The Astrophysical Journal, 447: L59–L63, 1995 July 10

COBE CONSTRAINTS ON BARYON ISOCURVATURE MODELS

Wayne Hu,[1] Emory F. Bunn,[1] and Naoshi Sugiyama[1,2]

Received 1995 January 12; accepted 1995 May 2

ABSTRACT

We consider Bayesian constraints on standard isocurvature baryon models from the slope and normalization of the anisotropy power spectrum detected by the *COBE* DMR experiment in their 2 year maps. In conjunction with either the amplitude of matter fluctuations σ_8 *or* its slope, all open models are ruled out at greater than 95% confidence, whereas cosmological-constant dominated models are constrained to be highly ionized. By including the *COBE* FIRAS 95% confidence upper limit on spectral distortion under the assumption of collisional ionization, we further reduce the available parameter space for Λ models by excluding these highly ionized models. These constraints define a single remaining class of standard models which makes definite and testable predictions for degree-scale anisotropies and large-scale structure.

Subject headings: cosmic microwave background — large-scale structure of universe

1. INTRODUCTION

The original baryon isocurvature scenario for structure formation (Peebles 1987a,b) presents a simple and attractive alternative to cold dark matter (CDM) inspired cosmogonies. It satisfies dynamical observations which suggest a low-density universe $\Omega_0 \simeq 0.2$–0.3, forms structure without the aid of hypothetical dark matter, and can alter light-element nucleosynthesis sufficiently to make an $\Omega_0 = \Omega_b$ baryonic universe acceptable (Gnedin & Ostriker 1992; Gnedin, Ostriker, & Rees 1995). Moreover, recent measurements of a large Hubble constant $H_0 = 100\ h$ km s^{-1} Mpc^{-1}, $h = 0.80 \pm 0.17$ (Freedman et al. 1994) would be easier to accommodate in such a low-density universe. Fixing H_0 and Ω_0 to lie in the observed range, the simplest class of isocurvature models involves only three free parameters: the normalization and slope of the initial fluctuations, and the ionization fraction after standard recombination. Of course, since the model is phenomenologically motivated, it is always possible to increase its complexity to satisfy observations, especially in the open-universe case.

When normalized to the *COBE* DMR detection (Smoot et al. 1992), the open-universe manifestations of the simple model generically predict small-scale power in significant excess of CDM (Efstathiou & Bond 1987; Chiba, Sugiyama, & Suto 1994; Hu & Sugiyama 1994, hereafter HS94). Yet given the present uncertain status of cosmic microwave background (CMB) anisotropy detections at degree to arcminute scales (see, *e.g.*, Wilkinson 1994), it is perhaps premature to constrain models on these grounds. The excess small-scale power also appears as a steeply blue slope in the large angle anisotropy spectrum (Sugiyama & Silk 1994), whereas the *COBE* DMR experiment prefers a flatter spectrum (Górski et al. 1994; Bunn, Scott, & White 1995). In this *Letter*, we perform a full analysis of the two-year *COBE* DMR maps to quantify this constraint using the complete anisotropy information in the model. Unlike previous treatments (Chiba et al. 1994; HS94), we also determine the normalization from the full data set as opposed to merely the rms fluctuation at 10°. This causes a 10% boost in the amplitude of fluctuations in open models. We further extend prior treatments by considering flat low Ω_0, cosmological constant Λ models whose predictions are somewhat more in accord with observations. The boost in amplitude can be up to 30% in these models and is of interest for simulations of large-scale structure formation. Finally, employing spectral distortion constraints from the *COBE* FIRAS experiment (Mather et al. 1994), we nearly close the parameter space available to these baryon isocurvature models. For the small class of models remaining, we present anisotropy and matter power spectra which can be used to make predictions for degree-scale anisotropies and large-scale structure.

2. GENERAL FEATURES

In the standard baryon isocurvature model, the universe consists of photons, baryons, and three families of massless neutrinos only. Initial entropy perturbations, *i.e.*, fluctuations in the baryon-photon and baryon-neutrino number densities, are assumed to take the form of a pure power law in $\tilde{k}$, $|S(\tilde{k})|^2 \propto \tilde{k}^n$, where the wavenumber $\tilde{k}$ is related to the eigenvalue of the Laplacian k as $\tilde{k}^2 = k^2 + K$, with $K = -H_0^2(1 - \Omega_0 - \Omega_\Lambda)$ as the curvature (Wilson 1983). Here Ω_Λ is the fraction of the critical density contributed by the cosmological constant. The Λ-dominated models which we consider here are flat for simplicity, *i.e.*, $K = 0$. In this case, $\tilde{k} = k$ and represents an ordinary Fourier mode of the perturbation.

Since there is no *ab initio* mechanism for generating the entropy perturbations, the index n is fixed by measurements of large-scale structure today. Isocurvature perturbations evolve such that below the photon diffusion scale, the initial entropy fluctuations become the density perturbations that seed large-scale structure. The observed power spectrum of approximately $P(k) \propto k^{-1}$ at large-scale structure scales (*e.g.*, Peacock & Dodds 1994) then implies an $n \simeq -1$ initial power law in the model. Numerical simulations which take into account nonlinearities confirm this result (Suginohara & Suto 1992). At the largest scales, however, isocurvature conditions prevent the formation of density perturbations leading to a steeply blue $P(\tilde{k}) \propto (\tilde{k}^2 - 4K)^2\tilde{k}^n$, *i.e.*, an $n + 4$ power spectrum below the curvature scale. This sharply rising spectrum of fluctuations is

[1] Departments of Astronomy and Physics and Center for Particle Astrophysics, University of California at Berkeley, 601 Campbell Hall, Berkeley, CA 94720.

[2] Department of Physics, Faculty of Science, The University of Tokyo, Tokyo 113, Japan.

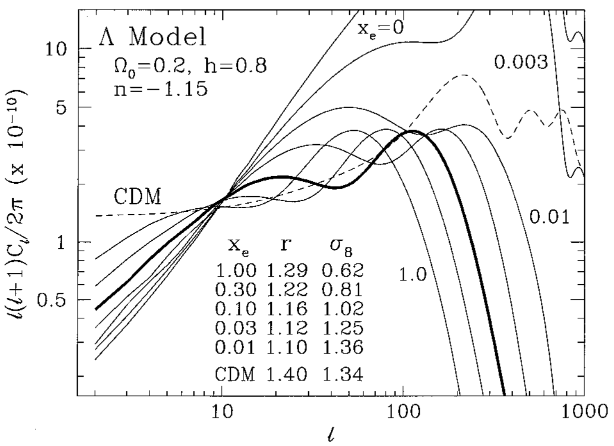

FIG. 1.—*COBE* normalized anisotropies in the Λ model as a function of ionization. As the ionization level increases from $x_e = 0$ to 1 as listed in the figure, the damping reaches to larger angles making the *COBE* slope shallower. Open universe models suffer less from this effect at large angles due to geodesic deviation. Fluctuations are also regenerated on the new last scattering surface. With high enough ionization, this can once again steepen the *COBE* slope. The *COBE* normalization also sets the level of matter fluctuations σ_8 for a fixed thermal history. The ratio r of amplitudes between the more complete likelihood analysis used here and the 10° rms normalization is shown. The most promising model, which currently escapes constraints from the *COBE* slope, normalization, and spectral distortion measurements, is shown (*thick line*, $x_e = 0.1$). This should be compared with the standard CDM ($h = 0.5$, $\Omega_b = 0.05$, $Q = 19.9$ μK) model (*dashed line*). Measurements in the range $l \simeq 20$–200 can further help to distinguish the models. These results agree with Peebles (1994) to better than 10% for a related model.

particularly dangerous when normalized at large scales by the *COBE* DMR measurement.

It may thus seem that the model can be ruled out by merely considering the implied amplitude of the matter power spectrum at the 8 h^{-1} Mpc scale. In an unbiased scenario of galaxy formation, which is expected in these baryon only models (Cen, Ostriker, & Peebles 1994), observations require $\sigma_8 \simeq 1$. However, the baryon isocurvature model has an additional degree of freedom to save it. Since Silk damping (Silk 1968) does not destroy entropy fluctuations, the large amount of small-scale power in the model allows for collapse of objects immediately following recombination. This could lead to sufficient energy input to reionize the universe (Peebles 1987a,b). Because Compton drag prevents the growth of structure, the ionization history can be tuned to provide the right ratio of matter to temperature fluctuations. Following Gnedin & Ostriker (1992), we assume that a fraction x_e of the electrons was reionized at $z \simeq 800$. For complications due to a multistaged ionization history and compact baryonic object formation, see HS94.

Reionization also leads to significant and observable consequences for the CMB. Large primary fluctuations from the acoustic oscillation phase (see, *e.g.*, Hu & Sugiyama 1995) are exponentially damped with optical depth below the horizon at the new last scattering surface. Secondary anisotropies are generated by Doppler shifts off moving electrons at last scattering. These are damped under the thickness of the last scattering surface due to redshift-blueshift cancellation as the photon travels across many wavelengths of the perturbation. Thus, higher ionization almost always implies smaller anisotropies under the angle that the horizon subtends at last scattering. We plot the anisotropies in a Λ model as a function of x_e in Figure 1, where the rms anisotropy is related to C_l via $\langle|\Delta T/T|^2\rangle = \Sigma(2l + 1)C_l/4\pi$, with l as the multipole number of the spherical harmonic decomposition of anisotropies on the sky. Open universe examples are displayed in HS94.

One exception to this damping rule is the second-order Doppler contributions from the Vishniac effect (Ostriker & Vishniac 1986; Vishniac 1987) which is not included in Figure 1. This effect is uncovered at arcminute scales, where other first order effects have suffered severe thickness damping, and is extremely sensitive to the amplitude of the matter perturbations. It thus is only important for highly ionized, late last scattering scenarios (Hu, Scott, & Silk 1994; HS94).

Finally, ionization also implies that the electrons have been heated to a temperature above that of the CMB. Compton scattering thus creates spectral distortions in the CMB as photons are upscattered in frequency by the electrons. The distortion is described by the Compton-y parameter defined as $y = \int d\tau\, k(T_e - T)/m_e c^2$, where T_e and T are the electron and CMB temperatures, respectively, and τ is the optical depth to Compton scattering.

3. MODEL CONSTRAINTS

When extended to large scales, the steep initial spectrum required by large-scale structure conflicts with the flatter anisotropy spectrum measured by *COBE*. Without reionization, the predicted *COBE* slope is approximately $n_{\rm eff} \simeq 2$ (Sugiyama & Silk 1994). $n_{\rm eff}$ is only weakly dependent on n and is somewhat shallower than one might expect from the $n + 4$ behavior of the matter power spectrum (Hu & Sugiyama 1995).

Reionization tends to suppress small-angle anisotropies and can mitigate a steep initial spectrum. However, if the ionization is too great, secondary anisotropies generated on the last scattering surface will counter this effect (see Fig. 1). Furthermore, reionization has no effect for angles much larger than that subtended by the horizon at last scattering. At the *COBE* scale, open models will thus be less affected by reionization than Λ models, since geodesic deviation carries the same physical scale at last scattering to a much smaller angle on the sky today. Lesser effects can be attributed to raising the baryon content through $\Omega_b h^2$ which delays last scattering and increases the physical scale of the horizon. However even for flat models, the projection from the last scattering surface depends strongly on Ω_0 and counters the Ω_b dependence in these $\Omega_0 = \Omega_b$ baryonic models. Furthermore, the late integrated Sachs-Wolfe effect (Sachs & Wolfe 1967; Hu & Sugiyama 1995) boosts the low-order multipoles slightly as Ω_0 decreases. In the range of interest, decreasing Ω_0 leads to a shallower *COBE* slope. High x_e, high h, low Ω_0, Λ models therefore offer the best prospects of bringing down the *COBE* slope.

Bunn et al. (1995) find that the observational constraints require $n_{\rm eff} = 1.3^{+0.24}_{-0.37}$ (with quadrupole). This agrees well with other analyses: Górski et al. (1994) obtain $n_{\rm eff} = 1.10 \pm 0.32$ with similar techniques, and Bennett et al. (1994) and Wright et al. (1994a) find $n_{\rm eff} = 1.53^{+0.49}_{-0.55}$ and $1.25^{+0.4}_{-0.45}$, respectively, using significantly different techniques. Tegmark & Bunn (1994) find $n_{\rm eff} = 1.10 \pm 0.29$ by inverting the entire pixel covariance matrix. The slight differences among these results are not surprising, given the range of statistical techniques and assumptions used; in particular, the results are in agreement in suggesting that models with $n_{\rm eff} \gtrsim 2$ are disfavored at greater than the 95% confidence level. (The result of Bennett et al.

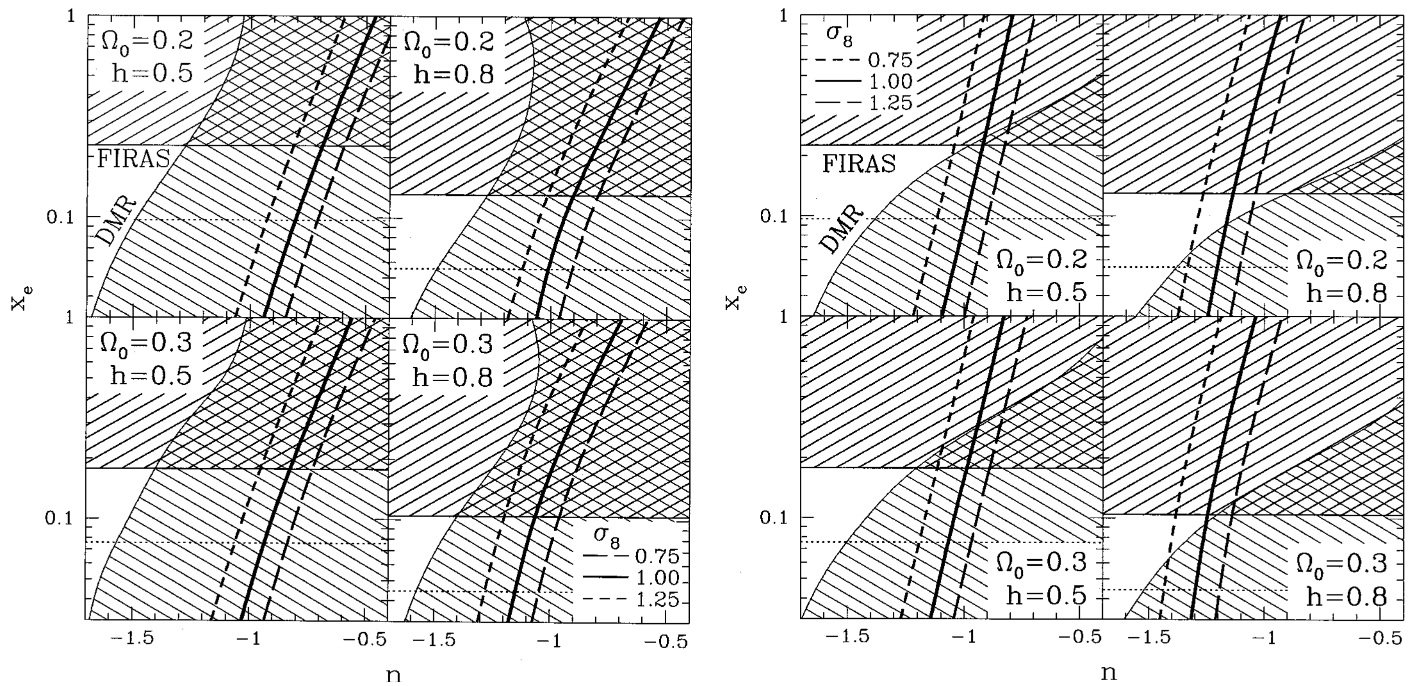

FIG. 2*a*

FIG. 2*b*

FIG. 2.—Constraints on the primordial spectral index n and ionization fraction x_e. The *COBE* DMR slope imposes a 95% upper confidence limit on n which is weakened as the ionization fraction increases, due to damping of the primary fluctuations, until a turning point at which fluctuations are significantly regenerated by the Doppler effect on the new last scattering surface. The *COBE* FIRAS constraint on spectral distortions through the Compton y parameter sets an upper limit on the ionization fraction. Here a conservative $T_e = 5000$ K is assumed. The effect of raising it to the more realistic $T_e = 10{,}000$ K is also shown (*dotted lines*). The *COBE* DMR normalization also sets the level of matter fluctuations at the $8\,h^{-1}$ Mpc scale σ_8. (*a*) No open model simultaneously satisfies all the observational constraints. We have not shown the $\Omega_0 = 0.1$, which is in better agreement with globular cluster ages for $h = 0.8$, since the whole class of models is unviable due to curvature cutoff effects which further raise $n_{\rm eff}$ (see Hu & Sugiyama 1995, Fig. 19). (*b*) For Λ models, a small region of parameter space is open for high h, low Ω_0 models. The full anisotropy spectrum for the most promising model $\Omega_0 = 0.2, h = 0.8, n = -1.15$ and $x_e = 0.1$ is displayed in Fig. 1 and the matter power spectrum in Fig. 3. Even this model is ruled out with the more realistic T_e.

1994 is the only one that does not strongly support this conclusion; however, the authors point out that their estimate of n may suffer from an upward bias of $\sim$0.3.) Of course, unknown systematic errors in the *COBE* DMR data set could weaken these conclusions.

To quantify this constraint, we undertake a full likelihood analysis of the 2 year *COBE* DMR sky maps for open and Λ isocurvature baryon models fixed by Ω_0, h, and x_e. We expand the two-year DMR data in a set of basis functions which are optimized to have the maximum rejection power for incorrect models (Bunn & Sugiyama 1995; Bunn et al. 1995). Employing the 400 most significant terms in this expansion, we then compute the likelihood functions for a variety of models. To set limits on n and the normalization Q, the rms quadrupole, we assume a prior distribution which is uniform for all Q and $n \leq 0$. Spectra with $n > 0$ are unphysical due to nonlinear effects which regenerate an $n = 0$, $P(k) \propto k^4$ large-scale tail to the fluctuations (Zel'dovich 1965; Peebles 1980). The constraint in the crucial $n \simeq -1$ regime is not sensitive to the details of this cutoff. Shown in Figure 2 are the 95% confidence upper limits imposed on n by integrating over the normalization Q to form the marginal likelihood in n. As expected, all open models with $n \simeq -1$ are ruled out regardless of ionization fraction, whereas highly ionized Λ models remain acceptable.

With the maximum likelihood value for the normalization $Q_{\mathcal{L}}$ of the model, we predict the amplitude of matter fluctuations σ_8. The likelihood value for the normalization tends to boost the amplitude over the *COBE* DMR 10° rms normalization value of 30 μK (Bennett et al. 1994) by a factor $r \equiv Q_{\mathcal{L}}/Q_{10°} \simeq 1.1$ for open models and low ionization Λ models. The difference is more significant in highly ionized Λ models due to the damping of the anisotropy spectrum. The boost is on the order $r \simeq 1.3$ for a fully ionized Λ model (see Fig. 1). This effect appears also in the CDM model with a greater magnitude in fact. The effect of the low quadrupole in the data on the 10° measure (Banday et al. 1994; Bunn et al. 1995) artificially suppresses its amplitude. In all cases, the likelihood analysis provides the better normalization by including the full data set and minimizing the effects of cosmic variance (Wright et al. 1994b). In Figure 2, we thus plot the value of σ_8 corresponding to this normalization as a function of ionization history and spectral index. The suppression of fluctuation growth in a highly ionized universe must be compensated by a steeper spectral index n. Notice that even ignoring limits on the large-scale structure slope, all open models which satisfy the *COBE* slope are ruled out.

Even though highly ionized Λ models can survive constraints on the *COBE* slope and the large-scale structure normalization, they run into difficulties with the low upper limit on spectral distortions imposed by the *COBE* FIRAS experiment, $y < 2.5 \times 10^{-5}$ (95% CL). If the intergalactic medium is collisionally ionized, the electron temperature must be $T_e \gtrsim 10{,}000$ K (see, *e.g.*, Gnedin & Ostriker 1992). The corresponding limit from the Compton y parameter may be avoided by more exotic ionization schemes which attempt to

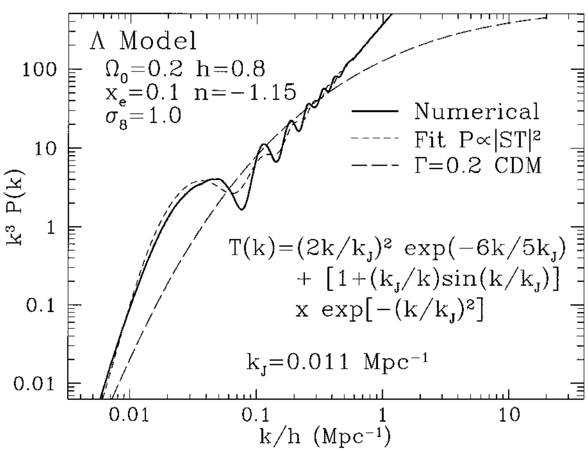

FIG. 3.—Power spectrum $k^3P(k)$ for an allowed Λ model ($\Omega_0 = 0.2, h = 0.8, x_e = 0.1, n = -1.15$). For comparison, a parameterized $\Gamma = 0.2$ CDM model (Efstathiou et al. 1992) is shown normalized to $\sigma_8 = 1$, which is known to fit the shape of the large scale structure data at $10^{-2} \lesssim k/h \lesssim 1$ Mpc^{-1}. To facilitate more detailed comparisons, we have also provided a simple fitting formula involving the isocurvature transfer function $P(k) \propto |T(k)S(k)|^2$ and the maximal Jeans scale k_J, which is motivated by the perturbation analysis of Hu & Sugiyama (1995).

inject as little energy as possible into the electrons (*e.g.*, neutrinos decaying via a 13.6 eV photon). However since we generically expect at least a few eV excess energy above the ionization threshold, we take $T_e \gtrsim 5000$ K as a reasonably conservative limit. With this constraint, even Λ models fall from favor. Only high h models have a small window of parameter space open, which in fact closes if a more realistic $T_e \gtrsim 10{,}000$ K is assumed. For these early ionized scenarios, this constraint largely obviates the need to impose limits from the Vishniac effect (HS94).

Finally, it is worthwhile to mention that once the observational situation at degree scales settles down, one can at the very least impose a lower limit on the ionization fraction (see Fig. 1 and HS94). Significant reionization ($x_e \gtrsim 0.01$) is necessary in these models to avoid large-degree and arcminute-scale fluctuations in the CMB. Moreover, with the rapidly increasing number of experiments, the sample variance (Scott, Srednicki, & White 1994) will decrease to the point where all baryon isocurvature models can be distinguished from the CDM model. In Figure 1, we have plotted the predictions for the most promising baryon isocurvature model (*thick solid line*) in comparison to the CDM model (*dashed line*). The shape of the rise to the prominent peak around $l \simeq 100$–200 and the fall-off thereafter may be used to help distinguish the models. In Figure 3, we plot the matter power spectrum for the same model and include a simple fitting formula that may facilitate the comparison with large-scale structure measurements. As a simple comparison, we also show a $\Gamma = 0.2$ CDM model which is known to fit the slope of the observed spectrum well (Efstathiou, Bond, & White 1992). These models may consequently run into problems with indications of a *smooth* bend in the large-scale structure measurements (Peacock & Dodds 1994). More work on nonlinear corrections in these models is necessary to quantify the constraint at the smallest scales. With these additional considerations, one may hope to close off the already small window of parameter space available to the model.

4. DISCUSSION

Standard baryon isocurvature models generically run into conflict with CMB observations even at *COBE* scales. Open models are ruled out at the 95% confidence level by a combination of the *COBE* spectral slope and implications of the normalization for the matter power spectrum. Whereas these two considerations leave a large window of acceptable Λ models, the inclusion of the *COBE* FIRAS constraint on spectral distortions even in a relatively conservative fashion is sufficient to drastically reduce the available parameter space such that a tuning of the ionization history, Ω_0 and h must be involved.

At present, none of the simplest models for structure formation fares well in comparison with the combined observations of the CMB and large-scale structure; it is, therefore, perhaps unwise to dismiss this scenario as entirely unviable. Indeed the standard CDM model suffers problems of comparable if not greater magnitude (see, *e.g.*, Ostriker 1993). Like CDM with its Λ, open, and mixed variants, the general idea of isocurvature-seeded fluctuations may of course be saved by introducing more free parameters.

The original model employs two simplifying assumptions: a power-law initial spectrum and a constant ionization fraction after reionization. Since open models run into difficulties by predicting a steep *COBE* slope, the former assumption must be dropped to save them. In fact, for the open models there is some reason to believe that the spectrum may possess nontrivial structure at the curvature scale (Lyth & Stewart 1990; Ratra & Peebles 1994; Bucher, Goldhaber, & Turok 1994). Note, however, that unlike the open adiabatic case, power-law behavior in gravitational potential fluctuations is equivalent to power-law behavior in the entropy fluctuation (Hu & Sugiyama 1995), which serves to eliminate a potential ambiguity of the open model.

On the other hand, more complicated ionization histories and compact baryonic object formation can be employed to help design a more favorable Λ, but not open, model. The ionization history can be fixed such that the relative normalization of the matter and radiation yields $\sigma_8 = 1$ (see HS94). However, since even the maximally damped open models violate the *COBE slope* constraint if $n \simeq -1$, no tuning of thermal histories alone can save the open baryon isocurvature scenario. For Λ models, thermal history effects can also be employed to escape the *COBE* FIRAS constraints without giving up the damping benefits of a highly ionized model. This is because spectral distortions are a function of the total optical depth, whereas the damping of anisotropies is determined at last scattering, where the optical depth equals unity. Thus late ionized scenarios may be more favorable. Unfortunately, the large amount of small-scale power may make delayed reionization impossible.

More radical solutions have also been proposed. Peebles (1994) suggests the addition of cold dark matter or defects and Gnedin et al. (1995) propose non-Gaussian fluctuations. Small admixtures of adiabatic fluctuations may also be added. Although these *ad hoc* patches on the model may provide sufficient freedom to save the model, they greatly reduce the appeal of the baryon isocurvature scenario.

We would like to thank D. Scott, J. Silk, and M. White for useful discussions. W. H. was supported by the NSF and N. S. by the JSPS.

REFERENCES

Banday, A. J., et al. 1994, ApJ, 436, L99
Bennett, C., et al. 1994, ApJ, 436, 423
Bucher, M., Goldhaber, A. S., & Turok, N. 1994, hep-ph-9411206, preprint
Bunn, E. F., Scott, D., & White, M. 1995, ApJ, 441, L9
Bunn, E. F., & Sugiyama, N. 1995, ApJ, 446, 49
Cen, R., Ostriker, J., & Peebles, P. J. E. 1994, ApJ, 415, 423
Chiba, T., Sugiyama, N., & Suto, Y. 1994, ApJ, 429, 427
Efstathiou, G., & Bond, J. R. 1987, MNRAS, 227, 33P
Efstathiou, G., Bond, J. R., & White, S. D. M. 1992, MNRAS, 258, 1P
Freedman, W. L., et al. 1994, Nature, 371, 757
Gnedin, N. Y., & Ostriker, J. P. 1992, ApJ, 400, 1
Gnedin, N. Y., Ostriker, J. P., & Rees, M. 1995, ApJ, 438, 48
Górski, K., et al. 1994, ApJ, 430, L89
Hu, W., Scott, D., & Silk, J. 1994, Phys. Rev. D, 49, 2
Hu, W., & Sugiyama, N. 1994, ApJ, 436, 456 (HS94)
———. 1995, Phys. Rev. D, 51, 2599
Lyth, D. H., & Stewart, E. D. 1990, Phys. Lett. B, 252, 336
Mather, J. C., et al. 1994, ApJ, 420, 439
Ostriker, J. P. 1993, ARA&A, 31, 689
Ostriker, J. P., & Vishniac, E. T. 1986, ApJ, 306, L51
Peacock, J. A., & Dodds, S. J. 1994, MNRAS, 267, 1020
Peebles, P. J. E. 1980, The Large Scale Structure of the Universe (Princeton: Princeton Univ. Press), 129
———. 1987a, Nature, 327, 210
———. 1987b, ApJ, 315, L73
———. 1994, ApJ, 432, L1
Ratra, B., & Peebles, P. J. E. 1994, 432, L5
Sachs, R. K., & Wolfe, A. M. 1967, ApJ, 147, 73
Scott, D., Srednicki, M., & White, M. 1994, ApJ, 421, L5
Silk, J. 1968, ApJ, 151, 459
Smoot, G., et al. 1992, ApJ, 396, L1
Suginohara, T., & Suto, Y. 1992, ApJ, 387, 431
Sugiyama, N., & Silk, J. 1994, Phys. Rev. Lett., 73, 509
Tegmark, M., & Bunn, E. F. 1994, astro-ph/9412005, preprint
Vishniac, E. T. 1987, ApJ, 322, 597
Wilkinson, D. T. 1995, Proc. 9th Lake Louise Winter Institute, (Singapore: World Scientific), in press
Wilson, M. L. 1983, ApJ, 273, 2
Wright, E., et al. 1994a, ApJ, 436, 443
———. 1994b, ApJ, 420, 450
Zel'dovich, Ya. B. 1965, Adv. Astron. Astrophys., 3, 241