Set 2:
Nature of Galaxies
Great Shapley-Curtis Debate

- History: as late as the early 1920’s it was not known that the “spiral nebula” were galaxies like ours

- Debate between Shapley (galactic objects) and Curtis (extragalactic, or galaxies) in 1920 highlighted the difficulties - distances in astrophysics difficult to measure - Shapley’s inferences based on star counts without extinction and too large a galaxy, novae as standard candles, proper motion

- Hubble in 1923 used Cepheids to establish that Andromeda (M31) is extragalactic at 285kpc - modern measurements say it is 770kpc from the sun.

- Our galaxy is just one of many. Copernican principle in cosmology - we do not occupy a special place in the universe
Galaxy Zoology

- Hubble’s tuning fork classification of galaxies
- A sequence going from ellipticals $E_n$, through regular $S0$ and barred $SB0$ lenticulars, to normal $S$ and barred spirals $SB$ ending in irregulars

The Hubble Tuning Fork with example galaxy images
Ellipticals are further distinguished by the degree of projected ellipticity: the projected major $\alpha$ and minor $\beta$ axes

$$\frac{n}{10} = \epsilon \equiv 1 - \beta/\alpha$$

Classification does not necessarily correspond to physical distinctions!
Galaxy Zoology: Ellipticals

- The actual ellipticity is 3 dimensional and the three axes ordered as $a \geq b \geq c$ determine the degree of oblateness: a sphere has $a = b = c$, perfectly oblate has $a = b$, perfectly prolate $b = c$

- In projection, a strongly prolate or oblate elliptical can have vanishingly small ellipticities

- Ellipticals are often called “early type” and spirals “late type” despite the fact that mergers of spirals can result in ellipticals
Galaxy Zoology: Ellipticals

- Ellipticals vary widely in physical properties from giants to dwarfs
- Absolute $B$ magnitude from -8 to -23
- Total mass from $10^7 M_\odot$ to $10^{13} M_\odot$
- Diameters from few tenths of kpc to hundreds of kpc
- Further classification
  - cD: high mass, high luminosity, high mass to light, in clusters
  - Normal elliptical: $B = -15$ to $-23$, $M = 10^8 - 10^{13} M_\odot$
  - Dwarf ellipticals: low surface brightness for a given $B = -13$ to $-19$, $M = 10^7 - 10^9 M_\odot$
  - Dwarf spheroidal: extremely low luminosity $B = -8$ to $-15$ and surface brightness can only be detected locally
  - Blue compact dwarf: small with vigorous star formation $B = -14$ to $-17$ and $M \sim 10^9$. 
Galaxy Zoology: Spiral NGC4414
Galaxy Zoology: Spirals

- Spirals are subdivided \( a, \ ab, \ b, \ bc, \ c \) in order of bulge prominence, tightly wound spiral arms, smoothest distribution of stars.

- The presence of a central bar is indicated with \( B \).

- Milky Way is a \( SBbc \), M31 is an \( Sb \).

- \( S(B)a − c \) smaller range of physical properties compared with ellipticals (table).

**TABLE 25.1** Characteristics of Early Spiral Galaxies.

<table>
<thead>
<tr>
<th></th>
<th>Sa</th>
<th>Sb</th>
<th>Sc</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_B ) (( M_\odot ))</td>
<td>(-17 to -23)</td>
<td>(-17 to -23)</td>
<td>(-16 to -22)</td>
</tr>
<tr>
<td>( M/L_\odot )</td>
<td>(10^9 - 10^{12})</td>
<td>(10^9 - 10^{12})</td>
<td>(10^9 - 10^{12})</td>
</tr>
<tr>
<td>( L_{\text{bulge}}/L_{\text{total}} ) ( B )</td>
<td>0.3</td>
<td>0.13</td>
<td>0.05</td>
</tr>
<tr>
<td>Diameter ( (D_{25}, \text{kpc}) )</td>
<td>5–100</td>
<td>5–100</td>
<td>5–100</td>
</tr>
<tr>
<td>( M/L_B ) ( (M_\odot/L_\odot) )</td>
<td>6.2 ± 0.6</td>
<td>4.5 ± 0.4</td>
<td>2.6 ± 0.2</td>
</tr>
<tr>
<td>( V_{\text{max}} ) (km s(^{-1}))</td>
<td>299</td>
<td>222</td>
<td>175</td>
</tr>
<tr>
<td>( V_{\text{max}} ) range (km s(^{-1}))</td>
<td>163–367</td>
<td>144–330</td>
<td>99–304</td>
</tr>
<tr>
<td>pitch angle</td>
<td>( \sim 6^\circ )</td>
<td>( \sim 12^\circ )</td>
<td>( \sim 18^\circ )</td>
</tr>
<tr>
<td>( (B - V) )</td>
<td>0.75</td>
<td>0.64</td>
<td>0.52</td>
</tr>
<tr>
<td>( (M_{\text{gas}}/M_{\text{total}}) )</td>
<td>0.04</td>
<td>0.08</td>
<td>0.16</td>
</tr>
<tr>
<td>( (M_{H_2}/M_{H_1}) )</td>
<td>2.2 ± 0.6 (Sab)</td>
<td>1.8 ± 0.3</td>
<td>0.73 ± 0.13</td>
</tr>
<tr>
<td>( (S_N) )</td>
<td>1.2 ± 0.2</td>
<td>1.2 ± 0.2</td>
<td>0.5 ± 0.2</td>
</tr>
</tbody>
</table>

**TABLE 25.2** Characteristics of Late Spiral and Irregular Galaxies.

<table>
<thead>
<tr>
<th></th>
<th>Sd/Sm</th>
<th>Im/IR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_B )</td>
<td>(-15 to -20)</td>
<td>(-13 to -18)</td>
</tr>
<tr>
<td>( M/\odot )</td>
<td>(10^8 - 10^{10})</td>
<td>(10^8 - 10^{10})</td>
</tr>
<tr>
<td>Diameter ( (D_{25}, \text{kpc}) )</td>
<td>0.5–50</td>
<td>0.5–50</td>
</tr>
<tr>
<td>( M/L_B ) ( (M_\odot/L_\odot) )</td>
<td>( \sim 1 )</td>
<td>( \sim 1 )</td>
</tr>
<tr>
<td>( V_{\text{max}} ) range (km s(^{-1}))</td>
<td>80–120</td>
<td>50–70</td>
</tr>
<tr>
<td>( (B - V) )</td>
<td>0.47</td>
<td>0.37</td>
</tr>
<tr>
<td>( (M_{\text{gas}}/M_{\text{total}}) )</td>
<td>0.25 (Scd)</td>
<td>0.5–0.9</td>
</tr>
<tr>
<td>( (M_{H_2}/M_{H_1}) )</td>
<td>0.03–0.3</td>
<td>( \sim 0 )</td>
</tr>
<tr>
<td>( (S_N) )</td>
<td>0.5 ± 0.2</td>
<td>0.5 ± 0.2</td>
</tr>
</tbody>
</table>
Galaxy Zoology: Irregulars

- Irregulars classed as $IrrI$ if there is any organized structure such as spiral arms
- Otherwise $IrrII$ otherwise
- Examples: Large Magellanic Clouds (LMC) is $IrrI$ and Small Magellanic Clouds (SMC) is $IrrII$
- Physical properties: tend to be small and faint
- Absolute $B$ magnitude from $-13$ to $-20$
- Masses from $10^8 M_\odot$ to $10^{10} M_\odot$
Galaxy Properties: Luminosity Function

- Abundance as a function of luminosity is called the “luminosity function”. Number of galaxies in $dL$ around $L$ and has a rough shape of a “Schechter function”

$$\phi_L dL \propto L^\alpha e^{-L/L_*} dL$$

$$\phi_M dM \propto 10^{-0.4(\alpha+1)M} e^{-10^{0.4(M_*-M)}}$$

with $\alpha \approx -1$
and $L_*$ from $M_* = -21$ in $B$
Galaxy Properties: Luminosity Function

- Luminosity function is to galaxies what the distribution in magnitudes of stars is to star counts.

- Galaxy counts probe the galaxy number density as a function of angular position (and redshift) to a limiting magnitude (a “redshift” survey).

- Luminosity function (determined locally) tells you how to interpret the observed counts in terms of a 3D distribution of galaxies.
Galaxy Properties: Surface Brightness

- Surface brightness profile defines the effective scale of the bulge and disk components
- Surface brightness $\mu$ measured in $B$-mag arcsec$^{-2}$
- Define $r_e$ as the radius within which $1/2$ the light emitted.
- Bulges of spirals and ellipticals follow a Sersic profile where the surface brightness in mag scales as a power law at $r \gg r_e$

$$\mu(r) = \mu_e + 8.3268 \left[ \left( \frac{r}{r_e} \right)^{1/n} - 1 \right]$$

where $n = 4$ is the de Vaucouleurs profile and $\mu_e$ is the surface brightness at $r_e$
Galaxy Properties: Surface Brightness

- Disks follow an exponential which in mag scales as

\[ \mu(r) = \mu_0 + 1.09 \left( \frac{r}{h_r} \right) \]

where \( h_r \) is the characteristic scale length
Galaxy Properties: Fundamental Plane

- Faber Jackson correlation between luminosity and velocity dispersion of stars (measured from the width of lines from aggregate unresolved stars) \( L \propto \sigma_0^4 \)

- Expected if mass to light and surface brightness a constant. Consider virial theorem

\[
-2\langle K \rangle = \langle U \rangle, \quad -2 \sum_{i}^{N} \frac{1}{2} m_i v_i^2 = U
\]
Galaxy Properties: Fundamental Plane

- Simplify as equal mass objects composing $M$

\[-\frac{m}{N} \sum_{i}^{N} v_i^2 = \frac{U}{N}\]

- Sum is the average $v^2$ and is an observable assuming that radial velocities reflect total $\langle v^2 \rangle = 3 \langle v_r^2 \rangle \equiv 3\sigma_r^2$

\[-3m\sigma_r^2 = \frac{U}{N}\]

- Potential energy for a constant density spherical distribution of mass $M = Nm$ and radius $R$

\[\frac{U}{N} = -\frac{3}{5} \frac{GM^2}{NR}, \quad M_{\text{vir}} = \frac{5R\sigma_r^2}{G}\]
Galaxy Properties: Fundamental Plane

- Eliminate $R$ by assuming constant surface brightness
  \[ \frac{L}{R^2} = C_{SB} \] eliminate $R = (L/C_{SB})^{1/2}$

  \[ M_{\text{vir}} = \frac{5\sigma_r^2}{G} (L/C_{SB})^{1/2} \]

- Eliminate $M_{\text{vir}}$ by assuming constant mass to light $M/L = 1/C_{ML}$

  \[ L = C_{ML} \frac{5\sigma_r^2}{G} (L/C_{SB})^{1/2} \]

  \[ L \propto \sigma_r^4 \]

- A tighter relation is obtained by introducing a second observable - either the effective radius

  \[ L \propto \sigma_r^{2.65} r_e^{0.65} \]

  which defines the fundamental plane of ellipticals
Galaxy Properties: SMBH

- A similar argument is used to measure the mass of the central black hole from the velocity dispersion of stars around it in both spirals and ellipticals.

- The inferred mass is also correlated with the velocity dispersion much further out in the bulge.

\[ M_{bh} \propto \sigma^\beta \quad (\beta = 4.86 \pm 0.43) \]

- Assembly of the bulge must be linked to the SMBH formation.
Galaxy Properties: $v_{\text{max}}$

- The maximum velocity in a rotation curve is a robust observable.
- The 21 cm line of the disk as a whole reflects the Doppler shifts of the HI participating in the rotation.
- Line has a double peaked profile with the peaks near $v_{\text{max}}$ since much of the gas is in the flat part of the rotation curve near the peak.
Galaxy Properties: Tully Fisher relation

- Correcting for the inclination from the observed radial velocity $v_r$
  \[
  \frac{\Delta \lambda}{\lambda_{\text{rest}}} = \frac{v_r}{c} = \frac{v_{\text{max}}}{c} \sin i
  \]

- Tully and Fisher established that $v_{\text{max}}$ is correlated with $B$ band luminosity as approximately $L_B \propto v_{\text{max}}^4$
Galaxy Properties: Tully Fisher relation

- Tully-Fisher relationship is expected if galaxies have a constant mass to light ratio and constant surface brightness
- Enclosed mass

\[ M = \frac{v_{\text{max}}^2 R}{G} \]

- Mass to light ratio \( M/L = 1/C_{ML} \)

\[ L = C_{ML} \frac{v_{\text{max}}^2 R}{G} \]

- Surface brightness \( L/R^2 = C_{SB} \) eliminate \( R = (L/C_{SB})^{1/2} \)

\[ L = C_{ML} \frac{v_{\text{max}}^2}{G} \left( \frac{L}{C_{SB}} \right)^{1/2} \]

\[ L \propto v_{\text{max}}^4 \]
Galaxy Properties: Tully Fisher relation

- In absolute magnitude

\[ M_B = -2.5 \log_{10} L_B + \text{const} \]

\[ M_B = -2.5 \log_{10} v_{\text{max}}^4 + \text{const} \]

\[ M_B = -10 \log_{10} v_{\text{max}} + \text{const} \]

- Tully Fisher relation is even tighter in IR bands such as $H$ band - less extinction and late type giant stars are better tracers of overall luminosity

- Tully Fisher relation can be used to measure distances: measure $v_{\text{max}}$, infer absolute magnitude and compare to apparent magnitude
Galaxy Properties: Spiral Structure

- Winding problem:
  if spiral structure were physical structures, a flat rotation curve would cause the arms to wind up tightly

- Lin-Shu density wave theory: spiral arms are quasistatic density waves - bunching is like cars in a traffic jam

- Stars pass through the wave/jam and do not cause a winding problem
Galaxy Properties: Spiral Structure

- Consider the orbital motion of a star in cylindrical coordinates \((R, \phi, z)\) where \(z\) is the coordinate out of the disk

\[
\frac{d^2 \mathbf{r}}{dt^2} = -\nabla \Phi
\]

where \(\Phi\) is the gravitational potential.

- Assuming axial symmetry for the potential this yields 3 equations for the three directions

\[
\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R}
\]

\[
\frac{1}{R} \frac{\partial (R^2 \dot{\phi})}{\partial t} = 0
\]

\[
\ddot{z} = -\frac{\partial \Phi}{\partial z}
\]
Galaxy Properties: Spiral Structure

- Second equation says that there is no force in the azimuthal direction or torque $\tau = r \times F$

$$L_z = M R v_\phi = M R^2 \dot{\phi} = \text{const}$$

where $M$ is the mass of the star. Defining $J_z = L_z / M = R^2 \dot{\phi}$ the angular momentum per unit mass

$$R \dot{\phi}^2 = \frac{J_z^2}{R^3}$$

- Radial equation becomes

$$\ddot{R} = -\frac{\partial \Phi}{\partial R} + \frac{J_z^2}{R^3}$$
Galaxy Properties: Spiral Structure

• The second term is an angular momentum barrier against radial infall or equivalently the centripetal acceleration required to keep $R$ constant $v_\phi^2$. It can be absorbed into an effective potential

$$\Phi_{\text{eff}} = \Phi + \frac{J_z^2}{2R^2}$$

so that the equations of motion becomes ($J_z$ is a constant in $z$)

$$\ddot{R} = -\frac{\partial \Phi_{\text{eff}}}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi_{\text{eff}}}{\partial z}$$

• Structure of $\Phi_{\text{eff}}(R, z)$ determines motion. In $z$ minimum is at the midplane. In $R$, minimum forms from the competition of gravity and angular momentum
Galaxy Properties: Spiral Structure

- Minimum found by seeing where slope vanishes (or equivalently where the gravitational and centripetal acceleration match)

\[
\frac{\partial \Phi_{\text{eff}}}{\partial R} = \frac{\partial \Phi}{\partial R} - \frac{J_z^2}{R^3} = 0
\]

- Orbits near this stable minimum \( m \) oscillate around it:

\[
\rho \equiv R - R_m
\]

\[
\Phi_{\text{eff}} \approx \Phi_{\text{eff},m} + \frac{1}{2} \kappa^2 \rho^2 + \frac{1}{2} \nu^2 z^2
\]

where \( \kappa^2 = \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2}|_m \) and \( \nu^2 = \frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2}|_m \)

- Equations of motion

\[
\ddot{\rho} = -\kappa^2 \rho
\]

\[
\ddot{z} = -\nu^2 z
\]
Galaxy Properties: Spiral Structure

- Star executes simple harmonic motion around minimum:

\[ \rho(t) = A_R \sin \kappa t \]

\[ z(t) = A_Z \sin(\nu t + \zeta) \]

where \( \zeta \) is a phase factor and we have defined \( t = 0 \) to eliminate the other phase factor.

- Azimuthal coordinate given in terms of radial motion

\[ \dot{\phi} = \frac{J_z}{R^2} \approx \frac{J_z}{R_m^2} \left(1 - 2 \frac{\rho(t)}{R_m} \right) \]

\[ \phi(t) = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m} A_R \cos \kappa t \]

where the unperturbed angular frequency \( \Omega = \frac{J_z}{R_m^2} \)}
Galaxy Properties: Spiral Structure

- Star executes epicyclic motion or rosette
- $\kappa$ also known as epicyclic frequency
- Relative to the unperturbed orbit (corrotating with the local angular speed $\Omega$, star executes a simple retrograde closed orbit around $R_m$)
Galaxy Properties: Spiral Structure

- If the epicyclic frequency $\frac{\kappa}{\Omega} = \frac{m}{n}$ integer ratio then the orbit is closed in the fixed frame: star executes $m$ epicycles during $n$ orbits.

- More generally, can always go into a rotating frame “local pattern speed” $\Omega_{lp}$ where this condition is true and orbits are closed:

  $$m(\Omega - \Omega_{lp}) = n\kappa$$

- An $(n = 1, m = 2)$ is shown for a case where $\Omega_{lp}$ is independent of $R$: if axis of orbit ovals are aligned then bar structure, if rotated then a two armed bar.
Galaxy Properties: Spiral Structure

- Only pattern is stationary - stars are continuously orbiting and piling up in the arms

- Non-constancy of the $\Omega_{lp}$ will still cause winding but of the pattern and typically at a slower rate for $(1, 2)$.

- Where the local pattern speed matches the global pattern speed Lindblad resonances occur where the epicyclic amplitude increases due to forcing from the local density enhancement - can destroy spiral pattern.

- $N$-body simulations show formation of transient $m = 2$ arm patterns and long lived bar instability.