An Acoustic Primer

Wayne Hu
Astro 448

CMB Anisotropies

COBE

Maxima

BOOMERanG

Hanany, et al. (2000)

de Bernardis, et al. (2000)
Ringing in the New Cosmology
Physical Landscape
Acoustic Oscillations
Thermal History

- $z > 1000; T_\gamma > 3000K$
  - Hydrogen ionized
  - Free electrons glue photons to baryons

\[ \gamma \longrightarrow e^- + p \]
  - Compton scattering
  - Coulomb interactions

Photons–baryon fluid
Potential wells that later form structure

Mean free path $\ll$ wavelength

Tightly coupled fluid
Cold
Hot
Hot
Thermal History

- $z > 1000; T_\gamma > 3000K$
  - Hydrogen ionized
  - Free electrons glue photons to baryons

- $z \sim 1000; T_\gamma \sim 3000K$
  - Recombination
  - Fluid breakdown

- $z < 1000; T_\gamma < 3000K$
  - Gravitational redshifts & lensing
  - Reionization; rescattering

Photon–baryon fluid
Potential wells that later form structure

$\gamma \rightarrow e^- p$
Compton scattering
Coulomb interactions

$\lambda \sim k^{-1}$

$\theta \sim l^{-1}$

last scattering surface

observer
Initial Conditions
Inflation and the Initial Conditions

- **Inflation**: (nearly) scale-invariant curvature (potential) perturbations
- Superluminal expansion $\rightarrow$ superhorizon scales $\rightarrow$ "initial conditions"
- Accompanying temperature perturbations due to cosmological redshift

- Potential perturbation $\Psi = \text{time-time metric perturbation}$
  \[ \frac{\delta t}{t} = \Psi \quad \rightarrow \quad \frac{\delta T}{T} = -\frac{\delta a}{a} = -\frac{2}{3}\frac{\delta t}{t} = -\frac{2}{3}\Psi \]

Sachs & Wolfe (1967); White & Hu (1997)
Initial Conditions & the Sachs-Wolfe Effect

- **Initial** temperature perturbation
- **Observed** temperature perturbation
  Gravitational redshift: $\Psi$
  + Initial temperature: $+ \Theta$
- Potential = time-time $\Psi = \delta t/t$
- Metric perturbations

![Diagram showing time and space with gravitational redshift and potential](image-url)
Initial Conditions & the Sachs-Wolfe Effect

- **Initial** temperature perturbation

- **Observed** temperature perturbation
  - Gravitational redshift: $\Psi$
  - + Initial temperature: + $\Theta$

- Potential = time-time $\Psi = \delta t/t$
  - Metric perturbations

- Matter-dominated expansion:
  - $a \propto t^{2/3}$, $\delta a/a = 2/3 \delta t/t$

- Temperature falls as:
  - $T \propto a^{-1}$

- Temperature fluctuation:
  - $\delta T/T = -\delta a/a$
Initial Conditions & the Sachs-Wolfe Effect

- **Initial** temperature perturbation

- **Observed** temperature perturbation
  
  Gravitational redshift: \( \Psi \)
  
  + Initial temperature: \( + \Theta \)

- Potential = time-time
  
  \( \Psi = \delta t/t \)

- Metric perturbations

- Matter–dominated expansion:
  
  \[ a \propto t^{2/3}, \quad \delta a/a = \frac{2}{3} \frac{\delta t}{t} \]

- Temperature falls as:
  
  \[ T \propto a^{-1} \]

- Temperature fluctuation:
  
  \[ \delta T/T = -\delta a/a \]

- Result

  - **Initial** temperature perturbation:
    
    \[ \Theta \equiv \delta T/T = -\delta a/a = -2/3 \frac{\delta t}{t} = -2/3 \Psi \]

  - **Observed** temperature perturbation:
    
    \( (\delta T/T)_{\text{obs}} = \Theta + \Psi = 1/3 \Psi \)

Sachs & Wolfe (1967)  
White & Hu (1997)
Acoustic Oscillations
Gravitational Ringing

- Potential wells = inflationary seeds of structure
- Fluid falls into wells, pressure resists: acoustic oscillations
Acoustic Oscillations

- Photon pressure resists compression in potential wells
- Acoustic oscillations

Peebles & Yu (1970)
Acoustic Oscillations

- Photon pressure resists compression in potential wells
- Acoustic oscillations
- Gravity displaces zero point
  \[ \Theta \equiv \frac{\delta T}{T} = -\Psi \]

Oscillation amplitude = initial displacement from zero pt.
\[ \Theta - (-\Psi) = \frac{1}{3} \Psi \]

\( \Delta T/T \)

\( \Theta + \Psi \)

\( -|\Psi|/3 \)

Peebles & Yu (1970)
Acoustic Oscillations

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- Gravitational redshift: observed
  \( (\delta T/T)_{\text{obs}} = \Theta + \Psi \)
  oscillates around zero

First Extrema

- \( \Theta + \Psi \)
- \( -|\Psi|/3 \)

Peebles & Yu (1970)
Acoustic Oscillations

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Second Extrema

Plane Waves

- Potential wells: part of a fluctuation spectrum
- Plane wave decomposition
Harmonic Modes

- Frequency proportional to wavenumber: $\omega = kc_s$
- Twice the wavenumber = twice the frequency of oscillation
Harmonic Peaks

- Oscillations frozen at last scattering
- Wavenumbers at extrema = peaks
- Sound speed $c_s$

$\frac{\Delta T}{T} = -\frac{1}{3} |\Psi|$ at last scattering

First Peak

$k_1 = \frac{\pi}{\text{sound horizon}}$

$\Theta + \Psi$

Harmonic Peaks

- Oscillations frozen at last scattering
- Wavenumbers at extrema = peaks
- Sound speed $c_s$

- Frequency $\omega = kc_s$; conformal time $\eta$
- Phase $\propto k$; $\phi = \int_0^{\eta_{\text{last scattering}}} d\eta \omega = k_{\text{sound horizon}}$
- Harmonic series in sound horizon $\phi_n = n\pi \rightarrow k_n = n\pi_{\text{sound horizon}}$

$\Delta T/T$

Last Scattering

First Peak

$k_1 = \pi/\text{sound horizon}$

Temporal Coherence in IC

Second Peak

$k_2 = 2k_1$

Hu & Sugyama (1995); Hu, Sugiyama & Silk (1997)
Seeing Sound

• Oscillations *frozen at recombination*

• Compression=hot spots, Rarefaction=cold spots
Harmonic Modes

- Frequency proportional to wavelength: $\omega = kc_s$
- Modes reaching extrema are integral multiples of harmonics $1:2:3$
Peaks in Angular Power

- Oscillations frozen at \textit{recombination}
- \textbf{Distant} hot and cold spots appear as temperature anisotropies
Projection into Angular Peaks

- Peaks in spatial power spectrum
- Projection on sphere
- Spherical harmonic decomposition

- Maximum power at $l = kd$
- Extended tail to $l \ll kd$
- Described by spherical bessel function $j_l(kd)$

$\text{Bond & Efstathiou (1987)}$
$\text{Hu & Sugiyama (1995); Hu & White (1997)}$
Projection into Angular Peaks

- Peaks in spatial power spectrum
- Projection on sphere
- Spherical harmonic decomposition

- Maximum power at $l = kd$
- Extended tail to $l \ll kd$
- 2D Transfer Function $T^2(k,l) \sim (2l+1)^2 [\Delta T/T]^2 j^2_l(kd)$

Hu & Sugiyama (1995)
Doppler Effect

- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature
Doppler Effect

- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature
- Zero point not shifted by baryon drag
- Increased baryon inertia decreases effect
  \[ m_{\text{eff}} V^2 = \text{const.} \quad V \propto m_{\text{eff}}^{-1/2} = (1+R)^{-1/2} \]
Doppler Peaks?

- Doppler effect has lower amplitude and weak features from projection.

\[ j_l(kd)Y_l^0 \]

Temperature peak

\[ (2l+1)j_l(100) \]

Doppler no peak

Hu & Sugiyama (1995)
Relative Contributions

Hu & Sugiyama (1995); Hu & White (1997)
Relative Contributions

Hu & Sugiyama (1995); Hu & White (1997)
Mechanics of the Calculation

- Radiation distribution: \( f(\mathbf{x}=\text{position}, t=\text{time}; \mathbf{n}=\text{direction}, v=\text{frequency}) \)
- Expand in basis functions: (local angular dependence \( \otimes \) spatial dependence)
  \[ s Y_l^m \otimes e^{i \mathbf{k} \cdot \mathbf{x}} \]

Sources
(radiation & metric: monopole, dipole, quadrupole)

Observables

- Addition of Angular Momentum
  \[ s Y_l''^m \otimes Y_1^0 \]
  \[ | l' \pm 1 | \]
  \[ \sum \text{ (clebsch-gordan) } j_l \]

- Integral
  \[ s Y_l''^m \otimes Y_1^0 \]
  \[ | l \pm l' | \]

Original codes
- Bond & Efstathiou (1984)
- Vittorio & Silk (1983)

- Seljak & Zaldarriaga (1996)
- Hu & White (1997)

- Seljak (1994)

CMBFast

Hierarchy
- plane wave \( \sim \) gradient
- \[ s Y_l''^m \otimes Y_1^0 \]
Semi-Analytic Calculation

• Treat Sources in the Tight Coupling Approximation
  • Expand the Boltzmann hierarchy equations in $1/(\text{optical depth per } \lambda)$
  • Also the trick to numerically integrating the stiff hierarchy equations
  • Closed Euler equation + Continuity equation = Oscillator equation

• Project Sources at Last Scattering
  • Integral equations
  • Visibility function of recombination
The First Peak
Shape of the First Peak

- Consistent with potential wells in place on superhorizon scale (inflation)
- Sharp fall from first peak indicates no continuous generation (defects)
Angular Diameter Distance

- A Classical Test
  Standard(ized) comoving ruler
  Measure angular extent
  Absolute scale drops out
  Infer curvature

- Upper limit 1st Peak Scale (Horizon)
  Upper limit on Curvature

- Calibrate 2 Physical Scales
  Sound horizon (peak spacing)
  Diffusion scale (damping tail)

Kamionkowski, Spergel & Sugiyama (1994)
Hu & White (1996)
Curvature in the Power Spectrum

- Features scale with angular diameter distance
- Angular location of the first peak
A Flat Universe!...

BOOMERanG
MAXIMA

Priors!

Riess et al. (1998)
Perlmutter et al. (1998)
First Peak Location

- BOOM's parabolic peak fit $185<l_1<209$ (2σ)
- MAX's value $l_1 \sim 220$
Are They Consistent?

- Using ΛCDM models: $184 < l_1 < 216$ (2σ; BOOM)
- Joint analysis: $194 < l_1 < 218$ (2σ; BOOM+MAX)

Hu, Fukugita, Zaldarriaga, Tegmark (2000)
Is the Scale too Large?

- Fiducial flat $\Omega_m=0.35, h=0.65$ model: $l_1=221$ (excluded at $\sim 2.5\sigma$)
- (a) positive curvature
- (b) high Hubble constant / matter
- (c) dark energy

$\Omega_b h^2 > 0.019$

Hu, Fukugita, Zaldarriaga, Tegmark (2000)
Flat solutions involve decreasing age of universe through $h$, $\Omega_m$ or dark energy equation of state $w = p/\rho$ (<13–13.5Gyr)

Hu, Fukugita, Zaldarriaga, Tegmark (2000)
Age and Dark Energy

- Region of consistency shrinking – headed to crisis?
- New physics? $w$, $m_\nu$, $\alpha$...

Hu, Fukugita, Zaldarriaga, Tegmark (2000)
The Second Peak
Baryon & Inertia

• **Baryons add inertia to** the fluid

• Equivalent to adding mass on a **spring**

• Same initial conditions

• Same null in fluctuations

• **Unequal** amplitudes of extrema
Baryon Drag

• Baryons provide **inertia**

• Relative momentum density

\[ R = \frac{(\rho_b + p_b)V_b}{(\rho_\gamma + p_\gamma)V_\gamma} \propto \Omega_b h^2 \]

• Effective mass \( m_{\text{eff}} = (1 + R) \)

Hu & Sugiyama (1995)
Baryon Drag

- Baryons provide inertia
- Relative momentum density
  \[ R = (\rho_b + p_b)V_b / (\rho_\gamma + p_\gamma)V_\gamma \propto \Omega_b h^2 \]
- Effective mass \( m_{\text{eff}} = (1 + R) \)
- Baryons drag photons into potential wells \( \rightarrow \) zero point
- Amplitude \( \uparrow \)
- Frequency \( \downarrow \) \( (\omega \propto m_{\text{eff}}^{-1/2}) \)

- Constant \( R, \Psi: \)
  \[ (1+R)\ddot{\Theta} + (k^2/3)\Theta = -(1+R)(k^2/3)\Psi \]
  \[ \Theta + \Psi = [\Theta(0) + (1+R)\Psi(0)] \cos \left[ k\eta / \sqrt{3}(1+R) \right] - R\Psi \]

High Baryon Content

Hu & Sugiyama (1995)
Baryon Drag

- Baryons provide inertia
- Relative momentum density
  \[ R = \frac{(\rho_b + p_b)V_b}{(\rho_\gamma + p_\gamma)V_\gamma} \propto \Omega_b h^2 \]
- Effective mass \( m_{\text{eff}} = (1 + R) \)

- Baryons drag photons into potential wells \( \rightarrow \) zero point ↑
- Amplitude ↑
- Frequency ↓ (\( \omega \propto m_{\text{eff}}^{-1/2} \))

- Constant \( R, \Psi \):
  \[ (1+R)\ddot{\Theta} + \left(\frac{k^2}{3}\right)\Theta = -(1+R)\left(\frac{k^2}{3}\right)\Psi \]
  \[ \Theta + \Psi = [\Theta(0) + (1+R)\Psi(0)] \cos \left[ \frac{k\eta}{\sqrt{3}} (1+R) \right] - R\Psi \]

Baryons in the CMB

- High odd peaks

- Additional Effects
  - Time–varying potential
  - Dissipation/Fluid imperfections

\[ \Omega_b h^2 \]
Baryons in the Power Spectrum
Height of the Second Peak

- **BOOM** and **MAX** both show a low power at $l_2 = 2.4 \ l_1$
- $H_2 = \text{power at 1st/2nd} = (\Delta T_{l_2}/\Delta T_{l_1})^2$
Height of the Second Peak

- BOOM: $H_2 = 0.37 \pm 0.044$ (1σ)
- BOOM+MAX: $H_2 = 0.38 \pm 0.04$ (1σ)

$75 < l < 400$
$400 < l < 600$
$(\Omega_\Lambda, \Omega_b h^2, n)$

$\Delta H_2 / H_2 = -0.65 \Delta \Omega_b h^2 / \Omega_b h^2$
$+ 0.88 \Delta n / n$
$+ 0.14 \Delta \Omega_m h^2 / \Omega_m h^2$
Height of the Second Peak

- Excludes fiducial LCDM ($n=1$, $\Omega_b h^2 = 0.02$, $H_2 = 0.51$) at $\sim 3.3\sigma$
- Requires $\Omega_b h^2 > 0.022n$ (if $\Omega_m h^2 > 0.16$, from $l_1$, flat, $h < 0.8$)
Height of the Second Peak

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- Requires $\Omega_b h^2 > 0.022n$ (if $\Omega_m h^2 > 0.16$, from $l_1$, flat, $h < 0.8$)

![Graph showing the height of the second peak and constraints on $\Omega_b h^2$ and $\Omega_m h^2$.]
Higher Peaks
Radiation and Dark Matter

- Radiation domination: potential wells created by CMB itself
- Pressure support $\Rightarrow$ potential decay $\Rightarrow$ driving
- Heights measures when dark matter dominates
Driving Effects and Matter/Radiation

- Potential perturbation: \( k^2 \Psi = -4\pi G a^2 \delta \rho \) generated by radiation
- Radiation \( \rightarrow \) Potential: inside sound horizon \( \delta \rho / \rho \) pressure supported \( \delta \rho \) hence \( \Psi \) decays with expansion

Hu & Sugiyama (1995)
Driving Effects and Matter/Radiation

- **Potential perturbation:** \( k^2 \Psi = -4\pi G a^2 \delta \rho \) generated by radiation
- **Radiation \( \rightarrow \) Potential:** inside sound horizon \( \delta \rho/\rho \) pressure supported \( \delta \rho \) hence \( \Psi \) decays with expansion
- **Potential \( \rightarrow \) Radiation:** \( \Psi \)–decay timed to drive oscillation
  \(-2\Psi + (1/3)\Psi = -(5/3)\Psi \rightarrow 5x \) boost
- **Feedback stops at matter domination**

\[ \Delta T/T \]
\[ \eta \]

Hu & Sugiyama (1995)
Driving Effects and Matter/Radiation

- Potential perturbation: \( k^2\Psi = -4\pi G a^2 \delta \rho \) generated by radiation
- **Radiation \rightarrow Potential:** inside sound horizon \( \delta \rho / \rho \) pressure supported, \( \delta \rho \) hence \( \Psi \) decays with expansion
- **Potential \rightarrow Radiation:** \( \Psi \)–decay timed to **drive oscillation**
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- Feedback stops at **matter domination**

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Hu & Sugiyama (1995)
Potential Envelope

- Similar to matter transfer function, acoustic oscillations, as a function of $k$, enveloped by a function that depends only on $k/\Omega_m h^2$
- Unlike matter transfer function, the potential envelope increases with $k$
- Combination distinguishes between dynamical effects and initial conditions: complementarity

![Graph](image)

Hu & White (1997)
Matter Density in the CMB

- Amplitude ramp across matter–radiation equality
- Radiation density fixed by CMB temperature & thermal history

- Measure $\Omega_m h^2$ from peak heights

![Graph showing $\Omega_m h^2$ and Power/Damping]
Dark Matter in the Power Spectrum
Damping Tail
Diffusion Damping

- Diffusion inhibited by baryons
- Random walk length scale depends on time to diffuse: horizon scale at recombination
Diffusion Damping

- Random walk during recombination
- Dissipation as hot meets cold
- Physical scale for standard ruler or calibration
Dissipation / Diffusion Damping

- Imperfections in the coupled fluid $\rightarrow$ mean free path $\lambda_C$ in the baryons
- Random walk over diffusion scale: geometric mean of mfp & horizon
  $\lambda_D \sim \lambda_C \sqrt{N} \sim \sqrt{\lambda_C \eta} \gg \lambda_C$
- Overtake wavelength: $\lambda_D \sim \lambda$; second order in $\lambda_C/\lambda$
- Viscous damping for $R<1$; heat conduction damping for $R>1$

$N = \eta / \lambda_C$

Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)
Dissipation / Diffusion Damping

- Rapid increase at recombination as mfp ↑
- Independent of (robust to changes in) perturbation spectrum
- Robust physical scale for angular diameter distance test ($\Omega_K, \Omega_\Lambda$)

Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)
Time Evolution

- Time evolution of an acoustic mode ($k=1$/Mpc, sCDM)

Hu & Sugiyama (1996)
Time Evolution

• Diffusion length compared with horizon at recombination

$\Omega_b h^2 = 0.0125$
$\Omega_m h^2 = 0.25$

Hu & Sugiyama (1996)
Acoustic Visibility

- Effective visibility for CMB and baryons accounting for damping

Hu & Sugiyama (1996)
Damping Length as Ruler

- Damping scale presents another physical scale for curvature test
- Independent of phase of oscillation (Lasenby's demon: curvature vs. novel isocurvature perturbations)
Damping Tail in the Power Spectrum

- **Calibrate** the standard **rulers** in **curvature** test
- **Depends on** baryon-photon ratio and **horizon scale** at recombination (matter-radiation ratio)
The Peaks
Physical Decomposition & Information

![Graph showing combined, temperature, and doppler power against l](image-url)
Physical Decomposition & Information

- Fluid + Gravity
  → alternating peaks
  → photon-baryon ratio
  → $\Omega_b h^2$
Physical Decomposition & Information

- Fluid + Gravity
  - alternating peaks
  - photon-baryon ratio
  - $\Omega_b h^2$
  - driven oscillations
  - matter–radiation ratio
  - $\Omega_m h^2$
Physical Decomposition & Information

- **Fluid + Gravity**
  - alternating peaks
  - photon-baryon ratio
  - $\Omega_b h^2$
  - driven oscillations
  - matter–radiation ratio
  - $\Omega_m h^2$

- **Fluid Rulers**
  - sound horizon
  - damping scale
Physical Decomposition & Information

- **Fluid + Gravity**
  - $\rightarrow$ alternating peaks
  - $\rightarrow$ photon-baryon ratio
  - $\rightarrow \Omega_b h^2$
  - $\rightarrow$ driven oscillations
  - $\rightarrow$ matter–radiation ratio
  - $\rightarrow \Omega_m h^2$

- **Fluid Rulers**
  - $\rightarrow$ sound horizon
  - $\rightarrow$ damping scale

- **Geometry**
  - $\rightarrow$ angular diameter distance $d(\Omega_\Lambda, \Omega_K)$
  - + flatness or no $\Omega_\Lambda$,
  - $\rightarrow \Omega_\Lambda$ or $\Omega_K$

\[ l_A = d_A \times \pi k_A \]
\[ l_D = d_A \times k_D \]