1 Problem 1: Green’s Method

- (a) From the continuity and Euler equations of the joint photon-baryon system (see notes) show

\[ \ddot{\Theta} + \frac{\dot{R}}{1+R} \dot{\Theta} + k^2 c_s^2 \Theta = F(\eta) \]  

(1)

\[ F(\eta) = -\ddot{\Phi} - \frac{\dot{R}}{1+R} \dot{\Phi} - \frac{k^2}{3} \Psi \]  

(2)

- (b) Take the solutions

\[ \theta_a = (1 + R)^{-1/4} \cos(ks) \]  

(3)

\[ \theta_b = (1 + R)^{-1/4} \sin(ks) \]  

(4)

and show that they solve the homogeneous $F = 0$ equation in the adiabatic approximation

- (c) Use the Greens method to construct the particular solution

\[ \Theta(\eta) = C_1 \theta_a(\eta) + C_2 \theta_b(\eta) + \int_0^\eta d\eta' \frac{\theta_a(\eta') \theta_b(\eta) - \theta_a(\eta) \theta_b(\eta')}{\theta_a(\eta') \theta_b(\eta) - \theta_a(\eta) \theta_b(\eta')} F(\eta') \]  

(5)

and give the expression in terms of $\Theta(0)$, $\dot{\Theta}(0)$, $R$, $\dot{R}(0)$, $s$. Think of this as taking a set of impulsive forces on the oscillator and propagating their effect into a temperature perturbation at a later time.

- (d) Evaluate the general solution for $R$, $\Psi$, $\Phi$ all constant.

- (e) What you expect to happen qualitatively to the acoustic oscillations for initial conditions where there are no gravitational potentials initially and $\Phi = \Psi$ only becomes substantial after horizon crossing $k \eta = 1$ (take $C_1 = C_2 = 0$). Remember that the Greens solution causally propagates an impulsive force. Argue that the appearance of a first peak that is consistent with adiabatic initial conditions is strong argument for inflation.