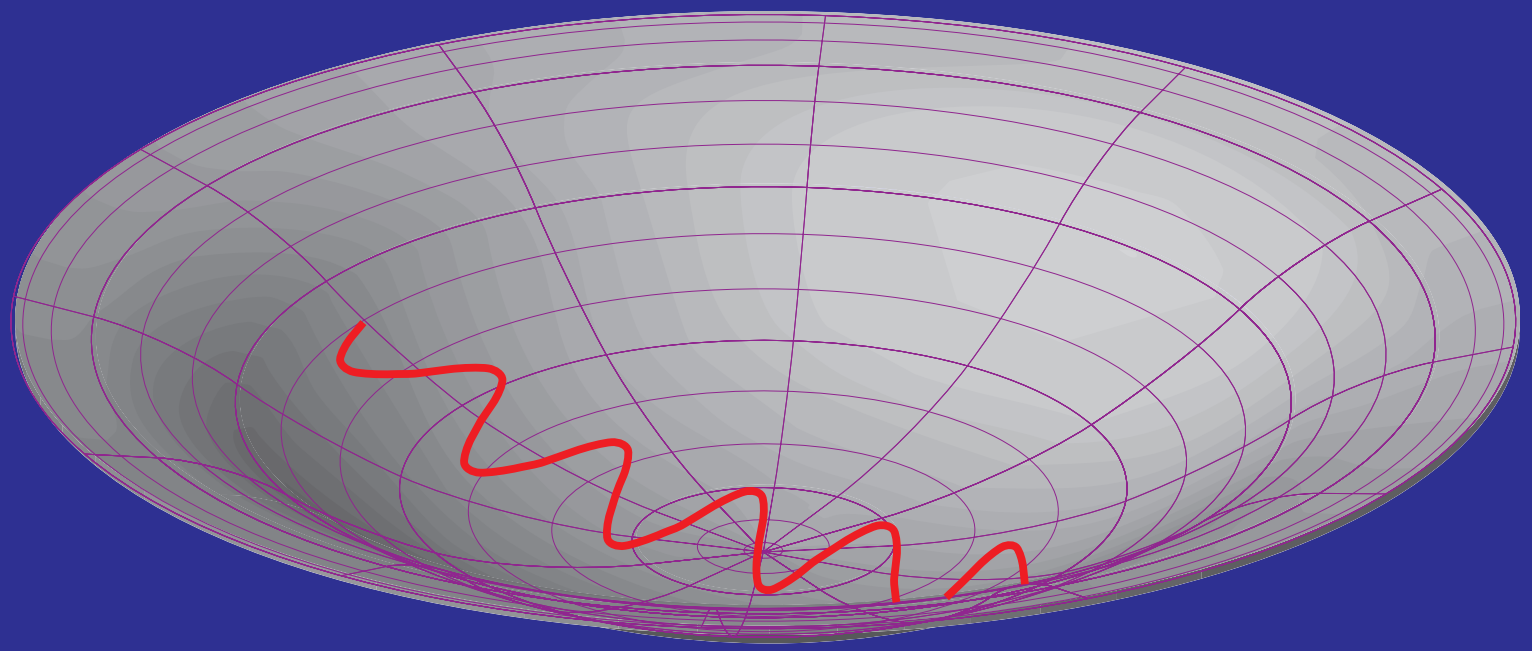
Secondary CMB Anisotropy



III: Dark Energy

Wayne Hu

Cabo, January 2009

Integrated Sachs-Wolfe Effect

Smooth Energy Density & Potential Decay

- Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature ζ)

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  but not

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- Imbalance causes potential to decay once smooth component dominates the expansion

Smooth Energy Density & Potential Decay

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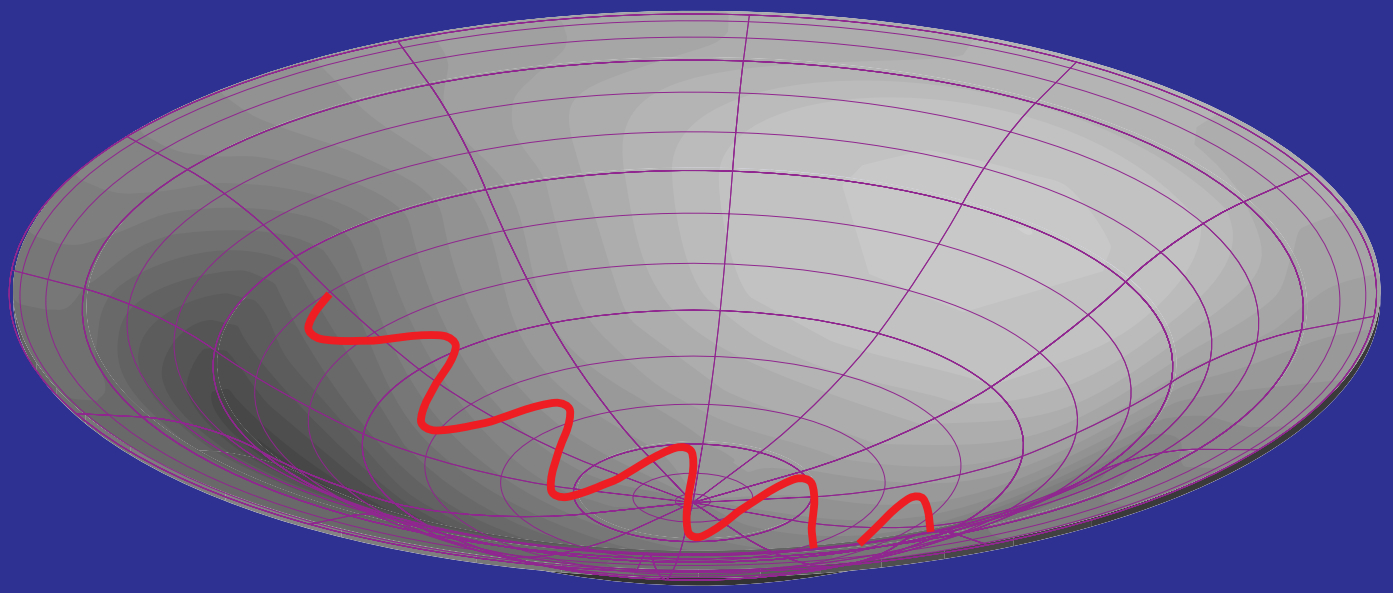
- A smooth component contributes
  density ρ to the expansion
  but not
  density fluctuation $\delta\rho$ to the Poisson equation
- Imbalance causes potential to decay once smooth component dominates the expansion
- Scalar field dark energy (quintessence) is smooth out to the horizon scale (sound speed c_s=1)
- Potential decay measures the clustering properties and hence the particle properties of the dark energy

ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: $\Delta T/T = 2\Delta\Phi$
- Effect from potential hills and wells cancel on small scales



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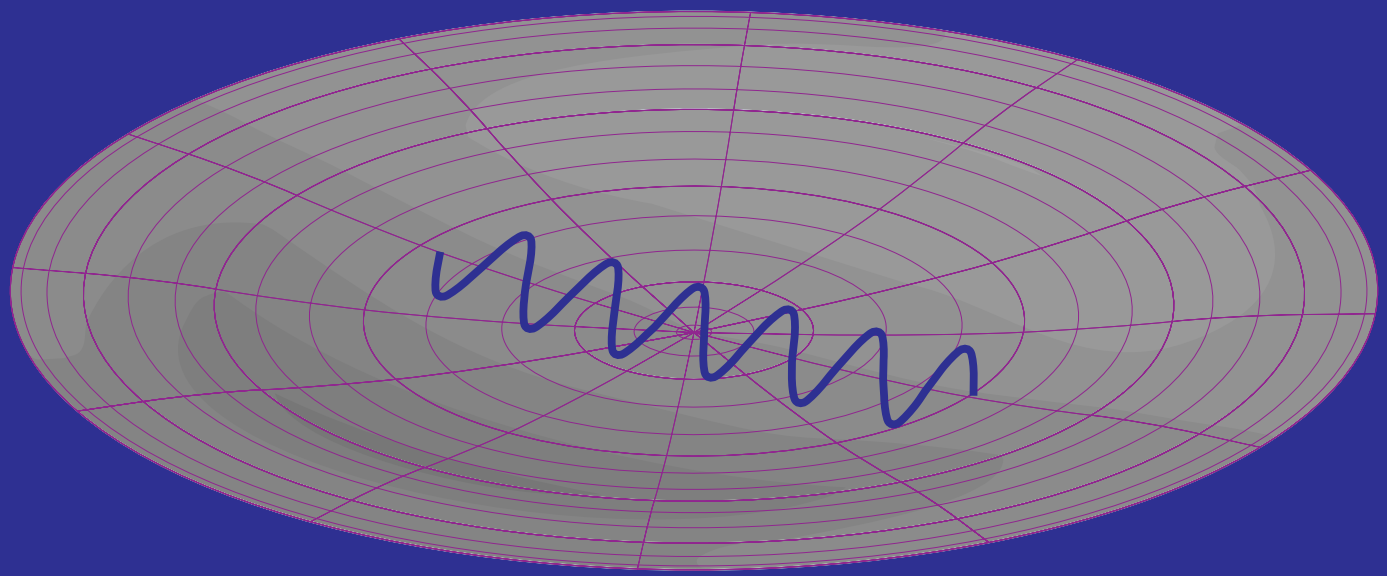
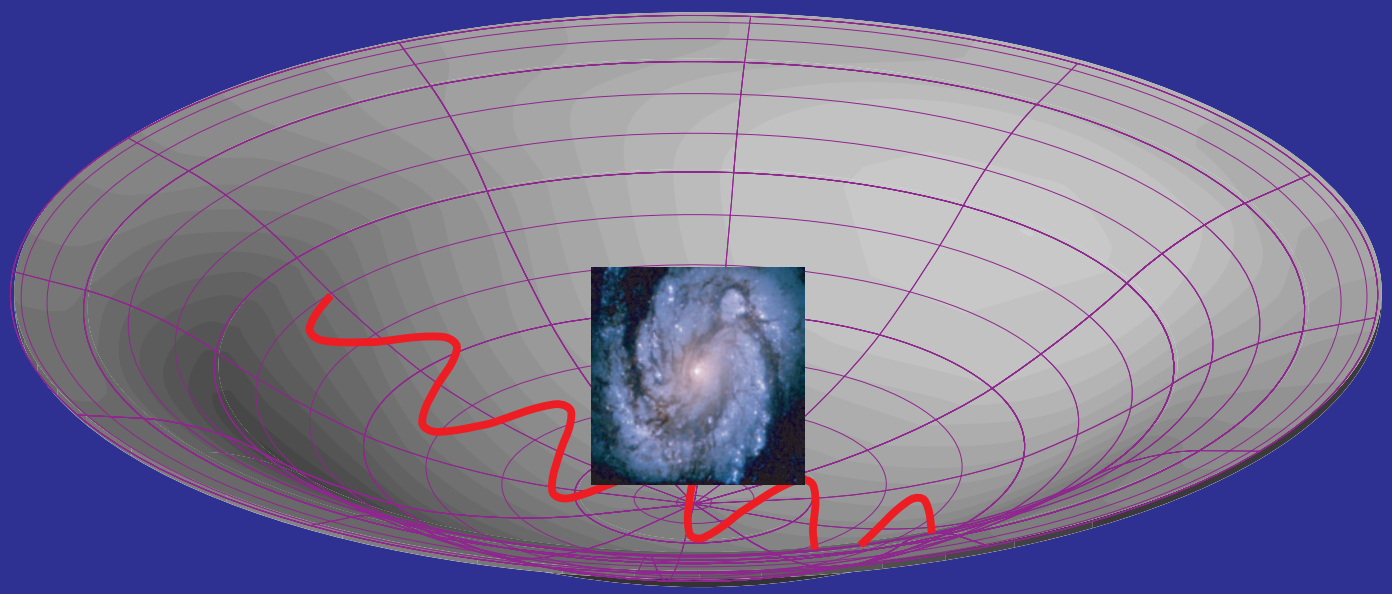


ISW-Galaxy Correlation

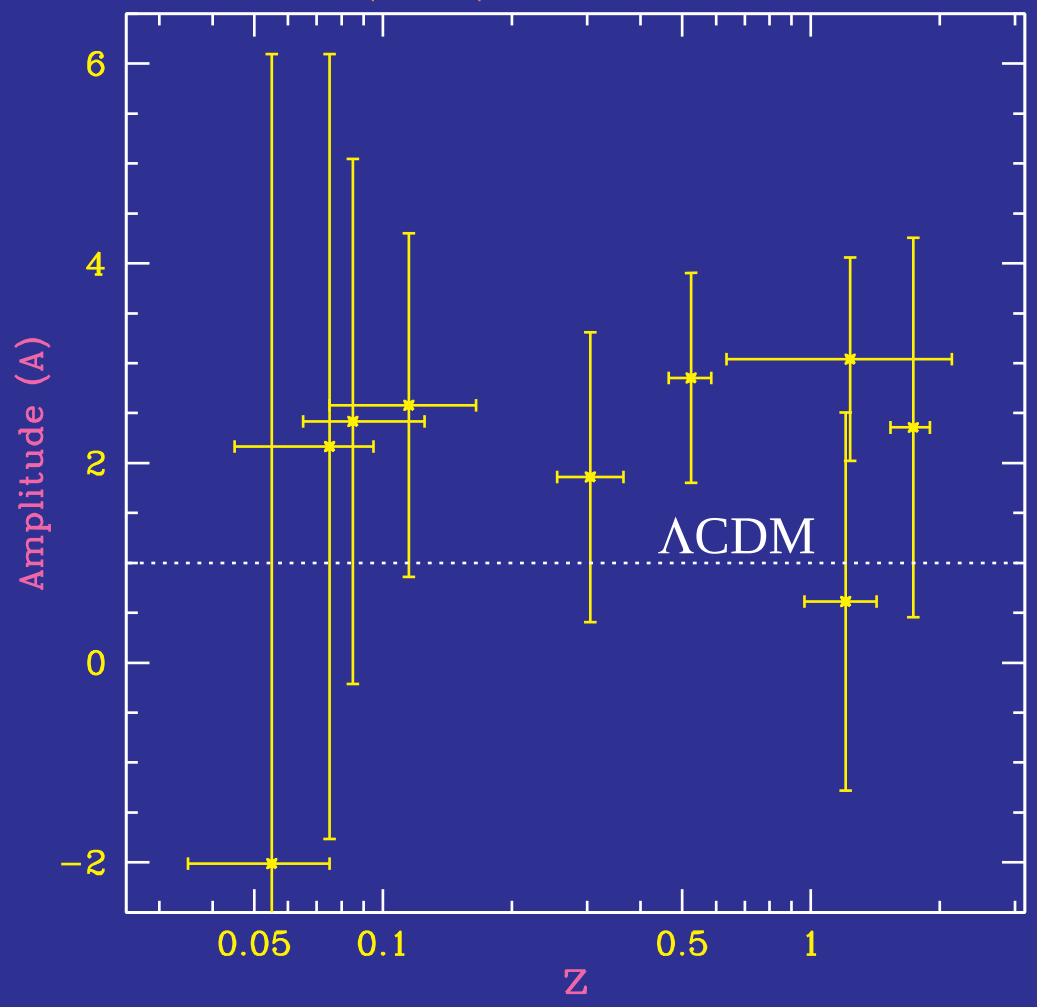
- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation



ISW-Galaxy Correlation

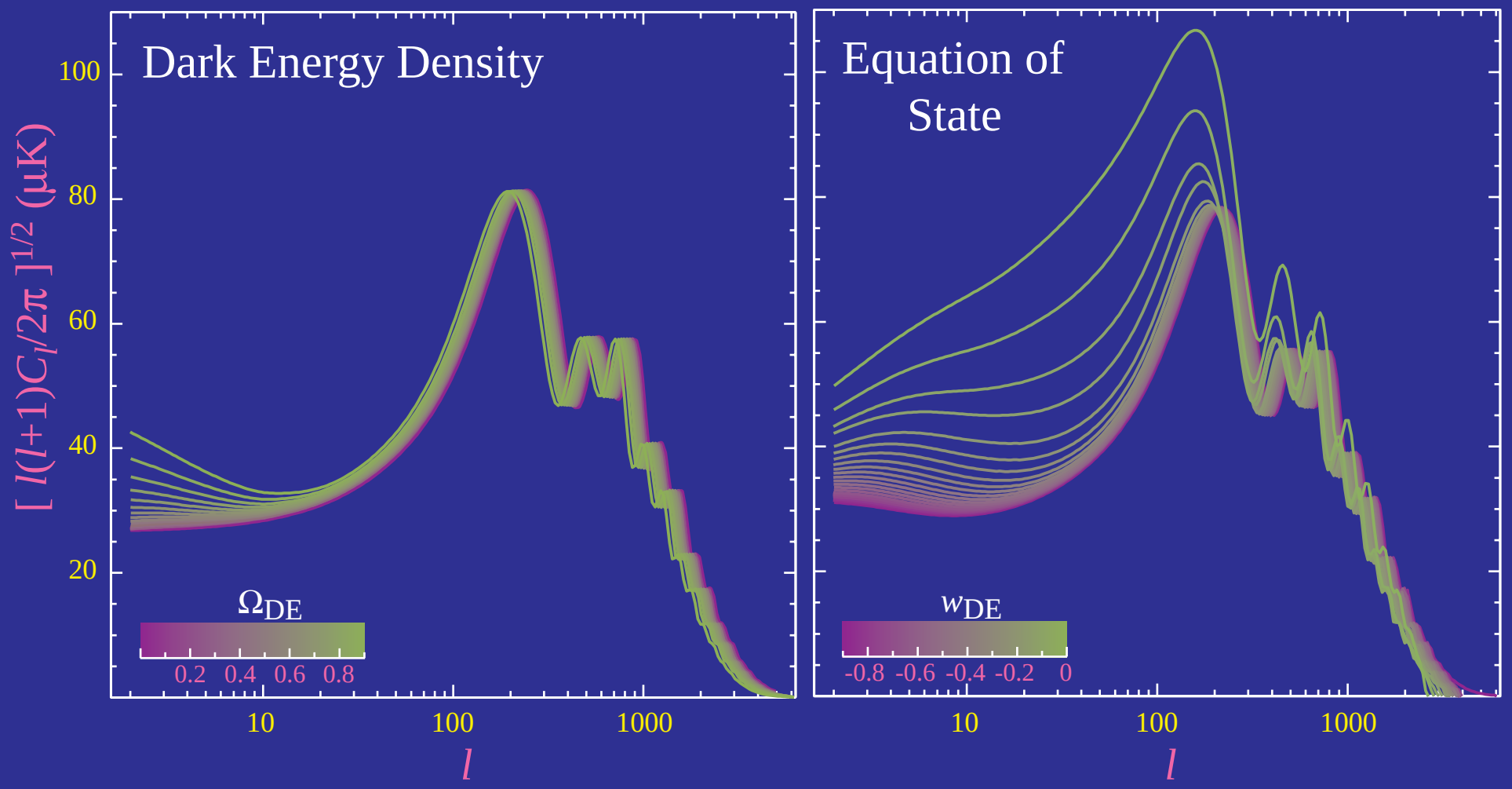
- ~4σ joint detection of ISW correlation with large scale structure (galaxies)
- ~2σ high compared with ΛCDM

Ho et al (2007) [Giannantonio et al 2008]



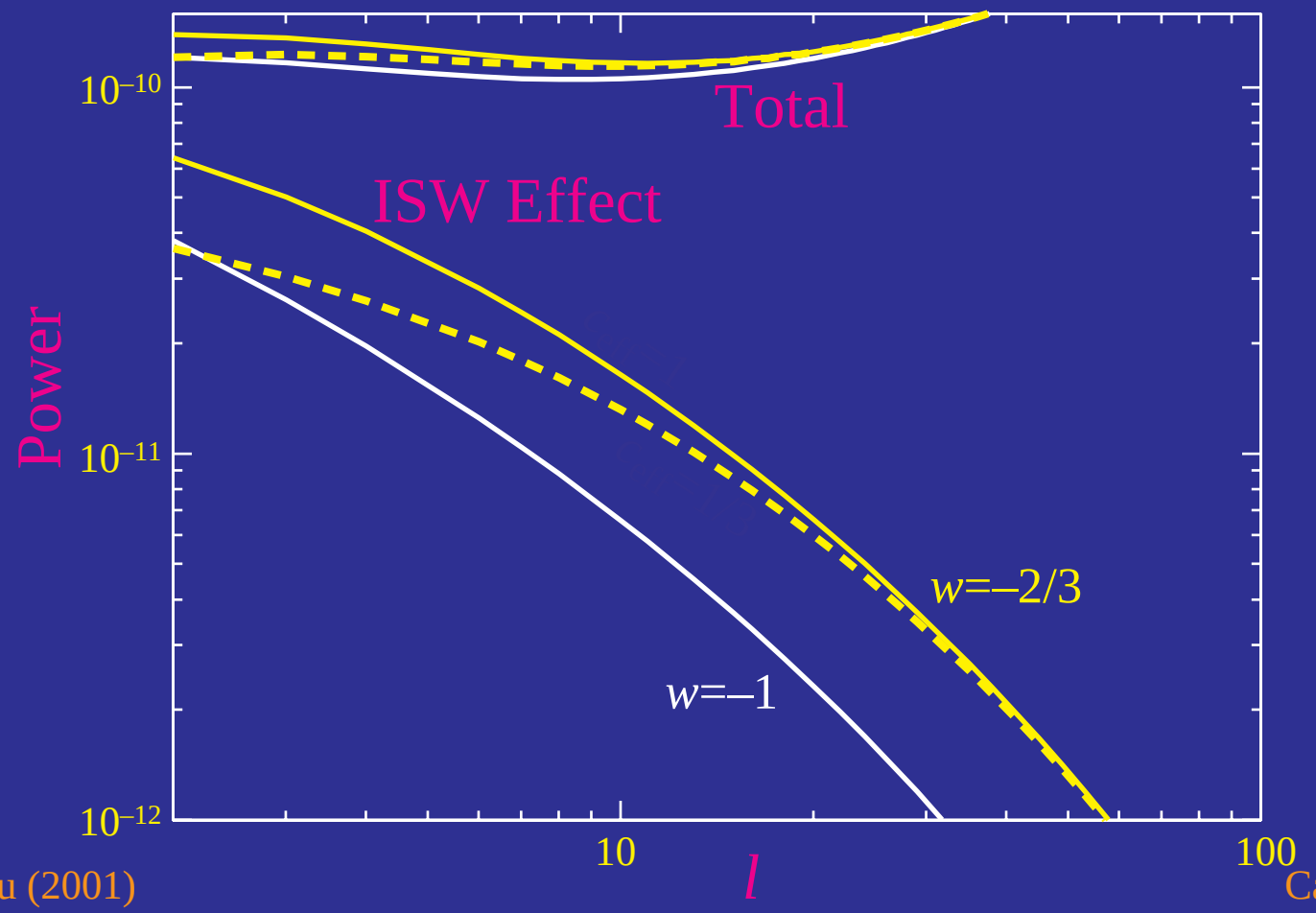
Dark Energy

- Peaks measure distance to recombination
- ISW effect constrains dynamics of acceleration



ISW Effect and Dark Energy

- Raising equation of state increases redshift of dark energy domination and raises the ISW effect
- Lowering the sound speed increases clustering and reduces ISW effect at large angles

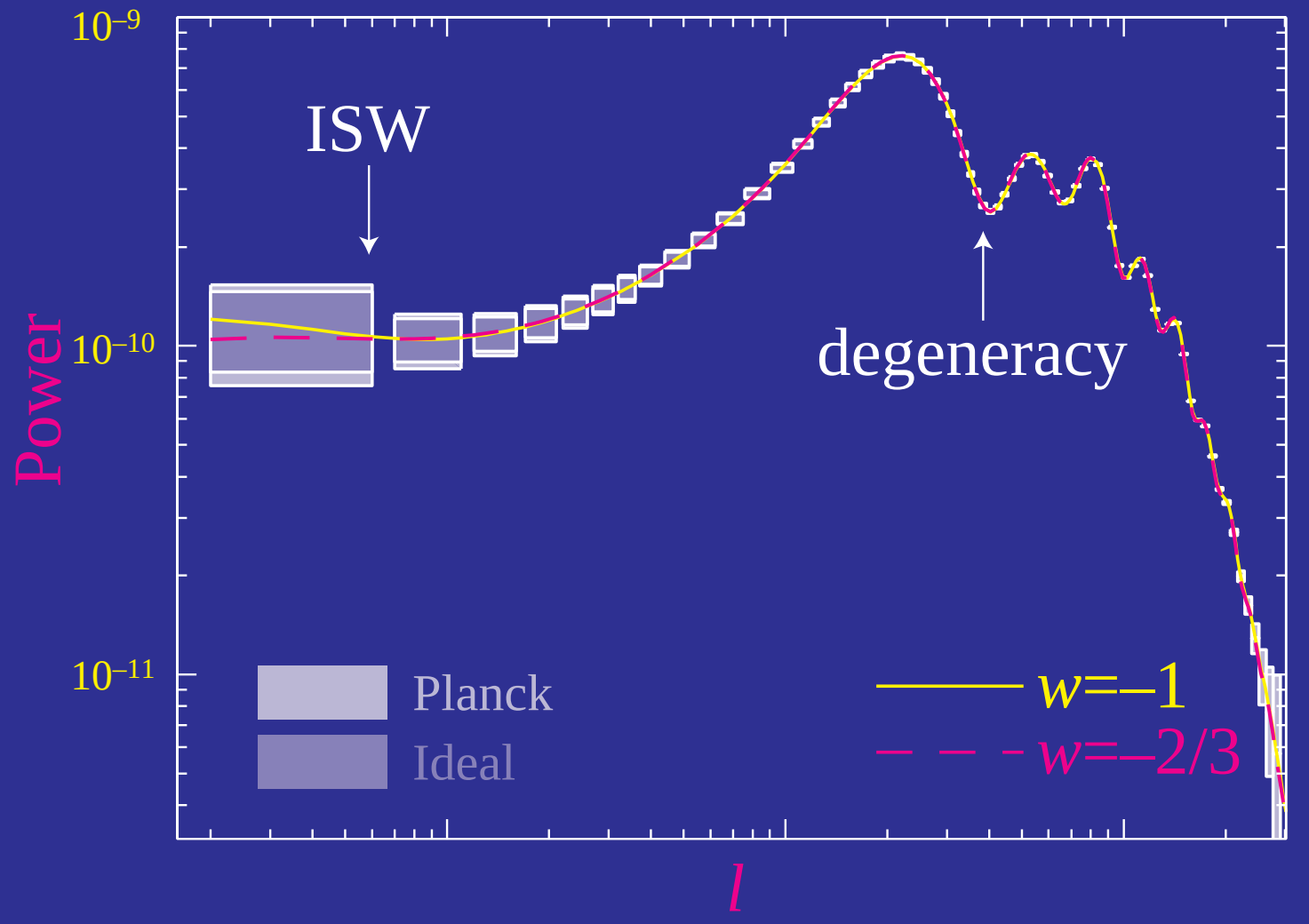


Hu (1998); Hu (2001)

Coble et al. (1997)
Caldwell et al. (1998)

Cosmic Variance Problem

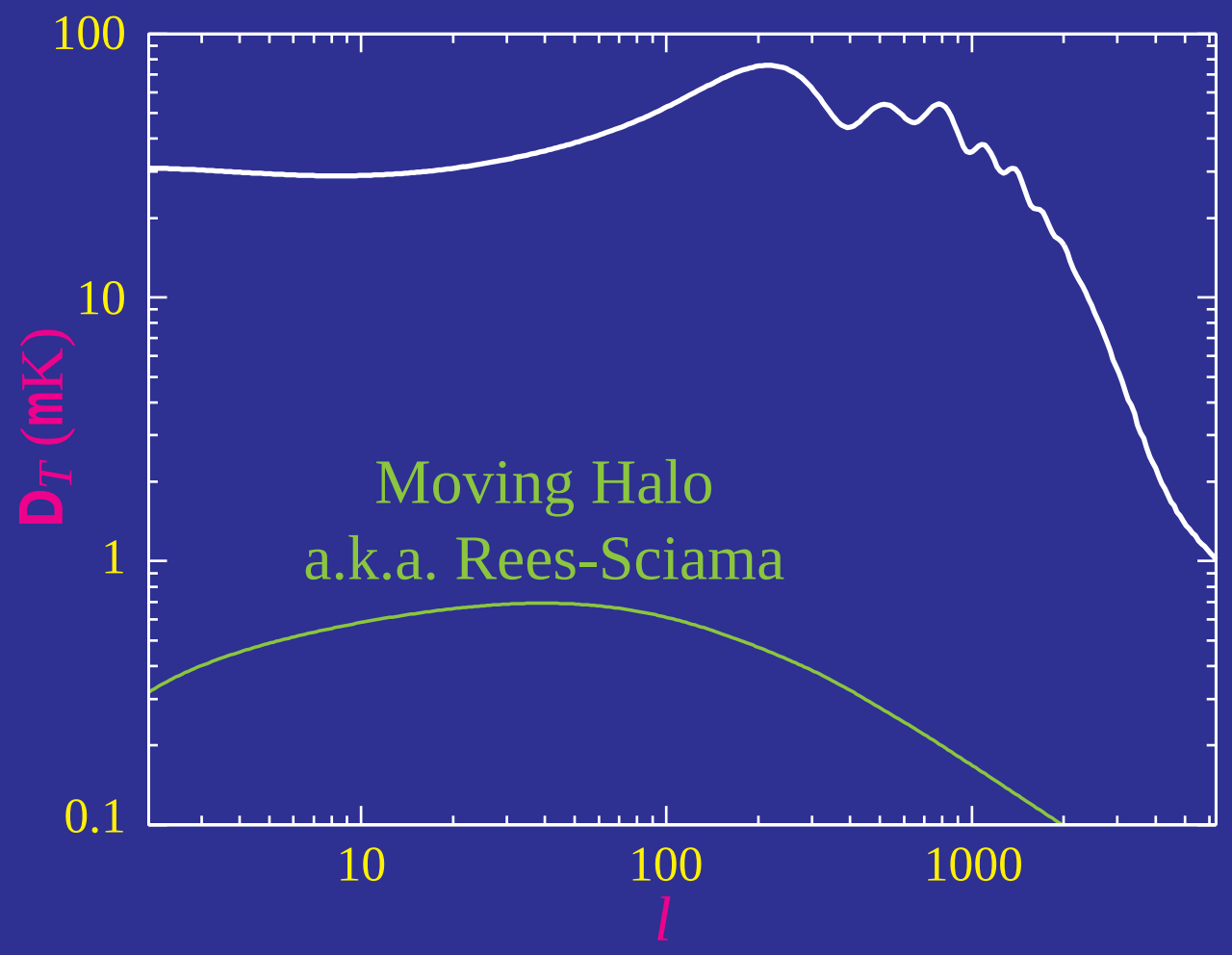
- Power spectrum sampling errors = $[(l+1/2)f_{sky}]^{-1/2}$
- Low multipole effects severely cosmic variance limited



Moving Halo Effect

Moving Halo Effect

- Change in potential due to halo moving across the line of sight



Modified Action $f(R)$ Model

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x\sqrt{-g}\left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_{\rm m}\right]$$

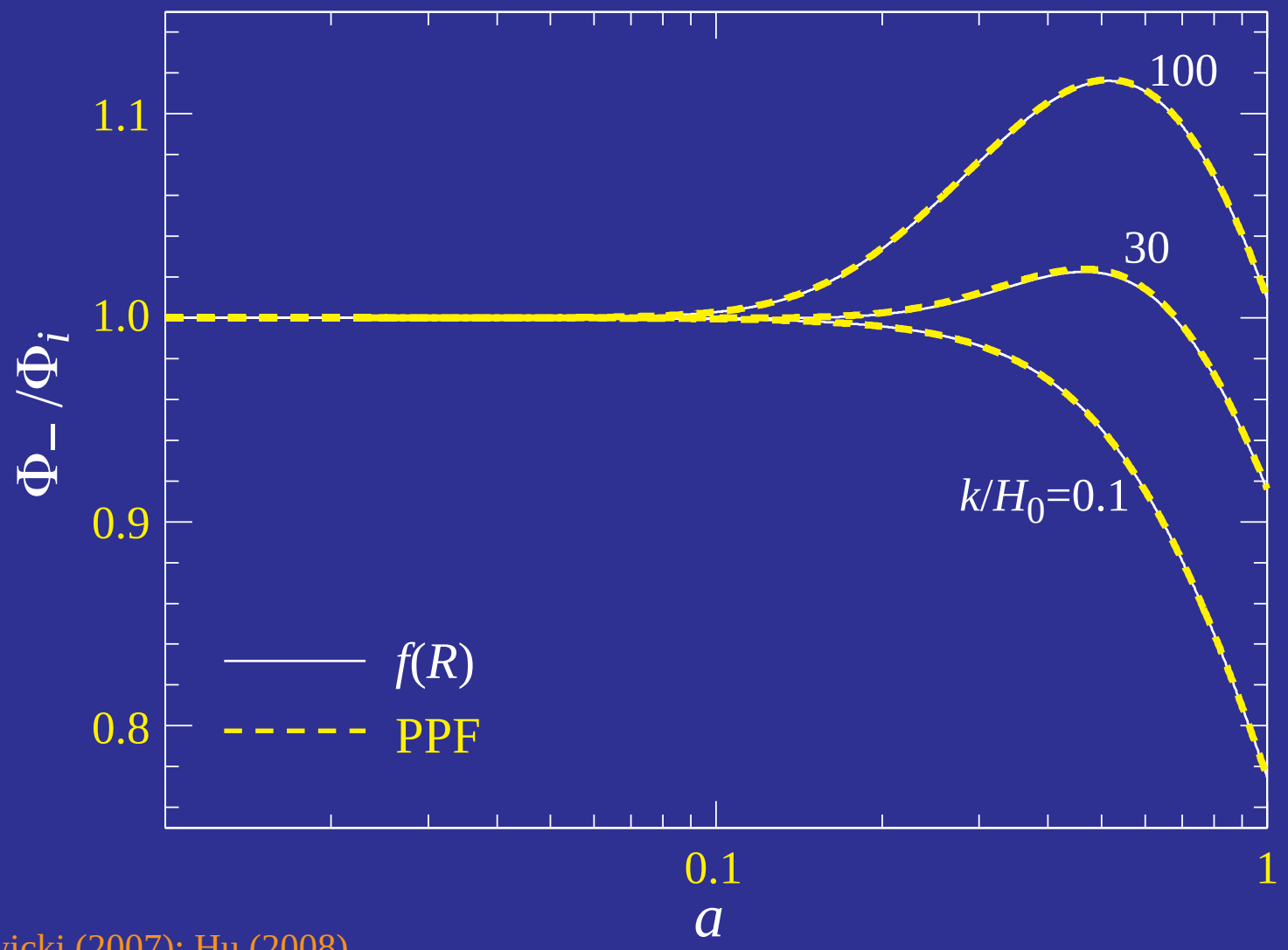
- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B: Compton wavelength of f_R squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate

PPF $f(R)$ Description

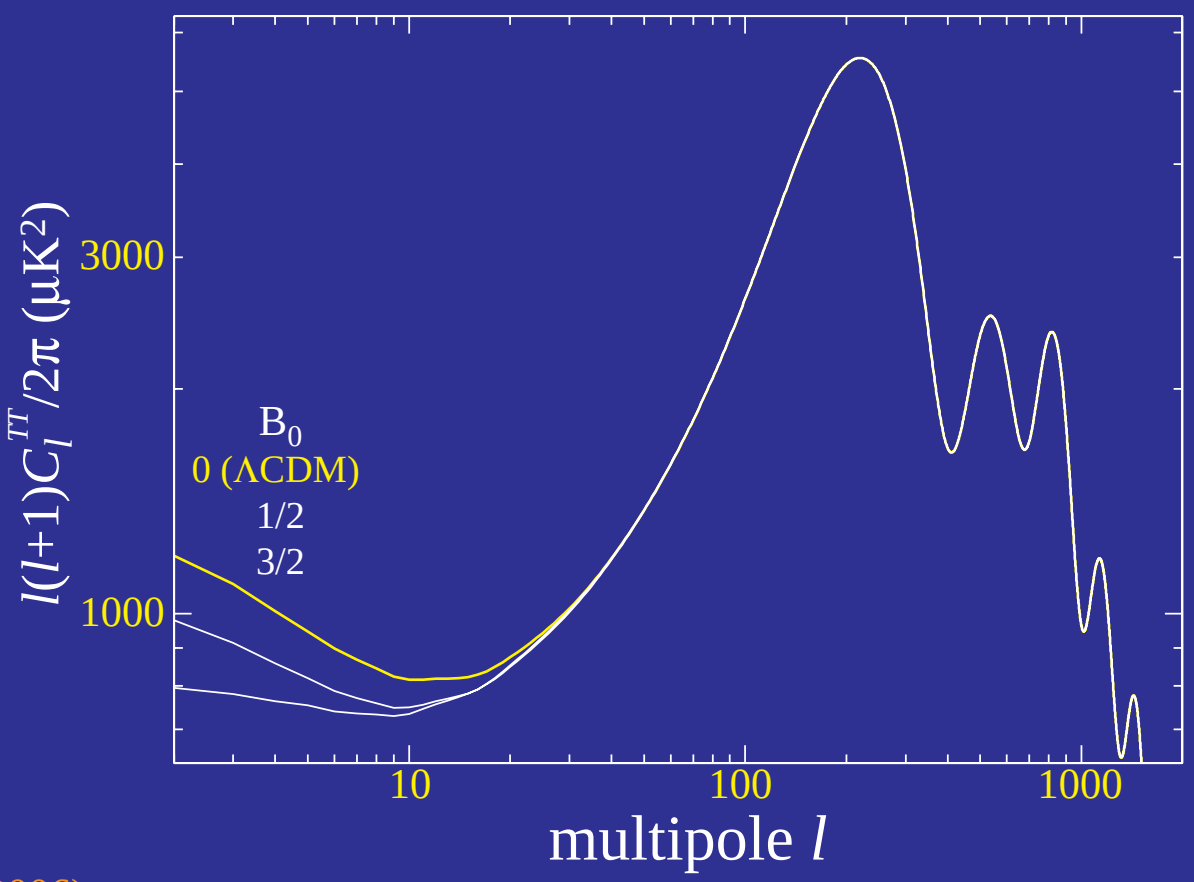
- Metric and matter evolution well-matched by PPF description
- Standard GR tools apply (CAMB), self-consistent, gauge invar.



Hu & Sawicki (2007); Hu (2008)

ISW Quadrupole

- Reduction of large angle anisotropy for B_0~1 for same expansion history and distances as ΛCDM
- Well-tested small scale anisotropy unchanged



Song, Hu & Sawicki (2006)

DGP Braneworld Acceleration

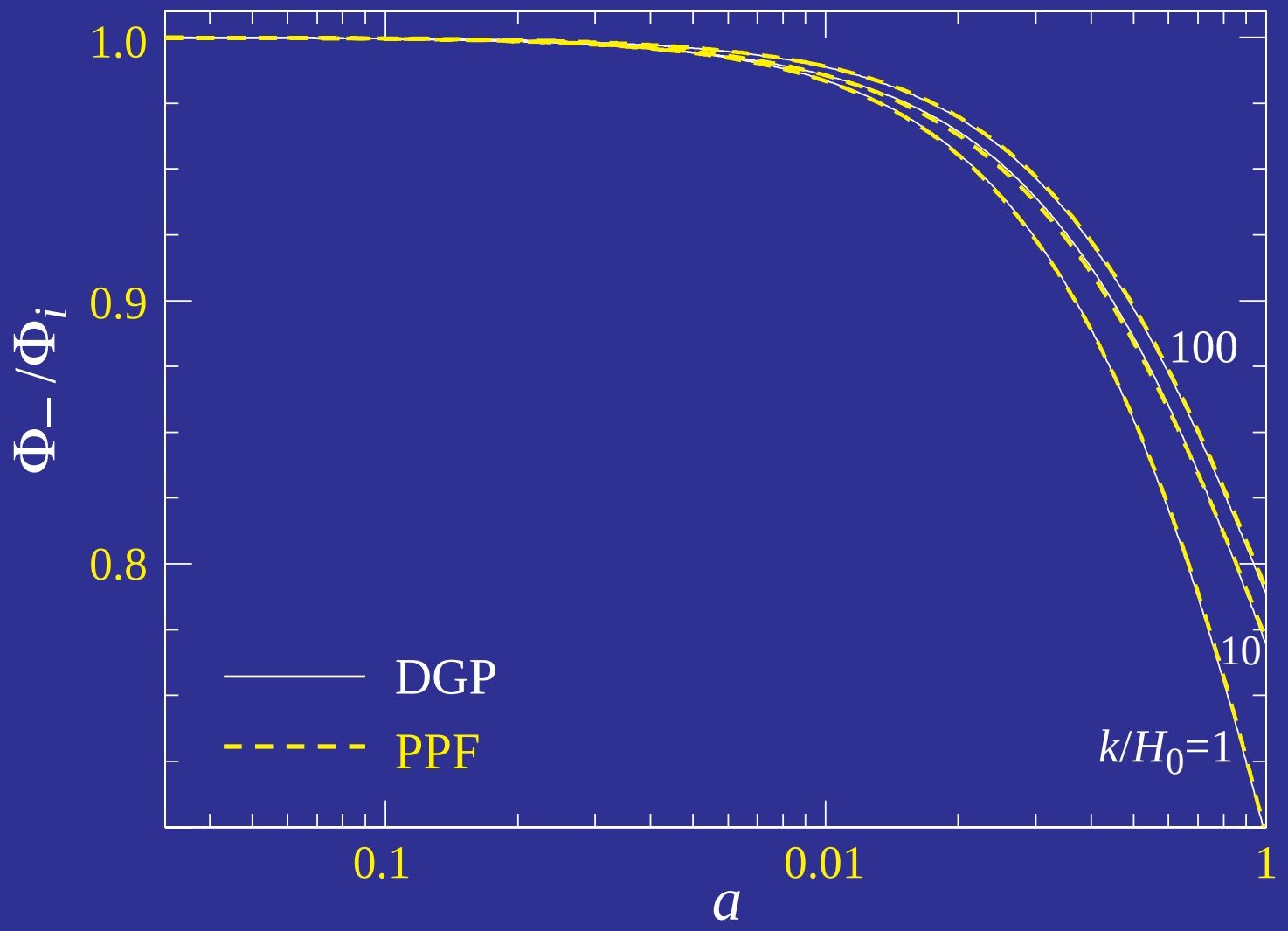
- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

$$S = \int d^5x \sqrt{-g} \left[\frac{{}^{(5)}R}{2\kappa^2} + \delta(\chi) \left(\frac{{}^{(4)}R}{2\mu^2} + \mathcal{L}_m \right) \right]$$

  with crossover scale $r_c = \kappa^2/2\mu^2$
- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)
- Matter still minimally coupled and conserved
- Exhibits the 3 regimes of modified gravity
- Weyl tensor anisotropy dominated conserved curvature regime $r > r_c$ (Sawicki, Song, Hu 2006; Cardoso et al 2007)
- Brane bending scalar tensor regime $r_* < r < r_c$ (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)
- Strong coupling General Relativistic regime $r < r_* = (r_c^2 r_g)^{1/3}$ where $r_g = 2GM$ (Dvali 2006)

DGP Horizon Scales

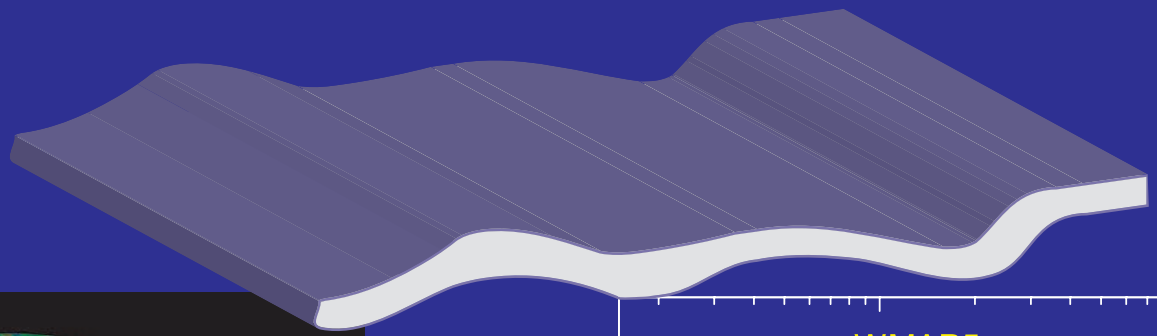
- Metric and matter evolution well-matched by PPF description
- Standard GR tools apply (CAMB), self-consistent, gauge invar.

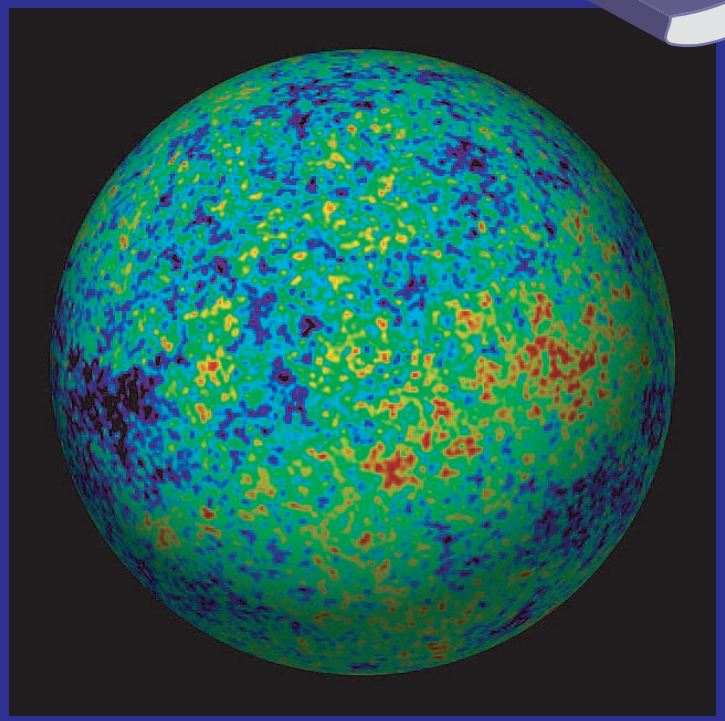


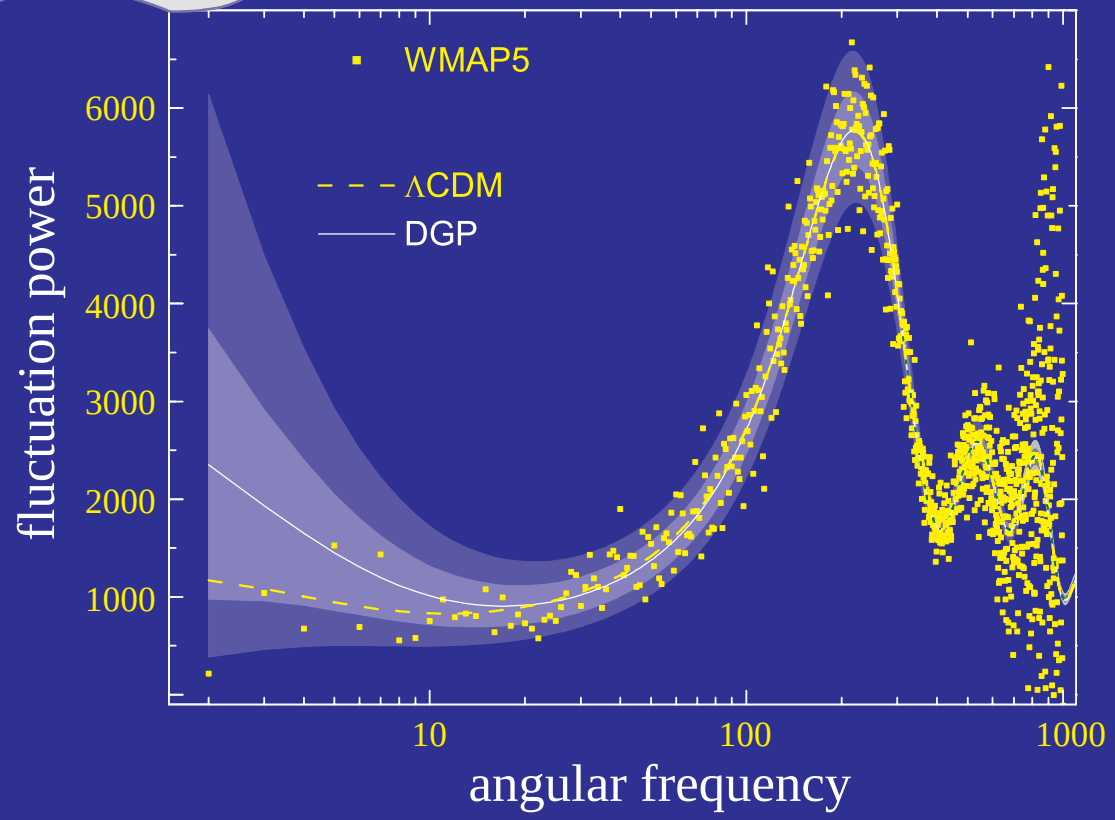
Hu & Sawicki (2007); Hu (2008)

DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background

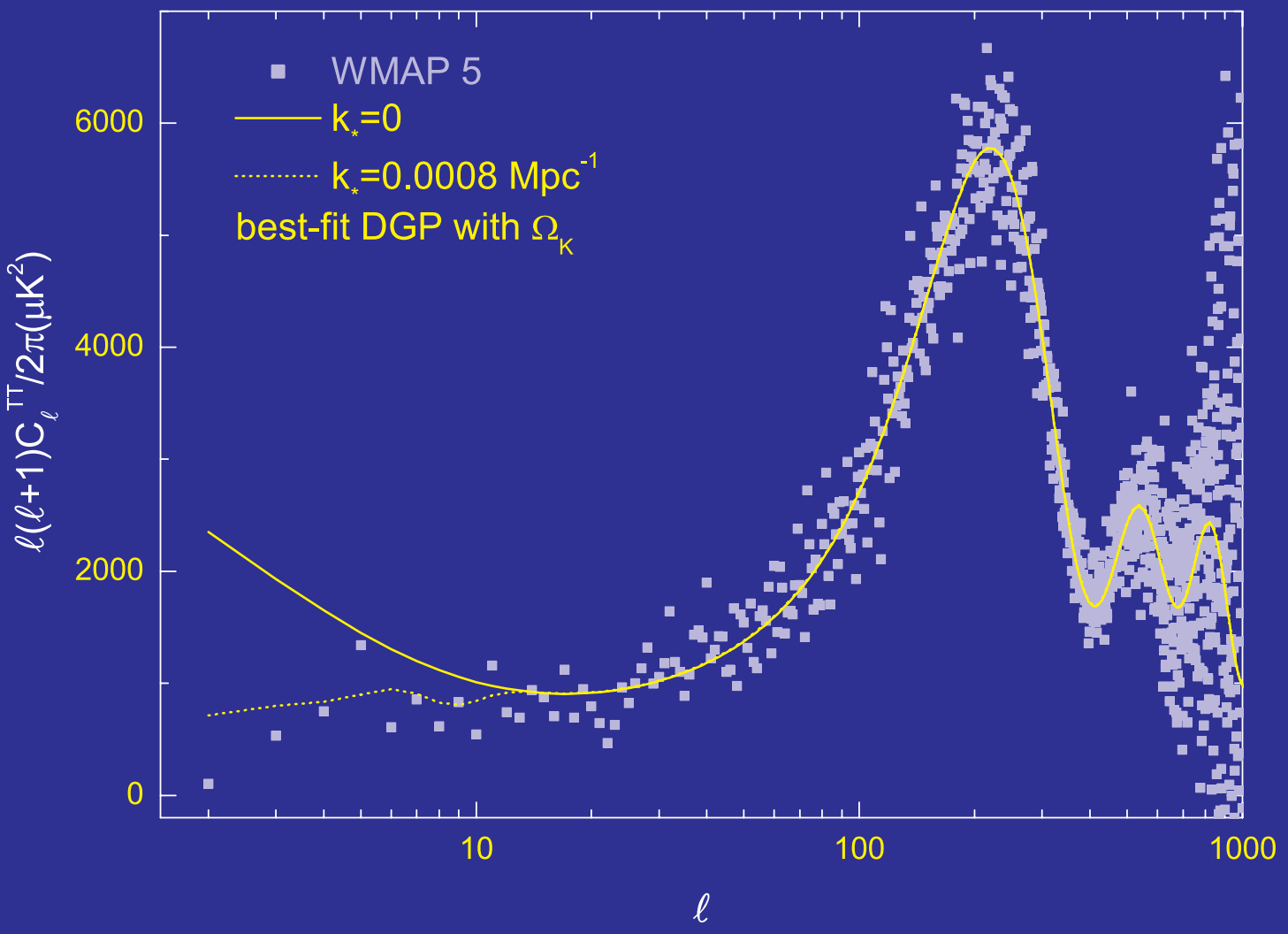






CMB in DGP

- Adding cut off as an epicycle can fix distances, ISW problem
- Suppresses polarization in violation of EE data - cannot save DGP!



Fang et al (2008)

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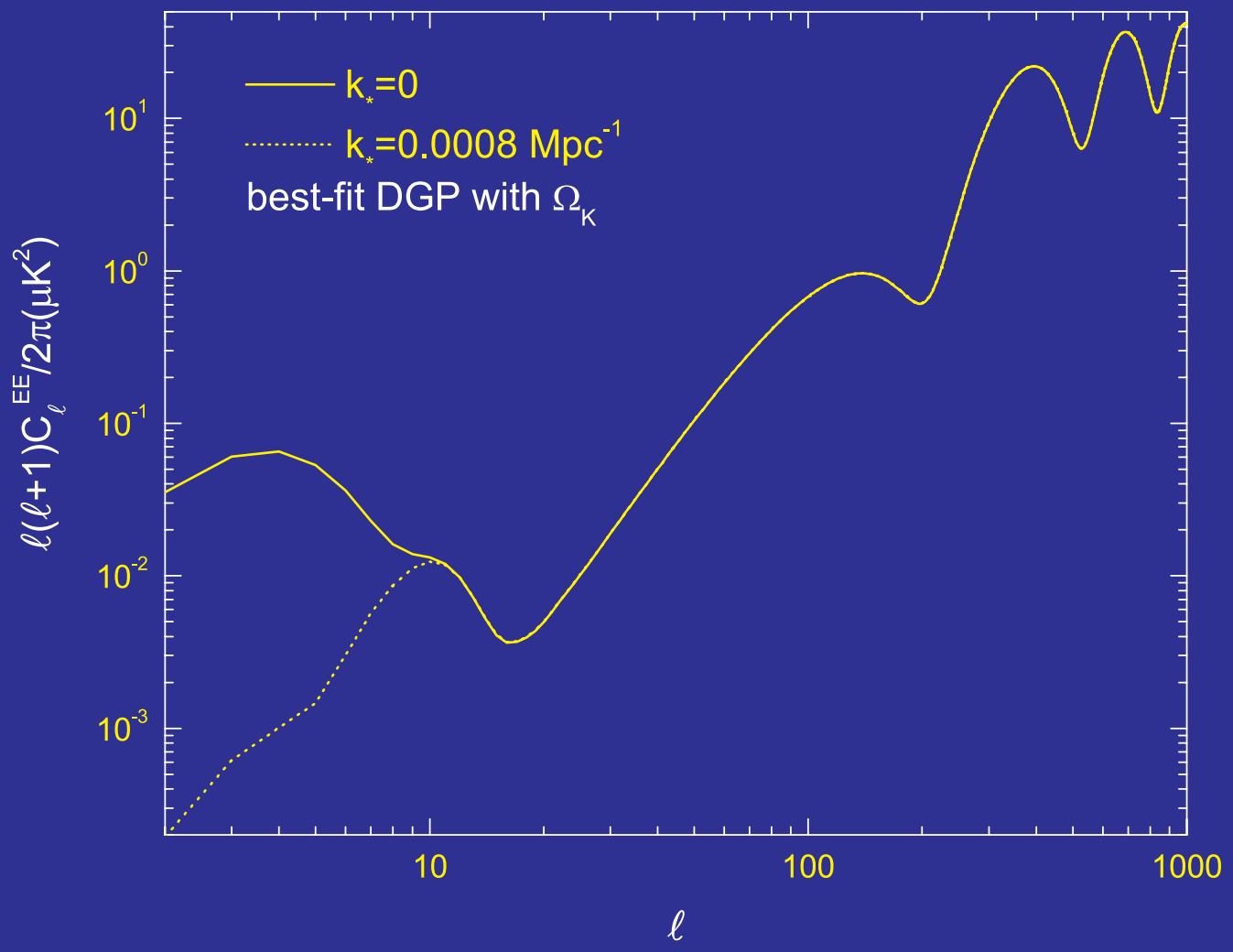
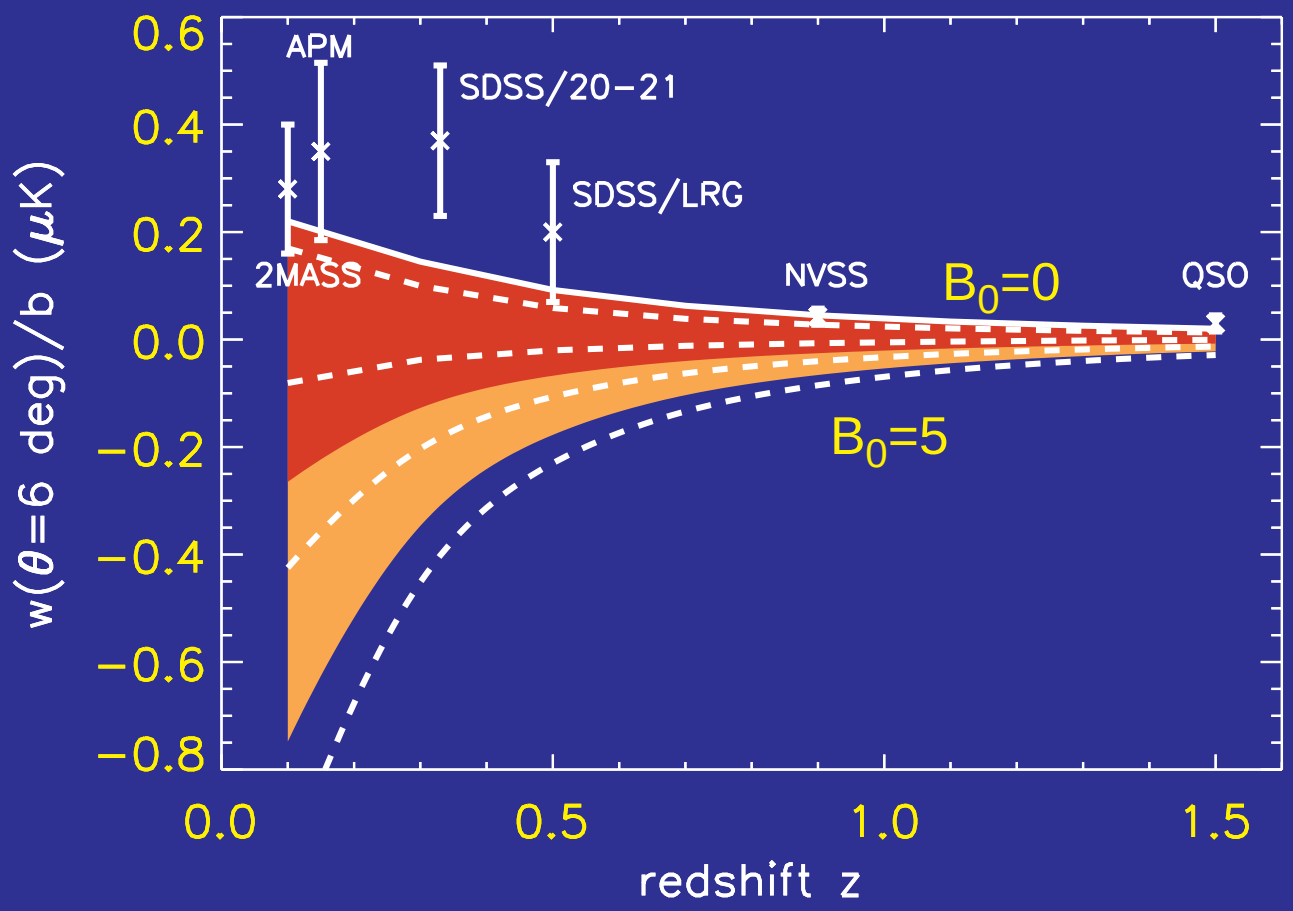


Fang et al (2008)

Galaxy-ISW Anti-Correlation

- Large Compton wavelength $B^{1/2}$ creates potential growth which can anti-correlate galaxies and the CMB
- In tension with detections of positive correlations across a range of redshifts



Song, Peiris & Hu (2007)

Thermal SZ Effect

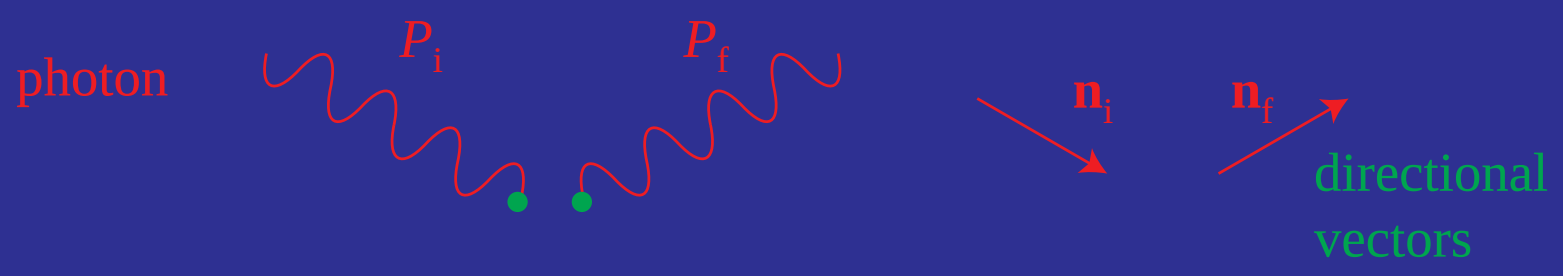
Counting Halos for Dark Energy

- Number density of massive halos extremely sensitive to the growth of structure and hence the dark energy
- Potentially %-level precision in dark energy equation of state

Haiman, Holder & Mohr (2001)

Beyond Thomson Limit

- Thomson scattering $e_i + \gamma_i \rightarrow e_f + \gamma_f$ in rest frame where the frequencies $\omega_i = \omega_f$ (elastic scattering) cannot strictly be true
- Photons carry off E/c momentum and so to conserve momentum the electron must recoil
- Doppler shift from transformation from rest frame contains second order terms
- General case (arbitrary electron velocity)



Energy-Momentum Conservation

- From energy-momentum conservation, the energy change is

$$\frac{E_f}{E_i} = \frac{1 - \beta_i \cos\alpha_i}{1 - \beta_i \cos\alpha_f + \frac{E_i}{\gamma m c^2}(1 - \cos\theta)}$$

  where $\hat{\mathbf{n}}_f \cdot \mathbf{v}_i = v_i \cos\alpha_f$ and $\hat{\mathbf{n}}_i \cdot \mathbf{v}_i = v_i \cos\alpha_i$

- Two ways of changing the energy: Doppler boost β_i from incoming electron velocity and E_i non-negligible compared to $\gamma m c^2$

- Isolate recoil in incoming electron rest frame $\beta_i = 0$ and $\gamma = 1$

$$\left.\frac{E_f}{E_i}\right|_{\text{rest}} = \frac{1}{1 + \frac{E_i}{mc^2}(1 - \cos\theta)}$$

Recoil Effect

- Since $-1 \leq \cos\theta \leq 1$, $E_f \leq E_i$, energy is lost from the recoil except for purely forward scattering
- The backwards scattering limit is easy to see

$$|\mathbf{q}_f| = m|\mathbf{v}_f| = 2\frac{E_i}{c}\,,$$

$$\Delta E = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{2E_i}{mc}\right)^2 = 2\frac{E_i}{mc^2}E_i$$

$$E_f = E_i - \Delta E = (1 - 2\frac{E_i}{mc^2})E_i \approx \frac{E_i}{1 + 2\frac{E_i}{mc^2}}$$

Second Order Doppler Shift

- Doppler effect: consider the limit of $\beta_i \ll 1$ then expand to first order

$$\frac{E_f}{E_i} = 1 - \beta_i \cos\alpha_i + \beta_i \cos\alpha_f - \frac{E_i}{mc^2}(1 - \cos\theta)$$

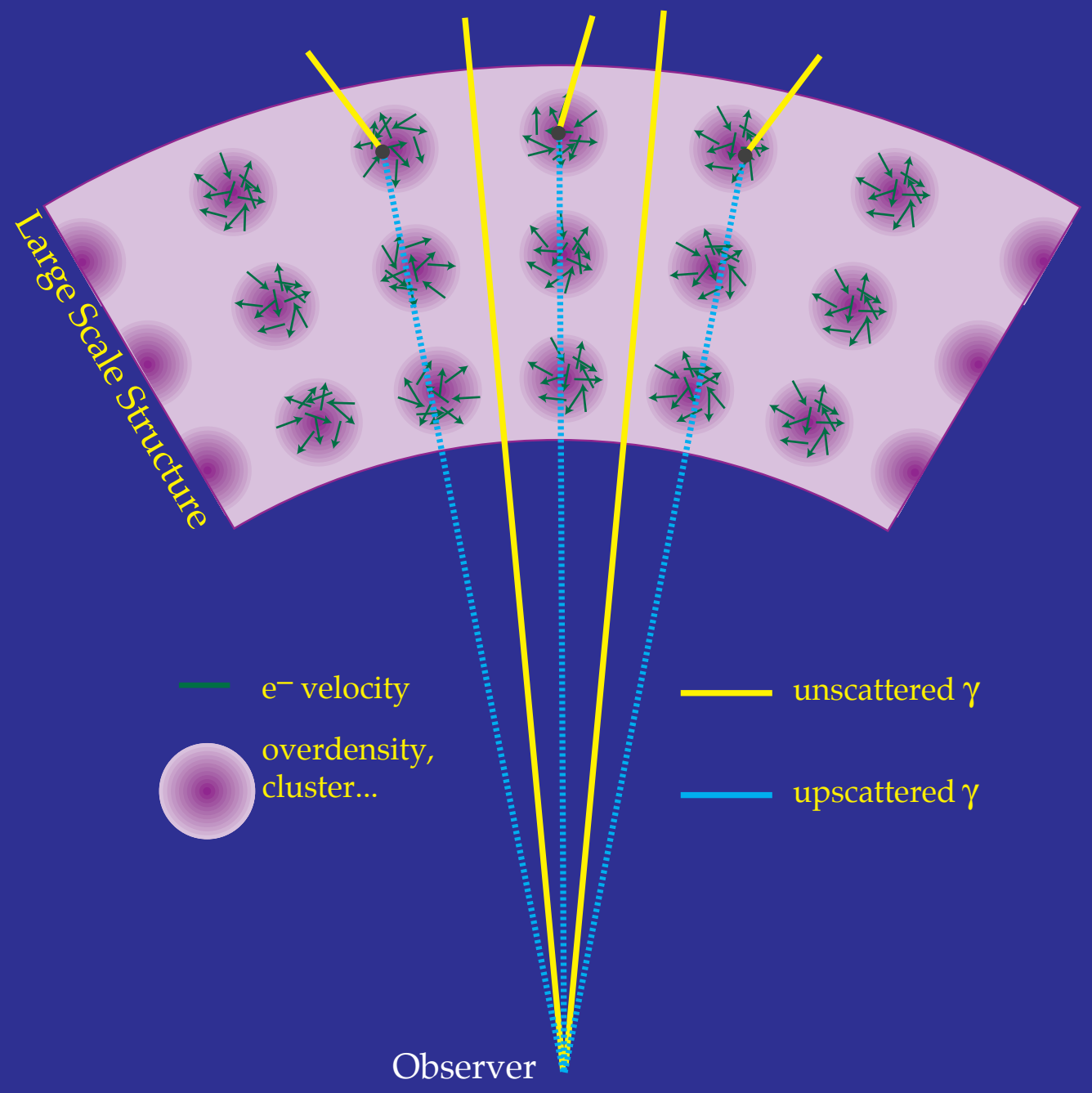
  however averaging over angles the Doppler shifts don't change the energies

- To second order in the velocities, the Doppler shift transfers energy from the electron to the photon in opposition to the recoil

$$\frac{E_f}{E_i} = 1 - \beta_i \cos\alpha_i + \beta_i \cos\alpha_f + \beta_i^2 \cos^2\alpha_f - \frac{E_i}{mc^2}$$

$$\langle\frac{E_f}{E_i}\rangle \approx 1 + \frac{1}{3}\beta_i^2 - \frac{E_i}{mc^2}$$

Thermal SZ Effect



Thermalization

- For a thermal distribution of velocities

$$\frac{1}{2}m\langle v^2\rangle = \frac{3kT}{2} \qquad \beta_i^2 \approx \frac{3kT}{mc^2} \rightarrow \langle \frac{E_f}{E_i} - 1\rangle \sim \frac{kT - E_i}{mc^2}$$

so that if $E_i \ll kT$ the photon gains energy and $E_i \gg kT$ it loses energy $\rightarrow$ this is a thermalization process

Kompaneets Equation

- Radiative transfer or Boltzmann equation

$$\frac{\partial f}{\partial t} = \frac{1}{2E(p_f)} \int \frac{d^3p_i}{(2\pi)^3} \frac{1}{2E(p_i)} \int \frac{d^3q_f}{(2\pi)^3} \frac{1}{2E(q_f)} \int \frac{d^3q_i}{(2\pi)^3} \frac{1}{2E(q_i)}$$
$$\times (2\pi)^4 \delta(p_f + q_f - p_i - q_i) |M|^2$$
$$\times \{ f_e(q_i) f(p_i)[1 + f(p_f)] - f_e(q_f) f(p_f)[1 + f(p_i)] \}$$

- Matrix element is calculated in field theory and is Lorentz invariant. In terms of the rest frame $\alpha = e^2/\hbar c$ (Klein Nishina Cross Section)

$$|M|^2 = 2(4\pi)^2 \alpha^2 \left[\frac{E(p_i)}{E(p_f)} + \frac{E(p_f)}{E(p_i)} - \sin^2 \beta \right]$$

with β as the rest frame scattering angle

Kompaneets Equation

- The Kompaneets equation ($\hbar = c = 1$)

$$\frac{\partial f}{\partial t} = n_e \sigma_T c \left(\frac{kT_e}{mc^2}\right) \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left(\frac{\partial f}{\partial x} + f(1+f) \right) \right] \qquad x = \hbar\omega / kT_e$$

takes electrons as thermal

$$f_e = e^{-(m-\mu)/T_e} e^{-q^2/2mT_e} \qquad \left[n_e = e^{-(m-\mu)/T_e} \left(\frac{mT_e}{2\pi}\right)^{3/2} \right]$$

$$= \left(\frac{2\pi}{mT_e}\right)^{3/2} n_e e^{-q^2/2mT_e}$$

and assumes that the energy transfer is small (non-relativistic electrons, $E_i \ll m$

$$\frac{E_f - E_i}{E_i} \ll 1 \qquad [\mathcal{O}(T_e/m, E_i/m)]$$

Kompaneets Equation

- Equilibrium solution must be a Bose-Einstein distribution since Compton scattering does not change photon number
- Rate of energy exchange obtained from integrating the energy $\times$ Kompeneets equation over momentum states

$$\frac{\partial u}{\partial t} = 4 n_e \sigma_T c \frac{kT_e}{mc^2} \left[1 - \frac{T_\gamma}{T_e} \right] u$$

$$\frac{1}{u} \frac{\partial u}{\partial t} = 4 n_e \sigma_T c \frac{k(T_e - T_\gamma)}{mc^2}$$

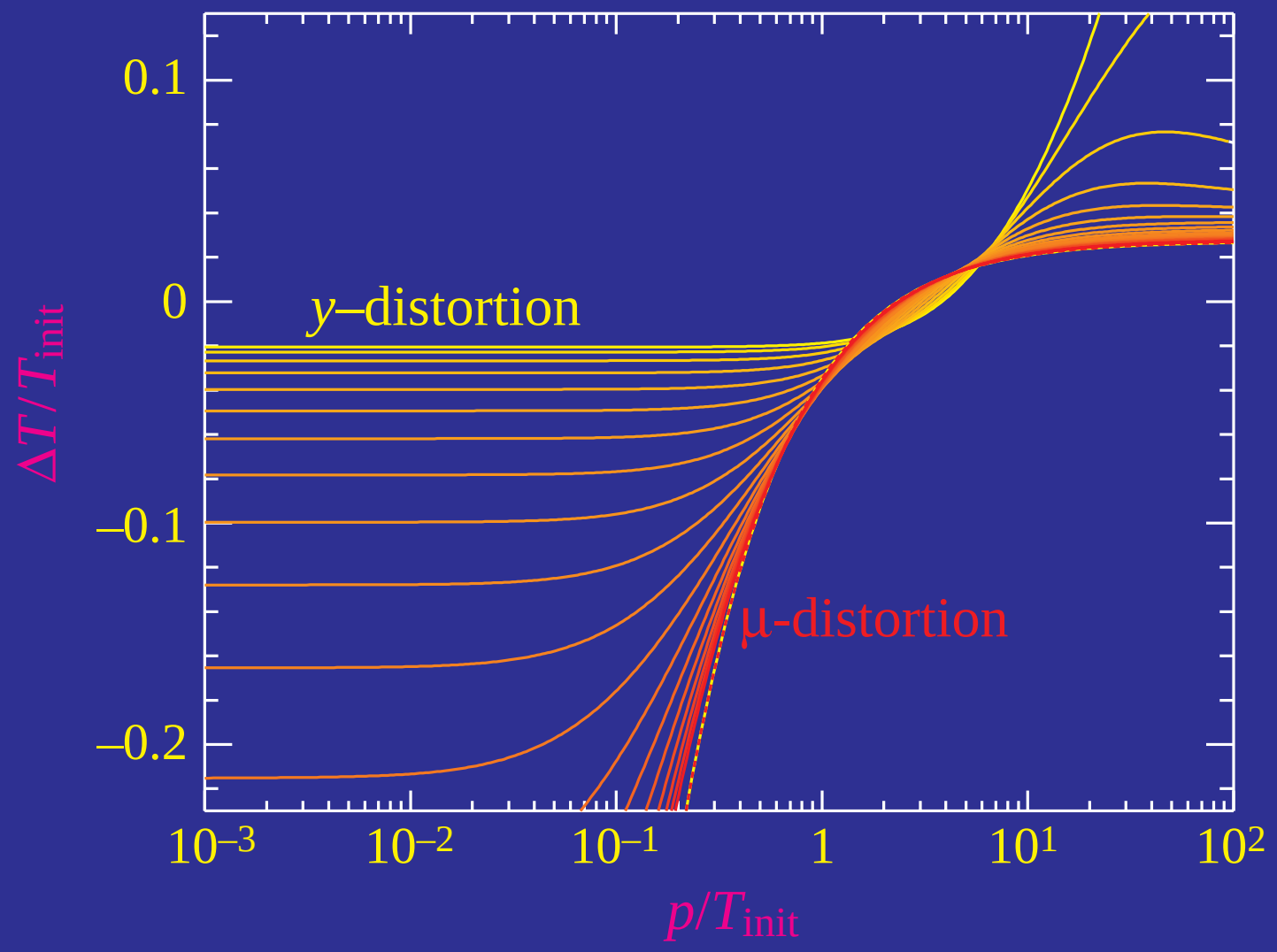
- The analogue to the optical depth for energy transfer is the Compton y parameter

$$d\tau = n_e \sigma_T ds = n_e \sigma_t c dt$$

$$dy = \frac{k(T_e - T_\gamma)}{mc^2} d\tau$$

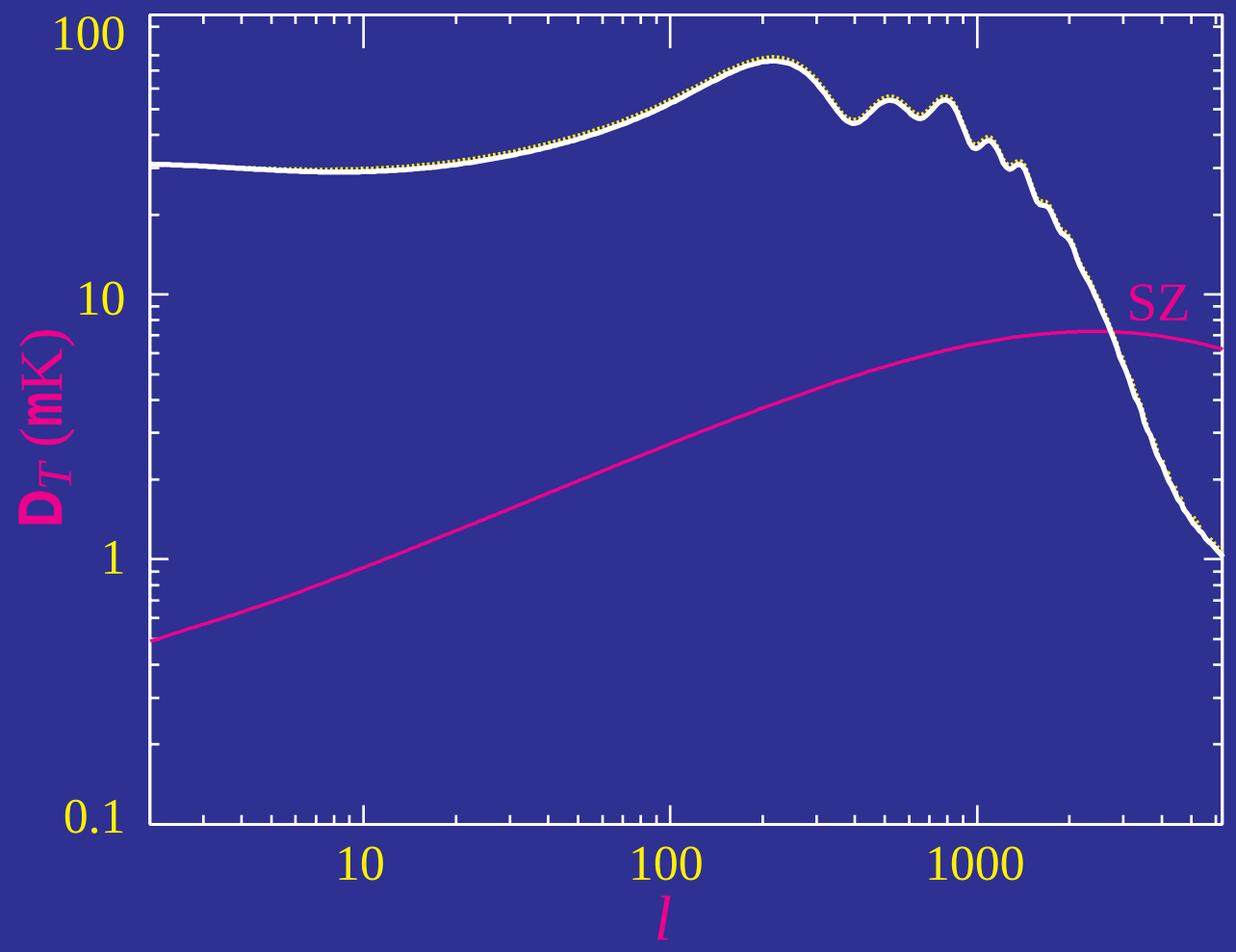
Spectral Distortion

- Compton upscattering: *y*–distortion
- Redistribution: μ-distortion

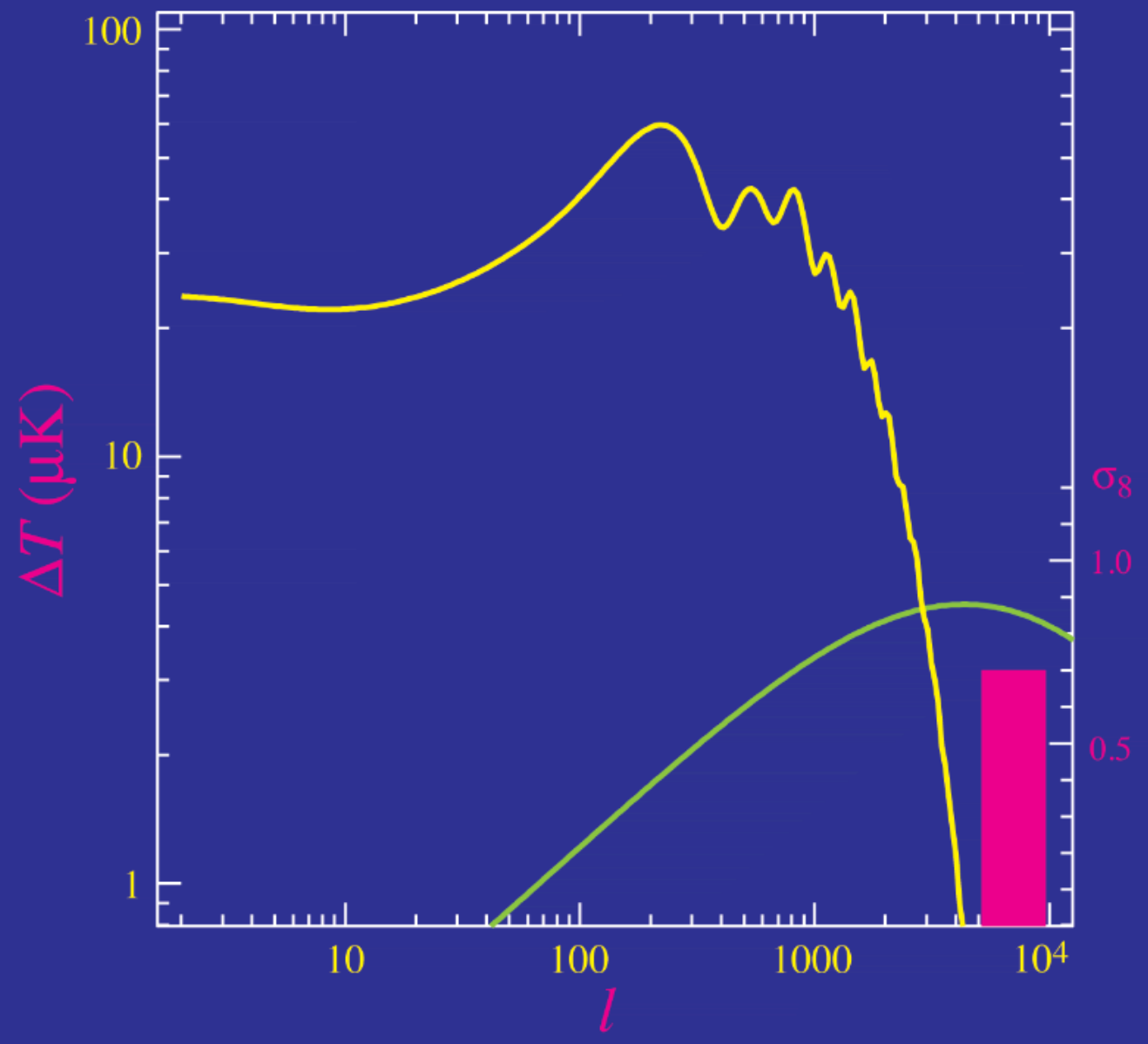


Thermal SZ Effect

- Second order Doppler effect escapes cancellation
- Velocities: thermal velocities in a hot cluster (1-10keV)
- Dominant source of arcminute anisotropies – turns over as clusters are resolved

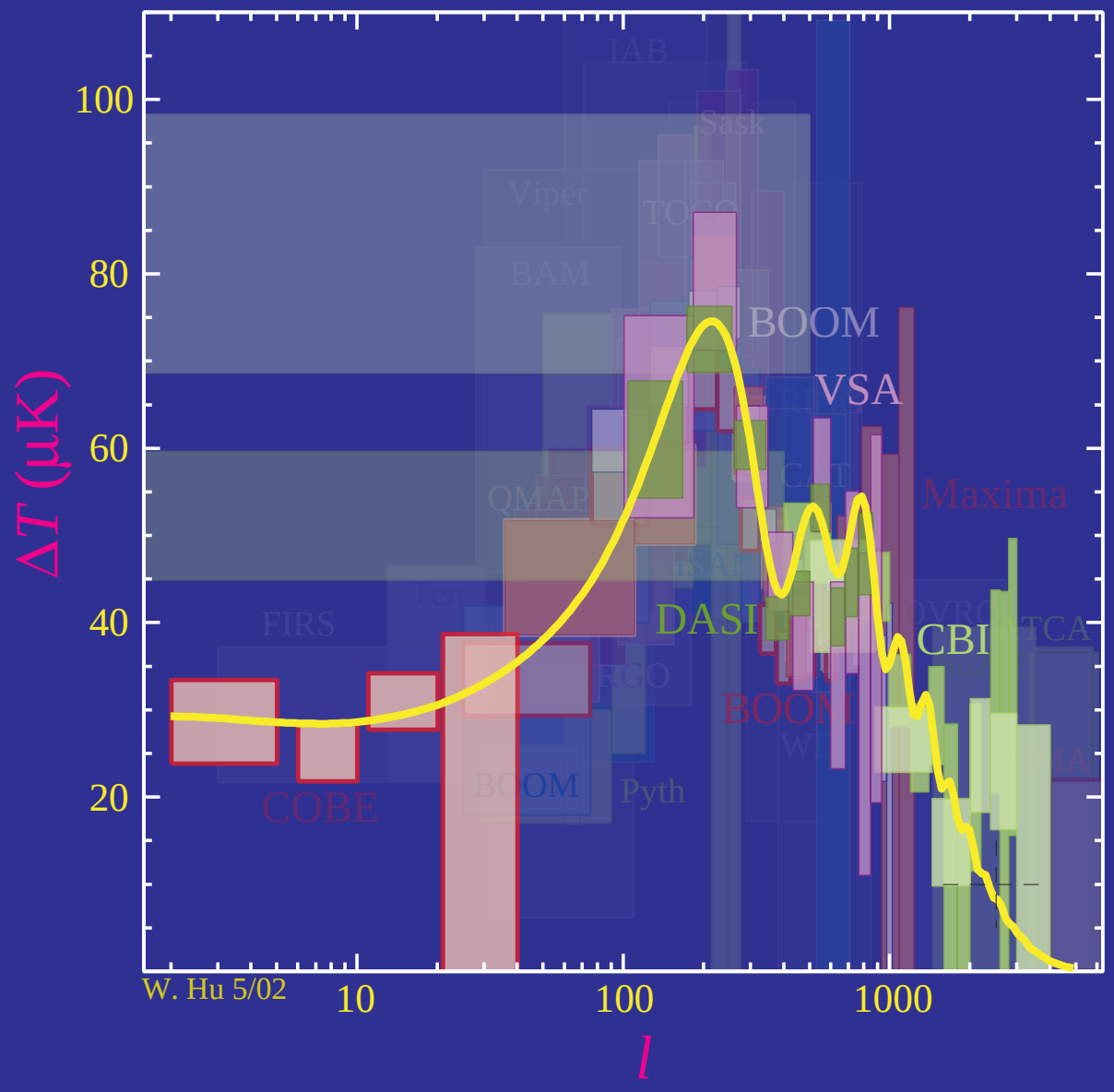


Amplitude of Fluctuations

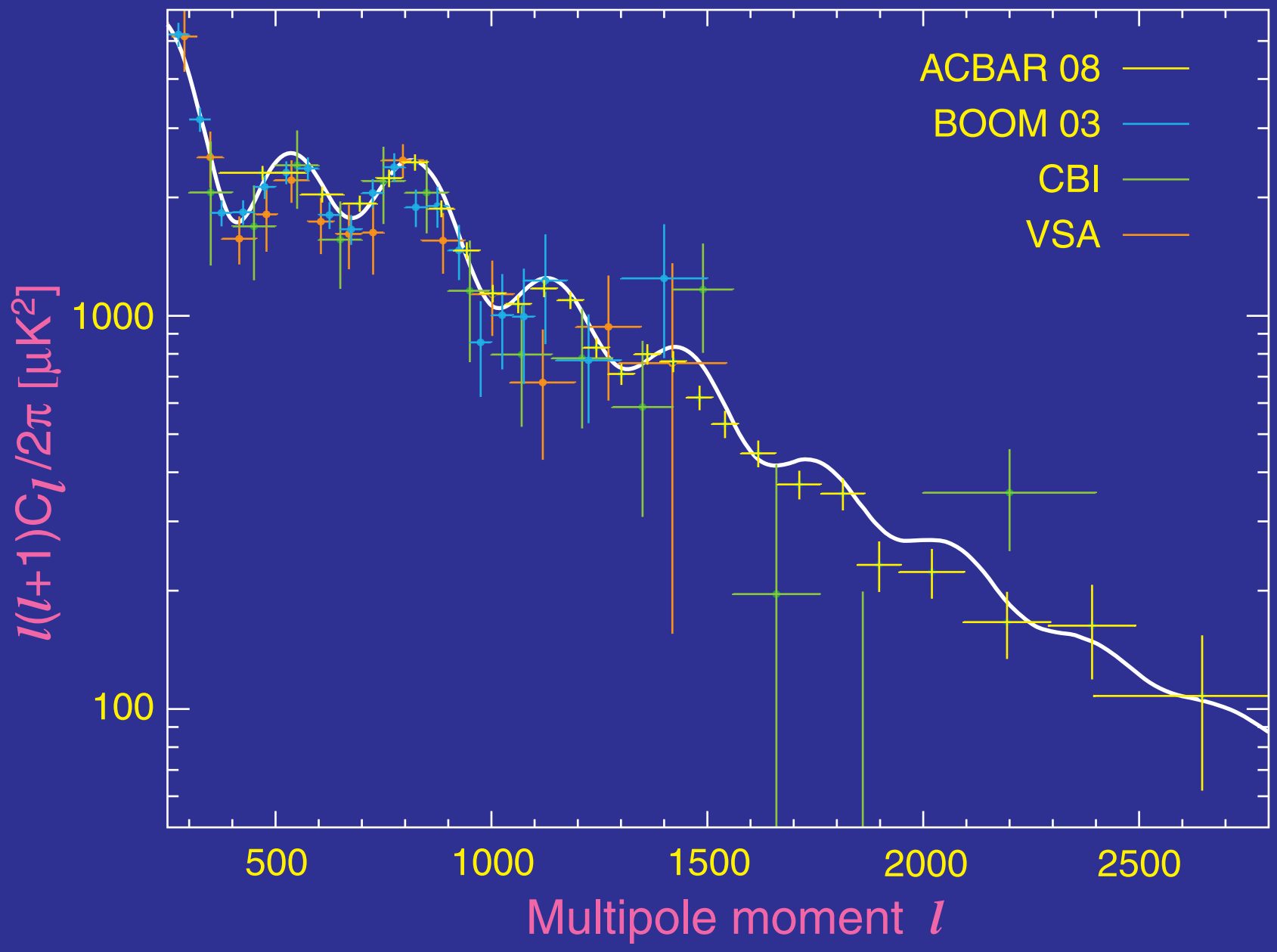


Clusters in Power Spectrum?

- Excess in arcminute scale CMB anisotropy from CBI

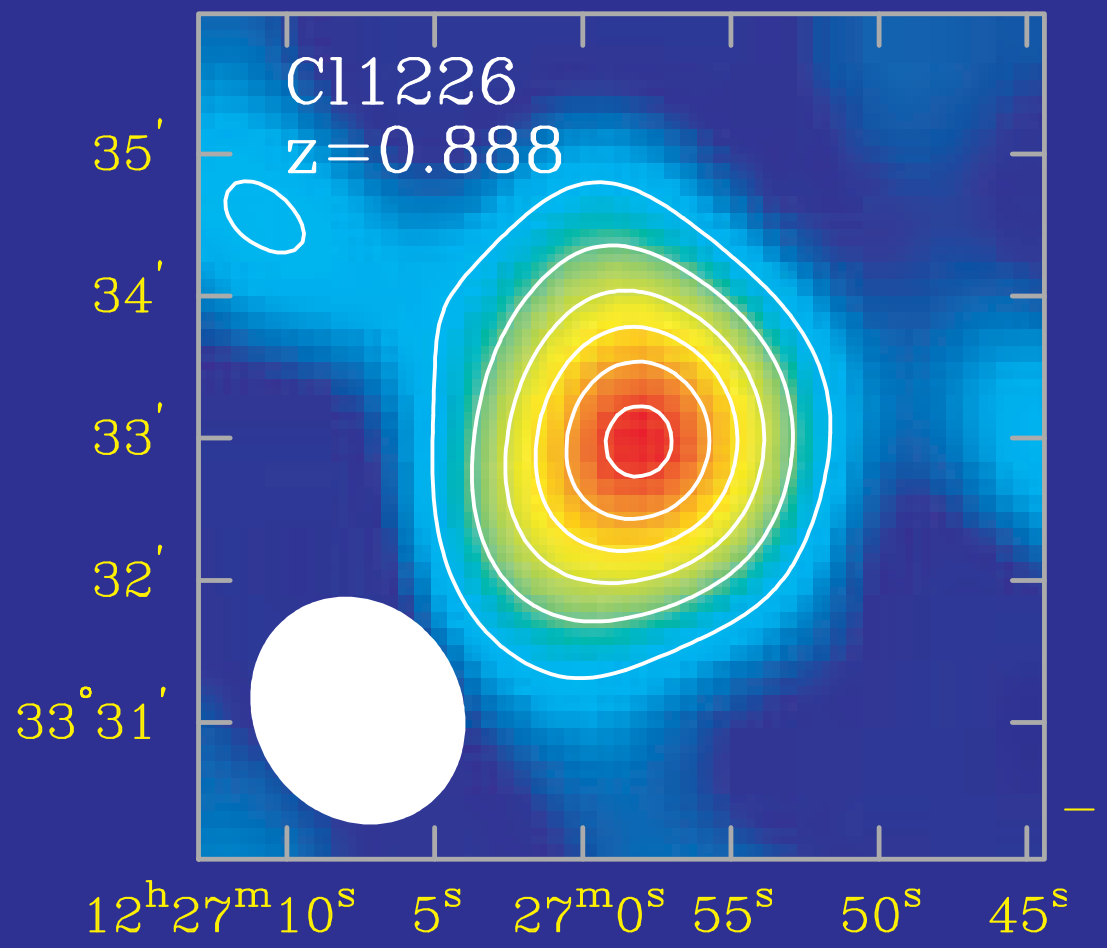


Power Spectrum Present



Counting Halos for Dark Energy

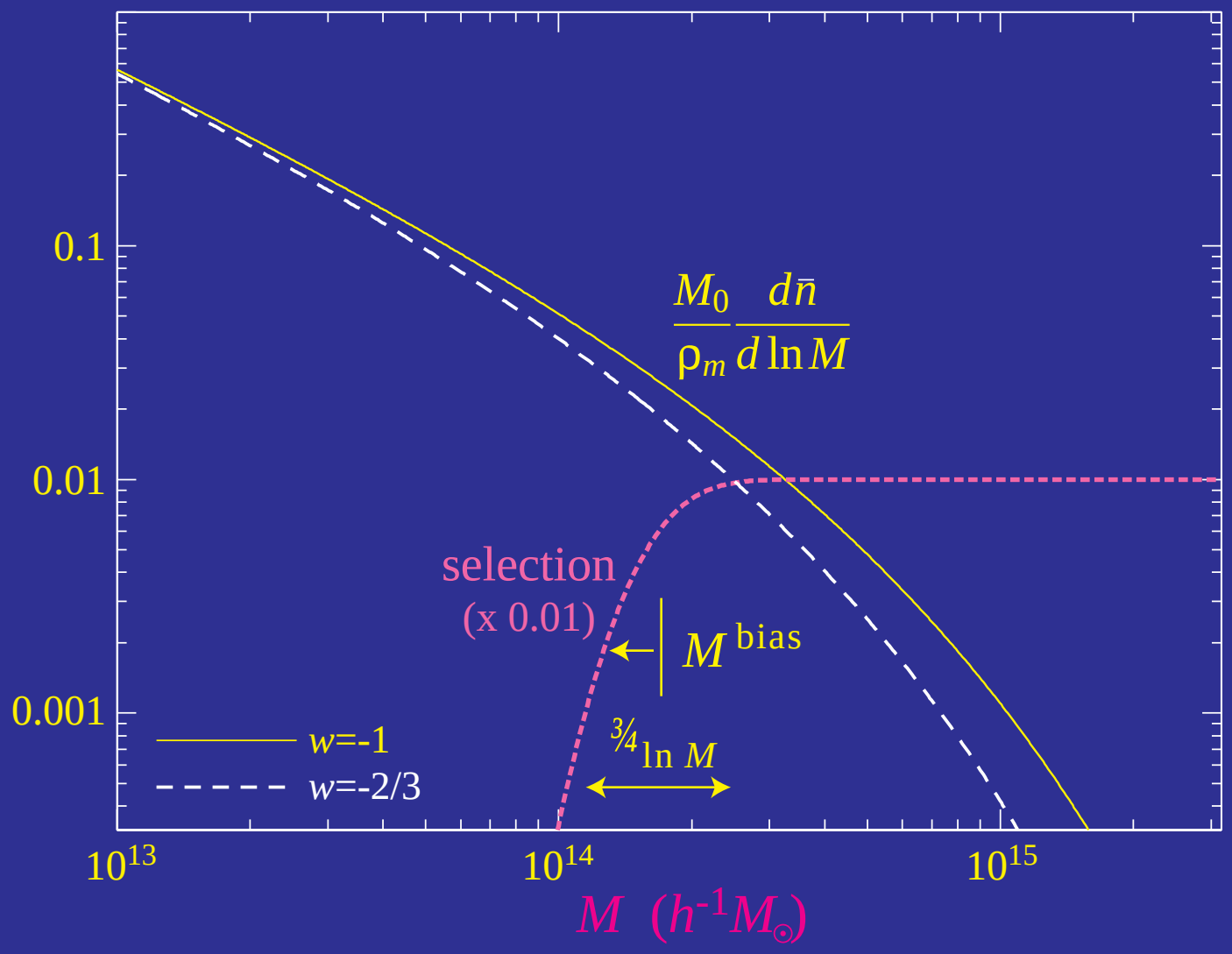
- Number density of massive halos extremely sensitive to the growth of structure and hence the dark energy
- Massive halos can be identified by the hot gas they contain



Carlstrom et al. (2001)

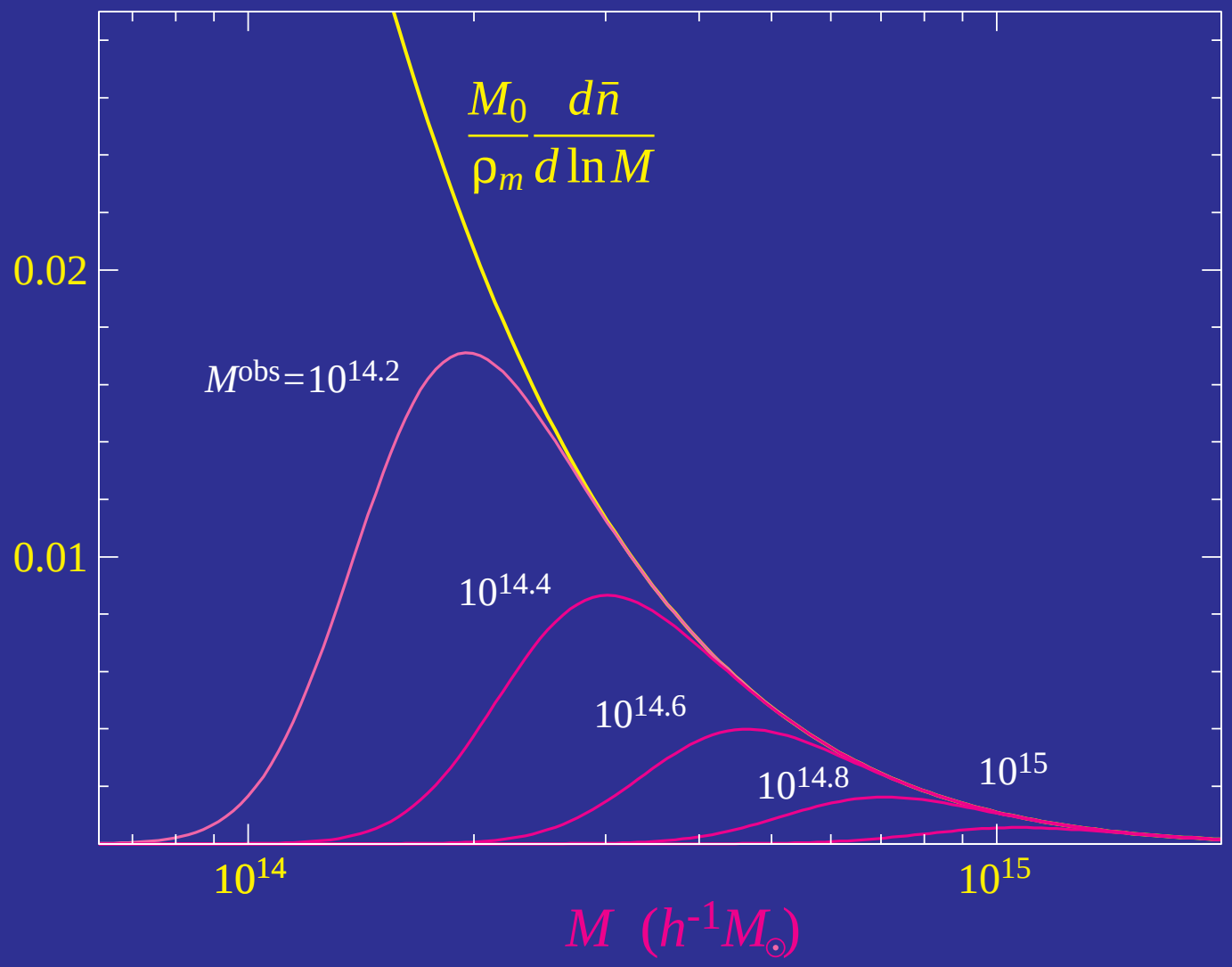
Degeneracy

- Uncertainties in bias and scatter cause degeneracies with dark energy



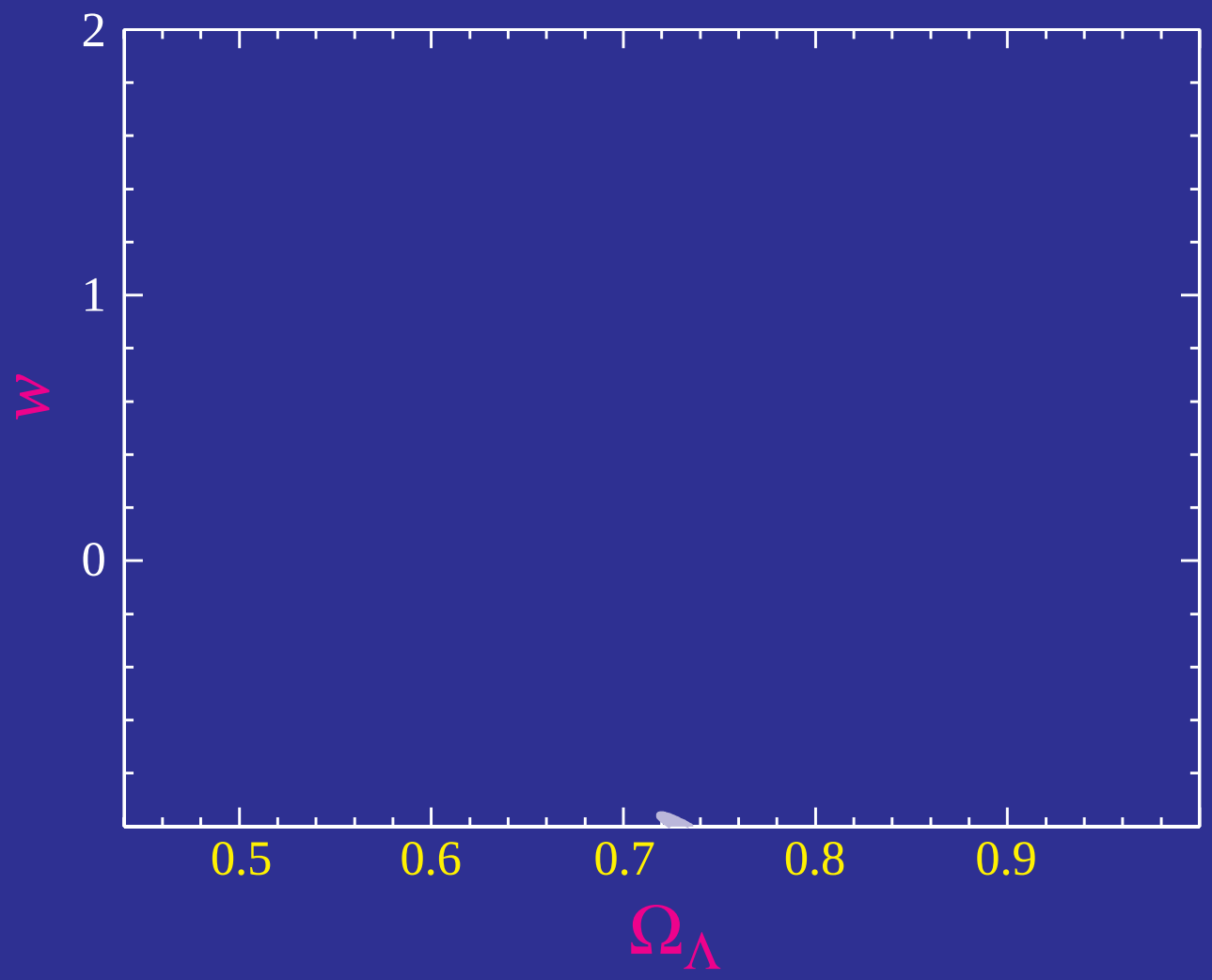
Selection Bias

- Exponential tail of mass function
- Threshold cut in the observable mass



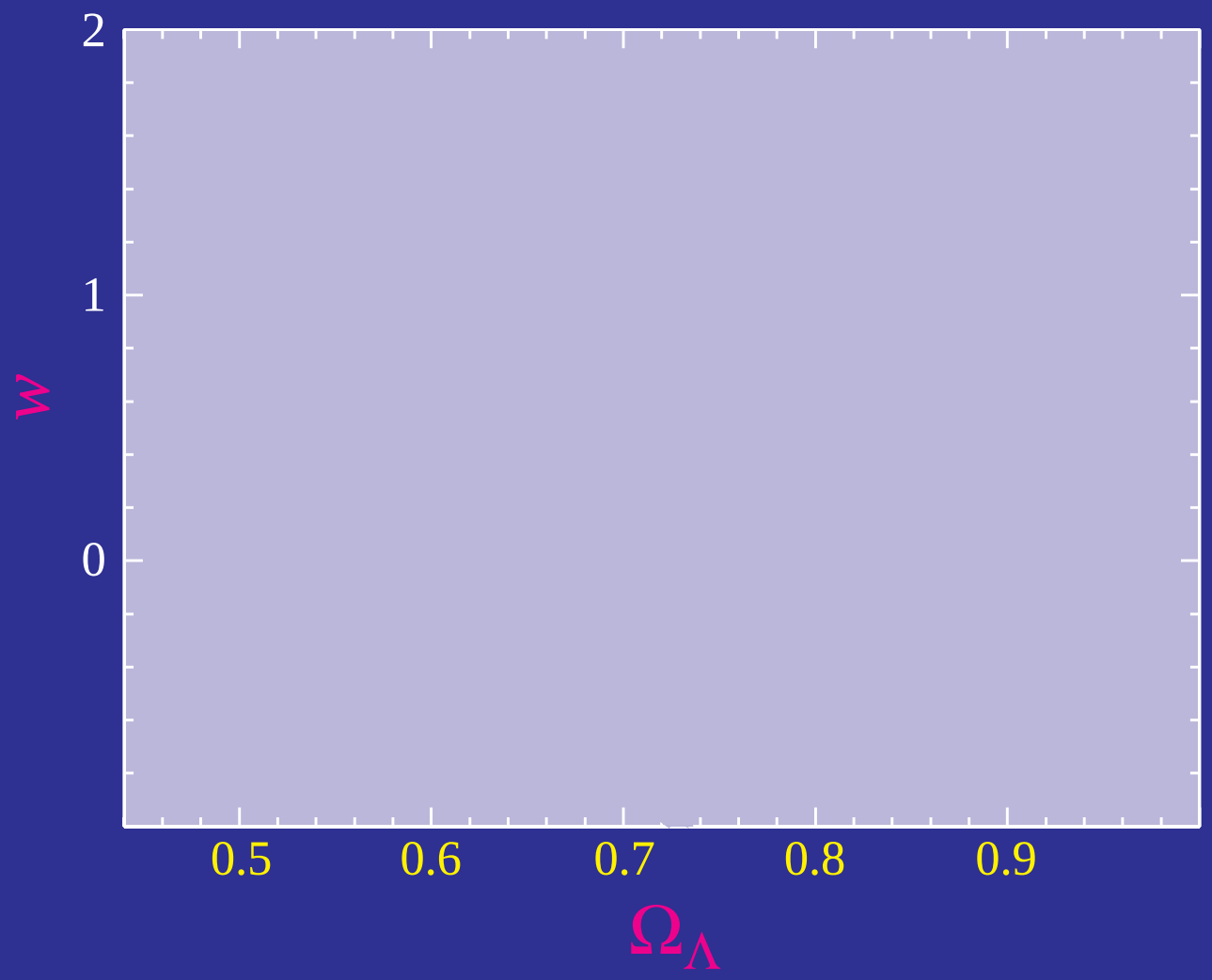
Fully Calibrated

- Given a completely known observable-mass distribution dark energy constraints are quite tight (4000 sq deg, $z<2$)



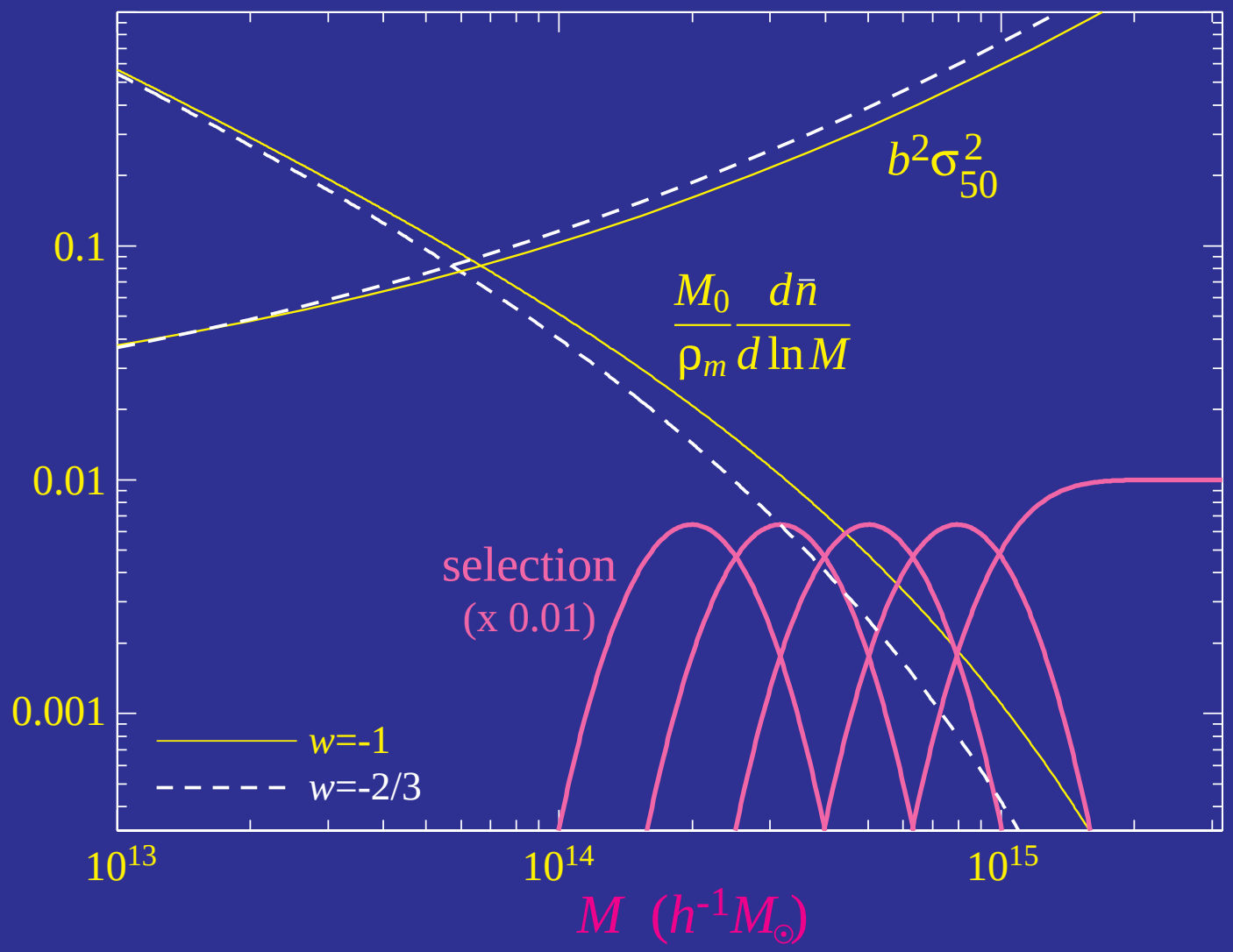
Un-Calibrated

- Marginalizing scatter (linear *z* evolution) and bias (power law evolution) destroys all dark energy information



Joint Self-Calibration

- Both counts and their variance as a function of binned observable
- Many observables allows for a joint solution of a mass independent bias and scatter with cosmology



Joint Self Calibration

- Power law evolution of bias and cubic evolution of scatter in *z*