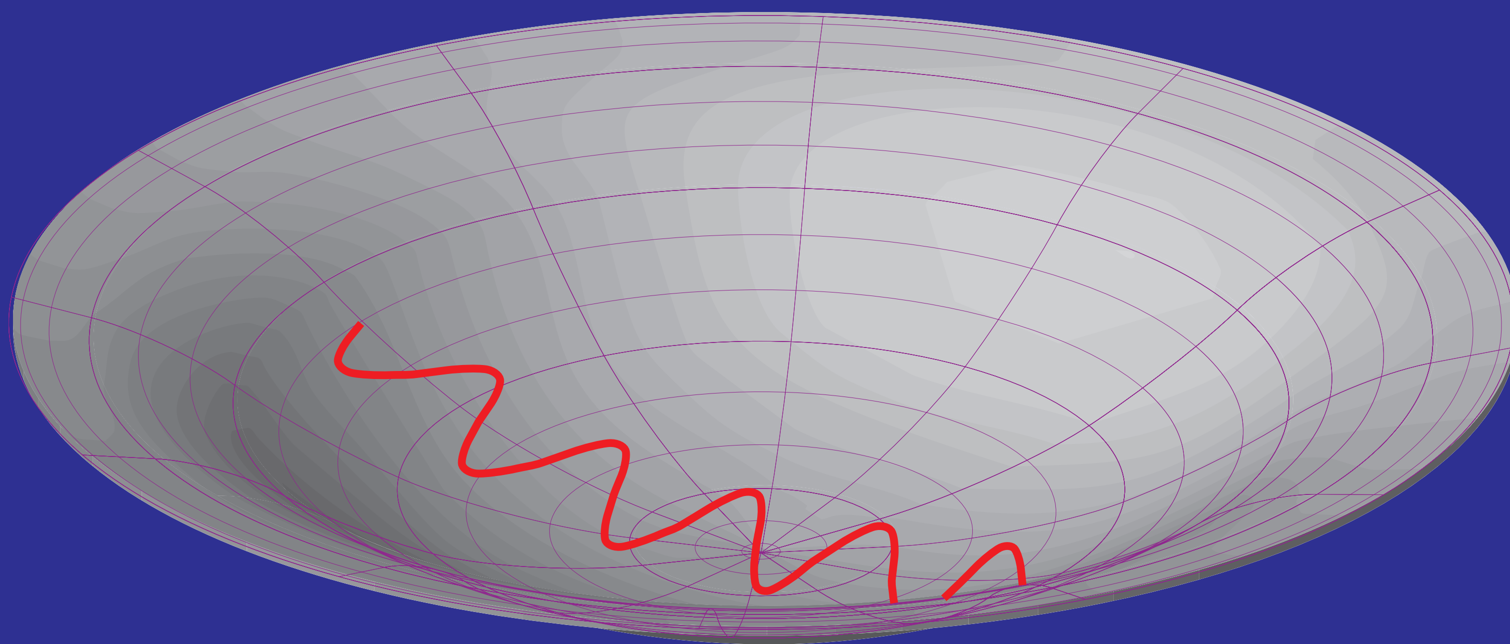
Secondary CMB Anisotropy



III: Cosmic Acceleration

Wayne Hu

Cabo, January 2009

Secondary CMB Anisotropy

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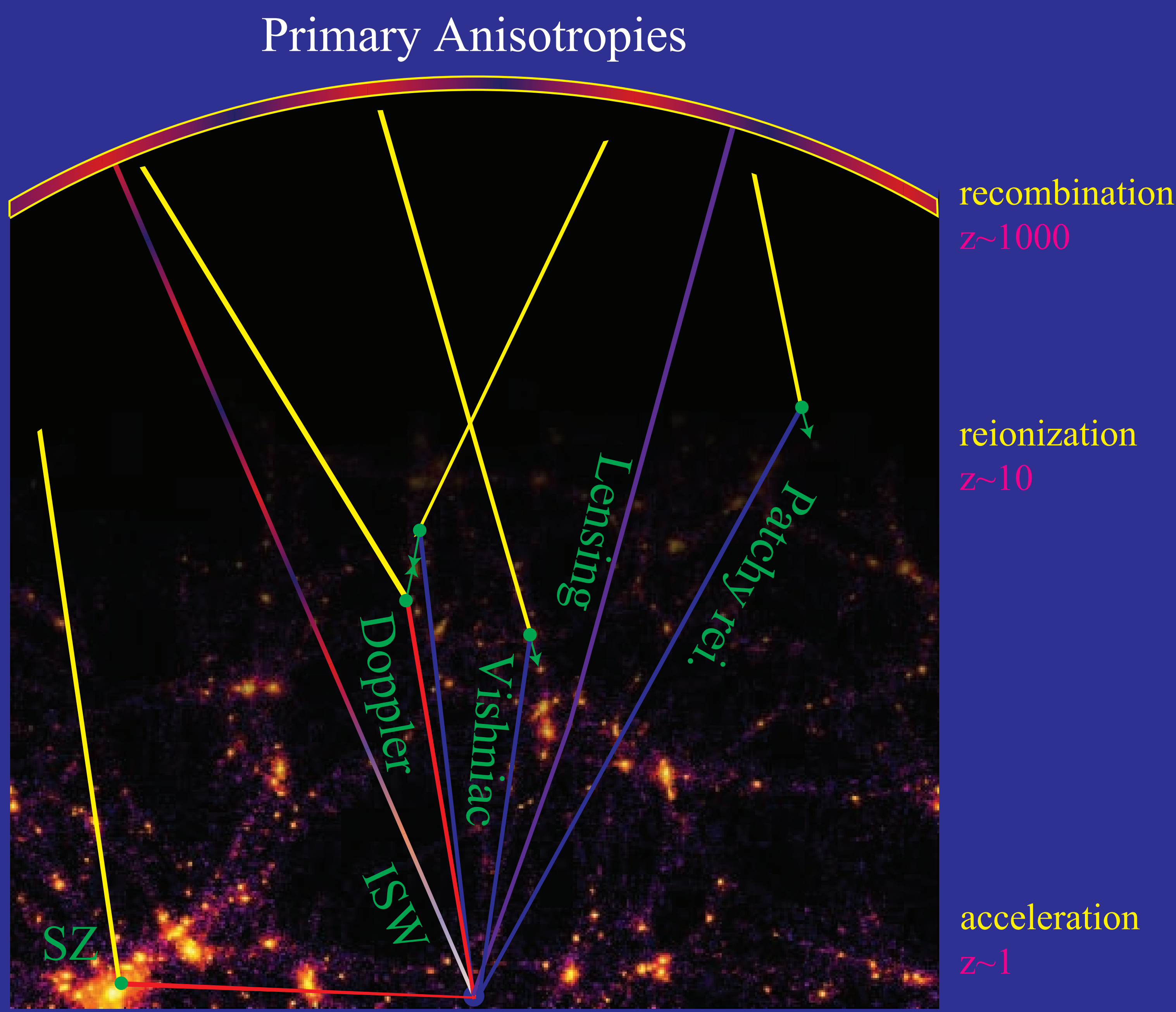
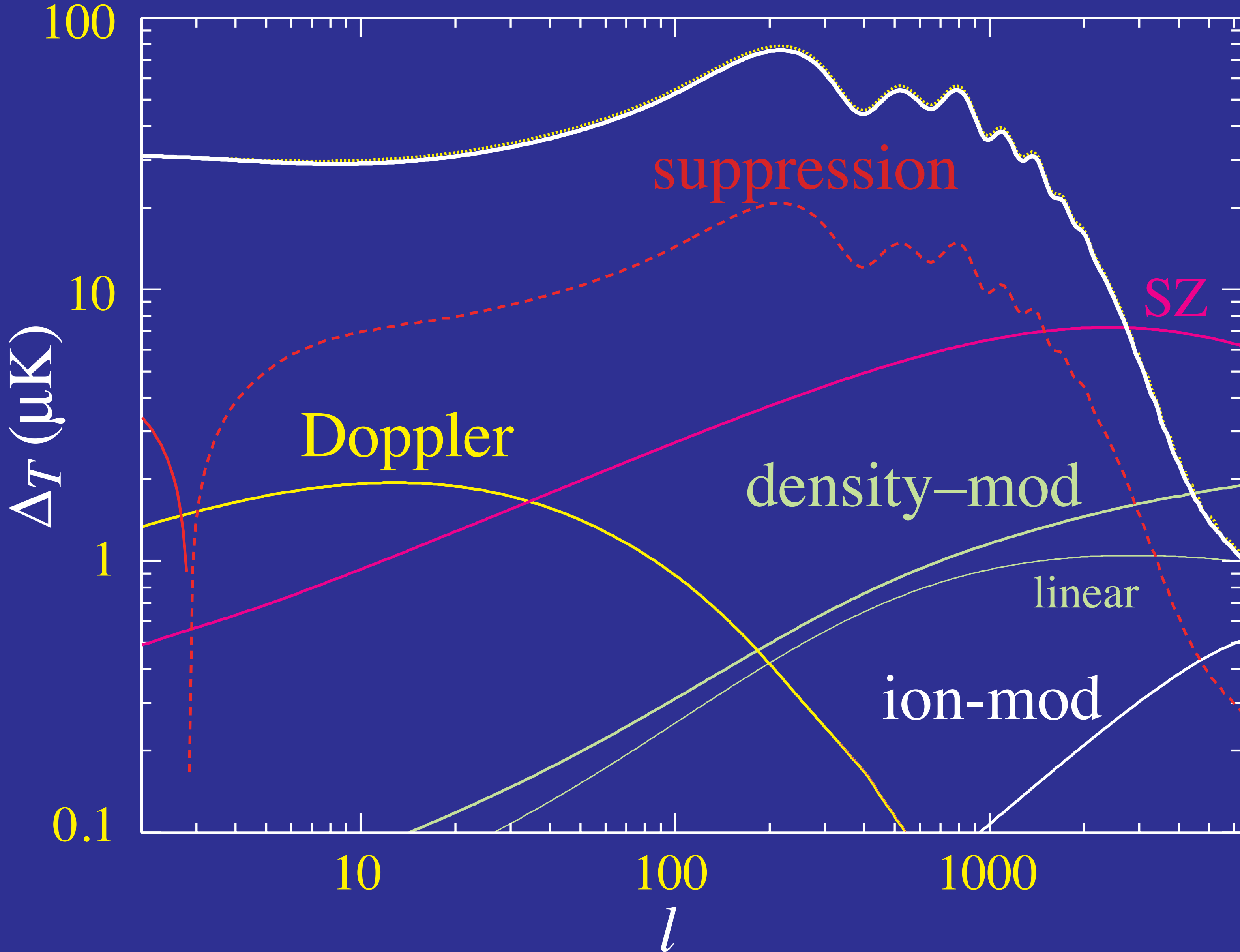
Cabo, January 2009

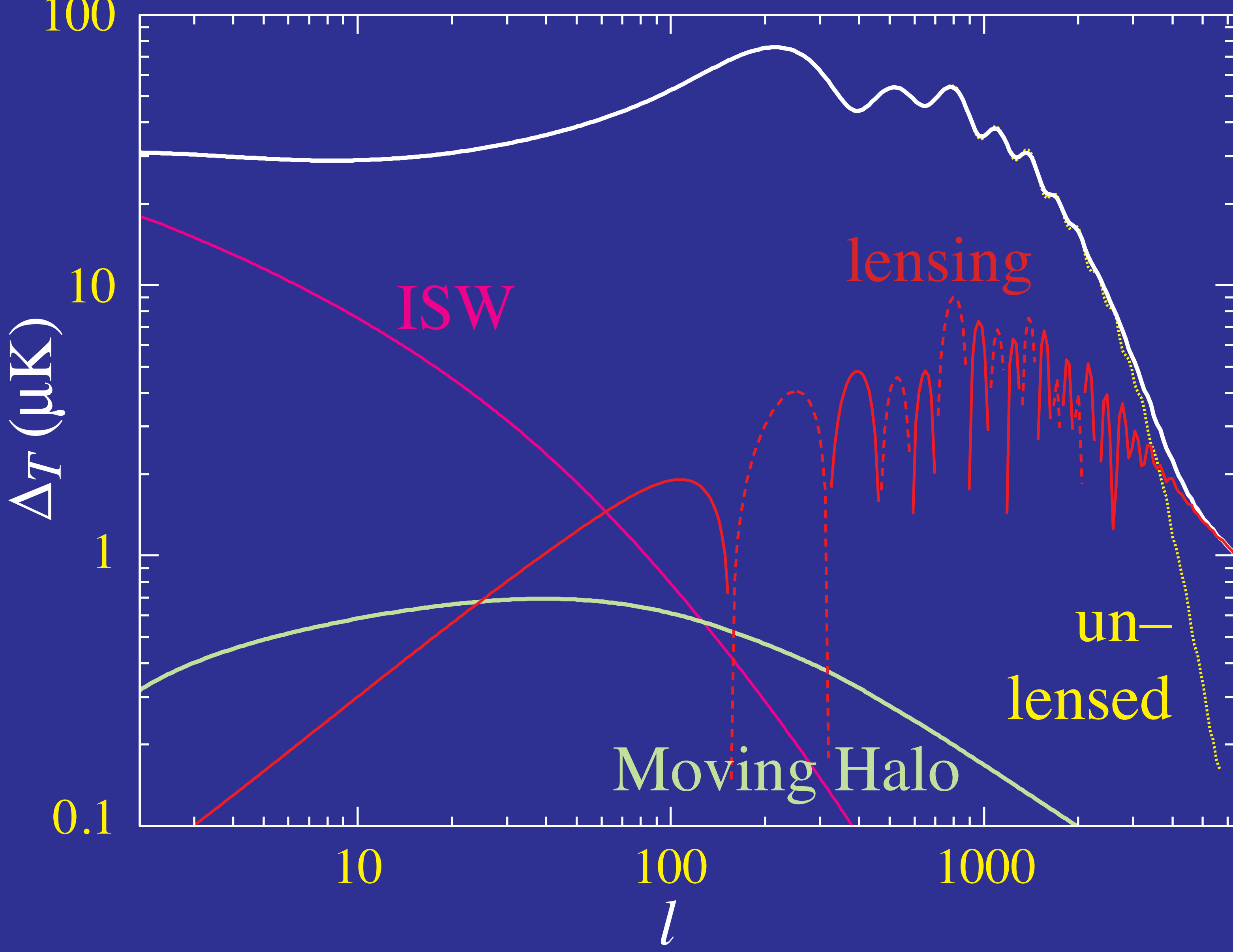
Physics of Secondary Anisotropies



Scattering Secondaries



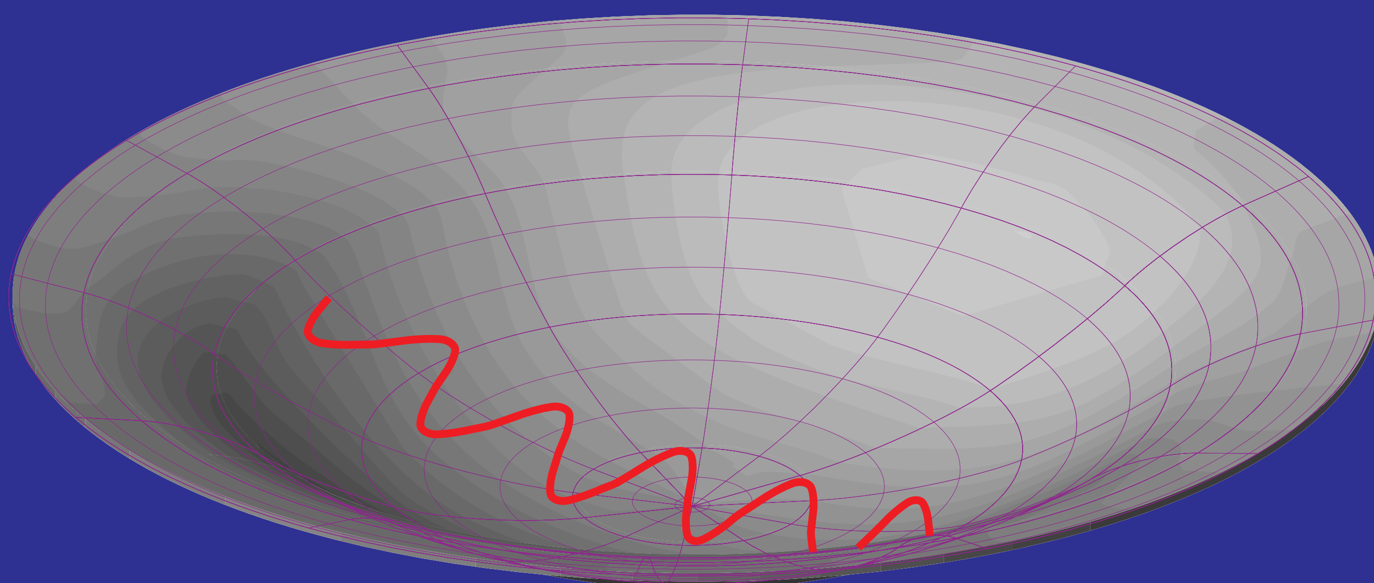
Gravitational Secondaries



Integrated Sachs-Wolfe Effect

ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: $\Delta T/T = 2\Delta\Phi$
- Effect from potential hills and wells cancel on small scales



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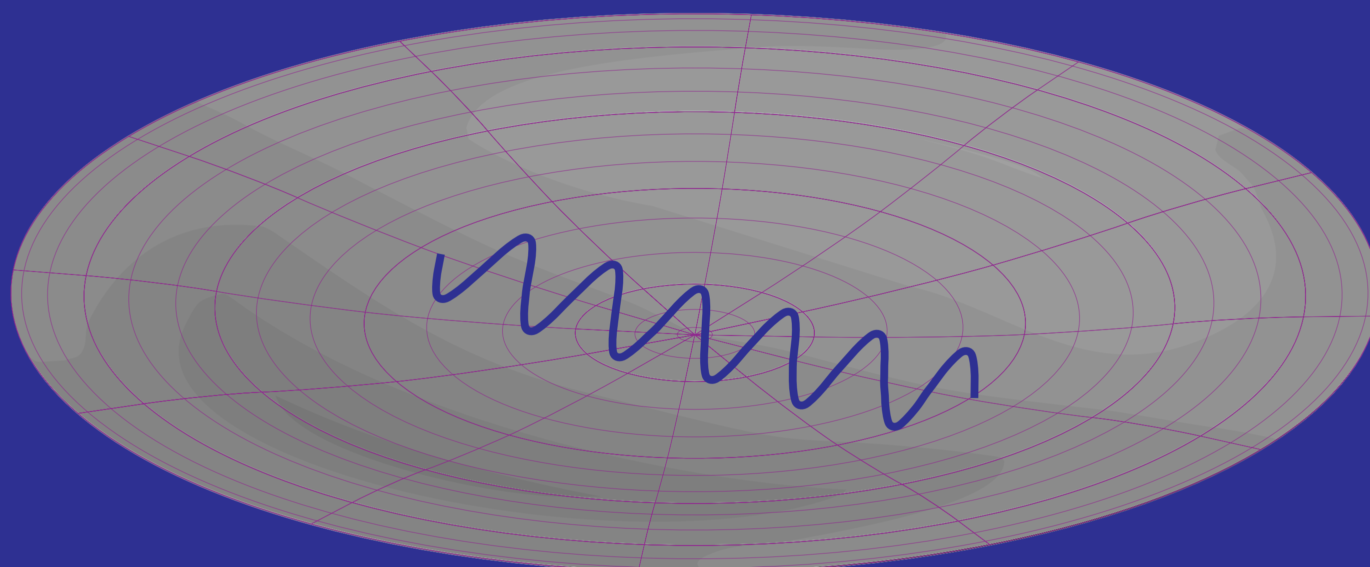

Smooth Energy Density & Potential Decay

- Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature ζ)

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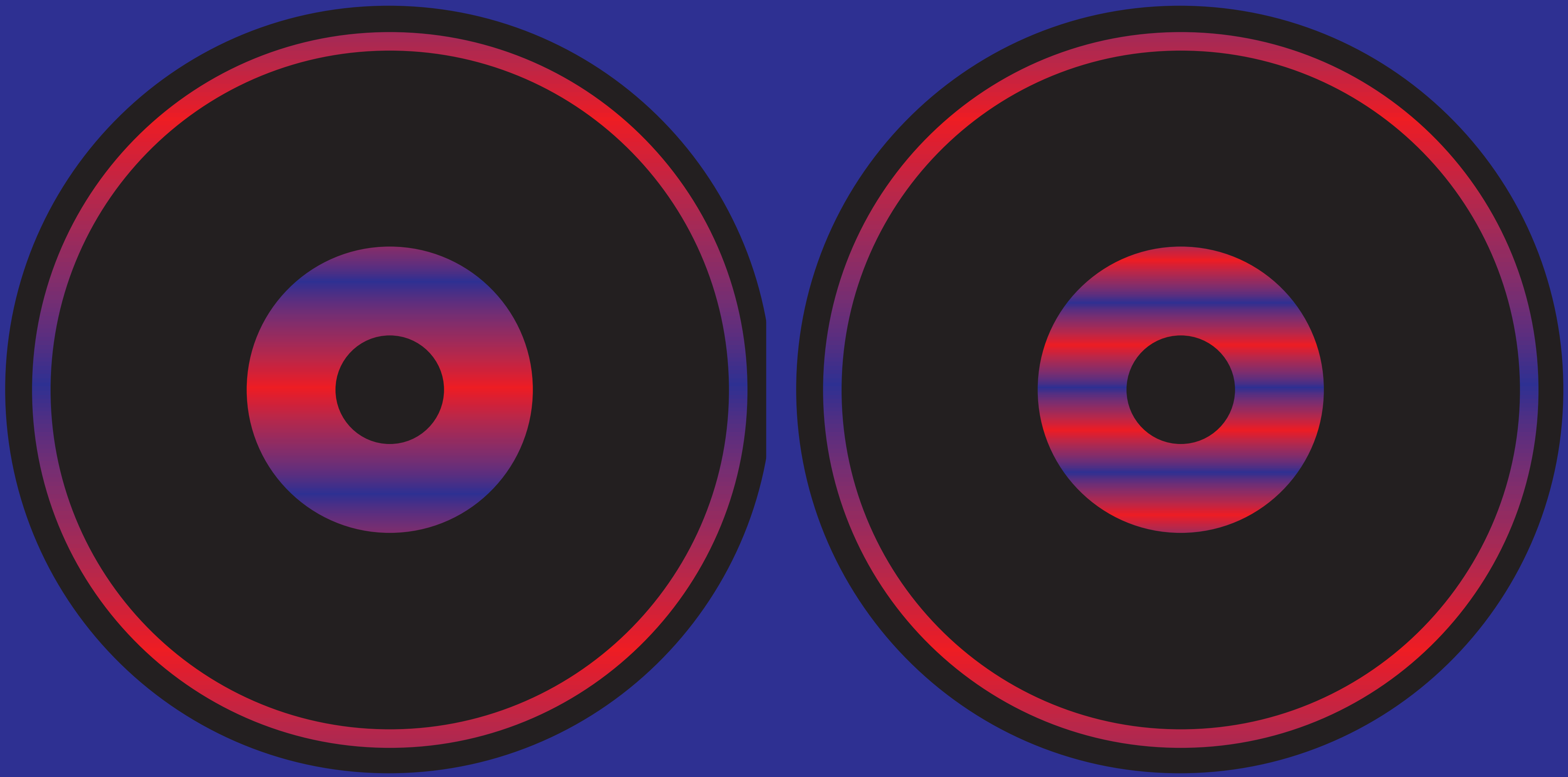
- A smooth component contributes
  density ρ to the expansion
  but not
  density fluctuation $\delta\rho$ to the Poisson equation
- Imbalance causes potential to decay once smooth component dominates the expansion

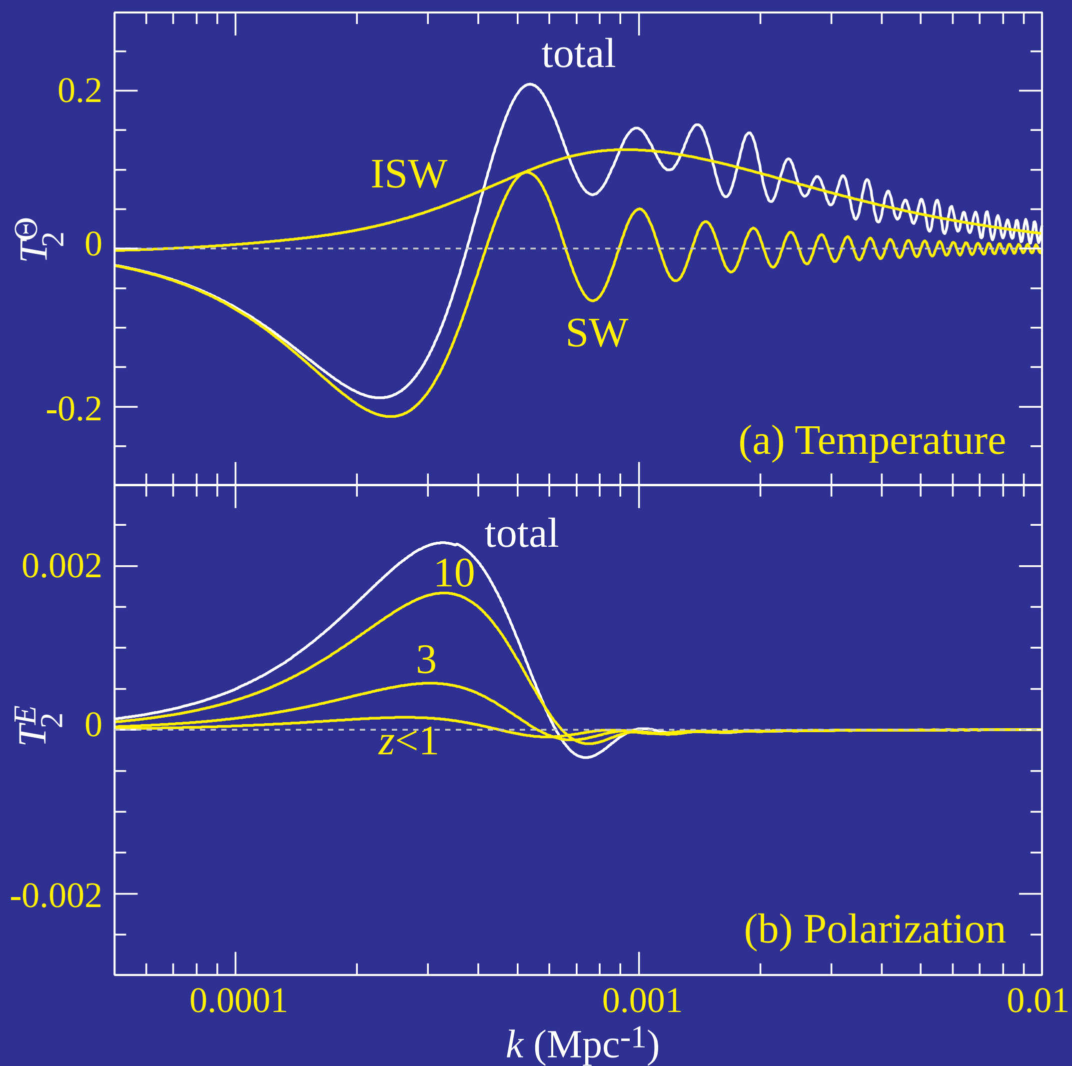
ISW Spatial Modes

- ISW effect comes from nearby acceleration regime
- Shorter wavelengths project onto same angle
- Broad source kernel: Limber cancellation out to quadrupole



Quadrupole Origins

- Transfer function for the quadrupole



Gordon & Hu (2004)

Smooth Energy Density & Potential Decay

- Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature ζ)
- A smooth component contributes

  density ρ to the expansion

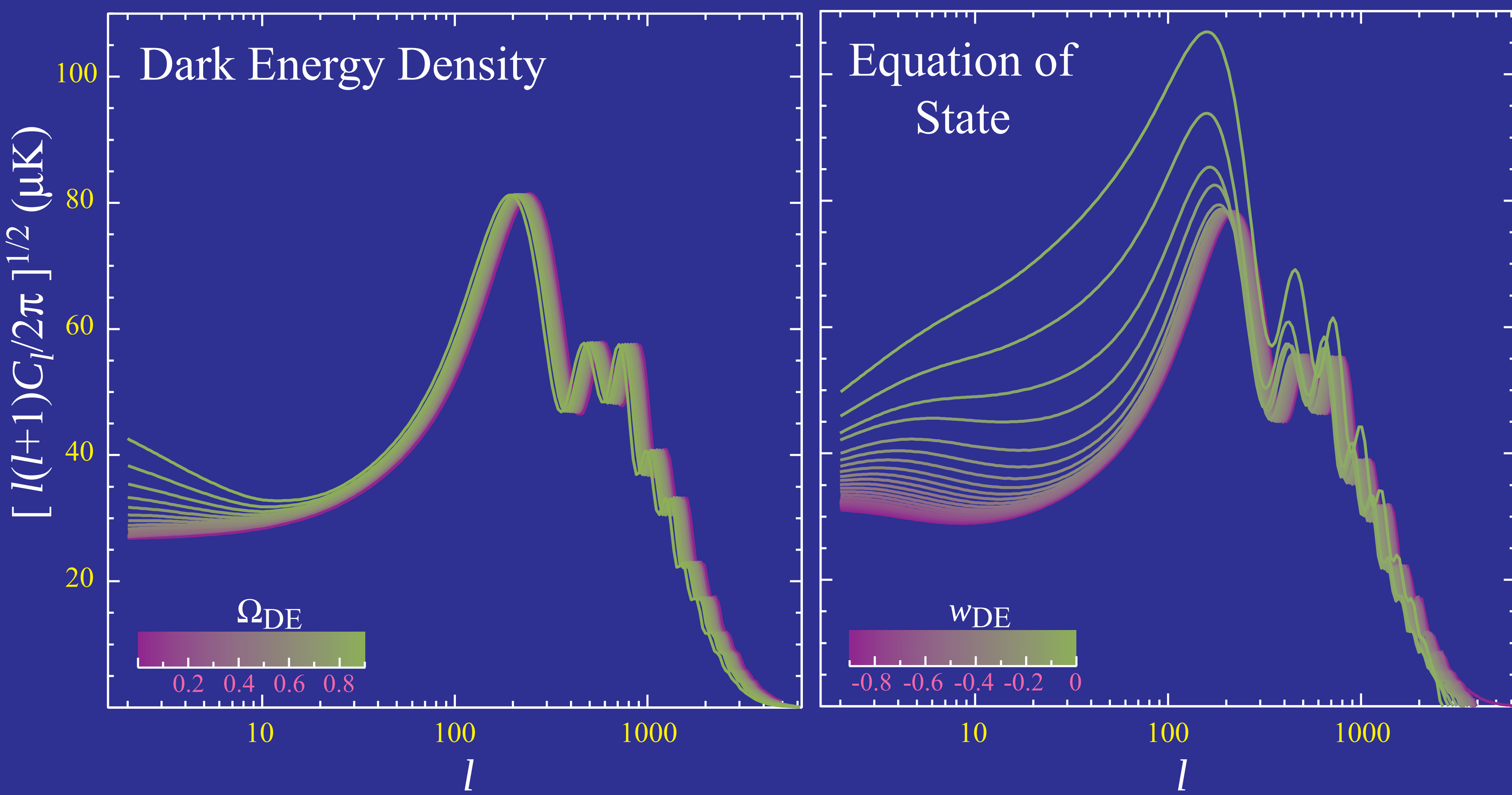
  but not

  density fluctuation $\delta\rho$ to the Poisson equation
- Imbalance causes potential to decay once smooth component dominates the expansion
- Scalar field dark energy (quintessence) is smooth out to the horizon scale (sound speed c_s=1)
- Potential decay measures the clustering properties and hence the particle properties of the dark energy

ISW & Dark Energy

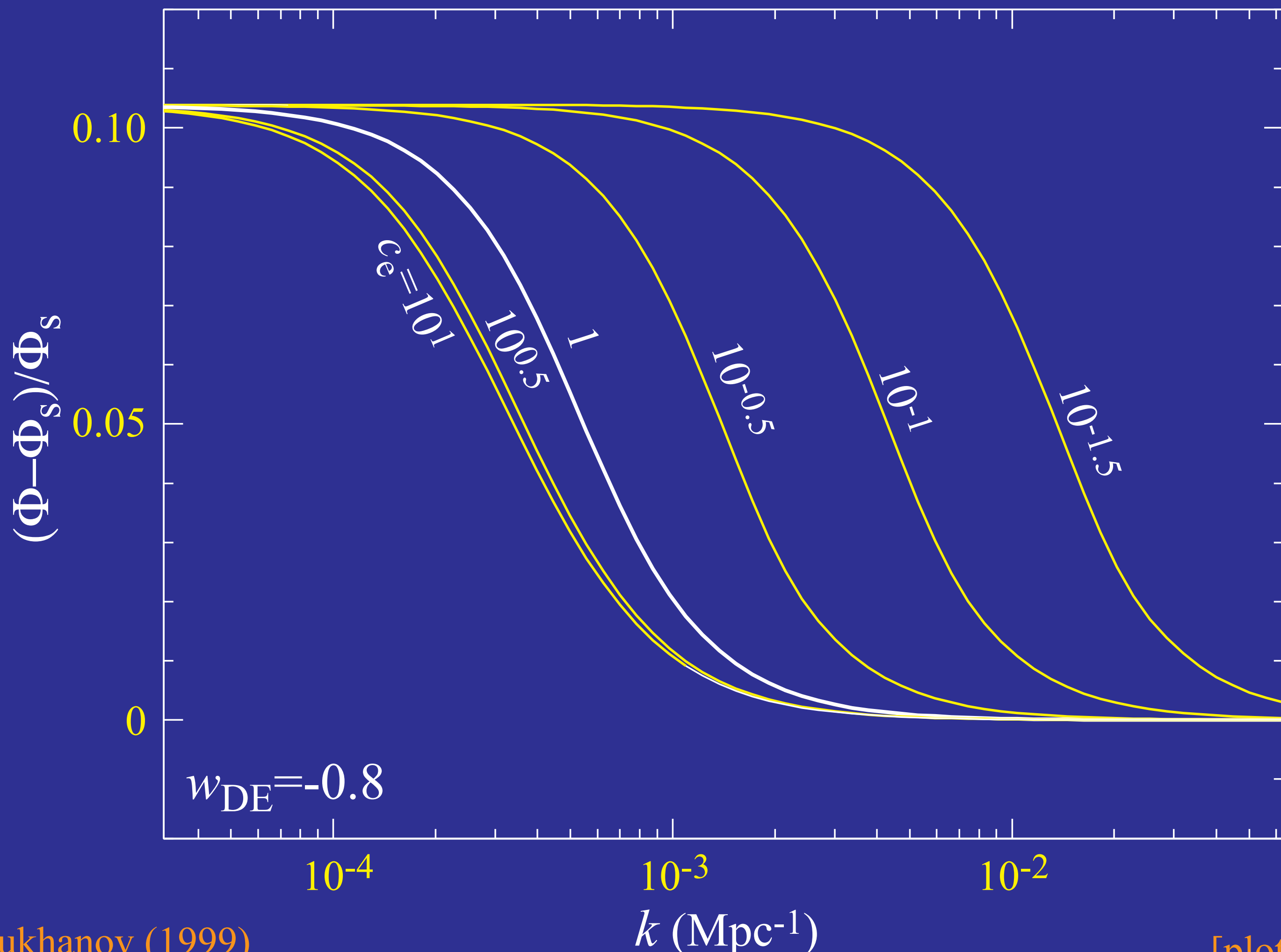
Dark Energy

- Peaks measure distance to recombination
- ISW effect constrains dynamics of acceleration



Dark Energy Sound Speed

- Smooth and clustered regimes separated by sound horizon
- Covariant definition: $c_e^2 = \delta p / \delta \rho$ where momentum flux vanishes
- For scalar field dark energy uniquely defined by kinetic term

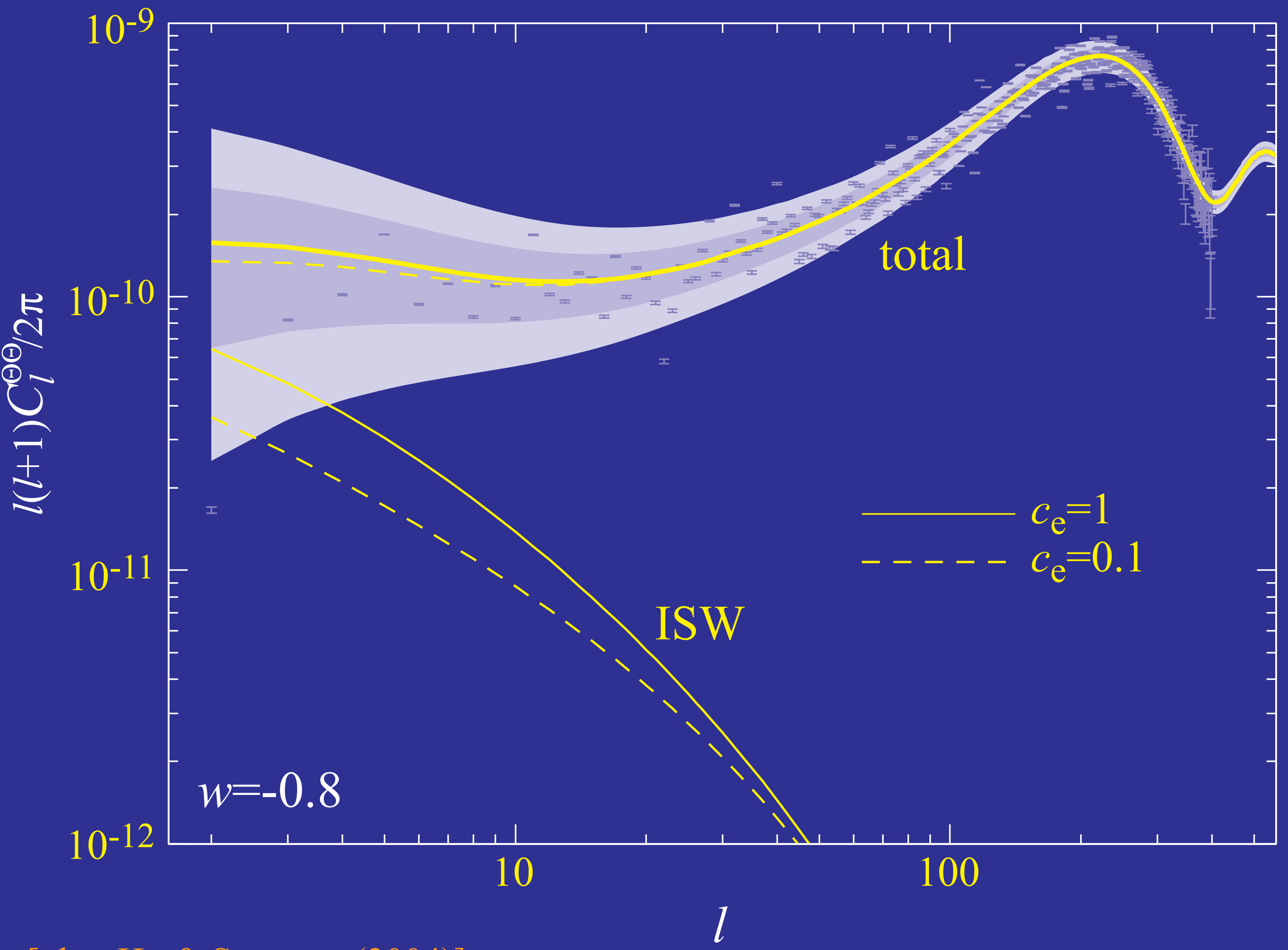


Hu (1998)
Garriga & Mukhanov (1999)

[plot: Hu & Scranton (2004)]

Dark Energy Clustering

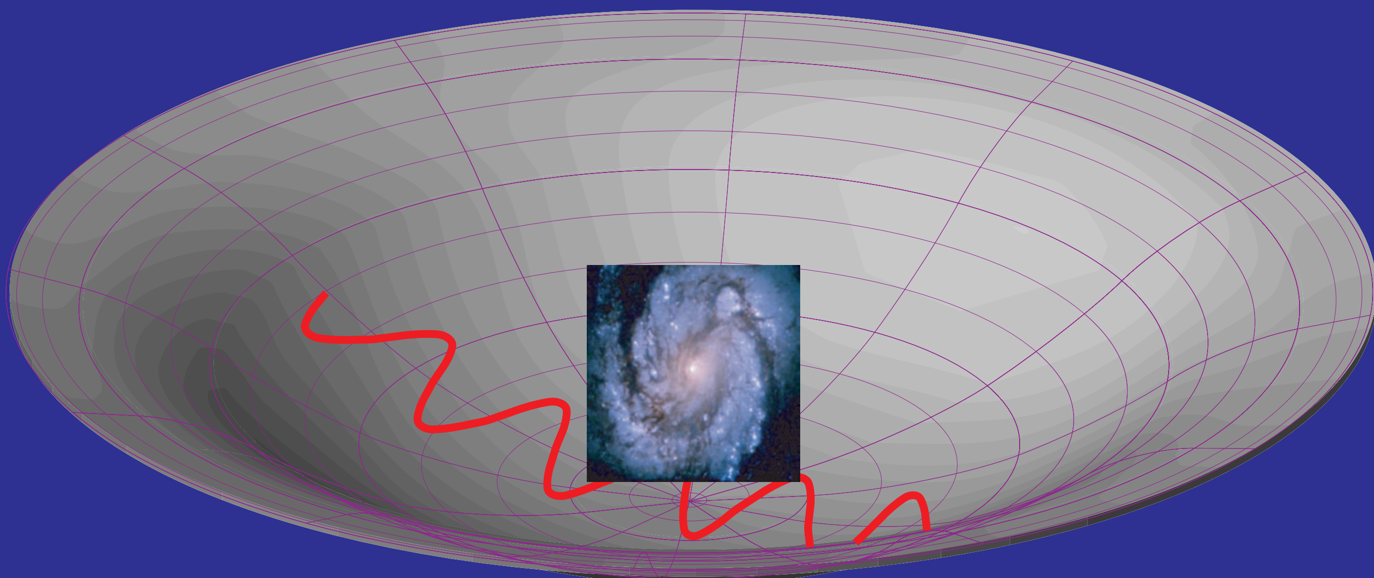
- ISW effect intrinsically sensitive to dark energy smoothness
- Large angle contributions reduced if clustered



Hu (1998); [plot: Hu & Scranton (2004)]

ISW-Galaxy Correlation

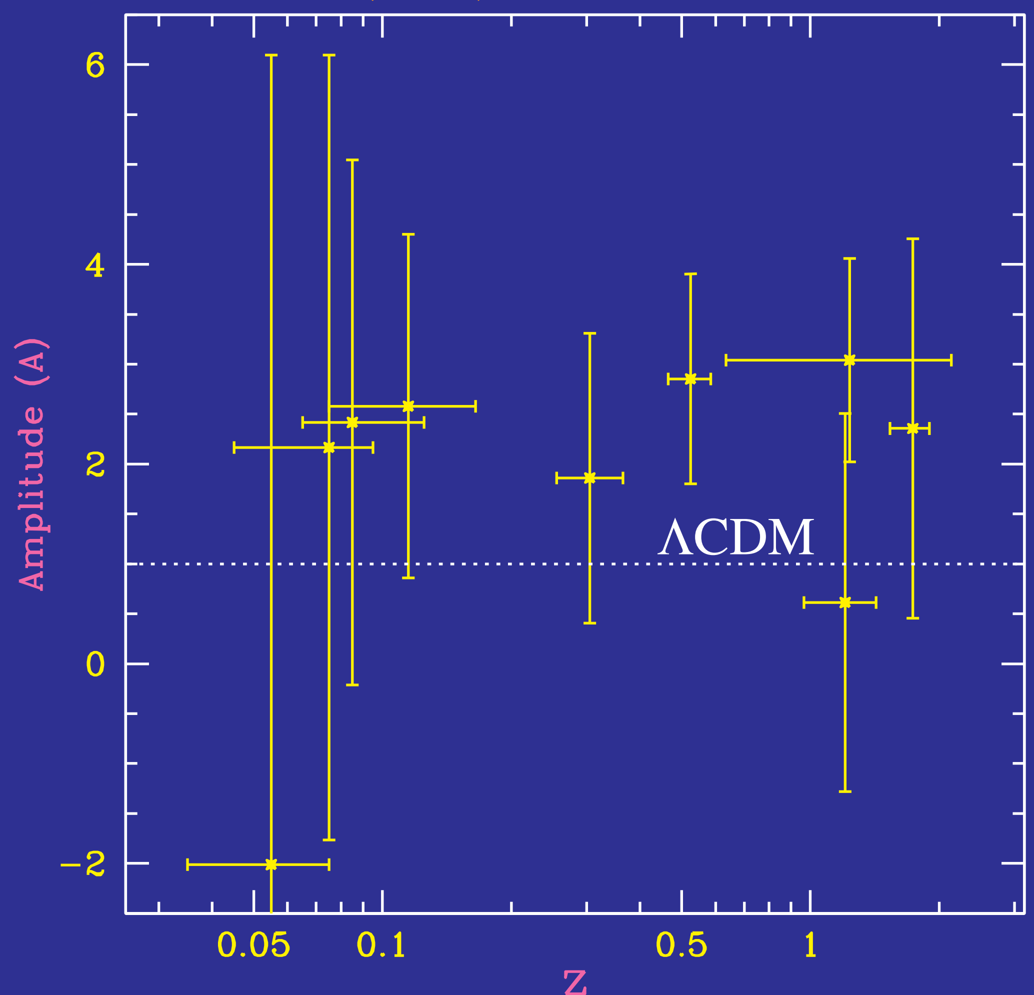
- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation



ISW-Galaxy Correlation

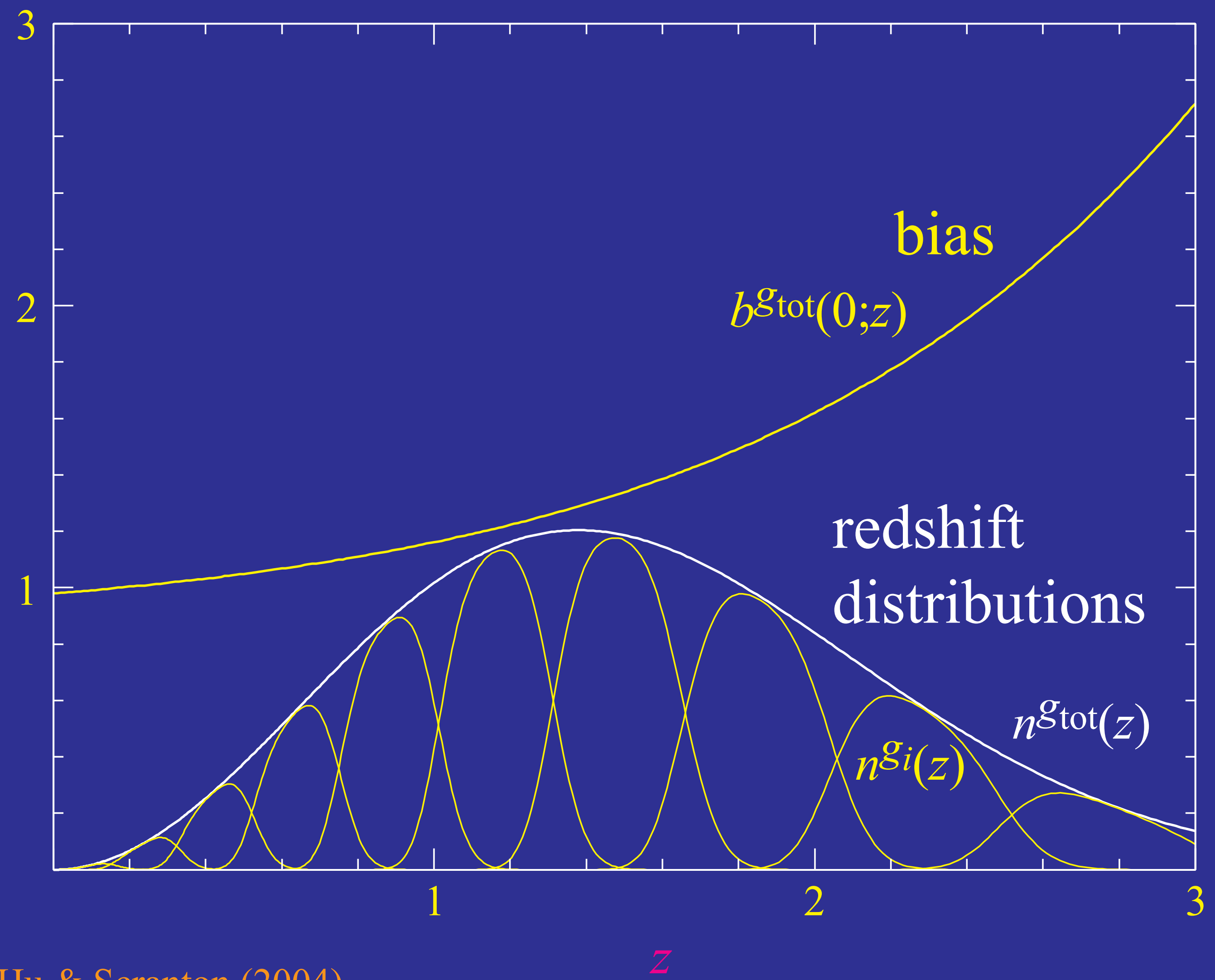
- ~4σ joint detection of ISW correlation with large scale structure (galaxies)
- ~2σ high compared with ΛCDM

Ho et al (2007) [Giannantonio et al 2008]



Ultra-Deep Wide Survey

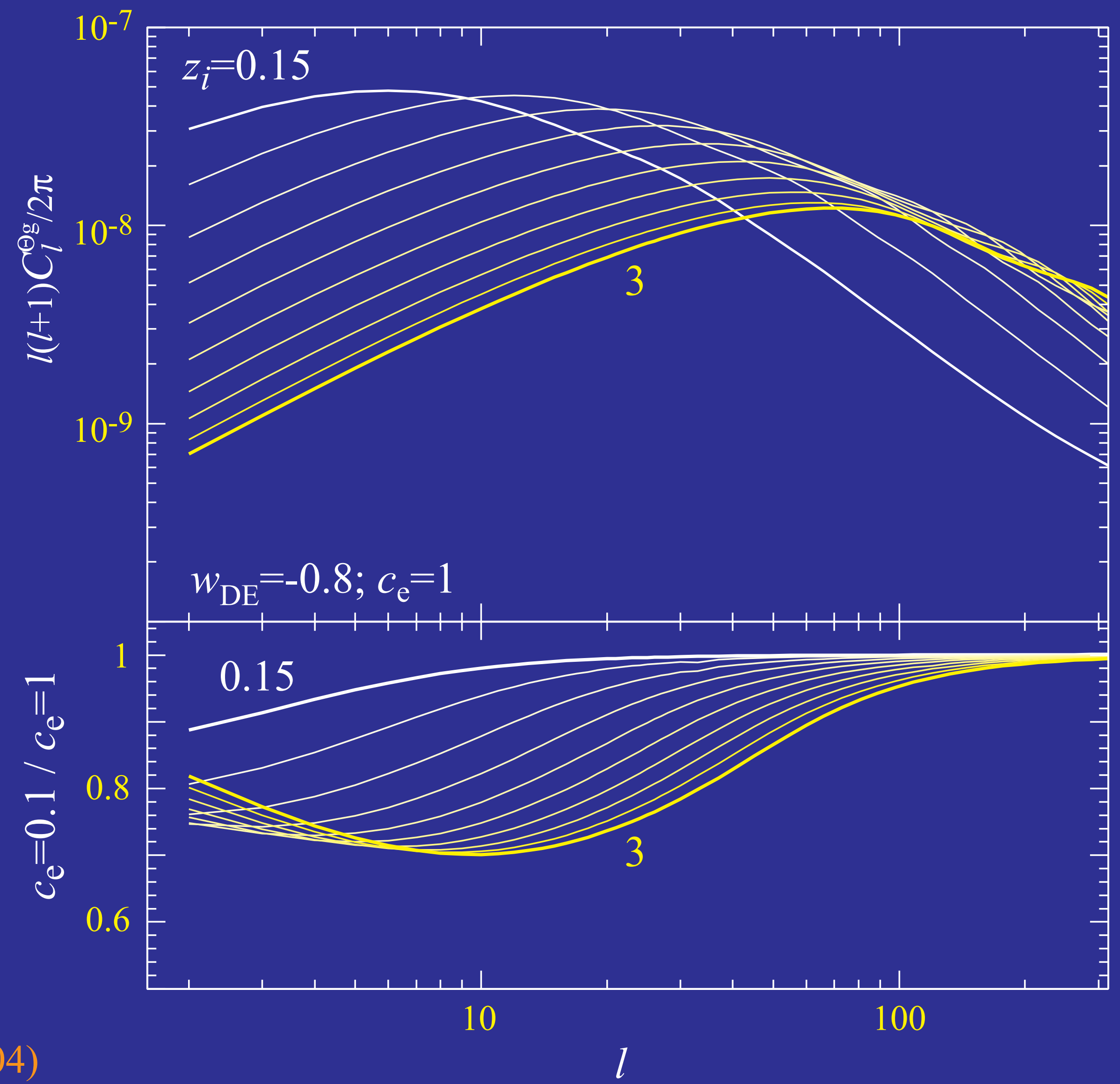
- Ultimate limit: deep wide-field survey with photometric redshift errors of $\sigma(z)=0.03(1+z)$, median redshift $z=1.5$, 70 gal/arcmin^2



Afshordi (2004); Hu & Scranton (2004)

Galaxy Cross Correlation

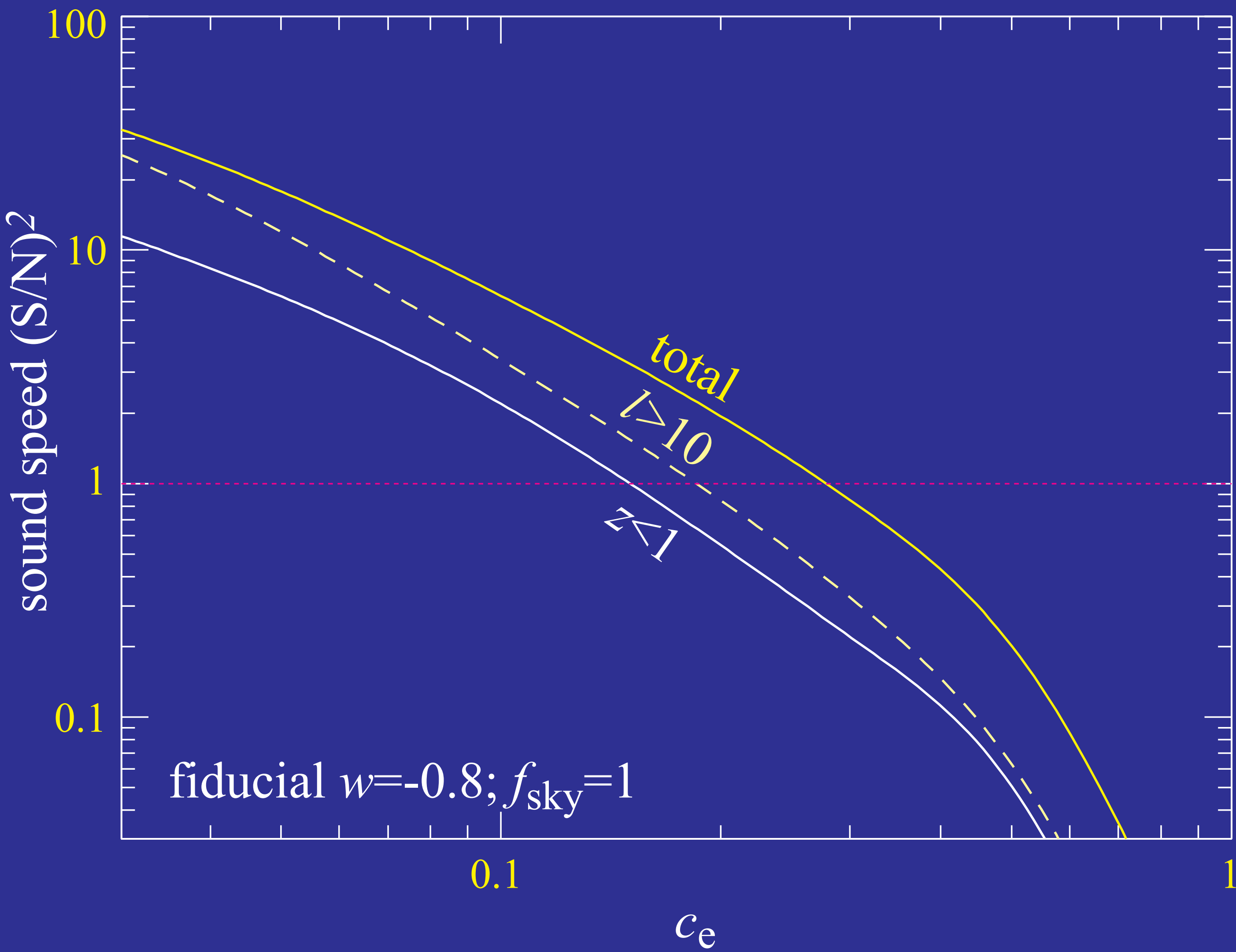
- Cross correlation highly sensitive to the dark energy smoothness (parameterized by sound speed)



Hu & Scranton (2004)

Galaxy Cross Correlation

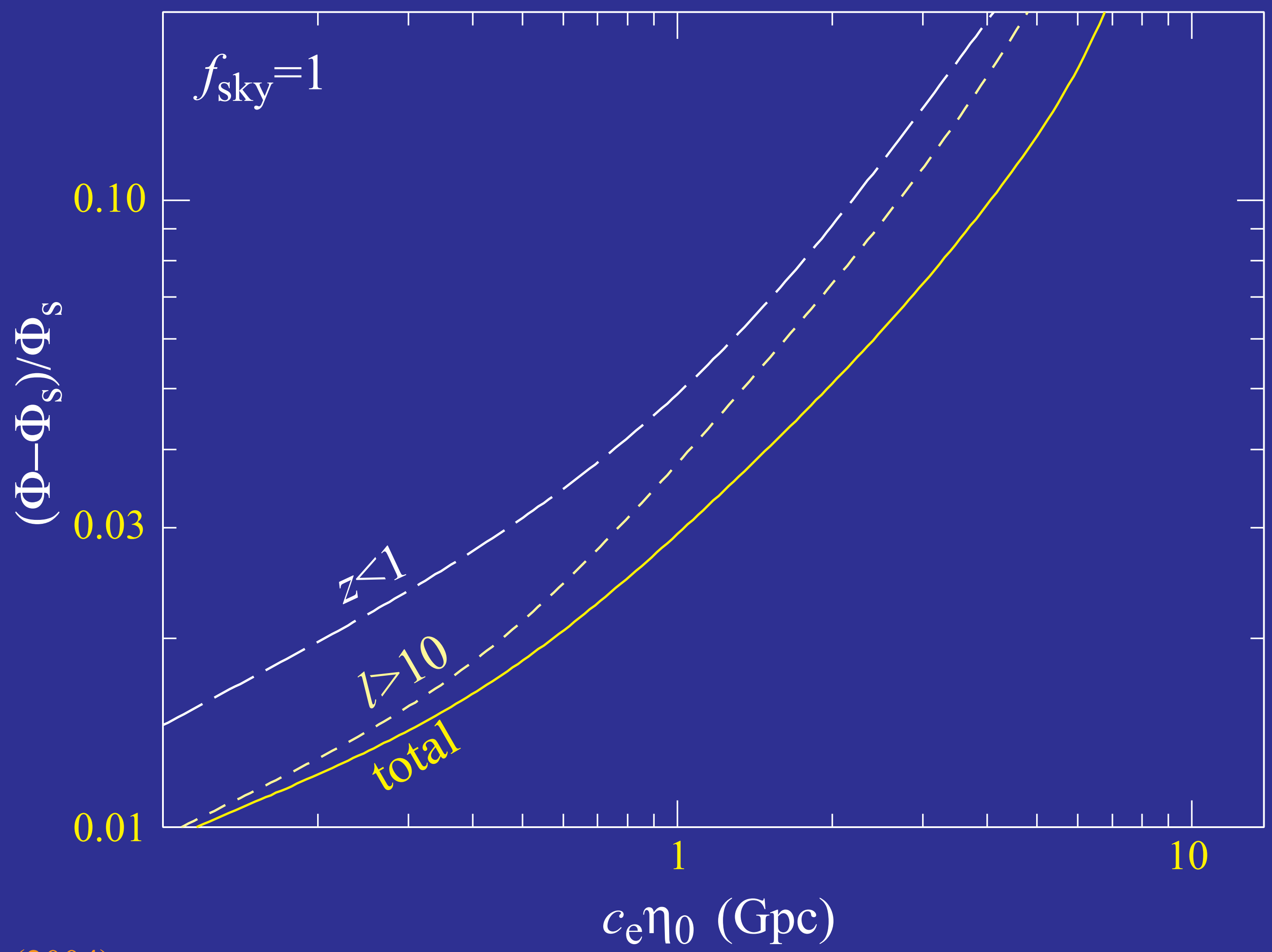
- Significance of the separation between quintessence and a more clustered dark energy with sound speed c_e



Hu & Scranton (2004)

Dark Energy Smoothness

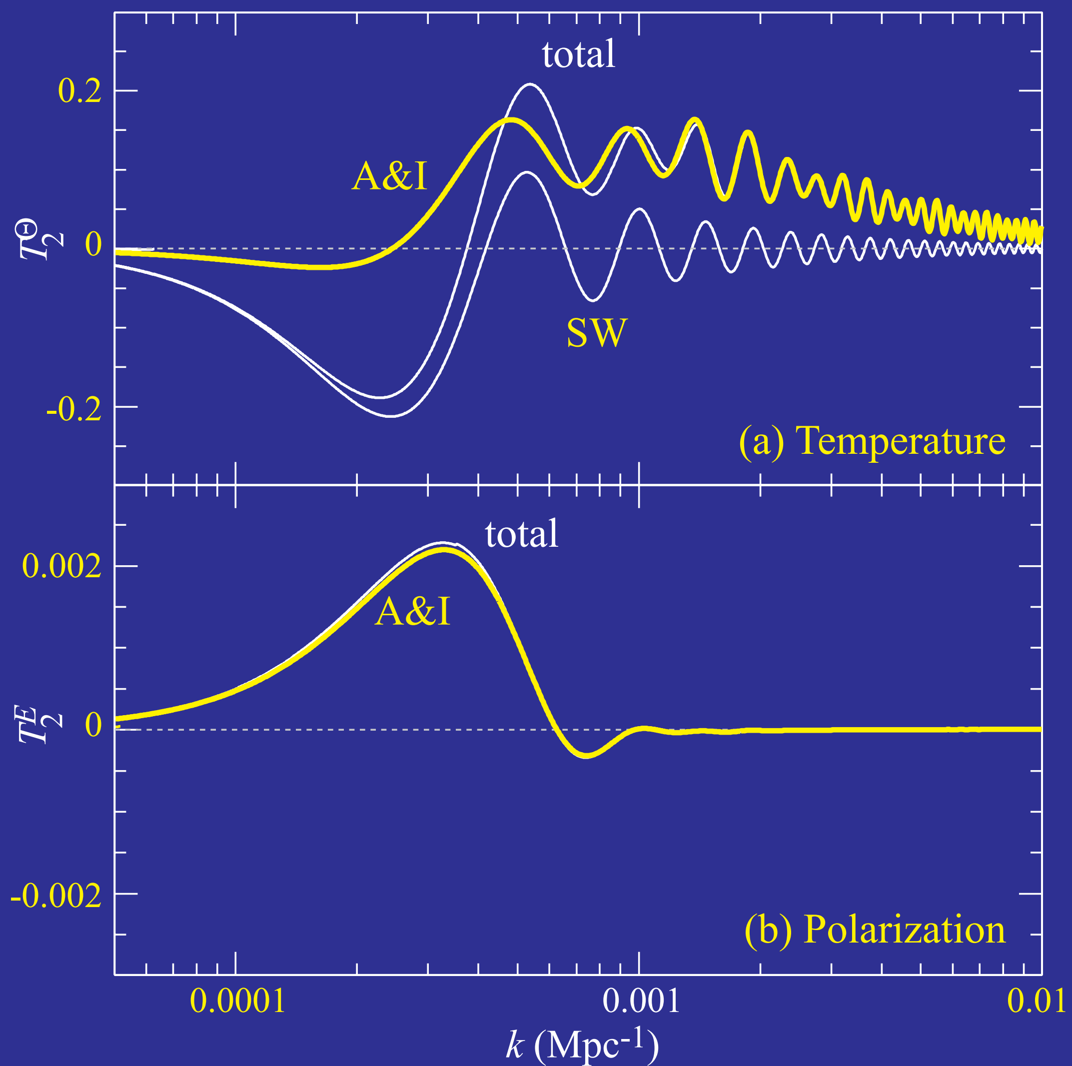
- More robust way of quoting constraints: how smooth is the dark energy out to a given physical scale:



Hu & Scranton (2004)

Isocurvature DE Perturbations

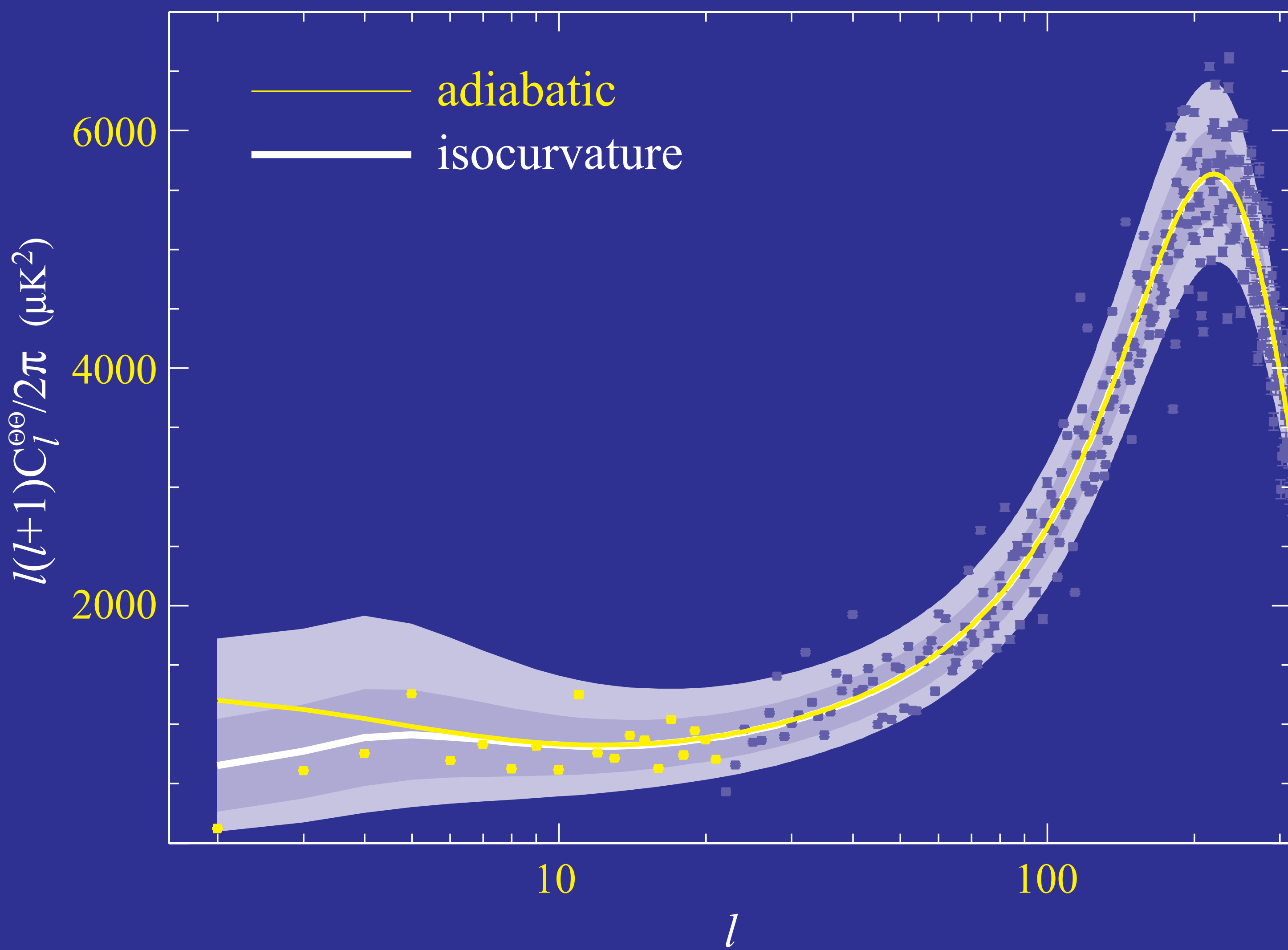
- Anti-correlated DE perturbations: ISW cancel SW effect



Moroi & Takahashi (2004); Gordon & Hu (2004)

Low Quadrupole Models

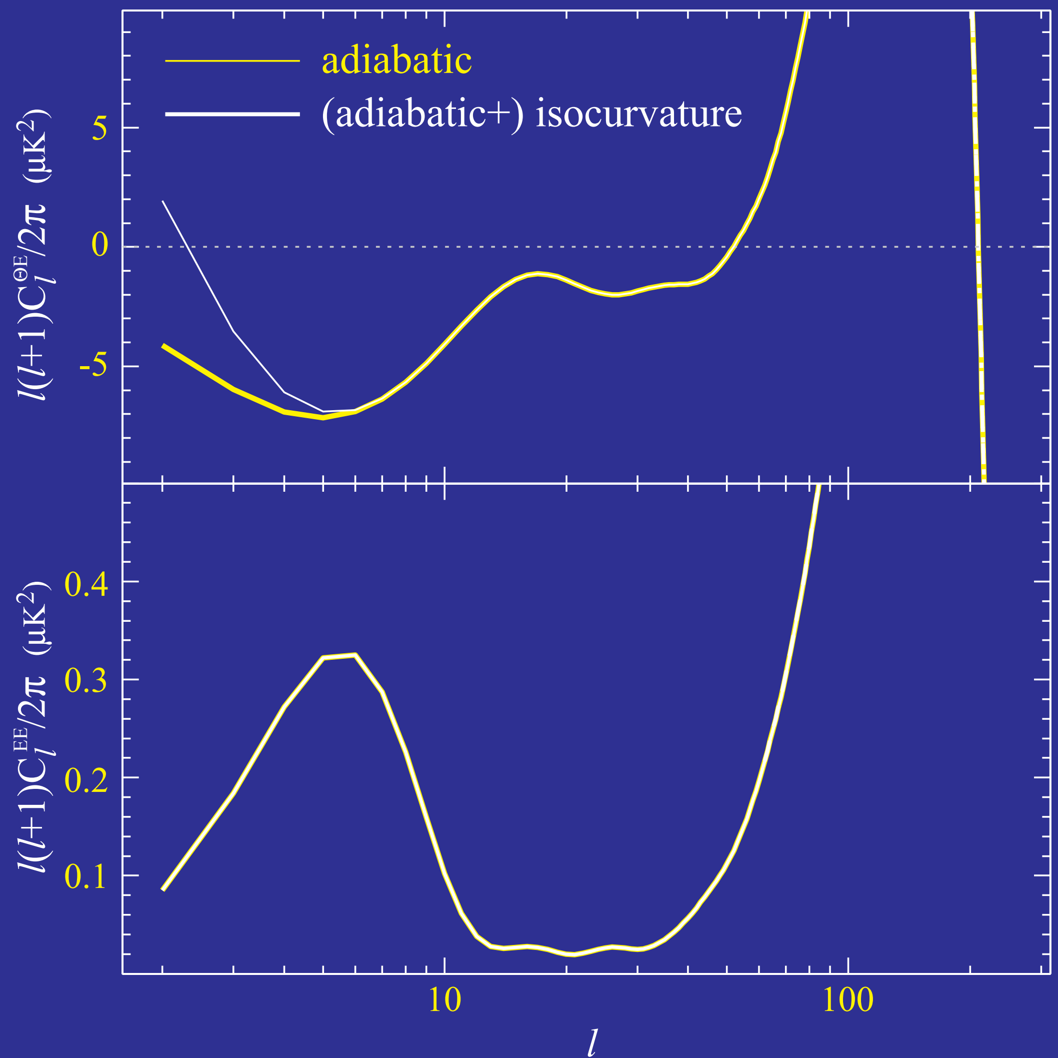
- Required isocurvature perturbation can be generated by variable decay reheating mechanism but overpredicts grav w.



Gordon & Hu (2004) [Dvali, Gruzinov, Zaldarriaga (2004) reheating]

Polarization Rejects ISW

- Polarization unchanged; cross correlation lowered



Gordon & Hu (2004)

ISW & Modified Gravity

Parameterizing Acceleration

- Cosmic acceleration, like the cosmological constant, can either be viewed as arising from

  Missing, or dark energy, with $w \equiv \bar{p}/\bar{\rho} < -1/3$

  Modification of gravity on large scales

$$
\begin{aligned}
G_{\mu\nu} &= 8\pi G \left(T^{\rm M}_{\mu\nu} + T^{\rm DE}_{\mu\nu}\right) \\
F(g_{\mu\nu}) + G_{\mu\nu} &= 8\pi G T^{\rm M}_{\mu\nu}
\end{aligned}
$$

- Proof of principle models for both exist: quintessence, k-essence; DGP braneworld acceleration, $f(R)$ modified action
- Compelling models for either explanation lacking
- Study models as illustrative toy models whose features can be generalized

DGP Braneworld Acceleration

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

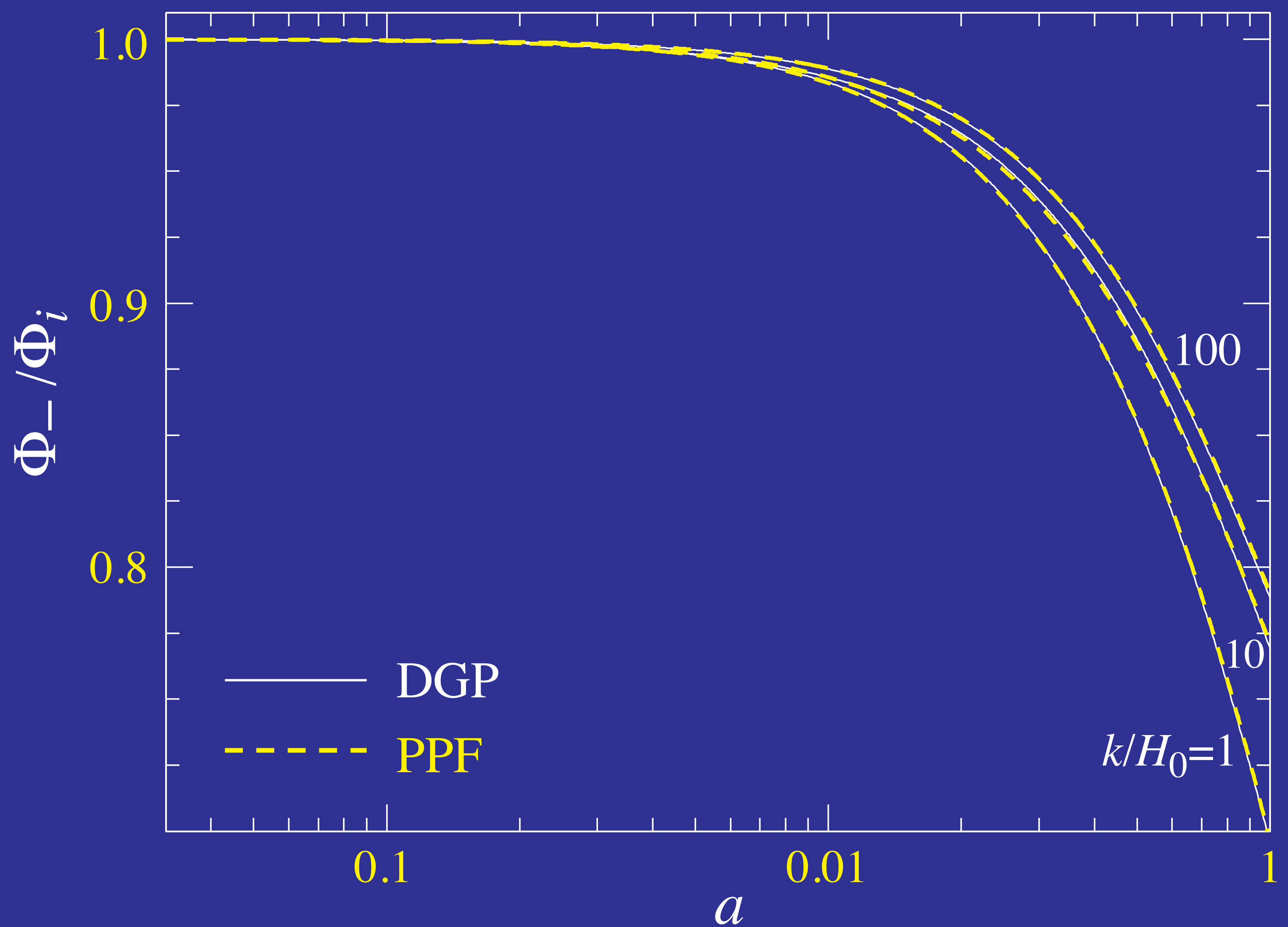
$$S = \int d^5x \sqrt{-g} \left[\frac{{}^{(5)}R}{2\kappa^2} + \delta(\chi) \left(\frac{{}^{(4)}R}{2\mu^2} + \mathcal{L}_m \right) \right]$$

  with crossover scale $r_c = \kappa^2 / 2\mu^2$

- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)
- Matter still minimally coupled and conserved
- Exhibits the 3 regimes of modified gravity
- Weyl tensor anisotropy dominated conserved curvature regime $r > r_c$ (Sawicki, Song, Hu 2006; Cardoso et al 2007)
- Brane bending scalar tensor regime $r_* < r < r_c$ (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)
- Strong coupling General Relativistic regime $r < r_* = (r_c^2 r_g)^{1/3}$ where $r_g = 2GM$ (Dvali 2006)

DGP Horizon Scales

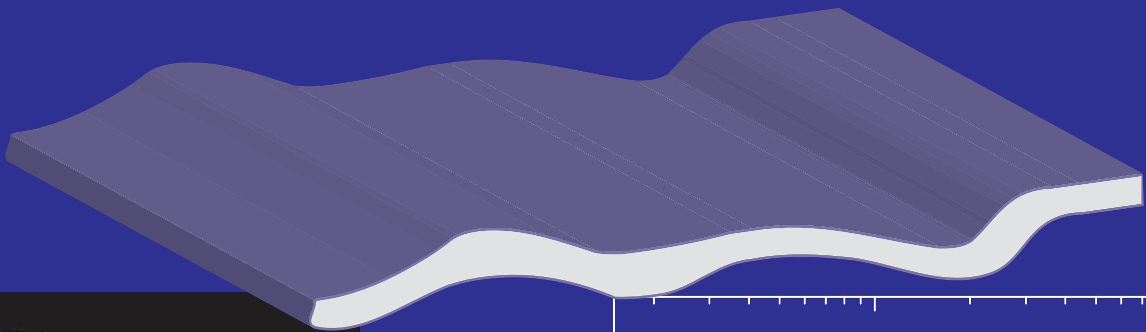
- Metric and matter evolution well-matched by PPF description
- Standard GR tools apply (CAMB), self-consistent, gauge invar.

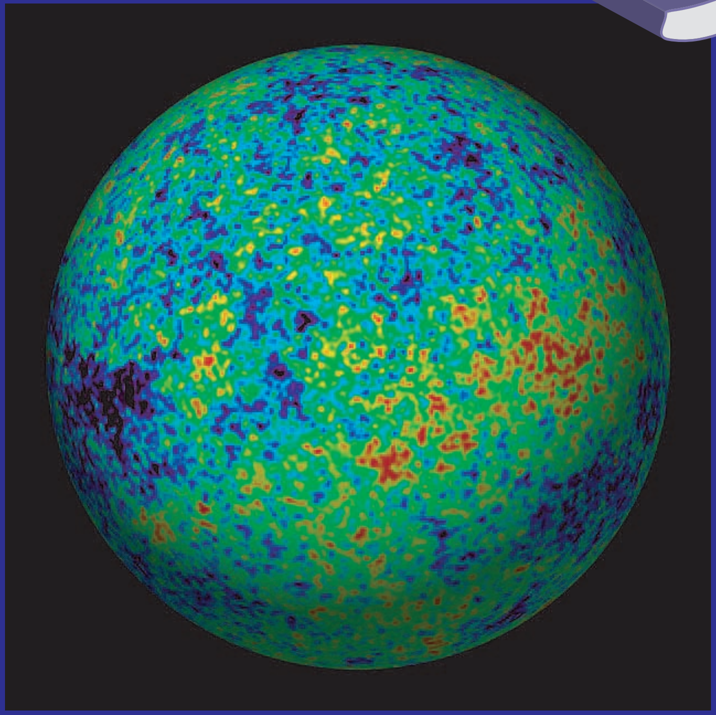


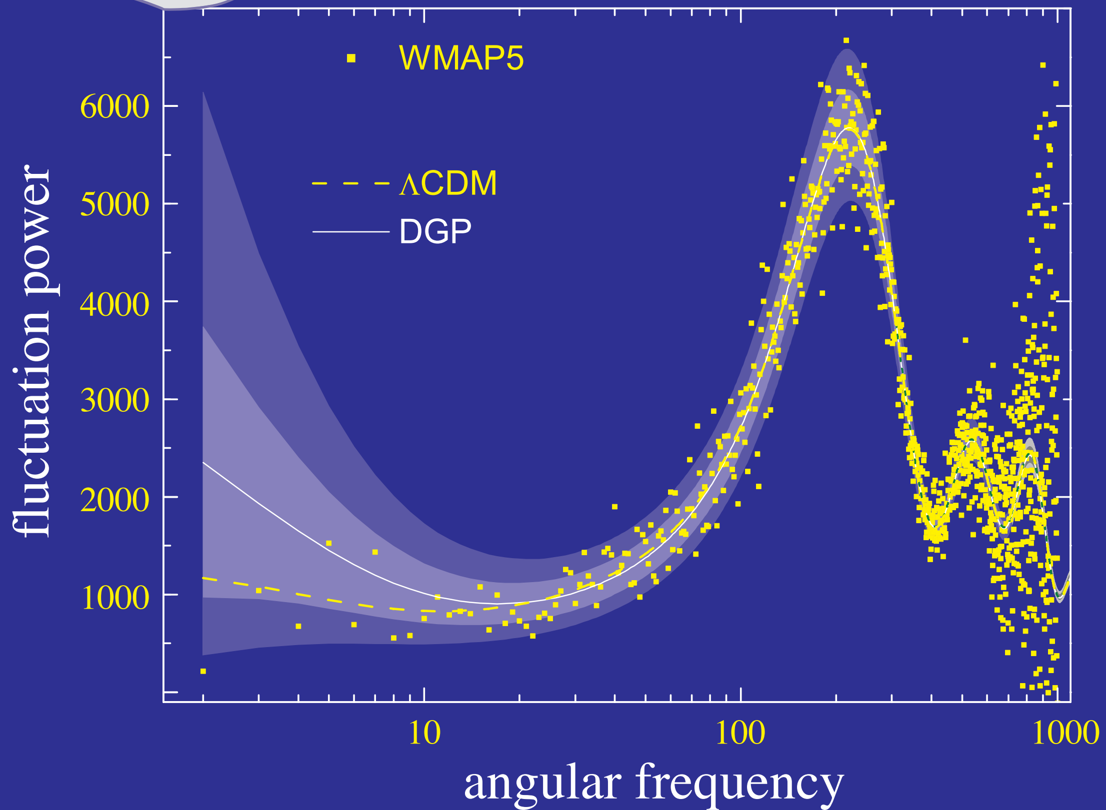
Hu & Sawicki (2007); Hu (2008)

DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background

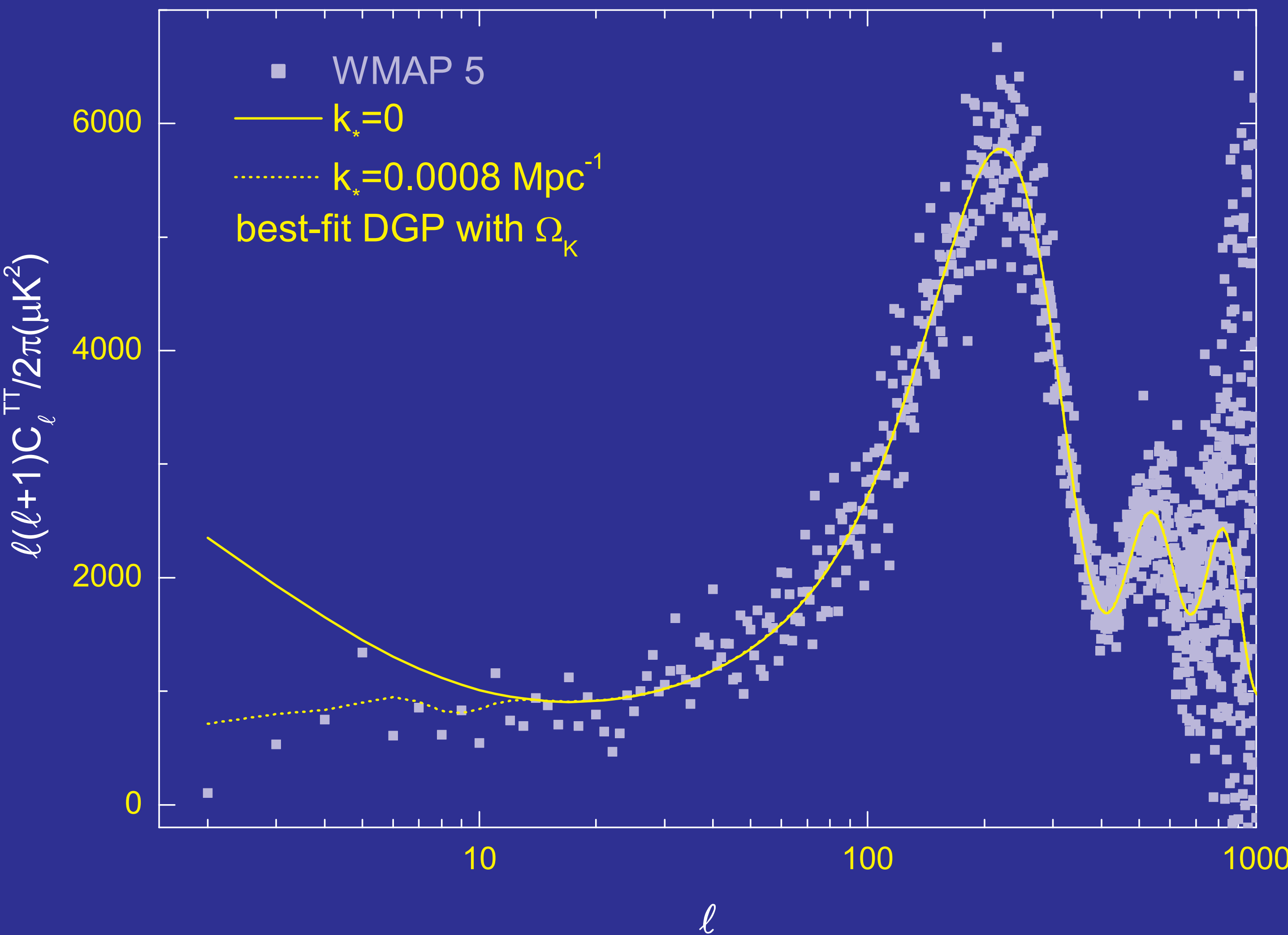






CMB in DGP

- Adding cut off as an epicycle can fix distances, ISW problem
- Suppresses polarization in violation of EE data - cannot save DGP!



Fang et al (2008)

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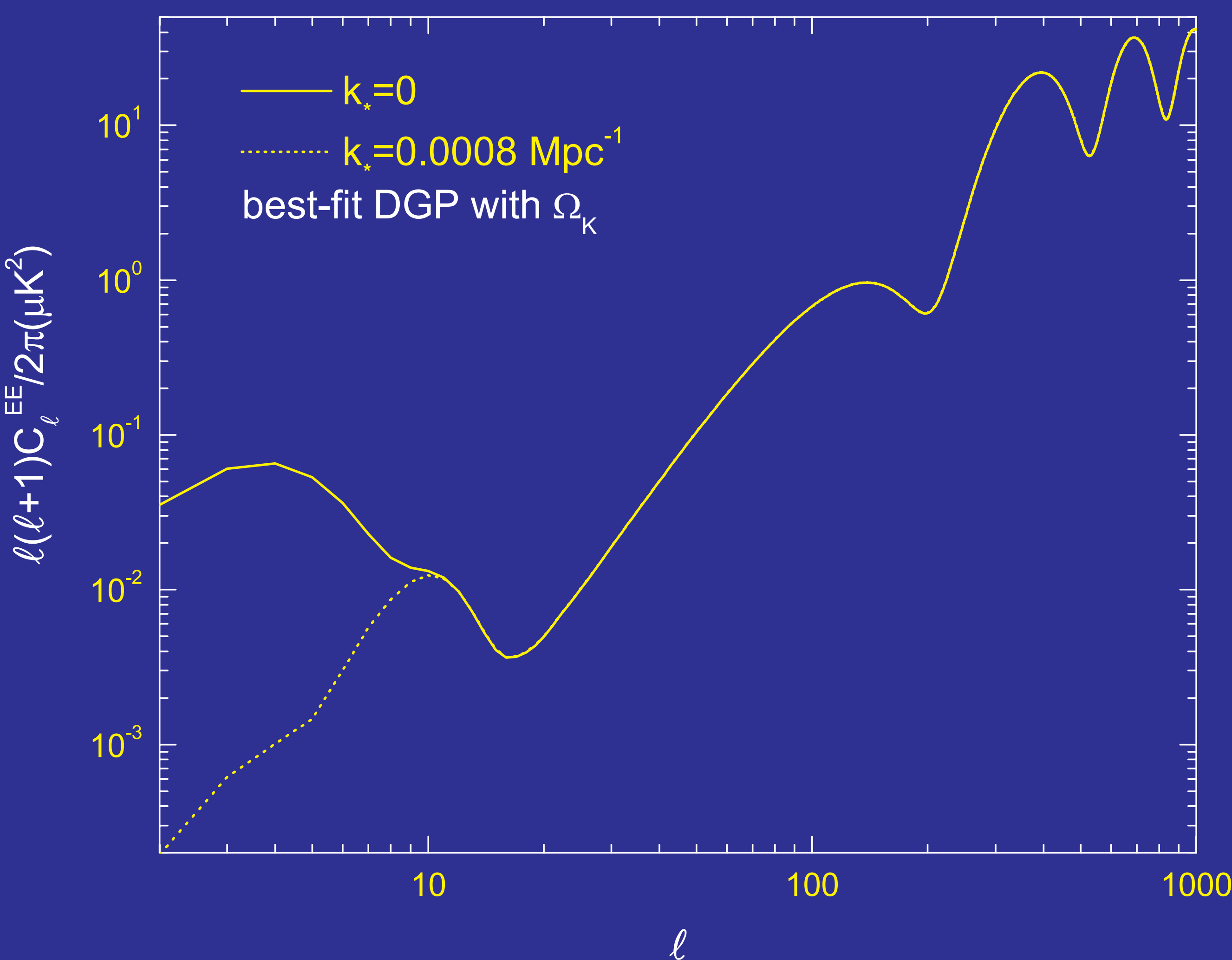


Fang et al (2008)

Modified Action $f(R)$ Model

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_{\rm m} \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B: Compton wavelength of f_R squared in units of the Hubble length

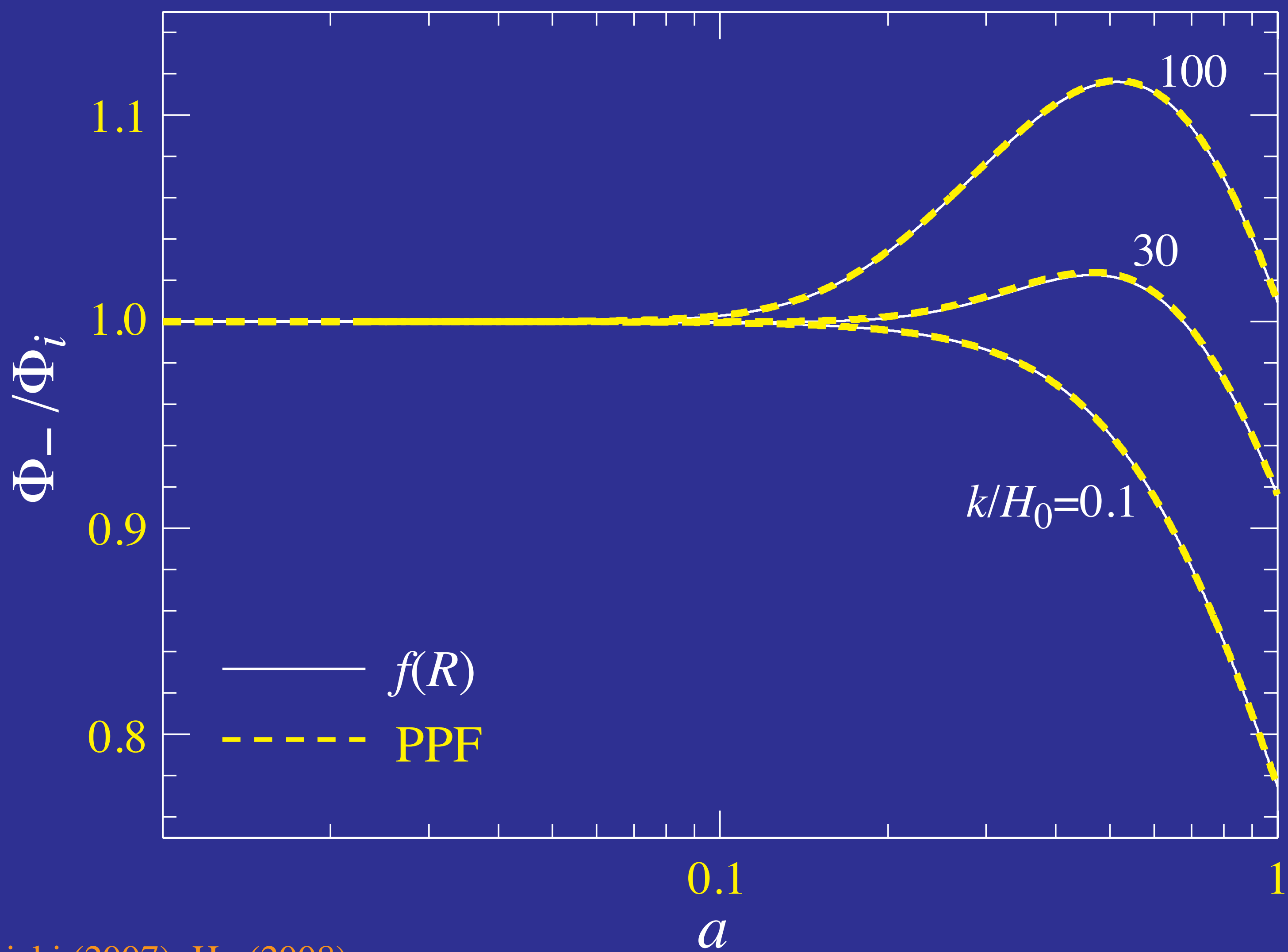
$$B \equiv \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

see Tristan Smith's talk

- $' \equiv d/d\ln a$: scale factor as time coordinate

PPF $f(R)$ Description

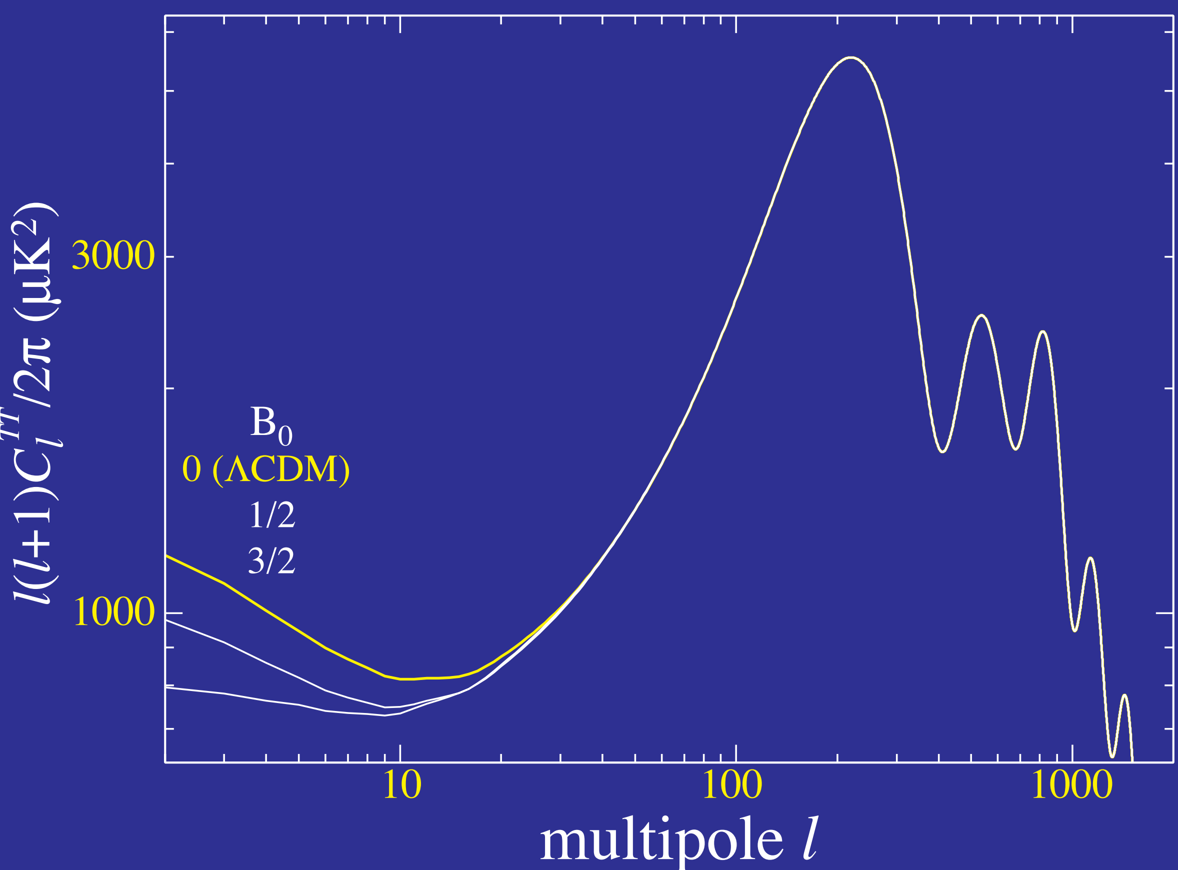
- Metric and matter evolution well-matched by PPF description
- Standard GR tools apply (CAMB), self-consistent, gauge invar.



Hu & Sawicki (2007); Hu (2008)

ISW Quadrupole

- Reduction of large angle anisotropy for B_0~1 for same expansion history and distances as ΛCDM
- Well-tested small scale anisotropy unchanged



Song, Hu & Sawicki (2006)

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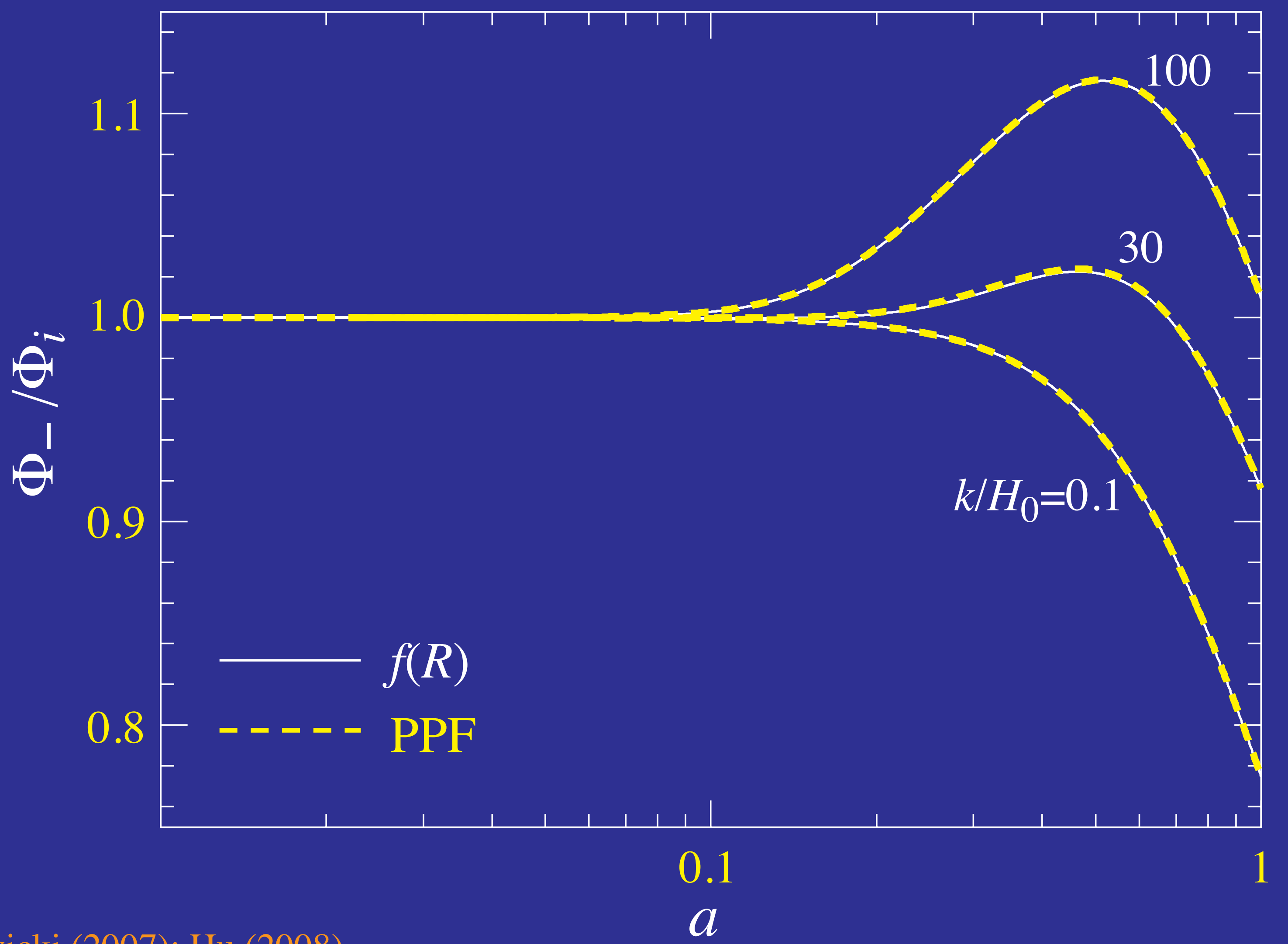
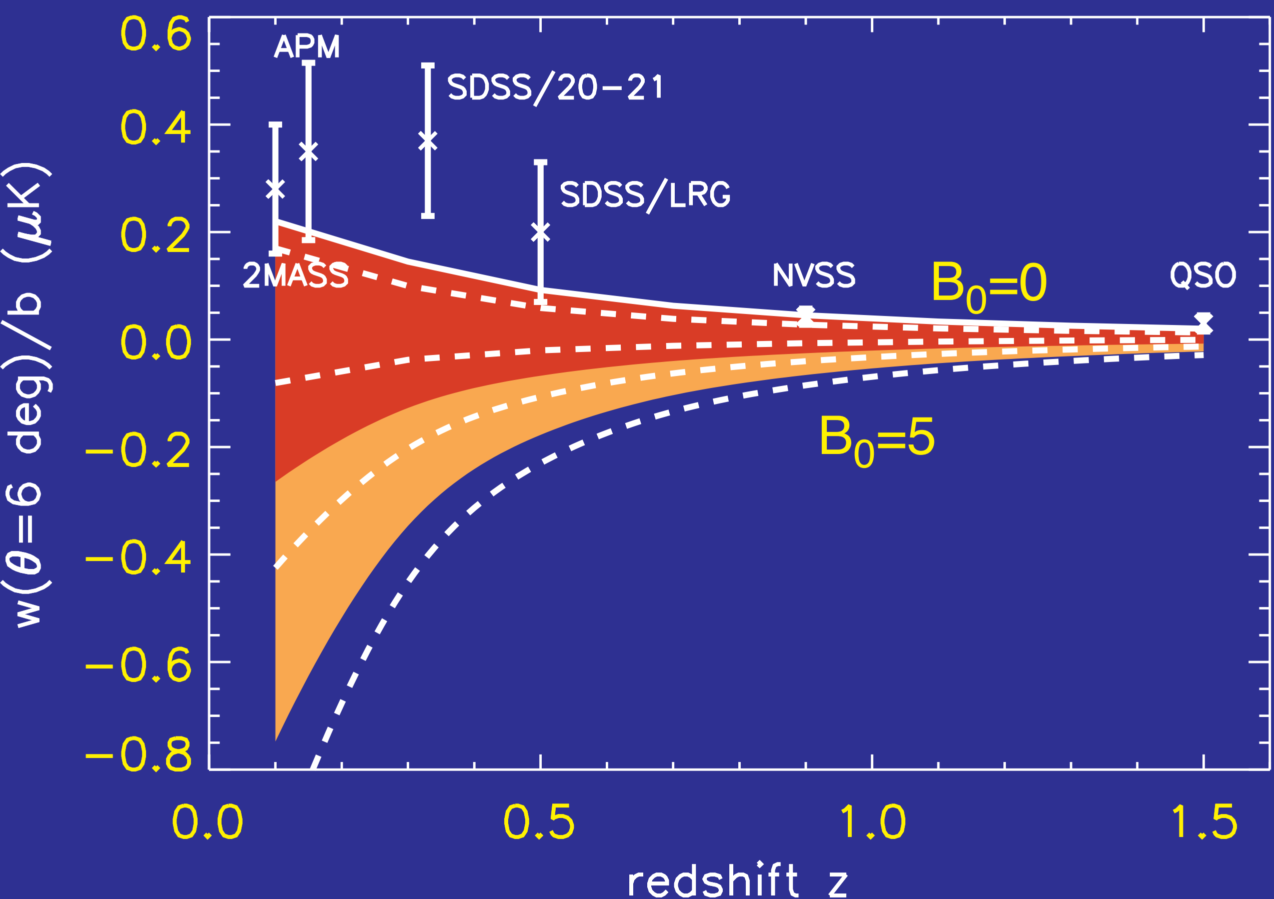


Hu & Sawicki (2007); Hu (2008)

Galaxy-ISW Anti-Correlation

- Large Compton wavelength $B^{1/2}$ creates potential growth which can anti-correlate galaxies and the CMB
- In tension with detections of positive correlations across a range of redshifts

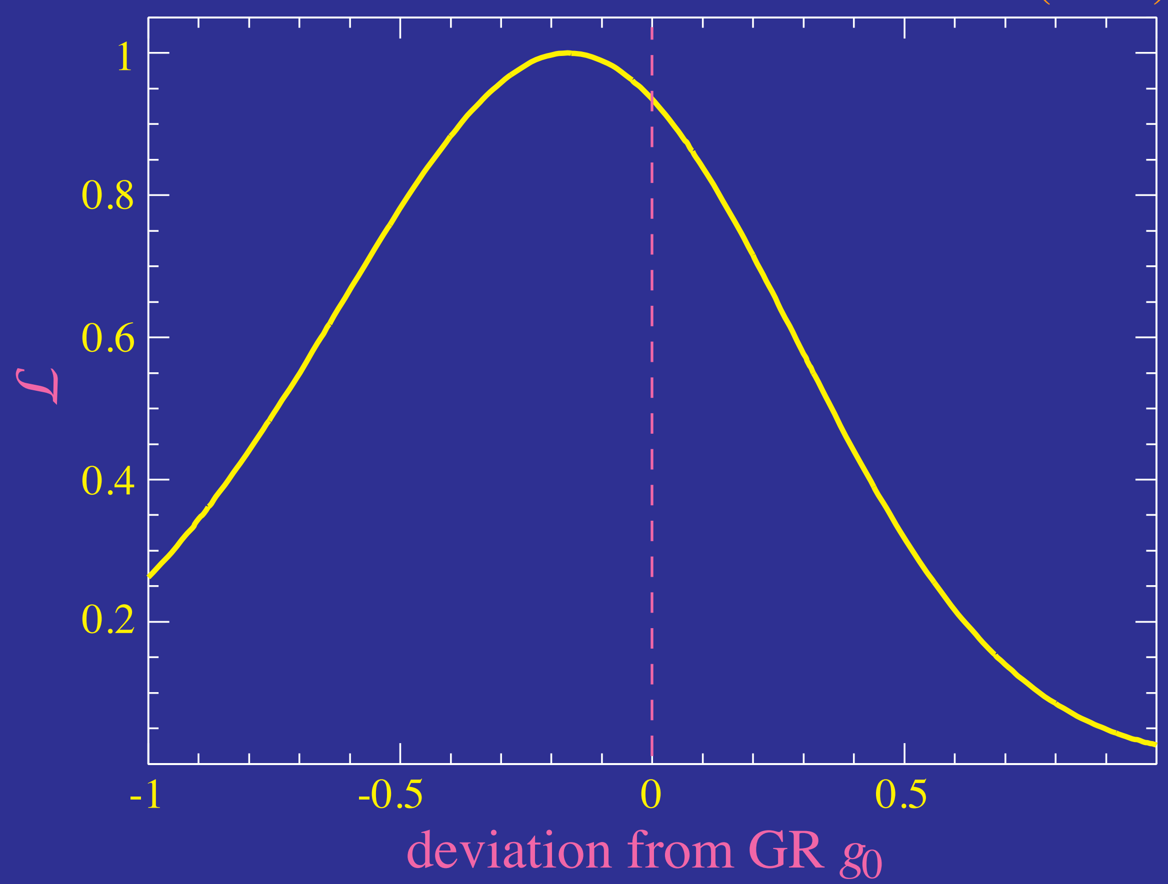


Song, Peiris & Hu (2007)

Parameterized Post-Friedmann

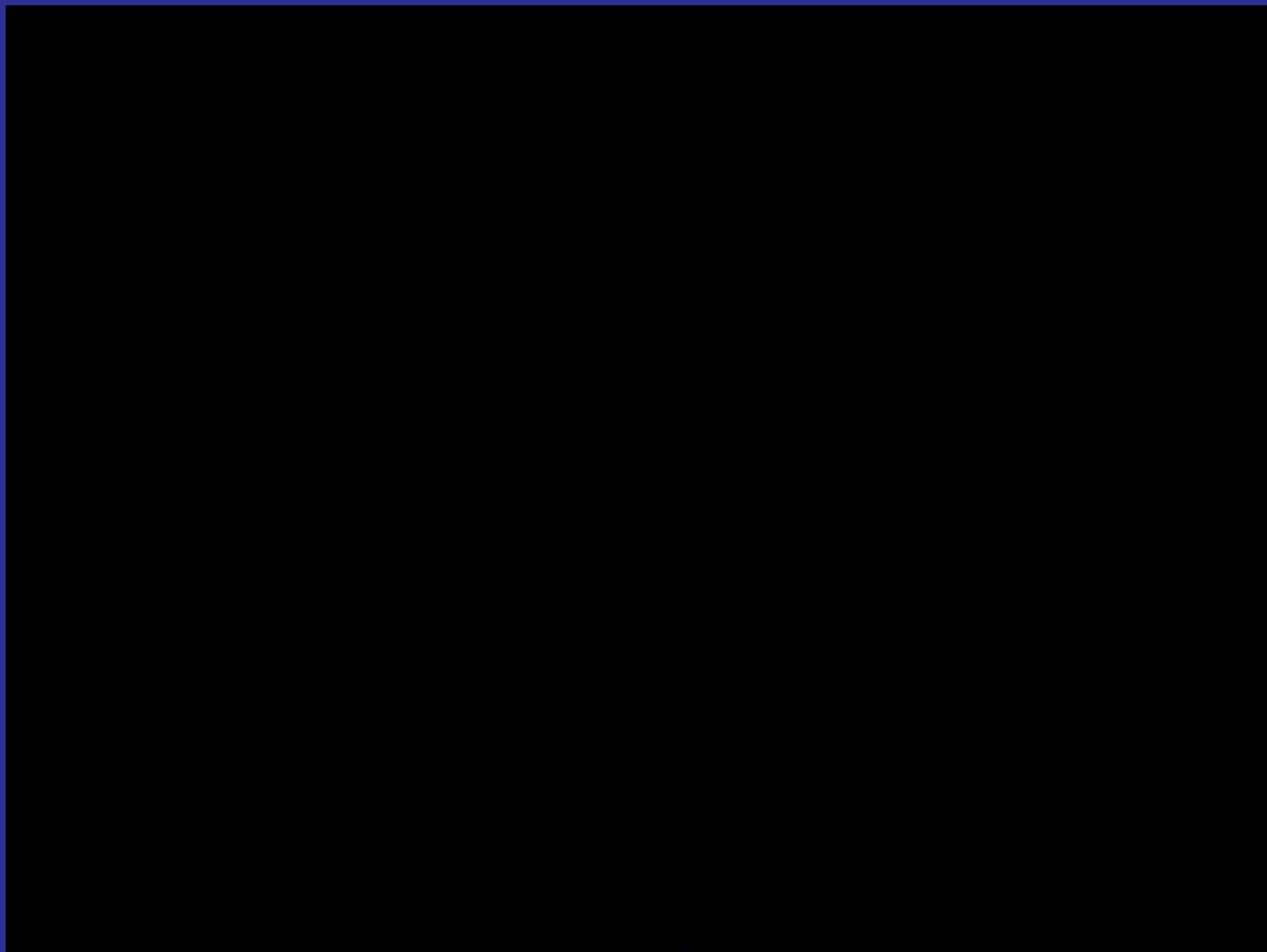
- Parameterizing the degrees of freedom associated with metric modification of gravity that explain cosmic acceleration
- Simple models that add in only one extra scale to explain acceleration tend to predict substantial changes near horizon and hence ISW

Hu (2008)



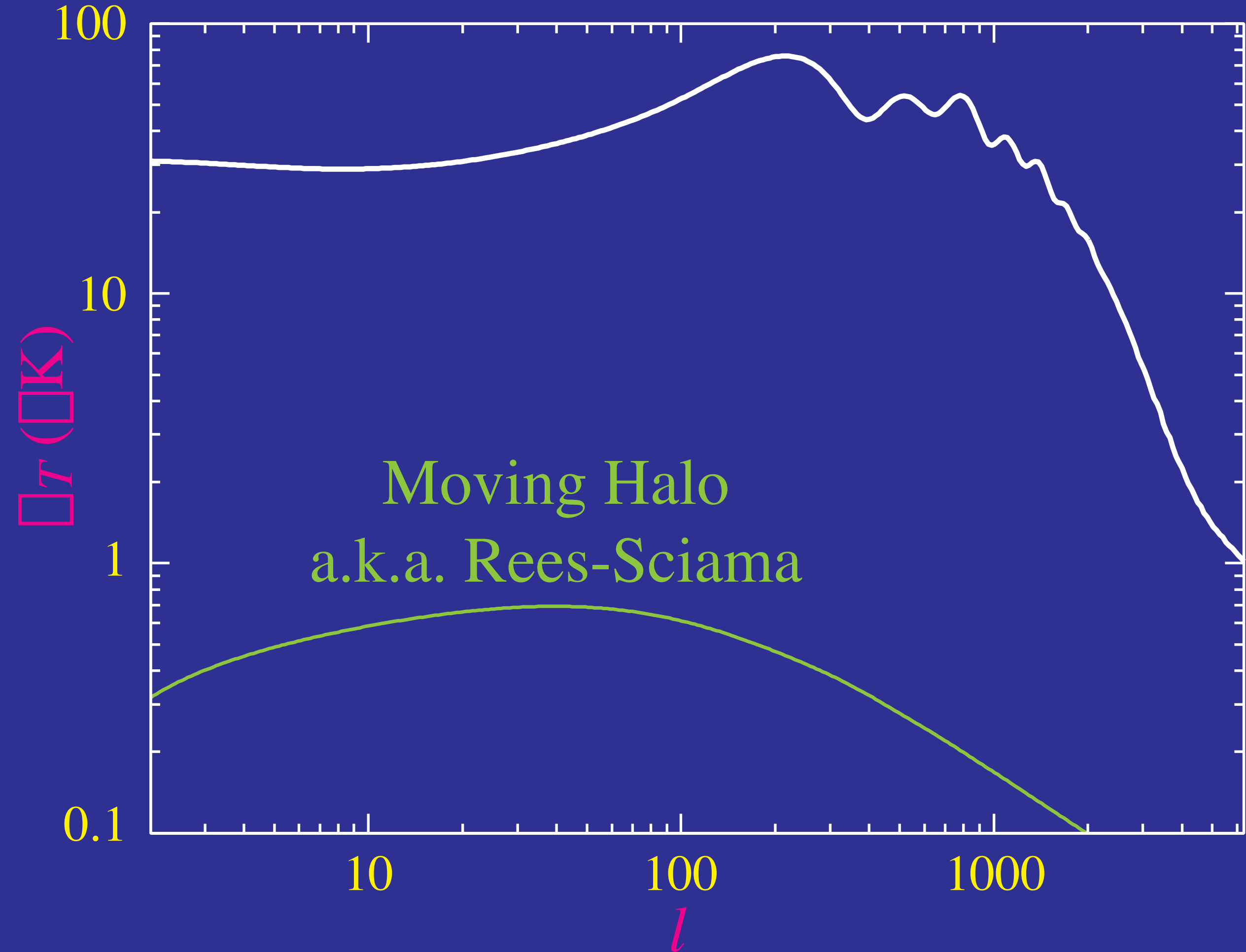
Non-linear ISW Effect

Moving Halo Effect



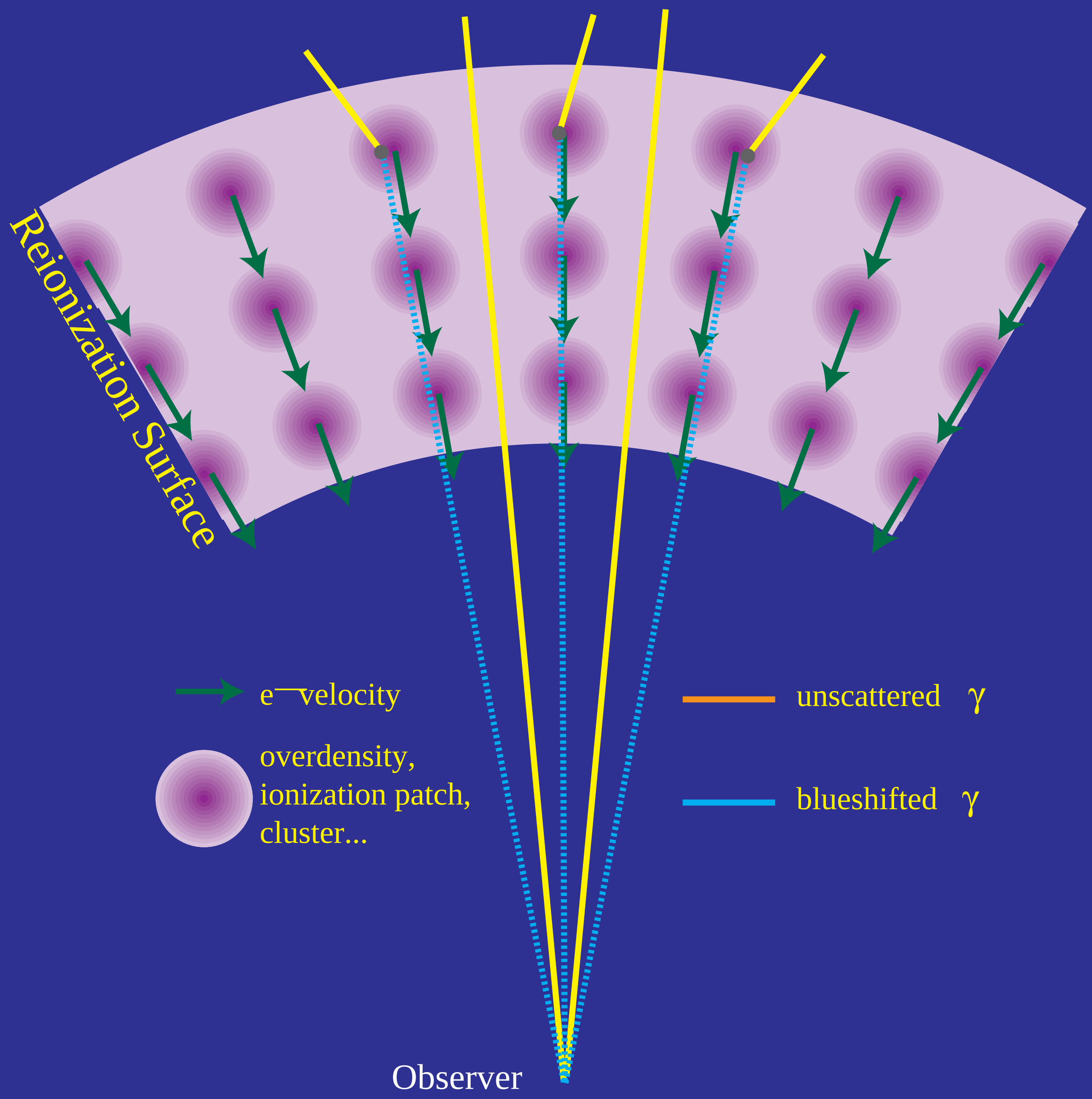
Moving Halo Effect

- Change in potential due to halo moving across the line of sight

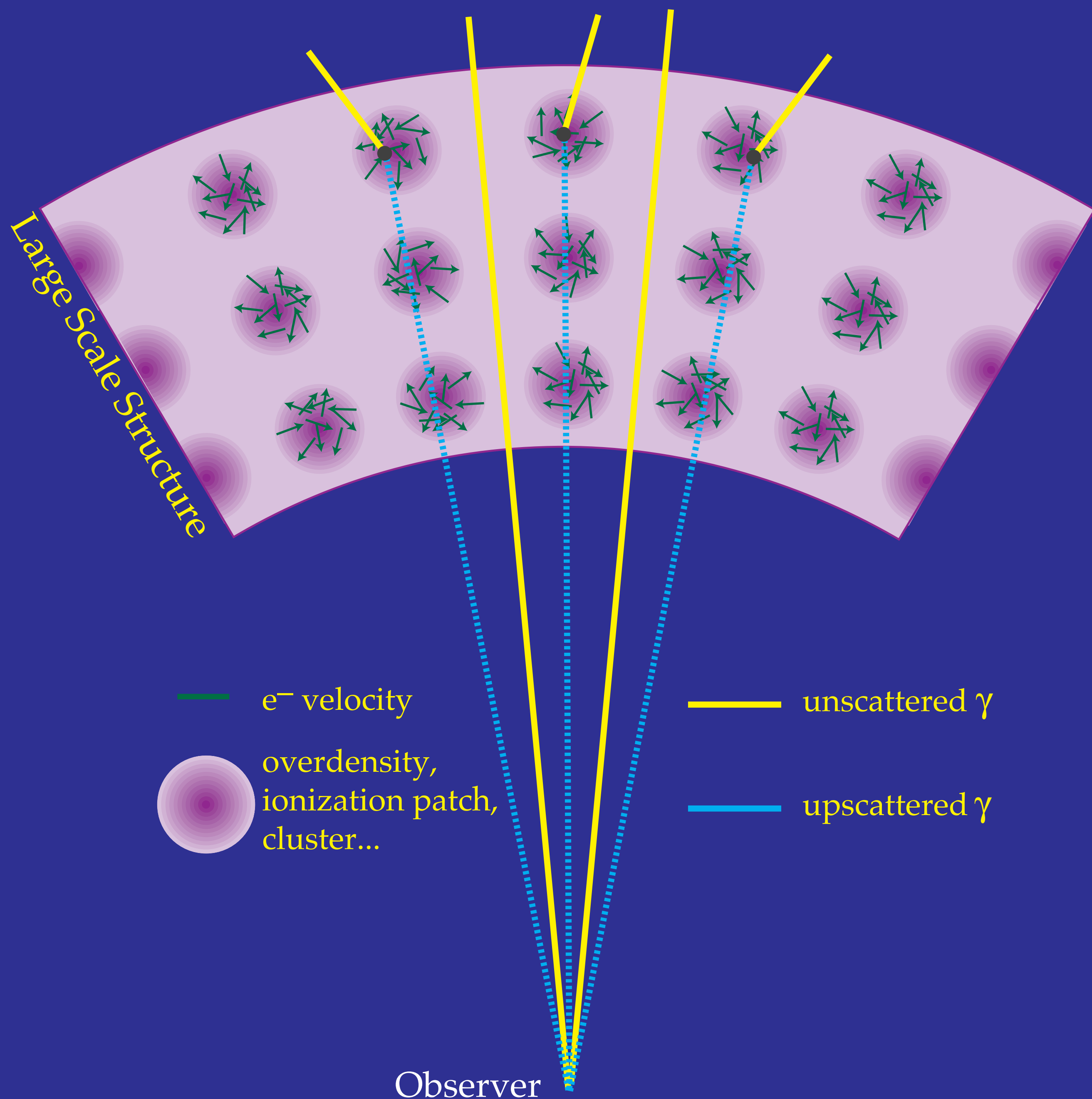


SZ Effect

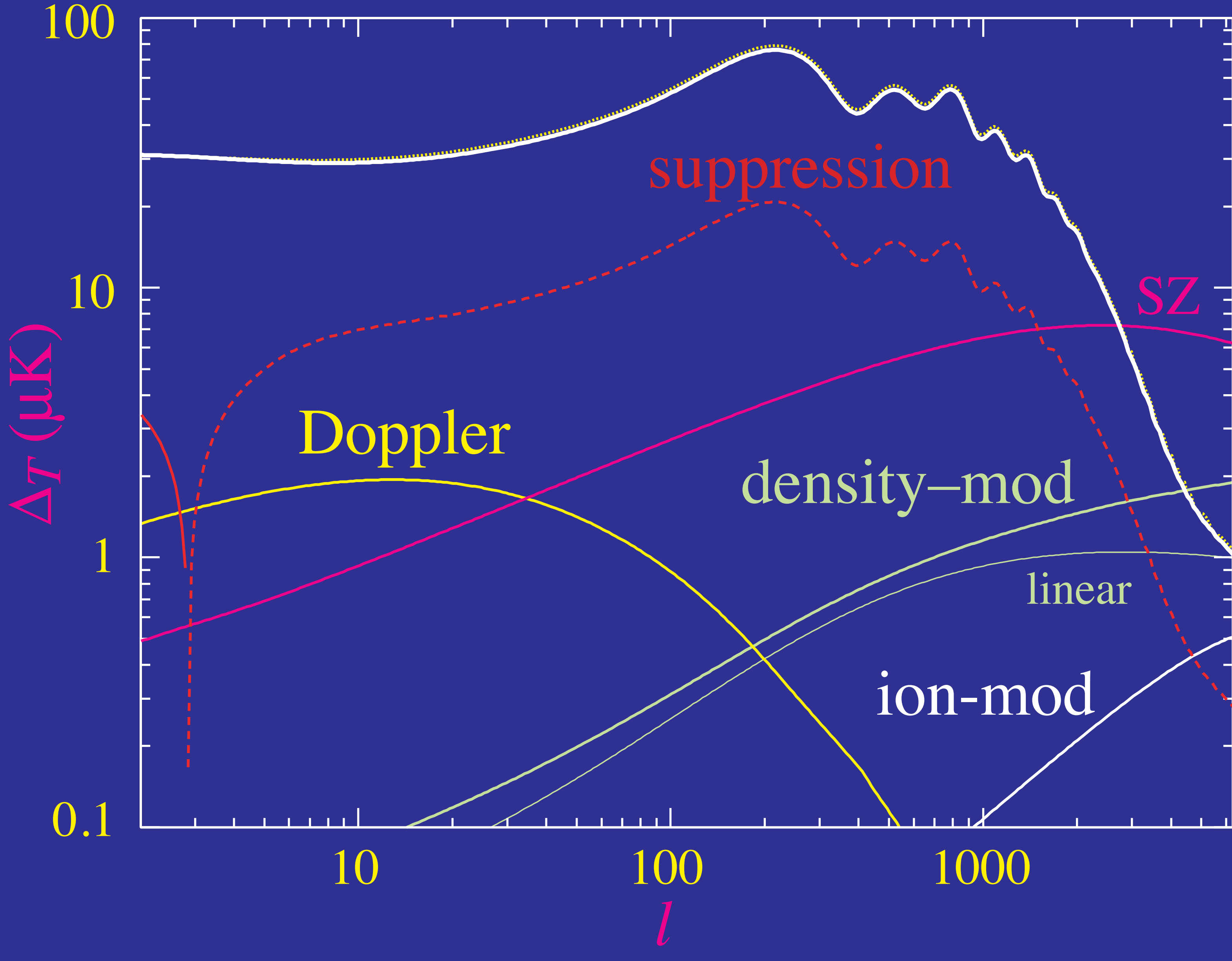
Modulated Doppler Effect



Thermal SZ Effect

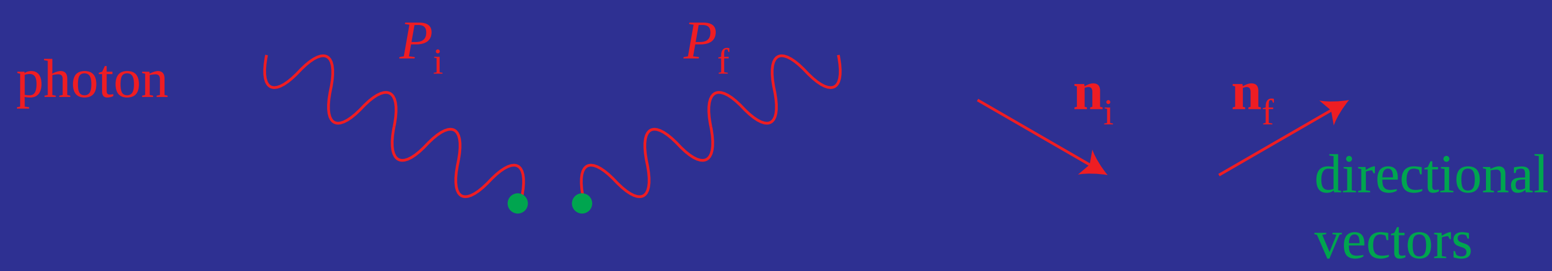


Scattering Secondaries



Beyond Thomson Limit

- Thomson scattering $e_i + \gamma_i \to e_f + \gamma_f$ in rest frame where the frequencies $\omega_i = \omega_f$ (elastic scattering) cannot strictly be true
- Photons carry off E/c momentum and so to conserve momentum the electron must recoil
- Doppler shift from transformation from rest frame contains second order terms
- General case (arbitrary electron velocity)



Energy-Momentum Conservation

- From energy-momentum conservation, the energy change is

$$\frac{E_f}{E_i} = \frac{1 - \beta_i \cos\alpha_i}{1 - \beta_i \cos\alpha_f + \frac{E_i}{\gamma m c^2}(1 - \cos\theta)}$$

  where $\hat{\mathbf{n}}_f \cdot \mathbf{v}_i = v_i \cos\alpha_f$ and $\hat{\mathbf{n}}_i \cdot \mathbf{v}_i = v_i \cos\alpha_i$

- Two ways of changing the energy: Doppler boost β_i from incoming electron velocity and E_i non-negligible compared to $\gamma m c^2$

- Isolate recoil in incoming electron rest frame $\beta_i = 0$ and $\gamma = 1$

$$\left.\frac{E_f}{E_i}\right|_{\rm rest} = \frac{1}{1 + \frac{E_i}{mc^2}(1 - \cos\theta)}$$

Recoil Effect

- Since $-1 \leq \cos\theta \leq 1$, $E_f \leq E_i$, energy is lost from the recoil except for purely forward scattering
- The backwards scattering limit is easy to see

$$|\mathbf{q}_f| = m|\mathbf{v}_f| = 2\frac{E_i}{c}\,,$$

$$\Delta E = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{2E_i}{mc}\right)^2 = 2\frac{E_i}{mc^2}E_i$$

$$E_f = E_i - \Delta E = (1 - 2\frac{E_i}{mc^2})E_i \approx \frac{E_i}{1 + 2\frac{E_i}{mc^2}}$$

Second Order Doppler Shift

- Doppler effect: consider the limit of $\beta_i \ll 1$ then expand to first order

$$\frac{E_f}{E_i} = 1 - \beta_i \cos\alpha_i + \beta_i \cos\alpha_f - \frac{E_i}{mc^2}(1 - \cos\theta)$$

however averaging over angles the Doppler shifts don't change the energies

- To second order in the velocities, the Doppler shift transfers energy from the electron to the photon in opposition to the recoil

$$\frac{E_f}{E_i} = 1 - \beta_i \cos\alpha_i + \beta_i \cos\alpha_f + \beta_i^2 \cos^2\alpha_f - \frac{E_i}{mc^2}$$

$$\langle \frac{E_f}{E_i} \rangle \approx 1 + \frac{1}{3}\beta_i^2 - \frac{E_i}{mc^2}$$

Thermalization

- For a thermal distribution of velocities

$$\frac{1}{2}m\langle v^2\rangle = \frac{3kT}{2} \qquad \beta_i^2 \approx \frac{3kT}{mc^2} \rightarrow \langle\frac{E_f}{E_i} - 1\rangle \sim \frac{kT - E_i}{mc^2}$$

so that if $E_i \ll kT$ the photon gains energy and $E_i \gg kT$ it loses energy $\rightarrow$ this is a thermalization process

Kompaneets Equation

- Radiative transfer or Boltzmann equation

$$\frac{\partial f}{\partial t} = \frac{1}{2E(p_f)} \int \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E(p_i)} \int \frac{d^3 q_f}{(2\pi)^3} \frac{1}{2E(q_f)} \int \frac{d^3 q_i}{(2\pi)^3} \frac{1}{2E(q_i)}$$
$$\times (2\pi)^4 \delta(p_f + q_f - p_i - q_i) |M|^2$$
$$\times \{ f_e(q_i) f(p_i) [1 + f(p_f)] - f_e(q_f) f(p_f) [1 + f(p_i)] \}$$

- Matrix element is calculated in field theory and is Lorentz invariant. In terms of the rest frame $\alpha = e^2/\hbar c$ (Klein Nishina Cross Section)

$$|M|^2 = 2(4\pi)^2 \alpha^2 \left[\frac{E(p_i)}{E(p_f)} + \frac{E(p_f)}{E(p_i)} - \sin^2 \beta \right]$$

with β as the rest frame scattering angle

Kompaneets Equation

- The Kompaneets equation ($\hbar = c = 1$)

$$\frac{\partial f}{\partial t} = n_e \sigma_T c \left(\frac{kT_e}{mc^2}\right) \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left(\frac{\partial f}{\partial x} + f(1+f)\right)\right] \qquad x = \hbar\omega/kT_e$$

takes electrons as thermal

$$f_e = e^{-(m-\mu)/T_e} e^{-q^2/2mT_e} \qquad \left[n_e = e^{-(m-\mu)/T_e} \left(\frac{mT_e}{2\pi}\right)^{3/2}\right]$$

$$= \left(\frac{2\pi}{mT_e}\right)^{3/2} n_e e^{-q^2/2mT_e}$$

and assumes that the energy transfer is small (non-relativistic electrons, $E_i \ll m$

$$\frac{E_f - E_i}{E_i} \ll 1 \qquad [\mathcal{O}(T_e/m, E_i/m)]$$

Kompaneets Equation

- Equilibrium solution must be a Bose-Einstein distribution since Compton scattering does not change photon number
- Rate of energy exchange obtained from integrating the energy $\times$ Kompeneets equation over momentum states

$$\frac{\partial u}{\partial t} = 4 n_e \sigma_T c \frac{kT_e}{mc^2}\left[1 - \frac{T_\gamma}{T_e}\right] u$$

$$\frac{1}{u}\frac{\partial u}{\partial t} = 4 n_e \sigma_T c \frac{k(T_e - T_\gamma)}{mc^2}$$

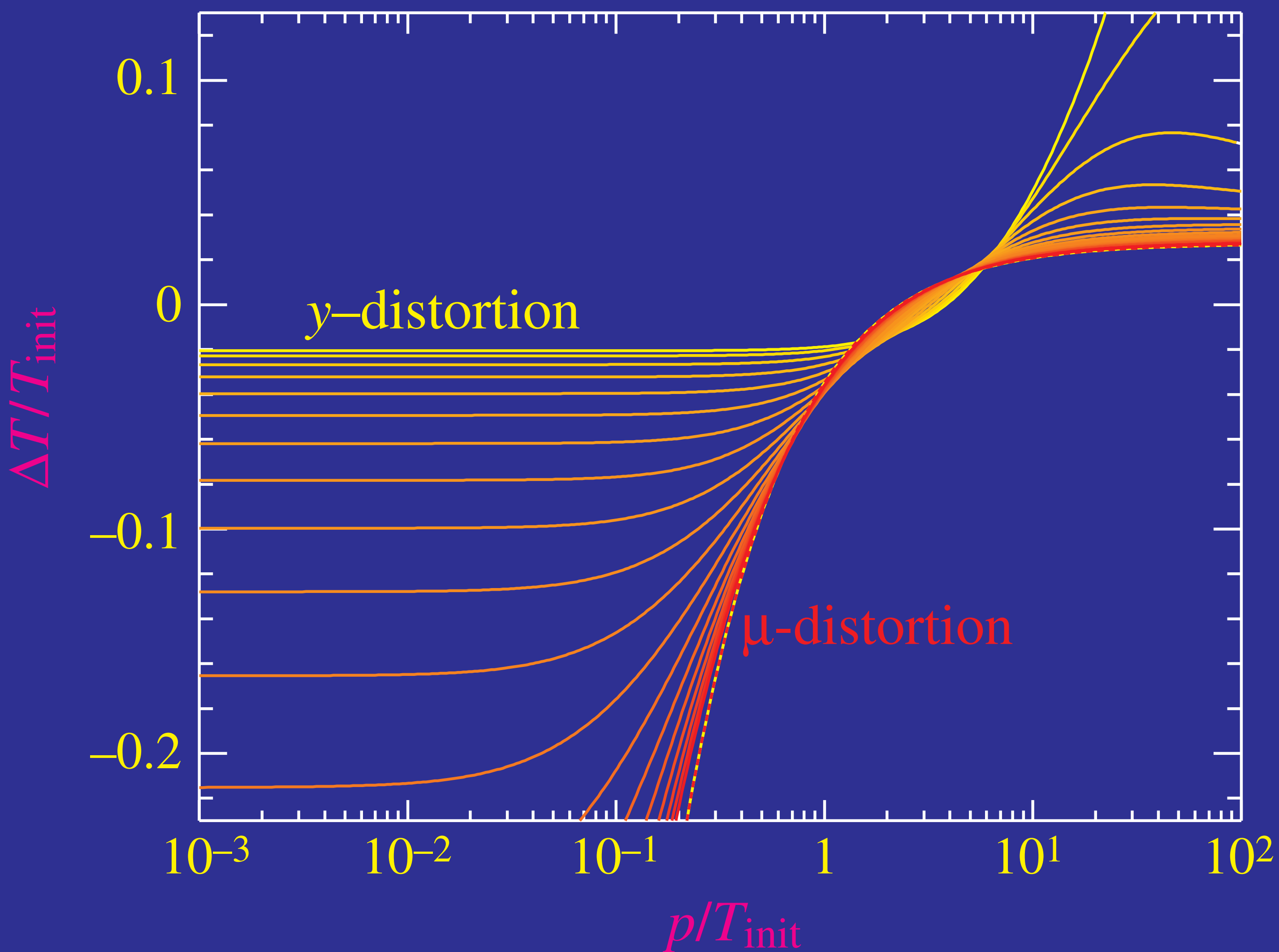
- The analogue to the optical depth for energy transfer is the Compton y parameter

$$d\tau = n_e \sigma_T ds = n_e \sigma_t c dt$$

$$dy = \frac{k(T_e - T_\gamma)}{mc^2} d\tau$$

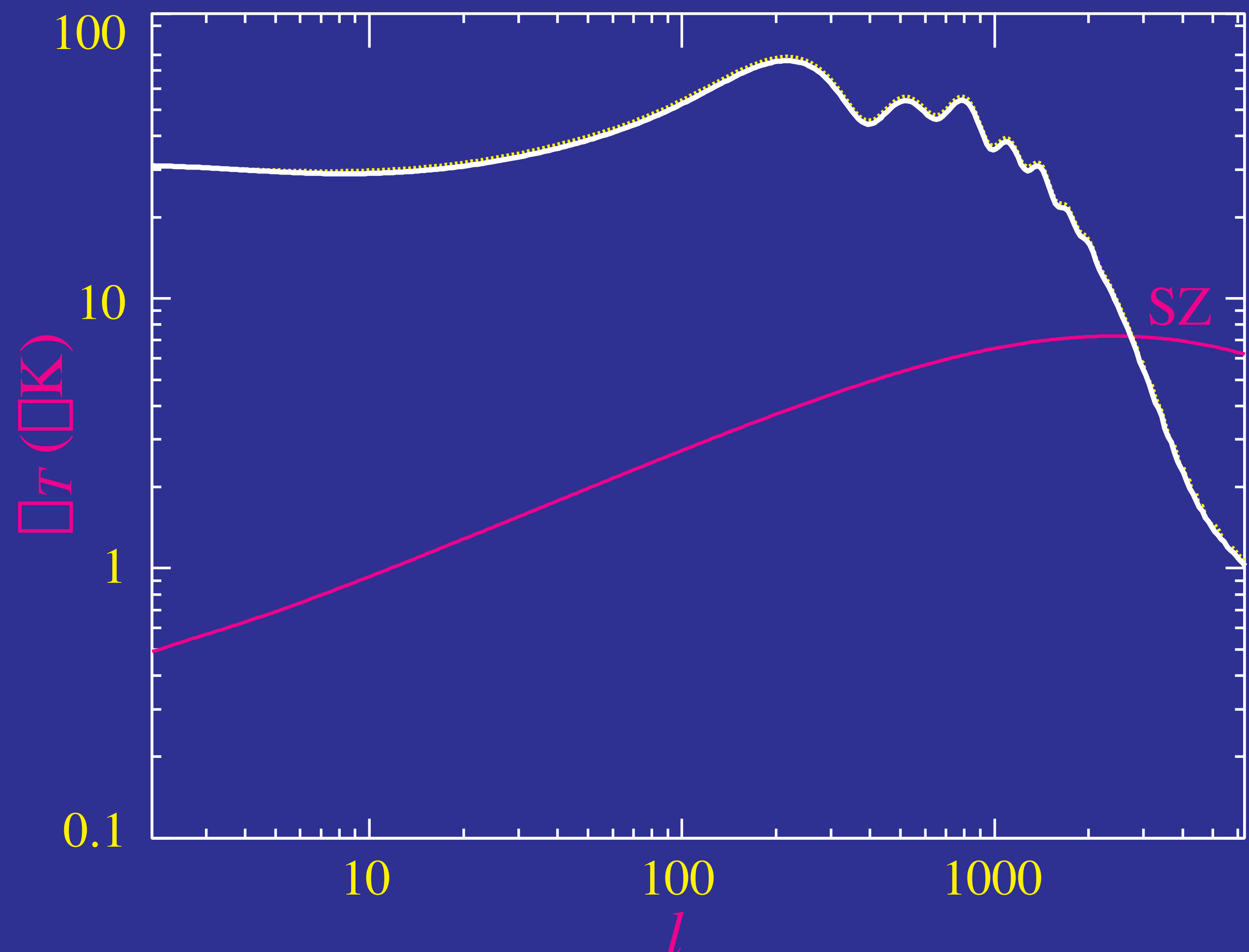
Spectral Distortion

- Compton upscattering: *y*–distortion
- Redistribution: μ-distortion

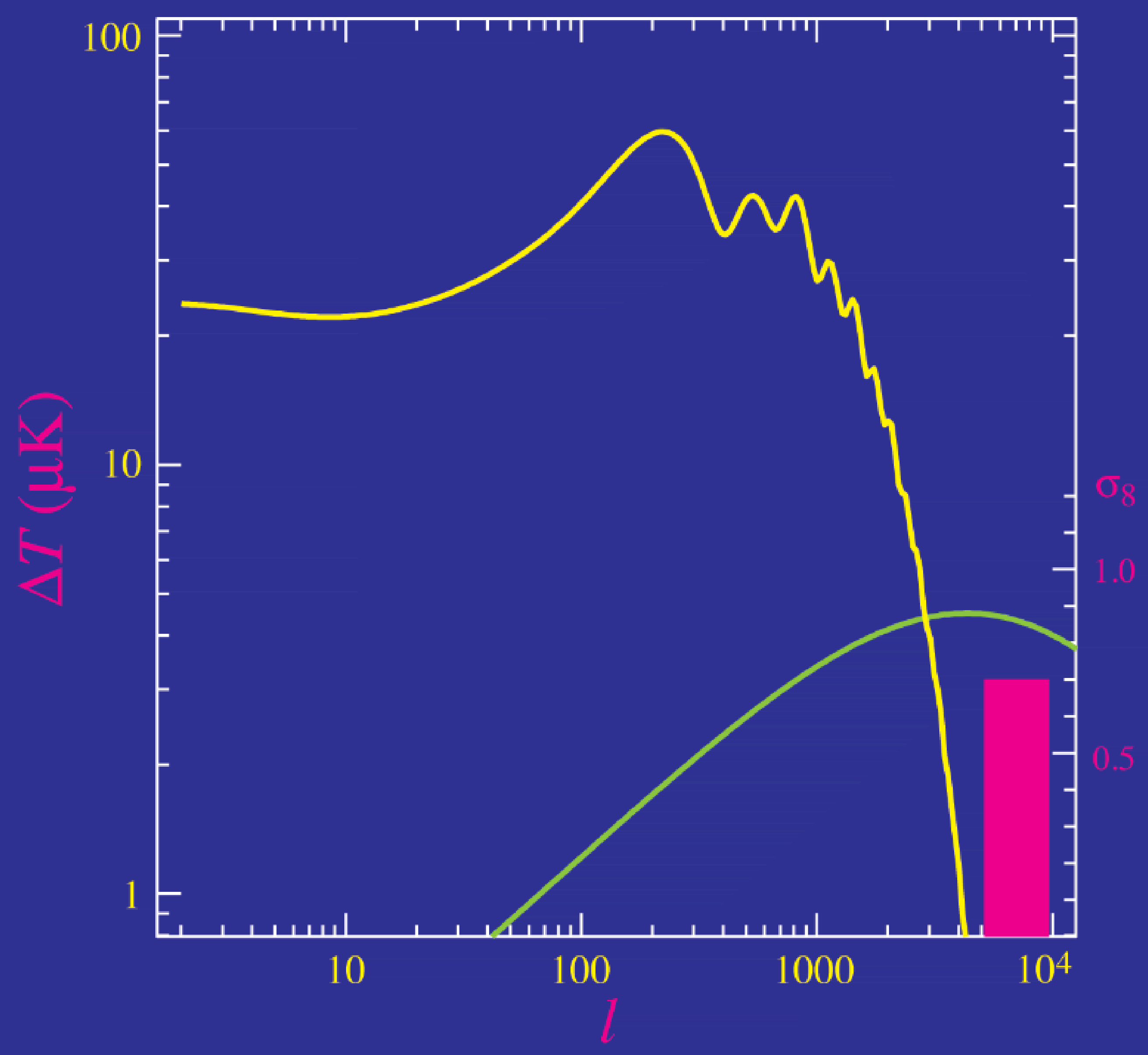


Thermal SZ Effect

- Second order Doppler effect escapes cancellation
- Velocities: thermal velocities in a hot cluster (1-10keV)
- Dominant source of arcminute anisotropies – turns over as clusters are resolved

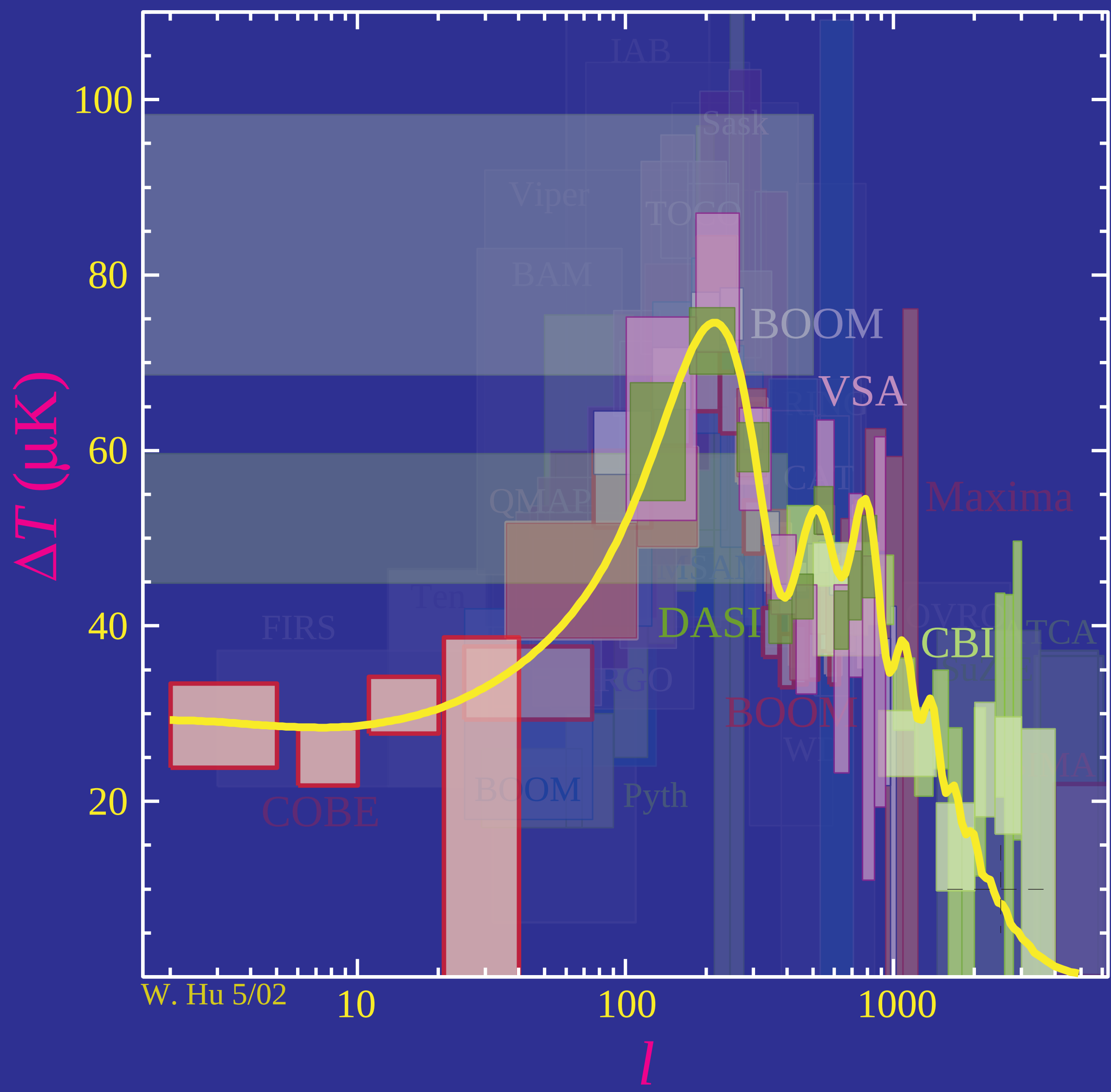


Amplitude of Fluctuations

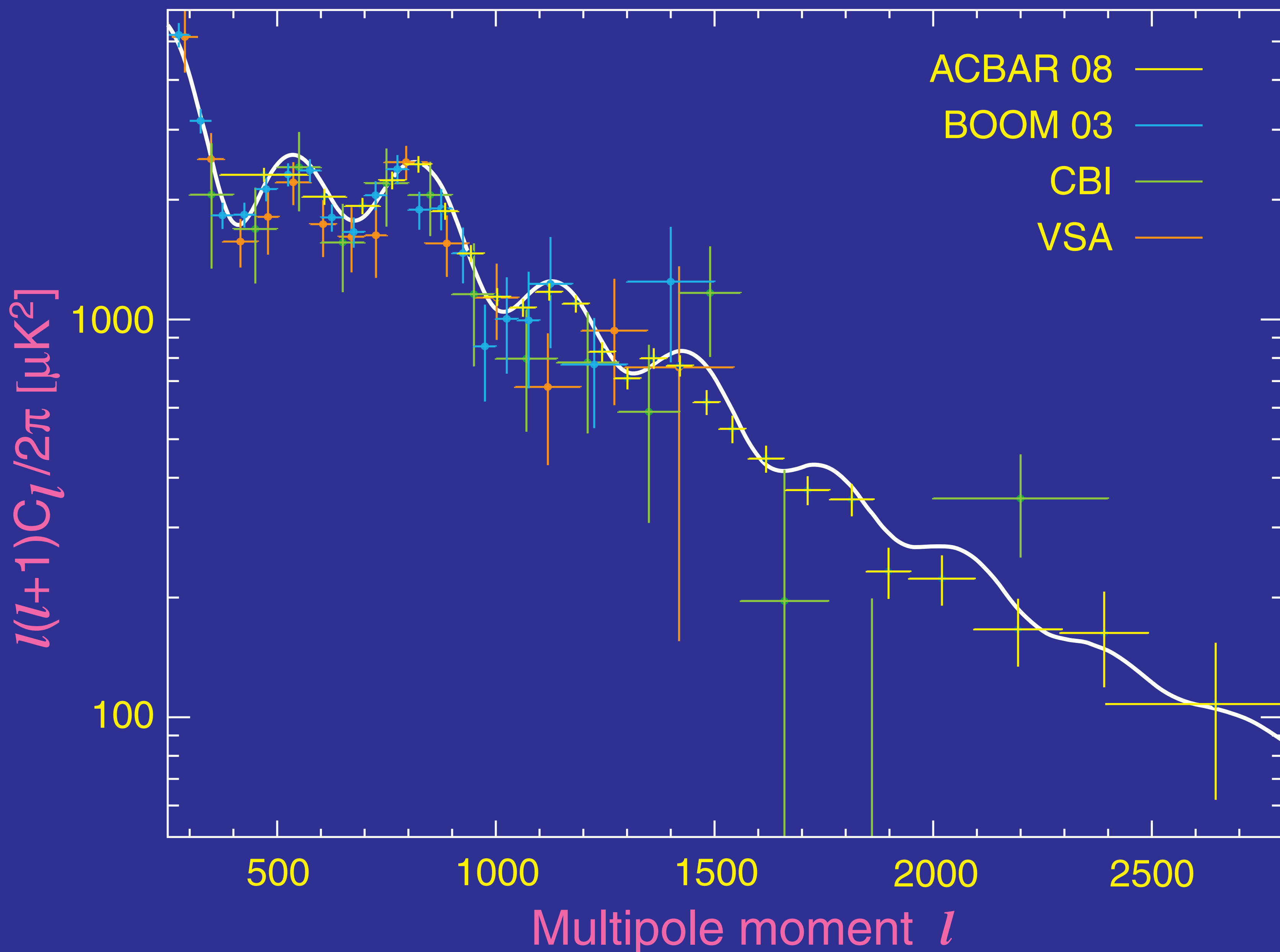


Clusters in Power Spectrum?

- Excess in arcminute scale CMB anisotropy from CBI

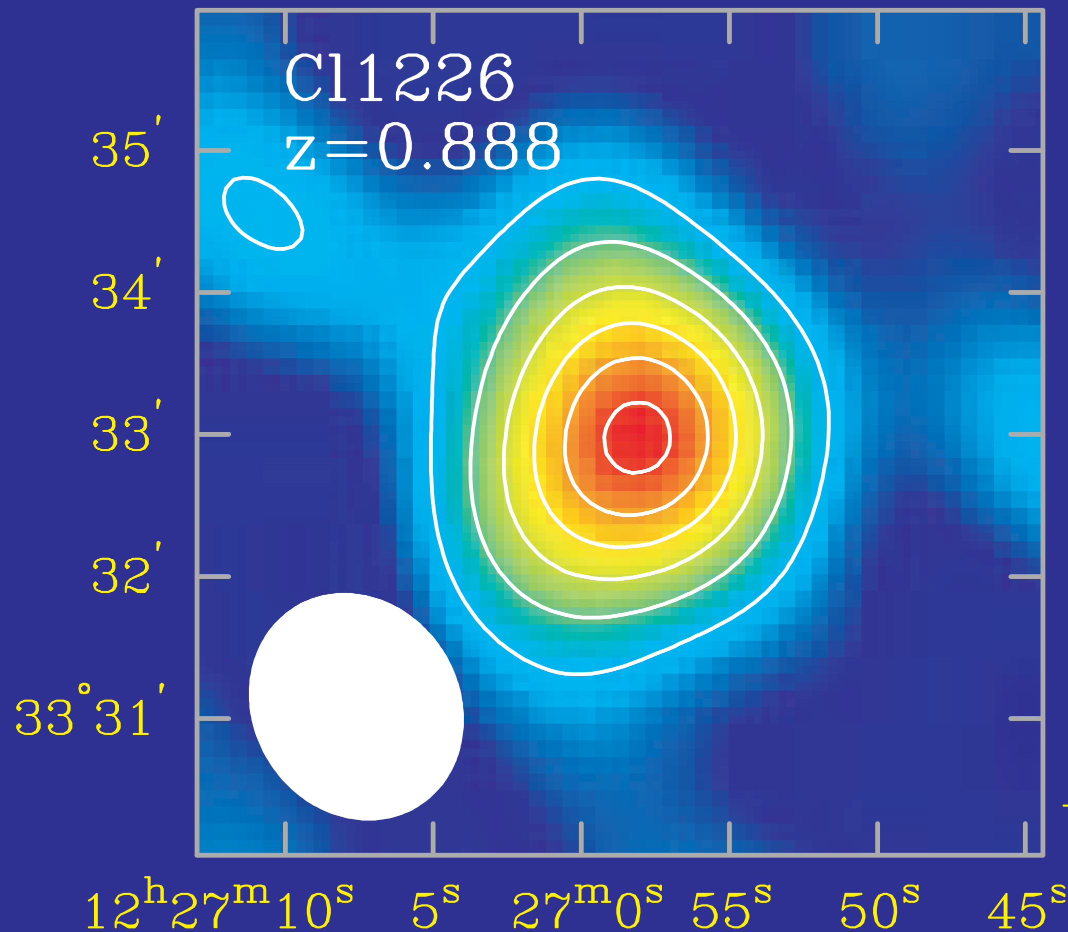


Power Spectrum Present



Counting Halos for Dark Energy

- Number density of massive halos extremely sensitive to the growth of structure and hence the dark energy
- Massive halos can be identified by the hot gas they contain

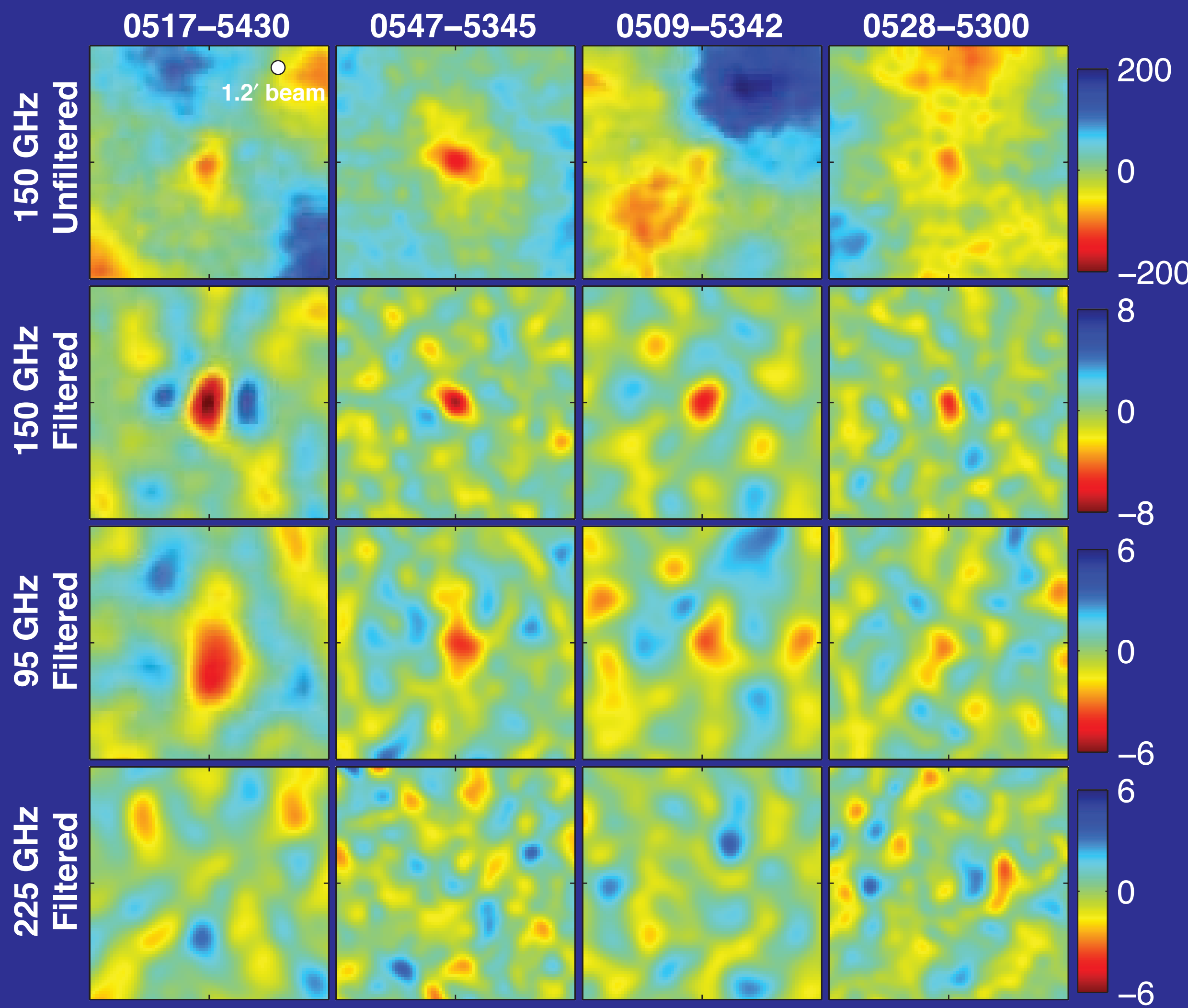


see Jack Sayers' talk

Carlstrom et al. (2001)

SPT Discovered Clusters

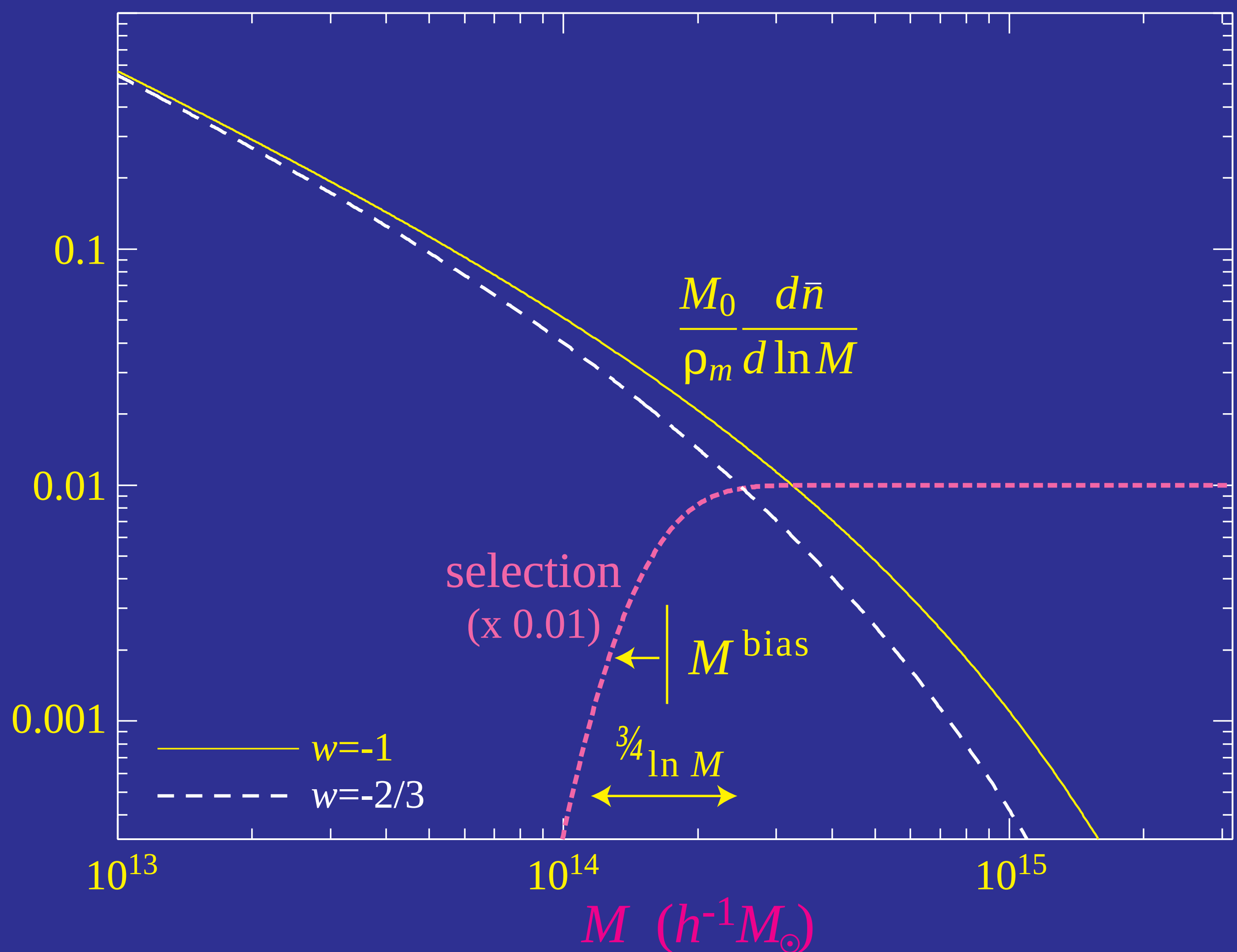
- Previously unknown clusters



Staniszewski et al (2008)

Mass-Observable Degeneracy

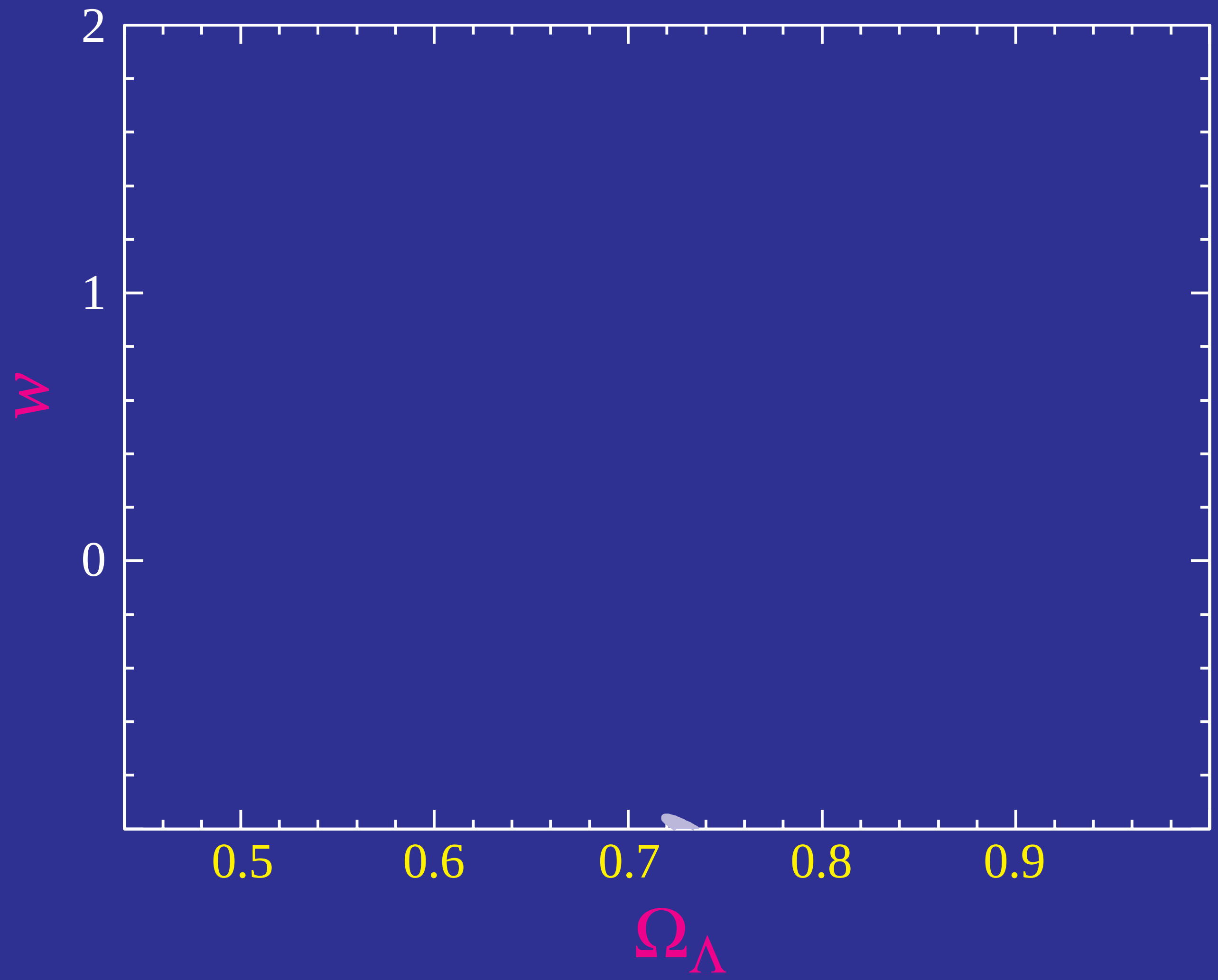
- Uncertainties in bias and scatter cause degeneracies with dark energy



see talks of
Carlos Cunha
Alexie Leauthaud

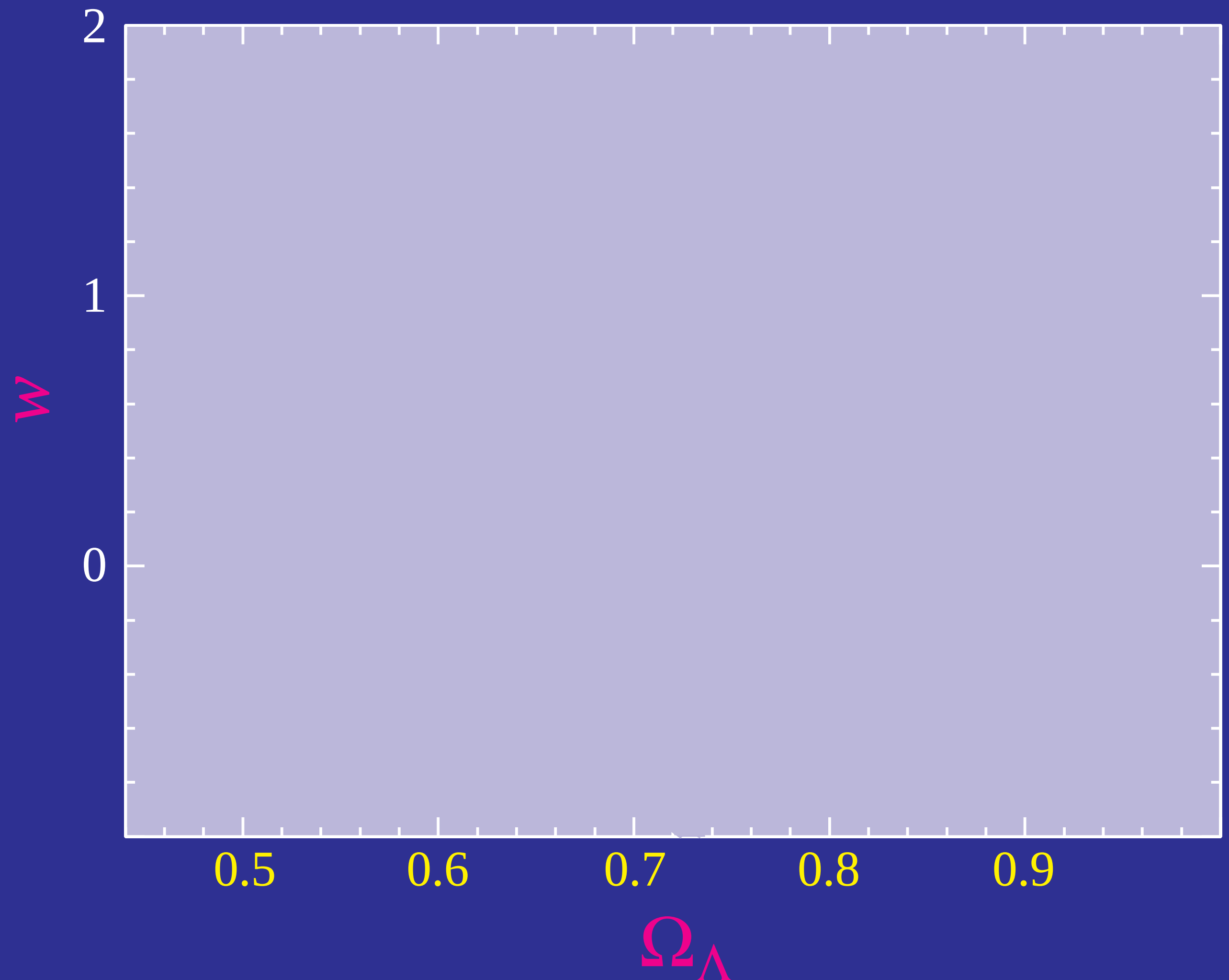
Fully Calibrated

- Given a completely known observable-mass distribution dark energy constraints are quite tight (4000 sq deg, z<2)



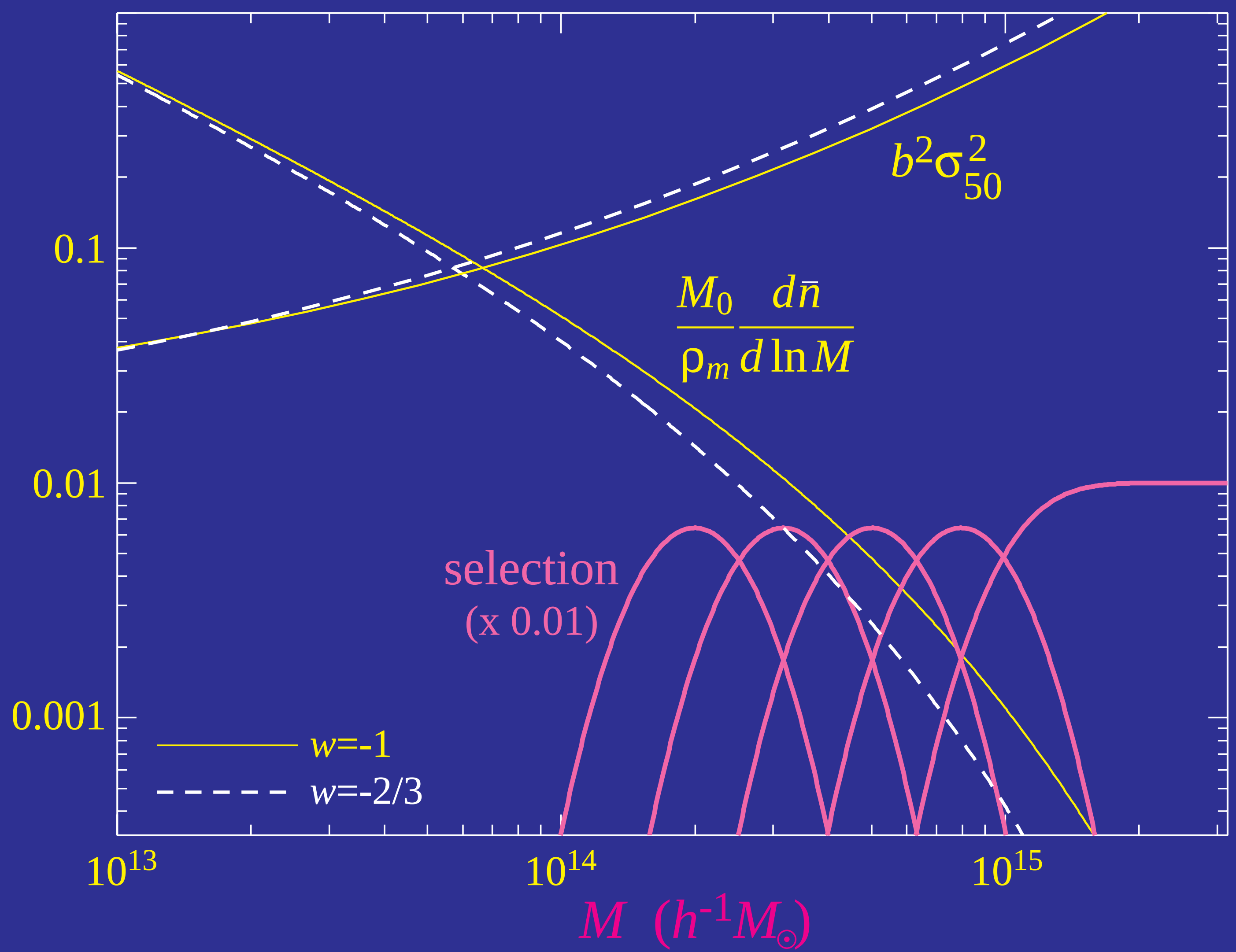
Un-Calibrated

- Marginalizing scatter (linear *z* evolution) and bias (power law evolution) destroys all dark energy information



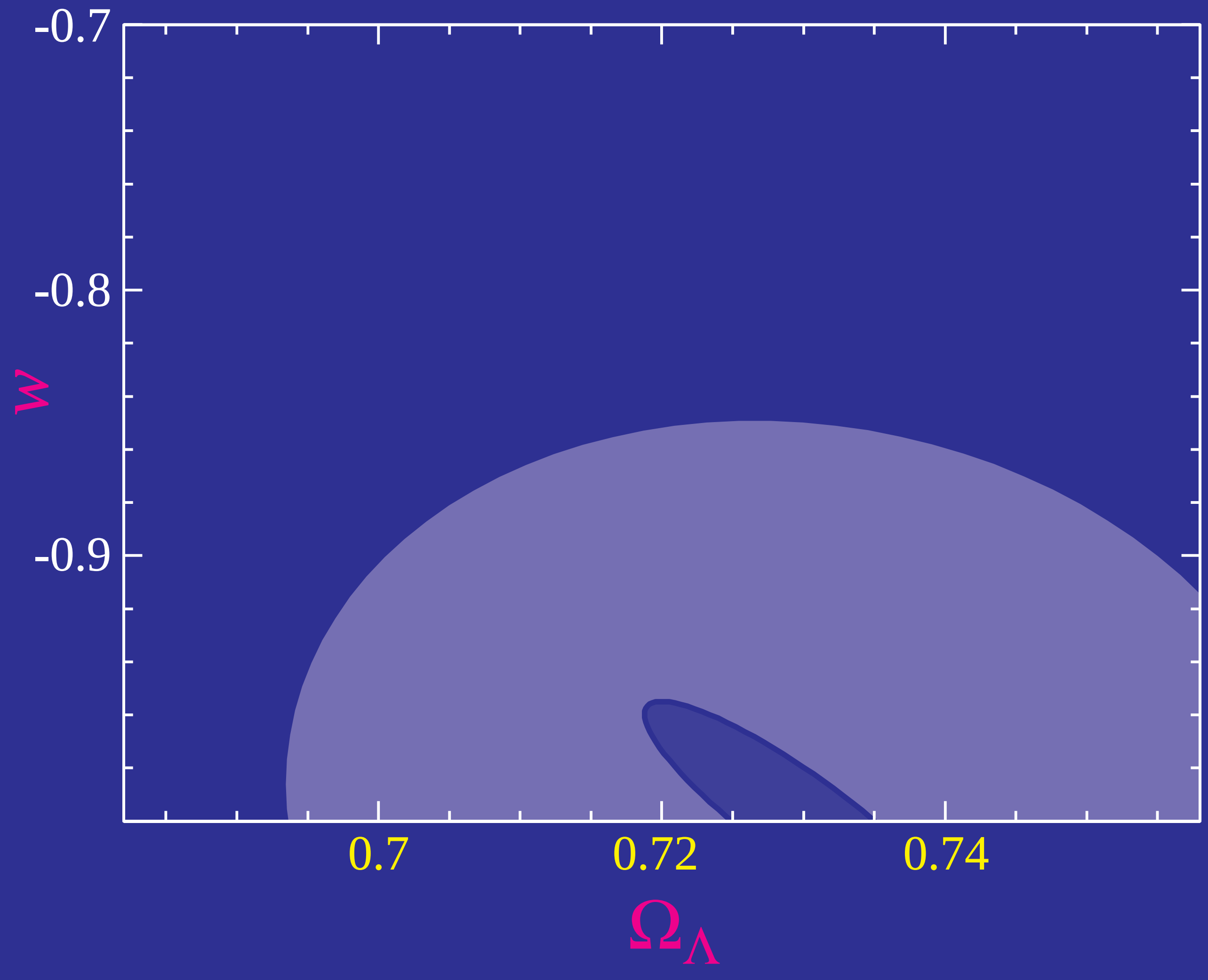
Joint Self-Calibration

- Both counts and their variance as a function of binned observable
- Many observables allows for a joint solution of a mass independent bias and scatter with cosmology



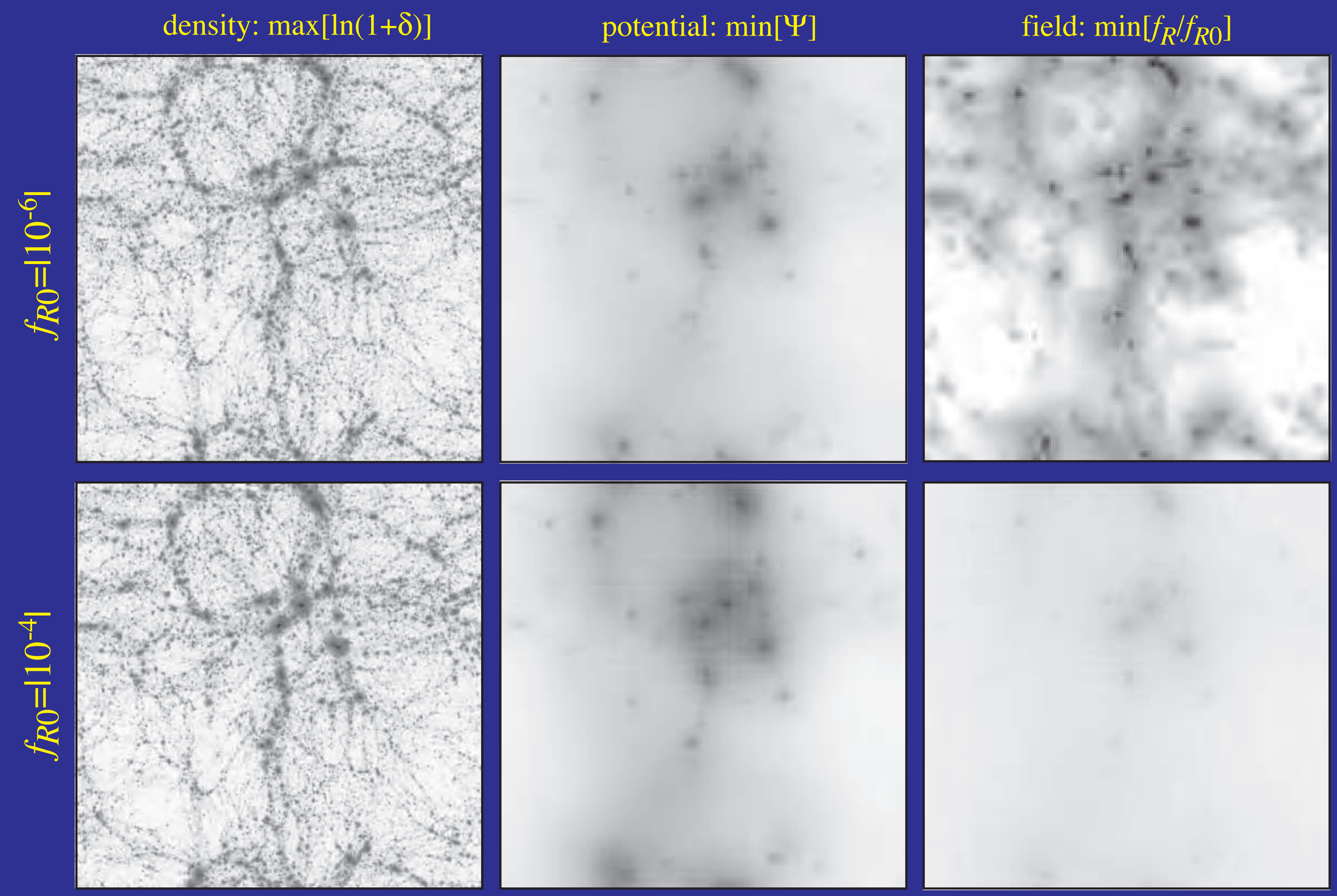
Joint Self Calibration

- Power law evolution of bias and cubic evolution of scatter in z



Modified Gravity $f(R)$ Simulations

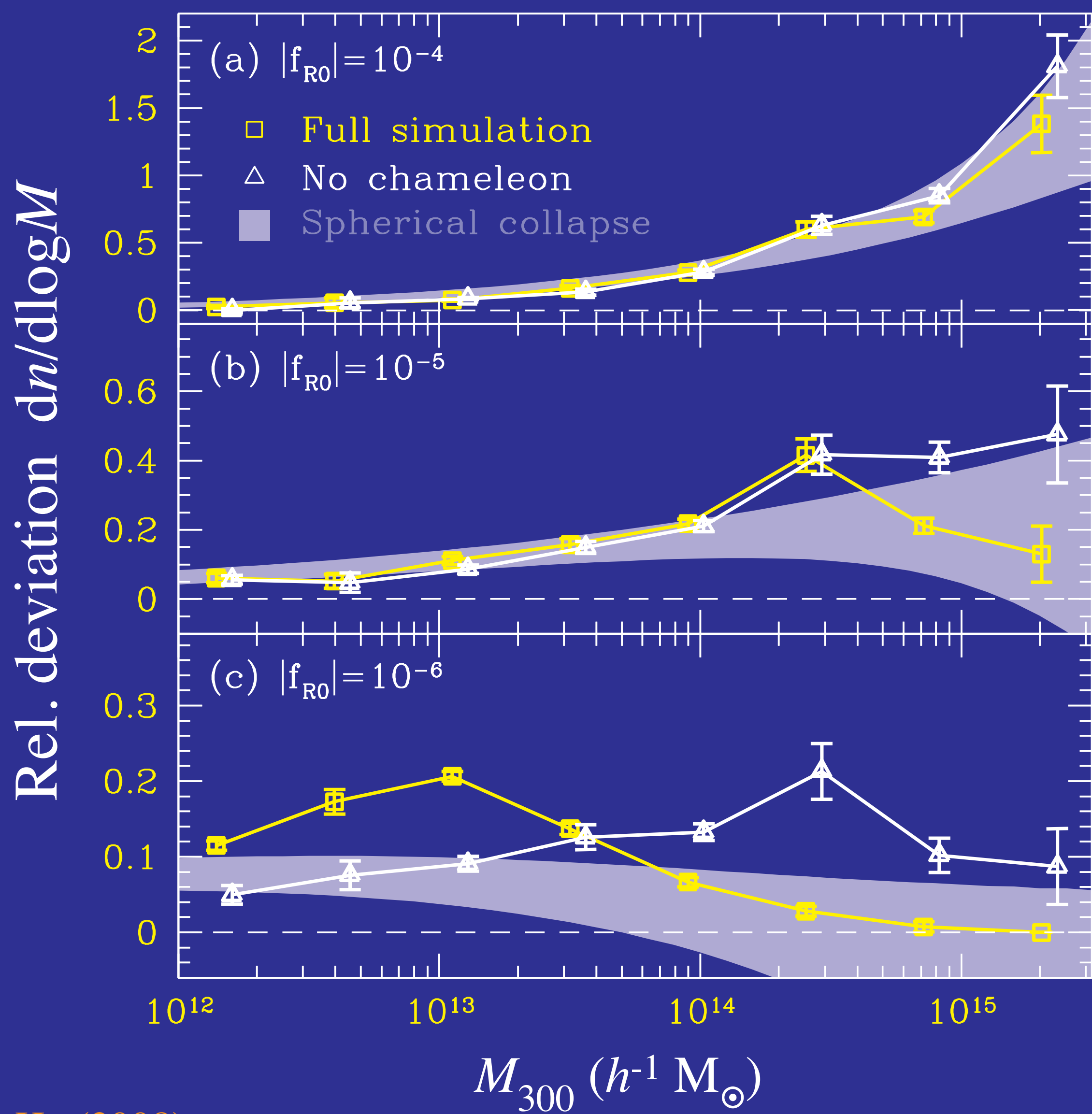
- For large background field, compared with potential depth, enhanced forces and structure



Oyaizu, Lima, Hu (2008)

Mass Function

- Enhanced abundance of rare dark matter halos (clusters) with extra force



Schmidt, Lima, Oyaizu, Hu (2008)

Summary: Lecture III

- Differential gravitational redshifts from evolving structure causes integrated Sachs-Wolfe (ISW) effect
- Appears on large angles and contributes to quadrupole comparably to primary
- Tests the microphysics of acceleration: clustering of dark energy, modified gravity, dark matter interactions
- Compton scattering leads to energy transfer and thermal SZ effect to second order in velocity
- Unresolved gas clumps generate excess arcminute power
- Resolved clusters provide sensitive test of microphysics of acceleration through counts if masses calibrated

Thanks to the Organizers

...setting sail for Cancun...