Testing GR on Cosmological Scales

$f(R)$ and DGP

Worked Examples

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Outline

- Testing Gravity Cosmologically
- $f(R)$ (chameleon) and DGP (Vainshtein) worked examples
- Collaborators

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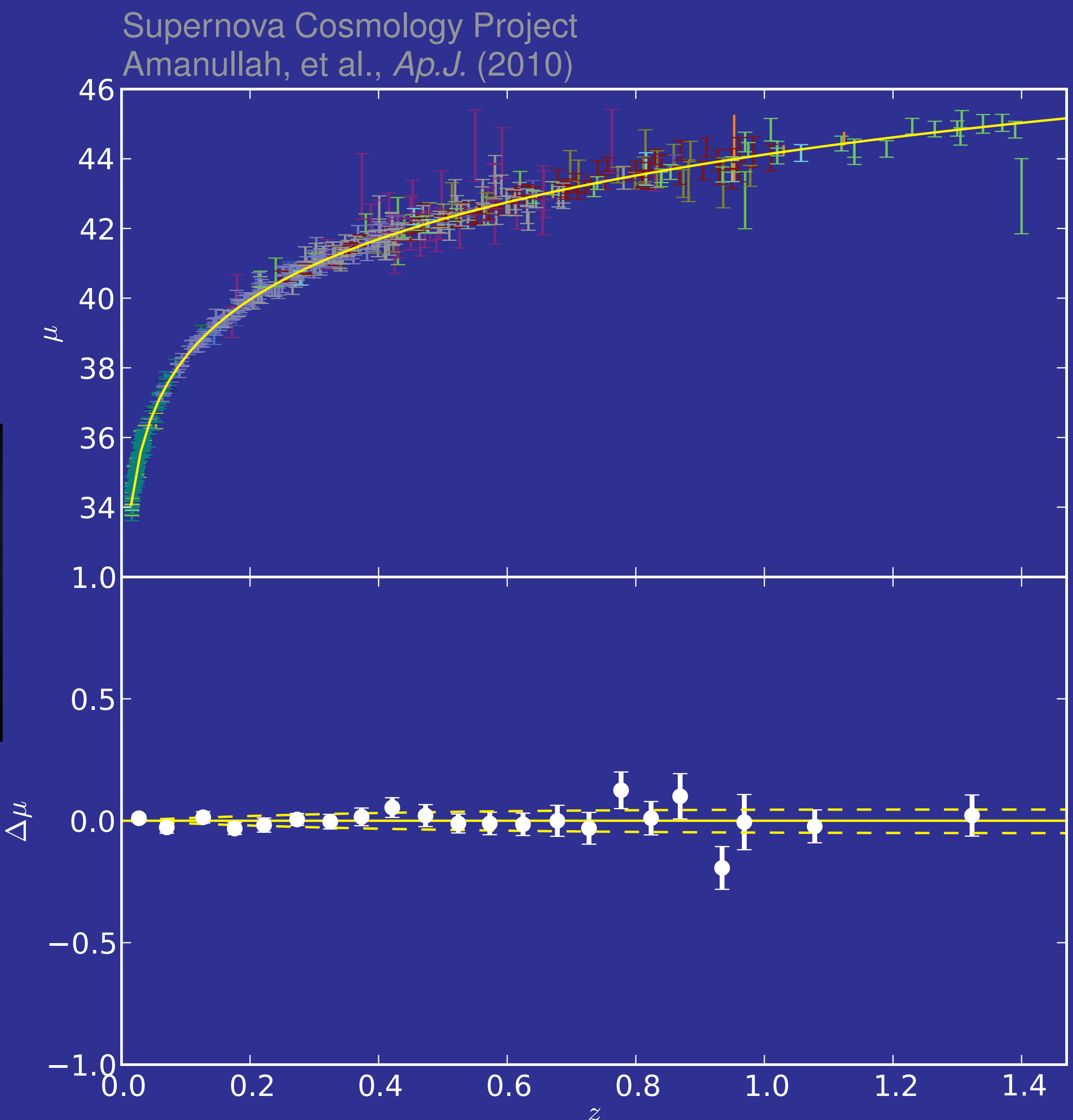
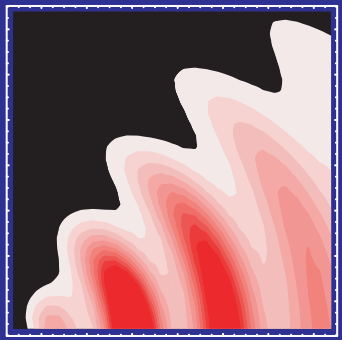
Cosmic Acceleration

- Geometric measures of distance redshift from SN, CMB, BAO

Standard(izable)
Candle
Supernovae
Luminosity v Flux

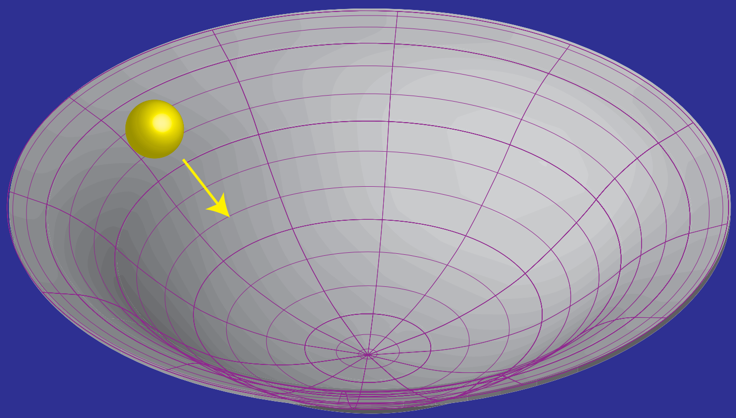




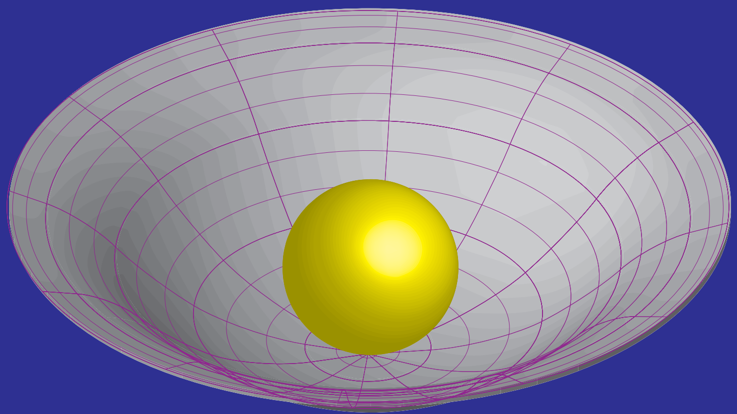
Standard Ruler
Sound Horizon
v CMB, BAO angular
and redshift separation

Mercury or Pluto?

- General relativity says Gravity = Geometry



- And Geometry = Matter-Energy



- Could the missing energy required by acceleration be an incomplete description of how matter determines geometry?

Two Potentials

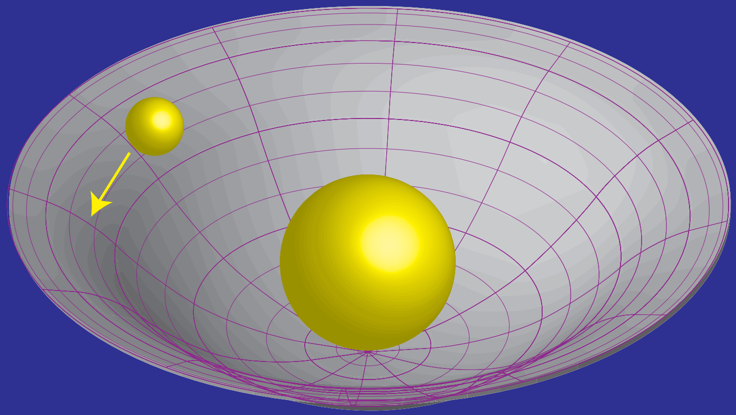
- Line Element

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2$$

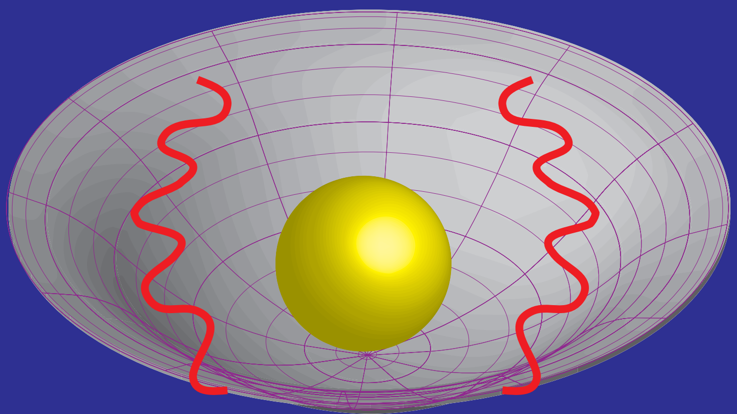
- Newtonian dynamical potential Ψ
- Space curvature potential Φ
- As in the parameterized post Newtonian approach, cosmological tests of the Φ/Ψ
- Space curvature per unit dynamical mass
- Given parameterized metric, matter falls on geodesics

Dynamical vs Lensing Mass

- Newtonian potential: $\Psi=\delta g_{00}/2g_{00}$ which non-relativistic particles feel



- Space curvature: $\Phi=\delta g_{ii}/2g_{ii}$ which also deflects photons



- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass

Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content
- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy

$$
\begin{aligned}
F(g_{\mu\nu}) + G_{\mu\nu} &= 8\pi G T^{\rm M}_{\mu\nu} \qquad \boxed{- F(g_{\mu\nu}) = 8\pi G T^{\rm DE}_{\mu\nu}} \\
G_{\mu\nu} &= 8\pi G [T^{\rm M}_{\mu\nu} + T^{\rm DE}_{\mu\nu}]
\end{aligned}
$$

and the Bianchi identity guarantees $\nabla^\mu T^{\rm DE}_{\mu\nu} = 0$

- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor
- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress

Modified Gravity $\neq$ “Smooth DE”

- Scalar field dark energy has $\delta p = \delta\rho$ (in constant field gauge) – relativistic sound speed, no anisotropic stress
- Jeans stability implies that its energy density is spatially smooth compared with the matter below the sound horizon

$$
\begin{aligned}
ds^2 &= -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2 \\
\nabla^2(\Phi - \Psi) &\propto \text{matter density fluctuation}
\end{aligned}
$$

- Anisotropic stress changes the amount of space curvature per unit dynamical mass

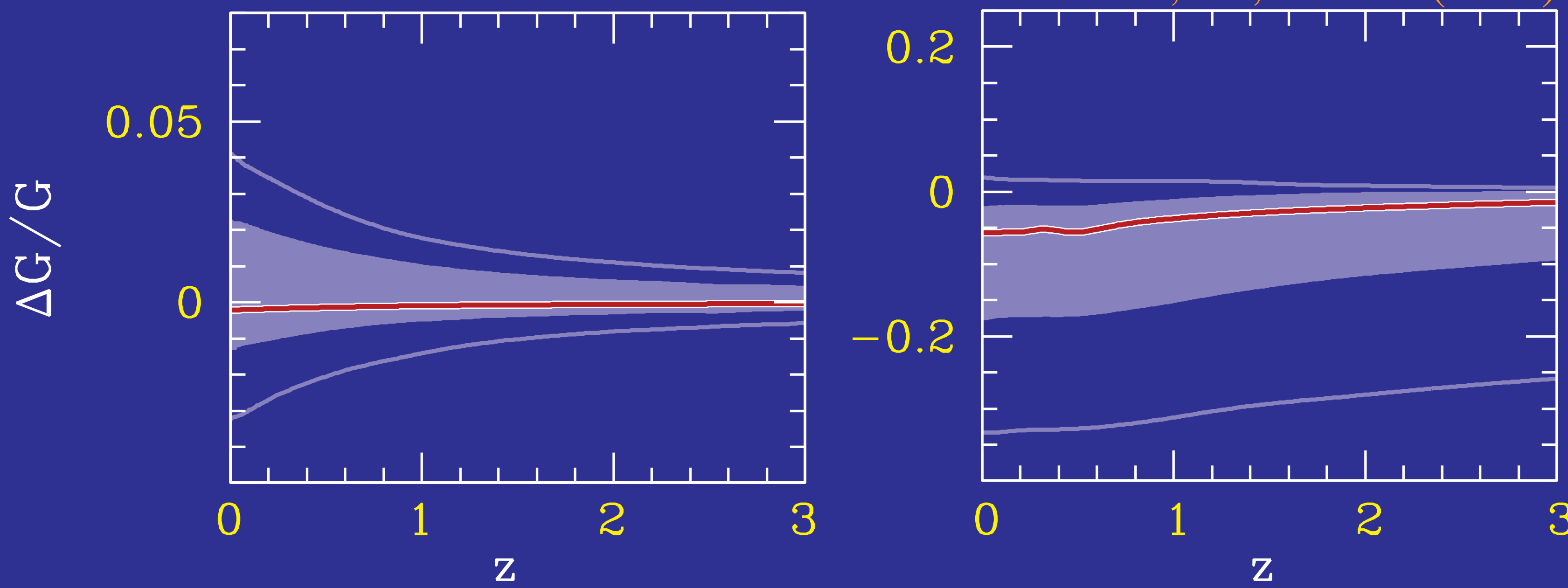
$$
\nabla^2(\Phi + \Psi) \propto \text{anisotropic stress}
$$

but its absence in a smooth dark energy model makes $g = (\Phi + \Psi)/(\Phi - \Psi) = 0$ for non-relativistic matter

Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)



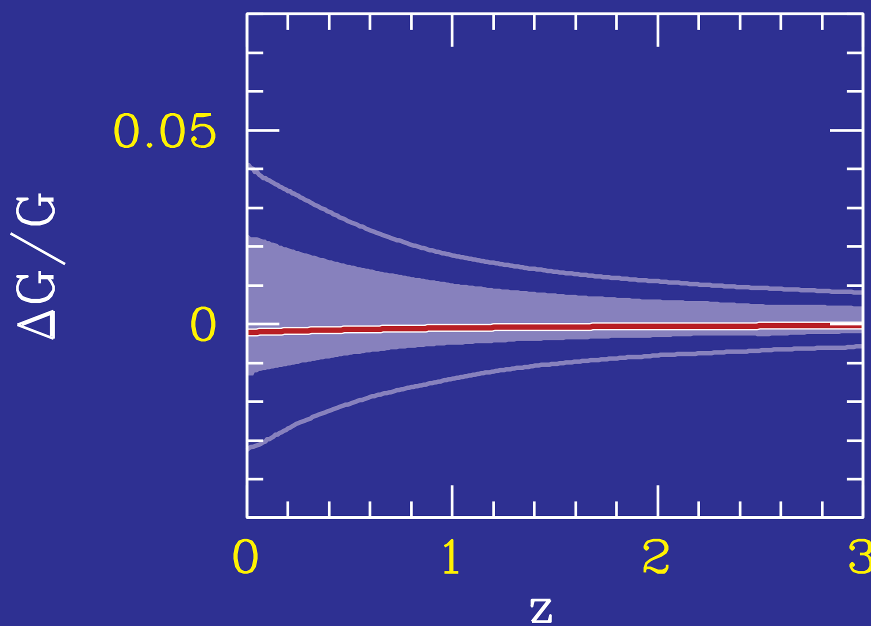
Cosmological Constant Quintessence

- Deviation significantly >2% rules out Λ with or without curvature
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in H_0]

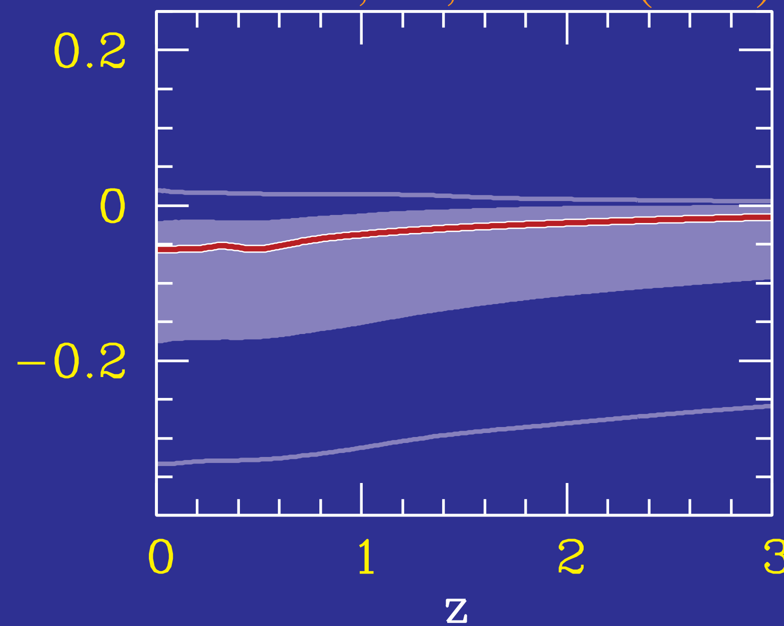
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

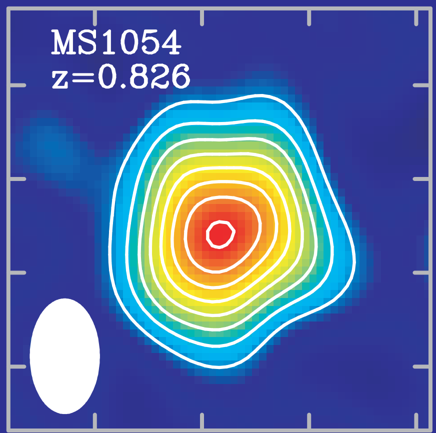
Mortonson, Hu, Huterer (2009)

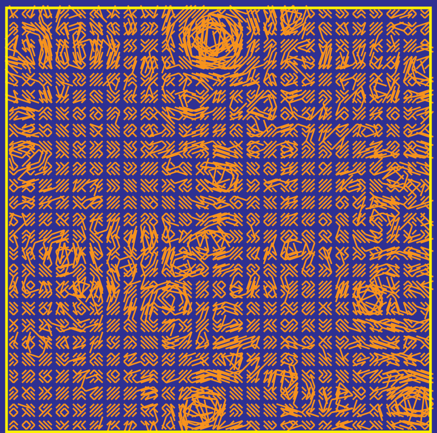


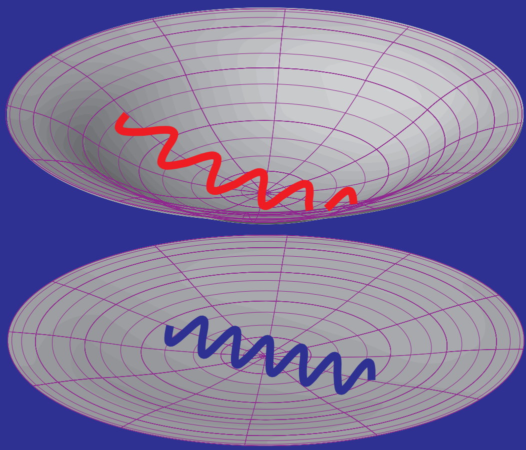
Cosmological Constant

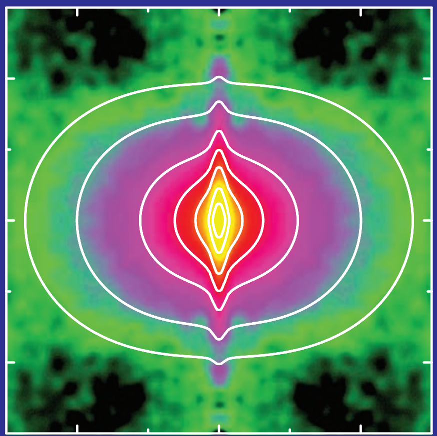


Quintessence







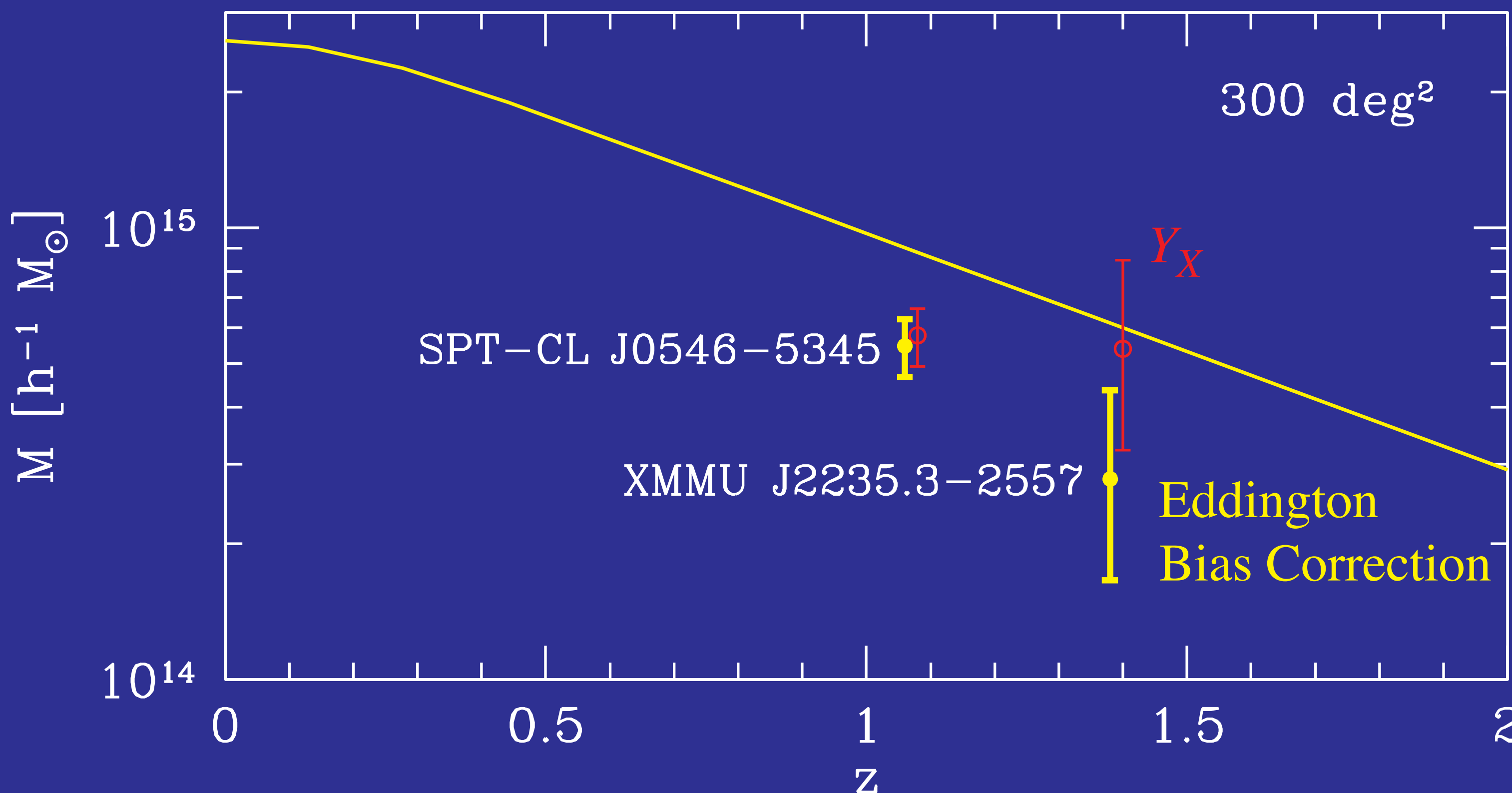


ΛCDM Falsified?

- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the $M(z)$ curve
- Convenient fitting formulae for future elephants:

  http://background.uchicago.edu/abundance

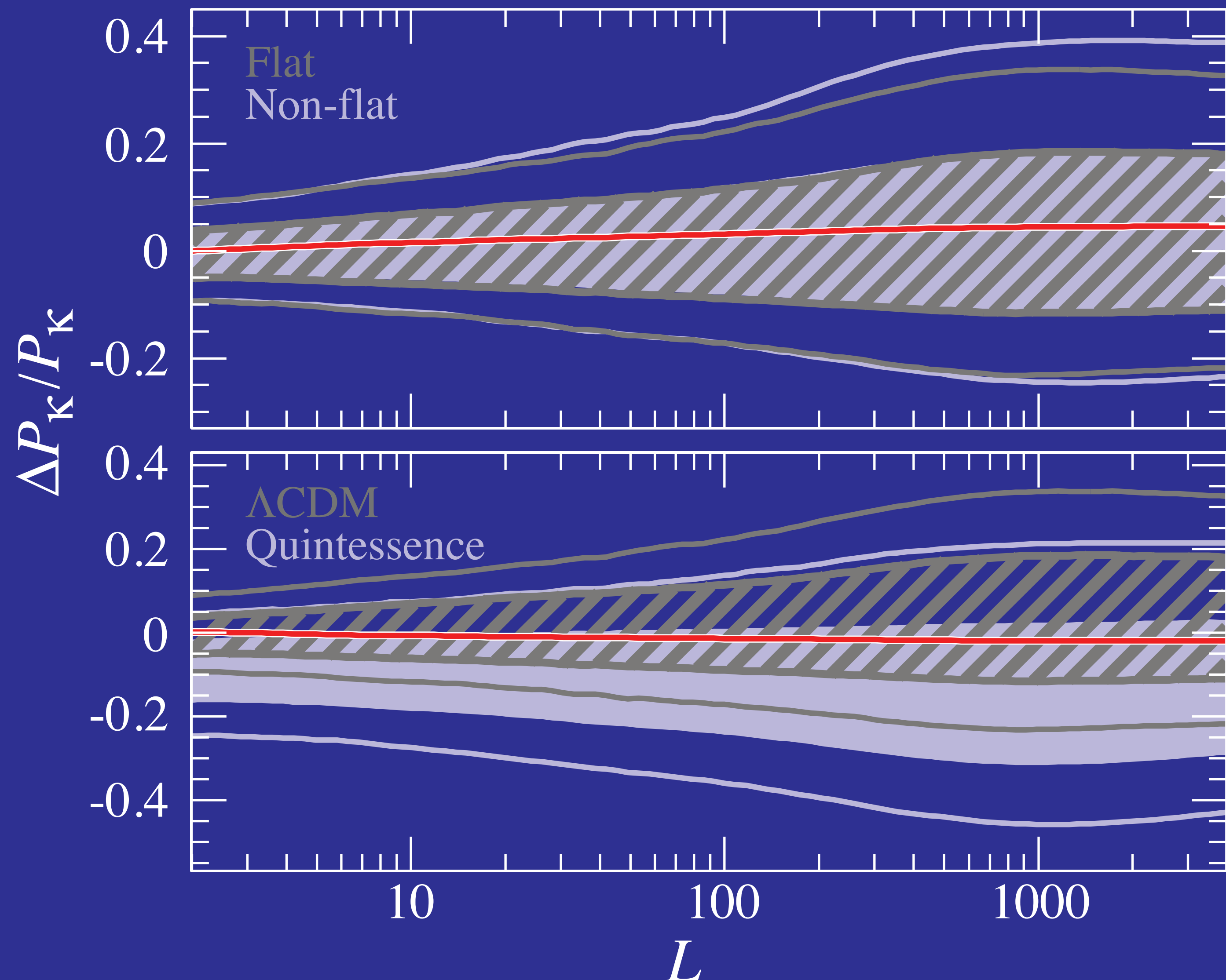
Mortonson, Hu, Huterer (2010)



Cosmic Shear Tests

- Convergence power spectrum of CFHLT-like survey; currently consistent with ΛCDM

Vanderveld et al (2012)



Falsify in Favor of What?
some toy examples

Nonlinearly Screened DOFs

- Modifications of gravity will introduce new propagating degrees of freedom (Weinberg)
- These DOFs mediate fifth forces and may lead to ghost and tachyon instabilities
- Even attempts to modify gravity on cosmological scales (IR) will have consequences for small scales (e.g. vDVZ discontinuity)
- Fifth forces are highly constrained in the solar system and lab
- Must be screened by a nonlinear mechanism in the presence of matter source: chameleon, symmetron, Vainshtein...
- Realization in models: $f(R)$, DGP, galileon, massive gravity
- $f(R)$, DGP examples solved from horizon scales through to dark matter halo scales with N-body simulations

Modified Action $f(R)$ Model

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_{\rm m} \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B: Compton wavelength of f_R squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate

Three Regimes

- Fully worked $f(R)$ example show 3 regimes
- Superhorizon regime: constant comoving curvature, $g(a)$
- Linear regime - closure $\leftrightarrow$ "smooth" dark energy density:

$$\begin{aligned} k^2(\Phi - \Psi)/2 &= 4\pi G a^2 \Delta\rho \\ (\Phi + \Psi)/(\Phi - \Psi) &= g(a, k) \end{aligned}$$

In principle $G(a)$ but conformal invariance: deviations order f_R

- Non-linear regime, scalar f_R:

$$\begin{aligned} \nabla^2(\Phi - \Psi)/2 &= -4\pi G a^2 \Delta\rho \\ \nabla^2\Psi &= 4\pi G a^2 \Delta\rho + \frac{1}{2}\nabla^2 f_R \end{aligned}$$

with non-linearity in the field equation

$$\nabla^2 f_R = g_{\rm lin}(a) a^2 \left(8\pi G \Delta\rho - N[f_R]\right)$$

Non-Linear Chameleon

- For $f(R)$ the field equation

$$\nabla^2 f_R \approx \frac{1}{3}(\delta R(f_R) - 8\pi G \delta\rho)$$

  is the non-linear equation that returns general relativity

- High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$

- Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if

$$\Delta f_R \le \frac{2}{3}\Phi \,,$$

  else required field gradients too large despite $\delta R = 8\pi G \delta\rho$ being the local minimum of effective potential

Non-Linear Dynamics

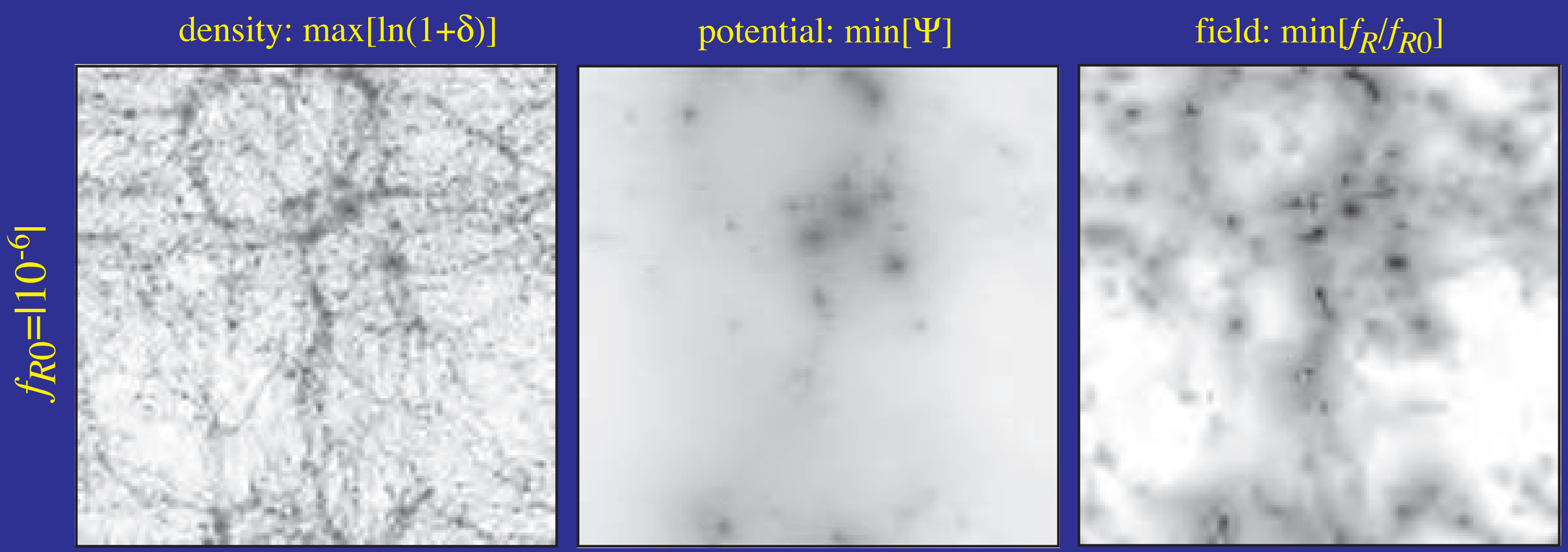
- Supplement that with the modified Poisson equation

$$\nabla^2 \Psi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R(f_R)$$

- Matter evolution given metric unchanged: usual motion of matter in a gravitational potential Ψ
- Prescription for N-body code
- Particle Mesh (PM) for the Poisson equation
- Field equation is a non-linear Poisson equation: relaxation method for f_R
- Initial conditions set to GR at high redshift

Environment Dependent Force

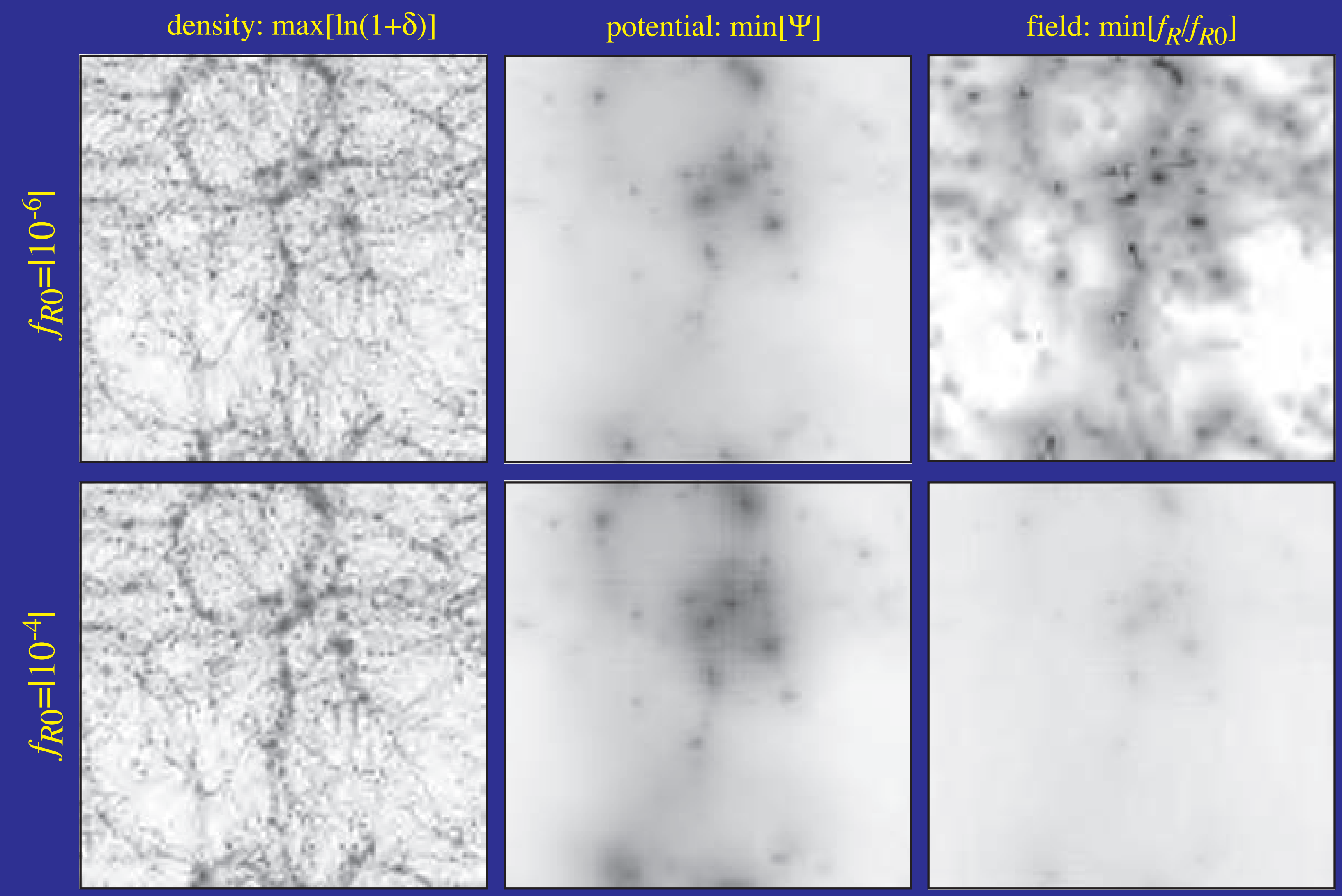
- Chameleon suppresses extra force (scalar field) in high density, deep potential regions



Oyaizu, Lima, Hu (2008)

Environment Dependent Force

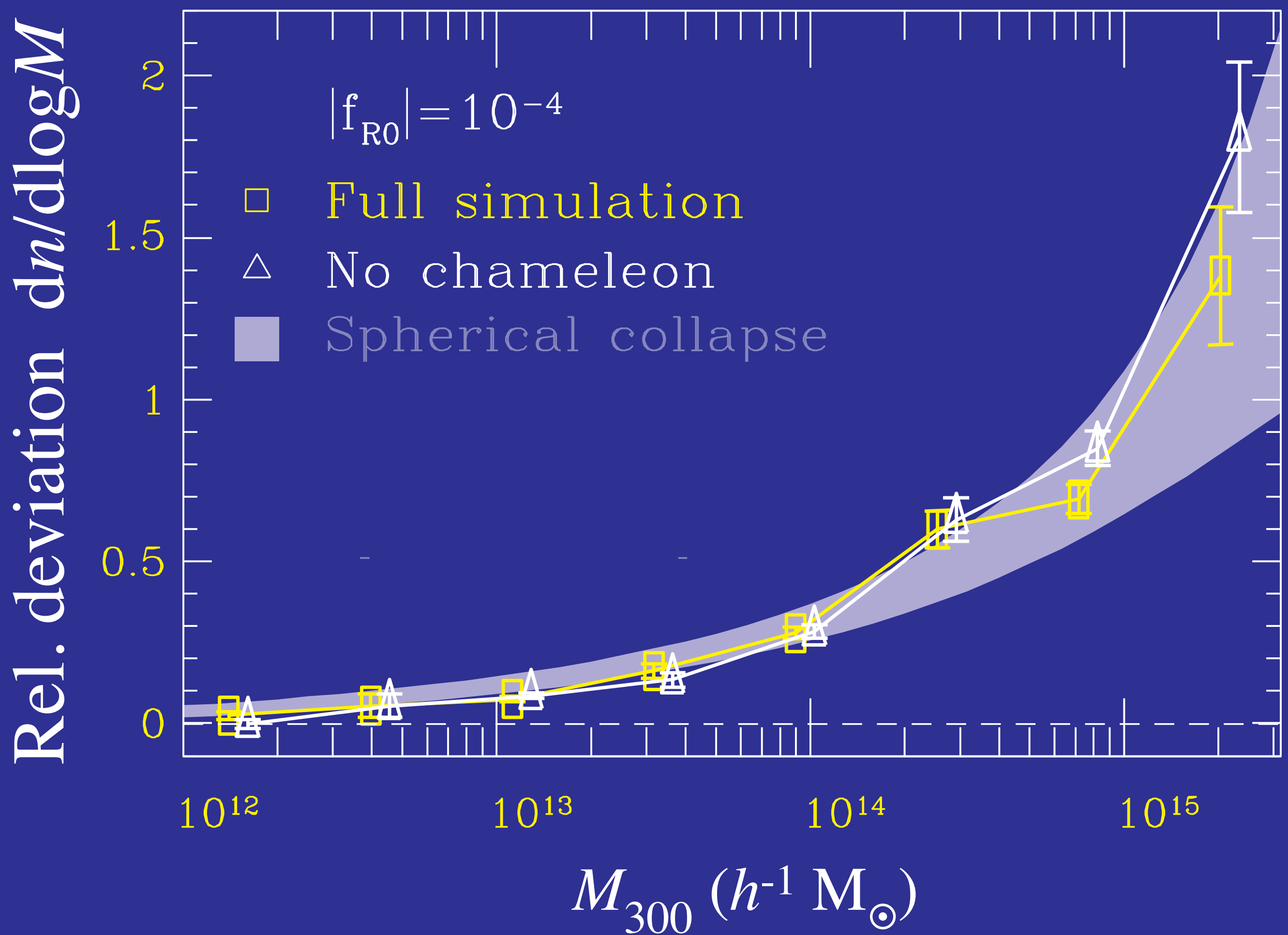
- For large background field, gradients in the scalar prevent the chameleon from appearing



Oyaizu, Lima, Hu (2008)

Cluster Abundance

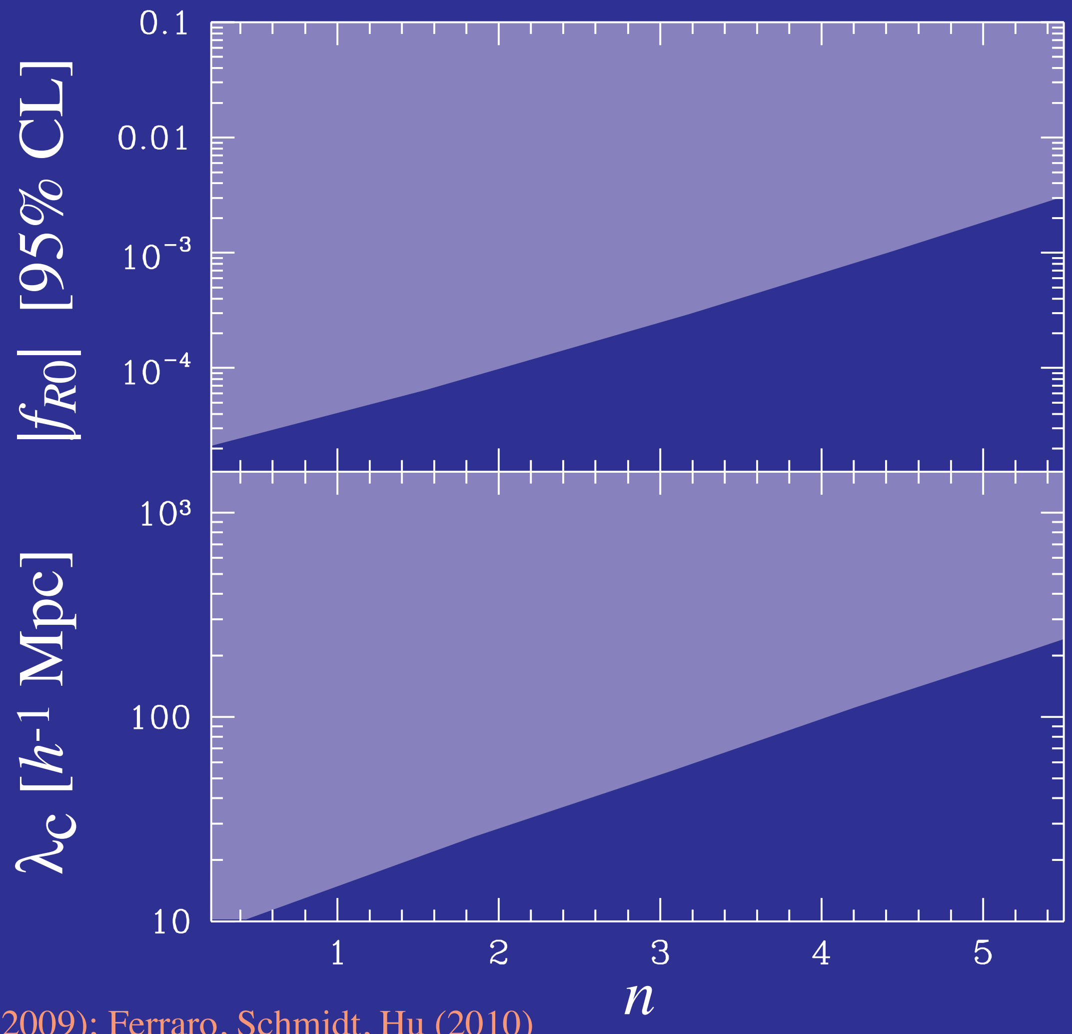
- Enhanced abundance of rare dark matter halos (clusters) with extra force



Lima, Schmidt, Oyaizu, Hu (2008)

Cluster $f(R)$ Constraints

- Clusters provide best current cosmological constraints on $f(R)$ models
- Spherical collapse rescaling to place constraints on full range of inverse power law models of index n



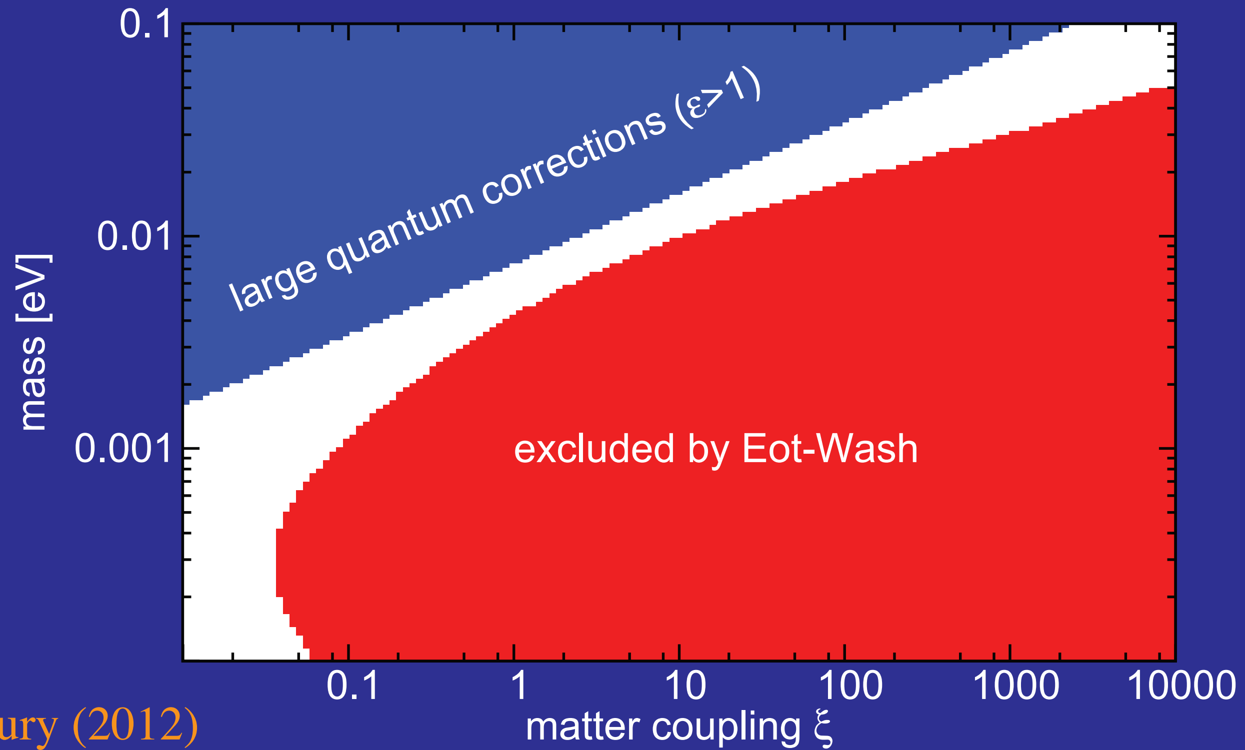
field amplitude

range of force

Schmidt, Vikhlinin, Hu (2009); Ferraro, Schmidt, Hu (2010)

Solar System & Lab

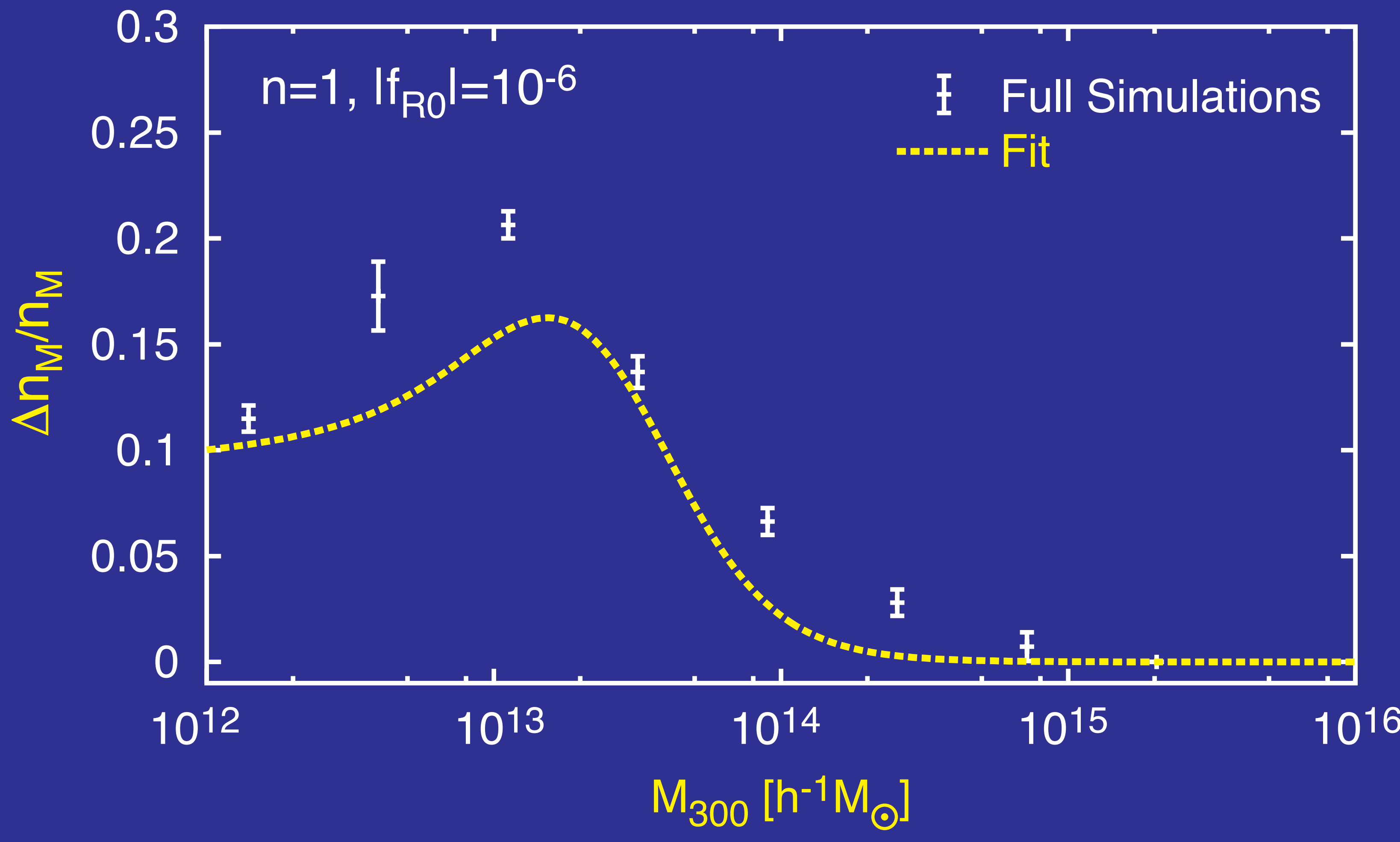
- Strictly valid for solar system / lab or are beyond effective theory?
- If former, solar system $f(R)$ tests of more powerful by at least 10
  (Hu & Sawicki 2009; exosolar tests: see Jain's talk)
- Laboratory tests: within factor of 2 of ruling out all gravitational strength chameleon models [$m < 0.0073(\xi\rho/10\text{g cm}^3)^{1/3}\text{eV}$]
  Already exceeded the vacuum scale (1000km) and earth (1cm) of Vainshtein models (Nicolis & Rattazzi 2004)



Upadhye, Hu, Khoury (2012)

Chameleon Mass Function

- Simple single parameter extention covers variety of models
- Basis of a halo model based post Friedmann parameterization of chameleon



Li & Hu (2011)

DGP Braneworld Acceleration

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

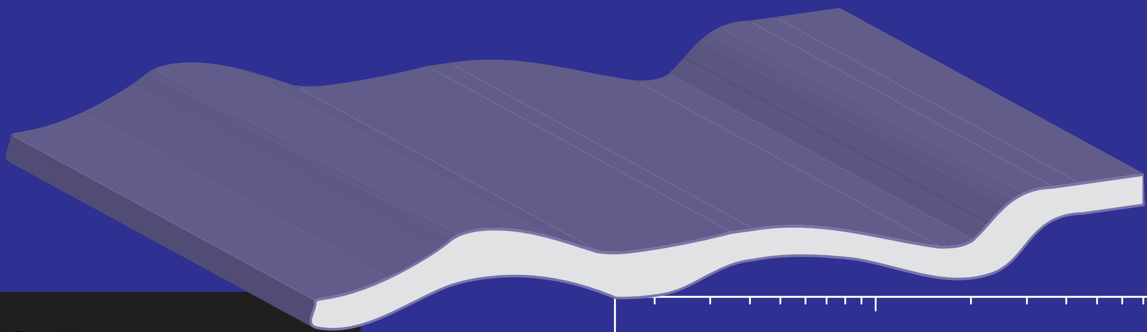
$$S = \int d^5x \sqrt{-g} \left[\frac{^{(5)}R}{2\kappa^2} + \delta(\chi) \left(\frac{^{(4)}R}{2\mu^2} + \mathcal{L}_m \right) \right]$$

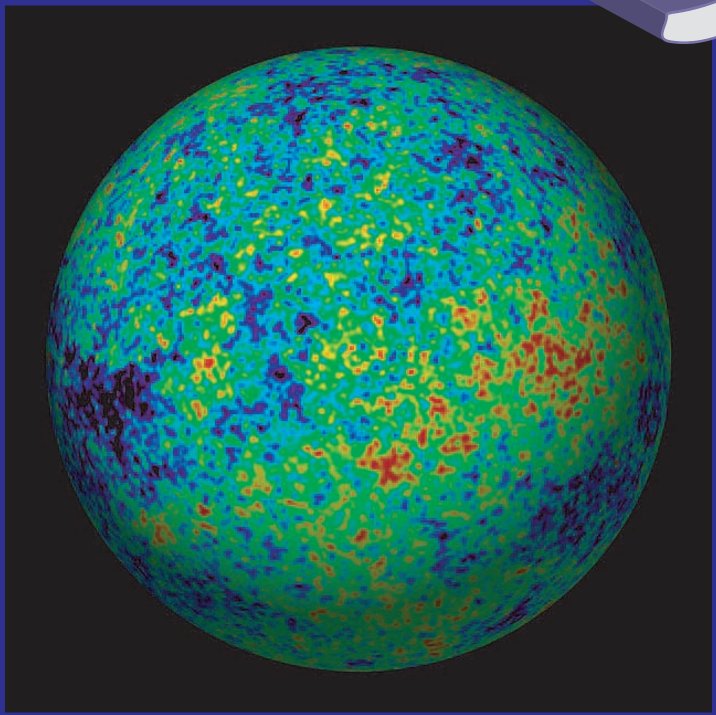
with crossover scale $r_c = \kappa^2 / 2\mu^2$

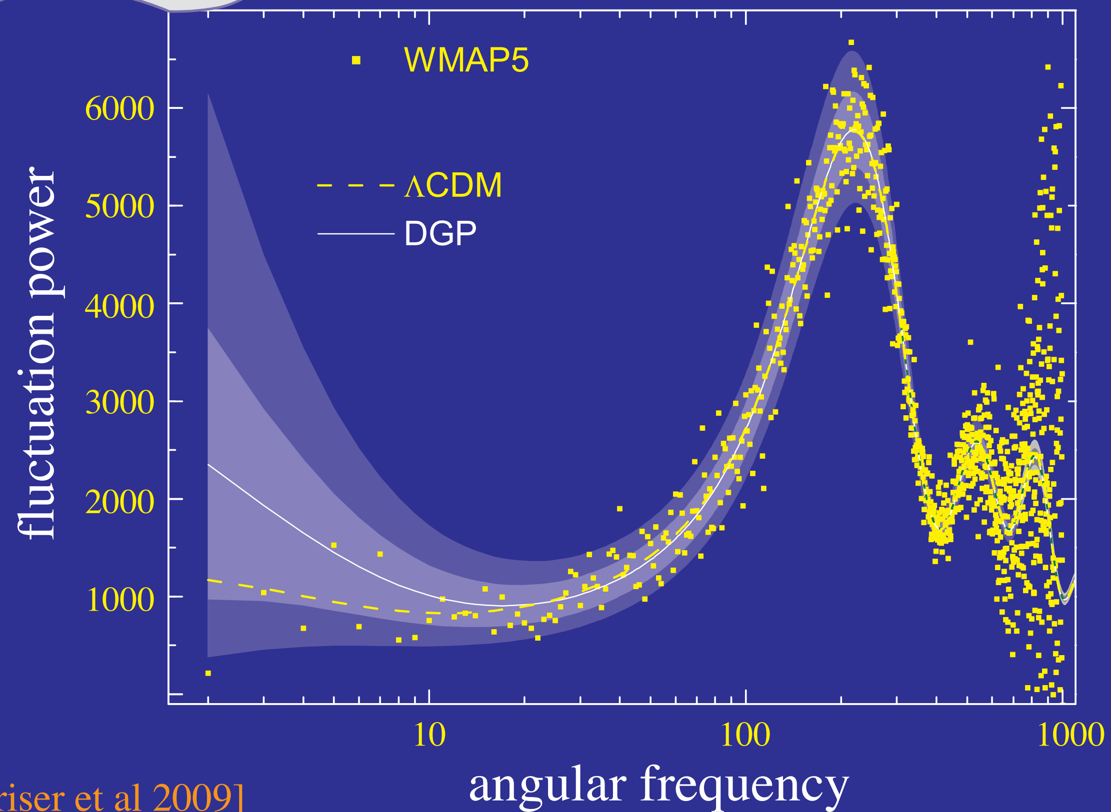
- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)
- Matter still minimally coupled and conserved
- Exhibits the 3 regimes of modified gravity
- Weyl tensor anisotropy dominated conserved curvature regime $r > r_c$ (Sawicki, Song, Hu 2006; Cardoso et al 2007)
- Brane bending scalar tensor regime $r_* < r < r_c$ (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)
- Strong coupling General Relativistic regime $r < r_* = (r_c^2 r_g)^{1/3}$ where $r_g = 2GM$ (Dvali 2006)

DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background







Fang et al (2008) [Seahra & Hu 2010; Lombriser et al 2009]

Massive Gravity

- DGP model motivated re-examination of massive gravity models
  [de Rham, Gabadadze, et al, Koyama et al, Hassan & Rosen, ... (2010-2012)]

- Graviton mass $\sim H_0$ provides self-acceleration

$$H^2 = \left(\frac{m}{2}\right)^2 + \frac{8\pi G}{3}\rho$$

  while also not seeing the cosmological constant contribution "degravitation"

- Key: nonlinearly complete Fierz-Pauli action: Vainshtein strong coupling (restoring vDVZ continuity), no Boulware Deser ghost

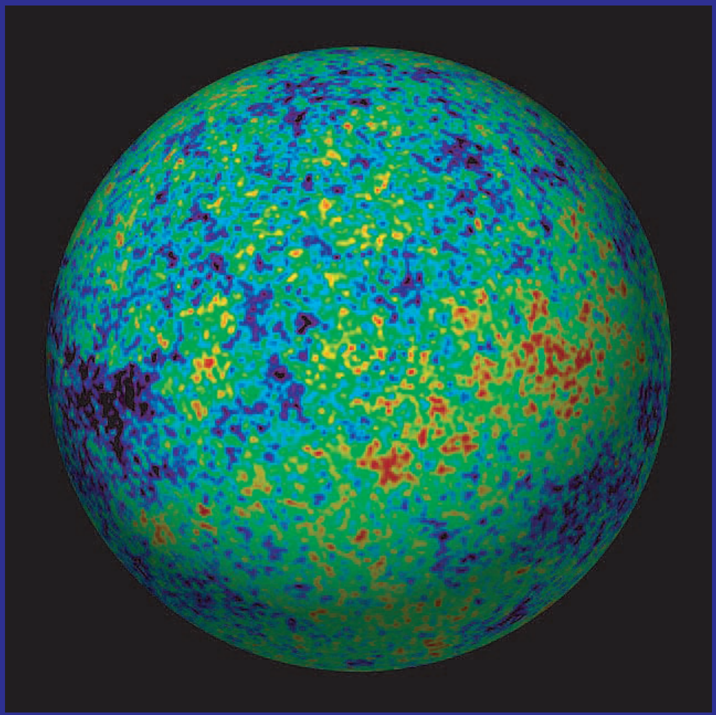
$$S = \int d^4x\sqrt{-g}\frac{1}{16\pi G}\left(R + m^2[\mathcal{L}^{(2)}(\mathcal{K}) + \alpha_3\mathcal{L}^{(3)}(\mathcal{K}) + \alpha_4\mathcal{L}^{(4)}(\mathcal{K})]\right)$$

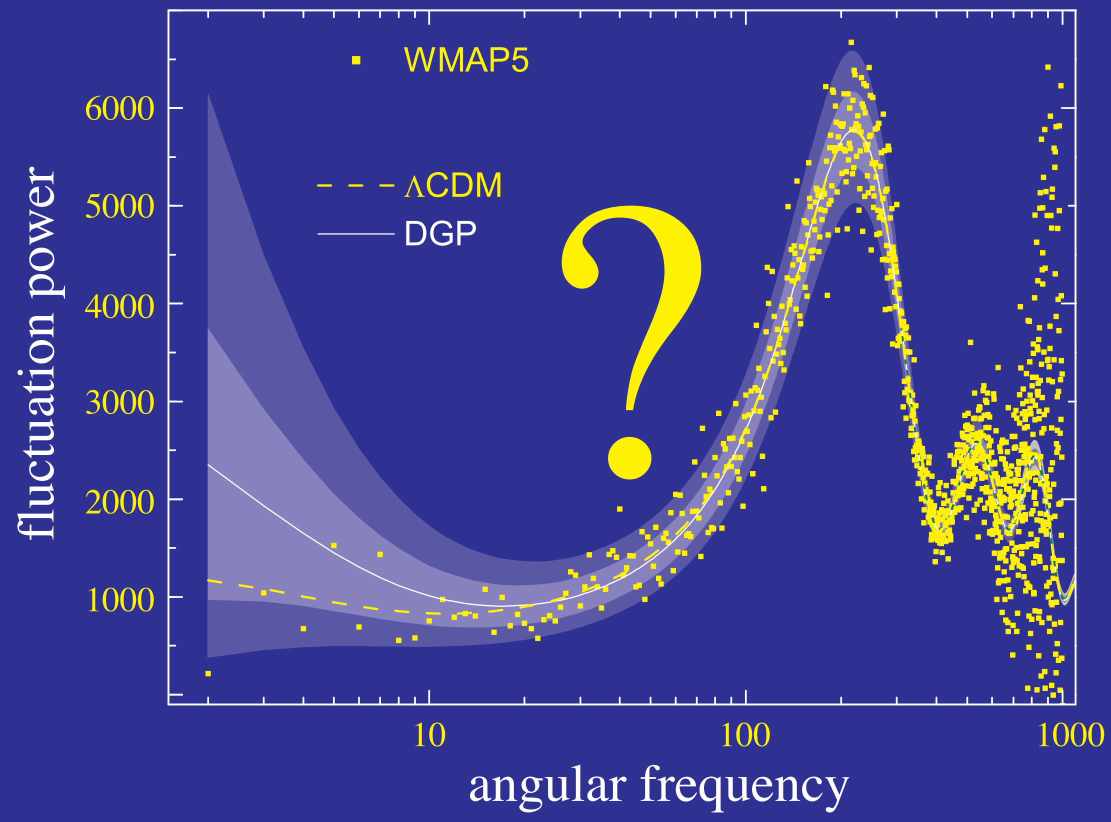
  with $\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{\partial^\mu\phi^a\partial_\nu\phi^b\eta_{ab}}$ in Stückelberg scalar language
  Arkani-Hamed, Georgi, Schwartz (2003)

- Much progress in the last year...stay tuned...

Massive Gravity

- What does ghost-free massive gravity predict on horizon scale?
- Indications that cosmology could be drastically different with horizon-scale domains (inhomogeneous or anisotropic... D'Amico et al 2011)
- But exact cosmological constant solution holds for any spherically symmetric configuration (MD to self-acceleration, radial perturbations...) (Gratia, Hu, Wyman 2012)





Nonlinear Interaction

Nonlinearity in field equation recovers linear theory if $N[\phi] \to 0$

$$\nabla^2\phi = g_{\rm lin}(a)a^2\left(8\pi G\Delta\rho - N[\phi]\right)$$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a nonlinear function of the field

Linked to gravitational potential

- For DGP, ϕ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4}\left[(\nabla^2\phi)^2 - (\nabla_i\nabla_j\phi)^2\right]$$

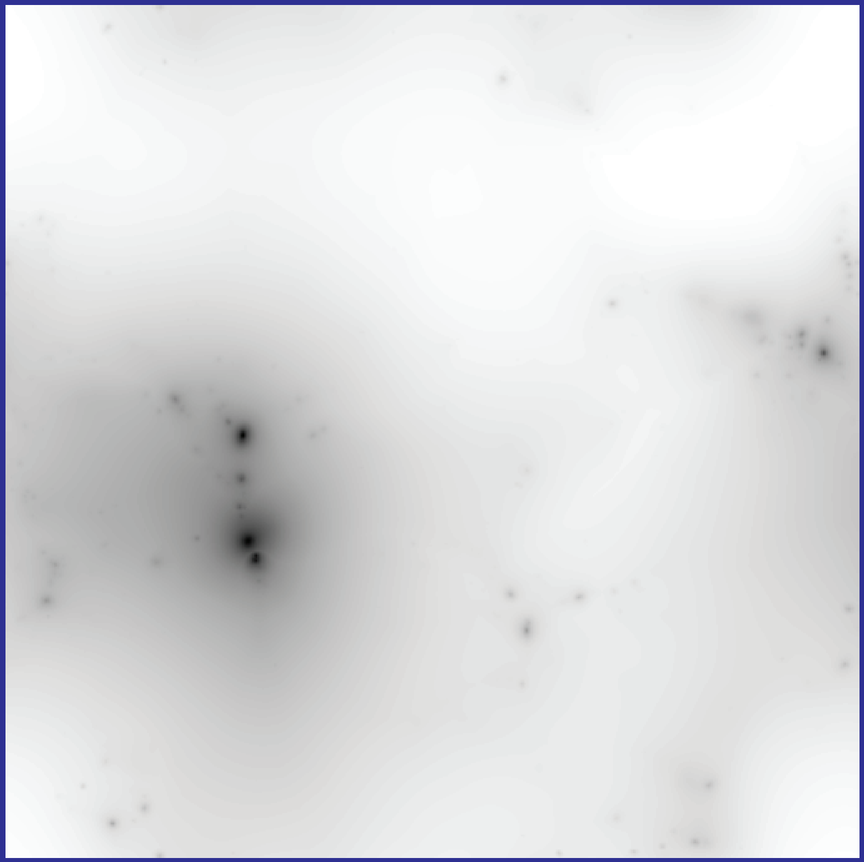
a nonlinear function of second derivatives of the field

Linked to density fluctuation - Galileon invariance - no self-shielding of external forces

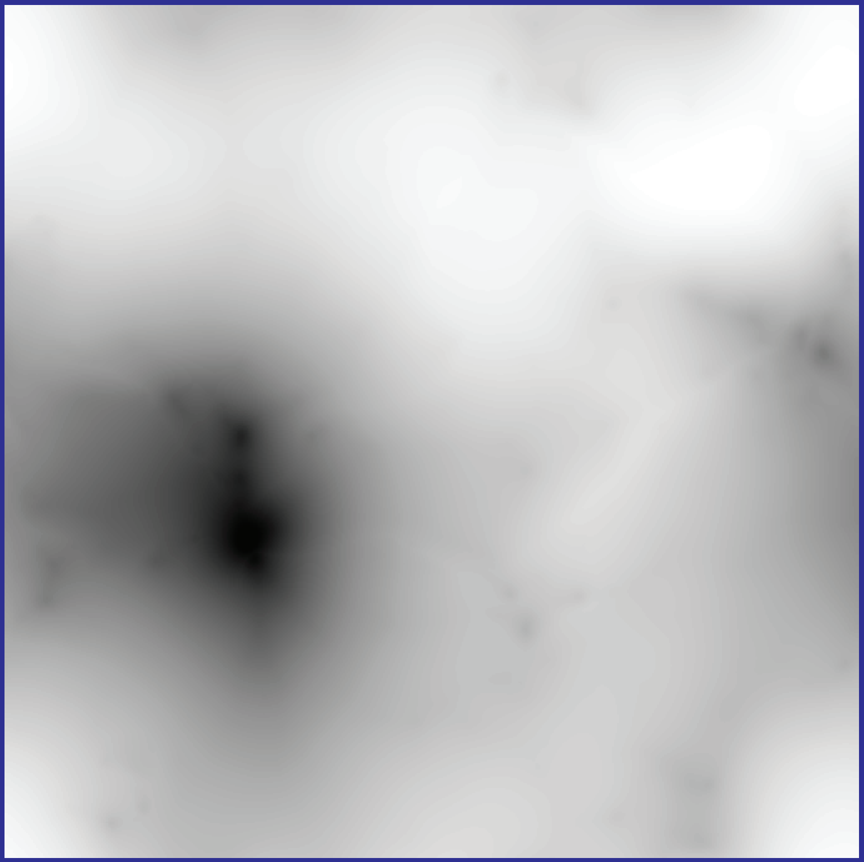
DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential



Brane Bending Mode



Schmidt (2009); Chan & Scoccimarro (2009) (cf. Khoury & Wyman 2009)

Summary

- Formal equivalence between dark energy and modified gravity
- Practical inequivalence of smooth dark energy and extra propagating scalar fifth force
- Appears as difference between dynamical mass and lensing mass or dark energy anisotropic stress
- Smooth dark energy (e.g. quintessence) highly falsifiable
- Three regimes of modified gravity
- Nonlinearity in field equations return to ordinary gravity

  Chameleon: deep potential well

  Vainshtein: high local density
- $f(R)$ modified action and DGP braneworld fully-worked examples
- Insights on how cosmology does (and does not) complement lab and solar system tests