Astro 282: Problem Set 2
Due April 19

1 Problem 1: Matter and Radiation

- Write down the Hubble parameter $H(a)$, where a is the scale factor normalized to unity today. Consider the universe to be composed of 3 types of density components parameterized by their density today in units of the critical density: Ω_m for matter (with $w_m = 0$), Ω_r for radiation (with $w_r = 1/3$) and a cosmological constant Ω_Λ (with $w_\Lambda = -1$).

- Take $\Omega_r = 1.68\Omega_\gamma$ and the photon density from the previous problem set. The prefactor comes from including neutrinos at $T_\nu = (4/11)T_\gamma$. Take $\Omega_m = 1/3$ which is close to the observed value. At what value of the scale factor $a = a_{\rm eq}$ were matter and radiation equal in density (keep the dependence on h)? Take a flat universe such that $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$. At what a were matter and the cosmological constant equal in density (you may ignore the small correction due to Ω_r here)? Argue that in at $a \ll 1$ one can neglect the contribution of dark energy to the Hubble parameter above.

- Using the fact that

$$H(a) \equiv \frac{1}{a}\frac{da}{dt}\,, \tag{1}$$

  solve for the coordinate time t as a function of $\Omega_m h^2$, $a_{\rm eq}$ (replacing $\Omega_r h^2$) and a, for $a \ll 1$ such that dark energy may be ignored. Argue that the coordinate time becomes independent of $\Omega_m h^2$ and $a_{\rm eq}$ for $a \ll a_{\rm eq}$. Why? From the redshifting of the CMB temperature $T_\gamma \propto a^{-1}$, what is the scale factor when the background temperature was $T_\gamma = 10^9$K. What is the energy scale $k_B T_\gamma$ in keV at that time? What is the age of the universe t at that scale? [This is the epoch of big-bang nucleosynthesis].

- Using the fact that the conformal time $\eta = \int dt/a$, solve for $\eta(a, a_{\rm eq}, \Omega_m h^2)$. When the universe is at an energy scale of 1/3 eV, what is the temperature T_γ and the scale factor a_*? What is the age of the universe in years then if $\Omega_m = 1/3$ and $h = 0.7$? What is the conformal time in Mpc then for the same parameters. Convert the age of the universe to light-years and then Mpc. Why are these two scales different. [This is the epoch of recombination.]