

Wandering in the Background:
A Cosmic Microwave Background Explorer

by

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We develop and examine the principles governing the formation of distortions in the cosmic microwave background. Perturbations in the frequency or spectral distribution of the background probe the thermal history of the universe, whereas those in the angular temperature distribution probe its dynamics and geometry. Stressing model independent results, we show how the microwave background can be used to extract information on the mass density, vacuum density, baryon content, radiation content, expansion rate and some aspects of structure formation in the universe. To address these issues, we develop elements of relativistic kinetic and perturbation theory as they become necessary for the description of the particle and gravitational interactions of the photons. Subtle issues such as fluctuation representation, or gauge, normal mode analysis in an open geometry, and second order effects are considered in detail. Employing analytic and numerical results, we construct anisotropies in a critical, open, and cosmological constant universe with adiabatic and/or isocurvature initial conditions allowing for possible early reionization. We find that anisotropy formation is a simple process governed by the Compton scattering of photons off electrons and their gravitational coupling to the other particle species in the universe.

Chair

Date

The thesis of Wayne T. Hu is approved:

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To Chuang-tzu,

Said the disciple, "After I heard your words, one year and I ran wild, two years and I was tame, three years and positions interchanged, four years and things settled down, five years and things came to me . . ."

—Chuang-tzu, 27

From Chuang-tzu,

I hear that there is a sacred tortoise which has been dead for three thousand years. His Majesty keeps it wrapped up in a box at the top of the hall in the ancestral shrine. Would this tortoise rather be dead, to be honored as preserved bones, or would it rather be alive and dragging its tail in the mud... Away with you! I shall drag my tail in the mud.

—Chuang-tzu, 17

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Preface

If you have a great tree and think it's a pity it's so useless, Why not plant it in the middle of nowhere in the wilds which spread out, and go rambling away aimlessly at its side, wander around and fall asleep in its shade?

–*Chuang-tzu, 1*¹

A mere three years ago when I started work on the cosmic microwave background (CMB) with Joe Silk, anisotropies had not yet been discovered. The theory of anisotropy formation was considered arcana and earned barely a mention in the standard textbooks of the time. With the number of detections now in the double digits, CMB anisotropies have joined spectral distortions, light element abundances, and large scale structure measurements as some of our most powerful observational probes of cosmology. The depth that even the interested non-specialist needs to understand the principles governing fluctuations in the CMB has consequently increased. This work begins the task of assembling the material necessary for a modern understanding of the CMB. Of course, the whole task is beyond the scope of a 200 some page dissertation assembled in a month's time! I make no claims of completeness. Rather, I develop a handful of general principles that seem to me may have lasting interest. As a consequence, I do not treat in any detail CMB constraints on specific cosmological models, except where necessary to illustrate general points. Moreover, important issues of statistical analysis related to the current generation of experiments are not covered here. I happily refer the interested reader to the excellent “companion thesis” by Emory Bunn [21].

Chapter 1 is provided as a qualitative and hopefully intuitive introduction to the subject. The formal development begins in chapter 2 with relativistic kinetic theory and continues in chapter 4 with relativistic perturbation theory. Readers who are familiar with these subjects should skip to their applications: spectral distortions in chapter 3 and density

¹Translations of the *Chuang-tzu* throughout are adapted from [65].

perturbation evolution in chapter 5. Given its importance, anisotropy formation occupies the rest of this work. Again, I stress robust features that may survive the current generation of models. I discuss how these features may be used to probe general cosmological issues such as the matter content, dynamics, and geometry of the universe. Advanced topics such as radiation feedback effects, polarization, and the details of recombination are saved for Appendix A. Appendix B gathers together useful material scattered throughout the text.

Some topics are covered at greater depth than others. Some will be of more interest to the specialist than to the general cosmologist. This thesis is nowhere near as homogeneous as the subject it purports to study (though it may be as directionless)! Wander through its pages and perhaps you will find it of some use – if nothing else, for its soporific qualities.

Wayne T. Hu

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