Secondary CMB Anisotropy

III: Dark Energy

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Integrated Sachs-Wolfe Effect
Smooth Energy Density & Potential Decay

- Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature $\zeta$)
Smooth Energy Density & Potential Decay

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• A smooth component contributes density $\rho$ to the expansion but not density fluctuation $\delta \rho$ to the Poisson equation

• Imbalance causes potential to decay once smooth component dominates the expansion
Smooth Energy Density & Potential Decay

• Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature $\zeta$).

• A smooth component contributes density $\rho$ to the expansion but not density fluctuation $\delta \rho$ to the Poisson equation.

• Imbalance causes potential to decay once smooth component dominates the expansion.

• Scalar field dark energy (quintessence) is smooth out to the horizon scale (sound speed $c_s=1$).

• Potential decay measures the clustering properties and hence the particle properties of the dark energy.
ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: $\Delta T/T=2\Delta \Phi$
- Effect from potential hills and wells cancel on small scales
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ISW-Galaxy Correlation

- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation
ISW-Galaxy Correlation

- $\sim 4\sigma$ joint detection of ISW correlation with large scale structure (galaxies)
- $\sim 2\sigma$ high compared with $\Lambda$CDM
Dark Energy

- Peaks measure distance to recombination
- ISW effect constrains dynamics of acceleration

\[ \Omega_{DE} \]

\[ w_{DE} \]

\[ \frac{1}{l(l+1)}C_l/2\pi^{1/2} \] (\(\mu\text{K}\))

\[ l \]

\[ l \]
ISW Effect and Dark Energy

- Raising equation of state increases redshift of dark energy domination and raises the ISW effect
- Lowering the sound speed increases clustering and reduces ISW effect at large angles

\[ w = -1 \]
\[ w = -2/3 \]
\[ c_{\text{eff}} = 1 \]
\[ c_{\text{eff}} = 1/3 \]
Cosmic Variance Problem

- Power spectrum sampling errors = \[\left(\frac{l+1/2}{f_{\text{sky}}}\right)^{-1/2}\]
- Low multipole effects severely cosmic variance limited

![Graph showing ISW and degeneracy with Planck and Ideal models with different w values: w=-1 and w=-2/3]
Moving Halo Effect
Moving Halo Effect

- Change in potential due to halo moving across the line of sight
Modified Action $f(R)$ Model

- $R$: Ricci scalar or “curvature”
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right] \]

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f / dR^2$: Compton wavelength of $f_R$ squared, inverse mass squared
- $B$: Compton wavelength of $f_R$ squared in units of the Hubble length

\[ B \equiv \frac{f_{RR}}{1 + f_R R'} \frac{H}{H'} \]

- $' \equiv d/d \ln a$: scale factor as time coordinate
PPF $f(R)$ Description

- **Metric and matter evolution well-matched by PPF description**
- **Standard GR tools apply (CAMB), self-consistent, gauge invar.**

Hu & Sawicki (2007); Hu (2008)
ISW Quadrupole

- Reduction of large angle anisotropy for $B_0 \sim 1$ for same expansion history and distances as $\Lambda$CDM
- Well-tested small scale anisotropy unchanged

$C_l \equiv l(l+1)C_l / 2\pi$ (µK$^2$)

$B_0 \ (\Lambda$CDM $)$

Song, Hu & Sawicki (2006)
DGP Braneworld Acceleration

- **Braneworld acceleration** *(Dvali, Gabadadze & Porrati 2000)*

\[
S = \int d^5x \sqrt{-g} \left[ \frac{(5)R}{2\kappa^2} + \delta(\chi) \left( \frac{(4)R}{2\mu^2} + \mathcal{L}_m \right) \right]
\]

with crossover scale \( r_c = \frac{\kappa^2}{2\mu^2} \)

- Influence of bulk through **Weyl tensor anisotropy** - solve **master equation** in bulk *(Deffayet 2001)*

- Matter still **minimally coupled** and conserved

- Exhibits the 3 regimes of modified gravity

- **Weyl tensor anisotropy** dominated conserved curvature regime \( r > r_c \) *(Sawicki, Song, Hu 2006; Cardoso et al 2007)*

- **Brane bending** scalar tensor regime \( r_* < r < r_c \) *(Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)*

- **Strong coupling** General Relativistic regime \( r < r_* = \left( r_c r_g \right)^{1/3} \)

where \( r_g = 2GM \) *(Dvali 2006)*
DGP Horizon Scales

- Metric and matter evolution well-matched by PPF description
- Standard GR tools apply (CAMB), self-consistent, gauge invar.

Hu & Sawicki (2007); Hu (2008)
DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background
CMB in DGP

- Adding cut off as an epicycle can fix distances, ISW problem
- Suppresses polarization in violation of EE data - cannot save DGP!

Fang et al (2008)
CMB in DGP

- Adding **cut off** as an epicycle can fix distances, ISW problem
- Suppresses **polarization** in violation of EE data - cannot save DGP!

Fang et al (2008)
Galaxy-ISW Anti-Correlation

- Large Compton wavelength $B^{1/2}$ creates potential growth which can anti-correlate galaxies and the CMB
- In tension with detections of positive correlations across a range of redshifts

Song, Peiris & Hu (2007)
Thermal SZ Effect
Counting Halos for Dark Energy

- Number density of massive halos extremely sensitive to the growth of structure and hence the dark energy
- Potentially %-level precision in dark energy equation of state

Haiman, Holder & Mohr (2001)
Beyond Thomson Limit

- Thomson scattering $e_i + \gamma_i \rightarrow e_f + \gamma_f$ in rest frame where the frequencies $\omega_i = \omega_f$ (elastic scattering) cannot strictly be true.

- Photons carry off $E/c$ momentum and so to conserve momentum the electron must recoil.

- Doppler shift from transformation from rest frame contains second order terms.

- General case (arbitrary electron velocity)
Energy-Momentum Conservation

- From energy-momentum conservation, the energy change is

\[
\frac{E_f}{E_i} = \frac{1 - \beta_i \cos \alpha_i}{1 - \beta_i \cos \alpha_f + \frac{E_i}{\gamma mc^2}(1 - \cos \theta)}
\]

where \( \hat{n}_f \cdot \mathbf{v}_i = v_i \cos \alpha_f \) and \( \hat{n}_i \cdot \mathbf{v}_i = v_i \cos \alpha_i \)

- Two ways of changing the energy: Doppler boost \( \beta_i \) from incoming electron velocity and \( E_i \) non-negligible compared to \( \gamma mc^2 \)

- Isolate recoil in incoming electron rest frame \( \beta_i = 0 \) and \( \gamma = 1 \)

\[
\left| \frac{E_f}{E_i} \right|_{\text{rest}} = \frac{1}{1 + \frac{E_i}{mc^2}(1 - \cos \theta)}
\]
Recoil Effect

- Since $-1 \leq \cos \theta \leq 1$, $E_f \leq E_i$, energy is lost from the recoil except for purely forward scattering.

- The backwards scattering limit is easy to see:

\[ |q_f| = m|v_f| = \frac{2E_i}{c}, \]

\[ \Delta E = \frac{1}{2}mv_f^2 = \frac{1}{2}m \left( \frac{2E_i}{mc} \right)^2 = 2 \frac{E_i}{mc^2} E_i \]

\[ E_f = E_i - \Delta E = \left( 1 - 2 \frac{E_i}{mc^2} \right) E_i \approx \frac{E_i}{1 + 2 \frac{E_i}{mc^2}} \]
Second Order Doppler Shift

- **Doppler effect:** consider the limit of $\beta_i \ll 1$ then expand to first order

$$\frac{E_f}{E_i} = 1 - \beta_i \cos \alpha_i + \beta_i \cos \alpha_f - \frac{E_i}{mc^2}(1 - \cos \theta)$$

however **averaging over angles** the Doppler shifts don’t change the energies

- **To second order** in the velocities, the Doppler shift transfers energy from the electron to the photon in opposition to the recoil

$$\frac{E_f}{E_i} = 1 - \beta_i \cos \alpha_i + \beta_i \cos \alpha_f + \beta_i^2 \cos^2 \alpha_f - \frac{E_i}{mc^2}$$

$$\langle \frac{E_f}{E_i} \rangle \approx 1 + \frac{1}{3} \beta_i^2 - \frac{E_i}{mc^2}$$
Thermal SZ Effect

Large Scale Structure

Observer

e⁻ velocity
overdensity, cluster...

unscattered γ
upscattered γ
Thermalization

- For a thermal distribution of velocities

\[
\frac{1}{2} m \langle v^2 \rangle = \frac{3kT}{2}, \quad \beta_i^2 \approx \frac{3kT}{mc^2} \rightarrow \left\langle \frac{E_f}{E_i} - 1 \right\rangle \approx \frac{kT - E_i}{mc^2}
\]

so that if \( E_i \ll kT \) the photon gains energy and \( E_i \gg kT \) it loses energy → this is a thermalization process.
Kompaneets Equation

- Radiative transfer or Boltzmann equation

\[
\frac{\partial f}{\partial t} = \frac{1}{2E(p_f)} \int \frac{d^3p_i}{(2\pi)^3} \frac{1}{2E(p_i)} \int \frac{d^3q_f}{(2\pi)^3} \frac{1}{2E(q_f)} \int \frac{d^3q_i}{(2\pi)^3} \frac{1}{2E(q_i)} \\
\times (2\pi)^4 \delta(p_f + q_f - p_i - q_i) |M|^2 \\
\times \{f_e(q_i)f(p_i)[1 + f(p_f)] - f_e(q_f)f(p_f)[1 + f(p_i)]\}
\]

- Matrix element is calculated in field theory and is Lorentz invariant. In terms of the rest frame \(\alpha = e^2/\hbar c\) (Klein Nishina Cross Section)

\[
|M|^2 = 2(4\pi)^2 \alpha^2 \left[ \frac{E(p_i)}{E(p_f)} + \frac{E(p_f)}{E(p_i)} - \sin^2 \beta \right]
\]

with \(\beta\) as the rest frame scattering angle.
Kompaneets Equation

- The Kompaneets equation \((\hbar = c = 1)\)

\[
\frac{\partial f}{\partial t} = n_e \sigma_T c \left( \frac{kT_e}{mc^2} \right) \frac{1}{x^2} \frac{\partial}{\partial x} \left[ x^4 \left( \frac{\partial f}{\partial x} + f(1 + f) \right) \right]
\]

\[ x = \frac{\hbar \omega}{kT_e} \]

takes electrons as thermal

\[
f_e = e^{-(m-\mu)/T_e} e^{-q^2/2mT_e}
\]

\[
n_e = e^{-(m-\mu)/T_e} \left( \frac{mT_e}{2\pi} \right)^{3/2}
\]

\[
= \left( \frac{2\pi}{mT_e} \right)^{3/2} n_e e^{-q^2/2mT_e}
\]

and assumes that the energy transfer is small (non-relativistic electrons, \(E_i \ll m\))

\[
\frac{E_f - E_i}{E_i} \ll 1 \quad [\mathcal{O}(T_e/m, E_i/m)]
\]
Kompaneets Equation

- Equilibrium solution must be a Bose-Einstein distribution since Compton scattering does not change photon number

- Rate of energy exchange obtained from integrating the energy \times Kompaneets equation over momentum states

\[ \frac{\partial u}{\partial t} = 4n_e\sigma_T c \frac{kT_e}{mc^2} \left[ 1 - \frac{T_\gamma}{T_e} \right] u \]

\[ \frac{1}{u} \frac{\partial u}{\partial t} = 4n_e\sigma_T c \frac{k(T_e - T_\gamma)}{mc^2} \]

- The analogue to the optical depth for energy transfer is the Compton \( y \) parameter

\[ d\tau = n_e\sigma_T ds = n_e\sigma_t c dt \]

\[ dy = \frac{k(T_e - T_\gamma)}{mc^2} d\tau \]
Spectral Distortion

- Compton upscattering: $\gamma$-distortion
- Redistribution: $\mu$-distortion
Thermal SZ Effect

- Second order Doppler effect escapes cancellation
- Velocities: **thermal velocities** in a hot cluster (1-10keV)
- **Dominant source** of arcminute anisotropies – turns over as clusters are resolved
Amplitude of Fluctuations

\[ \Delta T \text{ (\(\mu\)K)} \]

\[ l \]

\[ \sigma_8 \]

\[ 0.5 \]

\[ 1.0 \]

\[ 10 \]

\[ 100 \]

\[ 1000 \]

\[ 10^4 \]
Clusters in Power Spectrum?

- Excess in arcminute scale CMB anisotropy from CBI
Power Spectrum Present

\[ \frac{l(l+1)C_l}{2\pi} \text{ [\(\mu\text{K}^2\)]} \]

\[ \text{Multipole moment } l \]
Counting Halos for Dark Energy

- Number density of massive halos extremely sensitive to the growth of structure and hence the dark energy
- Massive halos can be identified by the hot gas they contain

Carlstrom et al. (2001)
Degeneracy

- Uncertainties in bias and scatter cause degeneracies with dark energy
Selection Bias

- Exponential tail of mass function
- Threshold cut in the observable mass

\[ M_{\text{obs}} = 10^{14.2} \]

\[ \frac{M_0 \, d\bar{n}}{\rho_m \, d\ln M} \]
- Given a completely **known** observable-mass distribution dark energy constraints are quite **tight** (4000 sq deg, $z<2$)
• Marginalizing scatter (linear $z$ evolution) and bias (power law evolution) destroys all dark energy information
Joint Self-Calibration

- Both \textit{counts} and their \textit{variance} as a function of \textit{binned observable}
- Many observables allows for a \textit{joint solution} of a mass independent bias and scatter with cosmology

\[
M_0 \frac{d\bar{n}}{\rho_m d\ln M}
\]

\[
b^2 \sigma^2_{50}
\]

\[
\frac{M_0}{\rho_m} d\bar{n} d\ln M
\]

\[
w = -\frac{2}{3}
\]

\[
w = -1
\]

\[
w = -\frac{2}{3}
\]

\[
M (h^{-1} M_\odot)
\]

Selection (x 0.01)
Joint Self Calibration

- Power law evolution of bias and cubic evolution of scatter in $z$