Constraints on supernovae dimming from photon-pseudo scalar coupling

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An alternative mechanism that dims high redshift supernovae without cosmic acceleration utilizes an oscillation of photons into a pseudo-scalar particle during transit. Since angular diameter distance measures are immune to the loss of photons, this ambiguity in interpretation can be resolved by combining CMB acoustic peak measurements with the recent baryon oscillation detection in galaxy power spectra. This combination excludes a non-accelerating dark energy species at the 4σ level regardless of the level of the pseudo-scalar coupling. While solutions still exist with substantial non-cosmological dimming of supernovae, they may be tested with future improvement in baryon oscillation experiments.

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I. INTRODUCTION

Cosmic acceleration is mainly inferred through geometrical measures of the expansion history, most notably from the dimming of high redshift supernovae [1, 2, 3, 4]. In order to be certain of this interpretation, all other plausible explanations of the dimming must be ruled out.

An alternative mechanism has been proposed for the dimming of supernovae [5]. Here photons are converted into an unseen particle during transit. This mechanism is based on a generic interaction between the photon and a pseudo-scalar field. Other cosmological consequences of this pseudo-scalar (p-p) coupling include Faraday-like rotations, spontaneous generation of polarization, and mordial magnetic fields generation [6, 7, 8, 9].

Mediated by an external magnetic field, this coupling also produces the conversion that dims supernovae. This mechanism is similar to that exploited by axion direct detection experiments [10] but in the reverse direction. Previous works examining the viability of this alternative dimming mechanism have largely focused on constraints from the observed achromaticity of the dimming [11, 12].

In this Paper, we reconsider this alternate mechanism in light of recent cosmological measurements. First, we show that there exists large regions in the parameter space that both satisfy current luminosity distance measurements and avoid achromaticity constraints by saturating the conversion. Hence exclusion of this alternate explanation solely on constraints from supernovae data is difficult.

On the other hand, cosmological probes of acceleration that do not involve luminosity distances are immune to dimming effects. Specifically, expansion history constraints based on angular diameter distances from the measurement of standard rulers can be used to directly test the dimming hypothesis. The acoustic oscillations set by the photon-baryon fluid before recombination imprint the sound horizon as a standard ruler in both the cosmic microwave background and galaxy power spectra. We combine measurements from WMAP [13] which give the angular diameter distance to recombination with the recent detection of baryon oscillations in the SDSS LRG galaxy survey [14] which provides the distance ratio between the galaxies and the CMB.

Our result exclude a non-accelerating species of dark energy at the 4σ level even allowing for the possibility of photon conversion. Furthermore, within the 95% confidence limits there exists separate solutions with no dimming and strong dimming of supernovae both of which require cosmic acceleration. These possibilities can be distinguished in the future with better constraints from the acoustic oscillations.

In an Appendix, we also consider spectral constraints from COBE FIRAS on the dimming of the CMB itself. These constraints, which are more robust than and equally powerful compared to CMB anisotropy constraints [3], are useful for eliminating astrophysically disfavored regions of the parameter space that involve large external magnetic fields with a short coherence length.

II. PHOTON-PSEUDO SCALAR CONVERSION

We study the interaction between photons and a pseudo-scalar field $\phi$ through

$$ S = \int d^{4}x \sqrt{-g} \left[ \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m^{2} \phi^{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\phi}{4 M} F_{\mu\nu} \tilde{F}^{\mu\nu} \right], $$

(1)

where $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ and $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita tensor.

In a small patch where the effect of the cosmic expansion is negligible, the equation of motion for $\phi$ is

$$ \frac{\partial^{2} \phi}{\partial t^{2}} - \nabla^{2} \phi = -m^{2} \phi - \frac{1}{M} \vec{E} \cdot \vec{B}. $$

(2)
The magnetic field $\vec{B}$ is a combination of the internal magnetic field of the radiation and any external magnetic field that might exist. We assume however that within a domain the external magnetic field dominates. The external magnetic field in the intergalactic medium is expected to be around $10^{-10}$ Gauss with 1Mpc comoving domain size [13, 14].

We split the electric Maxwell equation into two components, $E_\parallel$ parallel to $\vec{B}$ and $E_\perp$ perpendicular to $\vec{B}$. The equation of motion for $E_\parallel$ is

$$\frac{\partial^2}{\partial t^2}E_\parallel - \nabla^2 E_\parallel = \frac{1}{M} \frac{\partial^2}{\partial t^2} B_\parallel.$$  

(3)

Here $E_\parallel$ is coupled to the scalar field by the external magnetic field, but there is no mixture between $E_\perp$ and $\vec{B}$. To summarize the coupled equations in matrix form, we have

$$\left[ \begin{array}{c} \frac{\partial^2}{\partial t^2} + \omega^2 \end{array} \right] \left( \begin{array}{c} A_\parallel \\ A_\perp \\ \phi \end{array} \right) = \left( \begin{array}{ccc} \omega_0^2 & 0 & \mu \omega \\ 0 & \omega_0^2 & 0 \\ \mu \omega & 0 & m^2 \end{array} \right) \left( \begin{array}{c} A_\parallel \\ A_\perp \\ \phi \end{array} \right),$$  

(4)

where $\vec{A} = \vec{E}/\omega$, $\mu = B_\parallel/M$ and $l$ is the physical distance traveled in each domain.

The off-diagonal interaction terms induce mixing between $A_\parallel$ and $\phi$. The eigenvalues and the eigenstates of this mixing are given by

$$\lambda_\pm = \frac{\omega_0^2 + m^2}{2} \pm \frac{1}{2} \sqrt{\left(\omega_0^2 - m^2\right)^2 + 4 \mu^2 \omega^2}$$  

(5)

and

$$\left( \begin{array}{c} \lambda_- \\ \lambda_+ \end{array} \right) = \left( \begin{array}{cc} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{array} \right) \left( \begin{array}{c} A_\parallel \\ \phi \end{array} \right),$$  

(6)

where the mixing angle is given by

$$\tan 2\theta = \frac{2 \mu \omega}{\omega_0^2 - m^2}.$$  

(7)

The conversion probability of a photon polarized in the $\parallel$ direction in a single external magnetic domain is

$$P_{\gamma \rightarrow \phi} = \sin^2 2\theta \sin^2 \left[ s \frac{\sqrt{(\omega_0^2 - m^2)^2 + 4 \mu^2 \omega^2}}{4 \omega} \right],$$  

(8)

where $s$ is the physical size of a coherent field domain and $\omega_p$ is the plasma frequency. As pointed out by [11], the finite plasma frequency of the intergalactic medium sets a lower limit on the effective mass scale. We will hereafter assume that $\omega_p \gg m$ throughout.

### III. SIMULATING DIMMING

We now simulate the dimming of supernovae through multiple field domains in a cosmological context. We assume passive evolution of the parameters such that the field strength scales with the scale factor $a$ as $B \propto a^2$, the domain size is fixed in comoving coordinates $s \propto a$, and the plasma frequency

$$\omega_p = 1.2 \times 10^{-14} \left( \frac{n_{e0}}{10^{-7} \text{cm}^{-3}} \right)^{1/2} a^{-3/2} \text{eV},$$  

(9)

where $n_{e0}$ is the free electron density today.

A relevant parameter to consider is the ratio of the coherence length $s$ to the plasma length $l_p \equiv 2 \omega_0^2 \omega_p$ at the observation frequency [11]. If this ratio is large, then the spatially oscillating piece in the conversion probability Eqn. (8) will be averaged to 1/2 across the many random domains such that

$$P_{\gamma \rightarrow \phi} \approx \frac{1}{2} \sin^2 2\theta.$$  

(10)

in each domain. For optical light at $\omega_{\text{SN}} \sim 1 \text{eV}$, the plasma length today $l_p \sim 100 (n_{e0}/10^{-7} \text{cm}^{-3})^{-1}$ kpc. With a typical magnetic domain size of order 1 Mpc, we expect $s \gg l_p$ is a good assumption. Since the mixing angle scales with the plasma frequency as given in Eqn. (7), we will take a fiducial value of $n_{e0} = 2.4 \times 10^{-7} \text{cm}^{-3}$ with the understanding that other cases may be obtained by replacing

$$\mu \rightarrow \mu \left( \frac{n_{e0}}{2.4 \times 10^{-7} \text{cm}^{-3}} \right)^{-1}.$$  

(9)
Note that in this limit, the conversion probability in a single domain is highly chromatic with a scaling of \( P_{\gamma \rightarrow \phi} \propto \omega^2 \). As an aside, there is also empirical evidence disfavoring dimming in the opposite limit of \( s \ll l_p \). Here \( P_{\gamma \rightarrow \phi} \approx 4 s^2 \mu^2 \) and becomes independent of frequency. Hence low frequency photons from cosmological sources can be substantially dimmed. The blackbody nature of the CMB places significant constraints on dimming in this regime. In the Appendix we show that

\[
P_{\gamma \rightarrow \phi}(\omega_{\text{CMB}}) < 2.5 \times 10^{-4} \quad (95\%\text{CL}),
\]

where \( \omega_{\text{CMB}} = 1.24 \times 10^{-5}\text{eV} \). If at \( \omega_{\text{CMB}} \) the coherence length is still \( s \ll l_p \) then substantial dimming of supernovae would be completely ruled out.

Under the assumption that the transition \( s \sim l_p \) lies somewhere between the optical and CMB frequencies, this constraint translates to one on \( l_p/s \) through

\[
P_{\gamma \rightarrow \phi}(\omega_{\text{SN}}) \approx 2 \left( \frac{l_p}{s} \right)^2 \frac{D(z_{\text{CMB}})}{D(z_{\text{SN}})} a_{\text{SN}}^2,
\]

where \( D(z) \) is the comoving distance to redshift \( z \), \( l_p/0 \) is evaluated at the SN frequency, and \( z_{\text{CMB}} \) is the highest redshift for which the magnetic field persists. Note that this \( 10^{-4} \) constraint from spectral distortions is more robust than and equally powerful compared with constraints based on the angular variation of the dimming. Although the latter lie at an amplitude of \( 10^{-5} \), anisotropy measurements are typically taken at a frequency that is up to an order of magnitude smaller (30GHz vs 300GHz). The strong frequency scaling of \( P_{\gamma \rightarrow \phi} \propto \omega^2 \) more than compensates the lower precision of the spectral measurements. With supernovae at \( z_{\text{SN}} \approx 0.5 \) dimmed by order unity and assuming \( z_{\text{CMB}} \) is high, e.g. comparable to the reionization redshift, \( s/l_p > 1/10 \). Hence a very small coherence length to the field is not allowed if the supernovae are to be dimmed by order unity. We hereafter assume that \( s \gg l_p \) for viable models.

To handle multiple domains, we employ a density matrix formulation. The initial conditions at emission are given by unpolarized photons, arbitrarily normalized to unity for convenience, and no excitation of the pseudoscalar field

\[
S_0^{ab}(s_0) = S_1^{ab}(0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

In our notation for the density matrix \( S_i^{ab}(l) \), \( a \) and \( b \) denote the components \( (||, \perp, \phi) \) with respect to the magnetic field of the domain, the subscript \( i \) denotes the \( i \)th magnetic field domain, and the argument \( l \) denotes the physical location in the domain, \( 0 \) for the beginning, \( s_i \) for the end.

Since the orientation of \( \vec{B} \) in each domain is not identical, the density matrix must be rotated between each domain. Defining \( \xi \) as the angle between \( \vec{B} \) and the plane perpendicular to the propagation \( B_1 = \cos \xi \vec{B} \). With \( \psi \) as the angle between \( B_1 \) in two neighboring domains, the density matrix transforms as

\[
S_i^{ab}(0) = \begin{pmatrix} \cos^2 \psi S_i^{||} + \sin^2 \psi S_i^{\perp ||} - \sin 2 \psi S_i^{||} \\ \frac{1}{2} \sin 2 \psi (S_i^{\perp ||} - S_i^{||}) + \cos 2 \psi S_i^{||} \\ \sin \psi S_i^{\perp ||} - \sin \psi S_i^{||} \end{pmatrix} \end{pmatrix}
\]

\[
\begin{pmatrix} \cos 2 \psi S_i^{||} + \sin 2 \psi S_i^{\perp ||} + \frac{1}{2} \sin 2 \psi (S_i^{\perp ||} - S_i^{||}) + \cos 2 \psi S_i^{||} \\ \sin \psi S_i^{\perp ||} - \sin \psi S_i^{||} \end{pmatrix} \end{pmatrix}
\]

\[
\begin{pmatrix} \cos \psi S_i^{\perp ||} - \sin \psi S_i^{||} \end{pmatrix} \end{pmatrix}
\]

where the right hand side is evaluated at position \( s_i \) of the previous domain. We chose random variates uniform over 2\( \pi \) for the angles in each domain.

To propagate the density matrix across \( i \), we employ the conversion probabilities implied by the mixing matrix

\[
S_i^{||}(s_i) = (1 - \frac{1}{2} \sin^2 2\theta) S_i^{||}(0) + \frac{1}{2} \sin^2 2\theta S_i^{\phi}(0) + \sin 2\theta \cos 2\theta S_i^{\phi}(0),
\]

\[
S_i^{\phi}(s_i) = \frac{1}{2} \sin 2\theta \cos 2\theta S_i^{||}(0) - \frac{1}{2} \sin 2\theta \cos 2\theta S_i^{\phi}(0) + \sin^2 2\theta S_i^{\phi}(0),
\]

\[
S_i^{\phi}(s_i) = \frac{1}{2} \sin^2 2\theta S_i^{||}(0) + (1 - \frac{1}{2} \sin^2 2\theta) S_i^{\phi}(0) - \sin 2\theta \cos 2\theta S_i^{\phi}(0).
\]

There is no conversion in other components.

The total conversion probability at the observer is determined by the appearance probability of the pseudoscalar field at the last domain. The total conversion probability is given by

\[
P_{\gamma \rightarrow \phi} = S_i = D(z)f/S(s),
\]

where \( f \) is the one dimensional fraction of the external
magnetic domain between the observer and the source at a comoving distance $D(z)$. Here $S_{s_j} / a_i$ is the assumed constant coherence length in comoving coordinates.

As can be seen in Eqn. (16), the total conversion is a function of $\mu \sqrt{f}$. In the top panel of Fig 1, we show the conversion as a function of $z$ for several choices of $\mu \sqrt{f}$. Given a limit on the mass scale of the coupling of $M \geq 2 \times 10^{19}$eV \cite{17, 18}, the magnetic field strength must be of order $10^{-10}$ Gauss to establish a substantial conversion across a cosmological distance. This is a plausible strength for the intergalactic field.

The bottom panel of Fig 1 shows that the dimming produced by an accelerating component of dark energy with an equation of state $w = p/\rho < -1/3$ and by the p-p coupling with $w > -1/3$ looks almost identical (see the following section for model details). We would expect, as shown explicitly in the next section, that both scenarios would be equally favored by the luminosity distance measures alone.

To mimic cosmic acceleration, the dimming must also be achromatic. Although the conversion probability in an individual domain is highly chromatic, the net dimming need not be so. The conversion probability equilibrates at 1/3 due to the oscillation of the pseudo-scalar back to a photon. Once the conversion probability is saturated, the dimming becomes achromatic.

With a sufficiently large $\mu \sqrt{f}$ achromaticity can be achieved at a sufficiently low redshift to avoid observational constraints. Observations limit the magnitude difference of the dimming between B(0.44$\mu$m) and V(0.55$\mu$m) bands to be at most 0.03 \cite{11}. But as we see in the middle panel of Fig 1 once the conversion probability reaches the saturation limit, the difference in dimming between the bands disappears.

Thus in terms of supernovae data alone, dimming by the p-p coupling and cosmic acceleration are equally favored.

**IV. MODEL CONSTRAINTS**

We now explore the allowed parameter space of p-p dimming and cosmic acceleration models. The cosmic acceleration is parametrized by the dark energy equation of state $w = p/\rho$ and the non-relativistic energy density relative to critical $\Omega_m$ in a flat universe. The p-p dimming model is parameterized by $\mu \sqrt{f}$. For simplicity, we will fix $f = 2/3$ and quote the allowed region in $\mu$ evaluated at the present epoch.

First, we use the gold set of the supernovae data alone \cite{4}. When we consider the supernovae dimming just by the cosmic acceleration, a phantom dark energy model with $w < -1$ is marginally favored as shown in the left panel of Fig 2 (see e.g. \cite{19}). It is interesting to note that the bolometric dimming of the supernovae most favored by the cosmic acceleration scenario is statistically indistinguishable from the best fit p-p coupling scenario with $w = -0.2$ and no acceleration, as shown in the bottom panel of Fig 2. This degeneracy is quantified in the top-left panel in Fig 3. It shows that the cosmic acceleration by the phantom dark energy model with the negligible p-p coupling is almost equally favored compared with strong p-p coupling with no cosmic acceleration \cite{20}. The two appear as discrete solutions with the $\Lambda$CDM model.
in the weakly disfavored intermediate regime.

Other cosmological tests are required to break this degeneracy. We next combine the luminosity distance measured by the supernovae with the angular diameter distance measured by CMB. The angular diameter distance at recombination, $D_A(z_*)$, is given by fitting the CMB acoustic structures. We can estimate $D_A(z_*)$ based upon the cosmological parameter $\omega_m = \Omega_m h^2 = 0.135 \pm 0.008$ which controls the physical extent of the sound horizon as well as the redshift of recombination and its measured angular scale $l_A = 301 \pm 1$ [13, 21].

The right panel in Fig 2 and the bottom left panel in Fig 3 show that the p-p coupling scenario with no cosmic acceleration is now more likely than either phantom models or ΛCDM at $w = -1$ but the latter is still allowed at 95% confidence.

V. CONCLUSION

We study an alternative explanation for the dimming of supernovae involving the loss of photons to a pseudo-scalar field $\phi$ in transit. With only supernovae measurements on the bolometric total dimming and the achromaticity of dimming, solutions without cosmic acceleration exist.

This ambiguity in the supernovae data can be resolved with angular diameter distance measures that are based on a standard length scale instead of a standard candle. We apply the CMB acoustic peaks and the recently detected baryon oscillations as constraints on the model. We also use CMB spectral constraints to eliminate a class of small magnetic field coherence length solutions. We show that the non-accelerating solutions persist when adding just the CMB information but are eliminated at the 4$\sigma$ level once the baryon oscillations are added.

Intriguingly solutions with substantial dimming accompanied by cosmic acceleration still exist and could potentially bias measurements of the equation of state in the negative direction. These models can be tested with future baryon oscillation experiments. Likewise astrophysical sources of non-cosmological dimming such as dust extinction can be tested by these means.

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APPENDIX A: CMB SPECTRAL CONSTRAINT

COBE FIRAS measured the CMB to be a pure blackbody with no significant distortions [22]. A dimming of cosmological sources would distort the blackbody spectrum and so in this Appendix we use the COBE FIRAS data shown in Fig. 4 to set limits on dimming scenarios.

The p-p coupling dims the CMB spectrum as well as the supernovae light. We introduce the generic parameter $A_\nu$ representing the amplitude of the dimming of the CMB spectrum as

$$S(\nu; T) = \left[ 1 - A_\nu \left( \frac{\nu}{10\text{cm}^{-1}} \right)^2 \right] \frac{2hc^2 \nu^3}{\exp(hc\nu/kT) - 1}$$

FIG. 3: $w - \mu$ constraints at the 68%, 95% and 99% CL. The magnetic field filling fraction $f$ is fixed to 2/3 for simplicity.
where the frequency $\nu$ is measured in cm$^{-1}$. Our linearized fit includes three unknown parameters, $\Delta T$ the temperature variation from $T_0 = 2.726K$, $A_\nu$ and the level of residual galaxy emission $G_0$. The form is

$$I_0(\nu) = \left[1 - A_\nu \left(\frac{\nu}{10}\right)^2\right] B_\nu(T_0) + \Delta T \frac{\partial B_\nu}{\partial T} + G_0 g(\nu),$$

where $B_\nu(T_0)$ is the blackbody spectrum and $g(\nu)$ is a galaxy emission spectrum. We assume a fixed galaxy emission spectrum over the sky as given in [23].

As shown in Fig. 4 the best fit model is a pure blackbody spectrum of $T_0 = 2.726K$ with a small high frequency galaxy emission contamination. However the shape of the linear distortion by a temperature uncertainty $\Delta T$ is mildly degenerate with $A_\nu$. Constraints on $A_\nu$ are limited by this degeneracy as shown in Fig. 4. We obtain $A_\nu = 1.0 \times 10^{-5} \pm 2.5 \times 10^{-4}$ at the 95% confidence level.

![FIG. 4: Residuals $I_0(\nu) - B_\nu(T_0)$ compared with model distortions. Thin solid line denotes the best fit, thin dotted line denotes a deviation at the $\Delta \chi^2 = 1$ level. This deviation is composed of compensating changes from the temperature variation $\Delta T$, the p-p dimming $A_\nu$, and the galactic component $G_0$ as shown.](image)