Unsolved Problems in Dark Energy
a.k.a. Everything

Wayne Hu
Benasque, February 2011
Said the great Spanish physicist Raul Jimenez (in Americanized paraphrase):

“Better Lame than Late”

– circa 24 hrs ago
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or the A delayed Session = Cut the F***ing Time correspondence
Prolific Participants: $p + \bar{p} \rightarrow ?$

Hitoshi says 2 in billion survive!

Observational to Do List

- Just 3 things (from a theorist’s perspective)
  1. Falsify (flat) ΛCDM
  2. Falsify (flat) ΛCDM
  3. Falsify (flat) ΛCDM
How to Falsify

Test the consistency of $\Lambda$CDM in parameter space

- Expand parameter space or take alternate models
- MCMC set of parameters, find evidence for tension with $\Lambda$CDM
- Good - well defined, optimal if models well motivated
- Bad - there are no well motivated alternatives to $\Lambda$ (cf. Sundrum’s IQ Test)

- **FOM** (aka FML) and other elements of the Dark Energy Canon (Andrew Liddle and the New Bayesian Testament)

Test consistency in observable space

- MCMC in $\Lambda$CDM (or other complete paradigm like quintessence) and predict posteriors of new observables
- Good - theory says what best to observe to falsify $\Lambda$CDM
- Bad - falsify in favor of what?
Falsifying $\Lambda$CDM

- Geometric measures of distance redshift from SN, CMB, BAO

Supernova Cosmology Project

Standard(izable)
Candle
Supernovae
Luminosity v Flux

Standard Ruler
Sound Horizon
v CMB, BAO angular and redshift separation
Falsifying ΛCDM

• Λ slows growth of structure in highly predictive way
Falsifiability of Smooth Dark Energy

• With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate.

• Hence geometric measurements of the expansion rate predict the growth of structure:
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations

• Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm:
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)
Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way

\[ \Delta G/G \]

\[ z \]

Cosmological Constant

- Deviation significantly >2% rules out \( \Lambda \) with or without curvature

Quintessence

- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in \( H_0 \)]
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)
Redshift Space Distortion

- Redshift space distortions measure $f_G$ or $f\sigma_8$
- Measurements in excess of $\sim 5\%$ of $\Lambda$CDM would rule out quintessence

\[ \Delta \left( \frac{f_G}{f_G} \right) \]

Cosmological Constant

Quintessence

Mortonson, Hu, Huterer (2009)
NonPC Caveats on PCs

- Principal component decomposition of $w(z)$ shows many components can in principle be measured (Euclid, LSST, WFirst)
- Yet even models like Albrecht-Skordis (oscillating $w$) are still dominated by first component - average $w$ or pivot - plus 1-2 weaker
- Should a multidimensional parameterization be the basis for optimizing an experiment?
Pink Elephant Parade

- Too early too soon? SPT catalogue on 2500 sq degrees

Williamson et al (2010)

Other Analyses
Hoyle, Jiminez, Verde (2010)
many others...

Falisification Criteria
Mortonson, Hu, Huterer (2010)
Holz & Perlmutter (2010)
Systematics, Systematics, Systematics

- E.g.: clusters - mass calibration from X-ray, SZ, optical, lensing
Systematics, Systematics, Systematics

- Coffee talk: discuss amongst yourselves
Phenomenological To Do List

- How best to parameterize consistency tests or define theoretically predicted observables?
- How to treat baryonic effects in the non-linear regime e.g.
  - Galaxy occupation of halos [BAO, velocity field tests]
  - Concentration of clusters for cosmic shear
- Extensive simulations + modelling of gastrophysics including star formation (see Brant, Licia, Volker’s talks)
- Calibrate non-linear mean and covariance of dark energy observables as function of cosmological parameters (Volker’s talk)
- Simulate alternate models
  - (Kill) inhomogeneous models
  - Interacting dark matter-energy models
  - Modified gravity models
Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content

- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy

\[
F(g_{\mu\nu}) + G_{\mu\nu} = 8\pi G T_{\mu\nu}^M - F(g_{\mu\nu}) = 8\pi G T_{\mu\nu}^{DE}
\]

\[
G_{\mu\nu} = 8\pi G [T_{\mu\nu}^M + T_{\mu\nu}^{DE}]
\]

and the Bianchi identity guarantees \( \nabla^\mu T_{\mu\nu}^{DE} = 0 \)

- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor

- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress
Modified Gravity $\neq \text{“Smooth DE”}$

- **Scalar field dark energy** has $\delta p = \delta \rho$ (in constant field gauge) –
  relativistic sound speed, **no anisotropic stress**

- **Jeans stability** implies that its energy density is spatially smooth compared with the **matter** below the sound horizon

\[
ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2
\]

\[
\nabla^2(\Phi - \Psi) \propto \text{matter density fluctuation}
\]

- **Anisotropic stress** changes the amount of **space curvature** per unit dynamical mass

\[
\nabla^2(\Phi + \Psi) \propto \text{anisotropic stress}
\]

but its absence in a **smooth dark energy** model makes

\[
g = (\Phi + \Psi)/(\Phi - \Psi) = 0 \text{ for non-relativistic matter}
\]
Dynamical vs Lensing Mass

- Newtonian potential: $\Psi = \delta g_{00}/2g_{00}$ which non-relativistic particles feel

- Space curvature: $\Phi = \delta g_{ii}/2g_{ii}$ which also deflects photons

- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass
Lensing v Dynamical Comparison

- Gravitational lensing around galaxies vs. linear velocity field (through redshift space distortions and galaxy autocorrelation)
- **Consistent** with GR + smooth dark energy beginning to test interesting models


Three Regimes

- Three regimes with different dynamics
- Examples $f(R)$ and DGP braneworld acceleration
- Parameterized Post-Friedmann description
- Non-linear regime return to General Relativity / Newtonian dynamics

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**General Relativistic Non-Linear Regime**

- $r_*$
- *halos, galaxy*

**Scalar-Tensor Regime**

- $r_c$
- *large scale structure*

**Conserved-Curvature Regime**

- $r$
- *CMB*
Three Regimes

- Fully worked $f(R)$ and DGP examples show 3 regimes
- **Superhorizon** regime: $\zeta = \text{const.}, \, g(a)$
- **Linear** regime - closure condition - analogue of “smooth” dark energy density:

\[
\nabla^2 (\Phi - \Psi) / 2 = -4\pi G a^2 \Delta \rho \\
g(a, x) \leftrightarrow g(a, k)
\]

$G$ can be promoted to $G(a)$ but conformal invariance relates fluctuations to field fluctuation that is small

- **Non-linear** regime:

\[
\nabla^2 (\Phi - \Psi) / 2 = -4\pi G a^2 \Delta \rho \\
\nabla^2 \Psi = 4\pi G a^2 \Delta \rho - \frac{1}{2} \nabla^2 \phi
\]
Nonlinear Interaction

Non-linearity in the field equation

\[ \nabla^2 \phi = g_{\text{lin}}(a)a^2 (8\pi G \Delta \rho - N[\phi]) \]

recovers linear theory if \( N[\phi] \rightarrow 0 \)

- For \( f(R) \), \( \phi = f_R \) and

\[ N[\phi] = \delta R(\phi) \]

a non-linear function of the field

Linked to gravitational potential

- For DGP, \( \phi \) is the brane-bending mode and

\[ N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right] \]

a non-linear function of second derivatives of the field

Linked to density fluctuation    Example of Galileon invariance
Environment Dependent Force

- For **large background field**, gradients in the scalar **prevent** the chameleon from appearing

DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential  Brane Bending Mode

**Mass Function**

- Enhanced **abundance** of rare dark matter halos (clusters) with extra force goes away as non-linearity increases.

Phenomenological To Do List

- How best to parameterize consistency tests or define theoretically predicted observables?
- How to treat baryonic effects in the non-linear regime e.g.
  - Galaxy occupation of halos [BAO, velocity field tests]
  - Concentration of clusters for cosmic shear

Extensive simulations + modelling of gas astrophysics including star formation (see Brant, Licia, Volker’s talks)

- Calibrate non-linear mean and covariance of dark energy observables as function of cosmological parameters (Volker’s talk)

- Simulate alternate models
  - (Kill) inhomogeneous models
  - Interacting dark matter-energy models
  - Modified gravity models
Cooling/Star Formation in Clusters

- Baryonic effects can lead to false falsification of $\Lambda$CDM

Rudd, Zentner, Kravtsov (2007)
Theoretical To Do List

• Develop (discuss here) and explore known alternatives

• **Dynamical** Dark Energy
  • Quintessence, k-essence, phantom, effective field theoretic
    quintessence, vanishing sound speed k-essence, extended
    quintessence, coupled quintessence, electrostatic dark energy,
    elastically scattering dark energy, and other **venial sins**...

• **Modified Gravity**
  • braneworld, $f(R)$, $f(G)$, cascading gravity, degravitation,
    galileon, massive gravity, kinetic gravity braiding, TeVeS and
    other **heresies**...
Theoretical To Do List

- Develop (discuss here) and explore known alternatives
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  - Q: e.g. do massive gravity and galileon ideas for self-acceleration without ghosts make sense beyond the decoupling limit
Theoretical To Do List

- Develop alternatives that are more than just illustrative toy models. Floor is open to hear about them now!

- If observationally indistinguishable from $\Lambda$ why does it have the value it does?
  
  If we fail to find something to argue about in this session we can always try landscape/anthropic.
Andrew Liddle on Model Selection
What are we trying to achieve?

Goal: to define the key model tests to be carried and, where possible, to optimize survey strategies to achieve them. First we need to figure out which are the interesting models.

- **$\Lambda$CDM**: The current baseline cosmological model.
- Parameterized dark energy models, eg CPL $w = w_0 + (1-a) w_a$
  The most common candidate alternative for dark energy studies.
- Fundamental physics dark energy models, eg inverse power-laws, Albrecht-Skordis, etc
  Many candidate models in the literature though many fail to fit current data. Not clear which are best motivated.
- Modified gravity models
  Determination of best candidate modified gravity models required.
Parameter estimation tests

\[ W_0, W_a \]

**WMAP7: Komatsu et al**

- **WMAP + H_0 + SN**
- **WMAP + BAO + H_0 + SN**
- **WMAP + BAO + H_0 + D_{\Delta t} + SN**
Parameter estimation tests

This graph answers the following question:

If we assume that the $w_0-w_a$ dark energy model is correct, how good will our constraints on those parameters be?
Model tests

However that wasn’t the question we wanted to answer, which was:

Between the ΛCDM model and the dark energy model, which is the better description of the data? [i.e., can one of them be ruled out with respect to the other?]

This question can only be answered with a model-level analysis, e.g. Bayesian model selection.
Model tests

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Another way of expressing this: do you think that the prior probabilities of \( w_0 = -1 \) and of \( w_0 = -0.9 \) are equal?

I would argue that they are not just different in magnitude, but that the former is finite while the latter is infinitesimal.
(Almost) current dark energy data

Liddle, Mukherjee, Parkinson, and Wang, PRD, astro-ph/0610126

CMB shift+BAO(SDSS)+SN

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LambdaCDM

Constant \( W \)

\( W_0-W_a \)

Conclusion: If the models were originally equally likely, the data now indicates about a 75% chance that \( \Lambda \text{CDM} \) is correct.
Likelihood of $w_0$ given all models
Likelihood of $w_0$ given all models

Low IQ version
Likelihood of $w_0$ given all models

Low IQ version

High IQ version
Model tests/inference

Model-level inference can be used at several levels:

- **Model-level tests**
  Deploy Bayesian model selection tools to compare model classes.

- **Model selection forecasting**
  Evaluate the capability of proposed experiments to answer model selection questions, by defining model selection FoMs. Explore outcomes contingent on each model class (including ΛCDM) being correct.

- **Survey optimization**
  Vary survey configurations in order to optimize ability to carry out identified model test priorities.
Additional Material
Modified Action $f(R)$ Model

- $R$: Ricci scalar or “curvature”
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2f/dR^2$: Compton wavelength of $f_R$ squared, inverse mass squared
- $B$: Compton wavelength of $f_R$ squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate
Modified Einstein Equation

- In the **Jordan frame**, gravity becomes 4th order but matter remains minimally coupled and separately conserved

\[ G_{\alpha\beta} + f_R R_{\alpha\beta} - \left( \frac{f}{2} - \Box f_R \right) g_{\alpha\beta} - \nabla_\alpha \nabla_\beta f_R = 8\pi G T_{\alpha\beta} \]

- Trace can be interpreted as a scalar field equation for $f_R$ with a density-dependent effective potential ($\rho = 0$)

\[ 3\Box f_R + f_R R - 2f = R - 8\pi G \rho \]

- For small deviations, $|f_R| \ll 1$ and $|f/R| \ll 1$,

\[ \Box f_R \approx \frac{1}{3} (R - 8\pi G \rho) \]

the field is sourced by the deviation from GR relation between curvature and density and has a mass

\[ m^2_{f_R} \approx \frac{1}{3} \frac{\partial R}{\partial f_R} = \frac{1}{3 f_{RR}} \]
DGP Braneworld Acceleration

- **Braneworld acceleration** (Dvali, Gabadadze & Porrati 2000)

\[ S = \int d^5x \sqrt{-g} \left[ \frac{(5) R}{2\kappa^2} + \delta(\chi) \left( \frac{(4) R}{2\mu^2} + \mathcal{L}_m \right) \right] \]

with crossover scale \( r_c = \kappa^2 / 2\mu^2 \)

- Influence of bulk through **Weyl tensor anisotropy** - solve master equation in bulk (Deffayet 2001)

- Matter still **minimally coupled** and conserved

- Exhibits the 3 regimes of modified gravity

  - **Weyl tensor anisotropy** dominated conserved curvature regime \( r > r_c \) (Sawicki, Song, Hu 2006; Cardoso et al 2007)

  - **Brane bending** scalar tensor regime \( r_* < r < r_c \) (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)

  - **Strong coupling** General Relativistic regime \( r < r_* = (r_c r_g)^{1/3} \) where \( r_g = 2GM \) (Dvali 2006)
DGP Field Equations

• DGP field equations

\[ G_{\mu\nu} = 4r_c^2 f_{\mu\nu} - E_{\mu\nu} \]

where \( f_{\mu\nu} \) is a tensor quadratic in the 4-dimensional Einstein and energy-momentum tensors

\[
f_{\mu\nu} \equiv \frac{1}{12} AA_{\mu\nu} - \frac{1}{4} A^\alpha_{\mu} A^\alpha_{\nu} + \frac{1}{8} g_{\mu\nu} \left( A_{\alpha\beta} A^{\alpha\beta} - \frac{A^2}{3} \right)
\]

\[ A_{\mu\nu} \equiv G_{\mu\nu} - \mu^2 T_{\mu\nu} \]

and \( E_{\mu\nu} \) is the bulk Weyl tensor

• Background metric yields the modified Friedmann equation

\[
H^2 \pm \frac{H}{r_c} = \frac{\mu^2 \rho}{3}
\]

• For perturbations, involves solving metric perturbations in the bulk through the “master equation”
Dynamical v Strong Lensing

- Comparison of strong lensing and dynamical mass assuming a density profile and velocity dispersion data
- Mean exhibits a bias from GR expectation with statistical errors only
- No mass trend detectable
Falsify in Favor of What?

- Modified gravity models change space curvature per unit dynamical mass - enhanced or reduced forces on matter
- Requires two closure relations - 1st an an effective anisotropic stress that distinguishes lensing from dynamical mass
- Viable induced modifications exhibit three separate regimes
  - Horizon Scale
  - Scalar-Tensor
  - General Relativistic
- Choice of lensing mass contribution as 2nd parameter in scalar-tensor regime favored by conformal invariance of E&M (Hu & Sawicki 2007; see also Caldwell et al 2007; Amendola et al 2007)

CAMB Package for Linearized PPF: http://camb.info/ppf

Other uses: phantom crossing dark energy (Fang, Hu, Lewis 2009), dark energy PCs (Mortonson, Hu, Huterer 2009) cascading gravity (Afshordi, Geshnizjani, Khoury 2008)
Non-Linear Chameleon

- For $f(R)$ the field equation

$$\nabla^2 f_R \approx \frac{1}{3}(\delta R(f_R) - 8\pi G\delta \rho)$$

is the non-linear equation that returns general relativity.

- High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$.

- Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if

$$\Delta f_R \leq \frac{2}{3} \Phi,$$

else required field gradients too large despite $\delta R = 8\pi G\delta \rho$ being the local minimum of effective potential.
Non-Linear Dynamics

- Supplement that with the modified Poisson equation
  \[ \nabla^2 \Psi = \frac{16\pi G}{3} \delta \rho - \frac{1}{6} \delta R(f_R) \]

- Matter evolution given metric unchanged: usual motion of matter in a gravitational potential \( \Psi \)

- Prescription for \( \mathcal{N} \)-body code

- Particle Mesh (PM) for the Poisson equation

- Field equation is a non-linear Poisson equation: relaxation method for \( f_R \)

- Initial conditions set to GR at high redshift
Environment Dependent Force

- Chameleon suppresses extra force (scalar field) in high density, deep potential regions

\[ f_{R0} = 10^{-6} \]

- density: \( \max[\ln(1+\delta)] \)
- potential: \( \min[\Psi] \)
- field: \( \min[f_R/f_{R0}] \)

Environment Dependent Force

- For large background field, gradients in the scalar prevent the chameleon from appearing.

---

• 512³ PM-relaxation code resolves the chameleon transition to GR: greatly reduced non-linear effect

\[
P(k)/P_{GR}(k) - 1
\]

![Graph showing the power spectrum ratio as a function of k (h/Mpc) with a transition point at \( |f_R| = 10^{-6} \)]
Artificially turning off the chameleon mechanism restores much of enhancement.
N-body Power Spectrum

- Models where the chameleon absent today (large field models) show residual effects from a high redshift chameleon

\[ \frac{P(k)}{P_{GR}(k)} - 1 \]

| \(|f_{R0}| = 10^{-4}\) |
|--------------------------|
| \(|f_{R0}| = 10^{-6}\) |

Mass Function

- Local **cluster abundance** (Chandra sample) current **best** cosmological constraint (~4 orders of magnitude better than ISW)

Schmidt, Vikhlinin, Hu (2009)
Halos at a fixed mass less rare and less highly biased

Halo Mass Correlation

- Enhanced forces vs lower bias

Halo Model

- Power spectrum trends also consistent with halos and modified collapse

Nonlinear Interaction

Non-linearity in the field equation

$$\nabla^2 \phi = g_{\text{lin}}(a)a^2 (8\pi G \Delta \rho - N[\phi])$$

recovers linear theory if $N[\phi] \rightarrow 0$

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Linked to gravitational potential

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a non-linear function of second derivatives of the field

Linked to density fluctuation
DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential

Brane Bending Mode

Apparent Equivalence Principle Violation

- Self-field of a “test mass” can saturate an external field (for $f(R)$ in the gradient, for DGP in the second derivatives)

Hui, Nicolis, Stubbs (2009); Hu (2009)
Future Improvements

- Future Stage IV (SNAP+Planck) predictions sharpened by 2-3 and more importantly provide control of systematic errors