What's the Matter
with the CMB
COBE Normalization
COBE Normalization

• Sachs-Wolfe Effect relates the COBE detection to the gravitational potential on the last scattering surface

$$[\Theta + \Psi](\hat{n}, x) = \frac{1}{3} \Psi(x + D\hat{n}, \eta_*)$$

$$D = \eta_0 - \eta$$

• Decompose the angular and spatial information into normal modes: spherical harmonics for angular, plane waves for spatial

$$G^m_\ell(\hat{n}, x, k) = (-i)^\ell \sqrt{\frac{4\pi}{2\ell + 1}} Y^m_\ell(\hat{n}) e^{ik\cdot x}.$$
COBE Normalization cont.

- **Multipole moment decomposition for each** \( k \)

\[
\Theta(n, x) = \int \frac{d^3 k}{(2\pi)^3} \sum_{\ell m} \Theta^{(m)}_{\ell}(k) G_{\ell}^{m}(x, k, n)
\]

- **Power spectrum is the integral over** \( k \) **modes**

\[
C_{\ell} = 4\pi \int \frac{d^3 k}{(2\pi)^3} \sum_{m} \frac{\langle \Theta^{(m)*}_{\ell} \Theta^{(m)}_{\ell} \rangle}{(2\ell + 1)^2}
\]

- **Fourier transform** Sachs-Wolfe source

\[
[\Theta + \Psi](\hat{n}, x) = \frac{1}{3} \int \frac{d^3 k}{(2\pi)^3} \Psi(k, \eta_*) e^{ik \cdot (D \hat{n} + x)}
\]

- **Decompose plane wave**

\[
\exp(ikD \cdot \hat{n}) = \sum_{\ell} (-i)^{\ell} \sqrt{4\pi (2\ell + 1) j_\ell(kD)} Y^0_{\ell}(n)
\]
COBE Normalization \textit{cont.}

- Extract multipole moment, assume a constant potential

\[
\frac{\Theta^{(0)}_\ell}{2\ell + 1} = \frac{1}{3} \Psi (k, \eta_*) j_\ell(kD) = \frac{1}{3} \Psi (k, \eta_0) j_\ell(kD)
\]

- Construct angular power spectrum

\[
C_\ell = 4\pi \int \frac{dk}{k} j_\ell^2(kD) \frac{1}{9} \Delta_\Psi^2
\]

- For scale invariant potential \((n=1)\), integral reduces to

\[
\int_0^\infty \frac{dx}{x} j_\ell^2(x) = \frac{1}{2\ell(\ell + 1)}
\]

- Log power spectrum = Log potential spectrum / 9

\[
\frac{\ell(\ell + 1)}{2\pi} C_\ell = \frac{1}{9} \Delta_\Psi^2 \quad (n = 1)
\]
COBE Normalization *cont.*

- Relate to density fluctuations: Poisson equation and Friedmann eqn.

\[
k^2 \Psi = -4\pi G a^2 \delta \rho
= -\frac{3}{2} H_0^2 \Omega_m^2 \delta
\]

- Power spectra relation

\[
\Delta^2_\Psi = \frac{9}{4} \left( \frac{H_0}{k} \right)^4 \Omega_m^2 \Delta^2_\delta
\]

- In terms of density fluctuation at horizon and transfer function

\[
\Delta^2_\delta \equiv \delta^2_H \left( \frac{k}{H_0} \right)^{n+3} T^2(k)
\]

- For scale invariant potential

\[
\frac{\ell(\ell + 1)}{2\pi} C_\ell = \frac{1}{4} \Omega_m^2 \delta^2_H \quad (n = 1)
\]
COBE Normalization cont.

• Some numbers

\[ \frac{\ell(\ell + 1)}{2\pi} C_\ell = \frac{1}{4} \Omega_m^2 \delta_H^2 \quad (n = 1) \]

\[ = \left( \frac{28 \mu K}{2.726 \times 10^6 \mu K} \right)^2 \approx 10^{-10} \]

\[ \delta_H \approx (2 \times 10^{-5}) \Omega_m^{-1} \]

• Detailed Calculation from Bunn & White (1997) including decay of potential in low density universe and tilt

\[ \delta_H = 1.94 \times 10^{-5} \Omega_m^{-0.785-0.05 \ln \Omega_m} e^{-0.95(n-1)-0.169(n-1)^2} \]
Normalization Caveats

• Why aren't models like **cosmological defects**, which have large scale power at last scattering, automatically ruled out by COBE?

• As the photons propagate through the large-scale structure of the universe, **gravitational redshifts** from **time-varying potentials** can generate large angle fluctuations

• Dark energy domination implies potential decay, linear effect is called the **Integrated Sachs Wolfe (ISW) effect**
Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00} = -(1 + \Psi)^2 \delta_{ij}$

Kofman & Starobinskii (1985)  
Hu & Sugiyama (1994)
Integrated Sachs–Wolfe Effect

- Potential redshift:  
  \[ g_{00} = -(1 + \Psi)^2 \delta_{ij} \]

- Perturbed cosmological redshift
  \[ g_{ij} = a^2 (1 + \Psi)^2 \delta_{ij} \]
  \[ \frac{\delta T}{T} = -\frac{\delta a}{a} = \Psi \]

Kofman & Starobinskii (1985)  
Hu & Sugiyama (1994)
Integrated Sachs–Wolfe Effect

- Potential redshift: \( g_{00} = -(1 + \Psi)^2 \delta_{ij} \)

- Perturbed cosmological redshift

\[
g_{ij} = a^2 (1 + \Psi)^2 \delta_{ij}
\]

\[
\frac{\delta T}{T} = -\frac{\delta a}{a} = \Psi
\]

- Time–varying potential

  Rapid compared with \( \lambda/c \)

\[
\frac{\delta T}{T} = -2\Delta\Psi
\]

  Slow compared with \( \lambda/c \)

  Redshift–blueshift cancel

- Imprint characteristic time scale of decay in angular spectrum

\[
(2\Psi)^2
\]

\[
-l_{\text{ISW}} \sim d/\Delta\eta
\]

Kofman & Starobinskii (1985)  
Hu & Sugiyama (1994)
Calculation of Secondary Anisotropies

- Addition of angular momentum gives

\[ \text{multipole moment} = \int \left( \text{clebsch gordan} \right) \left( \text{bessel function} \right) \text{Source} \ d(\text{line of sight}) \]

- Primary anisotropies: source sharply peaked at last scattering

Tight Coupling Approximation:

\[ \text{multipole moment} \sim \left( \text{clebsch gordan} \right) \left( \text{bessel function} \right) \int \text{Source} \ d(\text{line of sight}) \]

- Secondary anisotropies: source slowly–varying in time

Weak Coupling Approximation:

\[ \text{multipole moment} \sim \text{Source} \left( \text{clebsch gordan} \right) \int \left( \text{bessel function} \right) d(\text{line of sight}) \]

- Log power spectrum of CMB \( \sim (\text{cg}) \) * Log power spectrum of source / \( l \)

- Scalar source and scalar field on sky: weak coupling = limber approx.

Hu & White (1996); Hu (2000)
ISW Effect in the Power Spectrum

- ISW effect cancelled on small scales
- Barely affects the COBE normalization
- Cosmic variance limited in detectability
- But... a unique probe of dark energy
- Cross correlation and Higher order statistics

\[
\begin{align*}
\text{Power} & \quad \text{Primary} \\
10^{-9} & \quad 10^{-10} \\
10^{-11} & \quad 10^{-12} \\
10^{-13} & \quad 10^{-14}
\end{align*}
\]

\[
l \quad \text{Power}
\]
Into the Non-Linear Regime
COBE Normalized Power Spectrum

- Non-linear scale at $k \sim 0.2 \, h/\text{Mpc}$
COBE Normalized Power Spectrum

- Fully non-linear power spectrum dilates scale and increases amplitude

![Graph showing non-linear and linear dilation](image)
HKLM / PD Scaling Relation

- Gravitational collapse implies that the density fluctuation at a given non-linear scale comes from a much larger region originally.

- Particle number conserved so density enhancement must come from a change in volume:

  \[ k = \left(1 + \Delta_\delta^2 \right)^{1/3} k_{\text{lin}} \]

- Ansatz: there is a universal mapping between the linear spectrum and non-linear spectrum

  \[ \Delta_\delta^2 (k) = f_{\text{nl}} \left[ \Delta_\delta^2 (k_{\text{lin}}) \right] \]

- Linear limit

  \[ f_{\text{nl}}[x \ll 1] = x \]

- Stable clustering limit in a flat matter dominated universe

  \[ f_{\text{nl}}[x \gg 1] = x^{3/2} \]

  if clustering is fixed in physical coordinates power scales with \( a^3 \)
Secondary Anisotropies: Power Spectra

- **Gravitational Effects**
  - ISW Effect
    (redshift from decaying potentials)
  - Weak Lensing
    (smooths peaks and generates power <1')

- **Scattering Effects**
  - Doppler Effect
  - Vishniac Effect
    (LSS kinetic SZ effect)
  - Patchy Reionization
    (LSS thermal)

![Power Spectra Graph]

<table>
<thead>
<tr>
<th>l</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>(10^{-13})</td>
</tr>
<tr>
<td>100</td>
<td>(10^{-11})</td>
</tr>
<tr>
<td>1000</td>
<td>(10^{-9})</td>
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</tbody>
</table>
Baryon Suppression
Small-Scale CDM Perturbations

- Modes which enter the horizon during radiation domination

- CDM perturbations get boosted at horizon crossing by the decay of the gravitational potentials associated with radiation

- Enter into logarithmically growing mode due to the presence of a dominant smooth radiation background

- If baryons are dynamically negligible, linear growth begins when the universe becomes matter dominated

- If baryons are a substantial fraction of total matter, they act as a smooth matter background and suppress growth to

\[ ap, \quad p = 1 - 3\Omega_b/5\Omega_m \]

- Likewise if there is a component of massive neutrinos
Growth Suppression from Baryons

- Before the end of the drag epoch, the smooth baryons suppress growth

\[ \frac{k}{k_{eq}} = \sqrt{2a_{eq}/a_H} = 220 \]

\[ \Omega_b/\Omega_m = 0.1 \]

\[ \delta_c \]

\[ \Omega_b/\Omega_m = 1 \]

Hu & Sugiyama (1996)
Compton Drag

• Momentum conservation in scattering causes a drag force on the baryons

• Relative momentum density $R = \frac{3\rho_b}{4\rho_\gamma}$ defines a drag rate related to the scattering rate by

$$\dot{\tau}_d = \frac{\dot{\tau}}{R}$$

• Compton drag epoch ends when

$$\tau_d(z_d) = 1$$

• "Visibility function" for the baryons

$$\dot{\tau}_d e^{-\tau_d}$$
Acoustic Visibility

- Effective visibility for CMB and baryons accounting for damping

\[ \Omega_b h^2 = 0.0125 \]
\[ \Omega_m h^2 = 0.25 \]

\( \eta / \eta_* \)

CMB

baryons

Hu & Sugiyama (1996)
Net Suppression from Baryons

- At the end of the drag epoch, match both onto linearly growing mode

\[
\frac{\Omega_b}{\Omega_m} \ll 1
\]

\[
\Omega_m = 1
\]

\[
k = 4 \text{ Mpc}^{-1}
\]

\[
\frac{\Omega_b}{\Omega_m} = \frac{2}{3}
\]

Hu & Sugiyama (1996)
Baryons in the Transfer Function

- Substantial *suppression* of small scale power
- Appearance of *oscillations* at high baryon fractions
Baryon Wiggles
Acoustic Peaks in the Matter

- Baryon density & velocity oscillates with CMB
- Baryons decouple at $\frac{\tau}{R} \sim 1$, the end of Compton drag epoch
- Decoupling: $\delta_b(\text{drag}) \sim V_b(\text{drag})$, but not frozen

End of Drag Epoch

Hu & Sugiyama (1996)
Acoustic Peaks in the Matter

- Baryon density & velocity oscillates with CMB
- Baryons decouple at $\tau/R \sim 1$, the end of Compton drag epoch
- Decoupling: $\delta_b(drag) \sim V_b(drag)$, but not frozen
- Continuity: $\dot{\delta}_b = -kV_b$
- Velocity Overshoot Dominates: $\delta_b \sim V_b(drag) \kappa \eta >> \delta_b(drag)$
- Oscillations $\pi/2$ out of phase with CMB
- Infall into potential wells (DC component)

Hu & Sugiyama (1996)
Velocity Overshoot

- Time evolution for baryon only universe

\[ \Omega_b = \Omega_m = 0.3 \]

\[
\Omega_b = \Omega_m = 0.3
\]

\[
\Omega_b = \Omega_m = 0.3
\]

Hu & Sugiyama (1996)
Infall into CDM Wells

- Infall into CDM potential wells after the drag epoch

Hu & Sugiyama (1996)
Time Evolution

- Diffusion length compared with horizon at recombination

\[ \Omega_b h^2 = 0.0125 \]
\[ \Omega_m h^2 = 0.25 \]

Hu & Sugiyama (1996)
Oscillations in the Transfer Function

- Transfer function in a baryon only universe

\[ \Omega_b = \Omega_m = 0.3 \]

Hu & Sugiyama (1996)
Wiggles in the Transfer Function

- Transfer function in a **CDM dominated universe** \((f_b=1/3)\)

\[ \frac{k}{k_{eq}} \]

\[ \Omega_b=0.1, \Omega_m=0.3 \]

- Hu & Sugiyama (1996)

Hu & Sugiyama (1996)
Features in the Power Spectrum

- **Features** in the linear power spectrum
- **Break** at sound horizon
- **Oscillations** at small scales; washed out by nonlinearities

\[ P(k) \text{ (arbitrary norm.)} \]

\[ k (h \text{ Mpc}^{-1}) \]

---

Eisenstein & Hu (1998)

numerical

---

<table>
<thead>
<tr>
<th>P(k)</th>
<th>k (h Mpc(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**nonlinear scale**
Features in the Power Spectrum

- Features in the linear power spectrum
- Break at sound horizon
- Oscillations at small scales; washed out by nonlinearities

$P(k)$ (arbitrary norm.)

$m_v = 0 \text{ eV}$
$m_v = 1 \text{ eV}$

$0.01 \quad 0.1$
$0.1 \quad 1$

$k (h \text{ Mpc}^{-1})$

W. Hu – Feb. 1998

Peacock & Dodds (1994)
Features in the Power Spectrum

- Features in the linear power spectrum
- Break at sound horizon
- Oscillations at small scales; washed out by nonlinearities

\[ P(k) \text{ (arbitrary norm.)} \]

\[ k (h \text{ Mpc}^{-1}) \]

\[ m_v = 0 \text{ eV} \]
\[ m_v = 1 \text{ eV} \]
Baryon Wiggles in Non-Linear Regime

- Mode coupling **washes out features** in the initial power spectrum
- (HKLM/PD mapping fails to describe this effect!)
- Relationship between dark matter and galaxies ("bias") non-linear

Better: think of the dark matter as being comprised of discrete virialized halos: the halo model

- DM power spectrum = correlations within halos + correlations between halos

**Ingredients:** halo number density (Press-Schechter)
halo profiles (NFW)
halo bias (Mo & White)
linear power spectrum (cosmology)

- Galaxy power spectrum modeled by assigning galaxies to halos
Halo Model of the Power Spectrum

\[ \Delta^2(k) \]

- within halos
- between halos
- Linear
- Total

\[ z = 0.5 \]

Seljak (2000); figure from Cooray & Hu (2000)
Complementarity
Combining Features in LSS + CMB

- Consistency check on thermal history and photon–baryon ratio
- Infer physical scale $l_{\text{peak}}(\text{CMB}) \rightarrow k_{\text{peak}}(\text{LSS})$ in $\text{Mpc}^{-1}$

$Eisenstein, Hu & Tegmark (1998)$
$Hu, Eisenstein, Tegmark & White (1998)$
Combining Features in LSS + CMB

- Consistency check on thermal history and photon–baryon ratio
- Infer physical scale $l_{\text{peak}}(\text{CMB}) \rightarrow k_{\text{peak}}(\text{LSS})$ in $\text{Mpc}^{-1}$
- Measure in redshift survey $k_{\text{peak}}(\text{LSS})$ in $h\ \text{Mpc}^{-1} \rightarrow h$

Eisenstein, Hu & Tegmark (1998)
Hu, Eisenstein, Tegmark & White (1998)
Combining Features in LSS + CMB

- Consistency check on thermal history and photon–baryon ratio
- Infer physical scale \( l_{\text{peak}}(\text{CMB}) \rightarrow k_{\text{peak}}(\text{LSS}) \) in \( \text{Mpc}^{-1} \)
- Measure in redshift survey \( k_{\text{peak}}(\text{LSS}) \) in \( h \text{ Mpc}^{-1} \rightarrow h \)
- Robust to low redshift physics (e.g. quintessence, GDM)

\[
\begin{align*}
P_m(k) &\rightarrow k^3P_\gamma(k) \\
P_m(k) &\rightarrow h
\end{align*}
\]

Eisenstein, Hu & Tegmark (1998)
Hu, Eisenstein, Tegmark & White (1998)

MAP +P +SDSS

$H_0 \pm 130 \pm 23 \pm 1.2$

$\Omega_m \pm 1.4 \pm 0.25 \pm 0.016$

CMB:

$\Omega_m H_0^2$

$\Omega_m + \Omega_\Lambda$

Classical Cosmology

SDSS

~line of constant

MAP +P +SDSS

$H_0 \pm 130 \pm 23 \pm 1.2$

$\Omega_m \pm 1.4 \pm 0.25 \pm 0.016$

$\Omega_\Lambda \pm 1.1 \pm 0.20 \pm 0.024$

Classical Cosmology

Any other measurement (including $H_0$) breaks degeneracy

Many opportunities for consistency checks!
(e.g. high-$z$ SNIa)