Outline

• **Theory**
  Why modified gravity?
  Ostrogradski, Horndeski and scalar-tensor gravity;
  Galileon gravity as generalized DGP;
  Galileon in Minkowski and curved spacetime.

• **Phenomenology and Constraints**
  background, de Sitter attractor
  linear perturbation, ghost and laplace instability;
  CMB, lensing, matter power spectrum, etc;
  tension between expansion and growth.
Part One

- Theory

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Why modified gravity?

- Cosmological Constant (CC)?

- No compelling field theory explanation for cosmic acceleration

- vacuum energy? 120 order of magnitude larger than CC
  In general there is high-order quantum correction

- A dynamical theory, protected against loop correction by symmetry
What qualifies?

- Modify GR in the infrared, acceleration without CC

- Nonlinearity, screening mechanism to pass solar system test
  - Vainshtein mechanism: DGP, Galileon, Massive Gravity
  - Chameleon mechanism: $f(R)$
  - Symmetron ...
Part One

• Theory

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Ostrogradski's theorem

- In general higher-derivative theories have extra DoF and usually plagued by instabilities
- Ostrogradski's theorem: linear instability in the Hamiltonians associated with Lagrangians which depend upon more than one time derivative non-degenerately
- 1D unrelativistic point particle example: \( L(q, \dot{q}, \ddot{q}) \)
- Euler-Lagrange eqn: 
  \[
  \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}} = 0
  \]

M. Ostrogradski, Mem. Ac. St. Petersbourg VI 4, 385 (1850)
Ostrogradski's theorem

- EoM: \( q^{(4)} = \mathcal{F}(q, \dot{q}, \ddot{q}, q^{(3)}) \)

- Canonical coordinates

  \[
  Q_1 \equiv q, \quad P_1 \equiv \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}},
  \]

  \[
  Q_2 \equiv \dot{q}, \quad P_2 \equiv \frac{\partial L}{\partial \ddot{q}}.
  \]

- inversion

  \[
  \frac{\partial L}{\partial \ddot{q}} \bigg|_{\substack{q = Q_1 \\ \dot{q} = Q_2 \\ \ddot{q} = \alpha}} = P_2
  \]

M. Ostrogradski, Mem. Ac. St. Petersbourg VI 4, 385 (1850)
Ostrogradski's theorem

- Legendre transformation

\[ H(Q_1, Q_2, P_1, P_2) \equiv \sum_{i=1}^{2} P_i q^{(i)} - L, \]

\[ = P_1 Q_2 + P_2 a(Q_1, Q_2, P_2) - L\left(Q_1, Q_2, a(Q_1, Q_2, P_2)\right) \]

- Hamiltonian eqns hold

- Hamiltonian linear in P1, thus unstable

M. Ostrogradski, Mem. Ac. St. Petersbourg VI 4, 385 (1850)
Horndeski's theory

- Suppress Ostrogradski's instability by requiring 2nd order E-L eqns
- Starting from the general form

\[ L = L(g_{ij}; g_{ij,i_1}; \ldots; g_{ij,i_1 \ldots i_p}; \phi; \phi,i_1; \ldots; \phi,i_1 \ldots i_q) \]

Horndeski proved the most general 2nd-order E-L eqn can be obtained from a Lagrange density of the above form with \( p=q=2 \), 10 free functions with 6 PDE's

- No new DoF, but not necessarily stable
- DGP and Galileon as subsets

Part One

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DGP

- 4D gravity in 5D Minkowski spacetime

- Galilean and shift symmetry in decoupling limit
  \[ \pi \rightarrow \pi + c \]
  \[ \partial_\mu \pi \rightarrow \partial_\mu \pi + b_\mu \]

- Vainshtein mechanism

- Self-accelerating branch, plagued by ghost (kinetic term has the wrong sign)
Motivation of Galileon gravity

- Most general 4D Lagrangian with galilean and shift symmetry. Protection from radiative loop correction?

- Vainshtein mechanism

- No-ghost de Sitter attractor solution

- Other motivation of Galilean symmetry: higher dimension symmetry massive gravity

Part One

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Galileon in flat spacetime

\[ \mathcal{L}_\pi = \sum_{i=1}^{5} c_i \mathcal{L}_i \]

\[ \mathcal{L}_1 = \pi \]
\[ \mathcal{L}_2 = -\frac{1}{2} \partial \pi \cdot \partial \pi \]
\[ \mathcal{L}_3 = -\frac{1}{2} [\Pi] \partial \pi \cdot \partial \pi \]
\[ \mathcal{L}_4 = -\frac{1}{4} (\Pi^2 \partial \pi \cdot \partial \pi - 2 [\Pi] \partial \pi \cdot \Pi \cdot \partial \pi - [\Pi^2] \partial \pi \cdot \partial \pi + 2 \partial \pi \cdot \Pi^2 \cdot \partial \pi ) \]
\[ \mathcal{L}_5 = -\frac{1}{5} ([\Pi]^3 \partial \pi \cdot \partial \pi - 3 [\Pi]^2 \partial \pi \cdot \Pi \cdot \partial \pi - 3 [\Pi] [\Pi^2] \partial \pi \cdot \partial \pi + 6 [\Pi] \partial \pi \cdot \Pi^2 \cdot \partial \pi + 2 [\Pi^3] \partial \pi \cdot \partial \pi + 3 [\Pi^2] \partial \pi \cdot \Pi \cdot \partial \pi - 6 \partial \pi \cdot \Pi^3 \cdot \partial \pi ) \]

\[ [\Pi] \partial \pi \cdot \partial \pi \equiv \Box \pi \partial_\mu \pi \partial^\mu \pi \]

Galileon in flat spacetime

- Only 2nd deriv appear in EoM's

\[
\begin{align*}
\mathcal{E}_1 &= 1 \\
\mathcal{E}_2 &= \Box \pi \\
\mathcal{E}_3 &= (\Box \pi)^2 - (\partial_\mu \partial_\nu \pi)^2 \\
\mathcal{E}_4 &= (\Box \pi)^3 - 3 \Box \pi (\partial_\mu \partial_\nu \pi)^2 + 2(\partial_\mu \partial_\nu \pi)^3 \\
\mathcal{E}_5 &= (\Box \pi)^4 - 6(\Box \pi)^2 (\partial_\mu \partial_\nu \pi)^2 + 8 \Box \pi (\partial_\mu \partial_\nu \pi)^3 + 3[(\partial_\mu \partial_\nu \pi)^2]^2 - 6(\partial_\mu \partial_\nu \pi)^4 \\
\mathcal{E}_i &\equiv \frac{\delta \mathcal{L}_i}{\delta \pi}
\end{align*}
\]

Covariant Galileon

• Minimally coupled Galileon

\[ S = \int d^4x \sqrt{-g} \left( R + \mathcal{L}_\pi + \mathcal{L}_{\text{matter}} \right) \]

• In curved spacetime, 3rd and 4th order derivatives appear in EoM and energy-momentum tensor

Covariant Galileon

- Unique non-minimal couplings to curvature eliminate higher derivatives, thus retain only one scalar

\[ \mathcal{L}_1 = M^3 \varphi, \]
\[ \mathcal{L}_2 = \nabla_\mu \varphi \nabla^\mu \varphi, \]
\[ \mathcal{L}_3 = \frac{2}{M^3} \Box \varphi \nabla_\mu \varphi \nabla^\mu \varphi, \]
\[ \mathcal{L}_4 = \frac{1}{M^6} \nabla_\mu \varphi \nabla^\mu \varphi \left[ 2(\Box \varphi)^2 - 2(\nabla_\mu \nabla_\nu \varphi)(\nabla^\mu \nabla^\nu \varphi) - R \nabla_\mu \varphi \nabla^\mu \varphi / 2 \right], \]
\[ \mathcal{L}_5 = \frac{1}{M^9} \nabla_\mu \varphi \nabla^\mu \varphi \left[ (\Box \varphi)^3 - 3(\Box \varphi)(\nabla_\mu \nabla_\nu \varphi)(\nabla^\mu \nabla^\nu \varphi) + 2(\nabla_\mu \nabla^\nu \varphi)(\nabla_\nu \nabla^\rho \varphi)(\nabla_\rho \nabla^\mu \varphi) - 6(\nabla_\mu \varphi)(\nabla^\mu \nabla^\nu \varphi)(\nabla^\rho \varphi)G_{\nu \rho} \right], \]

\[ M^3 \equiv M_{\text{Pl}} H_0^2 \]

Part Two

- Phenomenology and Constraints
  
  background, de Sitter attractor;
  
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  tension between expansion and growth.
Einstein and Jordan Frames

- Action in Einstein frame

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_{pl}^2 R}{2} - \frac{c_2}{2} (\partial \pi)^2 - \frac{c_3}{M^3} (\partial \pi)^2 \Box \pi - \frac{c_4 \mathcal{L}_4}{2} - \frac{c_5 \mathcal{L}_5}{2} - \mathcal{L}_m - \frac{c_G}{M_{pl} M^3} T^{\mu\nu} \partial_\mu \pi \partial_\nu \pi - \frac{c_0}{M_{pl}} \pi T \right] \]

- In the weak field limit, the action can be transformed into a linearized form in Jordan frame, after which we can promote it to its full, nonlinear form

\[ S = \int d^4x \sqrt{-g} \left[ \left( 1 - 2c_0 \frac{\pi}{M_{pl}} \right) \frac{M_{pl}^2 R}{2} - \frac{c_2}{2} (\partial \pi)^2 - \frac{c_3}{M^3} (\partial \pi)^2 \Box \pi - \frac{c_4 \mathcal{L}_4}{2} - \frac{c_5 \mathcal{L}_5}{2} - \frac{M_{pl} c_G G^{\mu\nu} \partial_\mu \pi \partial_\nu \pi - \mathcal{L}_m}{M^3} \right] \]

- Einstein field eqn and EoM of scalar by variation

Background

- Assuming flat FLRW

\[ ds^2 = -dt^2 + a^2 \delta_{i,j} dx^i dx^j \]

- Homogeneous evolution

\[
\begin{align*}
    x' &= -x + \frac{\alpha \lambda - \sigma \gamma}{\sigma \beta - \alpha \omega} \\
    \bar{H}' &= -\frac{\lambda}{\sigma} + \frac{\omega}{\sigma} \left( \frac{\sigma \gamma - \alpha \lambda}{\sigma \beta - \alpha \omega} \right) \\
    y' &= x
\end{align*}
\]

\[ x = \frac{\pi'}{M_{\text{pl}}} \quad y = \frac{\pi}{M_{\text{pl}}} \]

- De Sitter attractor

\[
\begin{align*}
    \lambda &= -\omega x \\
    \gamma &= -\beta x
\end{align*}
\]

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  tension between expansion and growth.
Linear Perturbation

• Consider only scalar mode in Newtonian gauge

• Subhorizon, quasi-static approximation


• Covariant Gauge Invariant perturbation using 3+1 decomposition, with modified CAMB


• No-ghost and Laplace stability constrains the parameter space
Part Two

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Weak lensing potential

Matter power spectrum

\[ P_{k}/h^{-3} \text{ Mpc}^3 \]

\[ k/h \text{ Mpc}^{-1} \]

\[ \rho_{\phi,i}/\rho_{m,i} = 10^{-4} \]
\[ \rho_{\phi,i}/\rho_{m,i} = 10^{-5} \]
\[ \rho_{\phi,i}/\rho_{m,i} = 5 \times 10^{-6} \]
\[ \rho_{\phi,i}/\rho_{m,i} = 10^{-6} \]

Galileon clustering

\[ k = 1.0 \, h\text{Mpc}^{-1} \]

\[ \log_{10} (10^5) \]

\[ \log_{10} (10^4) \]

\[ \log_{10} (10^3) \]

\[ \log_{10} (10^2) \]

\[ \log_{10} (10^1) \]

\[ \log_{10} (10^0) \]

\[ \log_{10} (10^{-1}) \]

\[ \log_{10} (10^{-2}) \]

\[ \log_{10} (10^{-3}) \]

\[ a \]

Quasi-static approximation

$k = 0.001 \ h\text{Mpc}^{-1}$

Part Two

- Phenomenology and Constraints

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Growth vs Expansion (uncoupled Galileon)

- Linear perturbation, scalar mode and quasi-static approx.
  \[ ds^2 = -(1 + 2\psi)dt^2 + a^2(1 - 2\phi)\delta_{ij}dx^i dx^j \]

- Modified Poisson eqn
  \[ \hat{\nabla}^2 \psi = \frac{4\pi a^2 G_{eff}(\psi)}{H_0^2} \rho_m \delta_m \]

- Effective Gravitational strengths

\[
\begin{align*}
G_{eff}(\phi) &= \frac{2 (\kappa_4 \kappa_6 - \kappa_5 \kappa_1)}{\kappa_5 (\kappa_4 \kappa_1 - \kappa_5 \kappa_3) - \kappa_4 (\kappa_4 \kappa_6 - \kappa_5 \kappa_1)} G_N \\
G_{eff}(\psi) &= \frac{4 (\kappa_3 \kappa_6 - \kappa_1^2)}{\kappa_5 (\kappa_4 \kappa_1 - \kappa_5 \kappa_3) - \kappa_4 (\kappa_4 \kappa_6 - \kappa_5 \kappa_1)} G_N \\
G_{eff}(\psi + \phi) &= \frac{\kappa_6 (2\kappa_3 + \kappa_4) - \kappa_1 (2\kappa_1 + \kappa_5)}{\kappa_5 (\kappa_4 \kappa_1 - \kappa_5 \kappa_3) - \kappa_4 (\kappa_4 \kappa_6 - \kappa_5 \kappa_1)} G_N
\end{align*}
\]

Trial of (uncoupled) Galileon

- CosmoMC

\[ \mathcal{L} = \mathcal{L}_{\text{CMB}} + \mathcal{L}_{\text{SN}} + \mathcal{L}_{\text{BAO}} + \mathcal{L}_{\text{growth}} \]

- CMB data from WMAP7 is applied in the form of the covariance matrix for the shift parameter, acoustic peak multipole, and redshift of decoupling

Distances from Type Ia supernovae in the Union2.1 data compilation constrain the expansion history at \( z \approx 0 - 1.4 \)

Distances from the baryon acoustic oscillation feature in the galaxy distribution, measured to 6 redshifts at \( z = 0.1 - 0.7 \)

- growth rate from the WiggleZ survey at four redshifts \( z = 0.2 - 0.8 \), and from the BOSS survey at \( z = 0.57 \), plus the EG growth probe

Trial of (uncoupled) Galileon

- Best fit yields

\[ \Delta \chi^2 = 31 \]

with respect to the best fit LCDM, despite having 4 extra fit parameters:

- \( \rho_{\pi,i} \)
- \( c_3, c_4, c_5 \)

Trial of (uncoupled) Galileon

- Best fit model
- Gravitational strengths diverges in the near future due to Laplace instability

Trial of (uncoupled) Galileon

- Narrow region of degeneracy

Other interesting aspects

- Tensor perturbation
- Generalized galileon
- G-inflation
- Connection to massive gravity
- Nonlinearity and Vainshtein mechanism
Thank you!