Slow Roll Relations

Recall the equation of motion for the unperturbed scalar field
\[ \ddot{\phi}_0 + 2 \frac{\dot{a}}{a} \dot{\phi}_0 + a^2 V' = 0, \]  \hspace{1cm} (1)

the definitions of the slow-roll parameters
\[ \epsilon = \frac{1}{16\pi G} \left( \frac{V'}{V} \right)^2, \]  \hspace{1cm} (2)
\[ \delta = \epsilon - \frac{1}{8\pi G} \frac{V''}{V}, \]  \hspace{1cm} (3)

where primes are derivatives with respect to the argument, \( \phi \) for \( V(\phi) \), and the formulae for the curvature and gravity wave power spectra
\[ \Delta^2_\zeta = \left( \frac{H}{m_{pl}} \right)^2 \frac{1}{\pi \epsilon}, \]  \hspace{1cm} (4)
\[ \Delta^2_h = \left( \frac{H}{m_{pl}} \right)^2 \frac{4}{\pi}. \]  \hspace{1cm} (5)

where \( m_{pl} = G^{-1/2} \).

1. Chaotic Inflation

Consider polynomial chaotic inflation where \( V = m^2 \phi^2/2 \).

- Write down \( \epsilon \) and \( \delta \). Inflation will occur if the initial field \( \phi_0(0) = \phi_i \) meets what conditions?
- Write down the slow roll equation in coordinate time \( (d^2 \phi_0/dt^2 = 0; \delta \ll 1) \) with \( H(\phi) \) \((\epsilon \ll 1)\) evaluated with the Friedmann equation.
- Solve for \( \phi_0(t) \).
- Solve for \( a(t) \) using the \( H(\phi) \) relation and assume \( a(t = 0) = a_i \).
- Take \( \epsilon = 1 \) to define the end of inflation. Show that the number of efoldings of inflation can be written as
\[ N = \ln(a_{end}/a_i) = 2\pi \frac{\phi_i^2}{m_{pl}^2} - \frac{1}{2}. \]  \hspace{1cm} (6)

what is the condition on \( \phi_i \) such that sufficient inflation occurs \((N > 70)\). Is it compatible with the slow roll conditions?

- Write down the curvature power spectrum \( \Delta^2_\zeta \) and gravity wave power spectrum \( \Delta^2_h \) for this model in terms of \( \phi \). Taking \( \phi = \phi_i \) defined now as \( N = 70 \) above, what is the condition on \( m \) such that the rms is \( \Delta_\zeta = 10^{-5} \). What is tensor-scalar ratio \( \Delta^2_h/\Delta^2_\zeta \) for such a model?