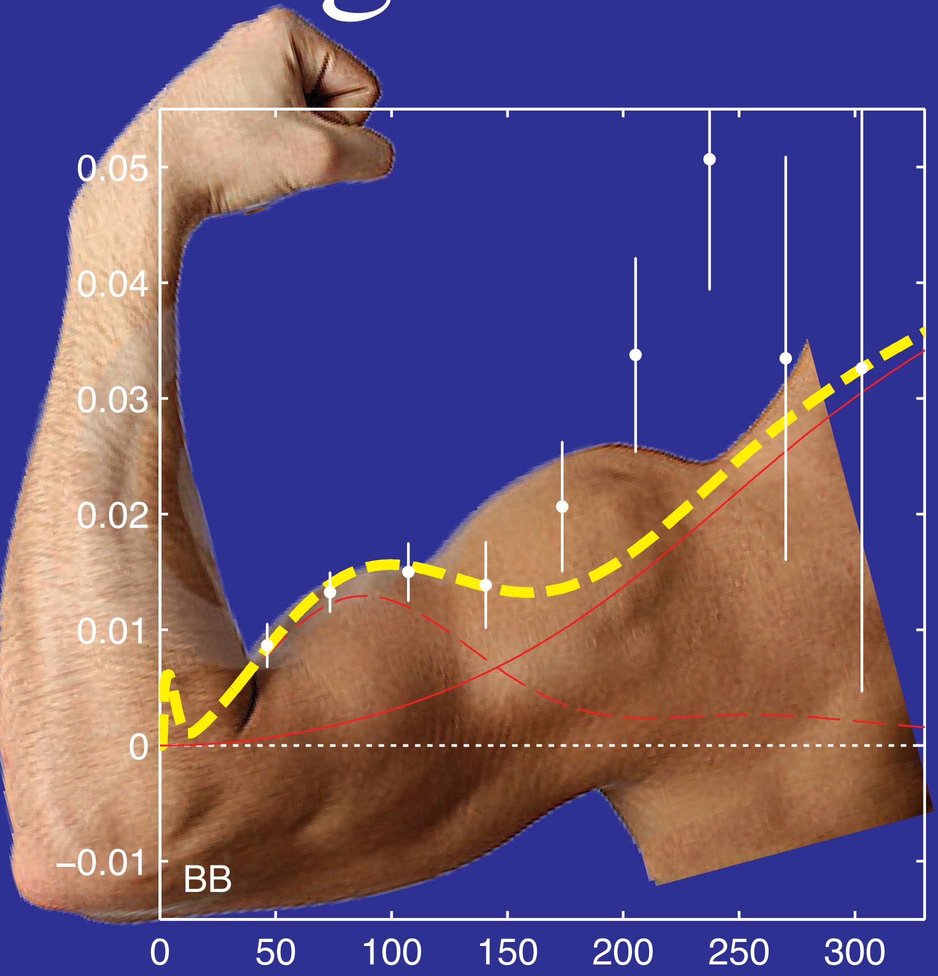
Priming the BICEP



Wayne Hu

Chicago, March 2014

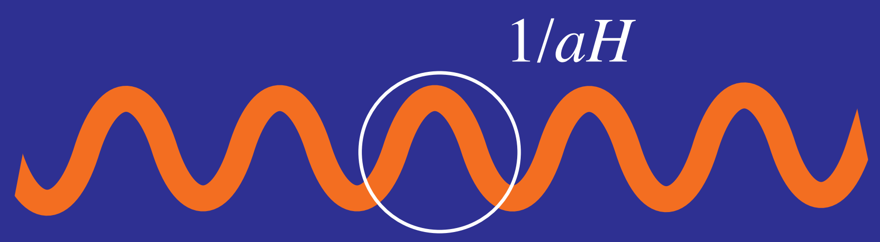
A BICEP Primer

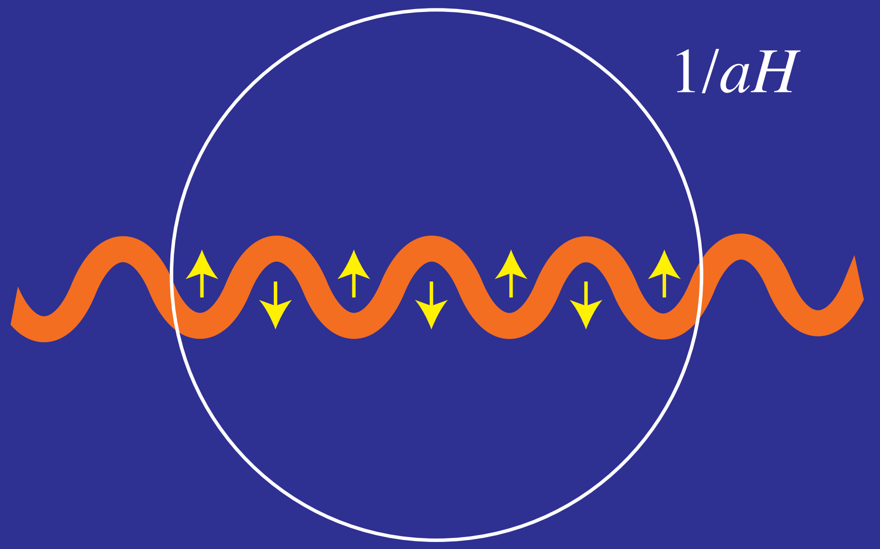
- How do gravitational waves affect the CMB temperature and polarization spectrum?
- What is a B mode?
- Why does inflation predict nearly scale invariant gravitational wave power?
- Why does measuring B determine the inflationary energy scale?
- What is r? and how is it related to tilt(s)
- Can the consistency relation be tested in the near future?
- Why does $r \sim 0.2$ imply that the inflaton rolls super-Planckian distances?
- Are the temperature and B mode power spectra compatible with inflation? simplest scale-free models?

CMB and Gravitational Waves

Gravitational Waves in Cosmology

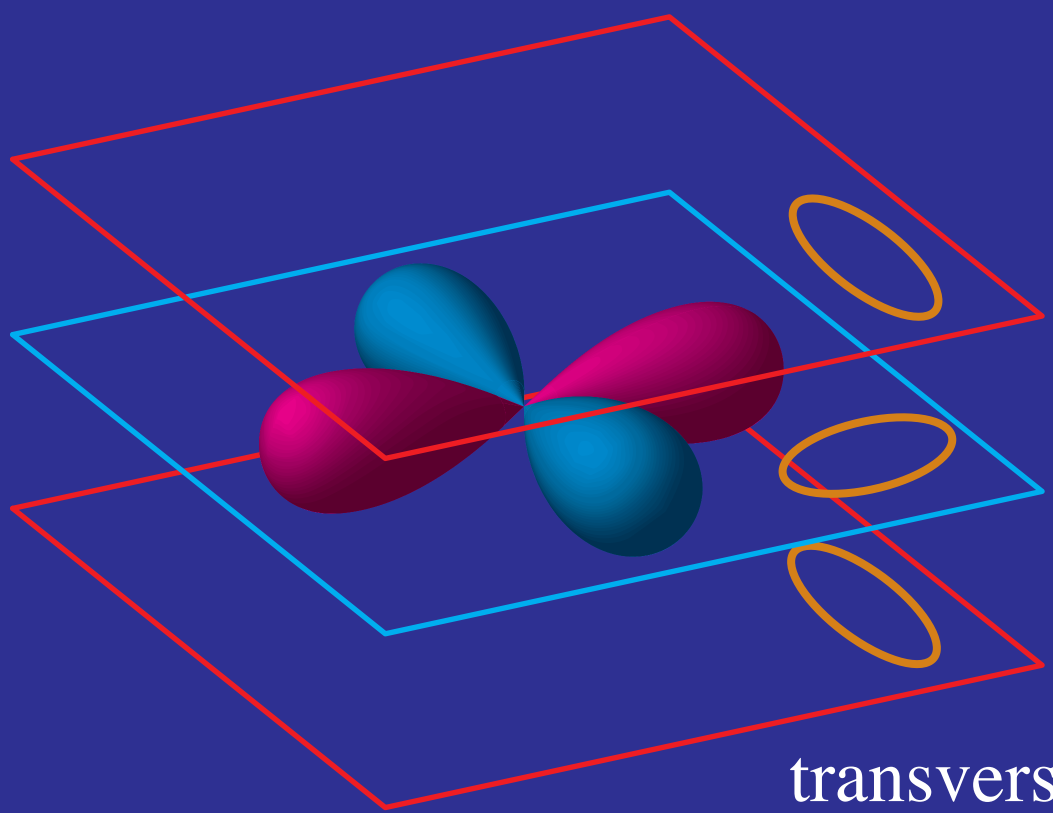
- During deceleration epoch gravity waves are frozen outside the horizon
- Oscillate inside the horizon and decay or redshift as radiation





Quadrupoles from Gravitational Waves

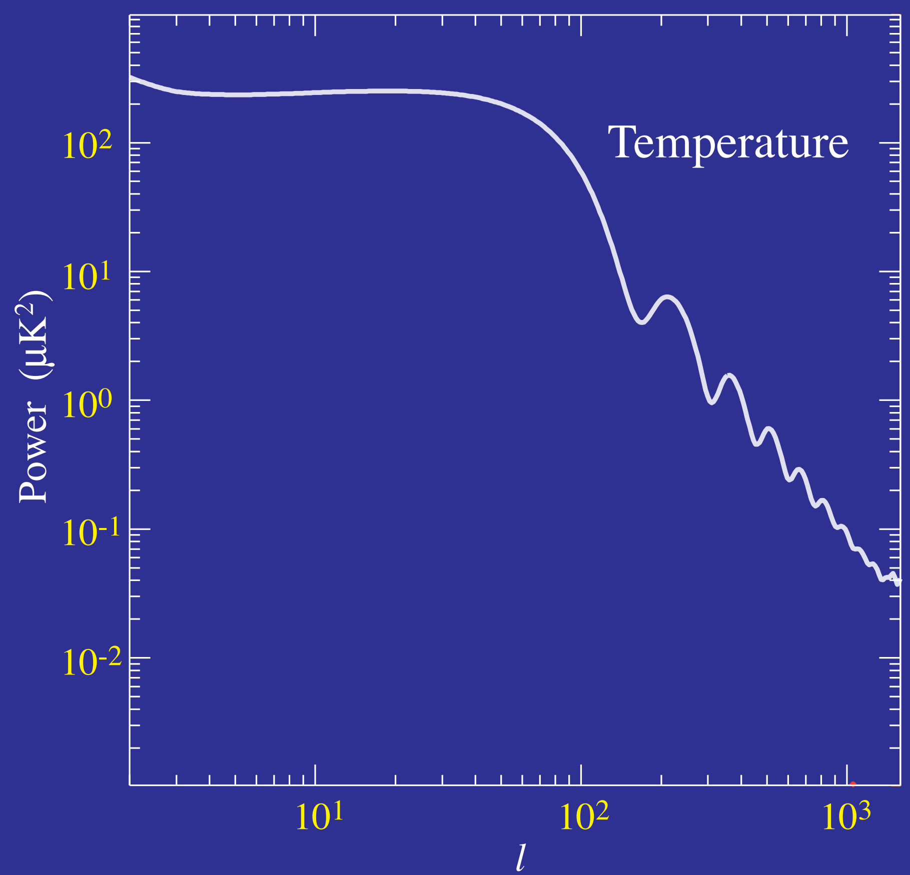
- Changing transverse traceless distortion of space, aka gravitational waves, creates quadrupole CMB anisotropy
- Gravitational waves are frozen when larger than the horizon and oscillate and decay as radiation inside horizon



transverse-traceless
tensor distortion

ISW Effect from Tensors

- Each tensor mode that crosses horizon imprints quadrupole temperature distortion
- Modes that cross before recombination: effect erased by rescattering
- Modes that cross after recombination: project along the line of sight - tensor ISW previously the best constraints on tensors



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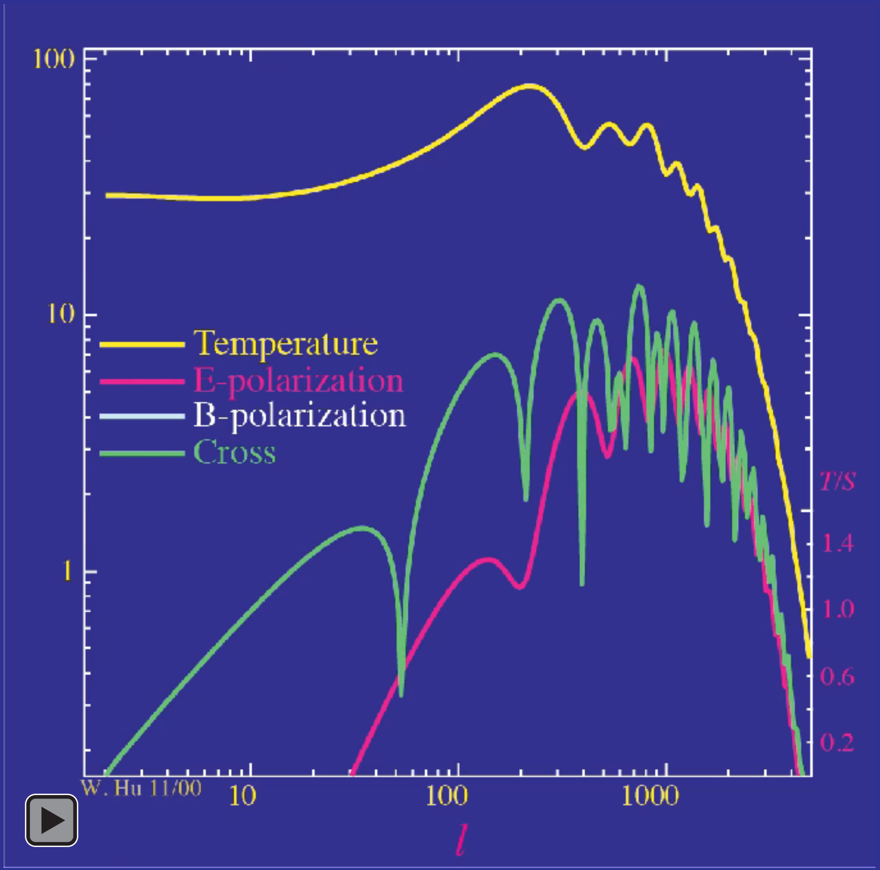
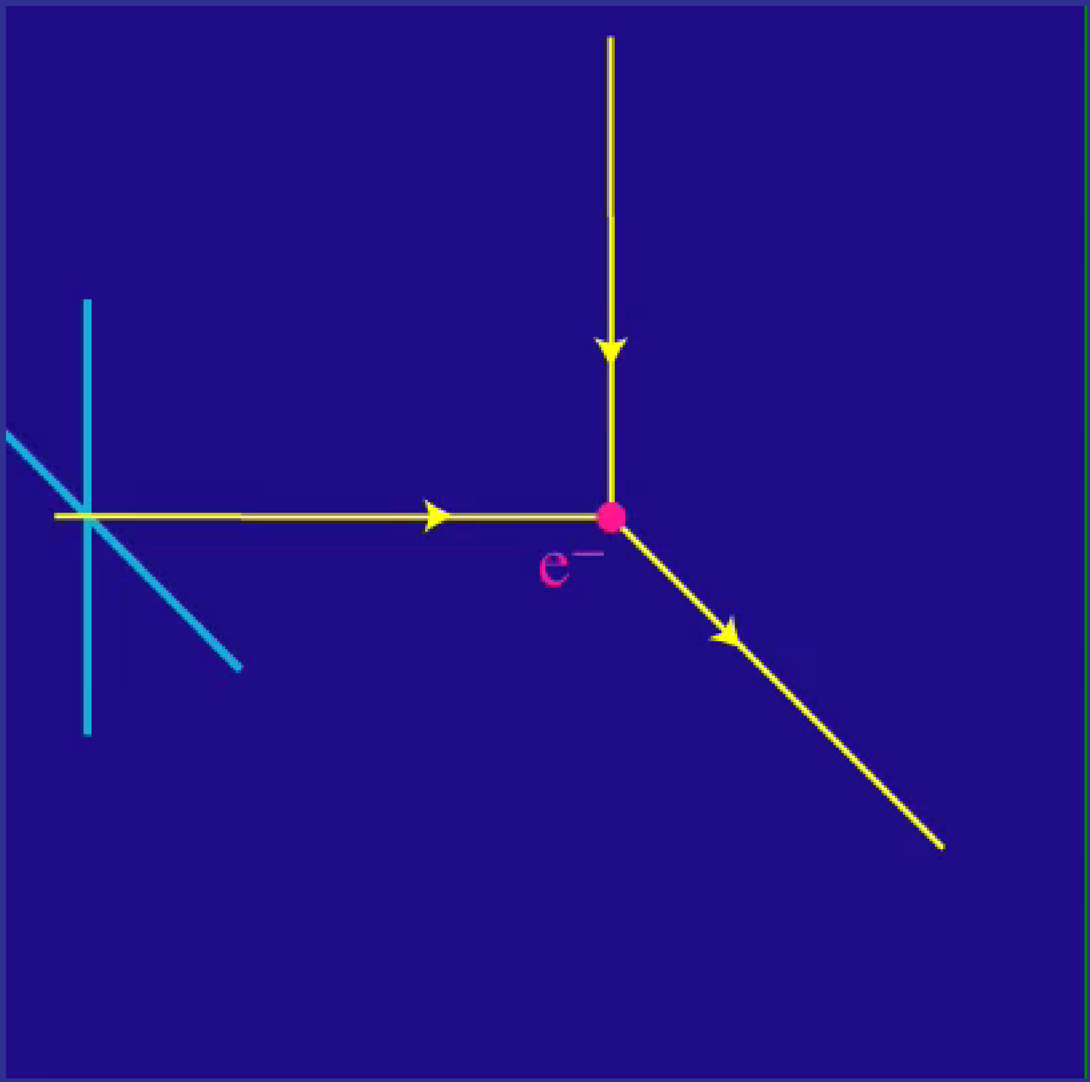


Polarization from Thomson Scattering

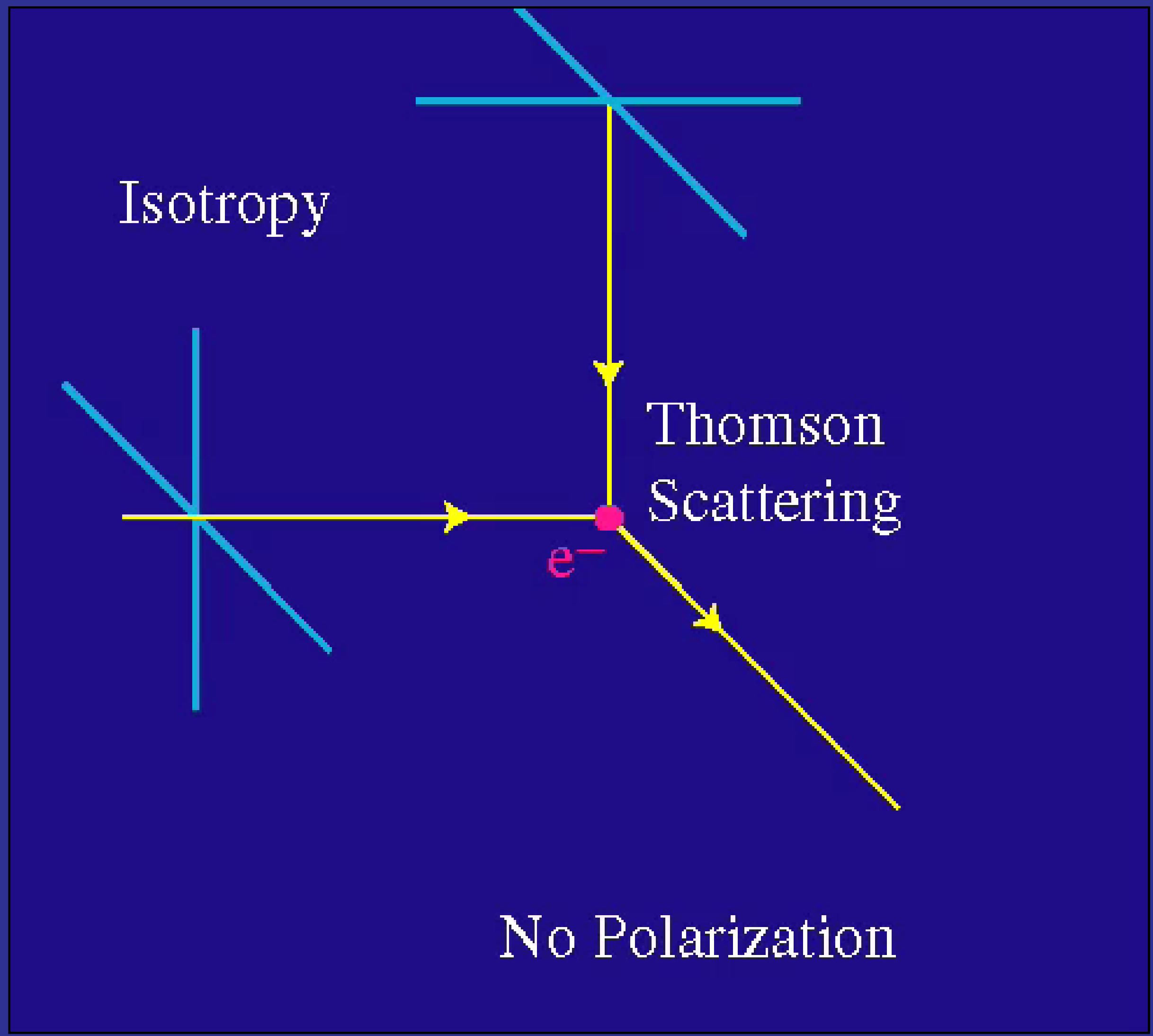
- Differential cross section depends on polarization and angle



$$\frac{d\sigma}{d\Omega} = \frac{3}{8\pi}|\hat{\varepsilon}' \cdot \hat{\varepsilon}|^2 \sigma_T$$

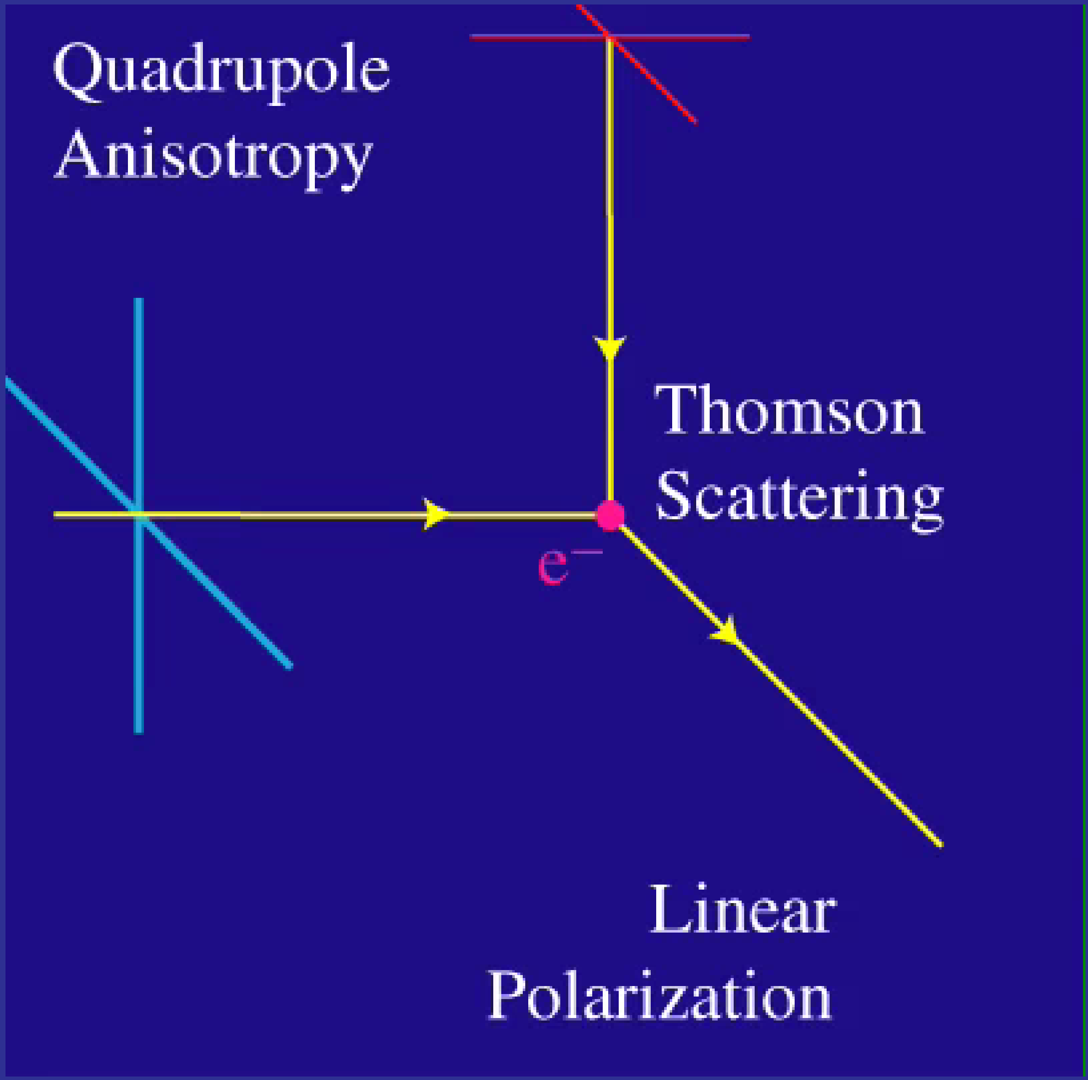
Polarization from Thomson Scattering

- Isotropic radiation scatters into unpolarized radiation



Polarization from Thomson Scattering

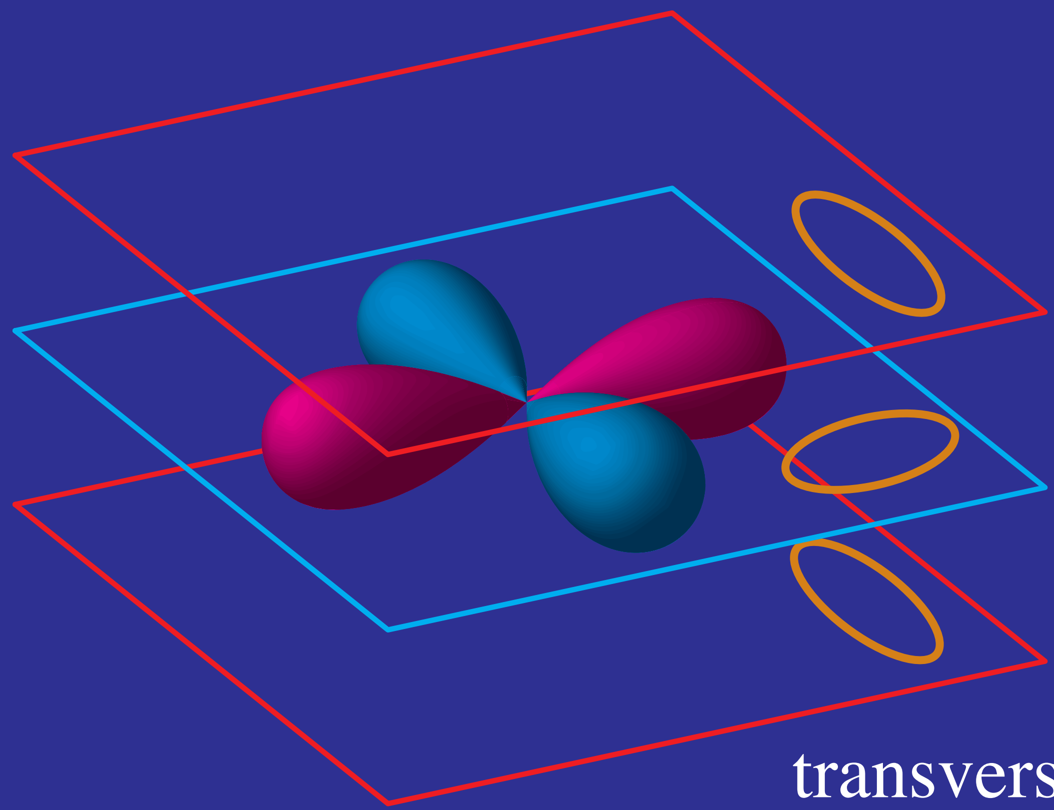
- Quadrupole anisotropies scatter into linear polarization



aligned with
cold lobe

Quadrupoles from Gravitational Waves

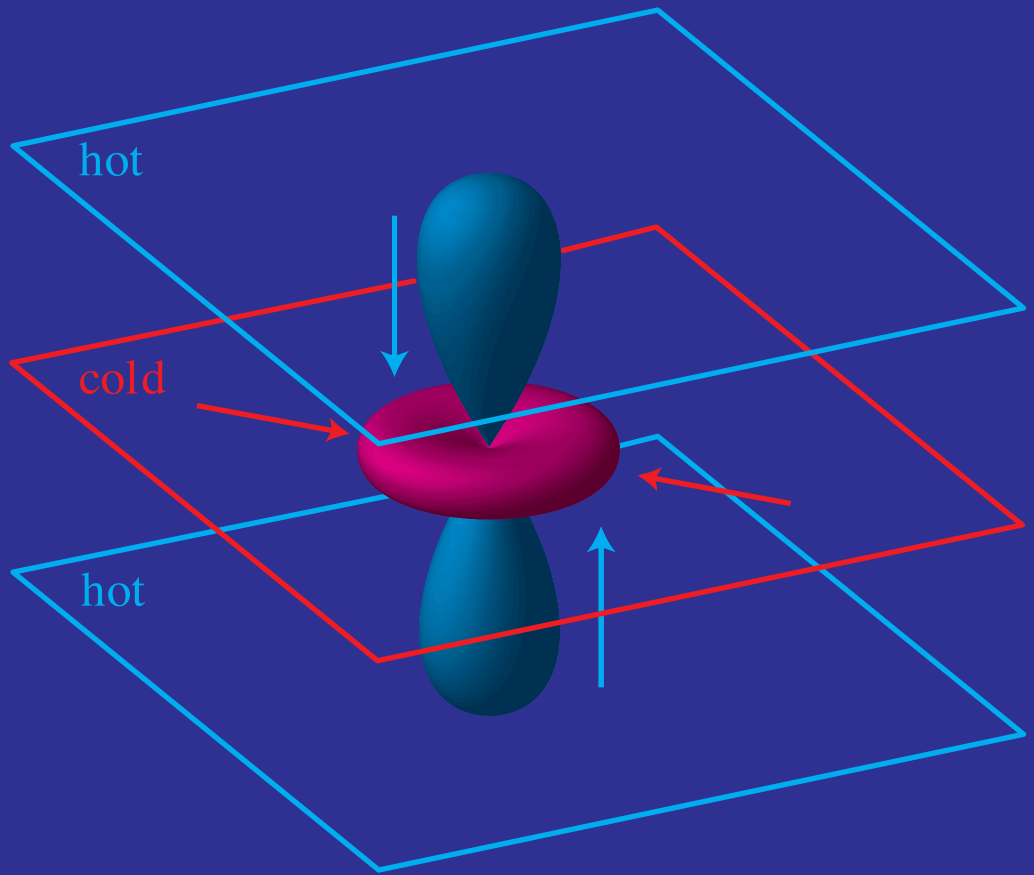
- Transverse-traceless distortion provides temperature quadrupole
- Gravitational wave polarization picks out direction transverse to wavevector



transverse-traceless
distortion

How do Scalars Differ?

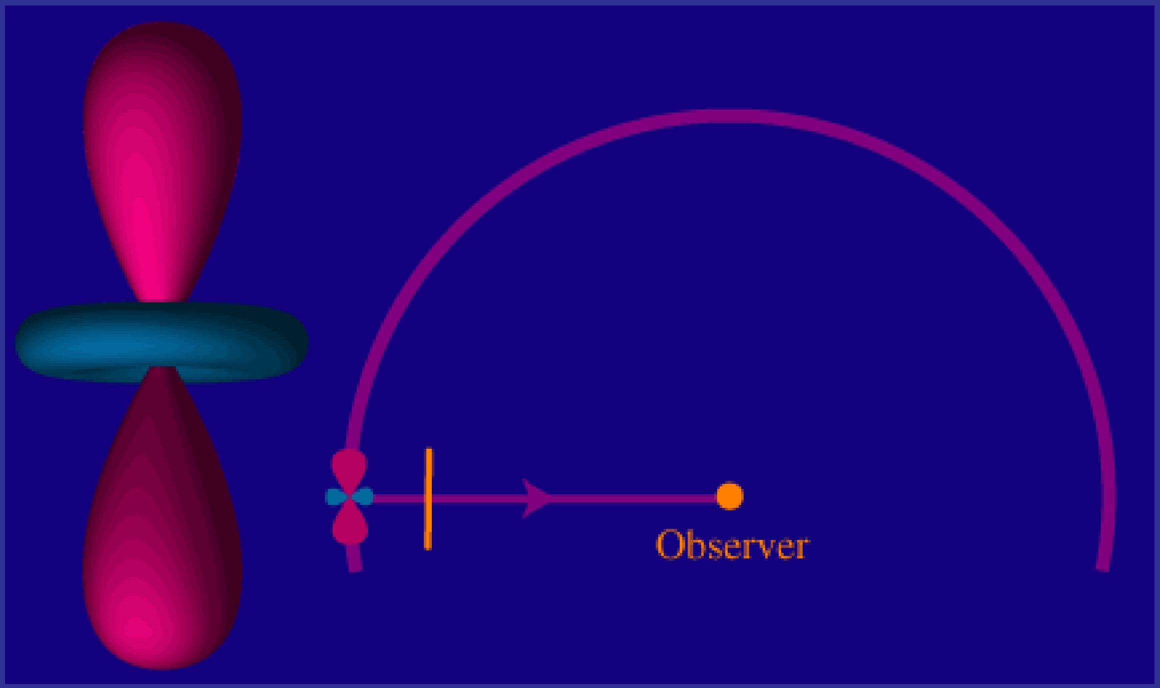
- Temperature inhomogeneities in a medium
- Photons arrive from different regions producing an anisotropy



Azimuthally symmetric around wavevector

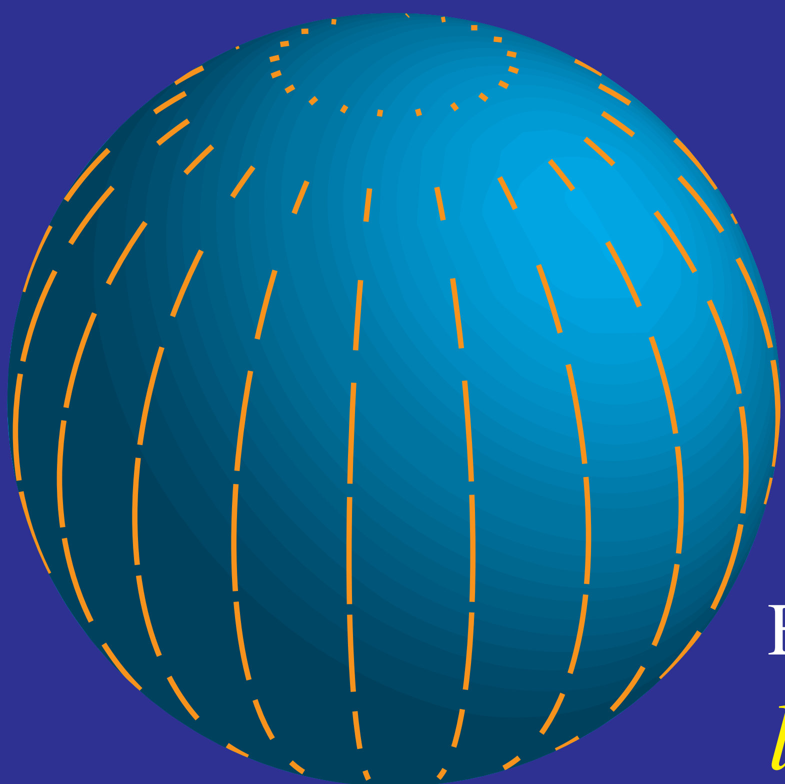
Whence Polarization Anisotropy?

- Observed photons scatter into the line of sight
- Polarization arises from the projection of the quadrupole on the transverse plane



Polarization Multipoles

- Mathematically pattern is described by the tensor (spin-2) spherical harmonics [eigenfunctions of Laplacian on trace-free 2 tensor]
- Correspondence with scalar spherical harmonics established via Clebsch-Gordon coefficients (spin x orbital)
- Amplitude of the coefficients in the spherical harmonic expansion are the multipole moments; averaged square is the power

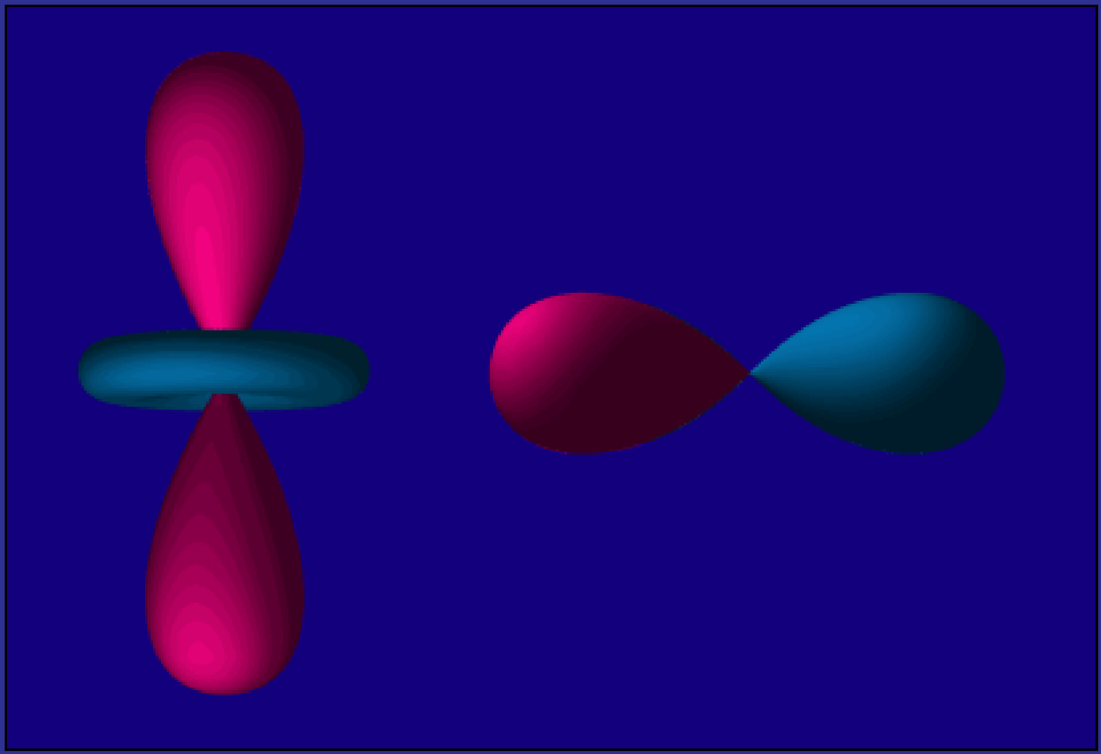


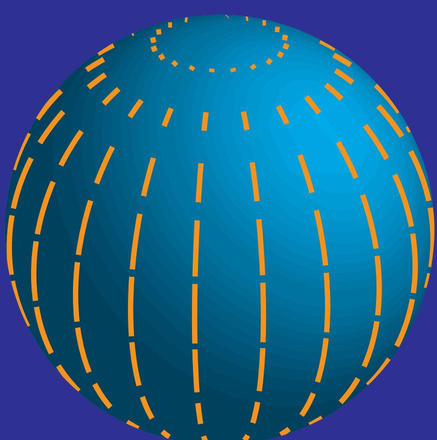
E-tensor harmonic

$l=2, m=0$

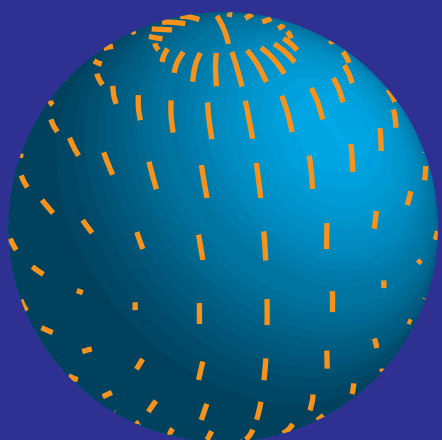
Gravitational Wave Pattern

- Projection of the quadrupole anisotropy gives polarization pattern
- Transverse polarization of gravitational waves breaks azimuthal symmetry





density perturbation

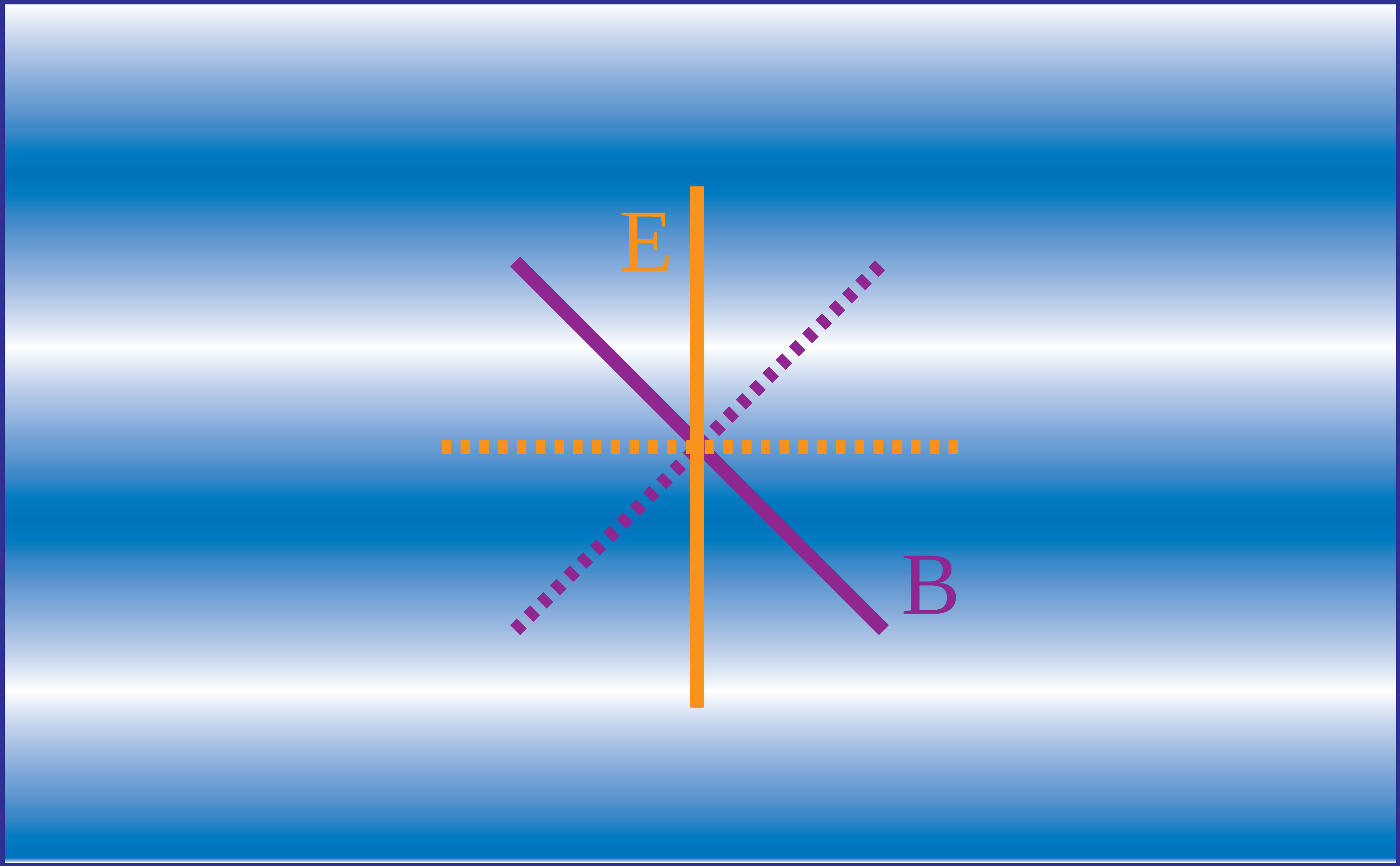


gravitational wave

Electric & Magnetic Polarization

(a.k.a. gradient & curl)

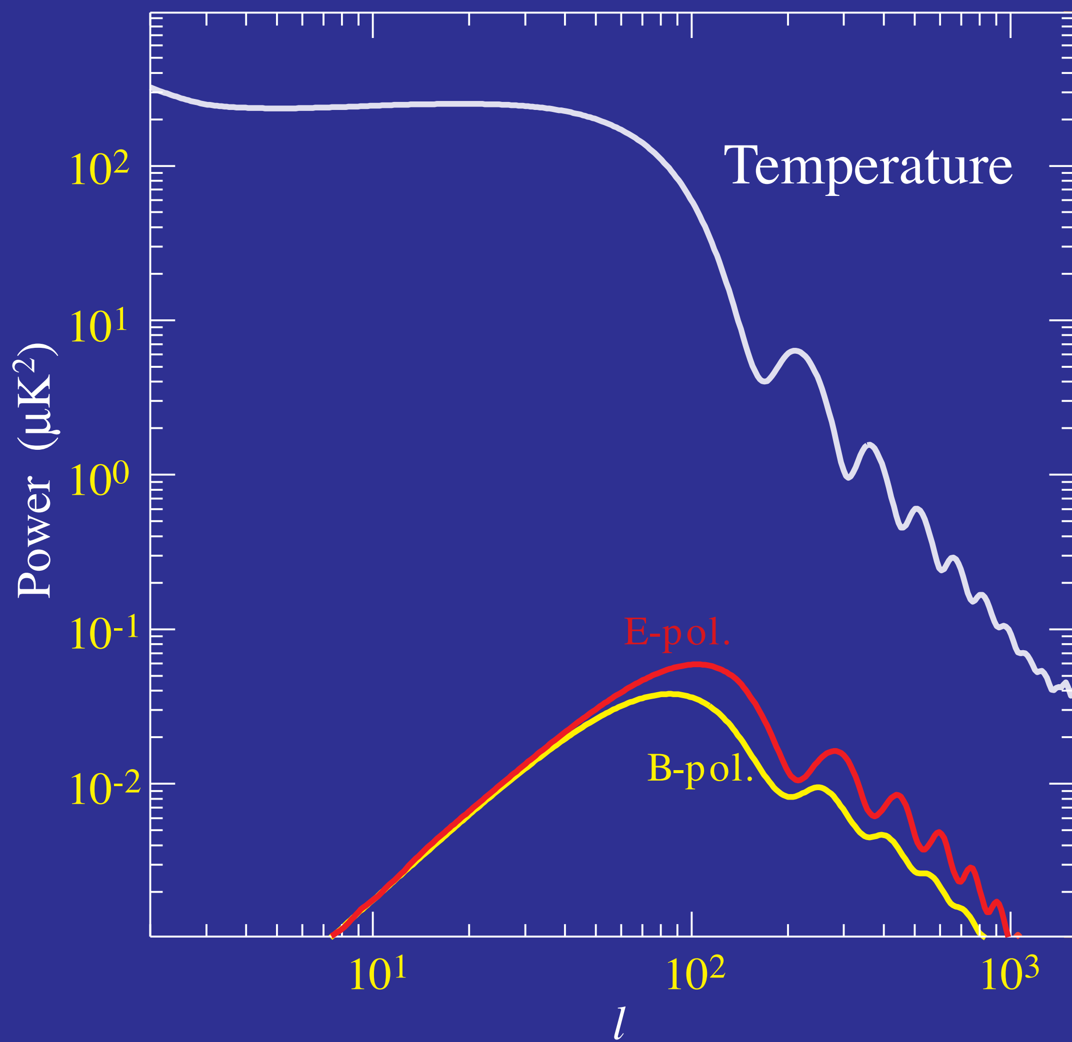
- Alignment of principal vs polarization axes (curvature matrix vs polarization direction)



Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)

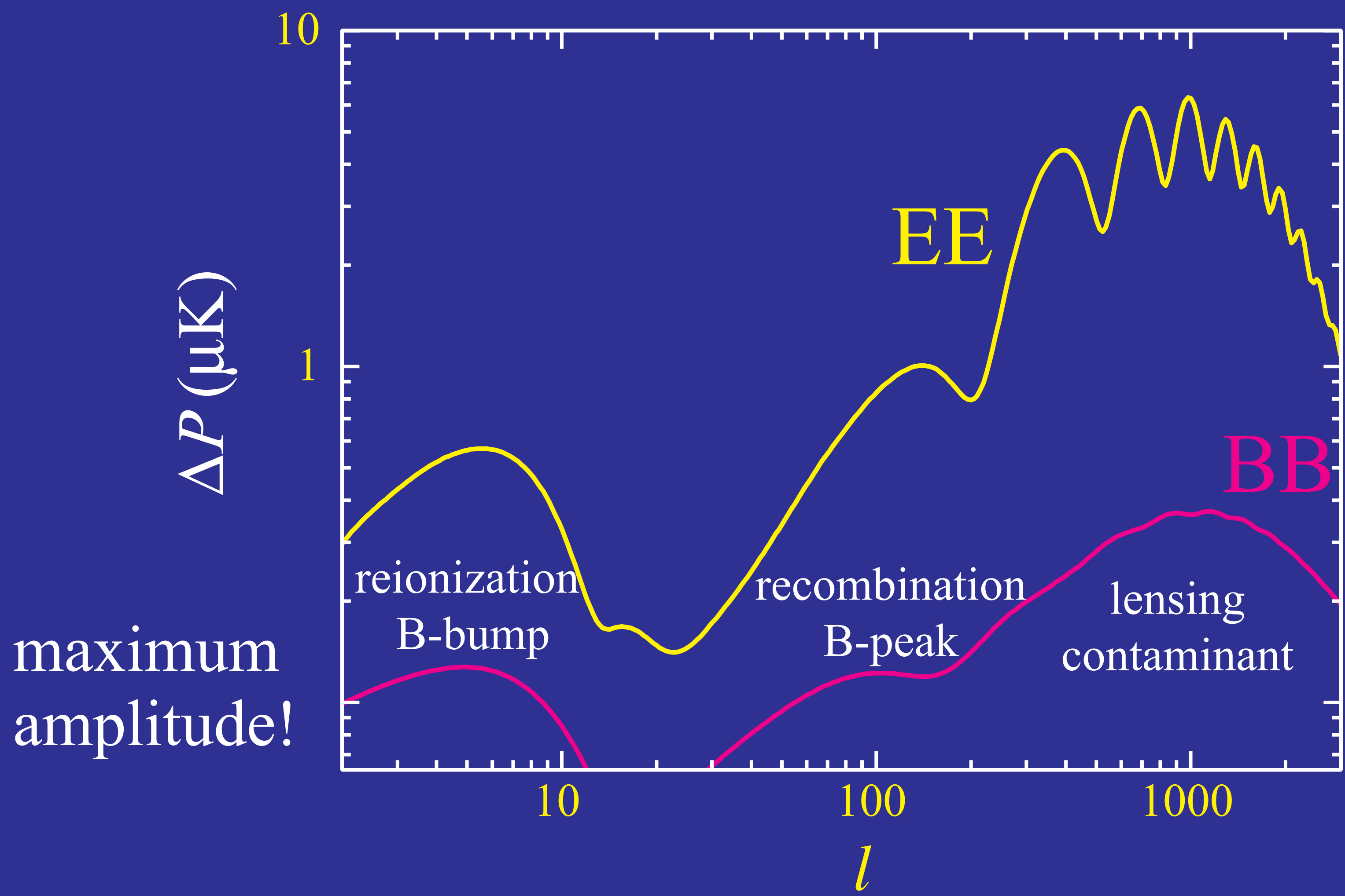
Recombination B-Modes

- Rescattering of quadrupoles at recombination yield a peak in B-modes



Polarized Landscape

- Two scattering epochs: recombination and reionization leave two imprints on B-modes



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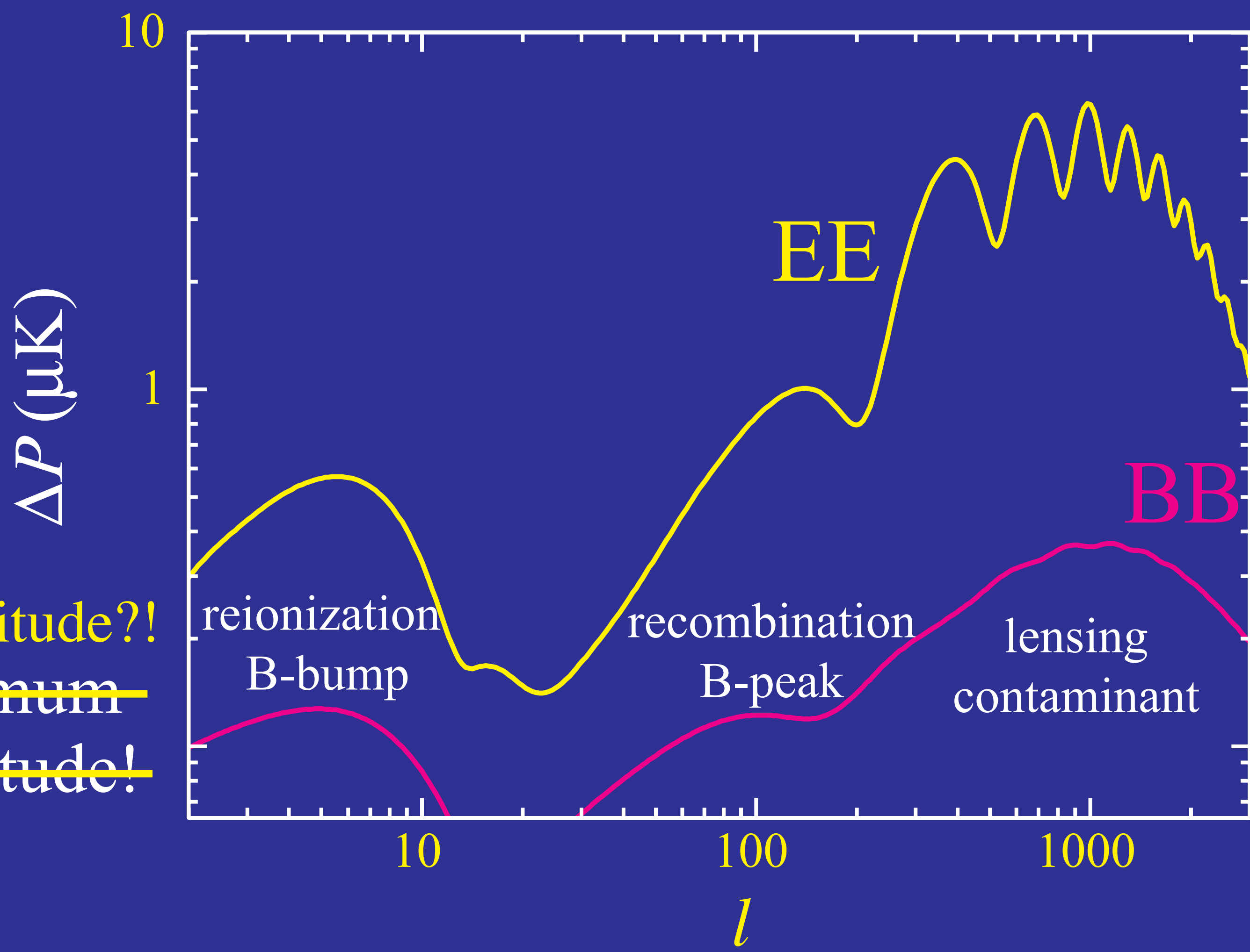
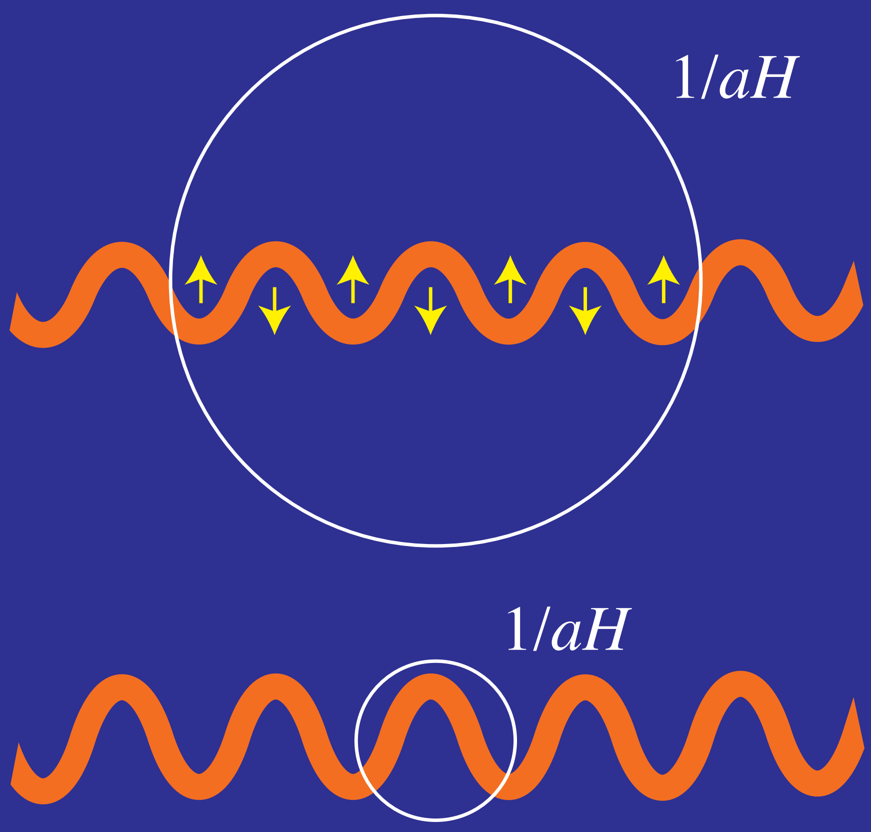


true amplitude?!
~~maximum~~
~~amplitude!~~

Gravitational Waves and Inflation

Gravitational Waves during Inflation

- During acceleration epoch gravity waves behave oppositely to deceleration epoch
- Oscillate inside the horizon and freeze when crossing horizon



Gravitational Waves

- Gravitational wave amplitude $h_{+,\times}$ satisfies same Klein-Gordon equation as scalars
- Just like inflaton ϕ, quantum fluctuations freeze out at horizon crossing with power per $\ln k$ given by the Hubble scale H

$$\Delta^2_{\delta\phi} = \frac{H^2}{(2\pi)^2}; \qquad \Delta^2_{+,\times} = \frac{2}{M^2_{\rm pl}}\frac{H^2}{(2\pi)^2}$$

- By the Friedmann equation

$$H^2 = \frac{\rho}{3M^2_{\rm pl}} \approx \frac{V(\phi)}{3M^2_{\rm pl}}$$

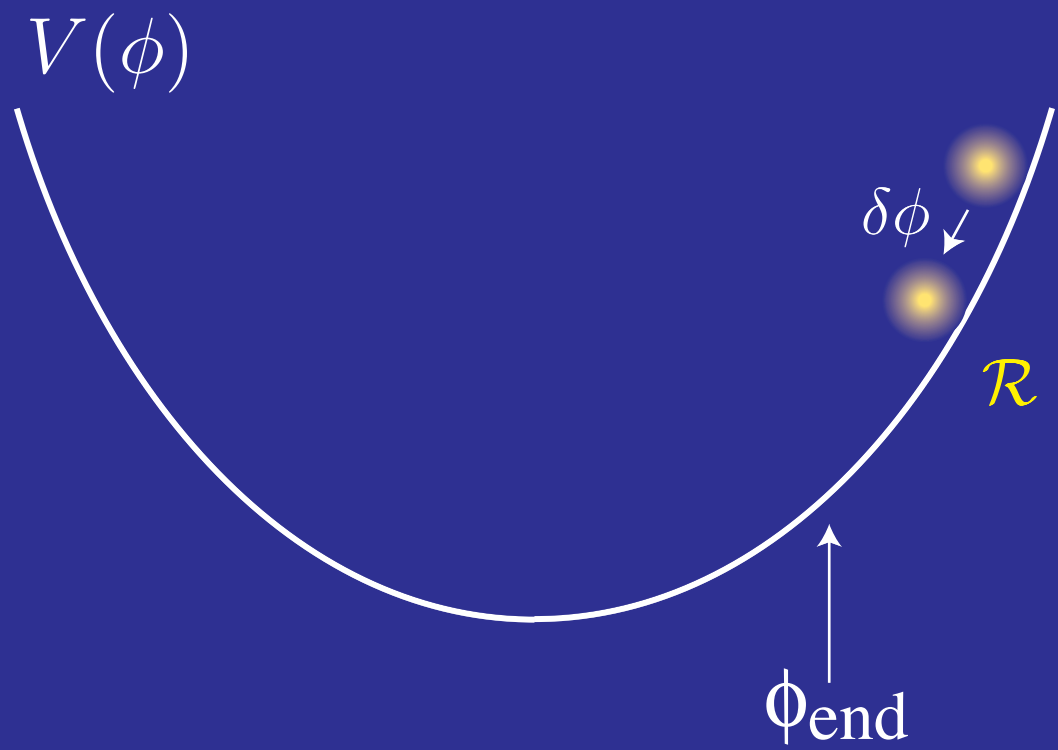
Measurement of B-modes determines energy scale $E_i = V^{1/4}$

$$B_{\rm peak} \approx 0.024 \left(\frac{E_i}{10^{16}{\rm GeV}} \right)^2 \mu{\rm K}$$

Tensor-Scalar Ratio

- Unlike gravitational waves, inflaton fluctuations determine when inflation ends in a given patch, changing the scale factor or curvature
- Curvature power is enhanced by the slowness of the roll

$$\varepsilon = \frac{\dot{\phi}^2}{2H^2 M_{\rm pl}^2} \qquad \Delta^2_{\mathcal{R}} = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \varepsilon}$$



$$\mathcal{R} = -\frac{\delta a}{a} = -H\frac{\delta\phi}{\dot{\phi}}$$

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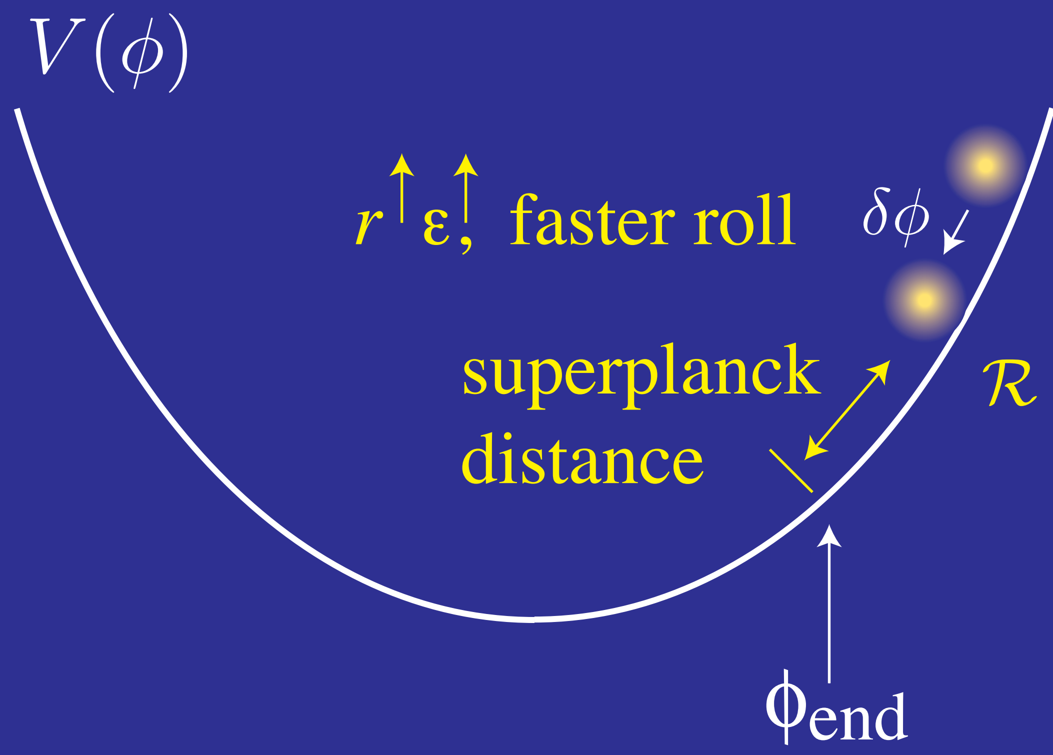

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Tensor-Scalar Ratio

- Tensor-scalar ratio r

$$r \equiv 4\frac{\Delta_+^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon$$

- A large r implies a large ϵ and a large roll

$$\epsilon = \frac{1}{2M_{\rm pl}^2}\left(\frac{d\phi}{d\ln a}\right)^2$$

- Observable scales span $d\ln a = d\ln k \sim 5$ so

$$\Delta\phi \approx 5\frac{d\phi}{d\ln a} = 5(r/8)^{1/2}M_{\rm pl} \approx 0.6(r/0.1)^{1/2}M_{\rm pl}$$

- For $r = 0.2$ the field must roll by at least $M_{\rm pl}$
- Difficult to protect the flat potential across this large a range in field space

$n_S - r$ Plane

- Scalar tilt

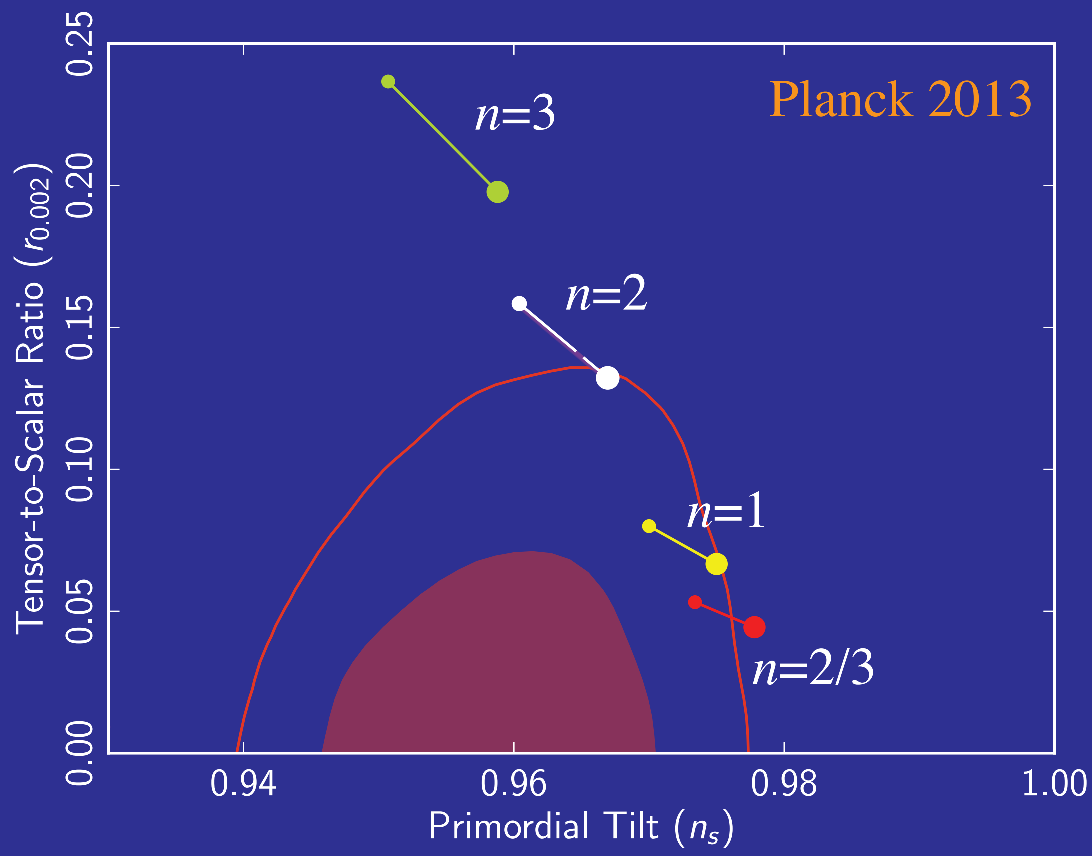
$$\frac{d\ln\Delta^2_{\mathcal{R}}}{d\ln k} \equiv n_S - 1$$
$$= 2\frac{d\ln H}{d\ln k} - \frac{d\ln\epsilon}{d\ln k} = -2\epsilon - \frac{d\ln\epsilon}{d\ln k}$$

- Measuring both $n_S - 1$ and r constrain the inflationary model
- In slow roll, related to derivatives of potential

$$\epsilon \approx \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2$$
$$\frac{d\ln\epsilon}{d\ln k} = 4\epsilon - 2M_{\rm pl}^2\frac{V''}{V}$$

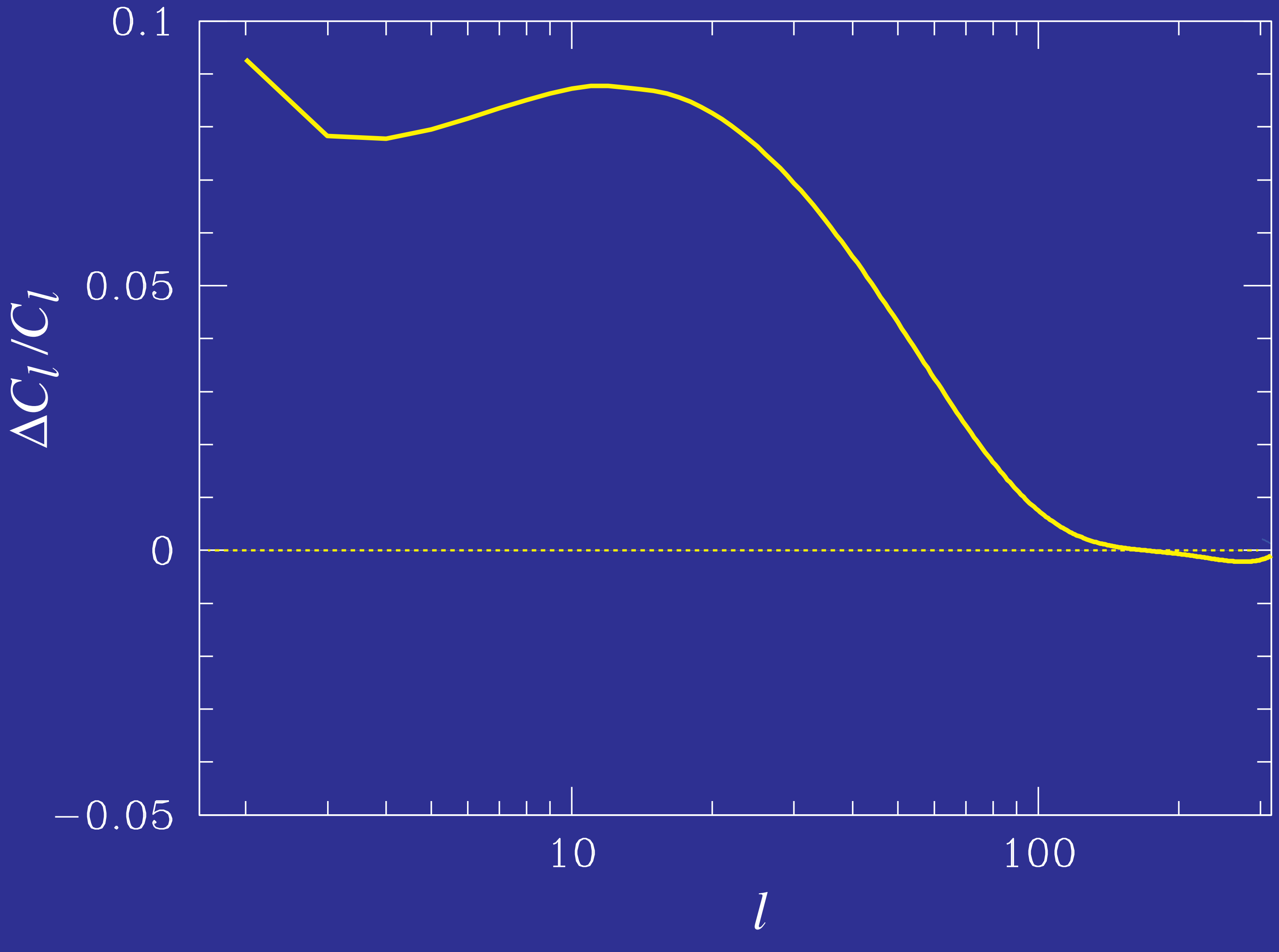
Tensor-Scalar Ratio

- Monomial, scale free, potentials $V(\phi) \sim \phi^n$ have both terms comparable and related
- Value depends position on potential efolds to end of inflation
- Planck temperature based constraints applicable to these models where scalar spectrum is a power law



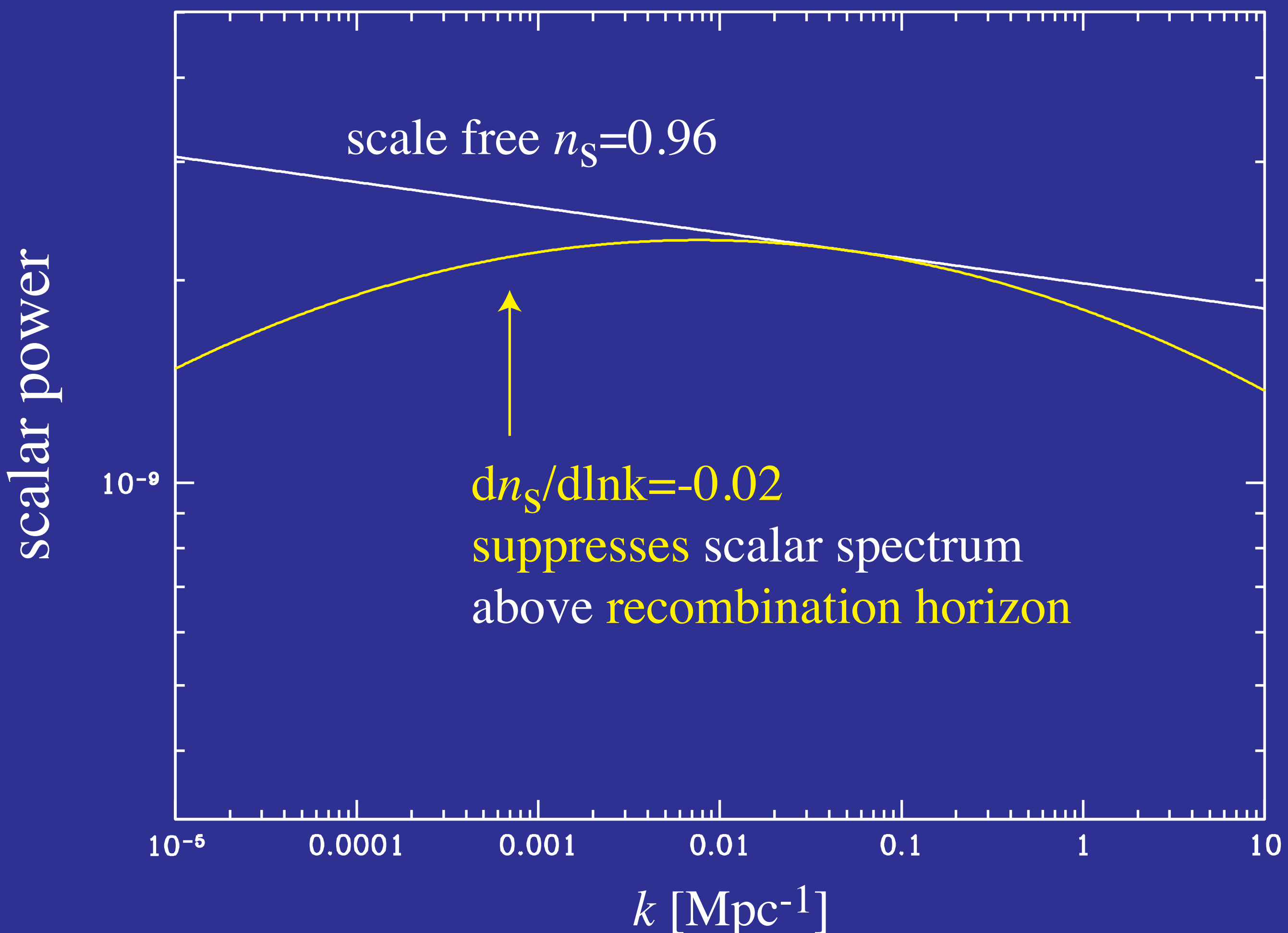
Tensor Temperature Excess

- r=0.2 and fixed acoustic peaks produces an excess in temperature power spectrum that is not observed (limits r<0.11 95% CL)



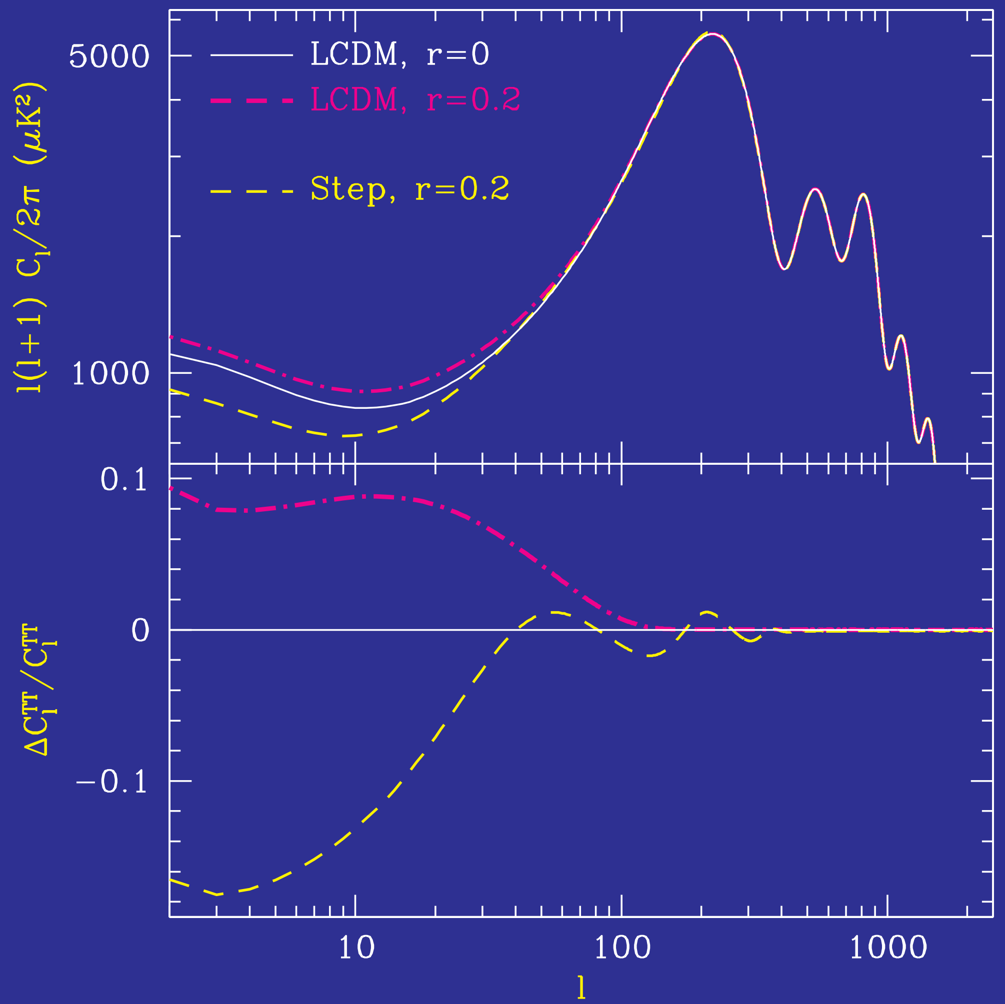
Running of the Tilt

- Introducing scale by running tilt changes inferences from temperature spectrum, weakening upper limit on r
- r=0.2 requires a large running of order the tilt, not compatible with rolling on simple scale-free potentials



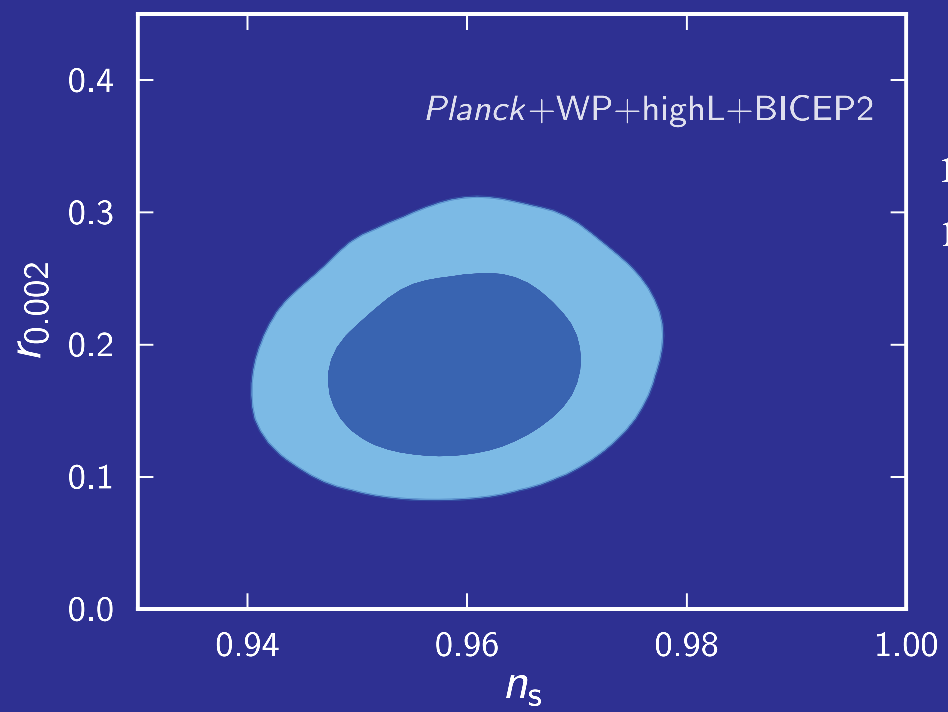
Tensor Temperature Excess

- Prefers sharper change than running, suppression over 1efold (excess exists even without tensors)
- Steps in power from steps in ε (or sound speed)



Tension with BICEP2

- Tension between temperature and polarization inferences can be alleviated if scalar spectrum is not scale free
- Large running tilt or feature that removes scalar power larger than the horizon at recombination



running marginalized

Polarized Landscape

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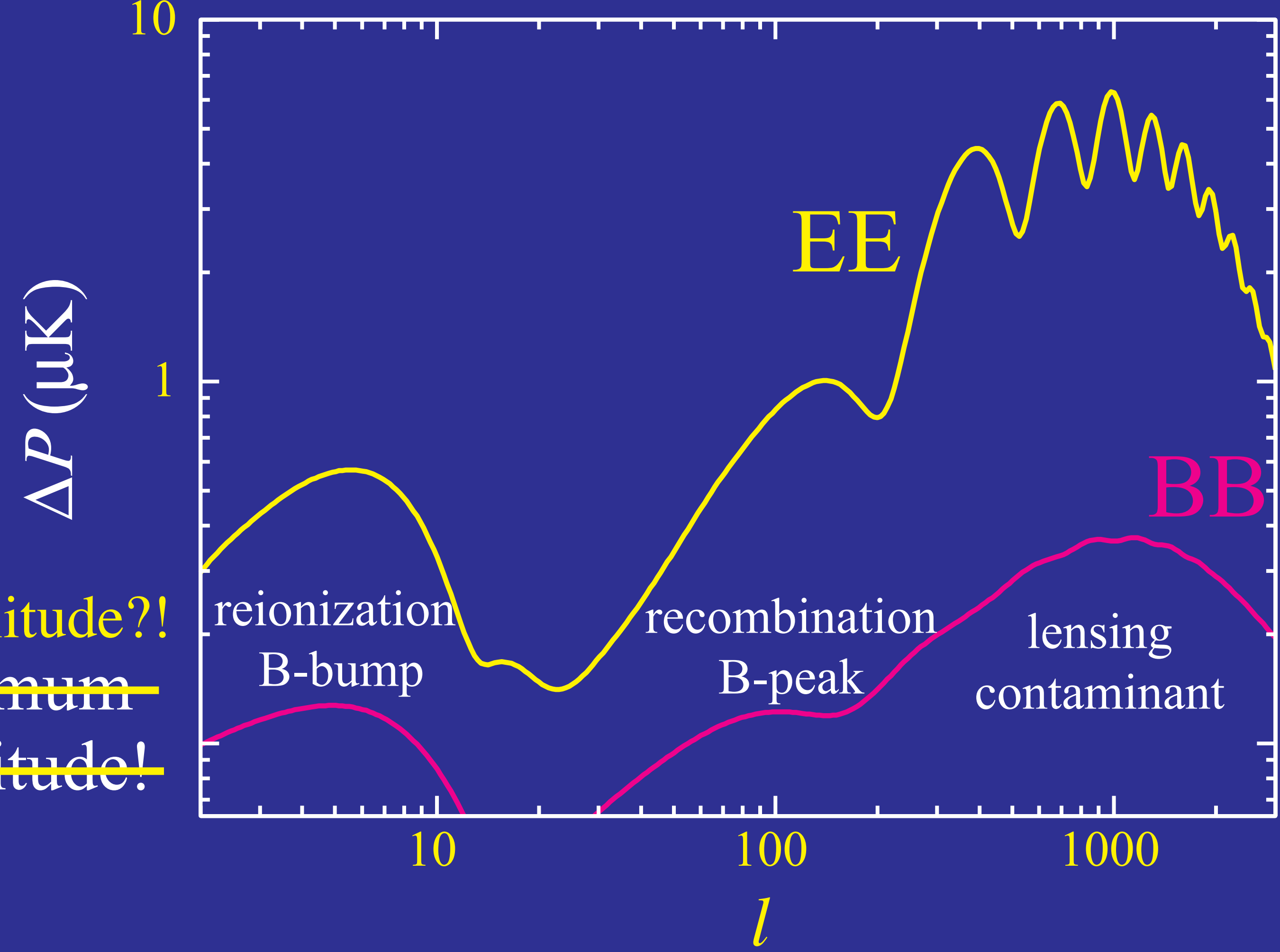
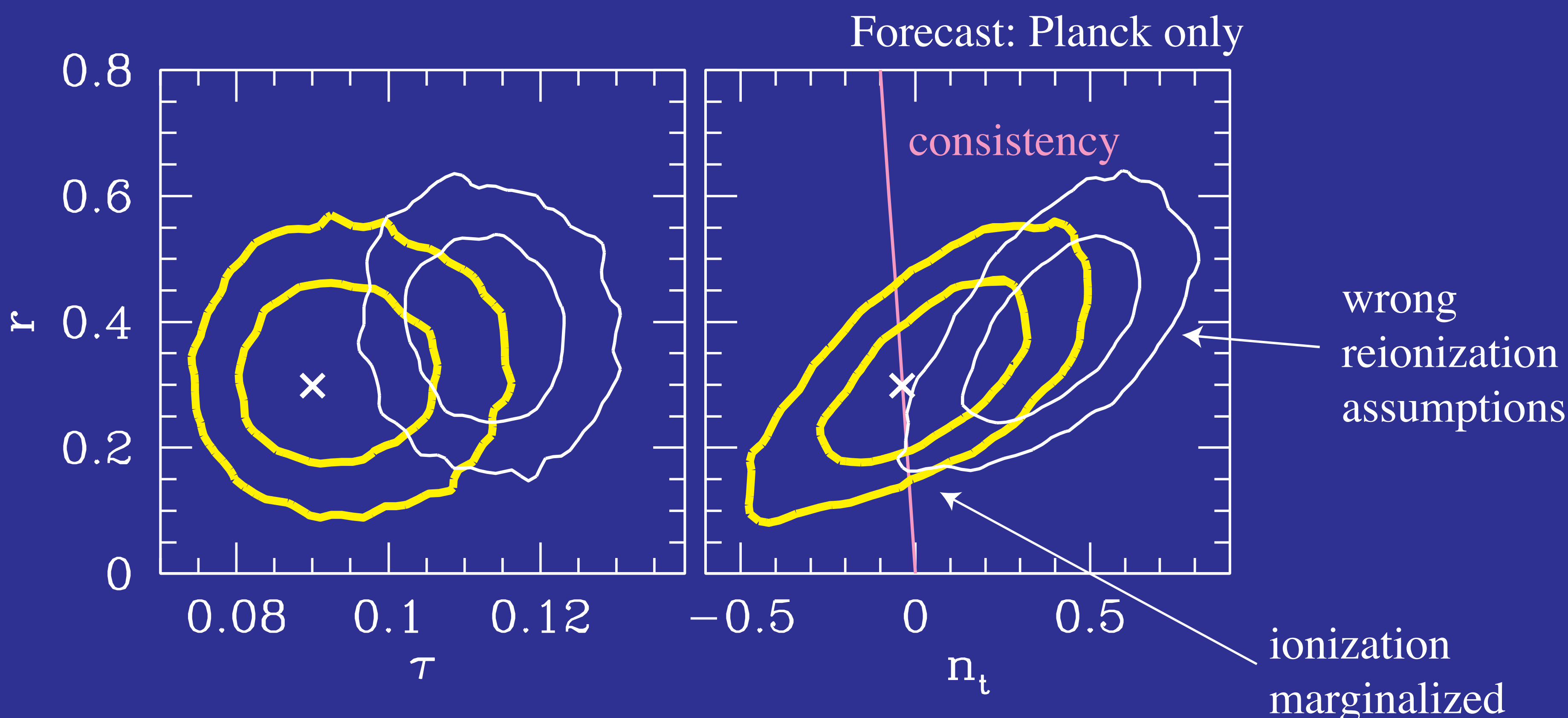


true amplitude?!
~~maximum~~
~~amplitude!~~

Reionization B-Bump

- r=0.2, Planck has sensitivity to detect reionization B, even internally, provide first check of scale invariance (consistency relation)
- Signal depends on reionization history but enough information to disentangle



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